Abstract

MIMO or Multiple Input Multiple Output; is a technique to increase the link reliability, the spectral efficiency, or both by using multiple antennas at both the transmitter and the receiver in wireless communication. In this assignment, the concept of an uncorrelated MIMO channel of the wireless link is applied to five RGB images of different objects. Firstly the images are converted into gray images and then resized into 16x16 matrices. These 16×16 image matrices are used as the MIMO channel matrix and the received signal vector under the one-to-one relation for each image that has sixteen eigenvalues is considered as the feature of the corresponding image.

The basic theory of Eigen decomposition under MIMO [1]

A MIMO system consists of several transmit and receive antennas. It is considered a system with n_T transmit and n_R receive antennas. The channel of this system is defined by an $n_R \times n_T$ matrix, which is denoted by H, as a complex matrix. The transmitted signal is represented by an $n_T \times 1$ column matrix, denoted by symbol x, whereas the received signal is represented by an $n_R \times 1$ column matrix, denoted by symbol r.

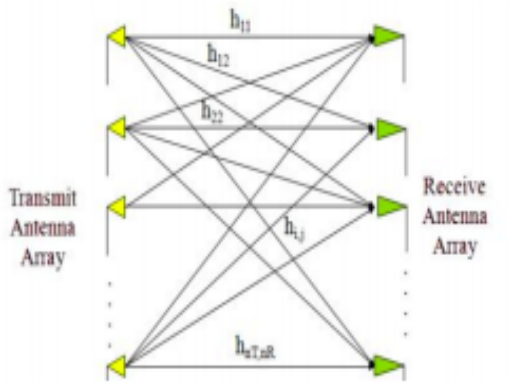

Figure 1: MIMO channel of a wireless network [1].

The complex channel matrix, $H(n)$ of Figure 1 is expressed by the following $n_R \times n_T$ a complex matrix is provided below.

$$\mathrm{H(n)} = \begin{bmatrix} h_{11}(n) & h_{12}(n) & \ldots\ldots\ldots & h_{Nr1}(n) \\ h_{21}(n) & h_{22}(n) & \ldots\ldots\ldots & h_{Nr2}(n) \\ \vdots & \vdots & & \vdots \\ h_{Nr1}(n) & h_{Nr2}(n) & \ldots\ldots\ldots & h_{NrNt}(n) \end{bmatrix}$$

The system equations can be expressed as the following matrix :

$$r(n) = H(n)\,x(n) + n(n)$$

The $n_R \times 1$ vector denotes the complex noise vector as:

$$\mathbf{n(n)} = \left[\tilde{n}_1(n), \tilde{n}_2(n), \ldots\ldots\ldots, \tilde{n}_{Nr}(n)\right]^T$$

To simplify the equation, we can suppress the dependence on time n by the following expression:

$$R = Hx + n$$

Let us define a matrix:

$$\mathbf{Q} = \begin{cases} \mathbf{H}\mathbf{H}^H & n_R < n_T \\ \mathbf{H}^H\mathbf{H} & n_R \geq n_T \end{cases}$$

The eigenvalues λ of Q is evaluated from:

$$det(\lambda I_m - Q) = 0$$

where,

$$m = min(n_R, n_T)$$

Let, matrix U and matrix V are two unitary matrices such that the columns of U are the eigenvectors of HH^H and that of V are the eigenvectors of $H^H H$. Now the received signal vector can be rewritten as follows:

$$R = U\Lambda V\, Hx + n$$

where Λ is an $n_R \times n_T$ non-negative diagonal matrix, U and U are $n_R \times n_T$ and $n_T \times nR$ unitary matrices respectively. The diagonal matrix Λ can be expressed as:

$$U^H H(V V^H)H^H U = \Lambda$$

Let us define a new $n_T \times n_T$ diagonal matrix $[D\ O]$. Here O is a null matrix and is added to maintain matching of matrix dimensions. We can write the diagonal matrix as:

$$\Lambda = [D\ 0][D\ 0]^H$$

Using the above Equations the following transformation can be achieved

$$r' = [D\ O]x' + n'r'$$

Where, $r' = U^H r$, $x' = V^H x$, and $n' = U^H n$. By doing these, the n_R uncoupled parallel channels are shown in Figure 2.

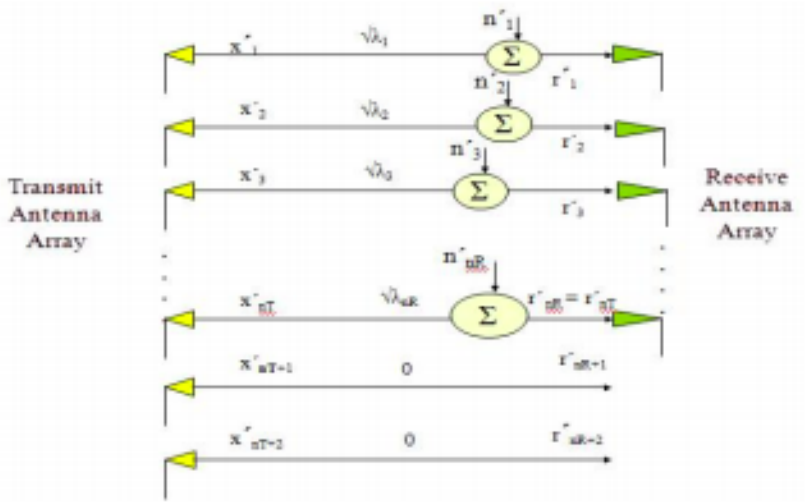

Figure 2: One to one equivalent mapping [1]

In this assignment x' and r' are considered as the image features.

Algorithm and Source code

1. Input an RGB image and convert it into a grayscale image, img_{gray}
2. $img \leftarrow resize\ img_{gray}\ into\ a\ 16 \times 16\ image$
3. Divide all the pixel values of img by the $img_{max} + 10^{-3}$ to make the values less than 1
4. $r \leftarrow H \times x$, $Q_1 \leftarrow H \times H^H$, $Q_2 \leftarrow H^H \times H$
5. $D_1, U \leftarrow eigen(Q_1)$, $D_2, V \leftarrow eigen(Q_2)$
6. $D \leftarrow sqrt(D_1)$, $x_p \leftarrow V^H \times x$, $r_p \leftarrow U^H \times r$
7. return D, x_p, r_p

The problem is solved using the python programming language. The source code link.

Results

Respective features of the five images x_p and r_p, Table 1.

```
image:  trump.jpg
xp
[ 2.9004+0.8479j -0.3862+0.5126j  0.2029+0.8723j  0.2209+1.0822j
  0.1249-0.7435j -0.2742+0.3471j  0.1685-0.6722j  0.4581+1.1474j
  0.2228-0.833j  -0.3487-0.2712j -0.0692+1.8797j -0.2532-0.2314j
  0.3111+0.2464j -0.9445-0.6775j  1.164 +0.2667j -0.0767-0.2857j]
rp
[ 2.4650e+01+7.2064e+00j -5.5050e-01+7.3070e-01j  2.7283e-01+1.1731e+00j
  1.3391e-01+6.5609e-01j  5.1366e-02-3.0588e-01j -9.3094e-02+1.1786e-01j
 -4.9562e-02+1.9776e-01j -1.1761e-01-2.9461e-01j  2.0651e-02-7.7209e-02j
 -3.7105e-02-2.8860e-02j  7.5210e-03-2.0432e-01j  1.1949e-02+1.0919e-02j
  1.0871e-02+8.6120e-03j -2.7243e-02-1.9541e-02j  7.2752e-04+2.7101e-03j
  1.8533e-02+4.2464e-03j]
```

```
image:  cat.jpg
xp
[ 2.8416+0.8181j -0.8122-0.645j   0.2981+0.1353j  0.1999+0.5992j
  0.1261+0.0385j  0.0344-0.8113j -0.2865-0.046j  -0.6139-0.3506j
  0.4483+2.0137j -0.4227-0.6716j  0.2536-0.2733j -0.1984+1.2638j
 -1.0474+0.5525j  0.5873-0.0353j  0.3862+1.2166j  0.3953-0.4127j]
rp
[ 2.3599e+01+6.7944e+00j  1.6303e+00+1.2948e+00j  3.6079e-01+1.6378e-01j
 -1.7227e-01-5.1633e-01j  6.4842e-02+1.9814e-02j  1.0695e-02-2.5203e-01j
  7.6787e-02+1.2336e-02j -1.1192e-01-6.3916e-02j  6.1480e-02+2.7615e-01j
  4.9752e-02+7.9044e-02j  2.2214e-02-2.3942e-02j -8.8859e-03+5.6595e-02j
 -3.5916e-02+1.8945e-02j -1.1169e-02+6.7217e-04j -1.9465e-03+2.0321e-03j
 -2.4728e-04-7.7900e-04j]
```

```
image:  rocket.jpg
xp
[ 2.7641e+00+1.0107j -9.9185e-01+0.0947j  1.2785e-02-1.2052j
 -7.6398e-01-0.6782j -1.4128e-01-0.5815j -2.0971e-03-0.1552j
  5.1875e-01-0.4163j  2.3006e-01+0.0641j  4.3159e-01+0.263j
 -3.2630e-01-0.7456j  1.4254e-02-0.4394j -8.5787e-02+0.9685j
  1.1881e-01+2.2208j  3.7570e-01-0.3076j  8.6371e-01-0.024j
 -9.0462e-01-0.3418j]
rp
[ 1.5690e+01+5.7372e+00j  8.1093e-01-7.7399e-02j  2.6153e-03-2.4653e-01j
 -5.2741e-02-4.6816e-02j  6.4609e-03+2.6593e-02j -5.5969e-05-4.1431e-03j
  6.9828e-03-5.6043e-03j  2.8798e-03+8.0281e-04j  2.4396e-03+1.4866e-03j
  1.1104e-03+2.5373e-03j  3.7235e-05-1.1479e-03j -1.1237e-04+1.2686e-03j
 -2.6273e-04+2.1508e-04j -5.1686e-06-9.6607e-05j  3.1938e-04-8.8782e-06j
  4.0680e-04+1.5369e-04j]
```

```
image:  traffic_light.jpg
xp
[ 2.8881+0.78j    0.6081+0.4822j  0.2323-0.006j  -0.1022-1.2325j
 -0.2003-1.0826j -0.4869-0.2607j  0.0457-0.1558j  0.6588-0.762j
 -0.4452+0.125j  -0.0478+0.8055j  0.4387+1.308j   0.1497-0.619j
 -0.1363-0.538j   0.8841-0.0194j -0.6535+0.4494j  0.6623+1.711j ]
rp
[ 4.0085e+01+1.0826e+01j  9.4013e-01+7.4544e-01j -2.1554e-01+5.5433e-03j
  4.6896e-02+5.6547e-01j  4.7389e-02+2.5616e-01j  4.1682e-02+2.2321e-02j
 -2.4260e-03+8.2688e-03j  1.1153e-02-1.2899e-02j -2.3470e-03+6.5868e-04j
 -1.8185e-04+3.0647e-03j -1.0164e-03-3.0302e-03j  1.6556e-04-6.8463e-04j
 -8.2151e-06-3.2421e-05j -3.0765e-04+6.7352e-06j -4.0922e-04+2.8144e-04j
  4.7012e-04+1.2145e-03j]
```

```
image:  wind-turbine.jpg
xp
[ 2.9434+0.8825j -0.1762-0.0999j -0.6448-0.2084j -0.0235+0.8288j
  0.2255-1.1568j -0.4963-0.8295j -0.3884-0.7345j  0.4713+0.7989j
  0.1166-0.1513j  0.0687+1.4925j  0.3213+1.5413j -0.7264+0.7544j
 -0.3516-0.0747j -0.8359-0.2426j  0.5564-0.1252j -0.0867-0.6195j]
rp
[ 3.5431e+01+1.0623e+01j -5.6328e-02-3.1926e-02j -1.4422e-01-4.6609e-02j
  3.3229e-03-1.1709e-01j  2.0331e-02-1.0428e-01j  3.3668e-02+5.6273e-02j
  2.3179e-02+4.3838e-02j  8.2751e-03+1.4026e-02j  1.0047e-03-1.3040e-03j
  2.7899e-04+6.0612e-03j  3.9551e-04+1.8972e-03j -5.4654e-04+5.6764e-04j
 -1.6924e-04-3.5942e-05j -2.1214e-04-6.1576e-05j -8.8246e-06-6.3042e-05j
 -8.0311e-05+1.8066e-05j]
```

Table 1: r_p *and* x_p values

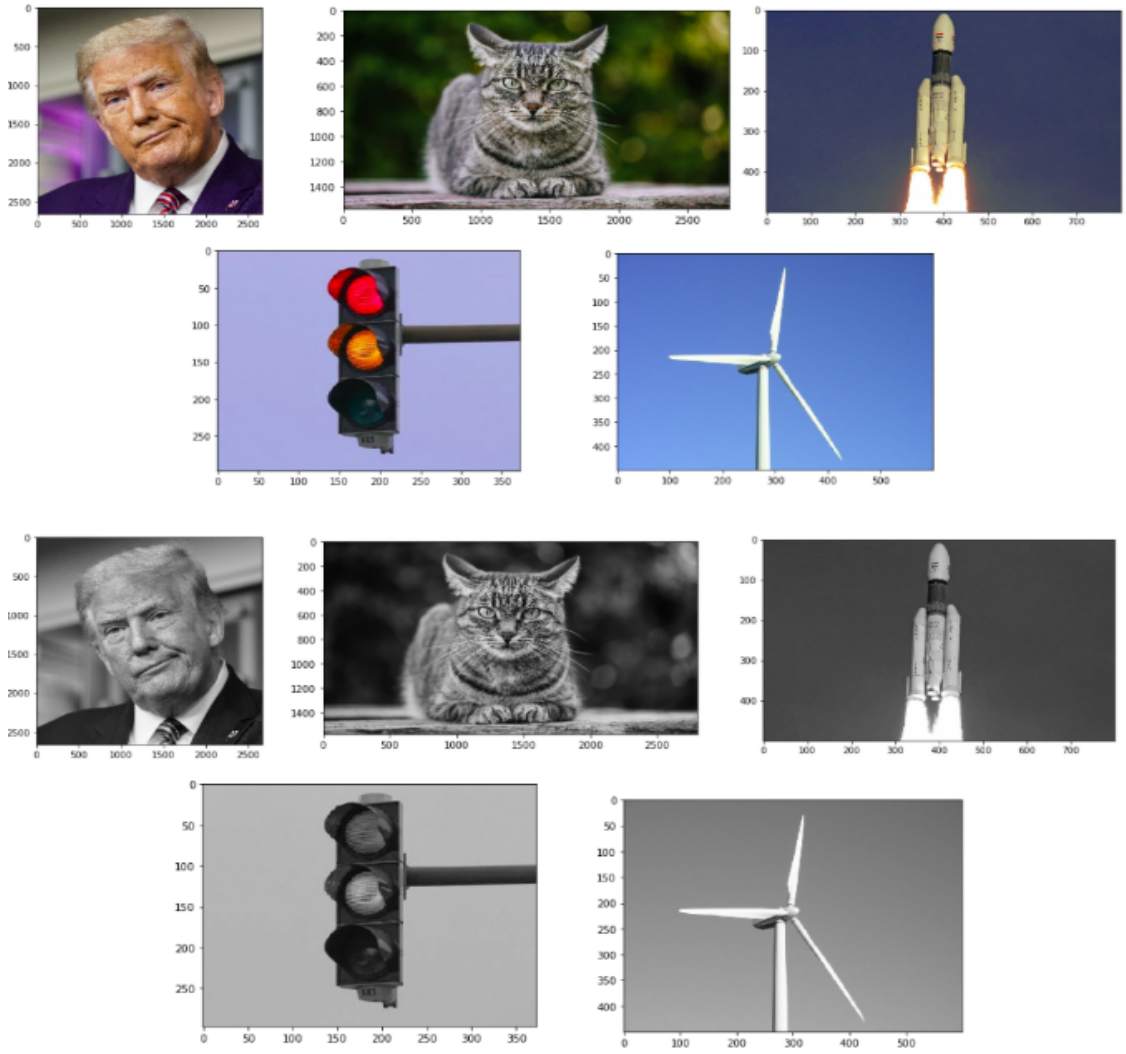

Figure 3: Five RGB images and their gray images

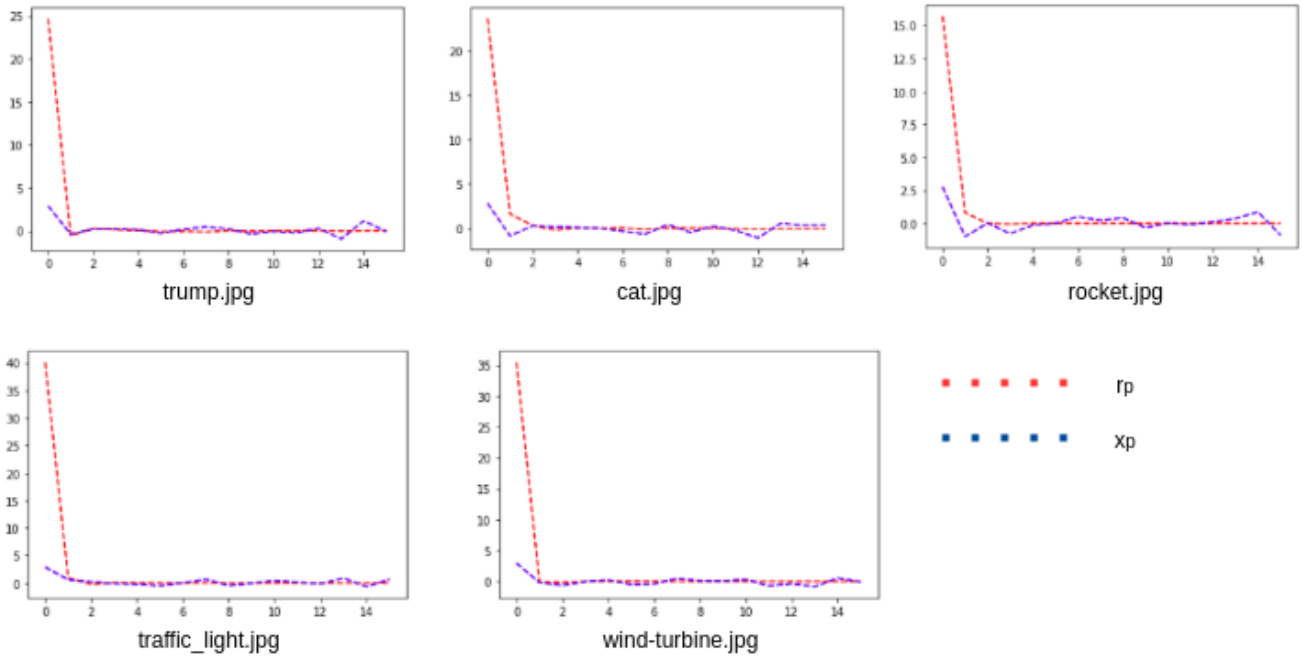

Figure 4: r_p *and* x_p plotting

Figure 3 shows the RGB images and their respective grayscale images which were used for the experiment. The graphs shows the amplitude of the signal y axis and the 16 eigenvalues x axis, Figure 4. These values can be used for detecting the originality of an image. For example, if we want to compare two images of a signature, we can figure it out by calculating the variance of the r_p *and* x_p values of the two images by fixing an error level.

Conclusion

The purpose of the MIMO system is primarily to provide additional channel capacity within the available bandwidth in wireless communication systems. In this assignment, five RGB images are used for extracting features. Using these features, the originality of an image can be tested. For example, false signatures, false fingerprints can be detected using these features. This approach can be as useful as other feature extraction algorithms like **PCA**. The experiment is conducted with python and open source libraries. Using Matlab may provide better results since the library for eigenvalue calculation is better in Matlab than the open-source libraries of python. Considering a higher dimension for the image matrix may provide better features. But it will consume huge computational time and resources.

References

[1] Islam, Mezbahul, et al. "Pattern Recognition Using the Concept of Disjoint Matrix of MIMO System."