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These techniques involve some important decisions about the **bias-variance** tradeoff, and the use of **(cross) validation** in checking model performance and selecting the best model.

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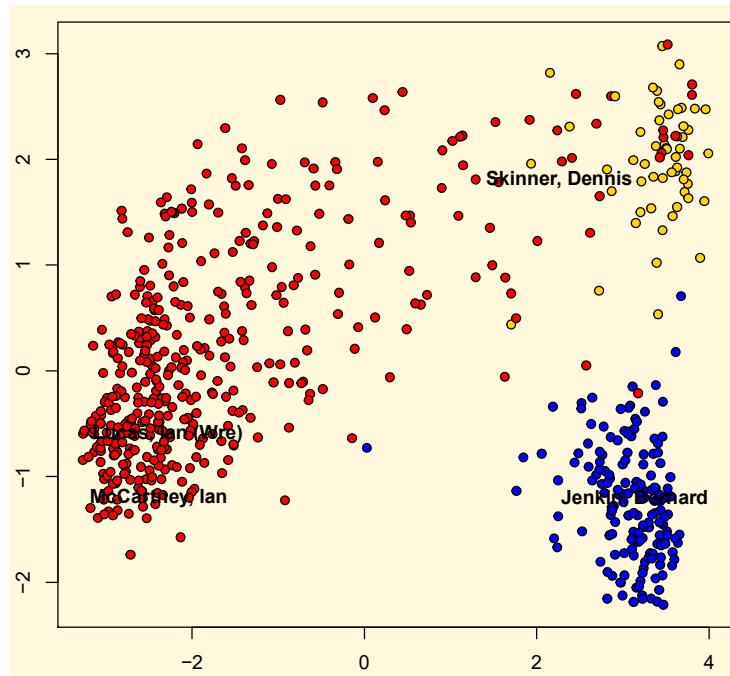
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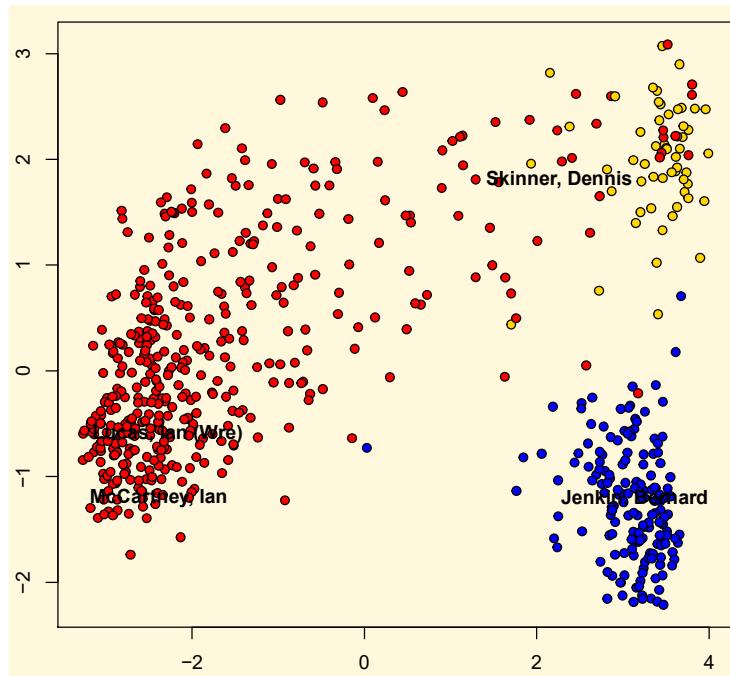
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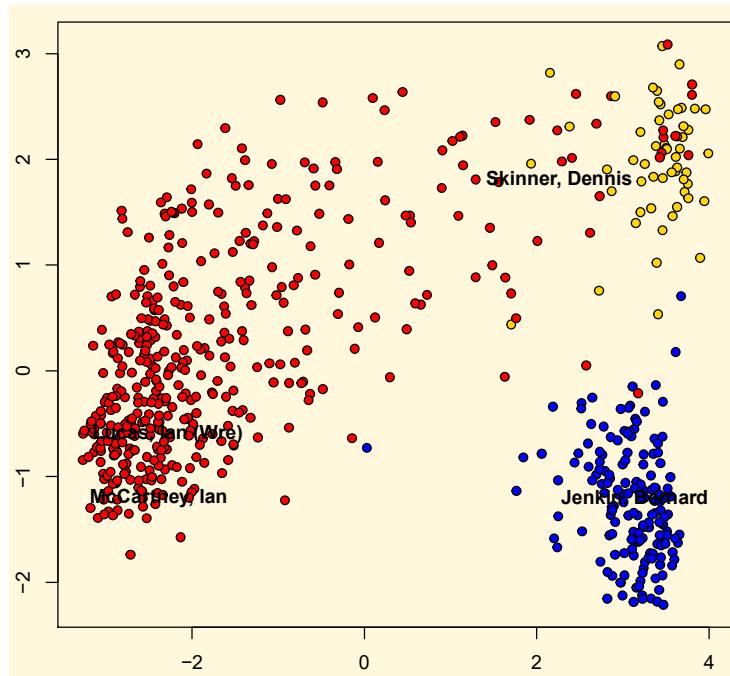


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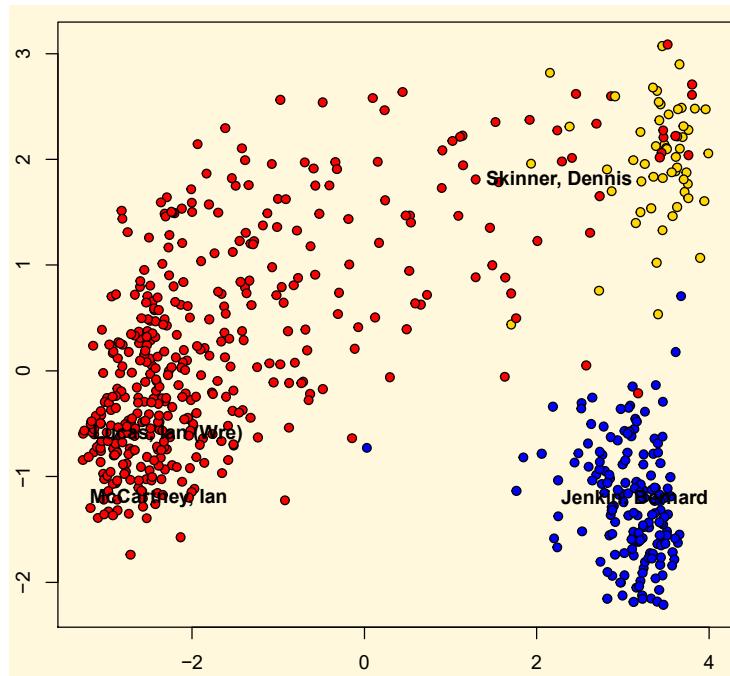


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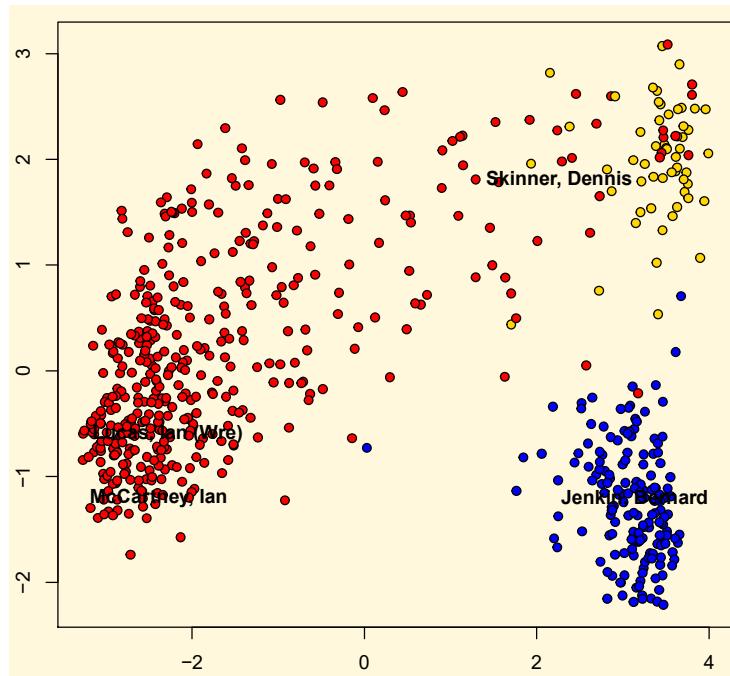
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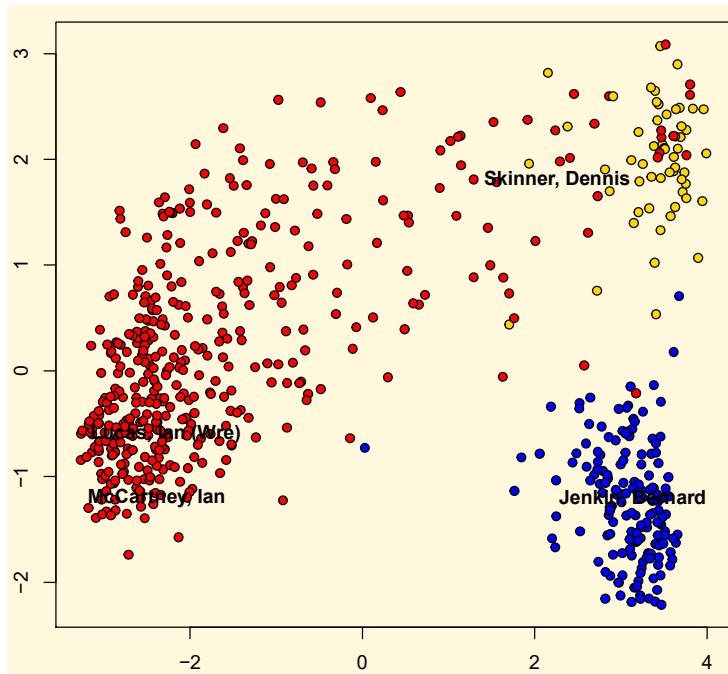
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CRITIC REVIEWS FOR STAR WARS: EPISODE VII - THE FORCE AWAKENS

All Critics (313) | Top Critics (48) | My Critics | Fresh (293) | Rotten (20)

The new movie, as an act of pure storytelling, streams by with fluency and zip.
Full Review... | December 21, 2015
 Anthony Lane
New Yorker
★ Top Critic

While Star Wars: The Force Awakens gets temporarily bogged down taking us back to the world that we left in 1983, it introduces us to the new and exciting torch-bearers of the franchise.
Full Review... | December 30, 2015
 Blake Howard
Graffiti With Punctuation

At the end The Force Awakens looks more like a nostalgic film that will work as a transition to the new Star Wars' age. [Full Review in Spanish]
Full Review... | December 29, 2015
 Salvador Franco Reyes

This film is a well-planned product that balances nostalgia with the capacity to attract new generations into the Star Wars universe. [Full Review in Spanish]
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- So techniques may require careful tuning of **regularization parameters** to obtain good performance.

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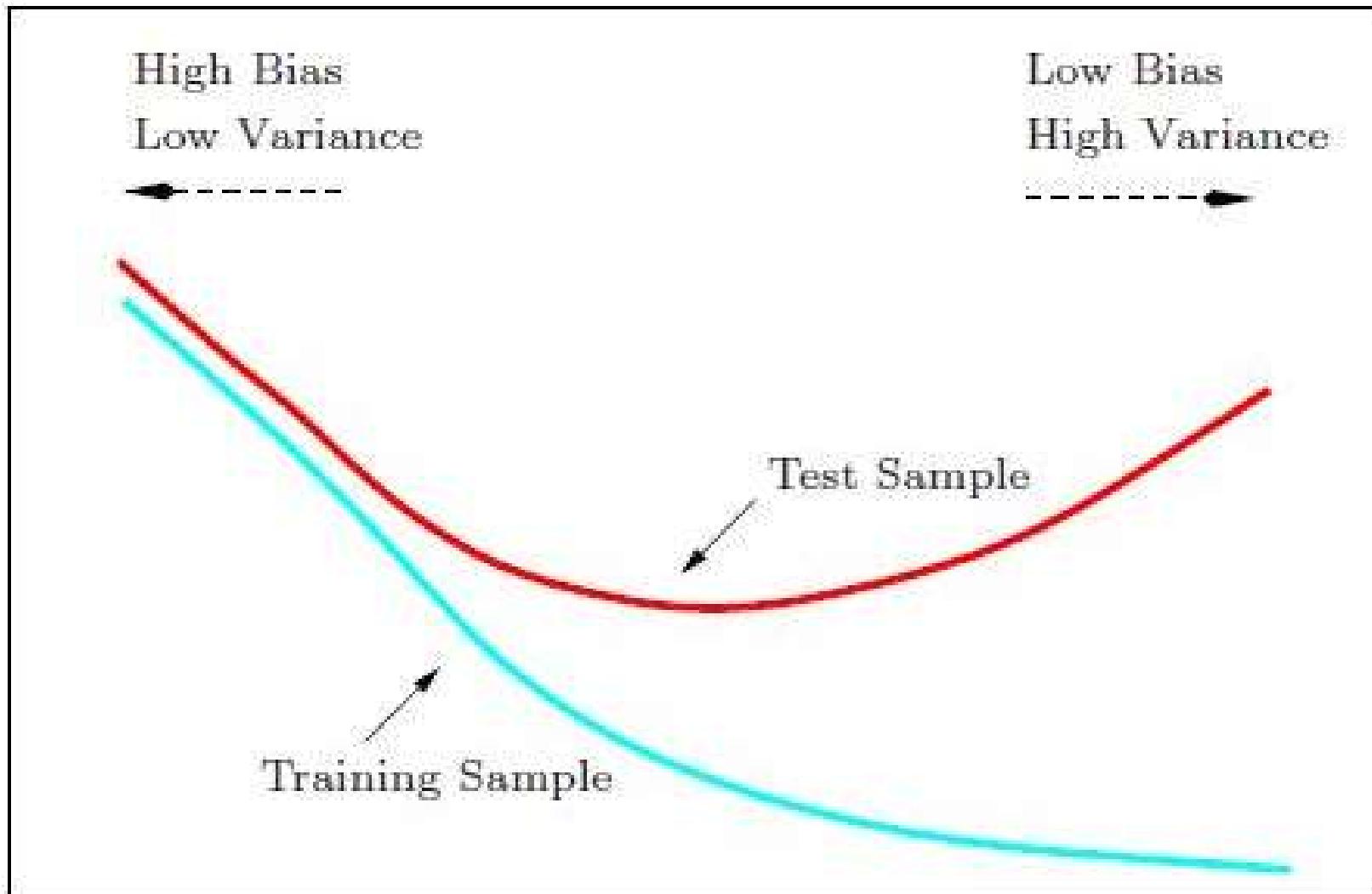
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Bias-Variance Tradeoff (Hastie et al, p38)

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Prediction Error



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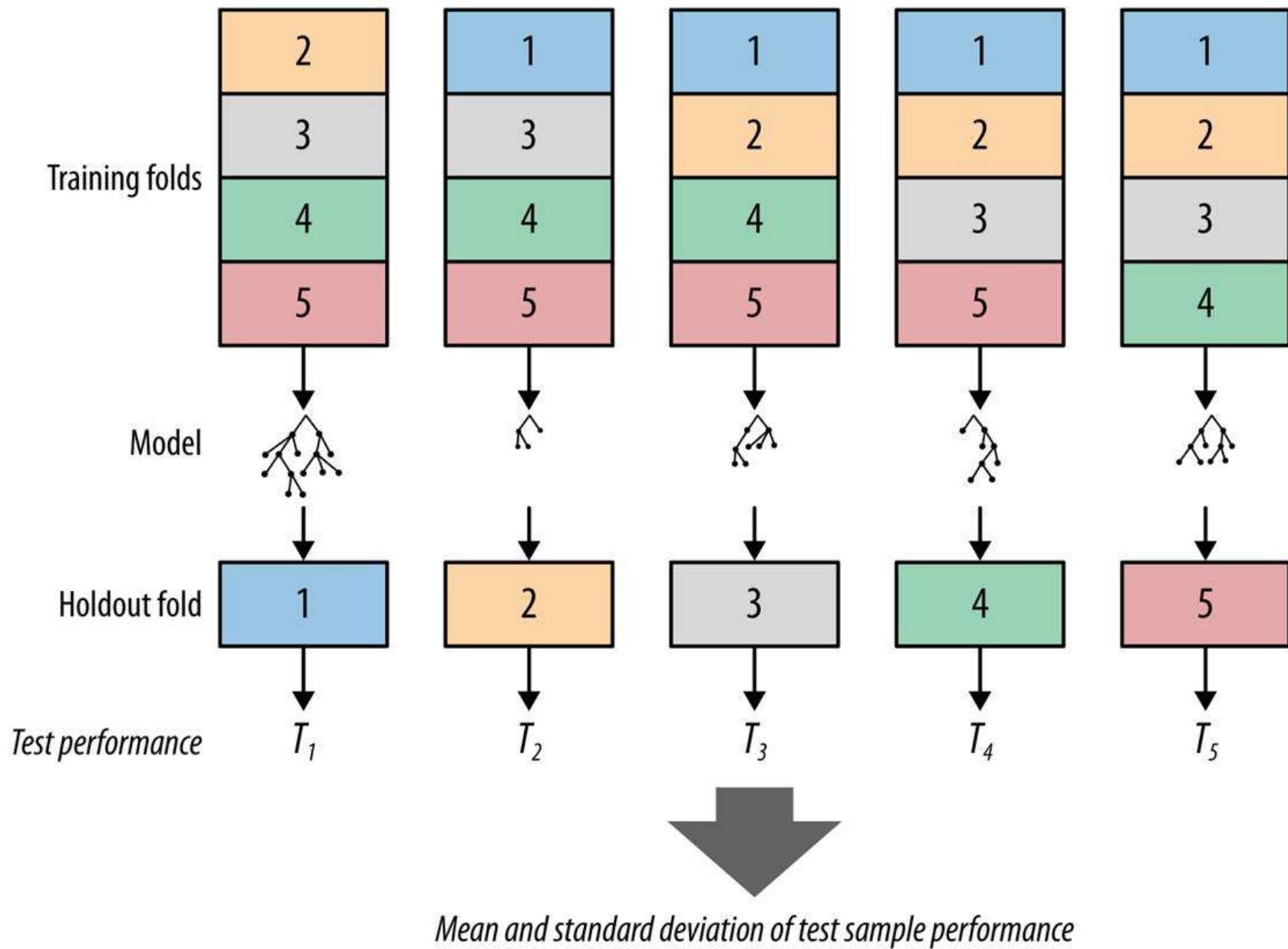
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Have the (stemmed, stopped, weighted etc) **speech term matrix** for each Senator as X .

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What method to use?

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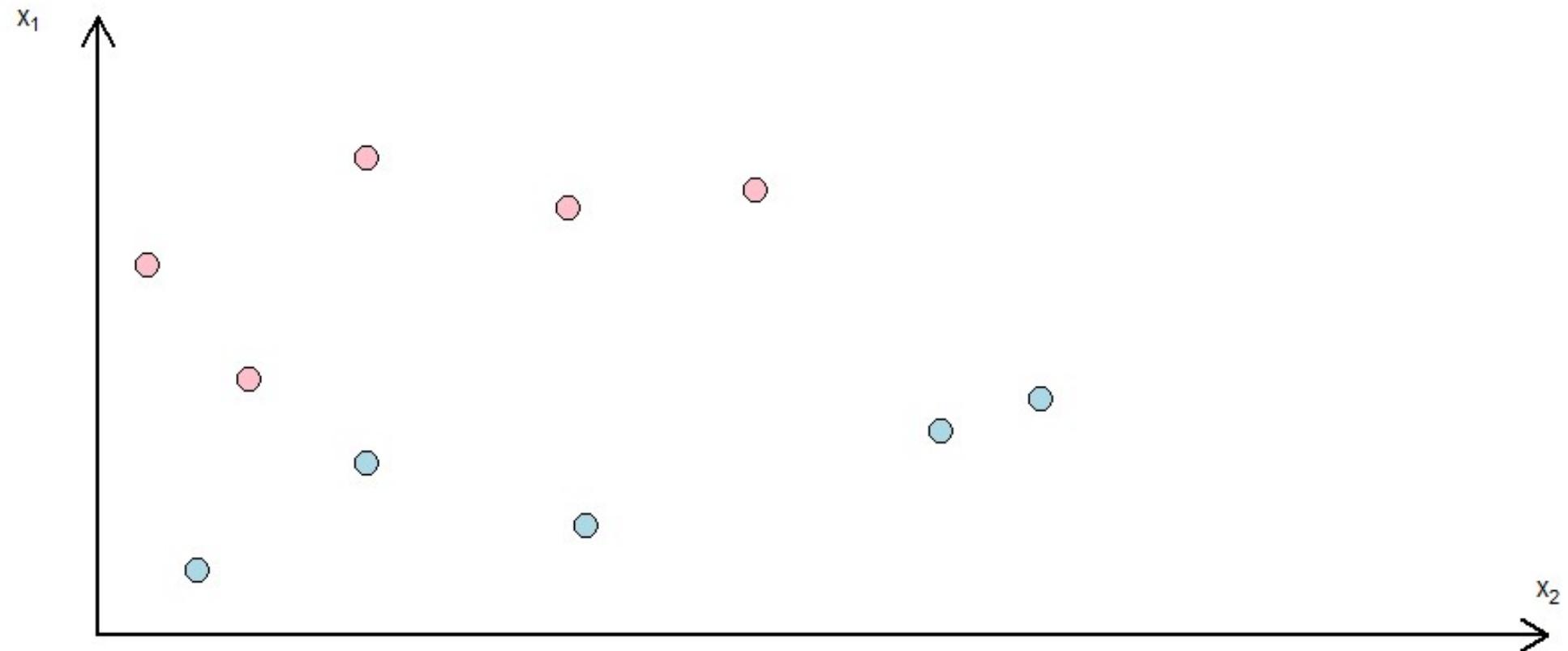
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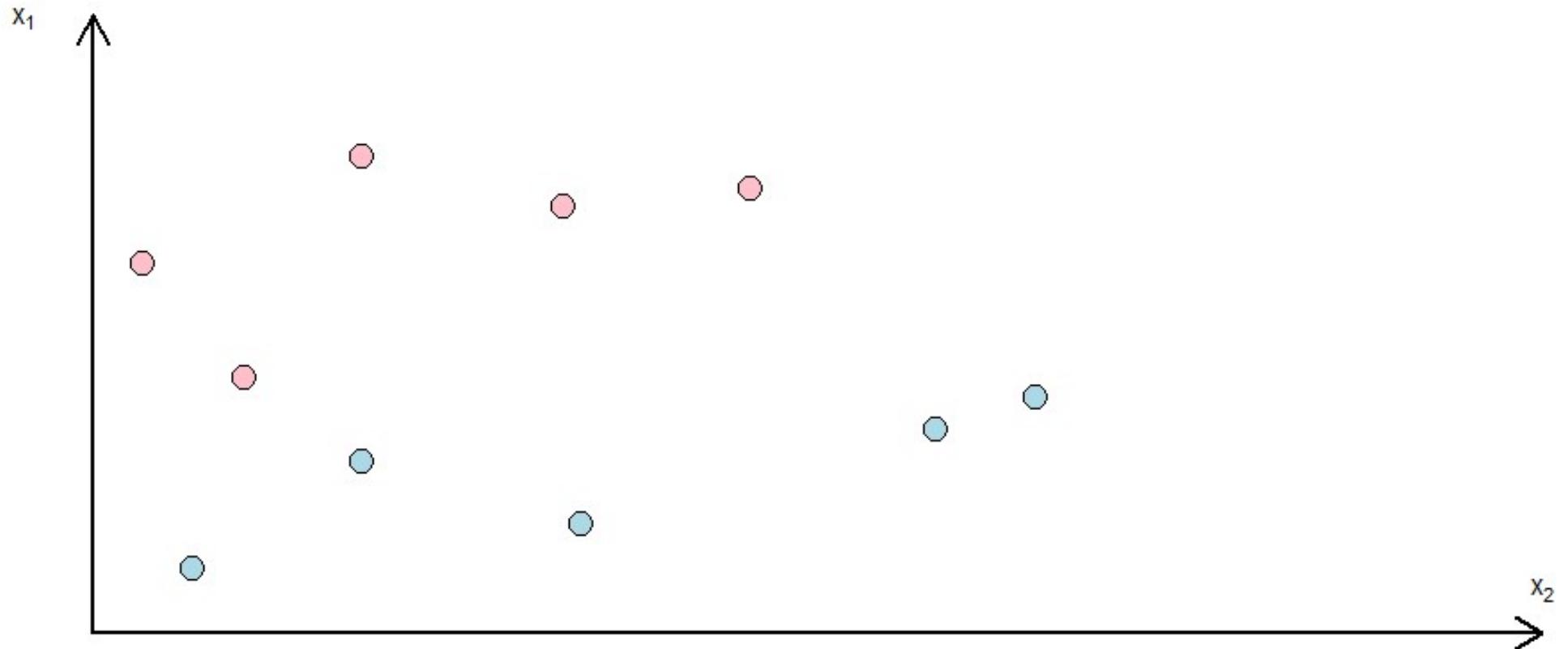
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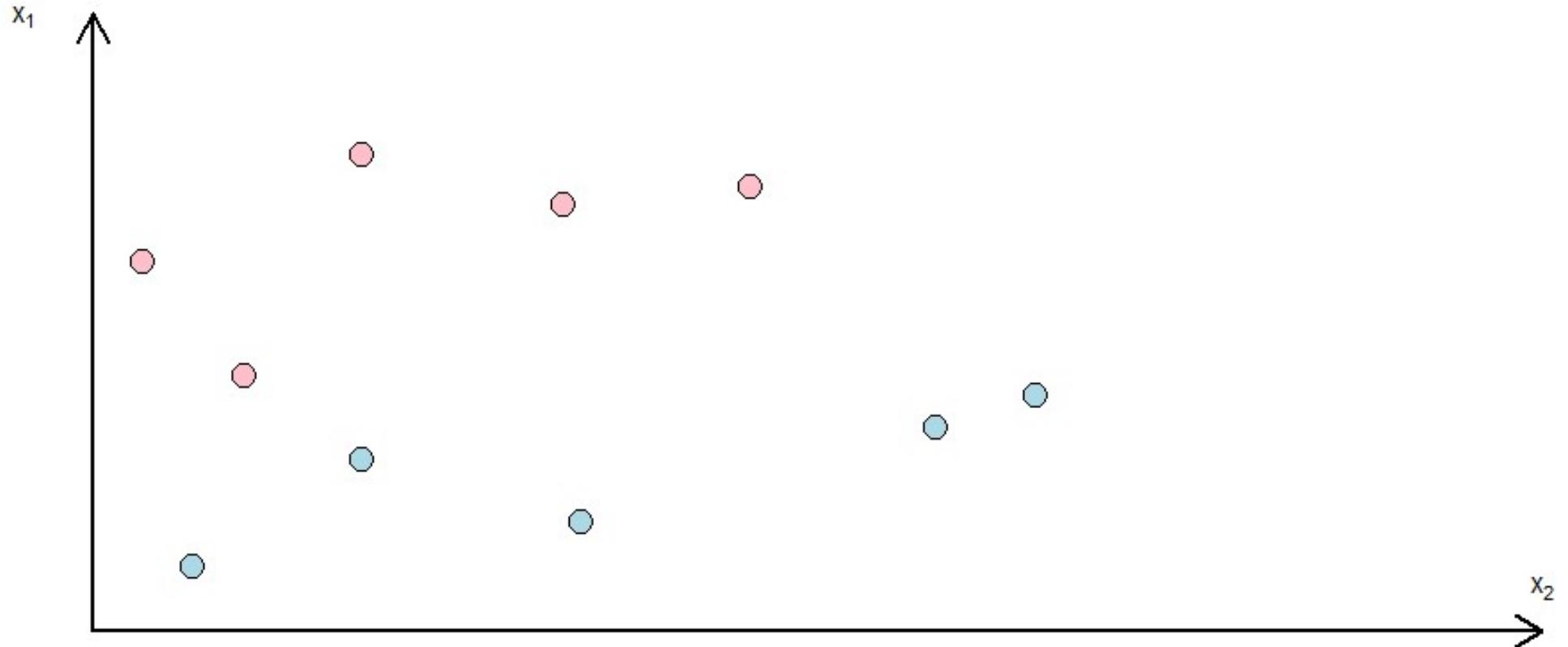


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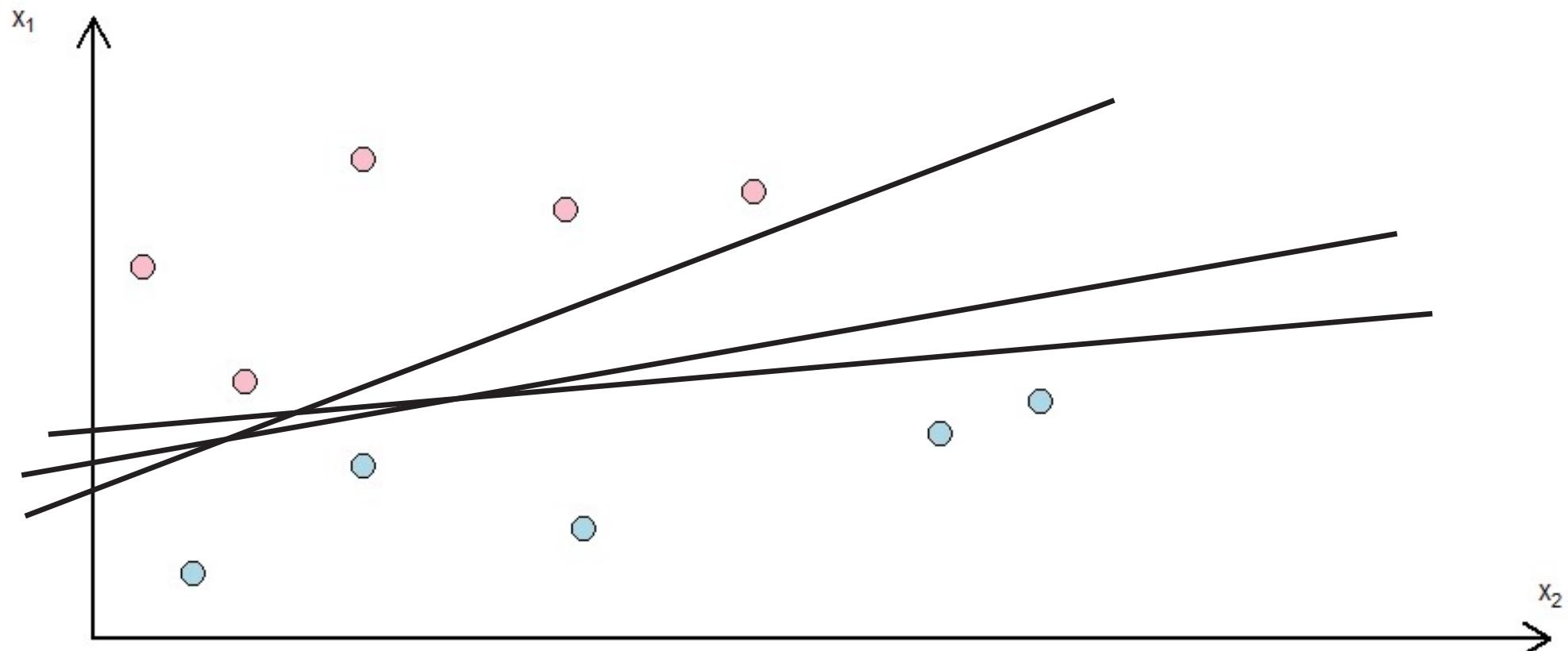
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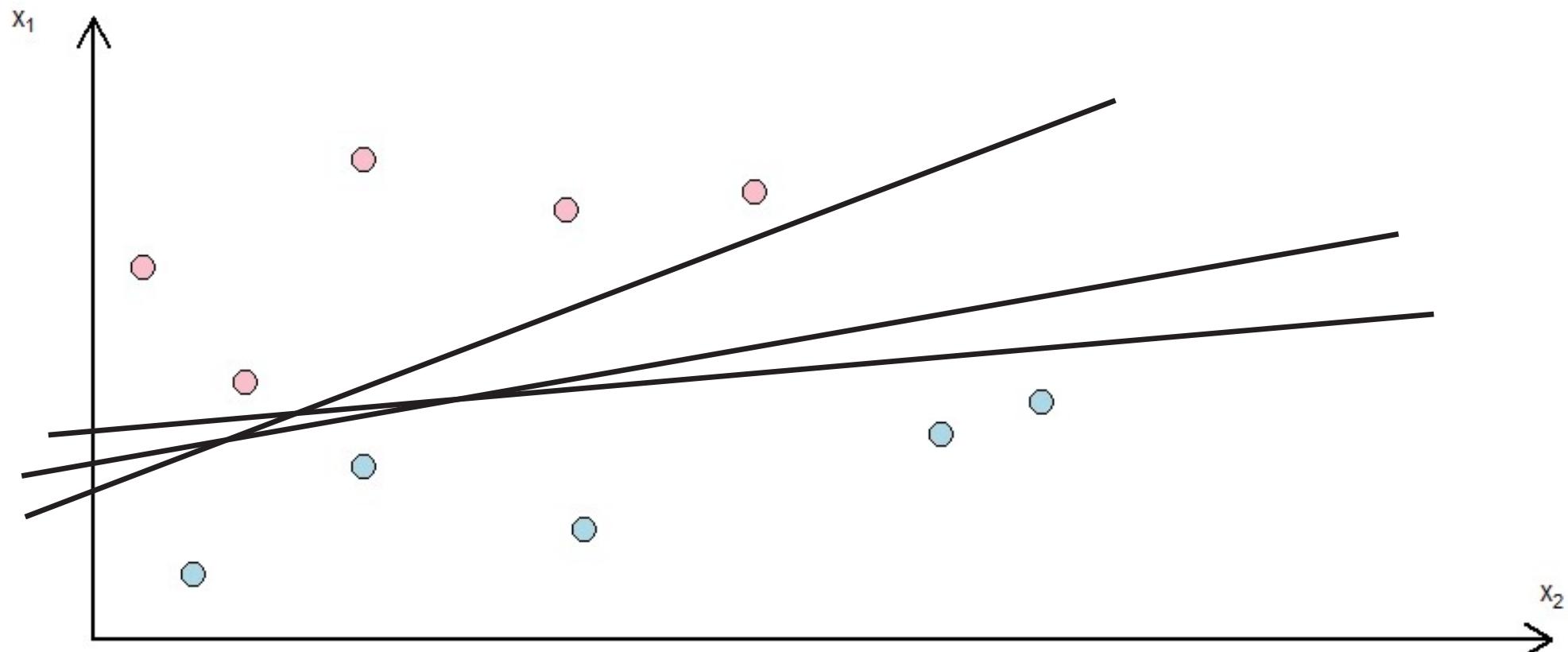


As the parties linearly separable? Where could you draw the line?

The 10 Senators

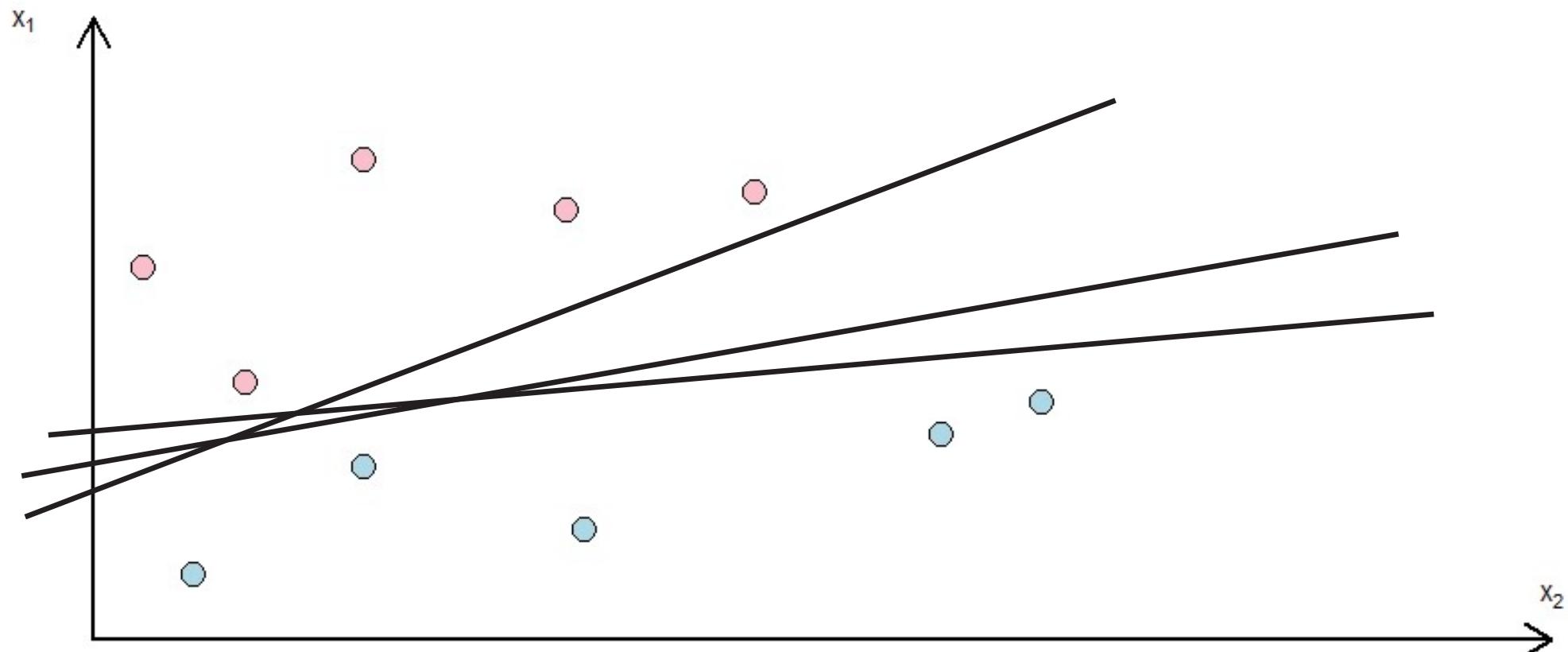


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- That optimal line—the separating hyperplane—is the **maximum margin** hyperplane. It will maximize the **margin** of the training data.

Partner Exercise

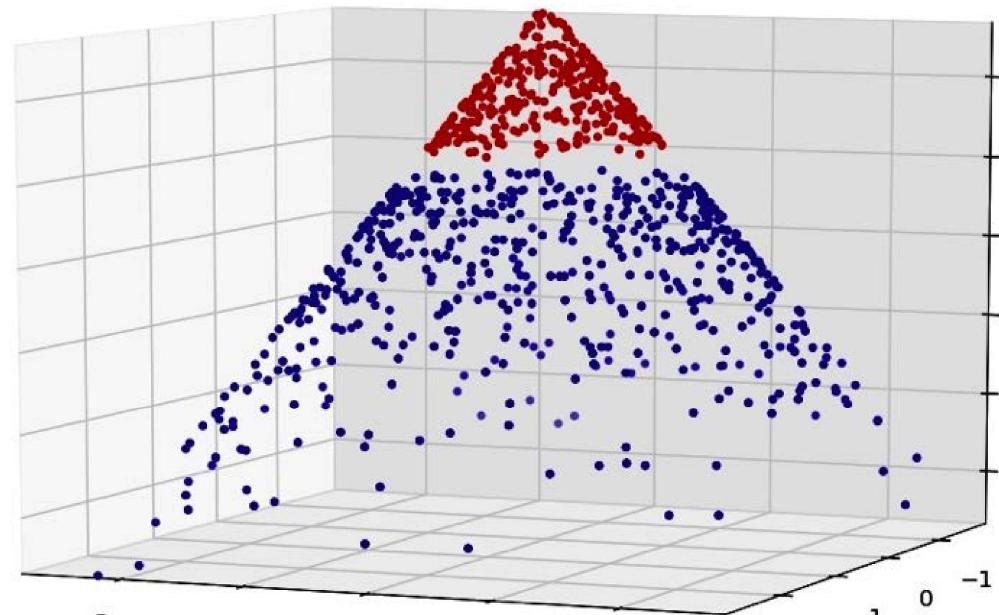
Partner Exercise

Consider the figure.

It's a situation where each Senator's features are of three dimensions (rather than two).

How could we (optimally) separate the data in a linear way?

Can we still use a line?



from <http://www.edvancer.in/>

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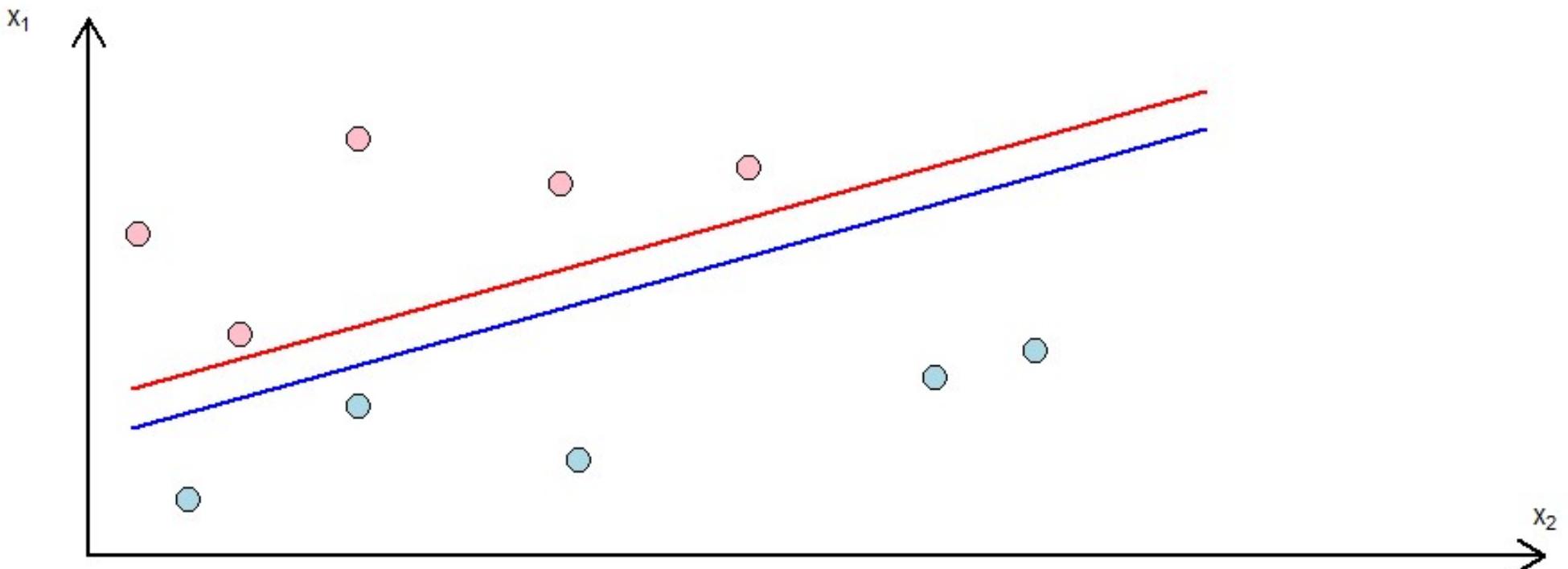
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NB The hyperplane cannot be anywhere other than **equidistant** because then it will break the rule about ensuring the **largest minimum distance**.

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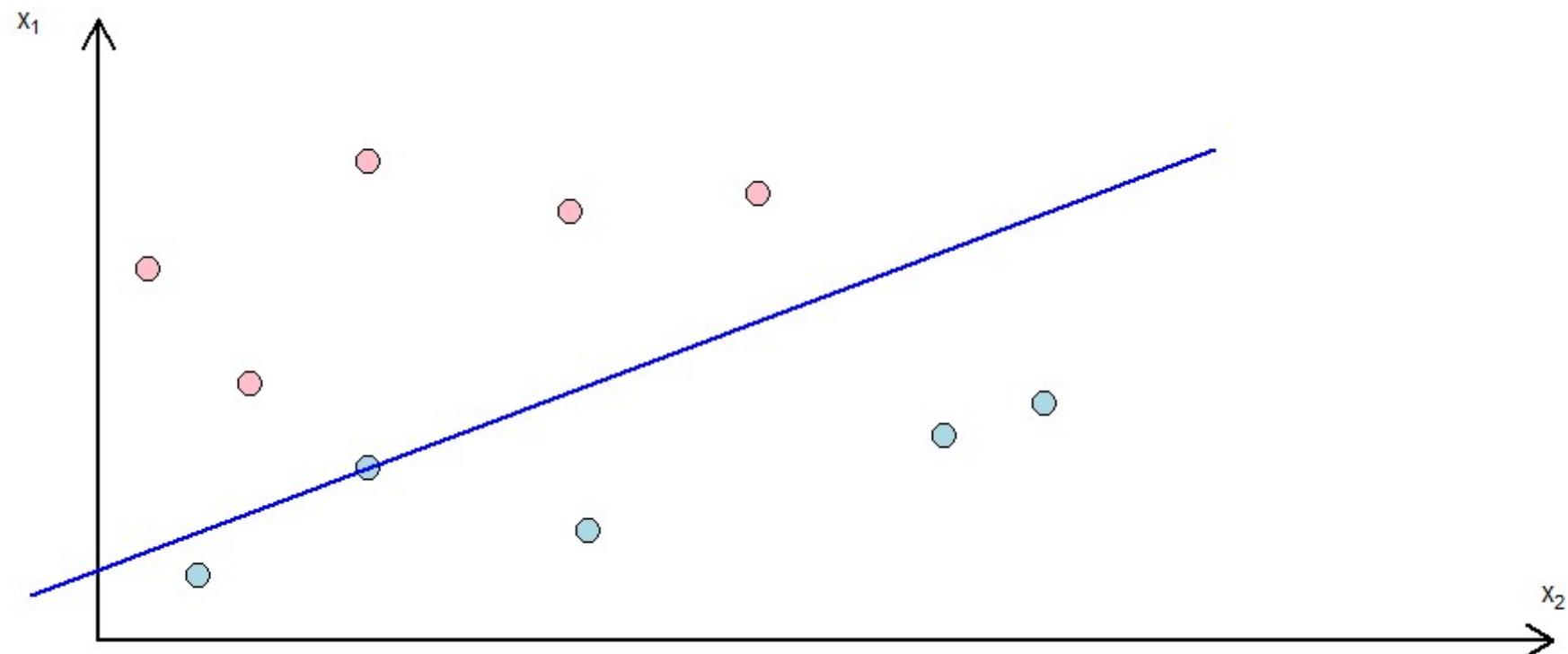
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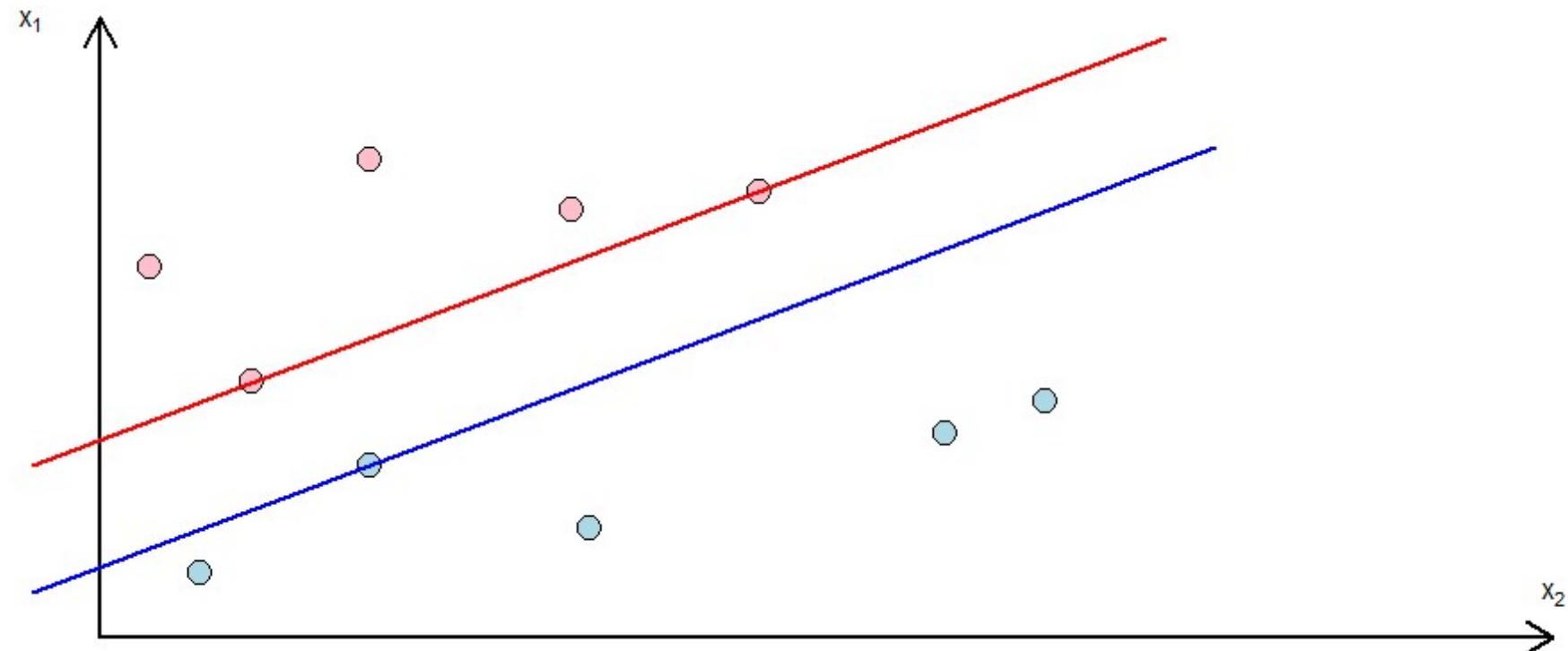
Turns out that the minimization of $\|\mathbf{w}\|$ is amenable to quadratic programming methods.

Graphically . . .

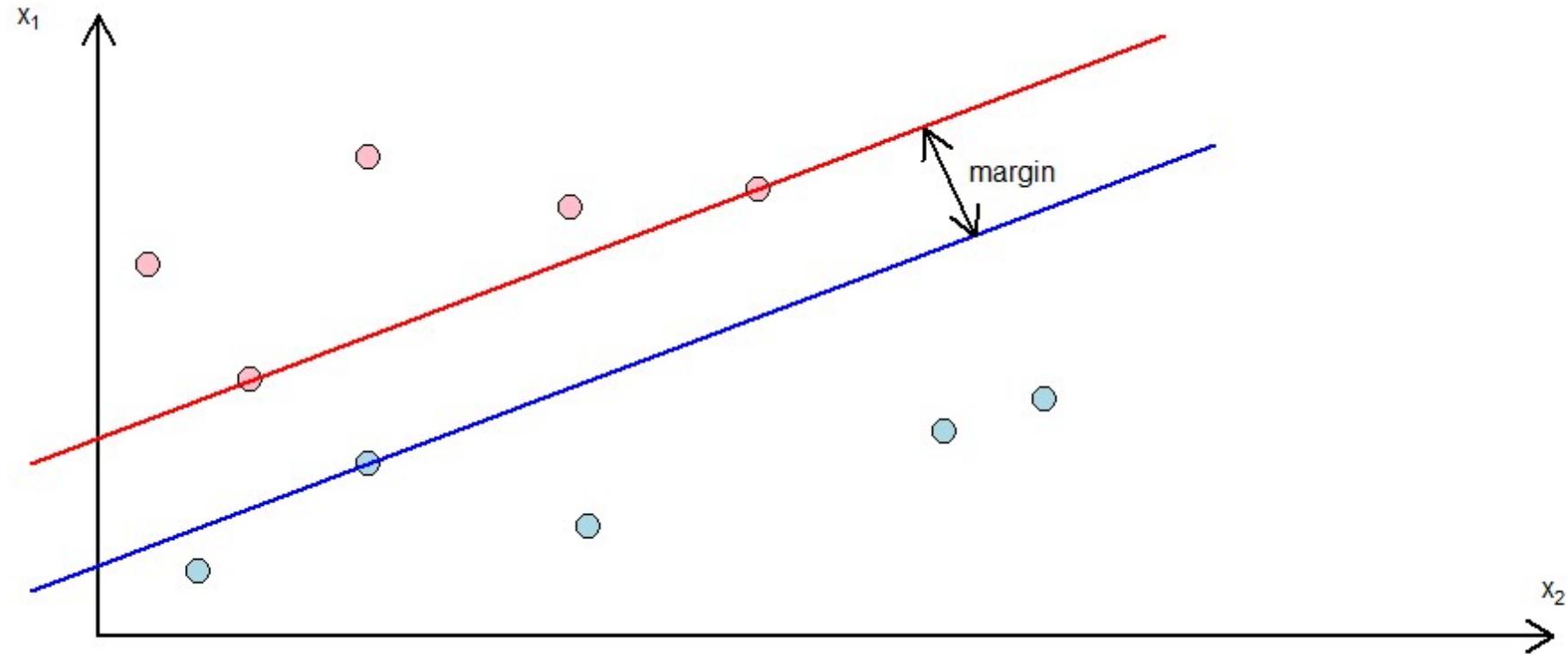
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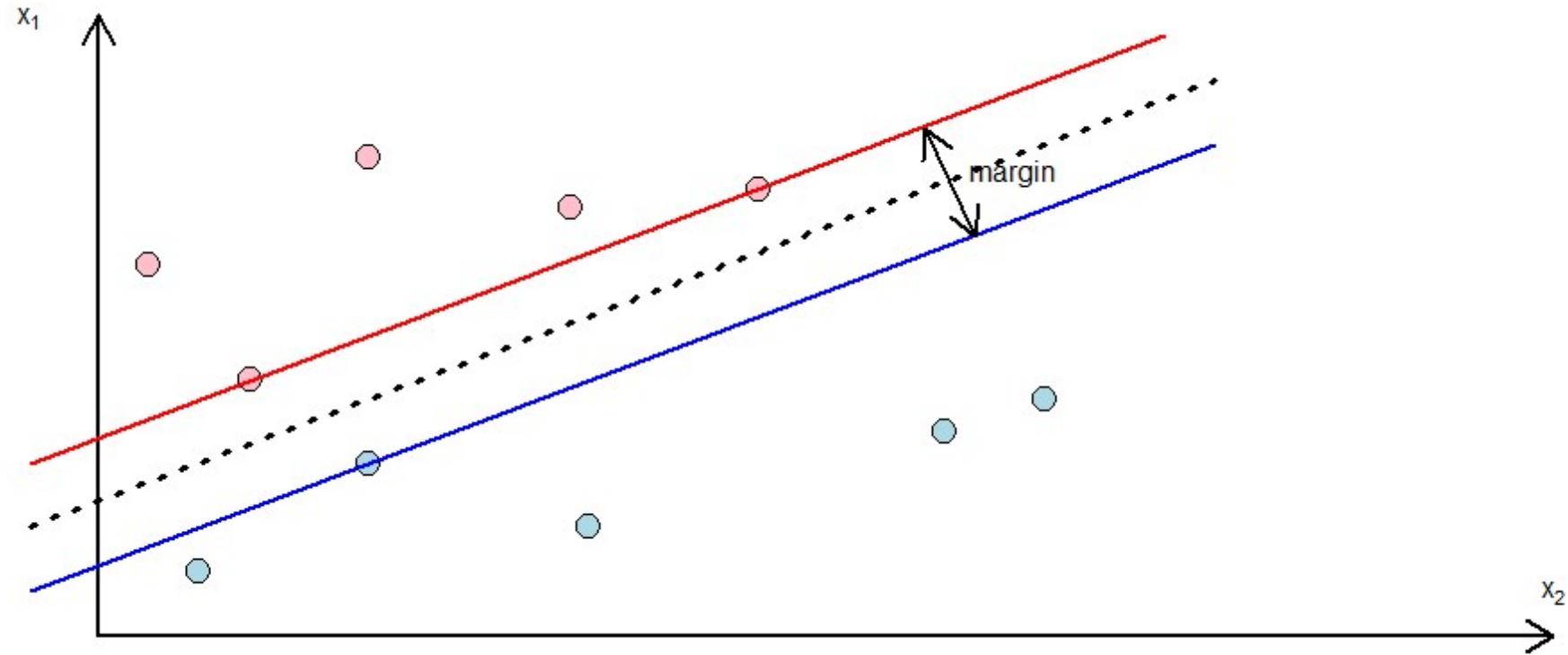
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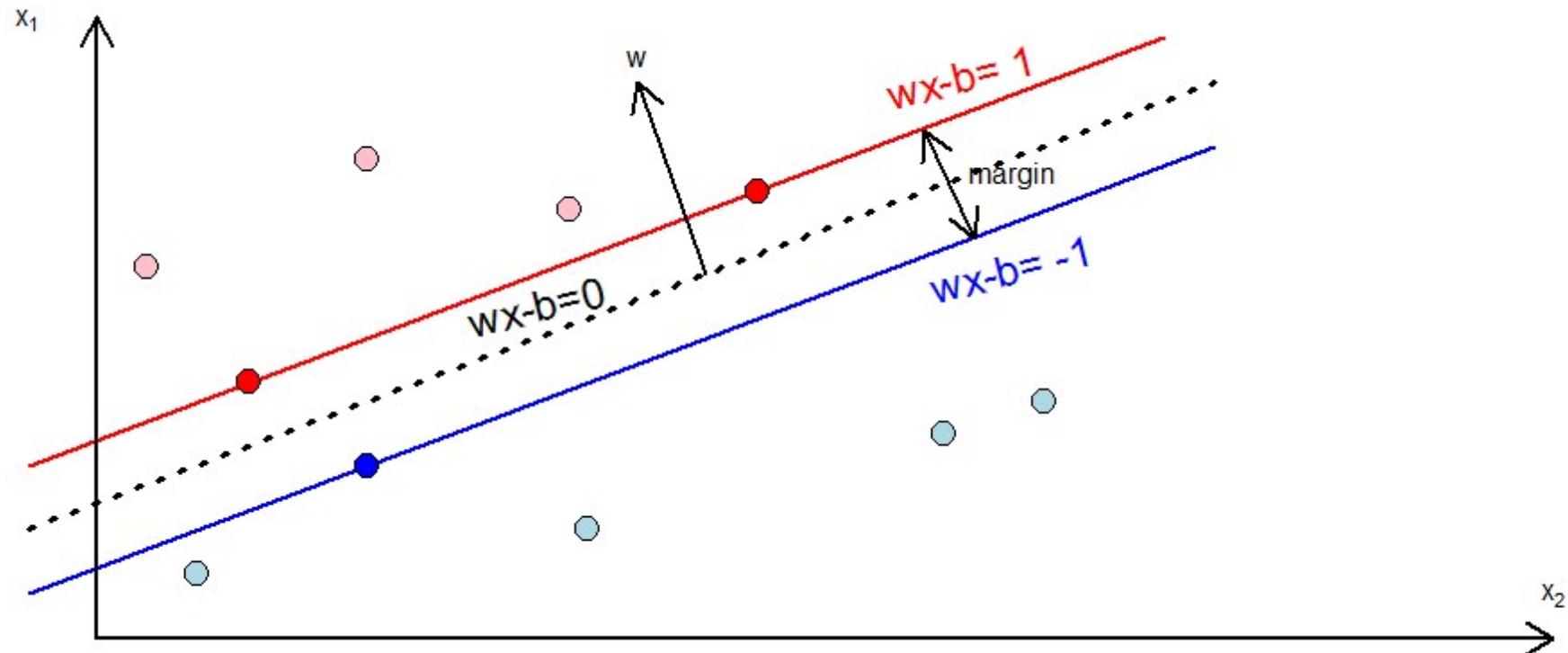
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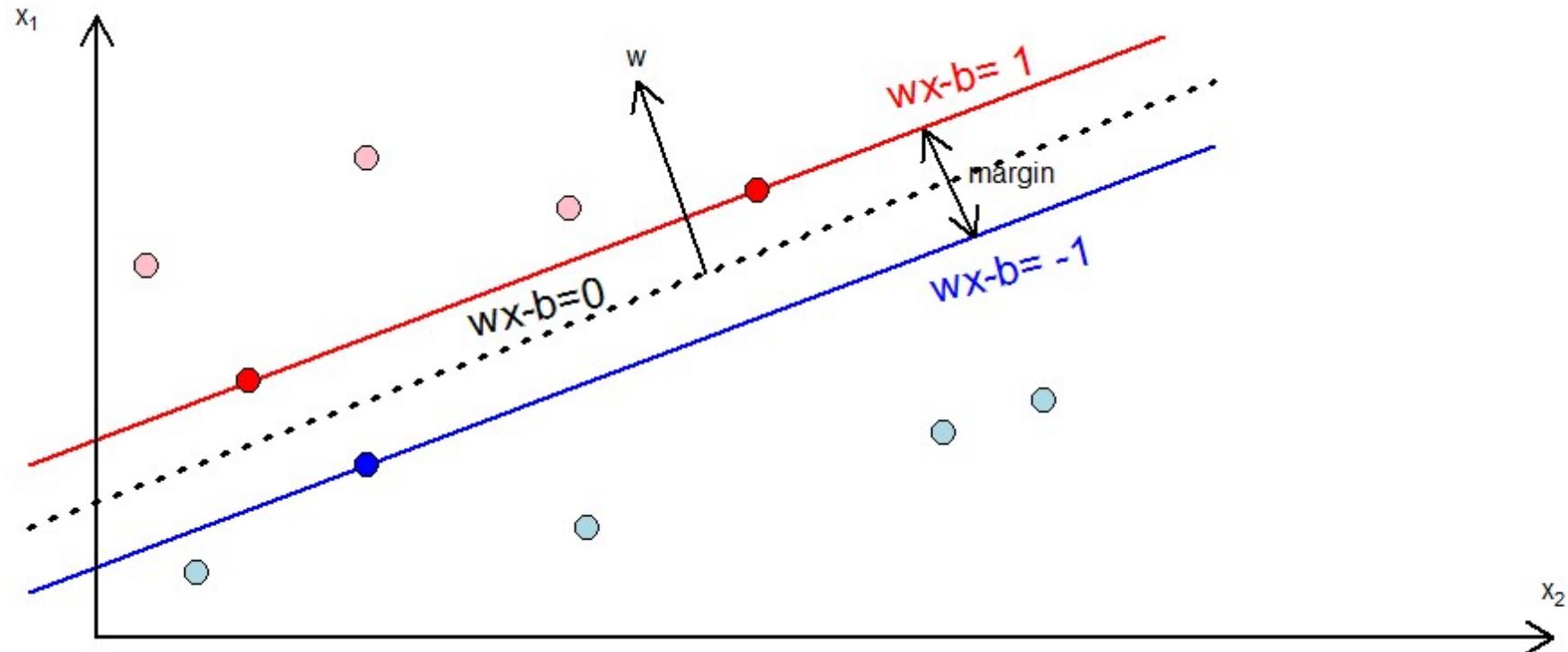
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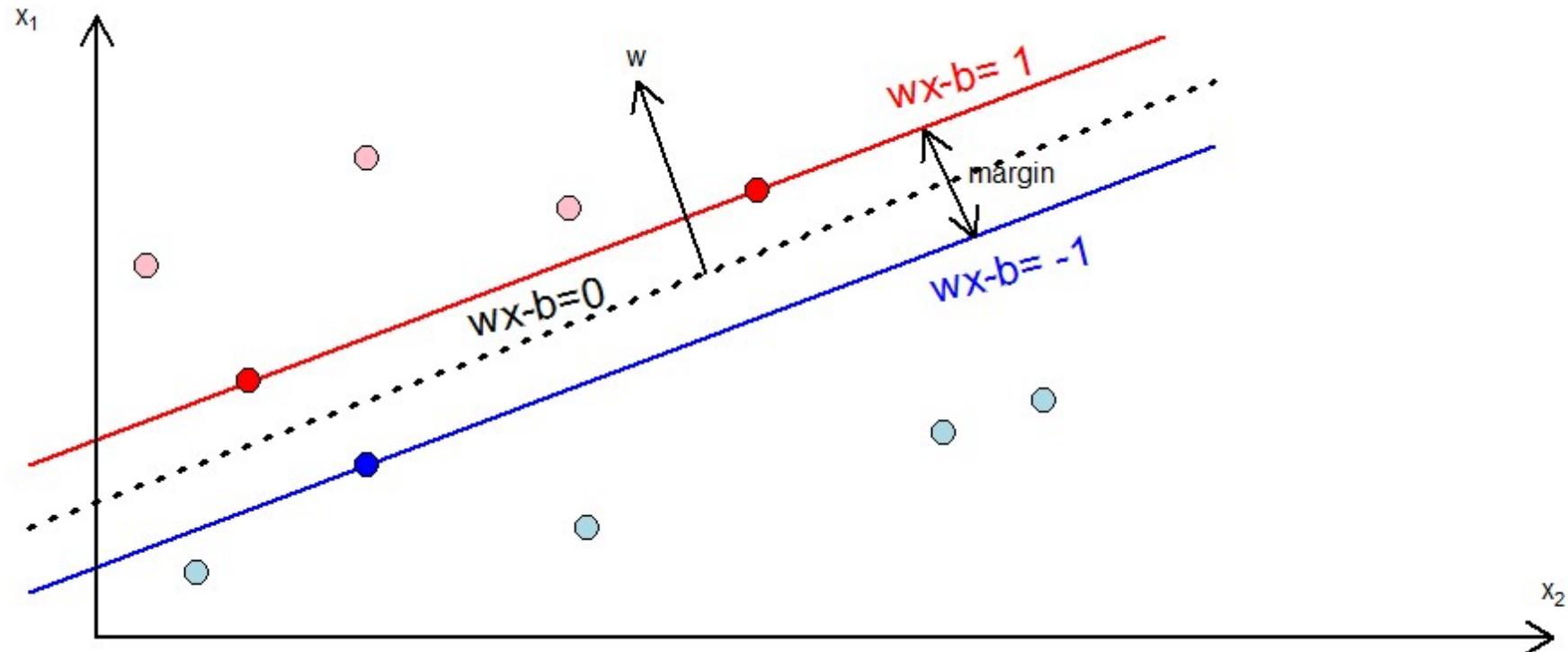


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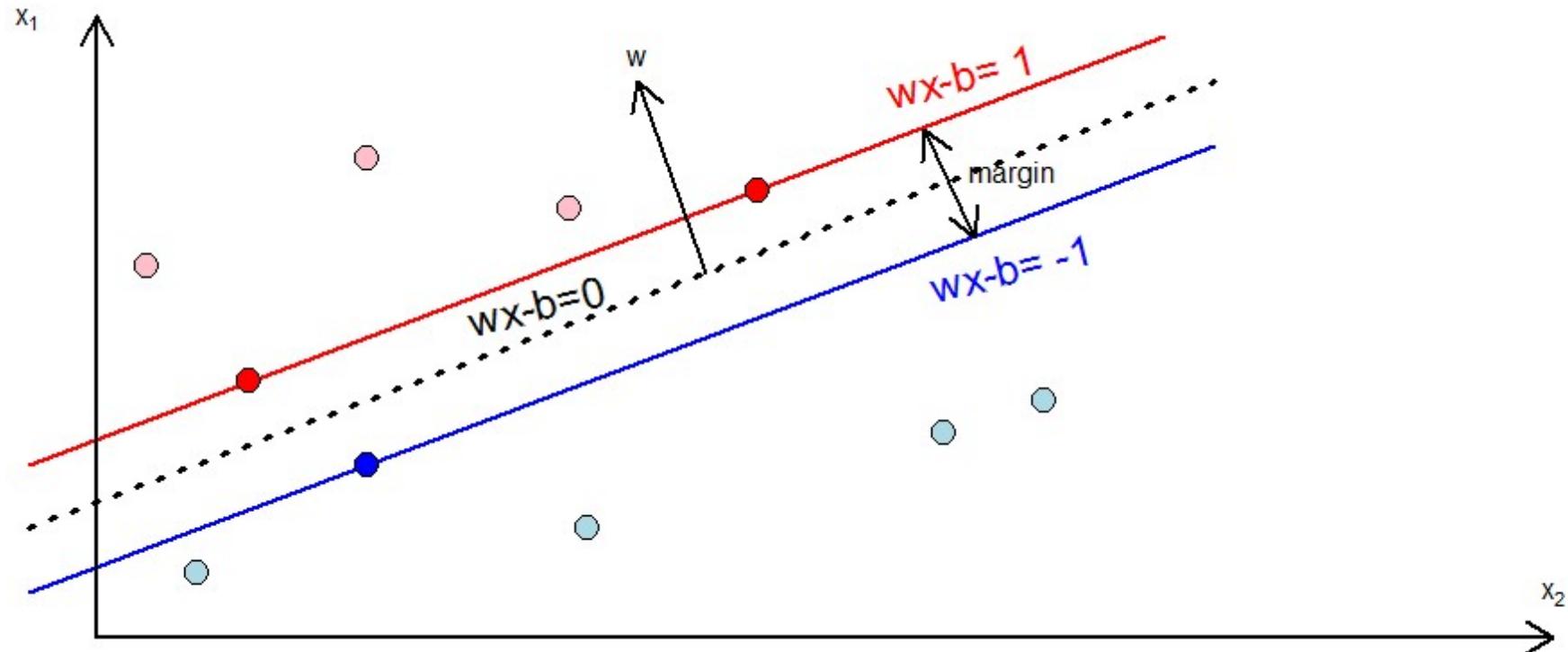
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Words			
Liberal		Conservative	
FAS: -199.49	SBA: -113.10	habeas: 193.55	homosexual: 103.07
Ethanol: -198.92	Nursing: -109.38	CFTC: 187.16	everglades: 102.87
Wealthiest: -159.74	Providence: -108.73	surtax: 151.81	tower: 101.67
Collider: -142.28	Arctic: -108.30	marriage: 145.79	tripartisan: 101.23
WIC: -140.14	Orange: -107.98	cloning: 141.71	PRC: 102.90
ILO: -139.89	Glaxo: -107.81	tritium: 133.49	scouts: 97.55
Handgun: -129.01	Libraries: -107.70	ranchers: 132.95	nashua: 99.32
Lobbyists: -128.95	Disabilities: -106.44	BTU: 121.92	ballistic: 97.22
Enron: -127.71	Prescription: -106.31	grazing: 121.59	salting: 94.28
Fishery: -127.30	NIH: -105.52	unfunded: 120.82	abortion: 91.94
Hydrogen: -122.59	Lobbying: -105.35	catfish: 120.82	NTSB: 93.81
Souter: -121.40	NRA: -105.20	IRS: 114.91	Haiti: 97.28
PTSD: -119.87	Trident: -104.15	unborn: 111.88	PAC: 92.85
Gun: -119.52	RNC: -103.46	Taiwan: 111.13	taxing: 90.39

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- 1 Does that imply that making conservative Senators use the word 'handgun' more often will make them more liberal? What does your answer suggest about **prediction** vs **explanation** with supervised techniques?
- 2 what is the (most likely) problem in the causal claim that $X \rightarrow Y$ in the Diermeier et al study?

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BTW RLR can cope well with **noise**, and (hard margin) SVM will struggle if there is no linear separability . . .

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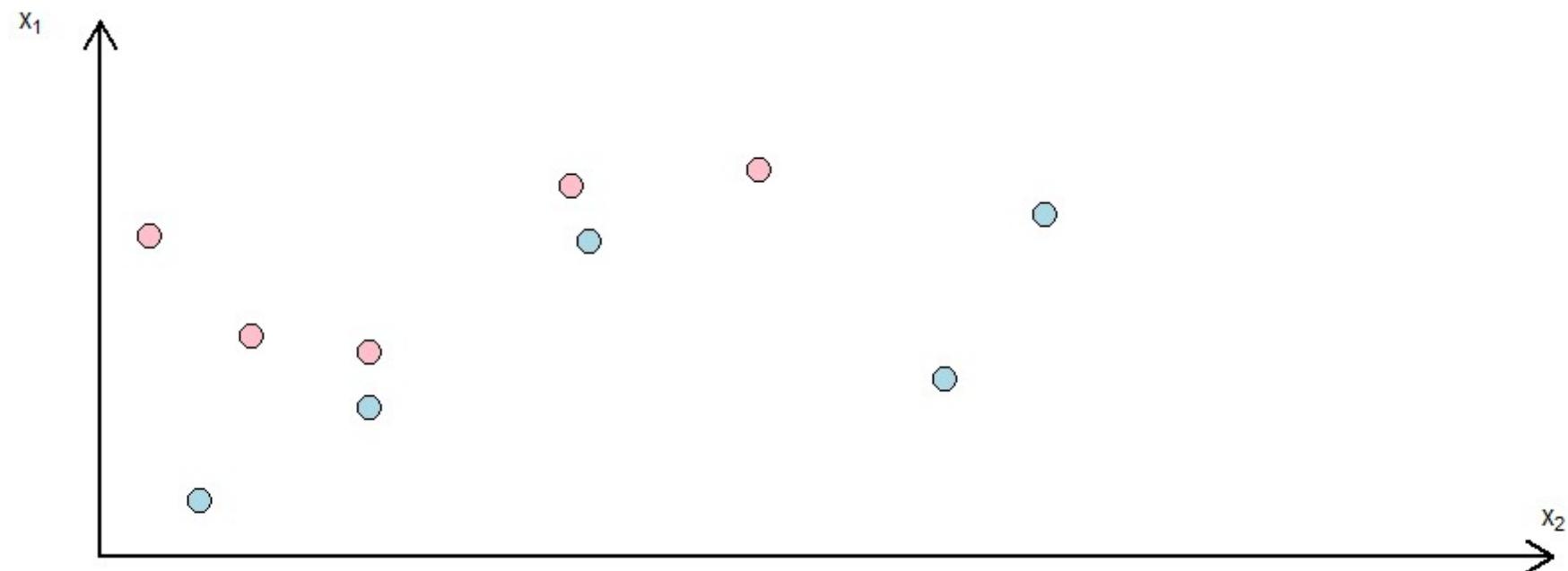
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Hyperplane(s) will be drawn in way that is more sensitive to 'bigger' mistakes in classification.

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 $1 - f(x)y$ is $1 - (-2)(+1)$ in first case and $1 - (-100)(+1)$ in second case! Hinge loss larger in second case!

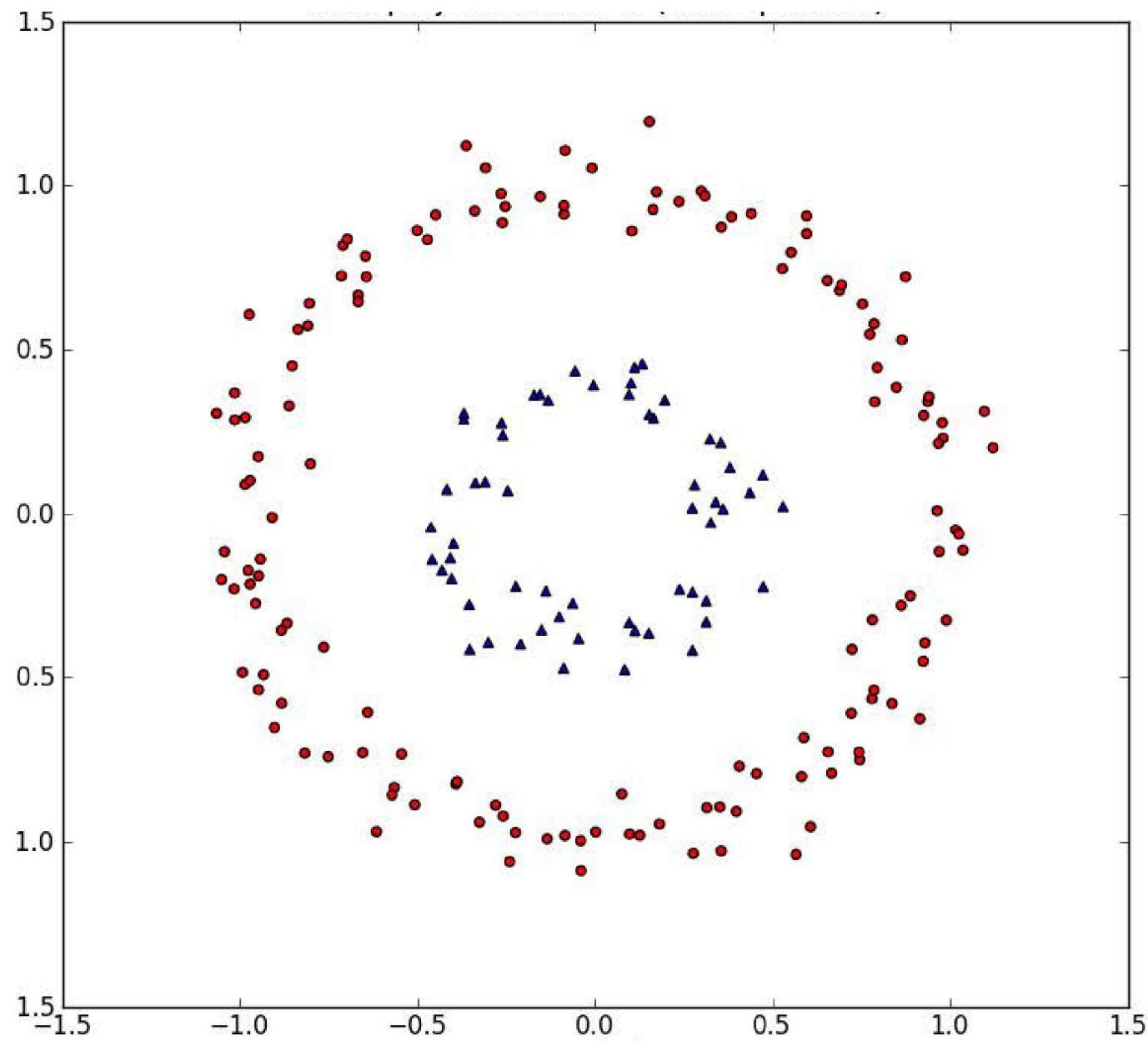
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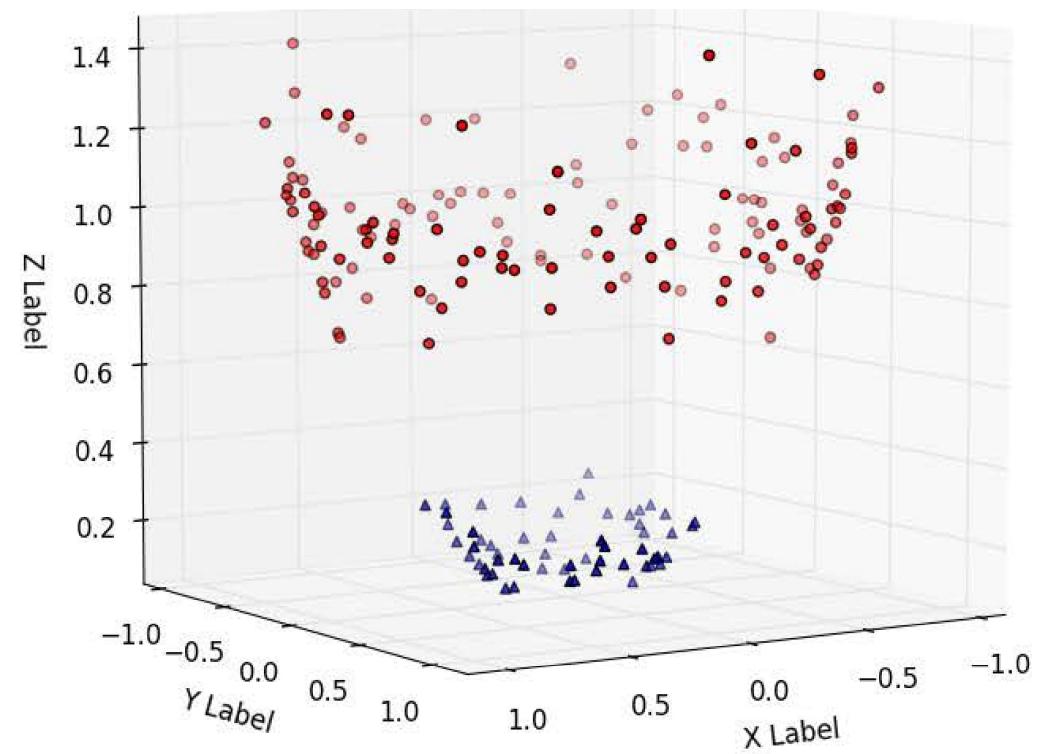
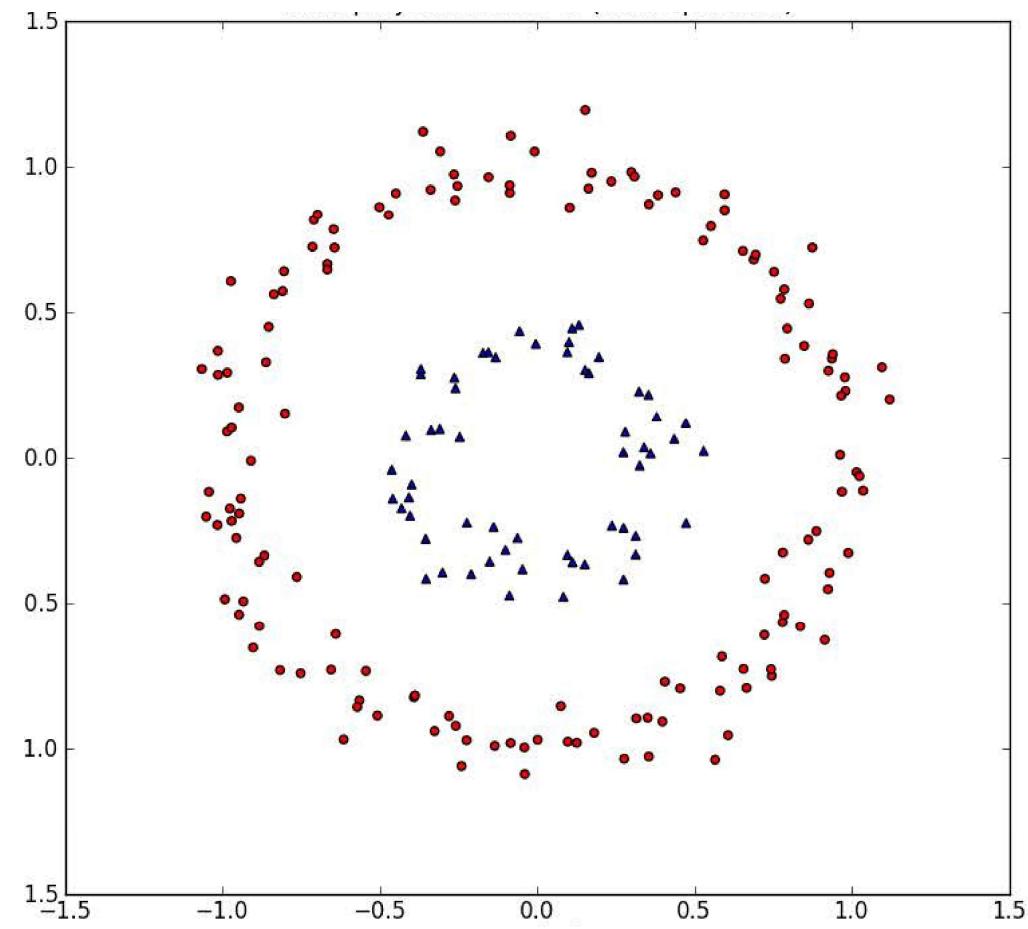
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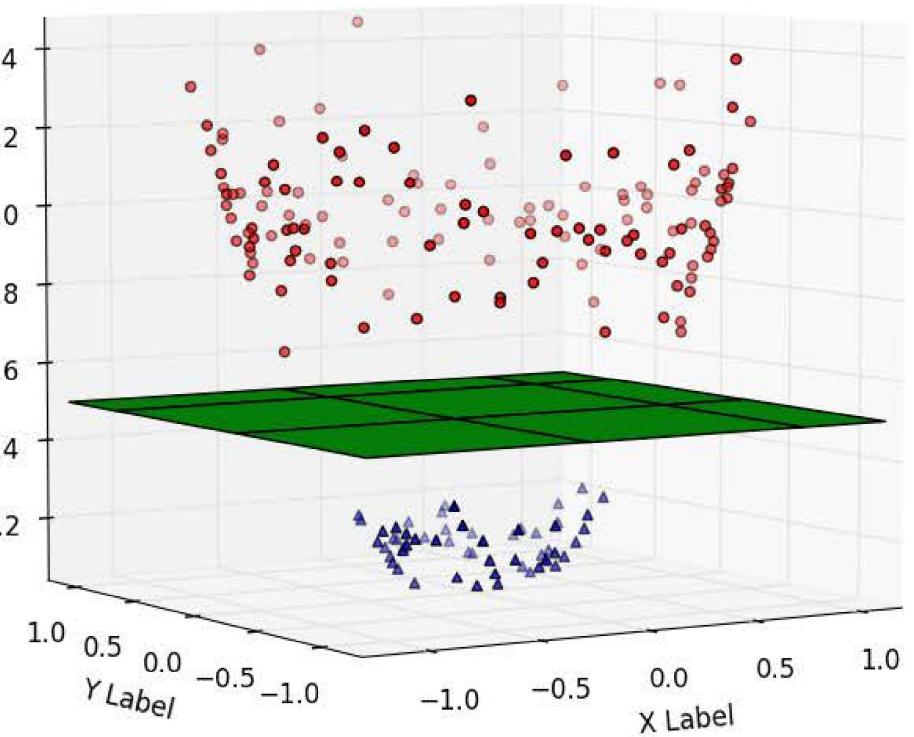
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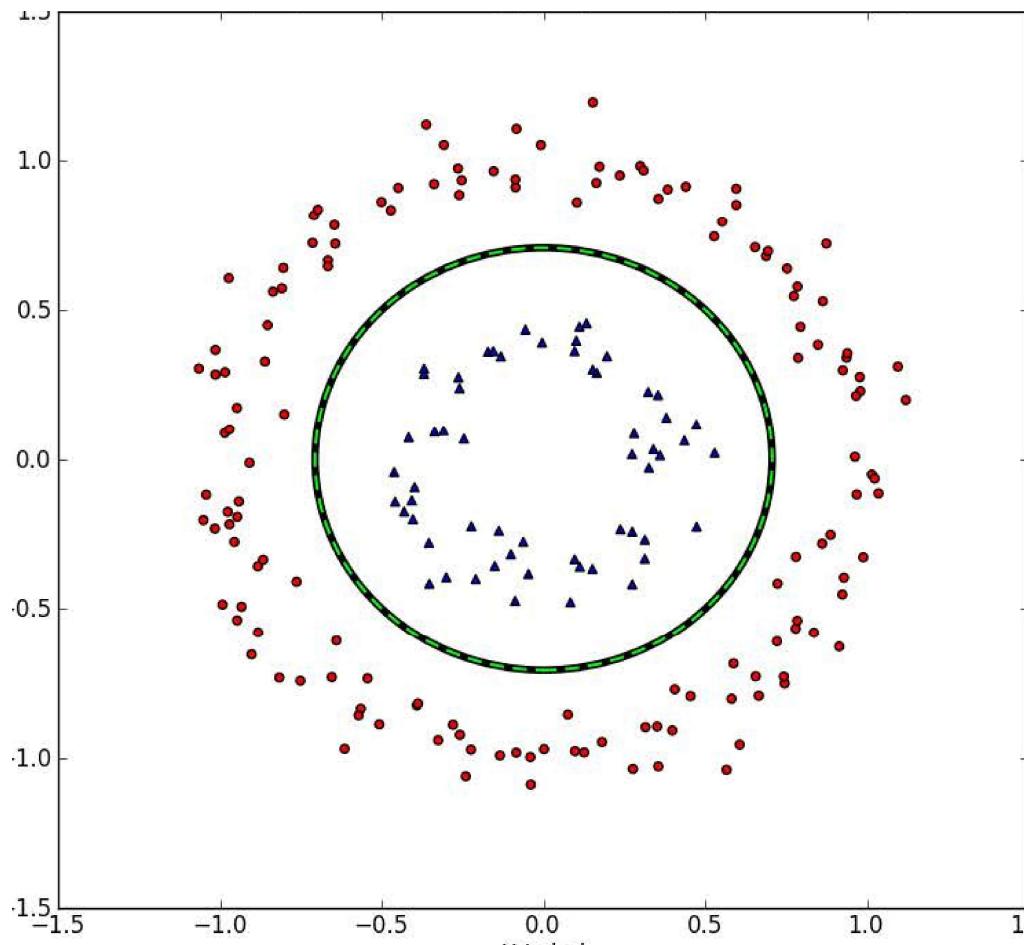
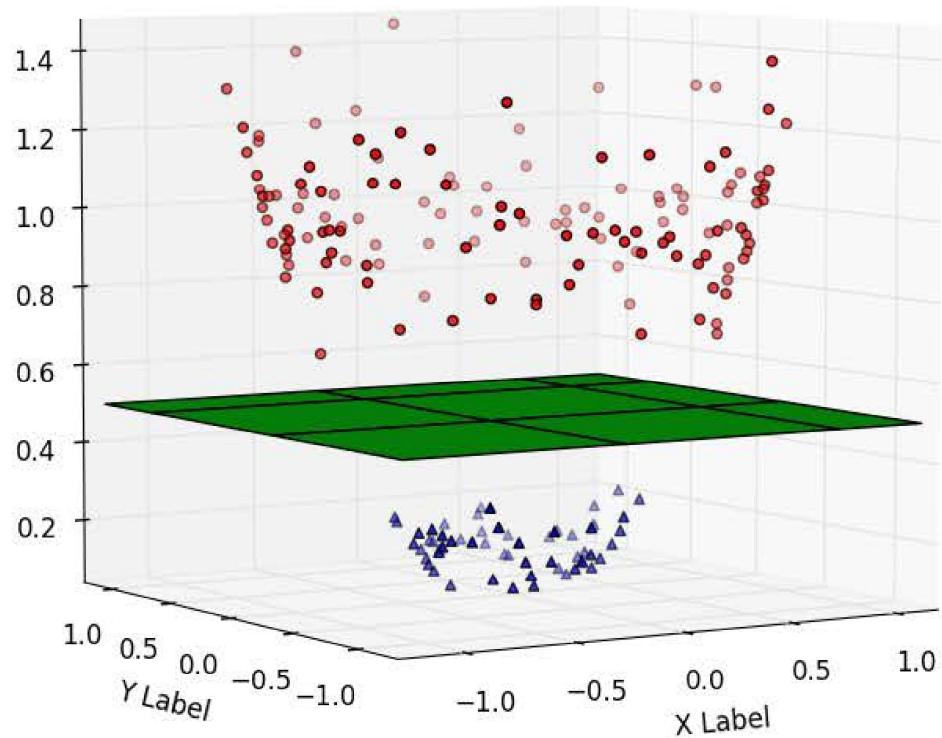
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→ Results in a **non-linear** hyperplane once back in 2 dimensions.



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For text analysis, **string kernels** use a function $K(a, b)$ to implicitly calculate the distance between strings of characters via the number of subsequences they have in common.

Partner Exercise

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Using the ideas we discussed at the start of lecture, how should one go about picking a kernel (from the large variety on offer) for the problem at hand?