Tutorial 6

AI2101 - Convex Optimization

Question (a)

The decision variables represented by x_1, x_2, x_3. Where,

x_1 = The proportion of time devoted each day to iPod covers production.
x_2 = The proportion of time devoted each day to iPhone covers production.
x_3 = The proportion of time devoted each day to iPad covers production.

Since x_1, x_2, x_3 are the proportion of time in each day. The sum of all should equal 1.
$i.e.,\ \Sigma x_i = 1.$

Now, the weekly production of each of these covers will be:

$w_1 = 5 * x_1 * 6000$; Production of iPod covers.
$w_2 = 5 * x_2 * 5000$; Production of iPhone covers.
$w_3 = 5 * x_3 * 3000$; Production of iPod covers.

Now, the space that each of these w_1, w_2, w_3 takes are:

$$S = (w_1 * 0.040) + (w_2 * 0.045) + (w_3 * 0.210) \quad ; S \leq 6000$$

Our goal is to maximize the net profit that we gain. Let P be the net profit gain. Then,

$$P = (4 * w_1) + (6 * w_2) + (10 * w_3)$$

After maximizing the profit function P, we get

$$x_1 \approx 0.1667$$
$$x_2 \approx 0.3073$$
$$x_3 \approx 0.5260$$
And $P \approx 145{,}000.00$

The decision variable P as a function of x_1, x_2, x_3 can be stated as follows:

$$P = (120000 * x_1) + (150000 * x_2) + (150000 * x_3)$$

Question (b)

The decision variables represented by y_1, y_2, y_3 . Where,

$$y_1 = \text{The number of iPod covers produced over the week.}$$
$$y_2 = \text{The number of iPhone covers produced over the week.}$$
$$y_3 = \text{The number of iPad covers produced over the week.}$$

Now, the space that each of these y_1, y_2, y_3 takes are:

$$S = (y_1 \ * \ 0.040) \ + \ (y_2 \ * \ 0.045) \ + \ (y_3 \ * \ 0.210) \qquad ; S \leq 6000$$

And, as per the constraints given in the Problem Statement,

$$5000 \leq y_1 \leq 10000, \qquad \text{Max and Min Production over the week for iPod.}$$
$$0 \leq y_2 \leq 15000, \qquad \text{Max and Min Production over the week for iPhone.}$$
$$4000 \leq y_3 \leq 8000. \qquad \text{Max and Min Production over the week for iPad.}$$

Our goal is to maximize the net profit that we gain. Let P be the net profit gain.

After maximizing the profit function P, we get

$$y_1 \approx 5000$$
$$y_2 \approx 7683$$
$$y_3 \approx 7889$$
$$\text{And} \quad P \approx 145{,}000.00$$

The decision variable P as a function of y_1, y_2, y_3 can be stated as follows:

$$P = (4 \ * \ y_1) \ + \ (6 \ * \ y_2) \ + \ (10 \ * \ y_3)$$

Question (c)

The decision variables represented by z_1, z_2, z_3. Where,

z_1 = The number of hours devoted to the production of iPod smart covers in one week.
z_2 = The number of hours devoted to the production of iPod smart covers in one week.
z_3 = The number of hours devoted to the production of iPod smart covers in one week.

Since z_1, z_2, z_3 are number of hours devoted in the week, the sum of all should equal 40.
$i.e.,\ \Sigma z_i = 40$.

Now, the weekly production of each of these covers will be:

$$w_1 = \frac{z_1 * 6000}{8} \qquad \text{; Production of iPod covers.}$$

$$w_2 = \frac{z_2 * 5000}{8} \qquad \text{; Production of iPhone covers.}$$

$$w_3 = \frac{z_1 * 3000}{8} \qquad \text{; Production of iPad covers.}$$

Now, the space that each of these w_1, w_2, w_3 takes are:

$$S = (w_1 * 0.040) + (w_2 * 0.045) + (w_3 * 0.210) \quad ; S \leq 6000$$

Our goal is to maximize the net profit that we gain. Let P be the net profit gain. Then,

$$P = (4 * w_1) + (6 * w_2) + (10 * w_3)$$

After maximizing the profit function P, we get

$$z_1 \approx 6.667$$
$$z_2 \approx 12.294$$
$$z_3 \approx 21.040$$
And $P \approx 145{,}000.00$

The decision variable P as a function of z_1, z_2, z_3 can be stated as follows:

$$P = (3000 * z_1) + (3750 * z_2) + (3750 * z_3)$$

Question (d)

Relationship between the variables x_1, x_2, x_3 and z_1, z_2, z_3 of part (a) and part (c) respectively.

As;

$$x_i = \text{The proportion of time devoted to each item in a day.}$$
$$z_i = \text{Total hours devoted to each item over a week.}$$

Since every day, the outcome is the same irrespective of the day of the week, the distribution of time each day , $i.e.,$ x_i is the same every day of the week (Working Days).

It is also stated that each day, the total hours of work is 8 hours. Therefore, the total hours devoted in a day when expressed in terms of x_i is $8 * x_i$.

A total of 5 working days per week. Therefore, the total hours devoted in a week would be $5 * (8 * x_i) = 40 * x_i$.

Thus, the mathematical formula to compute z_i from x_i is;

$$z_i = 40 * x_i$$