

# Package ‘SharedAS’

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**Type** Package

**Title** Shared active subspace for multi-variate vector-valued functions

**Version** 0.1.0

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## Description

Implements several baseline methods to compute a shared active subspace for multi-variate vector-valued functions. It also provides the implementation of the method of Zahm, O., P. G. Constantine, C. Prieur, and Y. M. Marzouk (2020). Gradient-based dimension reduction of multivariate vector-valued functions. *SIAM Journal on Scientific Computing* 42 (1), A534–A558.

**License** GPLv2

**Encoding** UTF-8

**LazyData** true

**Imports** cpc, geigen, ggplot2, mvtnorm, numDeriv, pracma, RRembo, stats, tidyr, dplyr, alphahull

**RoxygenNote** 7.2.3

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methods_grad	<i>Computes representative vectors for the Jacobians evaluated at data points and returns the matrix of eigenvectors obtained from the SPD matrix build from such vectors.</i>
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## Description

Computes representative vectors for the Jacobians evaluated at data points and returns the matrix of eigenvectors obtained from the SPD matrix build from such vectors.

## Usage

```
methods_grad(
  grads,
  n,
  m,
  d,
  method = c("AG", "LP", "MCH"),
  is_grad_norm = FALSE
)
```

## Arguments

grads	Gradient matrix, where the columns are the concatenation of the objective-wise gradients
n	Sample size
m	Number of objectives or function outputs
d	Number of input dimensions
method	The method to compute a representative vector, either <ul style="list-style-type: none"> <li>• 'AG' computes the average of gradient vectors</li> <li>• 'LP' for each input variable, computes the rank-1 linear projection of the objective-wise derivatives wrt that variable (based on the leading eigenvector of the SPD matrix of these same derivatives).</li> <li>• 'MCH' computes the minimum-norm element of the convex hull computed from the Jacobian</li> </ul>
is_grad_norm	If set to TRUE, performs the L2 normalization of the gradients. This is used only for AG and MCH.

## Value

Matrix of eigenvectors (column vectors)

## Examples

```
fn <- problem_car_side_impact
d <- 7
n <- 1000
X <- matrix(runif(d*n), n)
Y <- t(apply(X, 1, fn))
grads <- t(apply(X, 1, function(x) c(t(numDeriv::jacobian(fn, x = x)))))
```

```
methods_grad(grads, n, ncol(Y), d, method="MCH")
```

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methods_SPD	<i>Returns the matrix of eigenvectors computed from the stepwise estimation of eigenvectors, joint diagonalization based on the FG algorithm, or from the eigendecomposition of the sum of SPD matrices.</i>
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## Description

Returns the matrix of eigenvectors computed from the stepwise estimation of eigenvectors, joint diagonalization based on the FG algorithm, or from the eigendecomposition of the sum of SPD matrices.

## Usage

```
methods_SPD(grads, n, m, d, method = c("SSPD", "SEE", "FG"))
```

## Arguments

grads	Gradient matrix, where the columns are the concatenation of the objective-wise gradients
n	Sample size
m	Number of objectives
d	Number of input dimensions
method	Name of the SPD method to choose from <ul style="list-style-type: none"> <li>• 'SSPD' to compute the eigenvectors from the sum of SPD matrices,</li> <li>• 'SEE' to compute the eigenvectors in a stepwise fashion, or</li> <li>• 'FG' to do simultaneous diagonalization using the Flury and Gautschi algorithm.</li> </ul>

## Value

Matrix of eigenvectors (column vectors)

## References

- Flury, B. N. and W. Gautschi (1986). An algorithm for simultaneous orthogonal transformation of several positive definite symmetric matrices to nearly diagonal form. *SIAM Journal on Scientific and Statistical Computing* 7 (1), 169–184
- Trendafilov, N. T. (2010). Stepwise estimation of common principal components. *Computational Statistics & Data Analysis* 54 (12), 3446–3457.

## Examples

```
fn <- problem_car_side_impact
d <- 7
n <- 1000
X <- matrix(runif(d*n), n)
Y <- t(apply(X, 1, fn))
grads <- t(apply(X, 1, function(x) c(t(numDeriv::jacobian(fn, x = x)))))
methods_SPD(grads, n, ncol(Y), d, method="FG")
```

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min_convex_hull	<i>A recursive algorithm to compute the minimum L2-norm point in a polytope.</i>
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### Description

A recursive algorithm to compute the minimum L2-norm point in a polytope.

### Usage

```
min_convex_hull(P)
```

### Arguments

P	Matrix for the convex hull based on which to compute the minimum L2-norm element
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### Value

Vector representing the minimum L2-norm point of the convex hull of P

### References

Sekitani, K. and Y. Yamamoto (1993). A recursive algorithm for finding the minimum norm point in a polytope and a pair of closest points in two polytopes. *Mathematical Programming* 61, 233–249

### Examples

```
X <- matrix(runif(3*10), 10)
min_convex_hull(X)
```

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problem_car_side_impact	<i>Car side-impact problem</i>
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### Description

Car side-impact problem

### Usage

```
problem_car_side_impact(
  x,
  params = list(p = NULL, mp = c(0.345, 0.192, 0, 0), sp = c(0.06, 0.06, 10, 10), fixed =
    TRUE)
)
```

**Arguments**

- x** a vector in  $[0, 1]^7$  corresponding to 1: Thickness of B-Pillar inner 2: Thickness of B-Pillar reinforcement 3: Thickness of floor side inner 4: Thickness of cross members 5: Thickness of door beam 6 : Thickness of door beltline reinforcement 7 : Thickness of roof rail
- params** List of additional parameters:
- **p** optional vector of values for the random parameter, if **fixed** = TRUE, no sampling is done and they are fixed at their mean: 8: Material of B-Pillar inner 9: Material of floor side inner 10: Barrier height 11: Barrier hitting position
  - **mp**, **sp** mean and standard deviation for **p**
  - **fixed** Logical

**Value**

Vector of size 11

**References**

Deb, K. et al. "Reliability-Based Optimization Using Evolutionary Algorithms." Evolutionary Computation, IEEE Transactions on 13.5 (2009): 1054-1074. 2009 Institute of Electrical and Electronics Engineers

**Examples**

```
d <- 7
n <- 1000
X <- matrix(runif(d*n), n)
Y <- t(apply(X, 1, problem_car_side_impact))

pairs(Y)
```

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problem_marinedes	<i>Conceptual marine design problem</i>
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**Description**

Conceptual marine design problem

**Usage**

```
problem_marinedes(x, params = list(constraints = FALSE))
```

**Arguments**

- x** Vector in  $[0,1]^6$
- params** List of additional parameters:
- **returnCST**, logical, indicating whether the constraints are returned in the output

**Value**

If returnCST is TRUE a vector of size 12, the first 3 are the objectives, next are the constraints. Otherwise, it returns three objectives.

**Note**

Adapted from Tanabe, R., & Ishibuchi, H. (2020). An easy-to-use real-world multi-objective optimization problem suite. Applied Soft Computing, 89, 106078.

**References**

- M. G. Parsons and R. L. Scott, "Formulation of Multicriterion Design Optimization Problems for Solution With Scalar Numerical Optimization Methods," J. Ship Research, vol. 48, no. 1, pp. 61-76, 2004.
- Tanabe, R., & Ishibuchi, H. (2020). An easy-to-use real-world multi-objective optimization problem suite. Applied Soft Computing, 89, 106078.

**Examples**

```
d <- 6
n <- 1000
X <- matrix(runif(d*n), n)
Y <- t(apply(X, 1, problem_marinedes))

pairs(Y[, 1:3])
```

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problem_penicillin	<i>Penicillin production test function based on the implementation <a href="https://github.com/HarryQL/TuRBO-Penicillin">github.com/HarryQL/TuRBO-Penicillin</a>.</i>
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**Description**

Penicillin production test function based on the implementation [github.com/HarryQL/TuRBO-Penicillin](https://github.com/HarryQL/TuRBO-Penicillin).

**Usage**

```
problem_penicillin(input, params = list(t = 100, returnCST = FALSE))
```

**Arguments**

input	Matrix of input data
params	List of additional parameters: <ul style="list-style-type: none"> <li>• t Reaction time in hours, defaulted to 100</li> <li>• returnCST Optional. If TRUE returns the constraints as part of the output.</li> </ul>

**Value**

Vector of objective values: if returnCST is set to TRUE a vector of size 5 where the last 3 columns correspond to the constraints.

## References

Liang, Q. and L. Lai (2021). Scalable bayesian optimization accelerates process optimization of penicillin production. In NeurIPS 2021 AI for Science Workshop.

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problem\_switch\_ripple *Switching ripple test function. Corresponds to the switching ripple suppressor design problem for voltage source inversion in powered system.*

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## Description

Switching ripple test function. Corresponds to the switching ripple suppressor design problem for voltage source inversion in powered system.

## Usage

```
problem_switch_ripple(x, params = list(returnCST = FALSE))
```

## Arguments

- |        |   |
|--------|---|
| x      | vector specifying the location where the function is to be evaluated, of size $k + 4$ , $k > 1$ , see Details.  |
| params | List of additional parameters: <ul style="list-style-type: none"> <li>• returnCST Optional. If set to TRUE the last 5 columns in the output correspond to the values of the constraints.</li> </ul> |

## Details

Columns of x correspond to L1, L2, L3, C1, ..., Ck, Cf where k is an arbitrary integer  $> 1$ .

Parameters of the problems follow Table 2 in (Zhang et al, 2019).

## Value

Vector of objectives. The first k values are related to the suppression of harmonics while the k+1 one is the sum of inductors.

## References

- Zhang, Z., He, C., Ye, J., Xu, J., & Pan, L. (2019). Switching ripple suppressor design of the grid-connected inverters: A perspective of many-objective optimization with constraints handling. Swarm and evolutionary computation, 44, 293-303.
- He, C., Tian, Y., Wang, H., & Jin, Y. (2019). A repository of real-world datasets for data-driven evolutionary multiobjective optimization. Complex & Intelligent Systems, 1-9.

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problem_synthetic	<i>Two-dimensional tri-objective problem based on Branin and Currin functions.</i>
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### Description

Two-dimensional tri-objective problem based on Branin and Currin functions.

### Usage

```
problem_synthetic(x, params = list(A = NULL, B = NULL, seed = 19))
```

### Arguments

x	Vector or matrix to evaluate at, takes values in $[0,1]^3$
params	List of additional parameters: <ul style="list-style-type: none"> <li>• A and B embedding matrices of size 3x3</li> <li>• seed random seed for reproducibility</li> </ul>

### Note

Problem generated by sampling 2 random matrices and extracting an orthonormal basis from them.

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SharedAS	<i>Computes shared active subspace for vector-valued functions.</i>
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### Description

Computes shared active subspace for vector-valued functions.

### Usage

```
SharedAS(
  fn,
  grads = NULL,
  X = NULL,
  distr = list(name = "norm", n = 1000),
  input_dim,
  num_obj,
  start_dim = NULL,
  end_dim = NULL,
  methods = NULL,
  nexps = 1,
  is_norm = TRUE,
  seed = 126,
  params = NULL
)
```



**Arguments**

<code>fn</code>	Vector-valued function for which to compute a shared active subspace
<code>grads</code>	Optional, matrix of objective-wise gradients concatenated together
<code>X</code>	Matrix of input data: to provide only if grads are provided
<code>distr</code>	List indicating the data distribution and its parameters: <ul style="list-style-type: none"> <li>• name either "unif" for the uniform or "norm" for a normal (default is the standard normal) distribution</li> <li>• n number of samples</li> <li>• sigma covariance matrix, if the distribution is set to "norm" (the mean is always zero)</li> </ul>
<code>input_dim</code>	Dimensionality of the input data
<code>num_obj</code>	Number of outputs of the test function
<code>start_dim</code>	Optional, start number of the dominating eigenvectors
<code>end_dim</code>	End number of the dominating eigenvectors: To be set if <code>start_dim</code> is set.
<code>methods</code>	Vector of method names to compute a shared subspace <ul style="list-style-type: none"> <li>• AG: average of gradients</li> <li>• LP: linear projection of gradients</li> <li>• MCH: minimum convex hull of gradients</li> <li>• SEE: stepwise estimation of eigenvectors</li> <li>• SSPD: sum of SPD matrices</li> <li>• FG: joint diagonalization of SPD matrices</li> <li>• Zahm: the method of Zahm</li> </ul>
<code>nexps</code>	Number of experiments
<code>is_norm</code>	Logical, indicates whether to normalize the function outputs in the gradient computation (default is TRUE)
<code>seed</code>	Random seed for reproducibility
<code>params</code>	List of additional parameters for the test function

**Details**

Depending on the problem, the method of MCH can be costly to compute.

**Value**

List of objective-wise root-mean-square error and its sum for each number of dimensions and for each method, over nexps number of experiments

**Examples**

```
fn <- problem_car_side_impact
input_dim <- 7
num_obj <- 11
nexps <- 3
distr <- list(name="norm", n=1000, sigma = diag(3, input_dim))
SharedAS(fn=fn, distr=distr, start_dim=1, end_dim=5, input_dim=input_dim, num_obj=num_obj,
methods=c("AG", "LP", "MCH", "SSPD", "SEE", "FG", "Zahm"),
nexps=nexps, is_norm=TRUE, seed = 126, params=NULL)
```

```
fn <- problem_marinedes
input_dim <- 6
num_obj <- 3
nexps <- 5
distr <- list(name="unif", n=1000)
SharedAS(fn=fn, distr=distr, input_dim=input_dim, num_obj=num_obj,
methods=c("AG", "LP", "MCH", "SSPD", "SEE", "FG", "Zahm"),
nexps=nexps, is_norm=TRUE, seed = 126, params=NULL)

fn <- problem_synthetic
input_dim <- 3
num_obj <- 2
nexps <- 5
distr <- list(name="norm", n=1000, sigma = diag(3, input_dim))
SharedAS(fn=fn, distr=distr, input_dim=input_dim, num_obj=num_obj,
methods=c("AG", "LP", "MCH", "SSPD", "SEE", "FG", "Zahm"),
nexps=nexps, is_norm=FALSE)

fn <- problem_switch_ripple
input_dim <- 8
num_obj <- 10
distr <- list(name="norm", n=1000, sigma=diag(1,input_dim))
nexps <- 10
params=list(returnCST = TRUE)
SharedAS(fn=fn, distr=distr, input_dim=input_dim, num_obj=num_obj,
methods=c("AG", "LP", "MCH", "SSPD", "SEE", "FG", "Zahm"), nexps=nexps,
is_norm=TRUE, seed = 126, params=params)

fn <- problem_penicillin
input_dim <- 7
num_obj <- 5
nexps <- 5
distr <- list(name="norm", n=1000, sigma=diag(1,input_dim))
params <- list(t=100, returnCST=TRUE)
SharedAS(fn=fn, distr=distr, input_dim=input_dim, num_obj=num_obj, methods=c("AG", "LP"),
nexps=nexps, is_norm=FALSE, params=params)
```

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