Updates

Ambrose Yim (University of Oxford)

15 May 2018

Rather than using aggregating agents in a coalition with

$$u_C(a) = \min_{i \in C} u_i(a), \tag{1}$$

we could relax the 'min' condition by taking the harmonic mean. Let $v_c(a;\epsilon) = 1/(u_c(a) + \epsilon)$ and $v_i(a;\epsilon) = 1/(u_i(a) + \epsilon)$;

$$v_c(a;\epsilon) = \sum_{i \in C} v_i(a;\epsilon). \tag{2}$$

Then the objective of each coalition is to minimise the inverse utility. Feature of the scaling parameter ϵ: - $\epsilon = 0$: standard harmonic mean. Compared to simple arithmetic mean, the harmonic mean is dominated by small terms; consistency guaranteed if we rescale $u \in [0, 1]$. - $\epsilon \to \infty$: harmonic mean converges to arithmetic mean, shifting behaviour away from 'socialist' tendencies

Using a harmonic mean maybe more amenable to tractable analytic calculations. Moreover we can recourse to standard multi-agent optimisation action selection algorithms in literature (Q-learning and variable elimination).

More generally we can weaken this further: Let $C_{ij} = 1$ if (i, j) are in the same coalition (then they are one-neighbours on a *communication network* C). Instead of assigning a common utility to each coalition, we let each agent retain an individual (anti-)utility that is the harmonic mean of its neighbours' utilities on the communication network:

$$w_i(a) = \sum_i C_{ij} v_j(a). \tag{3}$$

Then the equilibrium condition is

$$w_i(a_i, a_{-i}) - w_i(a'_i, a_{-i}) = \sum_i C_{ij}(v_j(a_i, a_{-i}) - v_j(a'_i, a_{-i})) \leq 0 \quad \forall i \in \mathcal{I}. \tag{4}$$

It is transparent that if i is in a coalition with agents j s.t. $(v_j(a_i, a_{-i}) - v_j(a'_i, a_{-i}) = 0$ (i.e. j has no dependency on actions of i) then the equilibrium condition is the Nash equilibrium of the whole system, which may or may not exist.