Assignment 1

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***Abstract*—This document contains the solution for Ex. 3.38 of Discrete-Time Signal Processing by Oppenheim and Wilsky.**

Problem 1. Consider a stable linear time-invariant system. The $z-$transform of the impulse response is

$$H(z) = \frac{z^{-1} + z^{-2}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})} \tag{0.1}$$

Suppose $x[n]$, the input to the system, is $2u[n]$. Determine $y[n]$ at n = 1.

Solution: The $z-$transform of the impulse response for the stable linear time invariant system is given by,

$$H(z) = \frac{Y(z)}{X(z)} \tag{0.2}$$

And $x[n] = 2u[n]$, we get $X(z) = \dfrac{2}{1 - z^{-1}}$.
Therefore $Y(z)$ can be written as,

$$Y(z) = \frac{z^{-1} + z^{-2}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})} \cdot \frac{2}{1 - z^{-1}} \tag{0.3}$$

Applying partial fractions to $Y(z)$,

$$Y(z) = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + \frac{1}{3}z^{-1}} + \frac{C}{1 - z^{-1}} \tag{0.4}$$

solving for A, B, C gives us

$$A = -\frac{36}{5} \quad B = \frac{6}{5} \quad C = 6 \tag{0.5}$$

$$\therefore Y(z) = -\frac{36}{5} \cdot \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{6}{5} \cdot \frac{1}{1 + \frac{1}{3}z^{-1}} + 6 \cdot \frac{1}{1 - z^{-1}} \tag{0.6}$$

Now, applying inverse Z-transform, gives us

$$y(n) = -\frac{36}{5}\left(\frac{1}{2}\right)^n u(n) + \frac{6}{5}\left(\frac{-1}{3}\right)^n u(n) + 6u(n) \tag{0.7}$$

The equation (0.7) comes from the fact that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{0.8}$$

Finally, we need to determine the value of $y[n]$ at input $n = 1$. Substituting the value of $n = 1$ in (0.7) gives us,

$$y(1) = 2. \tag{0.9}$$