Setting up differential equations: solutions

ISSYP 2016

Question 5 "Radioactive decay, part 1":

$$\frac{dM}{dt} = -10^{-4}M \tag{1}$$

Question 6 "Radioactive decay, part 2":

$$\frac{dA}{dt} = -r_1 A \qquad \frac{dB}{dt} = r_1 A - r_2 B \qquad \frac{dC}{dt} = r_2 B \tag{2}$$

Question 7 "Mixing":

$$\frac{dM}{dt} = (0.1)(100) - (0.1)\frac{M}{100} \tag{3}$$

Question 8 "Free-fall with air resistance", solution 1:

$$\frac{dy}{dt} = v \qquad \frac{dv}{dt} = 9.8 - kv \tag{4}$$

Question 8 "Free-fall with air resistance", solution 2. Compared to solution 1, this just uses the opposite sign convention for v (either is correct):

$$\frac{dy}{dt} = -v \qquad \frac{dv}{dt} = -9.8 - kv \tag{5}$$

Question 9 "Roadrunner and coyote", solution 1. This uses three dependent variables: the coordinates (x_c, y_c) of the coyote and the x-coordinate x_r of the roadrunner.

$$\begin{aligned} \frac{dx_c}{dt} &= 12\frac{x_r - x_c}{\sqrt{(x_r - x_c)^2 + y_c^2}} \\ \frac{dy_c}{dt} &= -12\frac{y_c}{\sqrt{(x_r - x_c)^2 + y_c^2}} \\ \frac{dx_r}{dt} &= 10 \end{aligned} \tag{6}$$

Question 9 "Roadrunner and coyote", solution 2. You can also notice that the roadrunner's position x_r is always equal to $10t$, so we don't need to introduce x_r as a dependent variable. Instead we get a system of differential equations with two dependent variables:

$$\begin{aligned} \frac{dx_c}{dt} &= 12\frac{10t - x_c}{\sqrt{(10t - x_c)^2 + y_c^2}} \\ \frac{dy_c}{dt} &= -12\frac{y_c}{\sqrt{(10t - x_c)^2 + y_c^2}} \end{aligned} \tag{7}$$