How Can Organisations Innovate?

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Motivation

We want to understand the conditions for agents in an organisation to cooperate.

Can a small fraction of ‘innovators’ in the system influence the entire population to adopt a new idea?

What is the interplay between the population’s **willingness** to accept new behaviour and the way they **communicate**?

What if there are **constraints** that agents are not aware of?

What we will learn today

Segregating agents into individual teams hinder innovation if new problems come up that require agents across teams to coordinate.

Duh!

Model

Key modelling assumptions:

- Agents are connected in a **social network** [10] [4]
- Each agent receives *information* via its social network
- An agent's decision making is *constrained* by the decisions of other agents (**dependency network**)

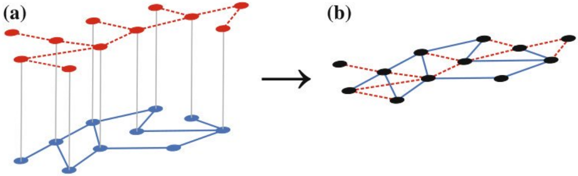


Fig. 2.1 **a** Two interdependent networks. A vertex in one network has a mutual dependence, represented by *grey vertical lines*, on zero or one vertex in the other network. **b** This can be reduced to a multiplex network by merging the mutually dependent vertices, and representing the edges of each network by different kinds or *colours* of edges

Agent behaviour

What is an agent? How do they coordinate? [4]

- Each agent has two states: default (0) or innovate (1)
- All agents start in the default state.
- Suppose there is a problem in the organisation that can only be solved if *all* agents coordinate.

Agent behaviour

- If too few neighbours on the dependency network decide to innovate, then it is not favourable for an agent to innovate
- Each agent has a **dependency threshold**: number of innovating dependency neighbours for it to ‘see’ the benefit of innovation

Agent behaviour

In game theory language, we say the agents play a **coordination game** (a.k.a. **Prisoners' Dilemma**) [9] [5].

- **positive pay-off** if it innovates *and* sufficiently many of its dependency neighbours innovate
- **negative pay-off** if it innovates *but* not enough agents that constrain you decide with innovate
- **neutral pay-off** if it remains in the default state.

We assume dependency is reflexive; if A influences B then B influences A.

Agent behaviour

- Agents must first be *aware* of innovation via their social network
- Agents have varying ‘social awareness’: number of ‘friends‘ required to persuade you to adopt new ideas

Agents must be aware and receive a positive pay-off from innovation to change its state from default to innovation.

Network Construction

- Initially, we assume social network $\mathcal{S}$ = dependency network $\mathcal{D}$
- Over time the business environment changes
- If not updated timely, $\mathcal{S} \neq \mathcal{D}$

Network Construction

We study two modes of 'information degradation' $\mathcal{S} \to \mathcal{D}$ [1]

1. Random edge swap:
   - $a - b$ and $c - d \Rightarrow a - d$ and $c - b$.
   - Agents preserve their degree (number of neighbours) from $\mathcal{S} \to \mathcal{D}$
   - Network topology (neighbourhood relations) is *altered* (loose memory of neighbours).
2. Random node reshuffle:
   - Agent degree in $\mathcal{D} \neq$ degree in $\mathcal{S}$.
   - Network topology is *preserved* (but loose memory of position in network).
   - In random degree distribution networks, same as rewiring edges.

Random Network Classes

Random graphs with nodes sampled from a degree distribution then randomly connected.

1. Erdős-Rényi Graph [2] [8]
    - Each pair of nodes has a probability p of forming an edge.
    - Average degree $z = Np$. Degree distribution is Binomial.
2. 'Planted Partition Model'[6]
    - Agents are segregated into C **communities** of equal size n.
    - Between pairs of nodes in the same community there is a probability p_i of forming an edge.
    - Between pairs of nodes in different communities there is a probability p_o of forming an edge.
    - Average degree $z = (n-1)p_i + (C-1)np_o$.
    - We fix $C = 100$ and $n = 100$, with $(C-1)np_o = 1/2$.

Thresholds

We assume social threshold and dependency thresholds are independent. [7]

We define the social/dependency threshold = fraction of neighbours above which node becomes aware/susceptible to infection

1. Agent randomly samples both thresholds independently from the same normal distribution (vary mean r and fix $\sigma = 0.2$) [3]
2. Dependency threshold sampled from normal distribution but social threshold = if one friend is innovating agent is aware.

Agents with a *negative* dependency and social thresholds are *innovators* who would innovate no matter what.

social network $\otimes$ social threshold $\otimes$ dependency layer $\otimes$ dependency threshold.

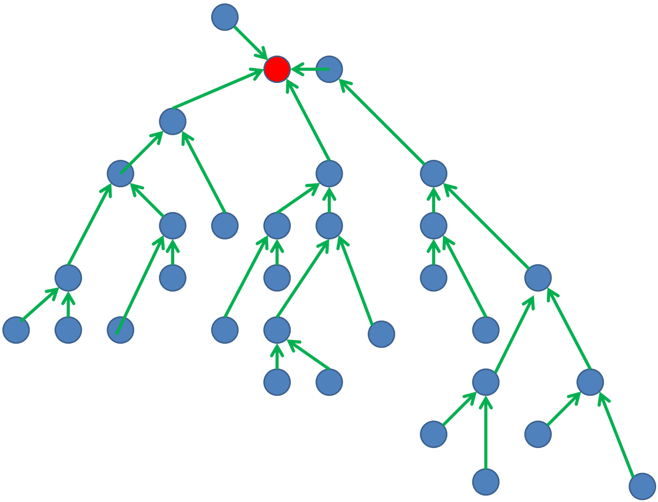
Single layer analysis: Watts-Gleeson Theory

Consider social layer = dependency layer and the thresholds are identical for each node.

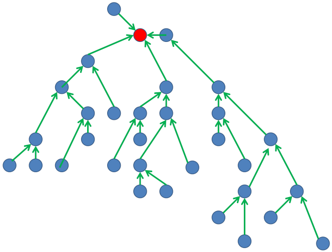
Watts-Gleeson theory [11] [3] applies to infinite random graphs with negligible clustering (probability of three connected nodes forming a triangle $\rightarrow 0$ as $N \rightarrow \infty$), or being *locally tree like*.

We can think of this process of synchronously updating the state of nodes as an **infection** from infinitely far away in the network.

Single layer analysis: Watts-Gleeson Theory



Single layer analysis: Watts-Gleeson Theory

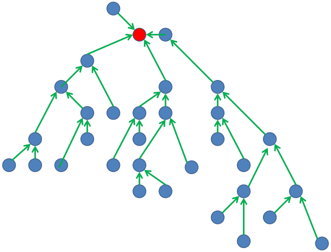


Initially all nodes are in the 0 state. Infection initiated on level $n = 0$ at time $t = 0$.

Probability that a node on layer $n + 1$ is infected at time $n + 1$ is dependent on the fraction of nodes infected in layer n:

$$q_{n+1} = g(q_n\ ; r) \tag{1}$$

Single layer analysis: Watts-Gleeson Theory



Brouwer fixed point theorem: there exists $q = g(q\,;r)$. Fixed point is either stable or unstable.

Final layer must be a (stable) fixed point. Let $q_\infty = \Pr(\text{infection on final layer} \mid \text{top node inactive})$.

Final fraction of infected nodes is

$$\rho = \rho(q_\infty\,;r) \tag{2}$$

Single layer analysis: Watts-Gleeson Theory

We look at Erdős-Rényi graphs of varying mean degree.

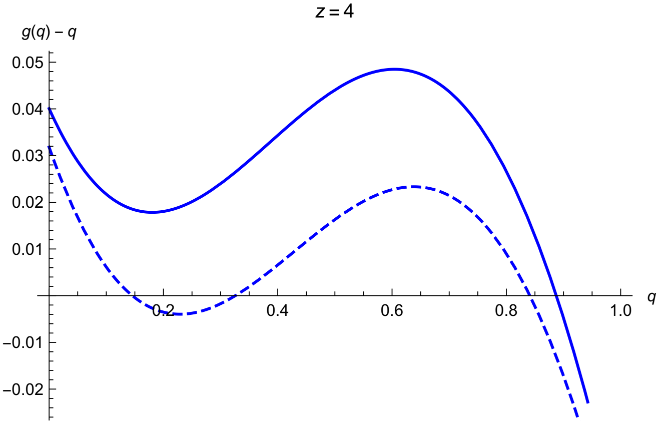


Figure 1: $g(q) - q$ for parameters $z = 4$; sold is $r = 0.35$, dashed is $r = 0.371$

Single layer analysis: Watts-Gleeson Theory

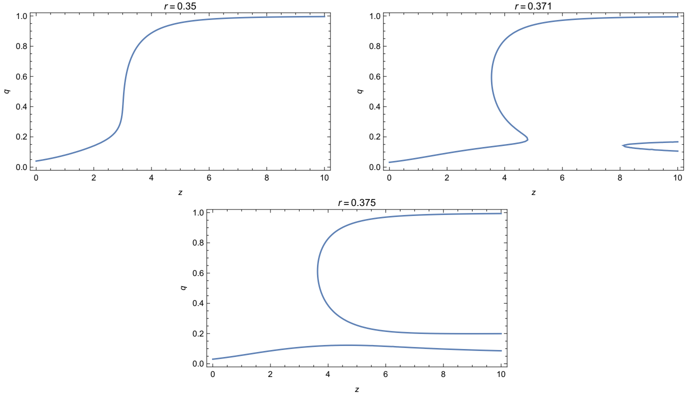


Figure 2: Fixed point plot for increasing r. Note Goldilocks zone in middle figure.

Single layer analysis: Watts-Gleeson Theory

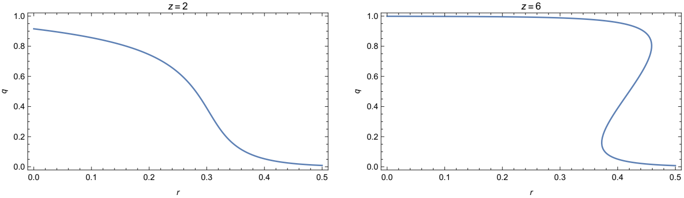
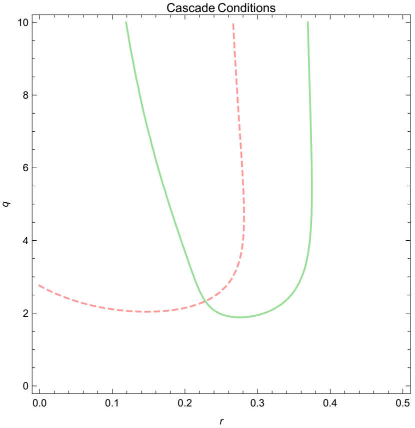


Figure 3: Fixed point plot for increasing z.

Single layer analysis: Watts-Gleeson Theory

First and Second order (approximate) cascade conditions: (z, r) values that induces a global cascade of infection contained in the union of both regions.



Double layer analysis: Watts-Gleeson Theory

Technically need two q probabilities for the two different layers respectively; however agent randomly samples both thresholds independently from the *same* normal distribution, so symmetry reduces the system to one variable.

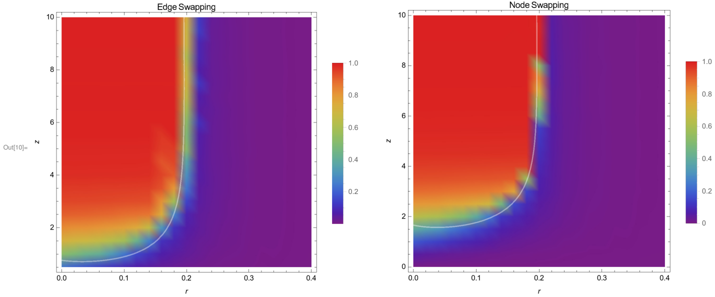


Figure 4: Final fraction size vs (z, r); white curve second order cascade condition

Double layer analysis: Watts-Gleeson Theory

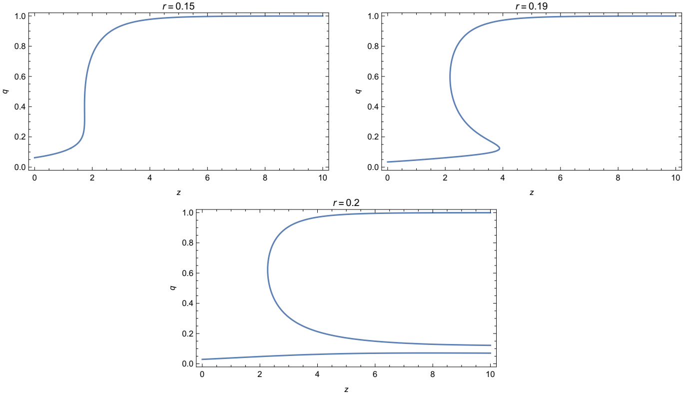


Figure 5: Edge swap: Fixed points cross section across different r values. Note that the Goldilocks zone disappears.

Double layer analysis: Watts-Gleeson Theory

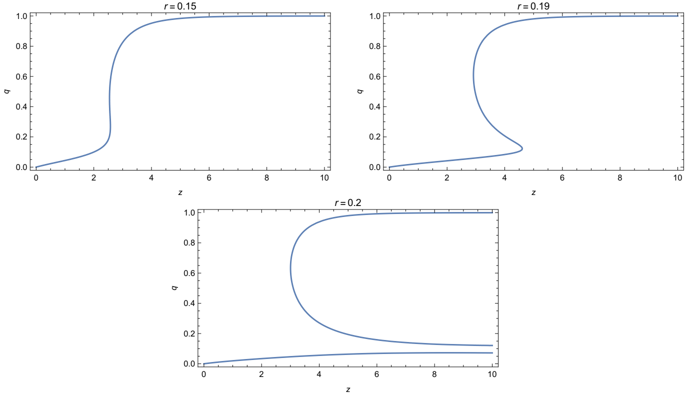


Figure 6: Node swap: Fixed points cross section across different r values. Note that the Goldilocks zone disappears.

Summary of Analytical Results

- The final infection size decreases as r increases
- The transition becomes steeper and steeper as z increases
- Transition becomes *discontinuous* for values of r and z greater than some critical value.
- Discontinuous transition separates two 'phases' where the global cascade is extensive across the population and the other where the infection does not spread.
- Adding an extra layer lowers the critical value of r where the discontinuity takes place.
- In single layer, there is a competition between the benefits of increasing node degree to increase the chance of exposure vs the increase in a agent's degree of dependency: **Goldilocks zone effect**
- In double layer, this zone is missing.

Simulation: clustering effects

'Planted Partition Model'

- Agents are segregated into C **communities** of equal size n.
- Between pairs of nodes in the same community there is a probability p_i of forming an edge.
- Between pairs of nodes in different communities there is a probability p_o of forming an edge.
- Average degree $z = (n-1)p_i + (C-1)np_o$.
- We fix $C = 100$ and $n = 100$, with $(C-1)np_o = 1/2$.

Simulation: clustering effects

'Planted Partition Model'

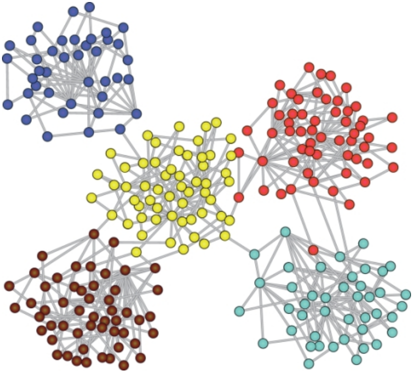


Figure 7: Image due to Preston Engstrom

Simulation: clustering effects

In our simulations we run the infection on networks of 100 communities, each with 100 members. Average number of edges outside the community is kept at 0.5 per node.

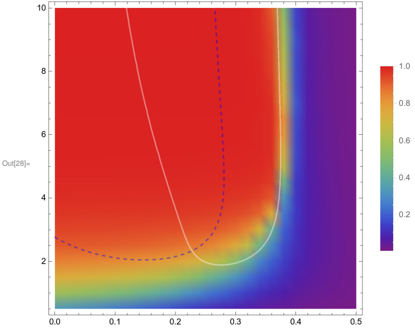


Figure 8: Single Layer simulation. Curve overlays are approximate theoretical thresholds of single layer ER graphs.

Simulation: clustering effects

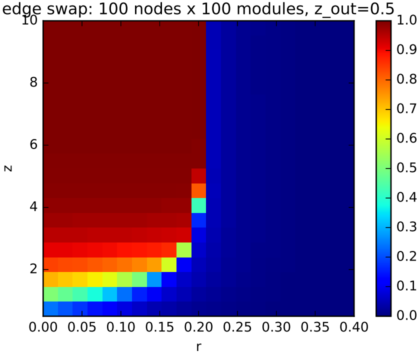


Figure 9: Phase plane with dependency network = edge swap, randomised thresholds.

N.B. in random edge swap, the dependency layer the network looses memory of its community structure and becomes an ER graph.

Simulation: clustering effects

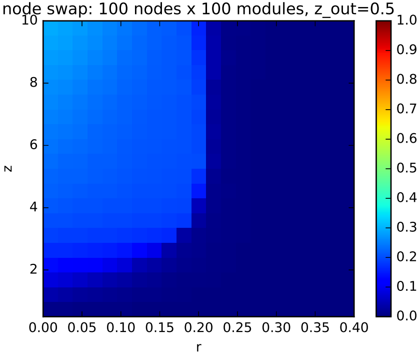


Figure 10: Phase plane with dependency network = node swap, randomised thresholds.

This scenario represents communities of agents trying to solve a *structured* problem which does not correspond to the same social community structure.

Contact Awareness

If the threshold to be socially aware is for only *one* friend to innovate...

Contact Awareness: Edge swap ER

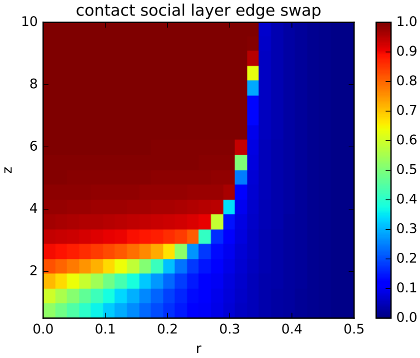


Figure 11: Phase plane with dependency network = edge swap of ER, contact threshold.

Very similar to situation without social network N.B. no Goldilocks.

Contact Awareness: Node swap ER

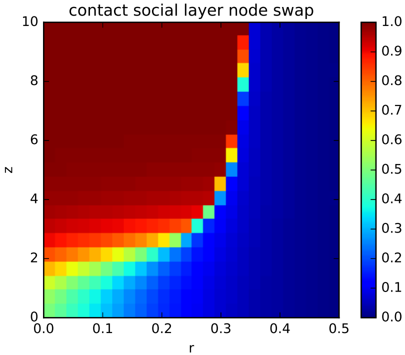


Figure 12: Phase plane with dependency network = node swap of ER, contact threshold.

Very similar to situation without social network N.B. no Goldilocks.

Contact Awareness: ER Dependency, PP Social

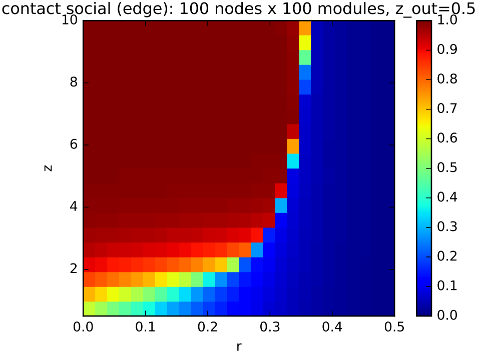


Figure 13: Dependency = ER, Social = PP, contact threshold.

Very similar to situation without social network N.B. no Goldilocks.

Contact Awareness: PP Dependency, ER Social

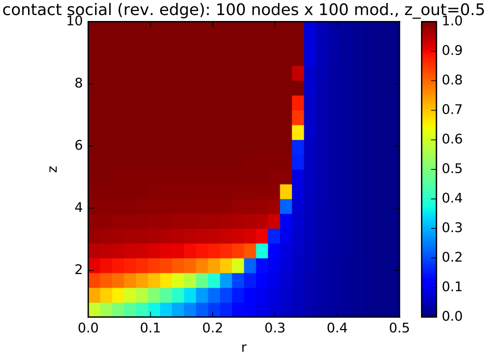


Figure 14: Dependency = PP, Social = ER, contact threshold.

Contact Awareness: node swap, PP Dependency, PP Social

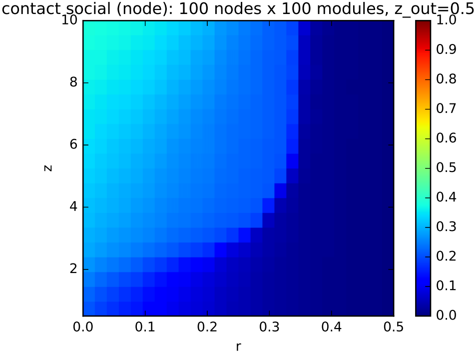


Figure 15: Dependency = PP, Social = ER, contact threshold.

Limited extent of infection appears again when PP swaps nodes.

Summary

- There is a **phase transition** between very little coordination to extensive, population wide coordination at critical values of *social awareness* and *degree of dependency*
- Highly socially aware agents are safe against decoupling of social and dependency network
- Community networks are inflexible against solving structured problems whose dependencies do not correspond to the same community structure. (*Limited horizon of infection*)
- Node swap is more dangerous because it disconnects agents with a high degree of dependency.
- Future extensions: different threshold distributions, different network topologies, adaptive social networks.

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