DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATION 2019/2020

BACHELOR OF SCIENCE IN CIVIL / MECHATRONIC/INDUSTRIAL CHEMISTRY/MECHANICAL /ELECTRICAL & ELECTRONIC ENGINEERING / GEGIS / GIS,BED ELECTRICAL ,BED MECHANICAL ,BED CIVIL ENGINEERING, COMPUTER SCIENCE/INFORMATION TECHNOLOGY, CHEMICAL ENGINEERING, T.I.E.

SMA 2119 CALCULUS III

Date: 17th NOVEMBER, 2020 **TIME: 8.30AM-10.30AM**

Answer QUESTION ONE and Any Other Two Questions

Question One (30marks)

a) Show that the series $\sum_{0}^{\infty} 3^{\frac{1}{\frac{1}{n}+1}}$ diverges (2marks)

b) Find a power series representation for $f(x) = \frac{1}{x+2}$ (4marks)

c) Find the centre of mass of the system of objects that has masses 5, 4 ,6 at the points (1,-3) , (-2, 2) and (3,4) respectively (5marks)

d) Evaluate the double integral $\int_0^2 \int_0^{1-x} x(y-1)\,dydx$ (4marks)

e) Determine whether the improper integral $\int_0^{\infty} \frac{dx}{1+x^2}$ converges or diverges (4marks)

f) Test the convergence of the series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ using the integral test(4 marks)

g) Find the total derivative $\frac{dz}{dt}$ given that $z = x^2 + 3xy + 7y^4$, $x = \cos 2t, y = \sin t$ (4marks)

h) Use the alternating series test to show that $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n^2}{5n^2+8}$ diverges(3marks)

Question2 (20marks)

a) Find the Maclaurin series for $f(x) = \ln(2+x)$ up to the term containing x^3. Hence evaluate ln 2.2 correct to 3 decimal places (6marks)

b) Find the value of the double integral $\iint_R (6x+2y^2)dA$ where R is the region bounded by the parabolas $x=y^2$ and $x+y=2$ (6marks)

c) Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+2)^n}{2^n(n)}$ (5marks)

d) Use double integration to find the volume under the plane $z=5x^2-3y$ and over the rectangle defined by $0\le x\le 3, -1\le y\le 2$ (3marks)

<u>Question 3(20marks)</u>

a) By reversing the order of integration evaluate $\int_0^1\int_y^{\sqrt{y}} 3y\,dx\,dy$ (5marks)

b) Use differentials to approximate the change in $f(x,y)=x^2y+2xy^3$ if (x,y) changes from (1,2) to (1.2,2.1) (5marks)

c) Using the method of partial sums, determine if the series $\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$ converges. (5marks)

d) Show that if x^4 and higher powers of x are neglected , then $f(x)=e^x\cos x=1+x-\frac{x^3}{3}$ (5marks)

<u>Question 4(20marks)</u>

a) Use the ratio test to determine if the series $\sum_{n=1}^{\infty}\frac{-10^n}{4^{n+2}(n+1)}$ converges or diverges (5marks)

b) Using double integrals, find the area of the region bounded by $y=x$ and $y=x^2$ in the first quadrant. (5marks)

c) Use the limit comparison test to determine the convergence of the series $\sum_{n=1}^{\infty}\frac{2n+1}{5n^4+7}$ (4marks)

d) Use the change of variable $x=u^2-v^2, y=2uv$ to evaluate $\iint_R dA$ where R is the region bounded by the $\iint_R dA$ x - axis and the parabolas $y^2=4-4x$ and $y^2=4+4x$, $y\ge 0$

(6marks)

<u>Question 5(20marks)</u>

a) Find the centre of mass of the region bounded by $y=x^2$ and the line $y\ge 0$ with density $^\circ(x,y)=x+y$ (8marks)

b) Show that if $z=\frac{1}{\sqrt{x^2+y^2+z^2}}$ then $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}=0$ (6marks)

c) Find the Taylor series for $f(x)=e^{-2x}$ about $x=-4$ (6marks)