Agenda

Mod

Count pairs

GCD intro

GCD properties

Delete One

Saturday till Dec 31

$\% \, m \Rightarrow [0, m-1]$

$\%$ remainder $17 \% 5 = 2$

$(a+b) \% m = (a \% m + b \% m) \% m$

$(7+8) \% 3 = (7 \% 3 + 8 \% 3) \% 3$

$(1+2) \% 3 = 3 \% 3 = 0$

$(a*b) \% m = (a \% m * b \% m) \% m$

$(8 \times 9) \% 5 = (8 \% 5 * 9 \% 5) \% 5$

2

$(3 \times 4) \% 5 = 12 \% 5 = 2$

$(a-b) \% m = (a \% m - b \% m + m) \% m$

$(6-4) \% 5 = (6 \% 5 - 4 \% 5 + 5) \% 5$

2

$= (1 - 4 + 5) \% 5$

$2 \% 5 = 2$

$$a^b \% m = [(a \% m)^b] \% m$$

$$(7^3) \% 5 = [(7 \% 5)^3] \% 5$$

$$343 \% 5 \qquad 2^3 \% 5 = 8 \% 5$$

$$= 3 \qquad = 3$$

Quiz $[(37)^{103} - 1] \% 12$

$$(37^{103} \% 12 - 1 + 12) \% 12$$

$$(37^{103} \% 12 + 11) \% 12$$

$$(1^{103} \% 12 + 11) \% 12$$

$$(1 + 11) \% 12 \qquad 12 \% 12 = 0$$

Q1 Given N +ve elements, calc number of pairs i,j st $(ar[i]+ar[j]) \% M = 0$

$i != j$ and (i,j) is the same as (j,i)

	0	1	2	3	4	5	
Eg-	4	7	6	5	8	3	M=3

Ans ⇒ (0,3) (0,4) (2,5)

(1,3) (1,4) ans = 5

Brute force: Check for all pairs

TC: $O(N^2)$

Idea: $(a+b) \% M = 0$

$$\Rightarrow (\underbrace{a \% M + b \% M}_{0 \quad M}) \% M = 0$$

$\Rightarrow a \% M + b \% M = 0 / M$

$a \% m$	$b \% m \rightarrow [0, m-1]$
1	$m-1$
2	$m-2$
3	$m-3$
⋮	
$M-1$	1

$0 \longrightarrow 0$

if $M/2 \longrightarrow M/2$

$M\%2=0$

	0	1	2	3	4	5	6	7	8	9	10	11
Eg	6	7	5	11	19	20	9	15	14	13	12	23
	1	2	0	1	4	0	4	0	4	3	2	3

$M = 5$

How many ways to form pairs of 0?

(2,5) (2,7) (5,7) $\Rightarrow \frac{3(3-1)}{2} = 3$

freq 1 to 4

1	2	3	4
2	2	2	3

1 match with 4 $\rightarrow$ 6

2 match with 3 $\rightarrow$ 4

3 match with 2 $\rightarrow$

will you count (3,2) $\Rightarrow$ NO

total = ? $3+6+4 = 13$

$M=6$

0	1	2	3	4	5	6	7	8	9	10	11	12
2	3	4	8	6	15	5	12	17	7	18	10	9
2	3	4	2	0	3	5	0	5	1	0	4	3

%6

0	1	2	3	4	5
3	1	2	3	2	2

} size $= M$

0 match with 0   3

1 match with 5 $1 \times 2 = 2$

2 match with 4 $2 \times 2 = 4$

3 match with 3

How to count (3,3) $\Rightarrow \frac{3(3-1)}{2} = 3$

4 match with 2

will you count ?   Nooo !!!

total $\Rightarrow 3+2+4+3 = 12$

Code

1) Create freq array of size m

```
ans = 0
// first handle 0 case
x = freq[0]
ans += x(x-1)/2

// now handle if M even, then M/2, M/2
if (M % 2 == 0) {
    x = freq[M/2]
    ans += x(x-1)/2
}

// now loop on the rest.
for (i=1 ; i < M/2 ; i++) {
    ans += freq[i] * freq[m-i]
}
```

TC: $O(N)$

return ans.

SC: $O(M)$

```
int freq[M] = {0}
for( i=0; i<N ; i++){
    freq[a[i]]++
}
```

break back at 10:20

0 0 1 1 0 1

	0	1
freq	$\cancel{0}$	$\cancel{0}$
	$\cancel{1}$	$\cancel{1}$
	$\cancel{2}$	$\cancel{2}$
	3	3

GCD

GCD : Greatest Common Divisor OR
HCF : Highest Common Factor

biggest number that divides both a & b

$gcd(a,b)$

$gcd(5,8) = 1$
$gcd(0,17) = 17$
$gcd(12,18) = 6$

Properties

- $gcd(a,b) = gcd(b,a)$
- $gcd(a,b,c) = gcd(a,(gcd(b,c)))$
- $gcd(0,x) = x$

Special property Say $gcd(a,b) = x$ $a > b$

then $gcd(\underbrace{a-b}, b) = x$

$$(a-b)\%x = (\underbrace{a\%x}_{0} - \underbrace{b\%x}_{0} + x)\%x \Rightarrow 0$$

$$\gcd(a,b) = \gcd(a-b,\ b)$$

Eg $\gcd(23,5) = \gcd(18,5) = \gcd(13,5)$
$= \gcd(8,5) = \gcd(3,5)$

$$\gcd(23,5) = \gcd(23\%5,\ 5)$$

Given $A, B > 0$ $a > b$

$$\gcd(a,b) = \gcd(a-b,\ b)$$
$$= \gcd(a-2b,\ b)$$
$$= \gcd(a-3b,\ b)$$
$$\vdots$$
$$= \gcd(a-xb,\ b)$$

subtracting the max divisor

$$= \gcd(a\%b,\ b)$$

as $a\%b < b$, write this as
$\gcd(b,\ a\%b)$

$$\gcd(a,b) = \gcd(b,\ a\%b)$$

```
int gcd(int a, int b){
    if (b==0) return a
    if (a==0) return b
    return gcd(b, a%b)
}
```

Euclidian GCD algorithm.

TC: $\log(\max(a,b))$

Q: Given N elements, calc gcd of entire array.

Eg - {6, 12, 15} ans = 3

{8, 16, 12, 10} ans = 2

- Idea : Take gcd one by one

```
int gcd_arr (int ar[], int N) {
    int ans = 0
    for (i=0; i<n; i++) {
        ans = gcd (ans, ar[i])
    }
    return ans
}
```

TC: $n \log(max)$

8, 16, 12, 10

ans = ~~8~~ ~~4~~ 2

Q3 Given N array elements, we have to delete 1 elem such that gcd of remaining is max.

	0	1	2	3	4
Eg –	24	16	18	30	15

ans = 3

- Brute force : Try deleting all elem one by one & calc the GCD.

TC: $N(N \log max)$

$= N^2 \log max$

Assume N=7 0 1 2 3 4 5 6

Delete	gcd
0	gcd [1,6]
1	gcd (gcd [0,0], gcd [2,6])
2	gcd (gcd [0,1], gcd [3,6])
3	gcd (gcd [0,2], gcd [4,6])
4	gcd (gcd [0,3], gcd [5,6])
5	gcd (gcd [0,4], gcd [6,6])
6	gcd [0,5]

Remember Prefix & Suffix Max

We can do same for gcd also.

Pfgcd [i] = gcd of all elem [0, i]

Sfgcd [i] = gcd of all elem [i, n-1]

Code

```
pfgcd[N] , sfgcd[N]

pfgcd[0] = ar[0]
for (i=1 ; i<N ; i++) {
    pfgcd[i] = gcd( pfgcd[i-1], ar[i])
}

sfgcd[n-1] = ar[n-1]
for ( i=n-2 ; i>=0 ; i--) {
    sfgcd[i] = gcd( sfgcd[i+1], ar[i])
}

// Now try deleting every elem.
```

	0	1	2	3	4
	24	16	18	30	15
pf	24	8	2	2	1
sf	1	1	3	15	15

0 1 2 . . . i-1 i i+1 . . . . . . n-1

```
ans = 0
for (i=1 ; i<=n-2; i++){
    //delete i^th
    left_gcd = pfgcd[i-1]

    right-gcd = sfgcd [i+1]


    ans = max (ans , gcd (left , right)
}
ans = max (ans, sfgcd[1] )
ans = max (ans, pfgcd [n-2] )

return ans
```

TC: $O(n \log max)$

SC: $O(n)$

	0	1	2	3	4
	24	16	18	30	15
bf	24	8	2	2	1
sf	1	1	3	15	15

0 7 8 20

13

$0 \rightarrow 5$

$1 \rightarrow \cancel{3}\ 2$

$2 \rightarrow 1$