

Count sort

$$A = [6 \ 3 \ 4 \ 2 \ 4 \ 2 \ 1]$$

if numbers are small [here 0-6]

create freq array

0	1	2	3	4	5	6
0	\emptyset_1	$\cancel{\emptyset}_2$	\emptyset_1	$\cancel{\emptyset}_2$	0	\emptyset_1

Now use this to generate sorted array

1 2 2 3 4 4 6

Does this work for large values like 10^9

No, because 10^9 size array I cannot make.

How to handle -ve numbers

$$A = [-3 \quad -2 \quad 2 \quad 0 \quad 1]$$

$$\min = -3$$

you want to represent min at idx 0

$$-3 \xrightarrow{+3} \overset{\text{idx}}{0}$$

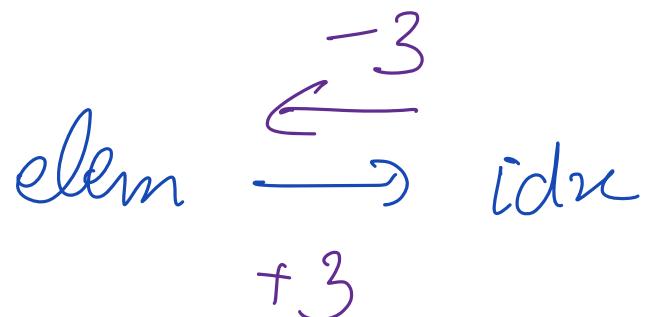
$$-2 \rightarrow 1$$

$$-1 \rightarrow 2$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$



Now freq array

0 1 2 3 4 5

1 1 1 1 1 1

Now sorted arr. $\text{idx}=0$ has 1

This means number = ? has 1

-3 -2 0 1 2

Q1) Given 2 sorted arrays A & B, merge them & return new sorted array.

Eg $a[3] = \{-1, 4, 8\}$

$b[2] = \{2, 9\}$

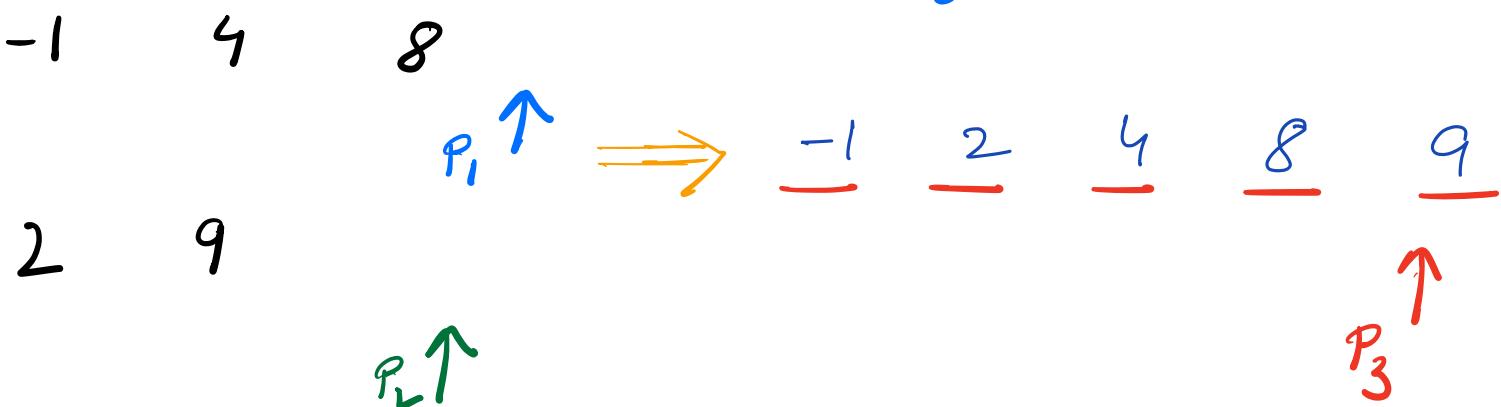
$c[5] = \{-1, 2, 4, 8, 9\}$

Brute: Simply append a after b & then sort.

$$\{-1, 4, 8, 2, 9\} \xrightarrow{\text{sort}} \{-1, 2, 4, 8, 9\}$$

TC: $O(n \log n)$

Idea: Use two pointers to merge.



Code

```
int[] merge(int A[], int B[], int n, int m) {
    int p1 = p2 = p3 = 0
    int C[N+M]
    while (p1 < N && p2 < M) {
        if (A[p1] ≤ B[p2]) {
            C[p3] = A[p1]
            // update pointers
            p1++
            p3++
        } else {
            C[p3] = B[p2]
            // update pointers
            p2++
            p3++
        }
    }
}
```

// Now one array is finished, other
should be copied as it is

```
while (P1 < N) do  
| c[P3] = A[P1]  
| P1 ++  
| P3 ++  
y
```

```
while (P2 < M) do  
| c[P3] = B[P2]  
| P2 ++  
| P3 ++  
y
```

return c

y

TC: $O(N+M)$
SC: $O(N+M)$

- O2 Given an array & 3 input idx s, m, e
- Subarray $[s:m]$ sorted $s < m < e$
 - Subarray $[m+1:e]$ sorted

Sort the subarray $[s:e]$

0 1 2 3 4 5 6 7 8 9 10 11

Eg: ar: 4 8 -1 2 6 9 11 3 4 7 13 0

s m e

2 6 9

How to use same funda as above ?

0 1 2 3 4 5 6 7 8 9 10 11

4 8 -1 2 6 9 11 3 4 7 13 0

P_1

P_2

$temp[8] = \underline{-1} \underline{2} \underline{3} \underline{4} \underline{6} \underline{7} \underline{9} \underline{11}$

P_3

Now copy temp back to arr $[s:e]$

Code

```
void merge (int a[], int s, int m, int e){
```

```
    int tmp [e-s+1]
```

```
    int p1 = s p2 = m+1 p3 = 0
```

```
    while (p1 ≤ m && p2 ≤ e) {
```

```
        if (a[p1] ≤ a[p2]) {
```

```
            tmp[p3] = a[p1]
```

```
            p1++ p3++
```

```
}
```

```
        else {
```

```
            tmp[p3] = a[p2]
```

```
            p2++ p3++
```

```
}
```

```
    while (p1 ≤ m) {
```

```
        tmp[p3] = a[p1] p1++ p3++
```

```
}
```

```
    while (p2 ≤ e) {
```

```
        tmp[p3] = a[p2] p2++ p3++
```

```
}
```

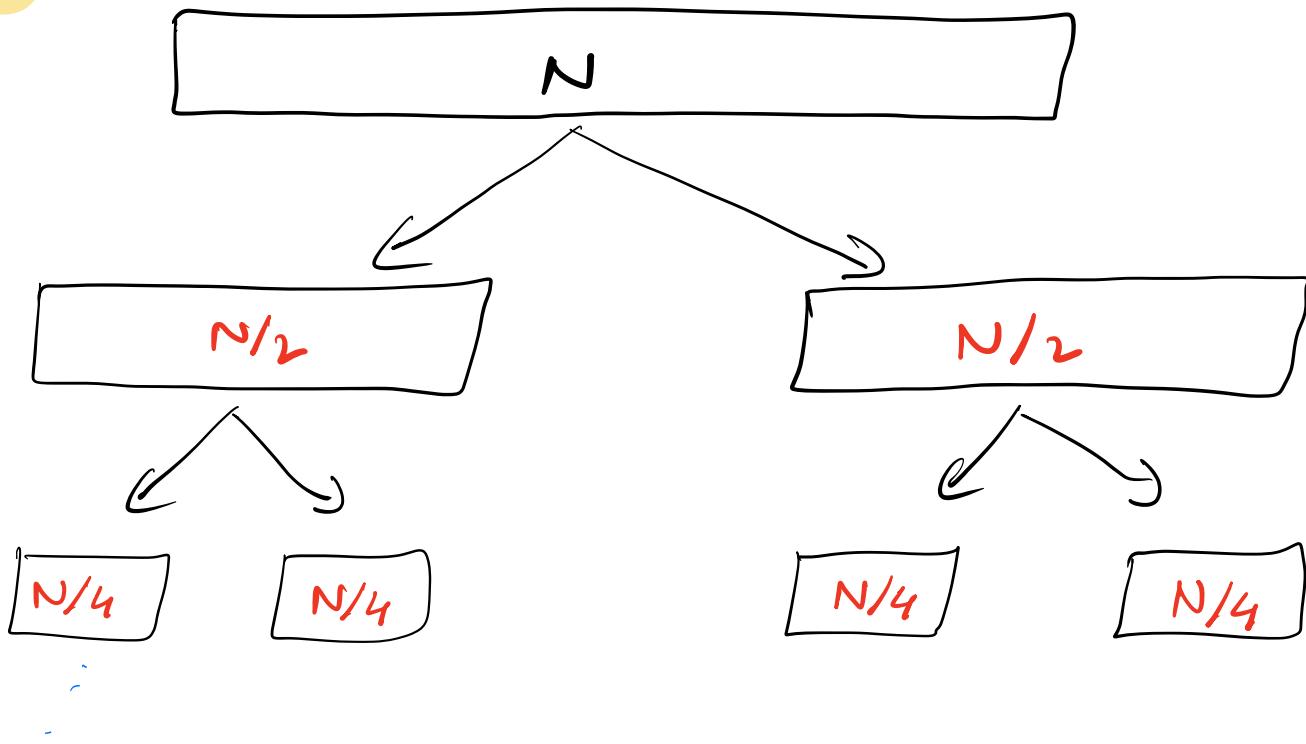
Now copy temp into the a.

```
for ( i=0 ; i < e-s+1 ; i++ ) {  
    a[ i+s ] = temp[ i ]  
}
```

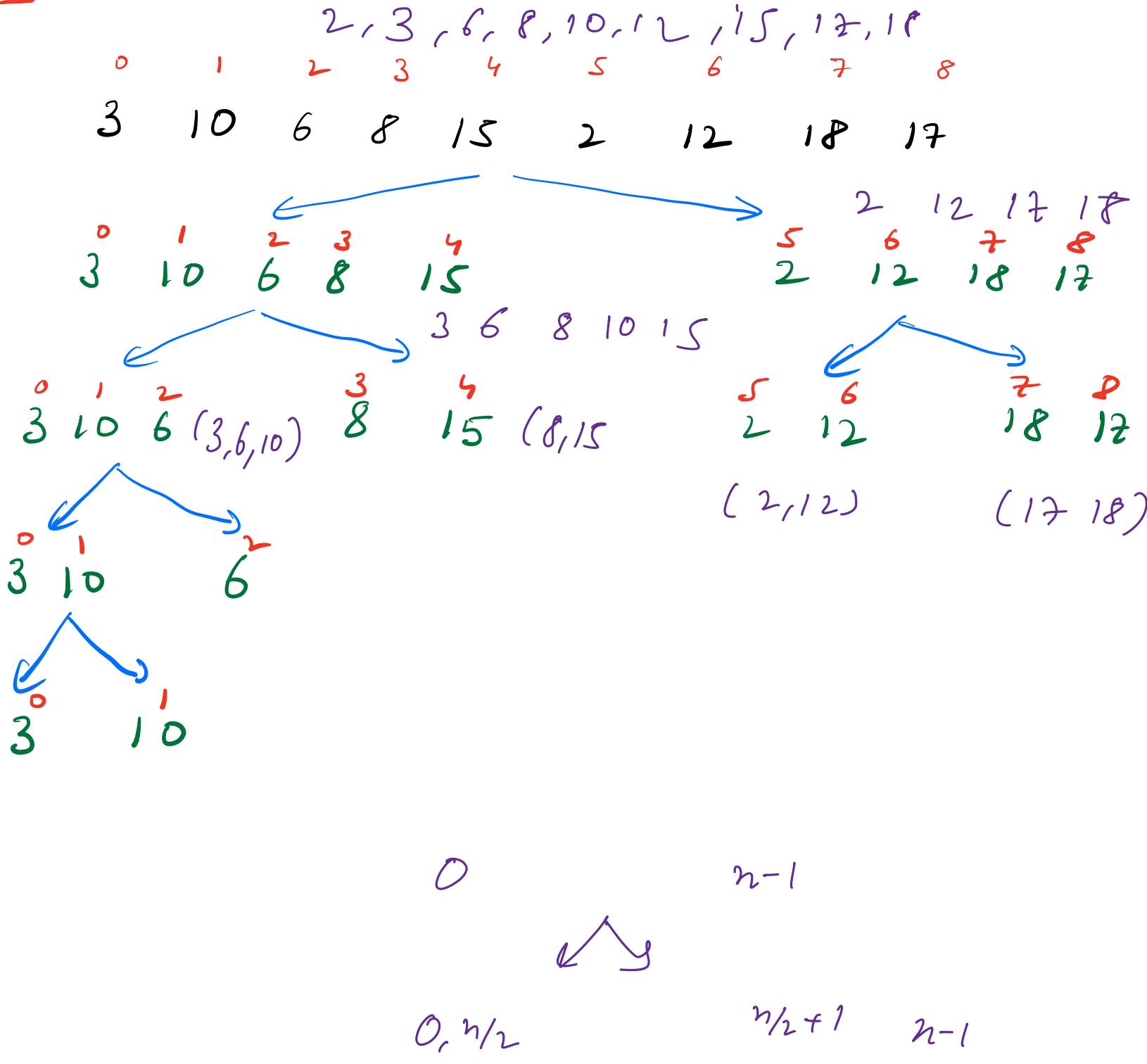
TC:
SC: } linear

Q3 Given an unsorted array, sort it
(Merge Sort)

Idea



when subarray size = 1, we cannot divide further

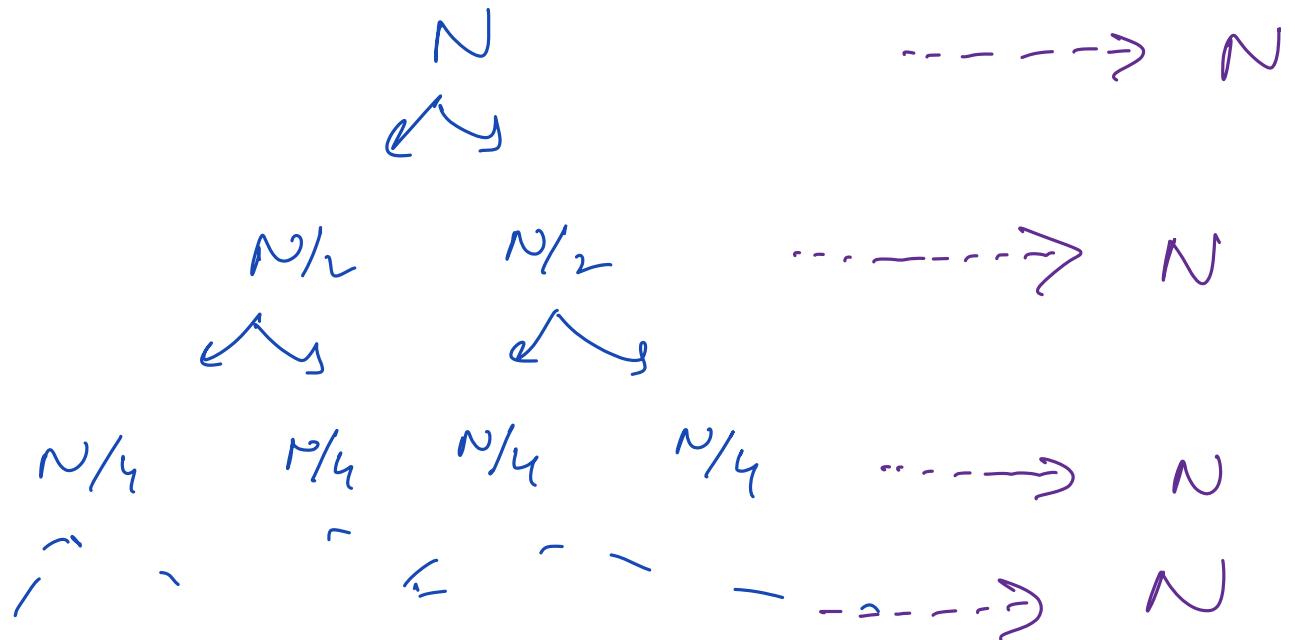


Code

```
void mergesort (int A[], int s, int e) {  
    if (s == e) return;  
  
    // Use recursion to sort the  
    // two halves  
    int m = (s+e)/2  
    mergesort (A, s, m)  
    mergesort (A, m+1, e)  
    // Now merge the two sorted  
    // halves  
    // Have we written code to merge  
    merge (A, s, m, e)
```

main () {

```
    mergesort (A, 0, n-1)
```



$\swarrow \searrow \dots \rightarrow N$

How many levels = $\log N$

$$\underbrace{N + N + N + \dots + N}_{\log N \text{ times}} = N \log N$$

$SC: O(N)$

5 3 1 4 2

53 51 54 52 31 32 42

Θ Given N elements, find no of pairs i, j st
 $i < j$ & $arr[i] > arr[j]$ } inversion pair

0 1 2

Eg 3 1 2 ans = 2

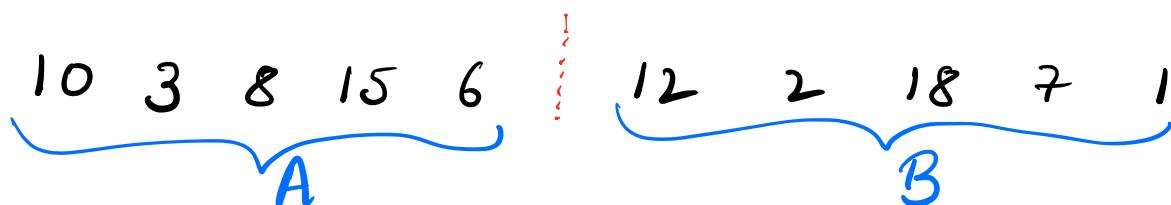
0 1 2 3 5,2 5,1

Eg 5 2 6 1 ans = 6,1 2,1

Brute: Consider all pairs TC: $O(n^2)$
Want to solve in $n \log n$

Idea: Mergesort is helpful

How?



IC means Inversion Count

$$IC(arr) = IC(A) + IC(B) + \begin{matrix} \text{one guy on A} \\ \text{one guy on B} \end{matrix}$$

Obs Changing the order of A & B will not affect $IC(A \& B)$.

So can I sort both the parts Yes!!!

10 3 8 15 6 | 12 2 18 7 1

$$IC(A) = 5$$

$$IC(B) = 4$$

Now sort both parts & perform merge

$\{ 3, 6, 8, 10, 15 \} \quad | \quad \{ 1, 2, 7, 12, 18 \}$

p_1 p_2

1 2 3 6 7 8 10 12 15

$b_2 - m - 1$

$$\begin{aligned} ansf &= 2 + 2 + 3 + 3 + 4 \\ &= 14 \end{aligned}$$

```

int merge (int a[], int s, int m, int e) {
    int cnt = 0
    int p1 = s    p2 = m+1    p3 = 0
    int temp [e-s+1]
    while (p1 ≤ m && p2 ≤ e) {
        if (a[p1] ≤ a[p2]) {
            temp[p3] = a[p1]    cnt += p2-m-1
            p1++    p3++
        } else {
            temp[p3] = a[p2]
            p2++
            p3++
        }
    }
    while (p1 ≤ m) {
        temp[p3] = a[p1]    p3++    p1++
        count += p2-m-1
    }
}

```

```

while (k2 ≤ e) {
    temp[k3] = a[k2]      k3++      k2++
}

for (i=0; i < e-s+1; i++) {
    a[i+d] = temp[i]
}

return cnt
}

```

```

int invcount (int arr, int l, int r) {
    if (l == r) return 0
    mid = (l+r)/2
    int x = invcount (arr, l, mid)
    int y = invcount (arr, mid+1, r)
    int z = merge (arr, l, mid, r)
    return x + y + z
}

```

TC: $O(N \log N)$
SC: $O(N)$

Stable sort \Rightarrow relative orders doesn't change

Inplace sort \rightarrow SC $O(1)$

```
sum(x, y) {  
    let z = x + y  
}
```

2 3 2 1 2 4 2 4 2 2
A B C D E

2 1 2 2 2 3 2 4 2 4

Unstable \Rightarrow B E A D C

2 1 2 2 2 3 2 4 2 4

B E A C D \Leftarrow Stable

{done}