# Monte-Carlo simulation methods

## Laboratory work 1

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# Part I

- Let us assume CEV model

$$dS_t = (r - q)S_t dt + \sigma S_t^{\gamma} dW_t$$

under the risk-neutral measure.

- Implement the program that takes as an input $(S_0, r, q, \sigma, \gamma, N, n, m, T)$, where $T$ is the maturity date, $N$ is the number of simulations, $n$ is the number of payoffs, $m$ is the number of discretization steps, and produces (using the MC method) the option price estimate of the cliquet contract with the the discounted payoff

$$\sum_{i=1}^{n} e^{-rt_i}(S_{t_i} - S_{t_{i-1}})^+$$

where $t_i = iT/n$.

- Select some parameters and generate the table of option prices for different pairs of $(N, m)$.

# Part II

- Assume the Merton jump model under the risk-neutral measure
$$dS_t = rS_{t-}dt + \sigma S_{t-}dW_t + S_{t-}dJ_t$$
where $W$ is a SBM, $r, \sigma$ are constant parameters, and
$$J_t = \sum_{j=1}^{N_t} (Y_j - 1)$$
where $Y_1, Y_2, \ldots$ are i.i.d. random variables and $N_t$ is a Poisson process with intensity parameter $\lambda$.
- Assume that $Y_i$ takes two values $y_- < y_+$ with probabilities $p$ and $1 - p$.

# Part II

- Implement the program that takes as an input $(S_0, r, \sigma, y_-, y_+, p, \lambda, N, n, T, K)$, where $T$ is the maturity date, $N$ is the number of simulations, $n$ is the number of observation dates, and produces the price estimate of the Asian put option with the fixed strike

$$\left(K - \frac{1}{n}\sum_{i=1}^{n} S_{t_i}\right)^+$$

  where $t_i = iT/n$.

- Select some parameter values and generate the table of option prices for different pairs of $(N, m)$.

# Part III

- Assume two correlated stock prices

$$dS_t^1 = rS_t^1 dt + \sigma_1 S_t^1 dW_t^1$$

$$dS_t^2 = rS_t^2 dt + \sigma_2 S_t^2 dW_t^2$$

  where $(W^1, W^2)$ are two standard Brownian motions under $Q$ with the correlation $\rho$. Suppose that $S_0^1 = S_0^2$.
- Let us consider an autocallable barrier reverse convertible which is an example of a structured note.
- According to this contract, the holder receives the coupon payments $c$ on dates $0 < t_1 < \ldots < t_n = T$ and principal payment $K$ at $T$, unless one of the two events below happen:

# Part III

- (i) if $S^1_{t_i} \geq TB$ and $S^2_{t_i} \geq TB$ for some $i < n$, where $TB \geq S^1_0 = S^2_0$ is the trigger barrier. In this case, the contract is terminated early, and the holder receives the final payment of $c + K$.

- (ii) if $S^1_t \leq B$ or $S^2_t \leq B$ for some $t < T$, where $B < S^1_0 = S^2_0$ is the lower barrier. In this case, the coupon payments stay the same but the terminal payment (in case the contract is not autocalled before) is given by

$$c + K - (K - \min(S^1_T, S^2_T))^+$$

- Assume that $S^1_0 = S^2_0 = K = TB = 100$, $t_i = iT/n$ and $t_i - t_{i-1} = 0.25$. Take some parameter values for $(r, \sigma_1, \sigma_2, \rho, TB, T, n)$. Also, take a reasonably large number $N$ of simulations and number $m$ of discretization steps. Write a program that determines the value of coupon $c$ that makes the contract value $V_0 = 100$ at time 0.