Monte-Carlo simulation methods

Laboratory work 2
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Part I

Let us assume the Heston model

$$dS_t = (r-q)S_t dt + \sigma_t S_t^{\gamma} dW_t \tag{1}$$
$$dv_t = \kappa(\theta - v_t)dt + \eta\sqrt{v_t}dB_t \tag{2}$$

under the risk-neutral measure, where W and B have some correlation ρ.

- 1) Apply the control variate method to price the forward start call option

  $$(S_{T_2} - S_{T_1})^+$$

  for some $T_1 < T_2$. As the control variate use the payoff of this option under the Black-Scholes model. Report the correlation between payoffs under two models.

Part I

- 2) Apply the conditional MC method to price the European call option. For this, generate trajectories of $\sigma_t^{(n)}$, $n = 1, \dots, N$ with m discretization steps, and conditional on them use the formula for call option under the BS model with time-dependent volatility. In this question, you can assume that $\rho = 0$. Then take average

$$C^{Heston} \approx \frac{1}{N} \sum_{n=1}^{N} C^{BS}(\sigma^{(n)})$$

- Select some parameters and generate the table of option prices for different pairs of (N, m).

Part II

- Let us consider d asset prices driven by geometric Brownian motions under the risk-neutral measure Q

$$dS_t^i = (r - \delta_i)S^i dt + \sigma_i S_t^i dW_t^i$$

  where r is the interest rate, δ_i is the dividend yield of asset i, σ_i is the volatility of asset i, and W^i are SBMs under Q with cross-correlations $\rho_{i,j}$.

- The parameters are: $r = 0.05$, $\delta_i = 0.02$, $\sigma_i = 0.3$, $\rho_{i,j} = 0.2$ for all i, j.

- Use the Monte-Carlo method to price the following multi-asset options:

Part II

- 1) European basket option with a payoff

$$(S_T^1 + S_T^2 - S_T^3 - K)^+$$

  where $S_0^1 = S_0^2 = S_0^3 = K = 100, \ T = 1.$
- 2) Bermudan basket option with payoff (using the least-squares Monte-Carlo method)

$$(S_\tau^1 + S_\tau^2 - S_\tau^3 - K)^+$$

  where $S_0^1 = S_0^2 = S_0^3 = K = 100, \ T = 1$. This contract can be exercised at $t_i = i/12, \ i = 1, \dots, 12$. Generate the lower bound, where the exercise policy is given by the regression-based method. Choose the set of basis functions.
- Report option prices and 95%-confidence intervals for different numbers of simulations.

Part III

- Assume two correlated stock prices given by CEV processes

$$dS_t^1 = rS_t^1 dt + \sigma_1 (S_t^1)^{\gamma_1} dW_t^1$$

$$dS_t^2 = rS_t^2 dt + \sigma_2 (S_t^2)^{\gamma_2} dW_t^2$$

  where (W^1, W^2) are two standard Brownian motions under Q with the correlation ρ. Suppose that $S_0^1 = S_0^2$.
- Let us consider a callable reverse convertible, which is an example of a structured note.
- According to this contract, the holder receives the coupon payments c on dates $0 < t_1 < \ldots < t_n = T$ and principal payment K at T, unless one of the two events below happen:

Part III

- (i) if the issuer decides to redeem the contract early at date t_i, $i = 1, \ldots, n-1$. In this case, the contract is terminated early, and the holder receives the final payment of $c + K$. This feature is similar to American options.

- (ii) if $\min(S_T^1, S_T^2) < K$. In this case, the terminal payment (in case the contract is not redeemed before) is given by

$$c + \min(S_T^1, S_T^2)$$

- Assume that $S_0^1 = S_0^2 = K = 100$, $T = 2$ years, $n = 8$, and $t_i = iT/n$ for all i. Take some parameter values for $(r, \sigma_1, \sigma_2, \gamma_1, \gamma_2, \rho)$. Also, a reasonably large number of N simulations and m discretization steps for the simulation of trajectories are taken. Write a program (based on the regression method for American options) that determines the value of coupon c that makes the contract value $V_0 = 100$ at time 0.