C501: Computer Architecture
Assessed Coursework 1

Question 1:

a) i) $E \equiv A \cdot B + B \cdot (A' + A \cdot B)$

$\equiv A \cdot B + B \cdot (A' + (A \cdot B))$ operator precedence

$\equiv A \cdot B + B \cdot ((A' + A) \cdot (A' + B))$ distributative rule

$\equiv A \cdot B + B \cdot (1 \cdot (A' + B))$ negation

$\equiv A \cdot B + B \cdot (A' + B)$ simplification rule

$\equiv A \cdot B + (B \cdot A') + (B \cdot B)$ distributative rule

$\equiv A \cdot B + (B \cdot A') + B$ idempotent law

$\equiv A \cdot B + B \cdot A' + B$ operator precedence

$\equiv B \cdot (A + A') + B$ distributative rule

$\equiv B \cdot 1 + B$ negation

$\equiv B + B$ simplification rule

$\equiv B$ idempotent law

ii) $E \equiv (A+B)' \cdot ((C+D+F)' + (A' \cdot B')$

$\equiv (A' \cdot B') \cdot (C' \cdot D' \cdot F') + (A' \cdot B')$ De Morgan's rule

$\equiv ((A' \cdot B') \cdot (C' \cdot D' \cdot F')) + (A' \cdot B')$ operator precedence

$\equiv ((A' \cdot B') + (A' \cdot B')) \cdot ((A' \cdot B') + (C' \cdot D' \cdot F'))$ distributative rule

$\equiv (A' \cdot B') \cdot ((A' \cdot B') + (C' \cdot D' \cdot F'))$ idempotent law

$\cancel{(\equiv ((A' \cdot B') + 0) \cdot ((A' \cdot B') \cdot (C' \cdot D' \cdot F')))}$

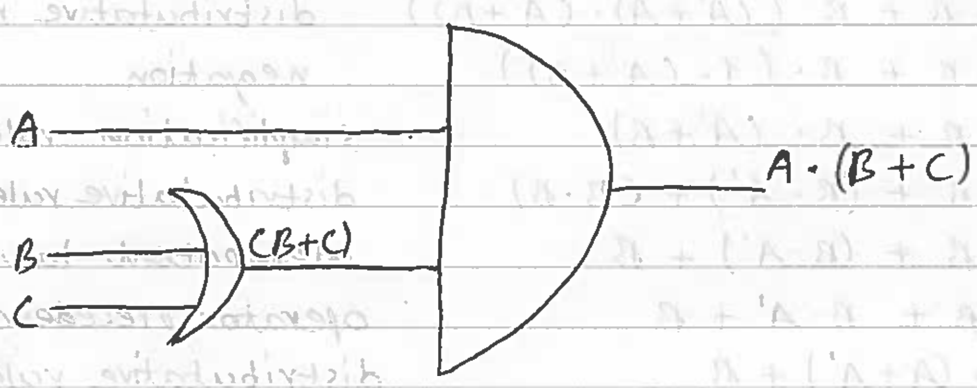
$\equiv (A' \cdot B') + (0 \cdot (C' \cdot D' \cdot F'))$ distributative rule

$\equiv (A' \cdot B') + 0$ simplification rule

$\equiv A' \cdot B'$ simplification rule

b) $E = A\cdot(B + A\cdot B) + A\cdot C$

$\equiv A\cdot(B\cdot(A+1)) + A\cdot C$	distributative rule
$\equiv A\cdot(B\cdot A) + A\cdot C$	simplification rule
$\equiv A\cdot(A\cdot B) + A\cdot C$	commutative rule
$\equiv (A\cdot A)\cdot B + A\cdot C$	associative rule
$\equiv A\cdot B + A\cdot C$	idempotent law
$\equiv A\cdot(B+C)$	distributative rule



c) $E = A\cdot B$

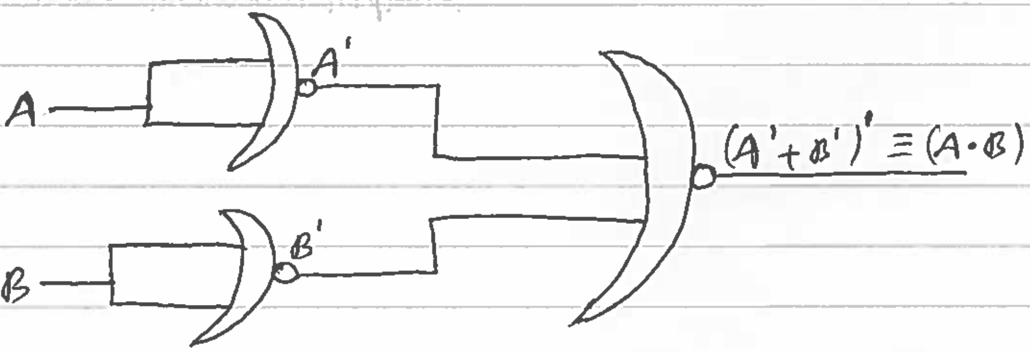
We can show that:

$(A' + B')' \equiv A'' \cdot B'' \equiv A\cdot B$

De Morgan's rule   negation rule

or:

A	B	A'	B'	A'+B'	(A'+B')'	A·B
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F



Question 2:

a) 5-bits

i) 9 - 12

For 9_{10}:

	Quotient	Remainder
9:2	4	1
4:2	2	0
2:2	1	0
1:2	0	1

$\rightarrow 1001_2$

so 9_{10} is 01001_2

For 12_{10}:

	Quotient	Remainder
12:2	6	0
6:2	3	0
3:2	1	1
1:2	0	1

$\rightarrow 1100_2$

so 12_{10} is 01100_2.

Two's Complement Subtraction: "negate the subtrahend and add".

$9 - 12 = 9 + (-12)$

-12 : $12 \rightarrow$ invert + 1 $\rightarrow 10011 + 1 \rightarrow 10100_2$

```
       0 1 0 0 1
     + 1 0 1 0 0
sum    1 1 1 0 1
carry  0 0 0 0 0
```

So the result is 11101_2.

We don't have overflow since we add operands of different signs.

We can check the result in decimal notation:
We need to turn 11101_2 from two's complement into decimal notation.

11101_2:

1. it is negative
2. magnitude $11101 - 1 = 11100$, invert $= 00011_2$

Therefore 11101_2 is -3 in decimal notation.

$9 - 12 = -3$, correct

ii) $-11-8$

Two's Complement Subtraction: "negate the subtrahend and add"

$-11 + (-8)$

-11_{10}:

	Quotient	Remainder
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1 → 1011_2

so -11_{10}: 01011 $(11_{10}) \rightarrow$ invert $+1 \rightarrow 10100 + 1 = 10101_2$

-8_{10}:

	Quotient	Remainder
8:2	4	0
4:2	2	0
2:2	1	0
1:2	0	1 → 1000_2

so -8_{10}: 01000 $(8_{10}) \rightarrow$ invert $+1 \rightarrow 10111 + 1 \rightarrow 11000_2$

```
      10101
    + 11000
sum   01101
carry 10000
```

↳ we discard carry-out bit

So the result is $\underline{01101_2}$

Overflow occurs in this operation, since we add two numbers of the same sign (negative) and the result has the opposite sign (positive).

We can check with decimal notation too:
01101_2 (two's complement) is $2^3+2^2+2^0=8+4+1=13_{10}$ in decimal notation.

$-11-8=-19 \neq 13$.

b) $\frac{189}{27}$

189_{10} is:

	Quotient	Remainder
189:2	94	1
94:2	47	0
47:2	23	1
23:2	11	1
11:2	5	1
5:2	2	1
2:2	1	0
1:2	0	1 → 10111101_2

So 189_{10} is 10111101_2 in 8-bit unsigned binary notation.

27_{10} is:

	Quotient	Remainder
27 : 2	13	1
13 : 2	6	1
6 : 2	3	0
3 : 2	1	1
1 : 2	0	1 → 11011_2

so 27_{10} is 11011 in 5-bit unsigned binary notation.

```
            0 0000111
   11011 ) 10111101
           -0
            10
           -00
            101
           -000
            1011
           -0000
            10111
           -00000
            101111
(1):      - 11011
            101000
(2):      -  11011
            0011011
          -   11011
            00000000
```

Therefore the result of this operation is:
quotient: 111 (3-bit unsigned binary)
remainder: 0

```
                                   A''  0 1 10
       A'  0 10                    A'   1 0 0 1 1 10
(1): A  1 0 1 1 1 1         (2): A   1 0 1 0 0 0
      -  1 1 0 1 1                 -   1 1 0 1 1
        0 1 0 1 0 0                  0 0 1 1 0 1
```

Question 3:

a) -31.01

31_{10} is:

	Quotient	Remainder
31:2	15	1
15:2	7	1
7:2	3	1
3:2	1	1
1:2	0	1 $\rightarrow 11111_2$

$$.01 = \frac{0.02}{2} = \frac{0.04}{4} = \frac{0.08}{8} = \frac{0.16}{16} = \frac{0.32}{2^5} = \frac{0.64}{2^6} = \frac{1.28}{2^7}$$

$$= \frac{1}{2^7} + \frac{0.28}{2^7} = \frac{1}{2^7} + \frac{0.56}{2^8} = \frac{1}{2^7} + \frac{1.12}{2^9} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{0.12}{2^9}$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{0.24}{2^{10}} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{0.48}{2^{11}} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{0.96}{2^{12}}$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{1.92}{2^{13}} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{0.92}{2^{13}} =$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1.84}{2^{14}} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{0.84}{2^{14}} =$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1.68}{2^{15}} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{0.68}{2^{15}}$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1.36}{2^{16}} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{0.36}{2^{16}}$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{0.72}{2^{17}} = \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{1.44}{2^{18}}$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{1}{2^{18}} + \frac{0.44}{2^{18}}$$

$$= \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{13}} + \frac{1}{2^{16}} + \frac{1}{2^{18}} + \frac{0.88}{2^{19}}$$

so $0.01 \simeq \frac{1}{2^7} + \frac{1}{2^9} + \frac{1}{2^{13}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{1}{2^{18}}$, or $.0000001010001111010$

so 31.01 is 11111.000000101000 1111010

$= 1.1111000000101000111101 0 \times 2^4$

IEEE Single Precision Format (32-bit)

1 bit is the sign bit

8 bit is the exponent

23 bit is the coefficient

Exponent is Excess-127

so actual value of 4 is 127+4 = 131 stored value

	Quotient	Remainder
131:2	65	1
65:2	32	1
32:2	16	0
16:2	8	0
8:2	4	0
4:2	2	0
2:2	1	0
1:2	0	1 → 10000011_2

Significand : 11110000001010001111010

Exponent : 10000011

Sign : 1 (negative number)

Sign	Exponent	Significand
1	10000011	11110000001010001111010

Hexadecimal Value:

1100	0001	1111	1000	0001	0100	0111	1010
C	1	F	8	1	4	7	A

so the Hexadecimal value is: 0x C1F8147A

b) 0X 40F108D4

Binary:

4	0	F	1	0	8	D	4
0100	0000	1111	0001	0000	1000	1101	0100

IEEE Single Precision Format

Sign	Exponent	Significand
0	10000001	11100010000100011010100

Decimal:

Exponent: 10000001_2 is 129 stored value

$129 - 127 = 2$ actual value in Excess-127.

Significand field + hidden bit: 1.11100010000100011010100

Sign 0 , so positive number.

Therefore: $1.11100010000100011010100 \times 2^2$

$= 111.100010000100011010100$

which is: $2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-5} + 2^{-10} + 2^{-14} + 2^{-15} + 2^{-17} + 2^{-19}$

$= 4+2+1+ \frac{1}{2} + \frac{1}{2^5} + \frac{1}{2^{10}} + \frac{1}{2^{14}} + \frac{1}{2^{15}} + \frac{1}{2^{17}} + \frac{1}{2^{19}}$

$= 7 + \frac{2^{18} + 2^{14} + 2^9 + 2^5 + 2^4 + 2^2 + 1}{2^{19}}$

$= 7 + 0.532327652 = 7.532327652$