I. SOME MATH QUESTIONS

1. What is the solution of the system of linear equations

$$\begin{aligned} 2x + y &= 6 \\ x - y &= -3 \end{aligned} \tag{1}$$

2. A fair coin is flipped four times. What is the probability of two or more heads?

3. What is the derivative of the function $f(x) = x \sin(2x)$?

4. What is the integral $\int x \sin(2x)\, dx$?

5. What is the general solution $y(x)$ of the second-order differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0 \tag{2}$$

6. What is the inverse of the 2-by-2 matrix

$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \tag{3}$$

   What are its eigenvalues and eigenvectors?

7. Suppose that a fair coin is flipped N times. Let H be the total number of heads obtained. This is an example of a random variable: H can take any value between 0 and N, and we can try to calculate various expectation values involving H. For example, the expectation value $\langle H \rangle$ of H itself is clearly equal to $(N/2)$.

   The variance of the random variable H is defined by

$$\mathrm{Var}(H) = \Big\langle (H - \langle H \rangle)^2 \Big\rangle \tag{4}$$

   The square root $\mathrm{Var}(H)^{1/2}$ is one measure of the "typical" size of fluctuations around the mean. Calculate $\mathrm{Var}(H)$ as a function of N.

8. What is the Fourier transform $\tilde{f}(k) = \int dx\, f(x) e^{ikx}$ of the function $f(x) = 1/(1 + x^2)$?