

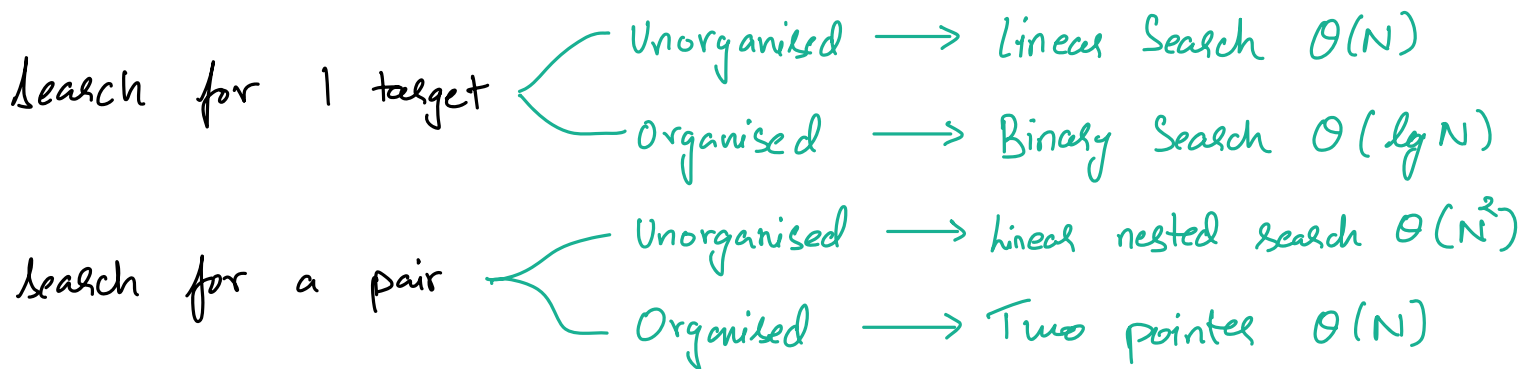
Two Pointers

Topics To Cover

- 1) Check Pair $\text{sum} = K$
- 2) Count pair $\text{sum} = K$
- 3) Pair diff. $= K$
- 4) Subarray with $\text{sum} = K$
- 5) Containers with most water.

Song: Girlfriend
- Avril Lavigne

Hi Everyone!!!



Q. Check pair sum
Given a sorted integer array & an int K . Check if there is any pair (i, j) such that $A[i] + A[j] = K$ & $i < j$ T/F

$A = \{1, 3, 5, 10, 20, 23, 30\}$

$K = 23$? True

$K = 30$? True

HS
3 1
5 10

BF: 1) Check for all pairs using 2 nested for loops
TC: $\Theta(N^2)$
SC: $\Theta(1)$

2) Hashset, save all values and try finding $K - A[i]$
TC: $\Theta(N)$
SC: $\Theta(N)$

3) Binary Search: Pick each ele and look for $K - A[i]$ using BS
TC: $\Theta(N \log N)$
SC: $\Theta(1)$

2 Pointers $\begin{cases} \rightarrow \text{Where to put pointers (both @ start/end or corners)} \\ \rightarrow \text{How to move pointers} \end{cases}$

$A = \{1, 3, 5, 10, 20, 23, 30\}$

\uparrow L
 \uparrow R

$K = 23$

① Put pointer at start and end.	L	R	Sum	
	1	30	31	$> K \rightarrow R--$
② if sum has to be decr than $R--$	1	23	24	$> K \rightarrow R--$
else $L++$	1	20	21	$< K \rightarrow L++$ <small>to incr sum</small>
	3	20	23	$\rightarrow \text{Ans} = \text{True}$

fn find - pair (Ar[N], K)

```

{
  L = 0 ; R = N-1;
  while (L < R)
  {
    if (Ar[L] + Ar[R] == K) return True;
    else if (Ar[L] + Ar[R] > K) R--;
    else L++;
  }
  return False;
}

```

TC: $\Theta(N)$
SC: $\Theta(1)$

Q. Check pair sum

Given a sorted integer array & an int K. Count all the pairs (i, j) such that

$$A[i] + A[j] == K \quad \&\& \quad i < j$$

$A = \{1, 3, 5, 10, 20, 23, 30\}$

$\underset{0}{1} \quad \underset{1}{3} \quad \underset{2}{5} \quad \underset{3}{10} \quad \underset{4}{20} \quad \underset{5}{23} \quad \underset{6}{30}$

$K = 30?$

Ans: 1
 $\{3, 27\}$

$K = 33?$

Ans: 2
 $\{1, 32\} \quad \{3, 30\}$

```

fn find-pair (Ar [N], K)
{
    count = 0;
    L = 0; R = N-1;
    while (L < R)
    {
        if (Ar[L] + Ar[R] == K) count++;
        if (Ar[L] + Ar[R] > K) R--;
        else L++;
    }
    return count;
}

```

// This only works for Non-duplicates.

A = { 1, 3, 3, 10, 20, 23, 30 } K = 33

count
111 = 3

A = { 1, 3, 10, 10, 20, 30, 30 } K = 33

1 = ~~1~~

① A = { 1, 3, 3, 10, 20, 30, 30 } K = 33

2 2
Count += 2 * 2 = 4

② Ar: [10, 10, 10, 10] so] K = 20

$${}^4C_2 \Rightarrow \frac{4 \times 3}{2} = 6$$

$${}^nC_2 = \frac{n(n-1)}{2}$$

```

fn count-sum (Ar [N], K)
{

```

L = 0; R = N-1;

Ans = 0;

while (L < R)

{

if (Ar[L] + Ar[R] == K)

{ if (Ar[L] == Ar[R])

cnt = (R - L + 1)

Ans += cnt * (cnt-1) / 2
return Ans;

(cnt C₂)

```
count-L = 0;  
for (i = L ; i < R ; i++)  
{  
    if (Ar[L] == Ar[i])  
        count-L++;  
    else  
        break;  
}
```

```
count-R = 0;  
for (i = R ; i > L ; i--)  
{  
    if (Ar[R] == Ar[i])  
        count-R++;  
    else  
        break;  
}
```

Ans += count-L * count-R;

```
R = R - count-R;  
L = L + count-L;
```

}

else if (Ar[L] + Ar[R] > K) R--;

else L++;

TC: $\Theta(N)$
SC: $\Theta(1)$

```
}  
return ans;  
}
```

Q. Given a sorted Array & an int K, Check if there is any pair (i, j) such that

$$A[j] - A[i] = K \quad \&\& (i < j) \quad (K > 0)$$

$A[j] > A[i]$

$A = \{-2, 0, 1, 3, 10, 20, 23\}$

$K = 9$?

Ans = True

$$10 - 1 = 9$$

$K = 15$?

Ans = False

Where to put the pointers

①

$A = \{-2, 0, 1, 3, 10, 20, 23\}$

$K = 9$

	\uparrow L		\uparrow R	
L	-2	R	23	diff 25
	-2		20	22
	0		23	23

observation $> K$

\swarrow R--

\swarrow L++

Ambiguity

②

pointers
① start

$A = \{-2, 0, 1, 3, 10, 20, 23\}$

$K = 9$

$R++ \rightarrow \text{diff} \uparrow$

$L++ \rightarrow \text{diff} \downarrow$

L	R	diff	observation
-2	0	2	$< K \rightarrow R++$
-2	1	3	$< K \rightarrow R++$
-2	3	5	$< K \rightarrow R++$
-2	10	12	$> K \rightarrow L++$
0	10	10	$> K \rightarrow L++$
1	10	9	$= K \rightarrow \text{TRUE}$

fn pair-diff (Ar[N], K)

L = 0 ; R = 1 ;

while (R < N)

if (Ar[R] - Ar[L] == K) return True ;

else if (Ar[R] - Ar[L] > K)

L++ ;

if (R == L)

R++ ;

else R++ ;

return False ;

TC: $O(N)$
SC: $O(1)$

Break : 10:22 - 10:30

Q. Given an array with +ve no. & integer K. Find a subarray with sum == K. If not possible return {-1, -1}.

A = { 1 3 10 5 23 3 }
0 1 2 3 4 5

K = 18 → { 1, 3 }

K = 20 → {-1, -1}

K > 0
Ar[i]

Approach → BF → check all subarray
TC: $O(N^3)$

↓
P/Sum → create p/sun and check all pairs
TC: $O(N^2)$
SC: $O(N)$

↓
Binary Search → P/Sun (sorted by default)

TC: $O(N \log N)$
SC: $O(N)$

$A = \{ \overset{L}{1} \underset{0}{}, \overset{L}{3} \underset{1}{}, \overset{L}{10} \underset{2}{}, \overset{R}{5} \underset{3}{}, \overset{R}{23} \underset{7}{}, \overset{R}{33} \underset{5}{} \}$
 $K = 18$

L	R	Sum	
0	0	1	$< K \rightarrow R++$
0	1	4	$< K \rightarrow R++$
0	2	14	$< K \rightarrow R++$
0	3	19	$> K \rightarrow L++$
1	3	18	$= K$

Ans

$\text{fn } \{ \text{subarray-sum} (\text{Ar}[N], \text{int } K)$
 $L = 0 ; R = 0 ; \text{Sum} = \text{Ar}[0];$

$\text{while } (R < N \ \&\& \ L < N)$
 $\{$
 $\quad \text{if } (\text{Sum} == K)$
 $\quad \quad \text{return } \{L, R\};$
 $\quad \text{else } \{ \text{if } (\text{Sum} < K)$
 $\quad \quad \{$
 $\quad \quad \quad R++;$
 $\quad \quad \quad \text{if } (R < N)$
 $\quad \quad \quad \quad \text{Sum} += \text{Ar}[R];$
 $\quad \quad \quad \}$
 $\quad \}$

else
 $\{$
 $\quad \text{Sum} -= \text{Ar}[L];$
 $\quad L++;$
 $\quad \}$

$\text{TC: } \Theta(N)$
 $\text{SC: } \Theta(1)$

$\}$
 $\text{return } \{-1, -1\}$
 $\}$

$$A = \{ \underset{0}{19} \underset{1}{3} \underset{2}{10} \underset{3}{5} \underset{4}{23} \overset{R}{\underset{5}{3}} \} \quad L$$

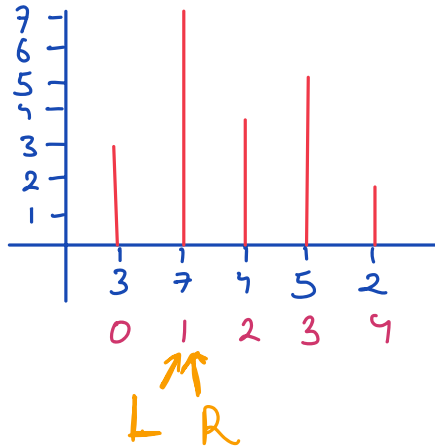
$$Sum = 19 \cancel{+ 3} \cancel{+ 10} \cancel{+ 5} \cancel{+ 23} \underset{18}{+ 3}$$

$$R = 18$$

Q. Containers with most water.

Given $ar[N]$ elements, $ar[i]$ represents height of a wall.
Find max water accumulated b/w any two walls.

$$Ar: [\underset{0}{3} \underset{1}{7} \underset{2}{4} \underset{3}{5} \underset{4}{2}]$$



select any 2 walls

$$\text{water cap} = \min(\overset{\text{height}}{Ar[L]}, Ar[R]) * R - L$$

$$Ans = 10$$

Index	L	R	Water Cap ($\min(Ar[L], Ar[R]) * R - L$)
→ 0	0	4	$2 * 4 = 8$
	0	3	$3 * 3 = 9$
	1	3	$5 * 2 = 10 \rightarrow Ans$
	1	2	$4 * 1 = 4$
	1	1	→ exit

$L = 0; R = N - 1; ans = 0;$
while ($L < R$)
{

$h = \min(Ar[L], Ar[R]);$

$w = R - L;$

$cap = h * w$

$ans = \max(ans, cap)$

TC: $\Theta(N)$
SC: $\Theta(1)$

if ($Ar[L] < Ar[R]$) $L++$;


```
    } else R--;  
return ans;
```