*Robust LP with polyhedral cost uncertainty.* We consider a robust linear programming problem, with polyhedral uncertainty in the cost:

$$\begin{array}{ll} \text{minimize} & \sup_{c \in \mathcal{C}} c^T x \\ \text{subject to} & Ax \succeq b, \end{array}$$

with variable $x \in \mathbf{R}^n$, where $\mathcal{C} = \{c \mid Fc \preceq g\}$. You can think of $x$ as the quantities of $n$ products to buy (or sell, when $x_i < 0$), $Ax \succeq b$ as constraints, requirements, or limits on the available quantities, and $\mathcal{C}$ as giving our knowledge or assumptions about the product prices at the time we place the order. The objective is then the worst possible (*i.e.*, largest) possible cost, given the quantities $x$, consistent with our knowledge of the prices.

In this exercise, you will work out a tractable method for solving this problem. You can assume that $\mathcal{C} \neq \emptyset$, and the inequalities $Ax \succeq b$ are feasible.

(a) Let $f(x) = \sup_{c \in \mathcal{C}} c^T x$ be the objective in the problem above. Explain why $f$ is convex.

(b) Find the dual of the problem

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Fc \preceq g, \end{array}$$

with variable $c$. (The problem data are $x$, $F$, and $g$.) Explain why the optimal value of the dual is $f(x)$.

(c) Use the expression for $f(x)$ found in part (b) in the original problem, to obtain a single LP equivalent to the original robust LP.

(d) Carry out the method found in part (c) to solve a robust LP with the data below. In MATLAB:

```
rand('seed',0);
A = rand(30,10);
b = rand(30,1);
c_nom = 1+rand(10,1);  % nominal c values
```

In Python:

```
import numpy as np
np.random.seed(10)
(m, n) = (30, 10)
A = np.random.rand(m, n); A = np.asmatrix(A)
b = np.random.rand(m, 1); b = np.asmatrix(b)
c_nom = np.ones((n, 1)) + np.random.rand(n, 1); c_nom = np.asmatrix(c_nom)
```

In Julia:

```
srand(10);
n = 10;
m = 30;
A = rand(m, n);
b = rand(m, 1);
c_nom = 1 + rand(n, 1);
```

Then, use $\mathcal{C}$ described as follows. Each $c_i$ deviates no more than 25% from its nominal value, *i.e.*, $0.75c_{\text{nom}} \preceq c \preceq 1.25c_{\text{nom}}$, and the average of $c$ does not deviate more than 10% from the average of the nominal values, *i.e.*, $0.9(\mathbf{1}^T c_{\text{nom}})/n \leq \mathbf{1}^T c/n \leq 1.1(\mathbf{1}^T c_{\text{nom}})/n$.

Compare the worst-case cost $f(x)$ and the nominal cost $c_{\text{nom}}^T x$ for $x$ optimal for the robust problem, and for $x$ optimal for the nominal problem (*i.e.*, the case where $\mathcal{C} = \{c_{\text{nom}}\}$). Compare the values and make a brief comment.