C12 - 3.1 - Long Division $R = 0$ Notes

$\boxed{\begin{array}{l} \textit{Goes Into} \\ \textit{Multiply} \\ \textit{Subtract} \\ \textit{Bring Down} \\ \textit{Repeat} \end{array}}$

$$\begin{array}{rl} 16 & \\ 4\overline{)64} & \\ -\underline{4} & \\ 24 & \textit{Bring down} \\ -\underline{24} & \\ 0 & \end{array}$$

remainder

$\boxed{\begin{array}{r} \textit{quotient} \\ \textit{divisor}\ \overline{)\ \textit{dividend}} \end{array}}$

$$\frac{64}{4} = 16 \qquad 64 = 4 \times 16$$

$\boxed{\dfrac{dividend}{divisor} = quotient}$ $\qquad$ $\boxed{dividend = (quotient)(divisor)}$

x *times what is* x^2

x *times what is* $2x$

$\boxed{\textit{The } 3 \textit{ is only for multiplication}}$

$$\begin{array}{rl} x + 2 & \\ x + 3\,\overline{)\,x^2 + 5x + 6} & \\ -\ \underline{x^2 + 3x\phantom{{}+6}} & \\ 2x + 6 & \\ -\ \underline{2x + 6} & \\ 0 & \end{array}$$

remainder

$x \times x = x^2$ $\qquad$ $3 \times x = 3x$

Bring down

$$\frac{x^2 + 5x + 6}{x + 3} = x + 2 \qquad x^2 + 5x + 6 = (x + 2)(x + 3)$$

$\boxed{\dfrac{P(x)}{x - a} = Q(x)}$ $\qquad$ $\boxed{P(x) = Q(x)(x - a)}$

C12 - 3.1 - Long Division Notes

$$\begin{array}{r}
16 \\
4\overline{)65} \\
-\ 4 \\
\hline
25 \\
-\ 24 \\
\hline
1
\end{array}$$

Bring down

remainder

$$\text{divisor}\ \overline{)\ \text{dividend}\ }^{\ \text{quotient}}$$

$$\frac{65}{4} = 16 + \frac{1}{4}$$

$$65 = 4 \times 16 + 1$$

$$\boxed{\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}}$$

$$\boxed{dividend = (quotient)(divisor) + remainder}$$

$$\begin{array}{r}
x + 2 \\
x + 3\ \overline{)\ x^2 + 5x + 9} \\
-\quad x^2 + 3x \\
\hline
2x + 9 \\
-\quad 2x + 6 \\
\hline
3
\end{array}$$

$(x + 3) \times x$

$(x + 3) \times 2$

remainder

$$\frac{x^2 + 5x + 9}{x + 3} = x + 2 + \frac{3}{x + 3}$$

$$x^2 + 5x + 9 = (x + 2)(x + 3) + 3$$

$$\boxed{\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}}$$

$$\boxed{P(x) = Q(x)(x - a) + R}$$

C12 - 3.1 - Synthetic Division $R = 0$ Notes

$$\frac{x^3 + x^2 - 8x + 4}{x - 2}$$

$x - 2 = 0$
$x = 2$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ + & & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3 + 1x^2 - 8x + 4$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ + & \downarrow & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

1) Bring down the first coefficient
2) $(2) \times 1 = 2$
3) $1 + 2 = 3$
4) Repeat last two steps.

$x^2 + 3x - 2 \quad R = 0$

$$\boxed{\frac{x^3 + x^2 - 8x + 4}{x - 2} = x^2 + 3x - 2}$$

$$\boxed{x^3 + x^2 - 8x + 4 = (x^2+3x - 2)(x - 2)}$$

$$\boxed{\frac{dividend}{divisor} = quotient}$$

$$\boxed{dividend = (quotient)(divisor)}$$

$$\boxed{\frac{24}{4} = 6}$$

$$\boxed{24 = 6 \times 4}$$

$$\frac{x^3 + 2x^2 - 5x - 6}{x + 1}$$

$x + 1 = 0$
$x = -1$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ + & & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3 + 2x^2 - 5x - 6$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ + & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$1x^2 + 1x - 6 \quad R = 0$

$x^2+x - 6$
$(x + 3)(x - 2)$

Factor

$$\frac{x^3 + 2x^2 - 5x - 6}{x + 1} = (x + 3)(x - 2)$$

$$x^3 + 2x^2 - 5x - 6 = (x + 3)(x - 2)(x + 1)$$

$$\boxed{\frac{P(x)}{x - a} = Q(x)}$$

$$\boxed{P(x) = Q(x)(x - a)}$$

C12 - 3.1 - Synthetic Division Notes

Graph tov

$$\frac{x^3 + x^2 - 8x + 7}{x - 2}$$

$x - 2 = 0$
$x = 2$

Set denominator equal to zero and solve.
$Denominator = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 7 \\ + & & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^3 + x^2 - 8x + 4$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 7 \\ + & & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

remainder

1) Bring down the first coefficient
2) $(2) \times 1 = 2$
3) $1 + 2 = 3$
4) Repeat last two steps.

$x^2 + 3x - 2 \qquad R = 3$

The remainder is the y value when $x = 2$
(2,3)

$f(2) = (2)^3 + (2)^2 - 8(2) + 7$
$f(2) = 8 + 4 - 16 + 7$
$f(2) = 3$

$$\boxed{\frac{x^3 + x^2 - 8x + 6}{x - 2} = x^2 + 3x - 2 + \frac{3}{x - 2}}$$

$$\boxed{x^3 + x^2 - 8x + 6 = (x^2 + 3x - 2)(x - 2) + 3}$$

$$\boxed{\frac{dividend}{divisor} = quotient + \frac{remainder}{divisor}}$$

$$\boxed{dividend = (quotient) \times (divisor) + remainder}$$

C12 - 3.1 - Synthetic Division Gap Notes

$$\frac{x^2 - 4}{x + 2}$$

$x + 2 = 0$ Set denominator equal to zero and solve.
$x = -2$ $Denominator = 0$

$$\begin{array}{r|rrr} -2 & 1 & 0 & -4 \\ + & & & \\ \hline \end{array}$$

Place that number to the left.
Write the coefficients. $1x^2 + 0x - 4$

$$\begin{array}{r|rrr} -2 & 1 & 0 & -4 \\ + & \downarrow & -2 & 4 \\ \hline & 1 & -2 & 0 \end{array}$$

1) Bring down the first coefficient
2) $(-2) \times 1 = -2$
3) $0 + (-2) = -2$
4) Repeat last two steps.

$1x - 2$ $R = 0$

$x - 2$

$$\boxed{\frac{x^2 - 4}{x + 2} = (x - 2)}$$

$$\boxed{\frac{x^2 - 4}{x + 2} = (x - 2)(x + 2)}$$

$$\frac{x^3 + 2x - 12}{x - 2}$$

$$\boxed{1x^3 + 0x^2 + 2x - 12}$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 2 & -12 \\ + & & & & \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 2 & -12 \\ + & \downarrow & 2 & 4. & 12 \\ \hline & 1 & 2 & 6 & 0 \end{array}$$

$1x^2 + 2x + 6$ $R = 0$

$$\boxed{\frac{x^3 + 2x - 12}{x - 2} = x^2 + 2x + 6}$$

$$\boxed{x^3 + 2x - 12 = (x^2+2x + 6)(x - 2)}$$

$$\frac{x^3 + 2x^2 - 6x - 12}{x + 2}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -6 & -12 \\ + & & & & \\ \hline \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -6 & -12 \\ + & \downarrow & -2 & 0 & 12 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

$1x^2 + 0x - 6$ $R: 0$

$x^2 - 6$ $R: 0$

$$\boxed{\frac{x^3 + 2x^2 - 4x + 8}{x + 2} = x^2 - 6}$$

$$\boxed{x^3 + 2x^2 - 4x + 8 = (x^2 - 6)(x + 2)}$$

C12 - 3.2 - Factor/Remainder Theorem Notes

$Factor\ Theorem$	If $(x-a)$ is a factor of $f(x)$, then:	$f(a) = 0$

Is $(x-2)$ a factor of $f(x) = x^3 + x^2 - 8x + 4$?

$f(x) = x^3 + x^2 - 8x + 4$
$f(x) = (2)^3 + (2)^2 - 8(2) + 4$
$f(2) = 8 + 4 - 16 + 4$
$f(2) = 0$ ← Remainder = 0
(2,0) ← $x - intercept$

$\boxed{\begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}}$

$Synthetic\ Division$

$\boxed{(x-2)\text{Is a Factor}}$

$$\frac{x^3 + x^2 - 8x + 4}{x - 2}$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 4 \\ & \downarrow & 2 & 6 & -4 \\ \hline + & 1 & 3 & -2 & 0 \end{array}$$

← Remainder = 0

$Remainder\ Theorem$	If $(x-a)$ is not a factor of $f(x)$, then:	$f(a) = remainder$

Is $(x-2)$ a factor of $f(x) = x^3 + x^2 - 8x + 5$?

$f(x) = x^3 + x^2 - 8x + 5$
$f(x) = (2)^3 + (2)^2 - 8(2) + 5$
$f(2) = 8 + 4 - 16 + 5$
$f(2) = 1$ ← Remainder = 1

$\boxed{\begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}}$

(2,1)

$\boxed{(x-2)\ is\ \text{Not a Factor!}}$

$Synthetic\ Division$

$$\frac{x^3 + x^2 - 8x + 4}{x - 2}$$

$$\begin{array}{r|rrrr} & 1 & 1 & -8 & 5 \\ 2 & \downarrow & 2 & 6 & -4 \\ \hline + & 1 & 3 & -2 & 1 \end{array}$$

← Remainder = 1

C12 - 3.2 - Potential Factors Notes $\pm \frac{d}{a}$

$f(x) = x^3 + x^2 - 8x + 4$ $\quad$ $Potential\ Factors$: $\pm 1, \pm 2, \pm 4$ $\quad$ $factors\ of$ "d"

Solve by inspection

$f(1) = (1)^3 + (1)^2 - 8(1) + 4 = -2$
$f(-1) = (-1)^3 + (-1)^2 - 8(-1) + 4 = 12$
$f(2) = (2)^3 + (2)^2 - 8(2) + 4 = 0$

$(x - 2)$ *is a factor* $\quad$ (2,0)

$$\begin{array}{c|cccc} 2 & 1 & 1 & -8 & 4 \\ + & \downarrow & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 0 \end{array}$$

$\boldsymbol{f(x) = 3x^2 + 5x - 2}$ $\quad$ $Potential\ Factors$: $\pm 2, \pm 1, \pm \frac{2}{3}, \pm \frac{1}{3}$ $\quad$ $factors\ of$ "c" $and\ \frac{factors\ of\ "c"}{factors\ of\ "a"}$

Solve by inspection

$f(-1) = 3(-1)^2 + 5(-1) - 2 = -4$
$f(1) = 3(1)^2 + 5(1) - 2 = 6$
$f(2) = 3(2)^2 + 5(2) - 2 = 20$
$f(-2) = 3(-2)^2 + 5(-2) - 2 = 0$

$(x + 2)$ *is a factor*

$$\begin{array}{c|ccc} -2 & 3 & 5 & -2 \\ + & \downarrow & -6 & 2 \\ \hline & 3 & -1 & 0 \end{array}$$

The other factor is: $(3x - 1)$

$(3x-1)(x+2)$

$x = \frac{1}{3}$ $\quad$ $x = -2$

C12 - 3.2 - Factoring Trinomials Notes

$\boldsymbol{f(x) = x^2 - 6x + 5}$ Potential Factors: Factors of c = $\pm 5\ and\ \pm\ 1$

$$\begin{array}{l|l} a & Synthetic = f(a) = x - int\ (a, 0) \\ \hline + & Factor = (x - a) \end{array}$$

Solve by inspection.

$f(1) = 1^2 - 6(1) + 5 \quad = 0$ $(x - 1)\, is\ a\ factor.$ Stop here if you want   (1,0) $x - int$

$f(-1) = (-1)^2 - 6(-1) + 5 = 12$

$f(5) = 5^2 - 6(5) + 5 \quad = 0$ $(x - 5)\ is\ a\ factor$ (5,0)

$f(-5) = (-5)^2 - 6(-5) + 5 = 60$

Do synthetic division with $\boldsymbol{1\ or\ 5}$ $x^2 - 6x + 5$

$$\begin{array}{c|ccc} 1 & 1 & -6 & 5 \\ & \downarrow & 1 & -5 \\ \hline + & 1 & -5 & 0 \end{array} \qquad \begin{array}{c|ccc} 5 & 1 & -6 & 5 \\ & \downarrow & 5 & -5 \\ \hline + & 1 & -1 & 0 \end{array}$$

$x - 5$ $x - 1$

$$\frac{x^2 - 6x + 5}{x - 1} = x - 5 \qquad \boxed{x^2 - 6x + 5 = (x - 5)(x - 1)} \qquad \frac{x^2 - 6x + 5}{x - 5} = x - 1$$

Calc: $\boldsymbol{x^2 - 6x + 5}$

1 Store x

TI84 up up Enter Enter

or

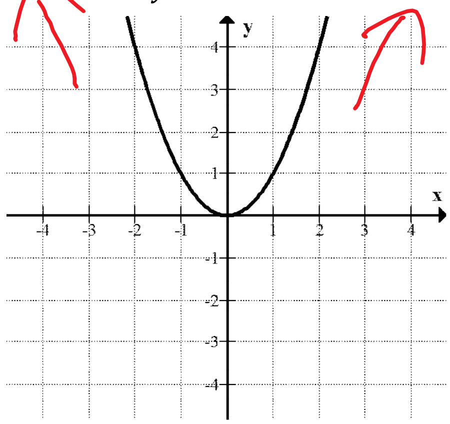
TI83 2nd Entry 2nd Entry Enter

Or

Graph and find x-intercepts

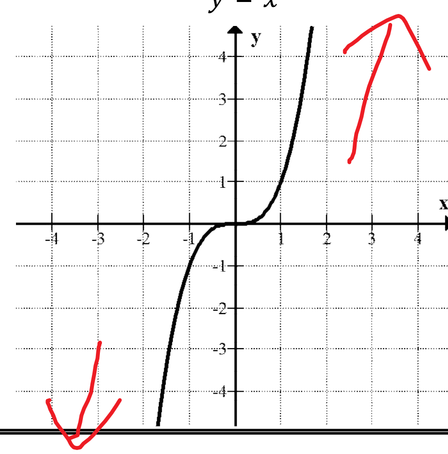
$+x^{even}$

$y = x^2$



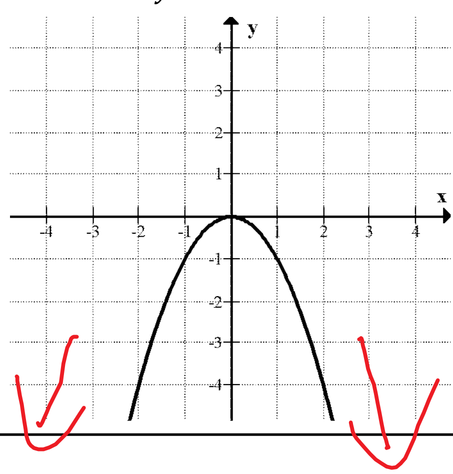
$+x^{odd}$

$y = x^3$



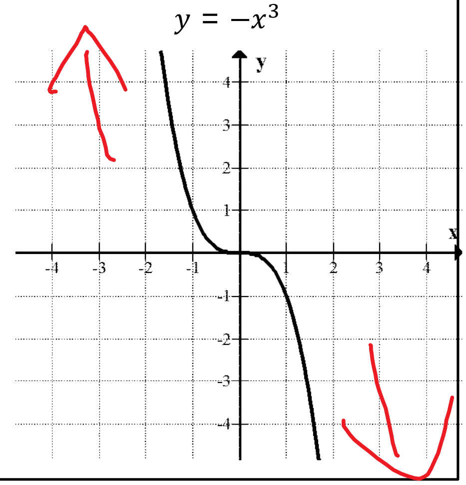
$-x^{even}$

$y = -x^2$



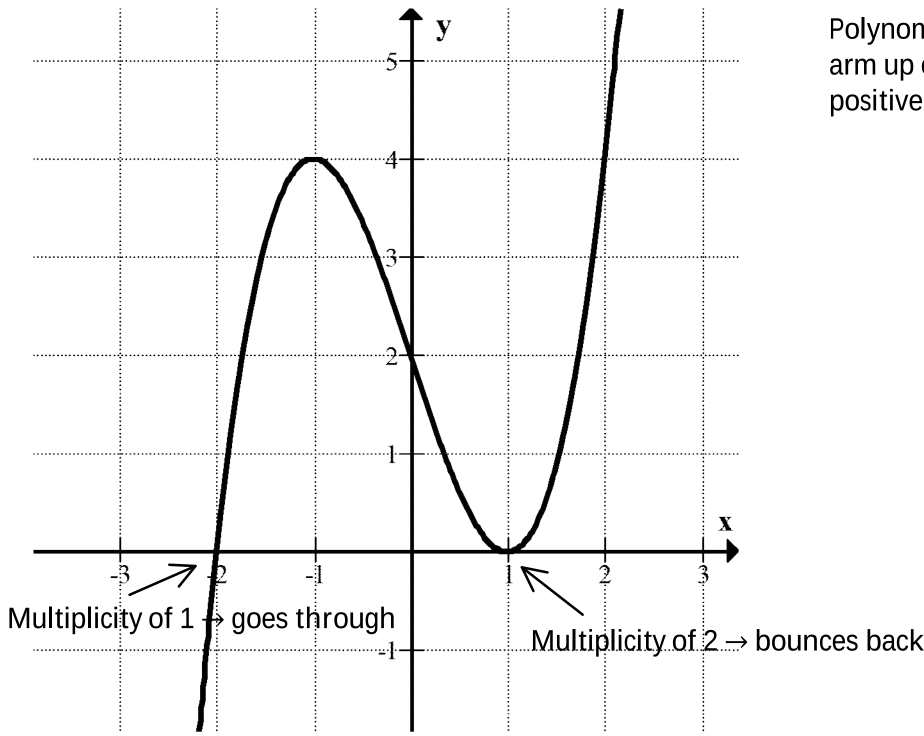
$-x^{odd}$

$y = -x^3$



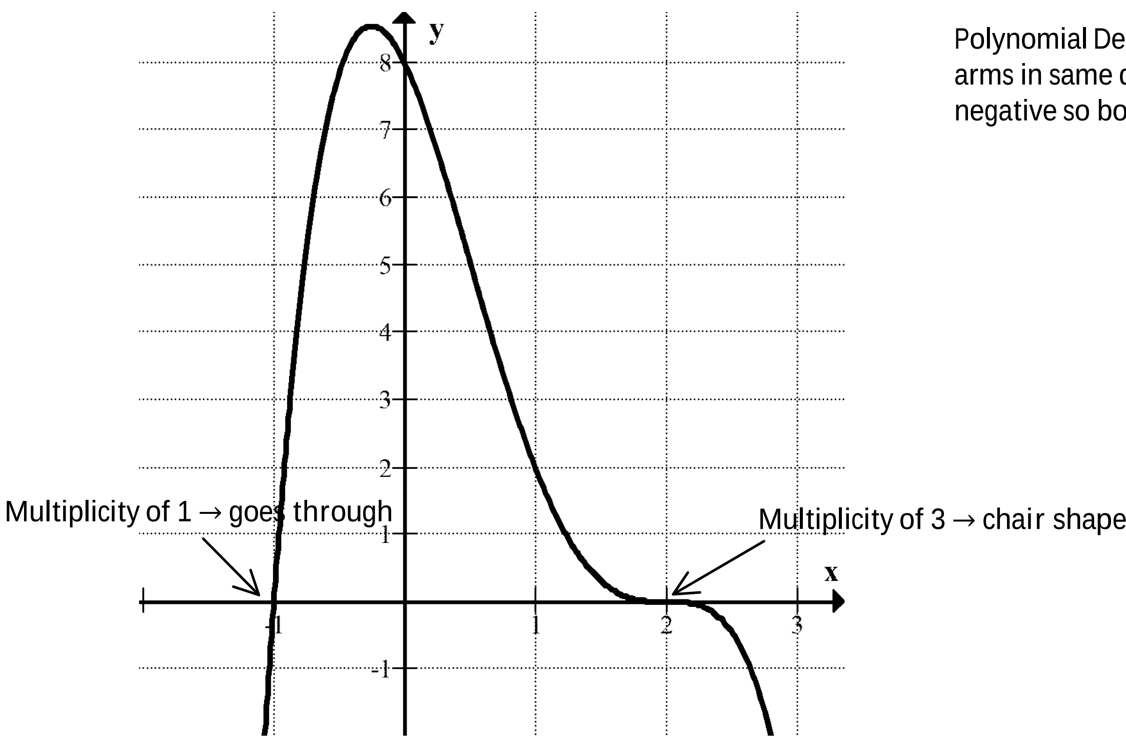
C12 - 3.4 - Multiplicity Graph Notes

$y = (x+2)^1(x-1)^2$



Polynomial Degree of 3, one arm up one arm down. Is positive so right arm up.

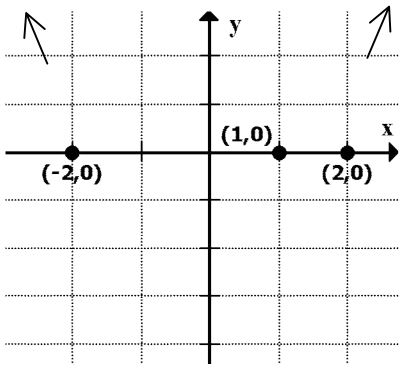
$-(x+1)(x-2)^3$



Polynomial Degree of 4, both arms in same direction. Is negative so both arms down.

C12 - 3.4 - Graph $(x-2)^2(x-1)(x+2)^3$ Notes

$(x-2)^2(x-1)(x+2)^3$



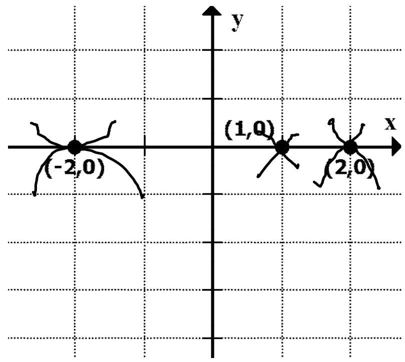
Steps to Graph

1. x-ints
2. End Behavior
3. Multiplicity
4. y-ints
5. Graph

1.

$x-2=0$	$x-1=0$	$x+2=0$
$x=2$	$x=1$	$x=-2$
$(0{,}2)$	$(0{,}1)$	$(0,-2)$

2. Positive: U opening upward $\quad (x)^2(x)(x)^3 = +x^6$

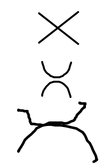


3.

$x = 1 \rightarrow degree\ 1 \quad Straight\ through$

$x = 2 \rightarrow degree\ 2 \quad U-shape$

$x = -2 \rightarrow degree\ 3 \quad Chair\ shape$



4.

$y = (x-2)^2(x-1)(x+2)^3$

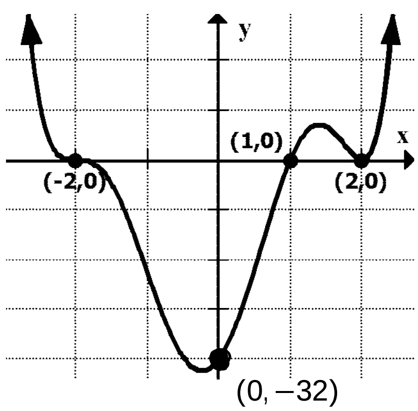
$y = (0-2)^2(0-1)(0+2)^3$

$y = (-2)^2(-1)(2)^3$

$y = 4(-1)(8)$

$y = -32$

$y\text{-}int$: $(0,-32)$



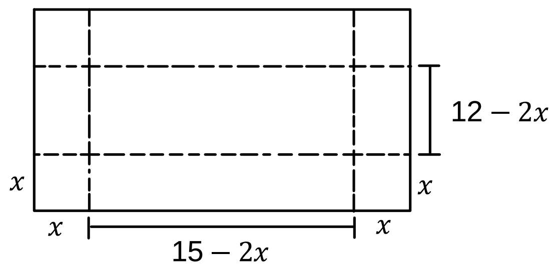
5. Graph

C12 - 3.5 - Open Rectangular Box Cut Side x Notes

Copied 102

An open rectangular box is made by cutting equal integer lengths from each corner of the 12 cm by 15 cm rectangular piece of cardboard, then folding up the sides. Find the length of the square that must be cut from each corner so the box has a volume of 162. And find Max Volume.

$let\ x = length\ to\ cut$



$$
\begin{aligned}
Volume &= length \times width \times height\\
162 &= (12 - 2x)(15 - 2x)(x)\\
162 &= 180x - 54x^2 + 4x^3\\
0 &= 4x^3 - 54x^2 + 180x - 162\\
0 &= 2x^3 - 27x^2 + 90x - 81
\end{aligned}
$$

Potential Factors: The factors of 81: ~~±27~~, ~~±9~~, ±3, ±1

Solve by inspection:

$Check: x = 1, 3$

$$
\begin{aligned}
f(x) &= 2x^3 - 27x^2 + 90x - 81\\
f(3) &= 2(3)^3 - 27(3)^2 + 90(3) - 81\\
f(3) &= 54 - 243 + 270 - 81 \qquad = 0
\end{aligned}
$$

$$
\begin{array}{r|rrrr}
3 & 2 & -27 & 90 & -81\\
+ & & 6 & -63 & 81\\
\hline
 & 2 & -21 & 27 & 0
\end{array}
$$

$2x^2 - 21x + 27$

$\therefore x = 3\ cm$

$2x^2 - 21x + 27$
$(2x - 3)(x - 9)$

$\cancel{x = 1.5}$ $\cancel{x = 9}$

We need to reject 6 and greater (and the negatives), so we don't get negatives lengths.