

# Bit Manipulation - 1

Decimal Number System  $\rightarrow$  <sup>base</sup> 10 digits  $[0, 9]$

$$\begin{array}{ccc} 7 & 3 & 4 \\ 2 & 1 & 0 \end{array} \quad 700 + 30 + 4 \quad 1) \text{ Next} \rightarrow 17 \text{ Nov} \\ 7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0 \quad \text{next Friday}$$

Solve all assign

$$\begin{array}{ccc} 6 & 5 & 9 & 4 \\ 3 & 2 & 1 & 0 \end{array} \quad 6000 + 500 + 90 + 4 \quad 2) \text{ 22 Nov} \quad 9 \text{ PM} - 10:30 \text{ PM} \\ 6 \times 10^3 + 5 \times 10^2 + 9 \times 10^1 + 4 \times 10^0$$

other systems  $\rightarrow$    
  $\rightarrow$  Octal   
  $\rightarrow$  Hexadecimal   
  $\rightarrow$  Binary

$[0, 7]$    
 Base = 8   
 Base = 16   
 Base = 2   
  $[0, 1]$

Binary  $\rightarrow$  Base is 2   
  $\rightarrow$  every digit is 0 or 1

$$1) (10110)_2 \quad 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 22 \\ 4 \quad 3 \quad 2 \quad 1 \quad 0$$

$$2) (1011010)_2 \quad 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ 6 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 0 \\ = 64 + 16 + 8 + 2 = 90$$

Convert decimal to binary.

{Repeated division}

37	1	
18	0	
9	1	
4	0	
2	0	
1	1	
0	STOP !!!	100101

bottom to top

45	1	
22	0	
11	1	
5	1	
2	0	
1	1	
0		101101

Addition of 2 decimal no.s.

$$\begin{array}{r} \textcircled{1} \quad \textcircled{1} \\ 7 \quad 8 \quad 9 \\ 1 \quad 4 \quad 2 \\ \hline 9 \quad 3 \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} \textcircled{1} \quad \textcircled{1} \\ 6 \quad 5 \quad 8 \\ 1 \quad 8 \quad 6 \\ \hline 8 \quad 4 \quad 4 \\ \hline \end{array}$$

Add 2 binary numbers.

$$\begin{array}{r} \textcircled{1} \quad \textcircled{1} \\ 1 \quad 0 \quad 1 \quad 1 \quad 0 \\ 0 \quad 0 \quad 1 \quad 1 \quad 1 \\ \hline 1 \quad 1 \quad 1 \quad 0 \quad 1 \end{array}$$

$$\begin{array}{r} 22 \\ 7 \\ \hline 29 \end{array}$$

$$\begin{array}{r} 2 \rightarrow 10 \\ 3 \rightarrow 11 \end{array}$$

$$\begin{array}{cccccc}
 & ① & & ① & ① & \\
 & 1 & 0 & 1 & 1 & 0 \\
 & \underline{1} & \underline{0} & \underline{1} & \underline{1} & \underline{1} \\
 1 & 0 & 1 & 1 & 0 & 1
 \end{array}$$

$$\rightarrow 101101$$

$$\begin{array}{lcl}
 \sim 0 & \rightarrow & 1 \\
 \sim 1 & \rightarrow & 0
 \end{array}$$

# Bitwise operators.

$\&$	$ $	$\wedge$	$\sim$	$\ll$	$\gg$
AND	OR	XOR	NOT	left shift	right shift
			toggle		
			<u>3mCT</u>		

$0 \rightarrow \text{false}$   
 $1 \rightarrow \text{true}$

same same  
 puppy thame  
 ↑

a	b	$a \& b$	$a   b$	$a \wedge b$	$\sim a$	$\sim b$
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	1
1	1	1	1	0	0	0

## Basic problems

$$29 \& 19 = 17$$

$$a = 29 \quad b = 19$$

4 3 2 1 0

a: 1 1 1 0 1

b: 1 0 0 1 1

$a \& b$  1 0 0 0 1  $\rightarrow 17$

$a | b$  1 1 1 1 1  $\rightarrow 31$

$a \wedge b$  0 1 1 1 0  $\rightarrow 14$

$$a = 20 \quad b = 45$$

a = 0 1 0 1 0 0

b = 1 0 1 1 0 1

1 1 1 0 0 1

$a \wedge b$

$\rightarrow 57$



Negative numbers  $\rightarrow$  2's complement

$-x$

- 1) get binary of  $x$
- 2) Take NOT of  $x$
- 3) Do  $+1$

$-5$

1) 0000101

2) 1111010 (1's complement)

3) 1111010

+ 0000001

---

1111011

2's complement

$$\begin{array}{r} \phantom{\rightarrow} \phantom{+1} \phantom{00000000} \\ \phantom{\rightarrow} \phantom{+1} \phantom{00000000} \\ \rightarrow \phantom{+1} \phantom{00000000} \\ +1 \phantom{00000000} \end{array}$$

-10

signed

unsigned (all true)

7	6	5	4	3	2	1	0
$-2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$

$$\begin{array}{ccccccc} & - & - & - & - & - & - \\ n-1 & n-2 & & & & & \\ -2^{n-1} & 2^{n-2} & & & & & \end{array}$$

$$2^{h-2} + 2^{h-3} + \dots + 2^2 + 2^1 + 2^0 = 2^{h-1} - 1$$



Range

$$[-2^{n-1}, 2^{n-1} - 1]$$

int (32 bit)

$$-2^{31}, 2^{31} - 1$$

$$\approx [-2 \times 10^9, 2 \times 10^9 - 1]$$

$-2147483648$

long (64 bit)

$$-2^{63}, 2^{63} - 1$$

$$\sim [-8 \times 10^{18}, 8 \times 10^{18} - 1]$$

## Constraints

int a =  $10^5$

int b =  $10^6$

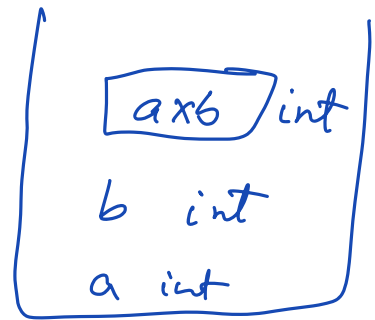
=  $10^{11}$

int c = a \* b

X

long c = a \* b

X

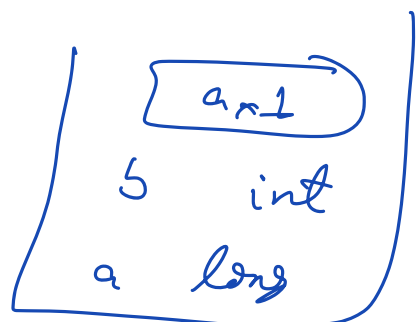


long c = long(a \* b)

X

long c = long(a) \* b

✓



Q Sum of array

$$1 \leq n \leq 10^5$$

$$1 \leq a[i] \leq 10^6$$

} sum  $\rightarrow 10^{11}$

$$\underbrace{10^6 \quad 10^6 \quad \dots \quad 10^6}_{10^5}$$

```
long sum = 0
for (i = 0 ; i < n ; i++) {
    sum += a[i]
}
return sum
```

long  
long int

{done}

0	0	4	5	6
0	1	2	3	4

$$\underbrace{17 \quad 17 \quad 17 \quad 17}_{11}$$

$$17 \times 11$$

$$\text{less} = 0$$

Pseudo code :

sort (arr)

// ascending order

int ans = 0

int less = 0

if (  $a[0] == 0$  ) // check  $a[0]$

ans++

for (  $i=1$  ;  $i < n$  ;  $i++$  ) {

if (  $a[i] == a[i-1]$  ) {

//  
else

less = i

if (  $a[i] == less$  )

ans++;

}

return ans;

1 1 4 5 6

TC:  $O(n \log n)$

SC:  $O(1)$