a	b	a&b	a^b	a|b	~a
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	0	1	0

Properties

15 Dec
next Friday
Contest
Arrays → 3 classes
Bit manip → 2 classes

1) A&1 → A is even 0
→ A is odd 1

A&A = A   17 & 17 = 17

A&0 = 0

0 01
0 00

2&1 =
11   3&1 =
01
01

2) A | 0 = A
A | A = A

3) A^0 = A
A^A = 0

A&B = B&A
A^B = B^A
A|B = B|A

(A&B) & C = A & (B&C)
(A|B) | C = A | (B|C)
(A^B) ^ C = A ^ (B^C)

~~a~~ ^ ~~b~~ ^ ~~a~~ ^ d ^ ~~b~~

~~1~~ ~~3~~ ~~5~~ ~~3~~ ② ~~1~~ ~~5~~

left shift <<

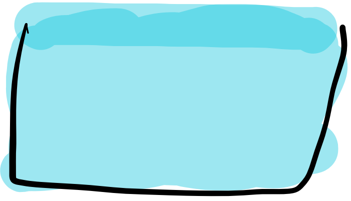
8 bit numbers.

	7	6	5	4	3	2	1	0	
a=10	0	0	0	0	1	0	1	0	10
a<<1 ig	0	0	0	1	0	1	0	0	20
a<<2 ig	0	0	1	0	1	0	0	0	40
a<<3	0	1	0	1	0	0	0	0	80
a<<4	1	0	1	0	0	0	0	0	160
a<<5	0	1	0	0	0	0	0	0	64

overflow : Exceeded what we can assign

→ We lose data

We exceeded capacity



General (assume no overflow)

$a << 1 = a \times 2^1$

$a << 2 = a \times 2^2$

$a << 3 = a \times 2^3$

$\vdots$

$a << N = a \times 2^N$

$5 << N = 5 \times 2^N$

- $1 << N = 1 \times 2^N = 2^N$

$1 << N$

2^N

$\downarrow$

$$\begin{array}{cccccccccc} & & & N & & \cdot\cdot & & \cdot\cdot 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$2^4 \quad \begin{array}{cccccc} 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array}$$

2^N → only N^{th} bit $= 1$

Right Shift

8 bit	7	6	5	4	3	2	1	0	
$x=10$	0	0	0	0	1	0	1	0	10
$x>>1$	0	0	0	0	0	1	0	1	5
$x>>2$	0	0	0	0	0	0	1	0	2
$x>>3$	0	0	0	0	0	0	0	1	1
$x>>4$									0
$x>>5$									0

Note: Here overflow (loss of data) does not happen

$$x>>1 = x/2^1$$
$$x>>2 = x/2^2$$
$$\vdots$$
$$x>>a = x/2^a$$

int ⇒ 4 bytes = 32 bits [0, 31]

long ⇒ 8 bytes = 64 bits [0, 63]

Power of <<

1) N | 1<<i

SET the ith bit

```
       ... i ... 2 1 0
     . ... x ........
OR     0001 000000
       ..... 1 ......
```

2) N ^ 1<<i

TOGGLE the ith bit

```
         ... i ... 2 1 0
xor    . ... x ........
         0001 000000
         ..... opp(x) ......
```

0^1 = 1
1^1 = 0

3) N & 1<<i

```
         ... i ... 2 1 0
and    . ... x ........
         0001 000000
         000x000000
```

if ith bit is OFF

N & 1<<i = 0

0&1 = 0
1&1 = 1

else

N & 1<<i = 1<<i

Q1 Given number (int) N, check if the i^{th} bit is ON or OFF

$N = 21$

$i = 2$

4	3	2	1	0
1	0	1	0	1

ON

$N = 34$

$i = 3$

5	4	3	2	1	0
1	0	0	0	1	0

OFF

● How to check if i^{th} bit is ON or OFF

Obs :

$$\begin{tikzcd}
 & N \,\&\, 1<<i \arrow[dl, "i^{th} \text{ bit OFF}"'] \arrow[dr, "i^{th} \text{ bit ON}"] & \\
0 & & 1<<i
\end{tikzcd}$$

```
bool checkBit (int N , int i) {

  if ( N & (1<<i) != 0)
       return true
  else
       return false
}
```

TC: O(1)
SC: O(1)

Left shift & right shift is O(1) operation

Q2 Count number of bits = 1 . Input is int

N = 12   1 1 0 0   ans = 2

Idea: int 32 bit [0, 31]

```
ans = 0
for ( bit = 0 ; bit ≤ 31 ; bit ++ ) {
        if ( checkbit ( N, bit ) == true )
                ans ++
}
return ans
```

TC: O(1)

i = 1

```
        3 2 1 0
N = 5   0 1 0 1        ans = 5
```

Q Unset the i^{th} bit

Eg N=6
i=1

$$\begin{array}{cccc} 3 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}$$

$\Rightarrow$ 0 1 0 0   ans = 4

Idea: If i^{th} bit = 0   dont do anything

If i^{th} bit = 1   TOGGLE i^{th} bit

How to make bit 1 → 0

^   1<<i

Code

```
if ( checkbit(N,i) == true )      //i^th bit =1
        N= N ^ 1<<i               // toggle

return N
```

Q Make a number ⇒
A 0's then B 1's then C 0's

Eg A = 4
B = 3
C = 2

$\underbrace{0\,0\,0\,0}_{A}\ \underbrace{1\,1\,1}_{B}\ \underbrace{0\,0}_{C}$

ans = 28

Obs 1 A is USELESS

$B-1 \qquad\qquad 0$

$C+B-1 \ldots\ \ldots\ C+1 \quad C \quad C-1 \ldots\ 2 \quad 1 \quad 0$

$\underbrace{1\ 1\ 1\ 1\ 1}_{B}\ \ \underbrace{0\ \ldots\ 0\ 0\ \ 0}_{C}$

0, 4
1, 5
5, 9

set bits from C to C+B-1 = 1

```
long ans = 0

  for( bit = C ; bit ≤ B+C-1 ; bit++)
  |     ans = ans | 1<<bit
  }
return ans
```

TC = O(1)

$$\underbrace{\overset{C+B-1}{1}\ \dots\ 1\ 1\ \overset{C+1}{1}\ \overset{C}{1}}_{B}\quad \underbrace{\overset{C-1}{0}\ \dots\ \overset{2}{0}\ \overset{1}{0}\ \overset{0}{0}}_{C}$$

$B \rightarrow 1's$

$C\ 0's$

$$\begin{array}{ccccccc} B & B-1 & --- & 3 & 2 & 1 & 0 \\ & 1 & & 1 & 1 & 1 & 1 \\ & & & & + & 1 & \\ \hline 1 & 0 & & 0 & 0 & 0 & 0 \end{array}$$

$N << C$

$5 \rightarrow 101$

$5 << 3$ 101000

$2^B - 1$

$(1 << B - 1) << C$

$mynum + 1 = 2^B$

<done>

$mynum = 2^B - 1$

$(1 << B - 1) << C$

(1<<B - 1) << C

$$\underbrace{1\,1\,1\,1\,1}_{B}\quad\underbrace{0\,0\,0\,0\,0\,0}_{C}$$

```
long solve (A, B, C) {
    return (1<<B -1) <<C
}
```