



DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2020/2021

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE

**DEGREE OF BACHELORS OF ELECTRICAL ENGINEERING, MECHANICAL
ENGINEERING, CIVIL ENGINEERING, BED CIVIL, BED EEE, BED
MECHANICAL, BSC GEGIS & GIS, BSC MATHEMATICAL MODELLING,
BSC INDUSTRIAL CHEMISTRY, BSC POLYMER TECHNOLOGY
CHEMICAL ENGINEERING, BSC. COMPUTER SCIENCE, BSC. I.T**

Instructions To Candidates

1. Answer Question ONE and ANY OTHER TWO Questions.
2. Show your working

QUESTION ONE (30 marks)

a) If

$$f(x) = \begin{cases} 3x & x < -1 \\ 2x-1 & -1 \leq x \leq 5 \\ x+3 & x > 5 \end{cases}$$

Find $f(3)$

(2 Marks)

b) Given that $f(x) = \frac{1}{3-x^2}$, $g(x) = \sqrt{x^2-1}$ determine the domain of $f \circ g(x)$

(4 Marks)

c) Find the derivative of $f(x) = (1+x^4 - \frac{1}{x})^{\frac{5}{3}}$ when $x=1$

(3 Marks)

d) Let

$$f(x) = \begin{cases} 3x^2-1, & x < 0 \\ cx+d, & 0 \leq x \leq 1 \\ \sqrt{x+8}, & x > 1 \end{cases}$$

Determine c and d so that f is continuous everywhere.

(4 Marks)

e) Find $\lim_{x \rightarrow \infty} \frac{4-3x^2}{\sqrt{16x^4-5}}$ (3 Marks)

f) Find the inverse of $f(x) = \frac{x-1}{1+4x}$ (4 Marks)

g) Use linear approximation to estimate $\sqrt[3]{8.5}$ (3 Marks)

h) $\int_1^2 \left(\frac{7-x^4+4x^7}{x^2} \right)$ (4 Marks)

i) A function is given by $f(x) = 3x^4 + 4x^3 - 12x^2$. Find the coordinates of the stationary points and distinguish between them. (3 Marks)

QUESTION TWO (20marks)

a) Evaluate the following limits

i) $\lim_{x \rightarrow 0} \frac{-3 + \sqrt{3+x}}{x}$ (4 Marks)

ii) $\lim_{x \rightarrow 2} \left(\frac{1}{x-2} - \frac{4}{x^2-4} \right)$ (5 Marks)

b) Find the domain and range of the function $f(x) = \sqrt{4-7x}$ (4 Marks)

c) Let $f(x) = \frac{5}{2}x^2 - e^x$. Find the value of x for which the second derivative is zero (4 Marks)

d) If the function $f(x) = \begin{cases} \frac{x^2-16}{x-4} & x \neq 4 \\ c & x = 4 \end{cases}$

is continuous, find the value of c (3 Marks)

QUESTION 3 (20marks)

Differentiate the following functions with respect to x

a) $y = \frac{1}{(3x^2 + 5)^4}$ (5 Marks)

b) $y = e^x \sin 2x$ (5 Marks)

c) $y = \frac{\ln x}{x}$ (5 Marks)

d) $y = \sqrt{\frac{(x-2)^2}{x^2 + 5}}$ (5 Marks)

QUESTION 4 (20MARKS)

a) Differentiate $f(x) = \sqrt{x^2 + 4}$ from first principles (5 Marks)

b) By using a suitable substitution find

$\int \frac{1}{(9-x)^4} dx$ (5 Marks)

c) Find the equation of the tangent to the curve given by the parametric equations

$x = 2 \cos t, y = 4 \sin t$ at the point $(\sqrt{2}, 2\sqrt{2})$ (5 Marks)

d) Oil is leaking onto a floor creates a circular pool with an area that increases at a rate of 5cm^2 per minute. How fast is the radius of the pool increasing when the radius is 20 cm? (5 Marks)

QUESTION 5 (20marks)

a) The distance x meters travelled by a vehicle in time t seconds after the brakes are applied

is given by $x = 20t - \frac{5}{3}t^2$. Determine

i) The speed of the vehicle at the instant the brakes are applied (3 Marks)

ii) The distance the car travels before it stops (2 Marks)

b) Determine the height and radius of a cylinder of volume 200cm^3 which has the least surface area. (5 marks)

c) Determine the coordinates of the maximum and minimum values of the curve $y = x^3 - 3x + 5$ (5 marks)

d) Find the equation of the normal to the curve $y = \frac{2x}{x^2 - 5}$ at the point $(2, -4)$. (5 Marks)