How Can Organisations Innovate?

Ambrose Yim, supervised by Stephen Cassidy, Stephen Brewis (BT); Renaud Lambiotte, Andrew Mellor (Oxford)

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InFoMM CDT, Mathematical Institute, University of Oxford

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What if there are **constraints** that agents are not aware of?

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Duh!

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- Agents are connected in a **social network** [10] [4]
- Each agent receives *information* via its social network
- An agent's decision making is *constrained* by the decisions of other agents (**dependency network**)

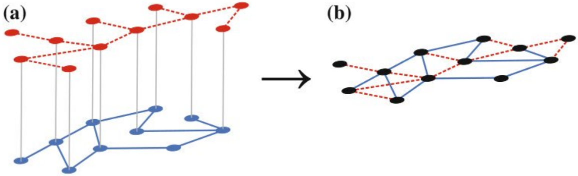

Fig. 2.1 **a** Two interdependent networks. A vertex in one network has a mutual dependence, represented by *grey vertical lines*, on zero or one vertex in the other network. **b** This can be reduced to a multiplex network by merging the mutually dependent vertices, and representing the edges of each network by different kinds or *colours* of edges

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- All agents start in the default state.
- Suppose there is a problem in the organisation that can only be solved if *all* agents coordinate.

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- Each agent has a **dependency threshold**: number of innovating dependency neighbours for it to 'see' the benefit of innovation

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- **positive pay-off** if it innovates *and* sufficiently many of its dependency neighbours innovate
- **negative pay-off** if it innovates *but* not enough agents that constrain you decide with innovate
- **neutral pay-off** if it remains in the default state.

We assume dependency is reflexive; if A influences B then B influences A.

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- Agents have varying ‘social awareness’: number of ‘friends‘ required to persuade you to adopt new ideas

Agents must be aware and receive a positive pay-off from innovation to change its state from default to innovation.

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- If not updated timely, $\mathcal{S} \neq \mathcal{D}$

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    - In random degree distribution networks, same as rewiring edges.

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1. Agent randomly samples both thresholds independently from the same normal distribution (vary mean r and fix $\sigma = 0.2$) [3]
2. Dependency threshold sampled from normal distribution but social threshold = if one friend is innovating agent is aware.

Agents with a *negative* dependency and social thresholds are *innovators* who would innovate no matter what.

social network $\otimes$ social threshold $\otimes$ dependency layer $\otimes$ dependency threshold.

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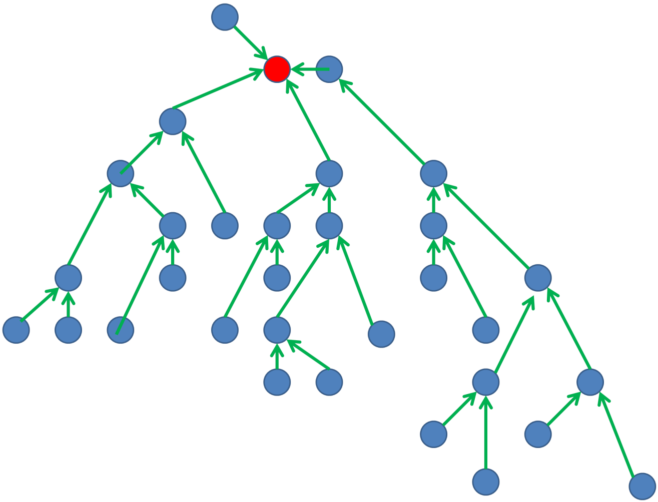
We can think of this process of synchronously updating the state of nodes as an **infection** from infinitely far away in the network.

Single layer analysis: Watts-Gleeson Theory



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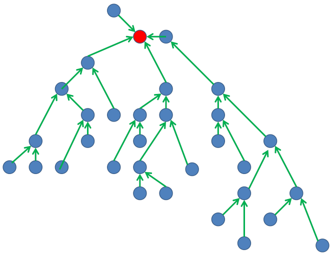

Initially all nodes are in the 0 state. Infection initiated on level $n = 0$ at time $t = 0$.

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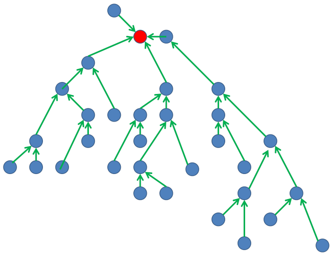

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Probability that a node on layer $n+1$ is infected at time $n+1$ is dependent on the fraction of nodes infected in layer n:

$$q_{n+1} = g(q_n \, ; r) \tag{1}$$

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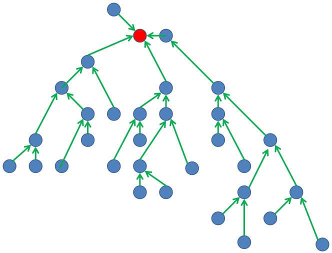

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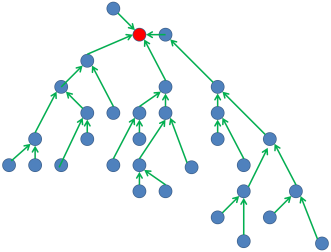

Brouwer fixed point theorem: there exists $q = g(q\ ; r)$. Fixed point is either stable or unstable.

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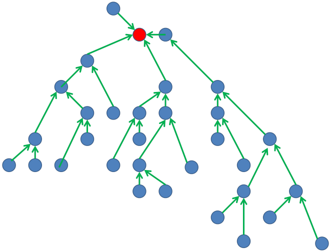

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Final layer must be a (stable) fixed point. Let $q_\infty = \Pr(\text{infection on final layer} \mid \text{top node inactive})$.

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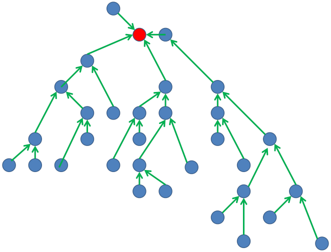

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Final fraction of infected nodes is

$$\rho = \rho(q_\infty\ ; r) \tag{2}$$

Single layer analysis: Watts-Gleeson Theory

We look at Erdős-Rényi graphs of varying mean degree.

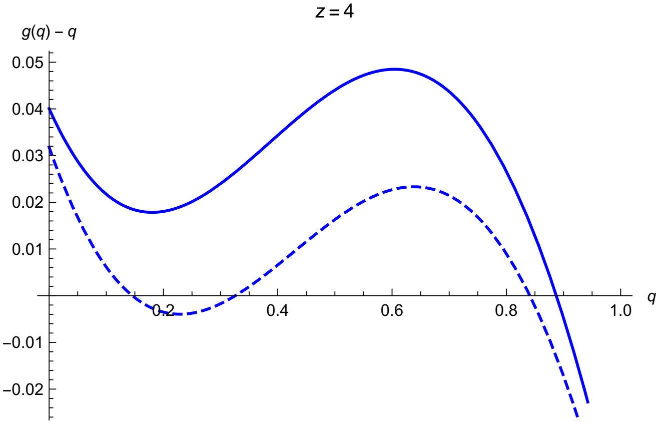

Figure 1: $g(q) - q$ for parameters $z = 4$; solid is $r = 0.35$, dashed is $r = 0.371$

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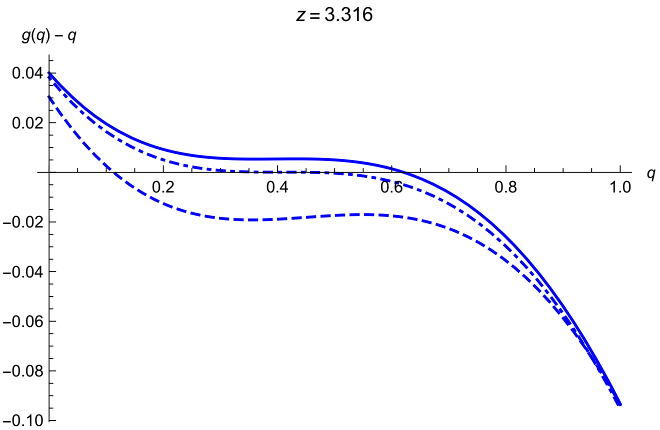

Figure 2: $g(q) - q$ for parameters $z = 4$; solid is $r = 0.35$, dot-dashed is $r = 0.3543$, dashed is $r = 0.375$

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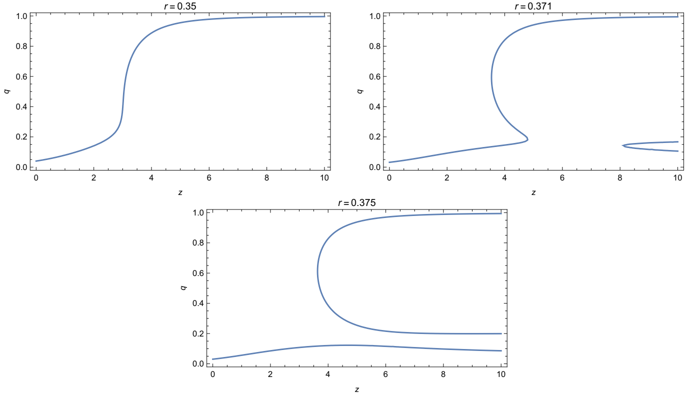

Figure 3: Fixed point plot for increasing r. Note Goldilocks zone in middle figure.

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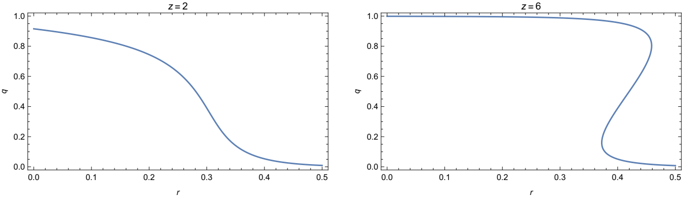
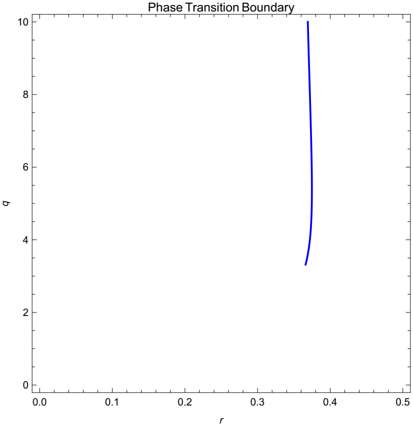


Figure 4: Fixed point plot for increasing z.

Single layer analysis: Watts-Gleeson Theory

Phase transition boundary: discontinuity in $q_\infty(z, r)$.



Double layer analysis: Watts-Gleeson Theory

Technically need two q probabilities for the two different layers respectively; however agent randomly samples both thresholds independently from the *same* normal distribution, so symmetry reduces the system to one variable.

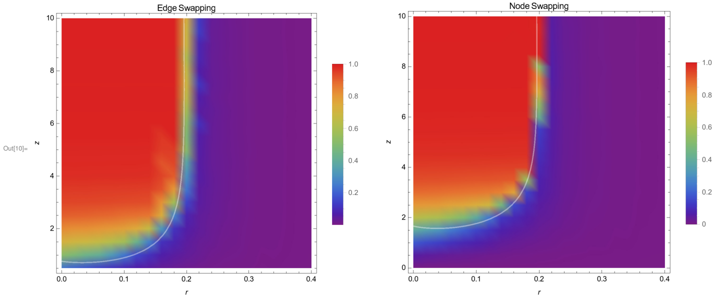


Figure 5: Final fraction size vs (z, r); white curve second order cascade condition

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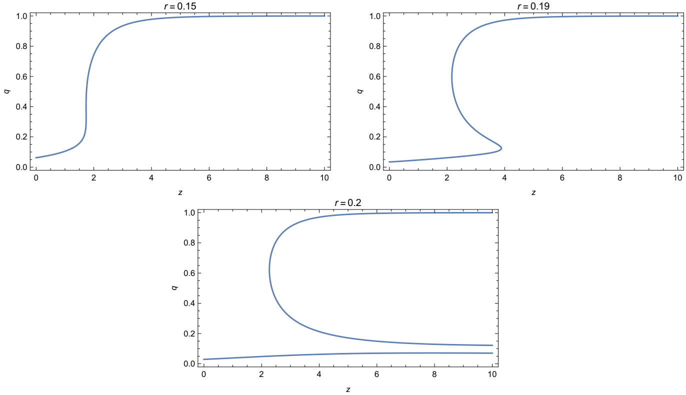

Figure 6: Edge swap: Fixed points cross section across different r values. Note that the Goldilocks zone disappears.

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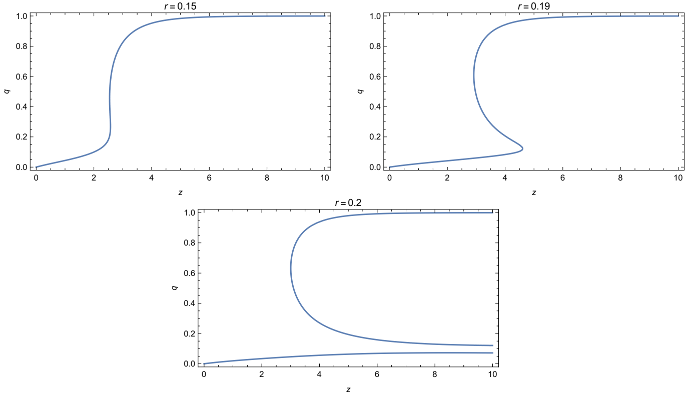

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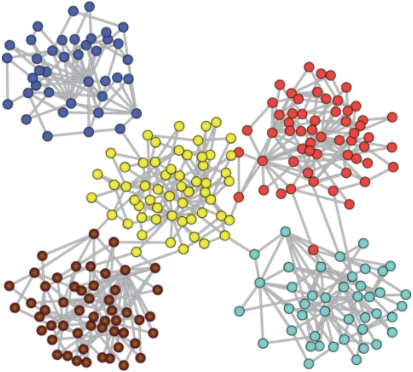

Figure 8: Image due to Preston Engstrom

Simulation: clustering effects

In our simulations we run the infection on networks of 100 communities, each with 100 members. Average number of edges outside the community is kept at 0.5 per node.

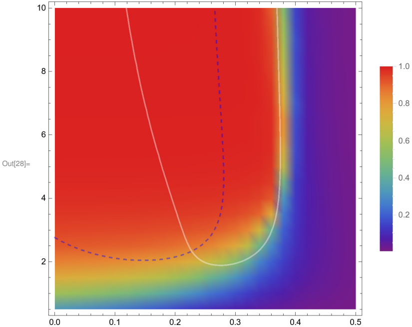

Figure 9: Single Layer simulation. Curve overlays are approximate theoretical thresholds of single layer ER graphs.

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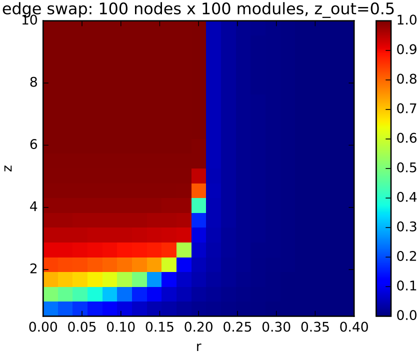

Figure 10: Phase plane with dependency network = edge swap, randomised thresholds.

N.B. in random edge swap, the dependency layer the network looses memory of its community structure and becomes an ER graph.

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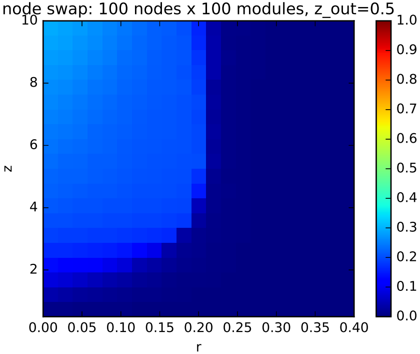

Figure 11: Phase plane with dependency network = node swap, randomised thresholds.

This scenario represents communities of agents trying to solve a *structured* problem which does not correspond to the same social community structure.

Contact Awareness

If the threshold to be socially aware is for only *one* friend to innovate...

Contact Awareness: Edge swap ER

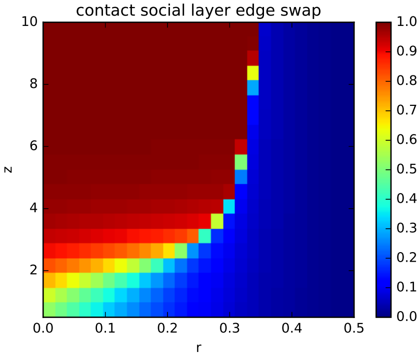

Figure 12: Phase plane with dependency network = edge swap of ER, contact threshold.

Very similar to situation without social network N.B. no Goldilocks.

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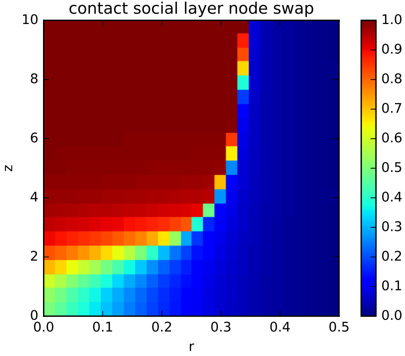

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Very similar to situation without social network N.B. no Goldilocks.

Contact Awareness: ER Dependency, PP Social

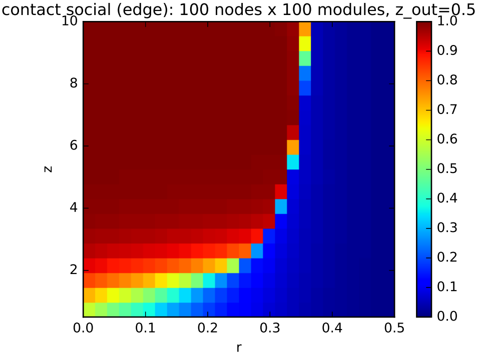

Figure 14: Dependency = ER, Social = PP, contact threshold.

Very similar to situation without social network N.B. no Goldilocks.

Contact Awareness: PP Dependency, ER Social

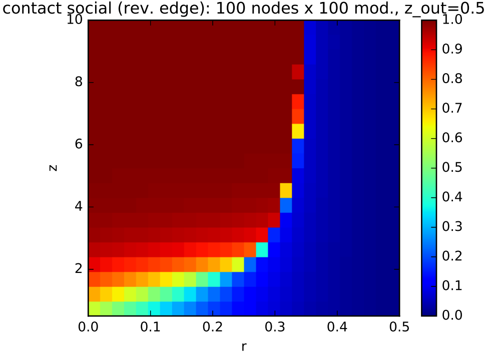

Figure 15: Dependency = PP, Social = ER, contact threshold.

Contact Awareness: node swap, PP Dependency, PP Social

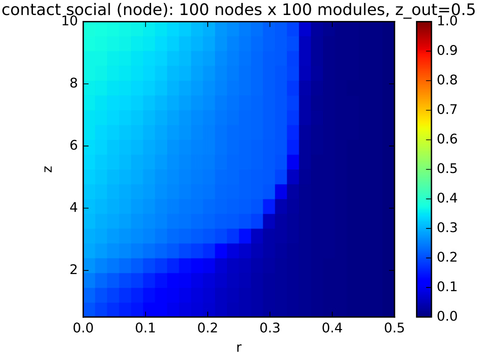

Figure 16: Dependency = PP, Social = ER, contact threshold.

Limited extent of infection appears again when PP swaps nodes.

Summary

- There is a **phase transition** between very little coordination to extensive, population wide coordination at critical values of *social awareness* and *degree of dependency*
- Highly socially aware agents are safe against decoupling of social and dependency network
- Community networks are inflexible against solving structured problems whose dependencies do not correspond to the same community structure. (*Limited horizon of infection*)
- Node swap is more dangerous because it disconnects agents with a high degree of dependency.
- Future extensions: different threshold distributions, different network topologies, adaptive social networks.

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