Monte-Carlo simulation methods

Quiz 1

Deadline: March 22, 23:00

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Part I

Assume that you are able to sample from the uniform distribution on $(0, 1)$. Show and explain how to sample from the following distributions (one way for each case is sufficient)

1. Arcsine law with cdf

$$F(x) = \frac{2}{\pi}\arcsin(\sqrt{x}), 0 \le x \le 1$$

   .

2. The distribution with pdf $f(x) = e^x/(e-1), \quad 0 \le x \le 1$

Part II

- Suppose X has a distribution with pdf

$$f(x) = \frac{1}{6}x^3e^{-x}, \qquad x \geq 0.$$

We need to apply the acceptance-rejection (AR) method to simulate X using exponential distribution $Exp(\lambda)$ with pdf

$$g(x) = \lambda e^{-\lambda x}, \qquad x \geq 0.$$

Find the optimal value of λ^* that makes the AR method most efficient.

Part III

- Write a pseudo-code for the Monte-Carlo method to price a min-call option with a payoff

$$\left(\min\left(S_T^1, S_T^2\right) - K\right)^+$$

with strike K and maturity $T > 0$ under correlated geometric Brownian motions

$$dS_t^1 = rS_t^1 dt + \sigma_1 S_t^1 dW_t^1$$
$$dS_t^2 = rS_t^2 dt + \sigma_2 S_t^2 dW_t^2$$

where $\left(W^1, W^2\right)$ are two standard Brownian motions with the correlation ρ.