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South East Technological University

BACHELOR OF SCIENCE (HONS) IN
- APPLIED COMPUTING
- COMPUTER FORENSICS & SECURITY
- ENTERTAINMENT SYSTEMS
- THE INTERNET OF THINGS

EXAMINATION:

DISCRETE MATHEMATICS (COMMON MODULE)

SEMESTER 1 - YEAR 1

AUGUST 2025 DURATION: 2 HOURS

INTERNAL EXAMINERS:	**DR DENIS FLYNN**	**DATE:**	**25 AUG 2025**
	DR KIERAN MURPHY	**TIME:**	**11:45 AM**
		VENUE:	**MAIN HALL**
EXTERNAL EXAMINER:	**DR JULIE CROWLEY**		

INSTRUCTIONS TO CANDIDATES

1. **ANSWER ALL QUESTIONS.**
2. **TOTAL MARKS = 100.**
3. **EXAM PAPER (5 PAGES EXCLUDING THIS COVER PAGE) AND FORMULA SHEET (1 PAGE)**

MATERIALS REQUIRED

1. **NEW MATHEMATICS TABLES.**
2. **GRAPH PAPER**

SOUTH EAST TECHNOLOGICAL UNIVERSITY

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OUTLINE MODEL ANSWERS & MARKING SCHEME

Course: BSc (H) in AC, in CF	Semester: 1	Page 1 of 6
Subject: MODULE NAME	Examiner: Dr D. Flynn, Dr K Murphy	

Question 1

(a) _____ **(4 marks)**

Partial marks if failed to make all propositions atomic.

Let's define the atomic propositions:

- A = "The alarm system is enabled"
- C = "The camera recognises an intruder"
- W = "A window is broken"
- G = "The Gardaí will be notified"

Then the given sentence as a propositional logic expression is:

$$(A \wedge (C \vee W)) \rightarrow G$$

(b) _____ **(4 marks)**

(i) Not well formed. Binary operator $\wedge$ is at the start of the formula and has only one operand after it.

(ii) Well formed.

(iii) Not well formed. Implication operator $\rightarrow$ has only one operand after it.

(iv) Not well formed. Implication operator $\rightarrow$ has no operand before it.

(c) _____ **(4 marks)**

(i) sum = 3 + 5 + 7 + 9 + 11 = 35

(ii) sum = 3 + 9 + 27 + 81 = 120

(iii) product = (1/2) * (2/3) * (3/4) * (4/5) = 1/5

(d) _____ **(4 marks)**

(i) $\binom{9-4+8-2}{5} = \binom{11}{5} = 462$. The paths all have length 11 (5 steps right and 6 steps up), we just select which 5 of those 11 should be to the right.

(ii) $\binom{6-4+5-2}{2} \times \binom{9-6+8-5}{3} = \binom{5}{2} \times \binom{6}{3} = 10 \times 20 = 200$. First, travel to $(6, 5)$, and then continue on to $(9, 8)$.

(iii) $\binom{11}{5} - \binom{5}{2} \times \binom{6}{3} = 462 - 200 = 262$. Remove all the paths found in the preceding question.

Question 2

(a) **(4 marks)**

(i) $f(g(2)) = f(3 \cdot 2 - 1) = f(5) = (5+2)^2 - 9 = 49 - 9 = 40$

(ii) $h(k(-9)) = h(4+9) = h(13) = 13//3 = -1$

(iii) $g(h(f(1))) = g(h((1+2)^2 - 9)) = g(h(0)) = g(0//3) = g(0) = 3 \cdot 0 - 1 = -1$

(iv) $j(k(j(12))) = j(k(12\%5)) = j(k(2)) = j(4-2) = j(2) = 2\%5 = 2$

(b) **(4 marks)**

(i) $a_0 = 8$ and $a_n = a_{n-1} + 16$ for $n \geq 1$.

(ii) $a_n = 16n + 8$.

(iii) $2024 = 16n + 8$ so $n = 126 \in \mathbb{N}$, therefore 2024 is a term in the sequence.

(iv) $16n + 8 < 1000$ so $n < 62$, therefore there are 62 terms less than 1000.

(v) $S_{99} = (99+1) \times \frac{8+(16\times 99+8)}{2} = 100 \times 800 = 80000$.

(c) **(4 marks)**

(i) The degree sequence is (3, 3, 4, 4, 4, 4).

(ii) Yes, this is a simple graph. No loops, or multiple edges.

(iii) The girth is 3, e.g. AC,CD,CA.

Question 3

(a) _____ **(4 marks)**

(i)

- $U = \{1, 2, \cdots, 11\}$
- $A = \{2, 5, 8\}$
- $B = \{1, 3, 5, 7, 9, 11\}$
- $C = \{3, 4, 5, 6, 7\}$.

(ii) Venn Diagram (as done in class, with the updated sets).

(iii) Equivalent mathematical expressions:

- D $= (A \cup B) \setminus C$
- E $= U \setminus (A \cap B)$
- F $= (B \setminus A) \cup (C \setminus B)$

(iv)

- D $= \{1, 2, 8, 9, 11\}$
- E $= \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11\}$
- F $= \{1, 3, 4, 6, 7, 9, 11\}$

(b) _____ **(4 marks)**

(i) $2^3 = \mathbf{8}$ subsets.

(ii) $\binom{4}{2} \cdot 2^3 = 6 \cdot 8 = \mathbf{48}$ subsets.

(iii) $2^7 - 2^3 - \binom{4}{1} \cdot 2^3 = 128 - 8 - 32 = \mathbf{88}$ subsets.

(iv) $\binom{4}{3} \times \binom{3}{2} = \mathbf{12}$ subsets.

(c) _____ **(4 marks)**

P	Q	R	$P \to Q$	$Q \to R$	$(\boldsymbol{P \to Q}) \wedge (\boldsymbol{Q \to R})$	$Q \wedge R$	$\boldsymbol{P \to (Q \wedge R)}$
T	T	T	T	T	**T**	T	**T**
T	T	F	T	F	**F**	F	**F**
T	F	T	F	T	**F**	F	**F**
T	F	F	F	T	**F**	F	**F**
F	T	T	T	T	**T**	T	**T**
F	T	F	T	F	**F**	F	**T**
F	F	T	T	T	**T**	F	**T**
F	F	F	T	T	**T**	F	**T**

$(P \to Q) \wedge (Q \to R)$ and $P \to (Q \wedge R)$ are not logically equivalent - outputs do not match.

Question 4

(a) _____ **(4 marks)**

(i) $\mathrm{V} \to \mathrm{P}$

(ii) $(\mathrm{P} \wedge \neg \mathrm{G}) \to \neg \mathrm{V}$

(iii) $\mathrm{G} \to (\mathrm{V} \vee \mathrm{P})$

(iv) $(\neg \mathrm{V} \wedge \neg \mathrm{P}) \to \neg \mathrm{G}$

(b) _____ **(4 marks)**

(i)

$$\underbrace{\frac{1}{2}}_{a_0}, \underbrace{1}_{a_1}, \underbrace{2}_{a_2}, \underbrace{4}_{a_3}, \underbrace{8}_{a_4}, \underbrace{16}_{a_5}, \ldots$$

$ar^2 = 2$, $ar^5 = 16 \implies r^3 = 8 \implies r = 2 \implies 4a = 2 \implies a = \frac{1}{2}$.

(c) _____ **(4 marks)**

(i) Starting with three fixed bits leaves $(12-3) = 9$ yes/no choices, giving $|B^9| = 2^9 = 512$ strings.

(ii) Weight 5 and starting with 110 (which has weight 2) means the remaining $(12-3) = 9$ bits must have weight $(5-2) = 3$. This gives us $|B_3^9| = \binom{9}{3} = 84$ strings.

(iii) Divisible by 8 means the bit string must end with 000. A weight of 6 then means the remaining $(12-3) = 9$ bits must have weight 6. This gives us $|B_6^9| = \binom{9}{6} = 84$ strings.

(d) _____ **(4 marks)**

p	q	r	$q \vee r$	$p \to (q \vee r)$	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \to r$	$(p \to (q \vee r)) \to ((p \wedge \neg q) \to r)$
F	F	F	F	T	T	F	T	T
F	F	T	T	T	T	F	T	T
F	T	F	T	T	F	F	T	T
F	T	T	T	T	F	F	T	T
T	F	F	F	F	T	T	F	T
T	F	T	T	T	T	T	T	T
T	T	F	T	T	F	F	T	T
T	T	T	T	T	F	F	T	T

or starting at T T T:

p	q	r	$q \vee r$	$p \to (q \vee r)$	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \to r$	$(p \to (q \vee r)) \to ((p \wedge \neg q) \to r)$
T	T	T	T	T	F	F	T	T
T	T	F	T	T	F	F	T	T
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	T	F	T
F	T	T	T	T	F	F	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	F	T	T
F	F	F	F	T	T	F	T	T

Since the final column is all true in both truth tables, the proposition is a tautology.

Question 5

(a) **(4 marks)**

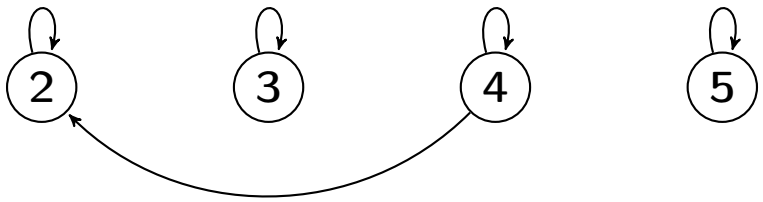
(i) The Python code gives the set $A = \{2, 3, 4, 5\}$, and the relation $R = \{(a, b)|a, b \in A, a \mod b = 0\}$
$= \{(2, 2), (3, 3), (4, 2), (4, 4), (5, 5)\}$.

(ii) The digraph is shown below.

(iii) The relation R is reflexive (every element in A is related to itself), antisymmetric (if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$), and transitive (if $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$).

(iv) R is not an equivalence relation as it's not symmetric (e.g., $(4, 2) \in R$ but $(2, 4) \notin R$).

(v) R is: not irreflexive ($\because (2, 2) \in R$ etc.); not asymmetric (e.g., R is reflexive) but is antisymmetric.



(b) **(4 marks)**

Returns True if sets A and B are equal, otherwise returns False. It uses a subset test and an equal cardinality test to achieve this.

(c) **(4 marks)**

The AP is $(a = 3, d = 5)$

$$\underbrace{3}_{a_0}, \underbrace{8}_{a_1}, \underbrace{13}_{a_2}, ..., \underbrace{a + d(n-1)}_{a_n}, ...$$

So the 12th element is $3 + 5(12 - 1) = 58$.