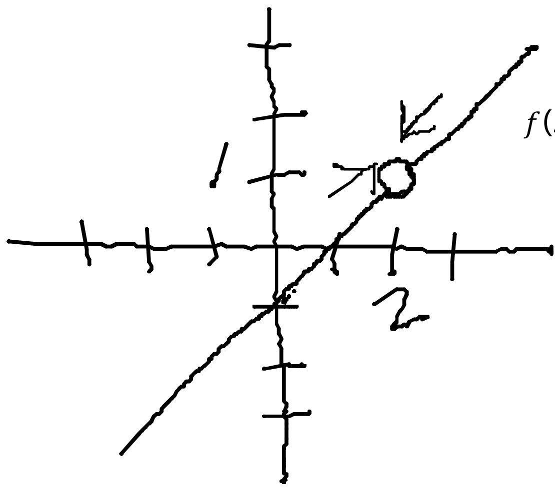
C12 - 1.1 - Limits Notes

$$f(x) = \frac{(x-1)(x-2)}{(x-2)}$$

Limit: What y is approaching.

What is y approaching as x approaches 2? $\lim_{x \to 2} f(x) = ?$



$f(x) = x - 1 \quad ; x \neq 2$

$f(2) = DNE$

x	y
1.9	.9
1.999	.999
2	DNE
2.001	1.001
2.1	1.1

$\lim_{x \to 2} f(x) = 1$

The Limit of $f(x)$, as x approaches 2, equals 1.

y *approaches* 1 *as x approaches* 2.

$\lim_{x \to 2^-} f(x) = 1$ *and* $\lim_{x \to 2^+} f(x) = 1$

Left Hand Limit = Right Hand Limit

$\lim_{x \to c} f(x) = L$ *The Limit of $f(x)$, as x approaches c, equals L*

One Sided Limits

$\lim_{x \to c^+} f(x) = L$ *The Limit of $f(x)$, as x approaches c, from the positive side (right), equals L.*

$\lim_{x \to c^-} f(x) = L$ *The Limit of $f(x)$, as x approaches c, from the negative side (left), equals L*

Limit Exists if and only if: *Left hand Limit = Right Hand Limit*

$\lim_{x \to c} f(x) = L$ $\lim_{x \to c^-} f(x) = L$ *and* $\lim_{x \to c^+} f(x) = L$

or

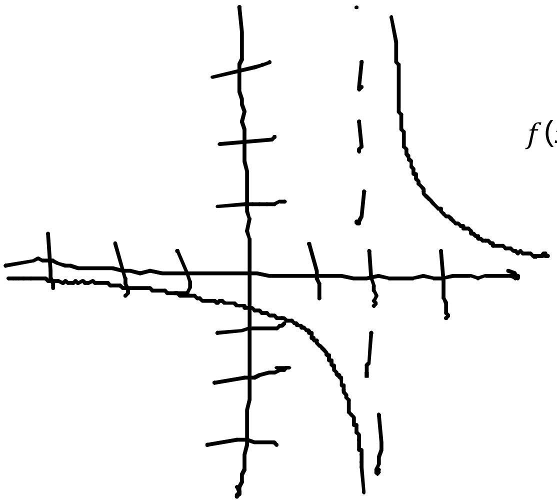
Limit Does Not Exist

$\lim_{x \to c} f(x) = DNE$

C12 - 1.1 - DNE Limit Notes

What is y approaching as x approaches 2?

$$\lim_{x \to 2} f(x) = ?$$



$$f(x) = \frac{1}{x-2}$$

x	y
1.9	-10
1.999	-1000
2	DNE
2.001	1000
2.1	10

DNE

$$\lim_{x \to 2} f(x) = DNE$$

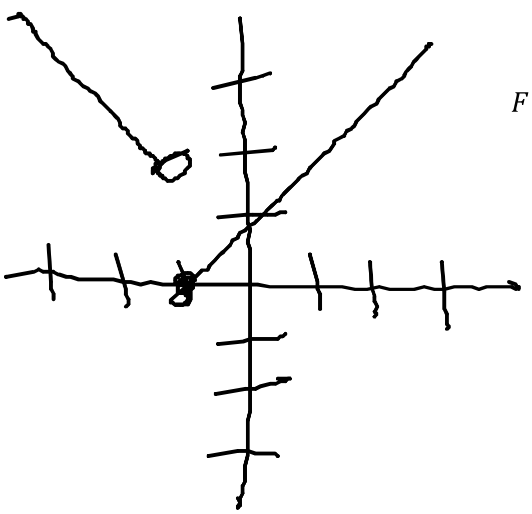
The Limit of $f(x)$, *as x approaches* 2, *Does Not Exist*

$$\lim_{x \to 2^-} f(x) = -\infty \quad and \quad \lim_{x \to 2^+} f(x) = +\infty$$

Left Hand Limit $\neq$ *Right Hand Limit*

What is y approaching as x approaches -1?

$$\lim_{x \to -1} f(x) = ?$$



$$F(x) = \begin{cases} -x + 1 & ; x < 1 \\ x + 1 & ; x \geq -1 \end{cases}$$

x	y
-1.1	2.1
-1.001	2.001
-1	0
-.999	.001
-.9	.1

DNE

$$\lim_{x \to -1} f(x) = DNE$$

The Limit of $f(x)$, *as x approaches* -1, *Does Not Exist*

$$\lim_{x \to -1^-} f(x) = 2 \quad and \quad \lim_{x \to -1^+} f(x) = 0$$

Left Hand Limit $\neq$ *Right Hand Limit*

$x \to -1$

$$\lim_{x \to -1^-} f(x) = 2 \quad and \quad \lim_{x \to -1^+} f(x) = 0$$

$$Left\ Hand\ Limit \neq Right\ Hand\ Limit$$

C12 - 1.2 - Limits Notes

Find the Limits

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}} \times \boxed{\frac{3+\sqrt{x}}{3+\sqrt{x}}}$$ ← Conjugate

$$\lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$$

$(3-\sqrt{x})(3+\sqrt{x}) =$

$9 + \cancel{3\sqrt{x}} - \cancel{3\sqrt{x}} - x =$

$9 - x$

Foil

$$\lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{9-x}$$

$$\lim_{x \to 9} \frac{(x-9)(3+\sqrt{x})}{-(x-9)}$$ GCF = -1

$$\lim_{x \to 9} \frac{\cancel{(x-9)}(3+\sqrt{x})}{-\cancel{(x-9)}}$$ Simplify

$$\lim_{x \to 9} \frac{(3+\sqrt{x})}{-1}$$

$$\lim_{x \to 9} -3-\sqrt{x}$$

$-3-3$ Substitute

-6

$$\lim_{x \to 0} \frac{\frac{1}{x+3}-\frac{1}{3}}{x}$$

$$\lim_{x \to 0} \frac{\frac{3-(x+3)}{3(x+3)}}{\frac{x}{1}}$$ $LCD = 3(x+3)$

$$\lim_{x \to 0} \frac{\frac{-x}{3(x+3)}}{\frac{x}{1}}$$ Simplify

$$\lim_{x \to 0} \frac{-x}{3(x+3)} \times \frac{1}{x}$$ Flip and Multiply

$$\lim_{x \to 0} -\frac{1}{3(x+3)}$$ Simplify

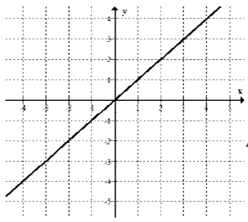
$-\frac{1}{9}$ Substitute

C12 - 1.3 - Horizontal Asymptotes Cases Notes

Case 1: (No Horizontal asymptote)

You may cross a horizontal asymptote

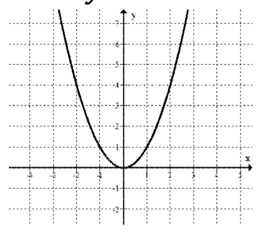
$\boldsymbol{y = x}$ $\quad y = \dfrac{x^2}{x}$



HA: None

$Range: y \in R$

$\boldsymbol{y = x^2}$



HA: None

$Range: y \geq 0$

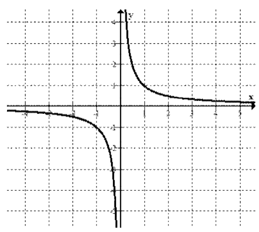
If the exponent of x is higher on the top than the bottom, no horizontal asymptote.

Unless there is a slant.

Case 2: (Horizontal Asymptote at $y = 0$)

$\boldsymbol{y = \dfrac{1}{x}}$

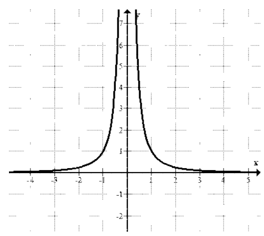
$y = \dfrac{x}{x^2}$



HA: $y = 0$

$Range: y \neq 0$

$\boldsymbol{y = \dfrac{1}{x^2}}$



HA: $y = 0$

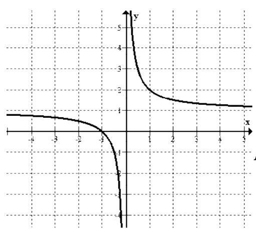
$Range: y > 0$

If the exponent of x is higher on the bottom, HA: $y = 0$

Modified Case 2: (Horizontal Asymptote at $y = c$)

$\boldsymbol{y = \dfrac{1}{x} + c}$

$y = \dfrac{1}{x} + 1$

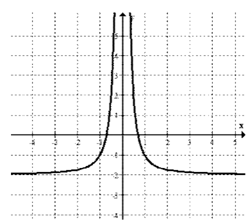


HA: $y = 1$

$Range: y \neq 1$

$\boldsymbol{y = \dfrac{1}{x^2} + c}$

$y = \dfrac{1}{x^2} - 2$



HA: $y = -2$

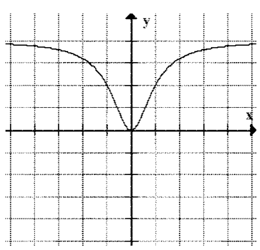
$Range: y > -2$

If case 2 is shifted up or down $= c, HA: y = c$

Case 3: (Horizontal Asymptote at $y = \frac{a}{b}$)

$y = \dfrac{ax^n}{bx^n}$

$y = \dfrac{4x^2}{(x^2 + 1)}$



$HA: y = \dfrac{4}{1}$

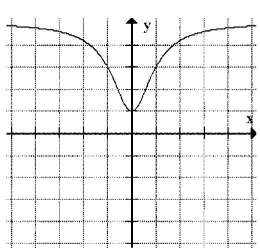
$Range: 0 \leq y < 4$

If the exponent of x is the same on the top as the bottom, HA: y = fraction of coefficients

Modified Case 3: (Horizontal Asymptote at $y = \frac{a}{b} + c$)

$y = \dfrac{ax^n}{bx^n} + c$

$y = \dfrac{4x^2}{(x^2 + 1)} + 1$



$HA: y = \dfrac{4}{1} + 1 = 5$

$Range: 1 \leq y < 5$

If case 3 is shifted up or down $= c$, $HA: y$ = fraction of coefficients + c

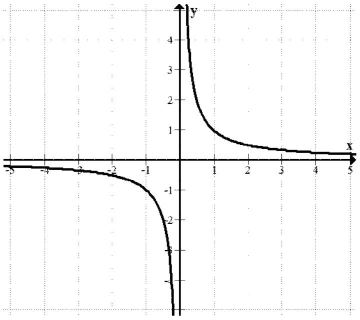
A horizontal asymptote by definition is the limit as x approaches ±infinity. Substitute ±infinity for x into a table of values.

Horizontal Asymptote	$\lim_{x \to \pm-\infty} f(x) = L$; HA

C12 - 1.4 - Vertical Asymptotes Notes

To find Vertical Asymptotes: Cannot have a denominator of 0. $\boldsymbol{VA: Vertical\ Asymptote}$

$$f(x) = \frac{1}{x}$$

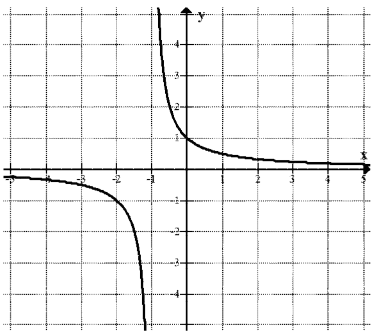


$VA: x = 0$ $\quad$ $Set\ denominator = 0, and\ solve.$

NPVs, Restrictions:
$x = 0$

Domain:
$x \neq 0$

$$f(x) = \frac{1}{x+1}$$



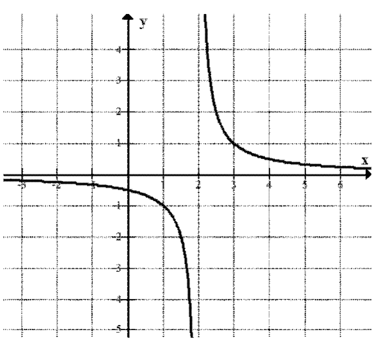
$VA: x + 1 = 0$ $\quad$ $Set\ denominator = 0, and\ solve.$
$x = -1$

NPVs, Restrictions:
$x = -1$

Domain:
$x \neq -1$

$Notice: The\ vertical\ asymptote\ has\ shifted\ 1\ to\ the\ left\ from\ \frac{1}{x}$

$$f(x) = \frac{1}{x-2}$$



$VA: x - 2 = 0$ $\quad$ $Set\ denominator = 0, and\ solve.$
$x = 2$

NPVs, Restrictions:
$x = 2$

Domain:
$x \neq 2$

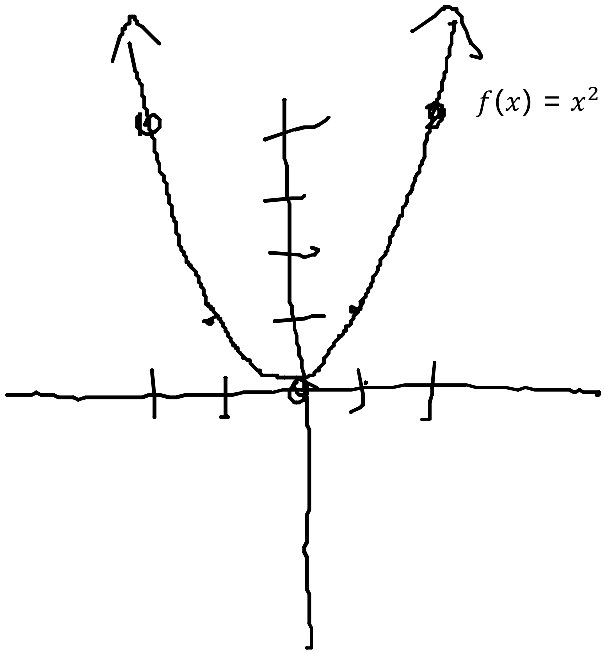
$Notice: The\ vertical\ asymptote\ has\ shifted\ 2\ to\ the\ right\ from\ \frac{1}{x}.$

A vertical asymptote by definition is the limit as x approaches $\pm x$ value of vertical asymptote. Substitute ± x values close to the vertical assympotoe into a table of values. If y equals $+infinity$ on one side and $-infinity$ on the other it is a vertical asymptote.

C12 - 1.5 - Even Odd Functions Symmetry Notes

Even and Odd Functions — Symmetry

Even: $f(-x) = f(x)$ *A horizontal flip over the y – axis is same as original*

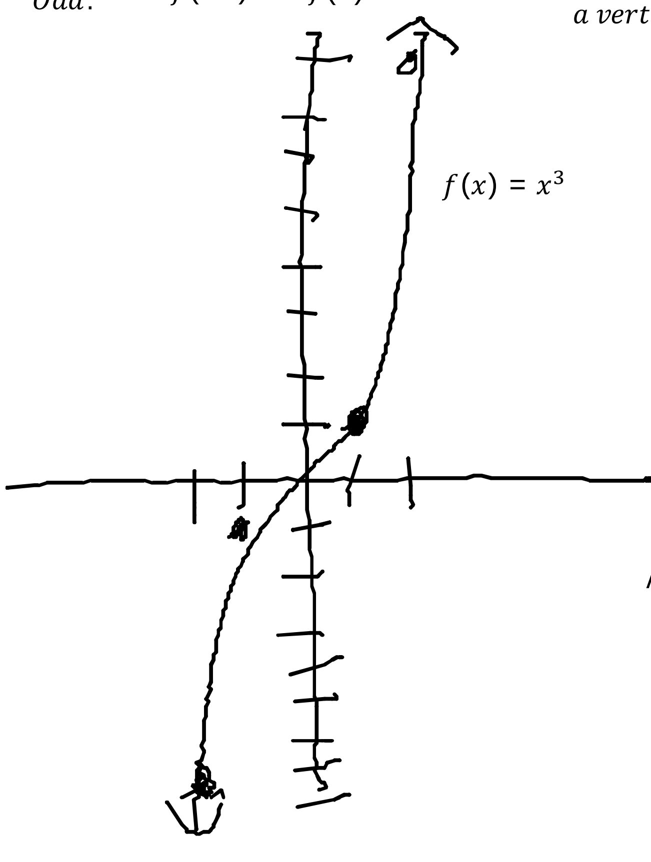


$f(x) = x^2$

$$\begin{aligned} f(-x) &= f(x) \\ (-x)^2 &= x^2 \\ x^2 &= x^2 \end{aligned}$$

✓

Odd: $f(-x) = -f(x)$ *A horizontal flip over the y – axis is same as a vertical flip over the x – axis.*



$f(x) = x^3$

$$\begin{aligned} f(-x) &= -f(x) \\ (-x)^3 &= -(x^3) \\ -x^3 &= -x^3 \end{aligned}$$

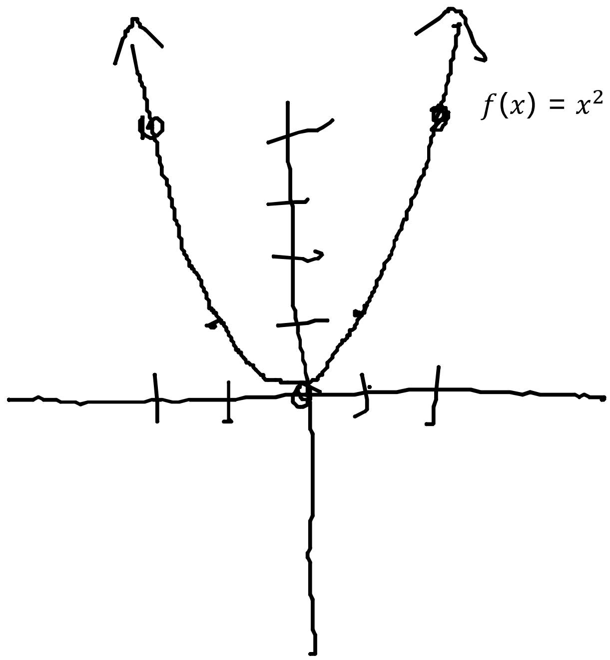
✓

A horizontal flip equal to a vertical flip!

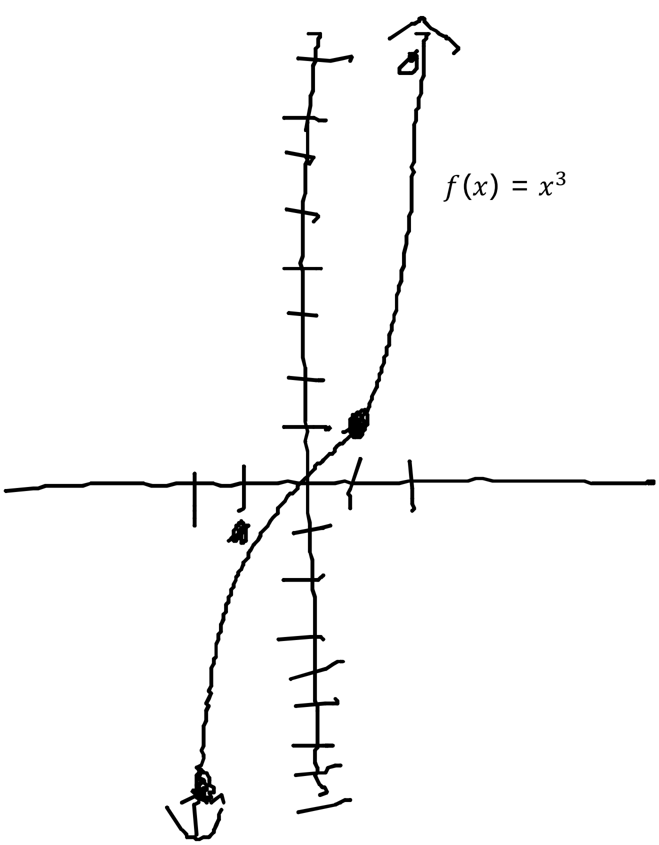
C12 - 1.6 - One-to-One Functions Notes

Inverse is a Function

$One - to - One\ Function$ $\quad f(a) \neq f(b)$ $\quad$ *Only one x value for every y value. The horizonal line test. Run your pencil horizonally down the page: your pencil can only ever hit the graph once.*



$Not\ One - to - One$



$One - to - One$

C12 - 1.7 - Inverse Function Notes

$f(x) = x + 2$
$y = x + 2$ — *Substitue y for f(x)*
$x = y + 2$ — *Switch x and y*
$x - 2 = y$ — *Solve for y*
$y = x - 2$ — *Mirror*
$f^{-1}(x) = x - 2$ — *Substitute f^(−1) (x)for y*

Inverse: A diagonal flip over the xy − axis

Check your answer

$f^{-1}(f(x)) = ?$

$f^{-1}(x) = x - 2$
$f^{-1}(x + 2) = (x + 2) - 2$
$f^{-1}(x + 2) = x$

$f^{-1}(f(x)) = x$

$f(f^{-1}(x)) = ?$

$f(x) = x + 2$
$f(x - 2) = (x - 2) + 2$
$f(x - 2) = x$

$f(f^{-1}(x)) = x$

A function has an inverse function if it is One − to − One,
Or if you restrict the domain.

$$\lim_{x\to 0} \frac{\sin 2x}{2x} \cdot 2$$

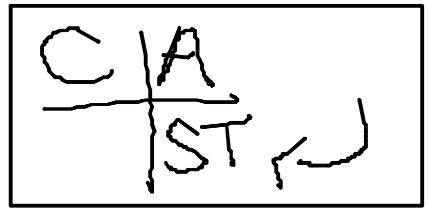
$$\frac{x}{2}$$

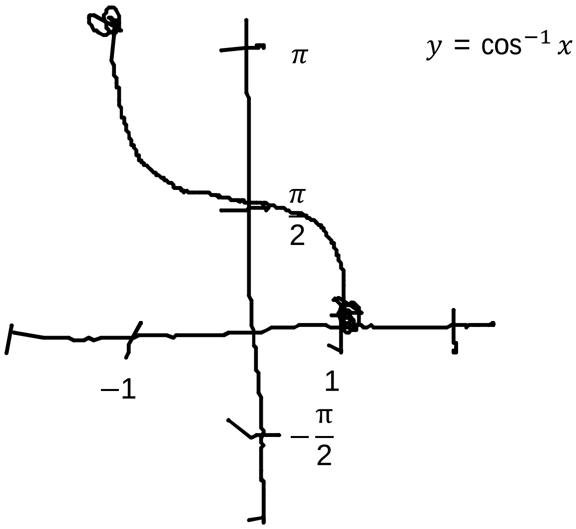
$$1 \cdot 2$$

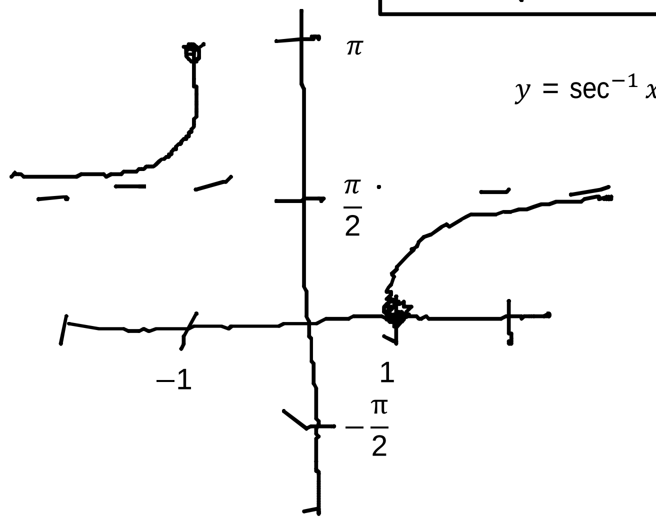
$$\frac{\frac{x}{1}}{\frac{2}{1}}$$

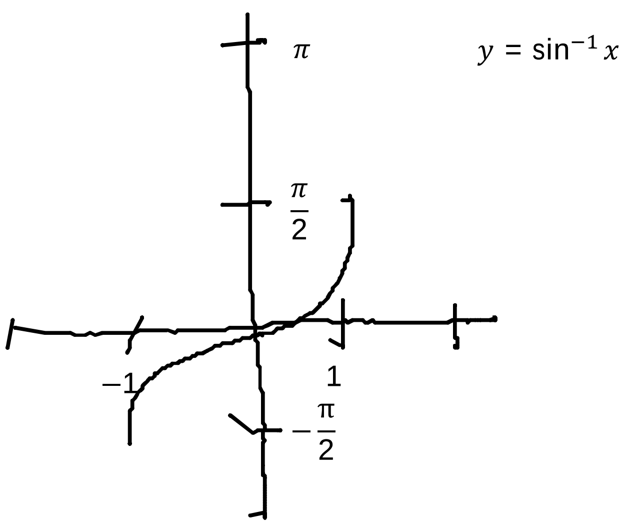
$$\frac{\cancel{\frac{\tan 4x}{4x}}\, 4x}{\cancel{\frac{\tan 3x}{3x}}\, 3x} = \frac{4}{3}$$

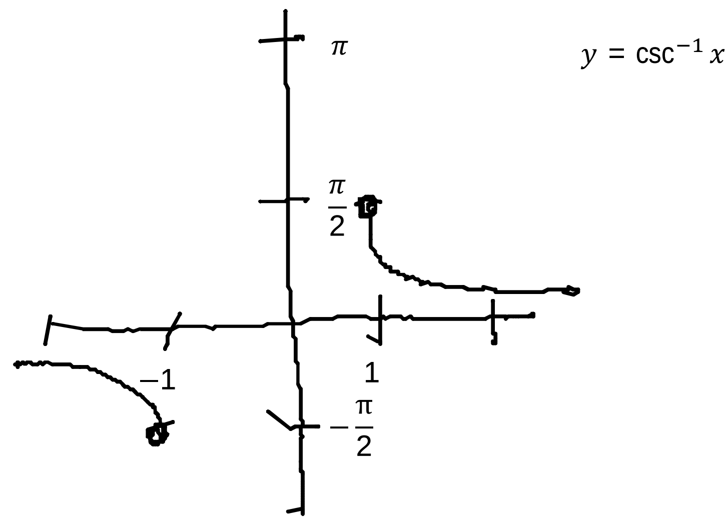
C12 - 1.8 - Inverse Trig Domain Notes

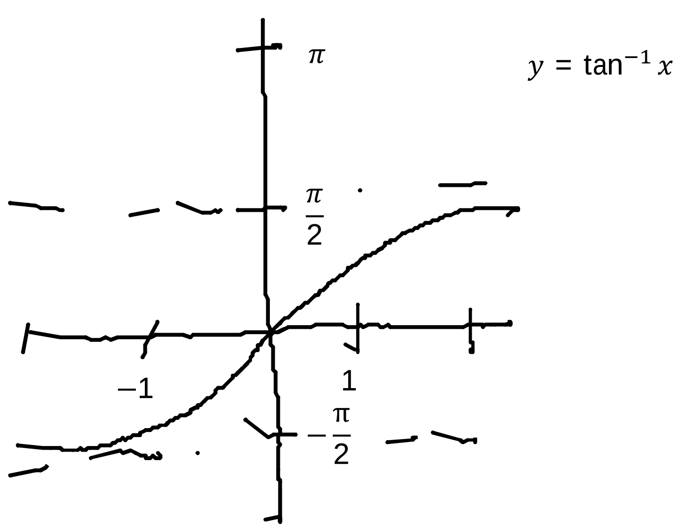


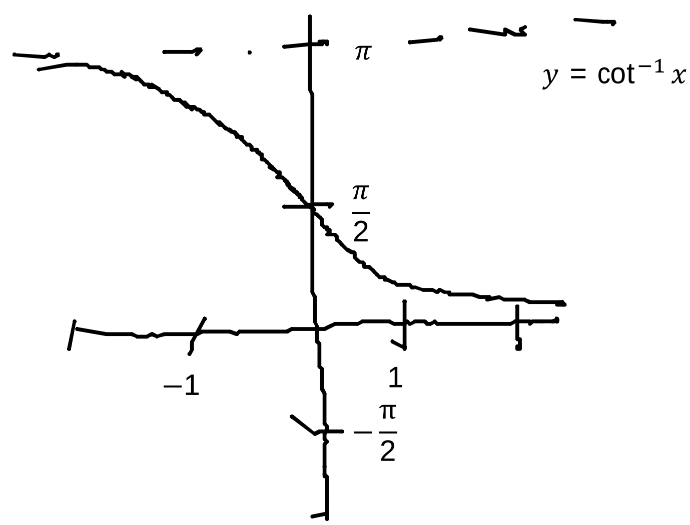




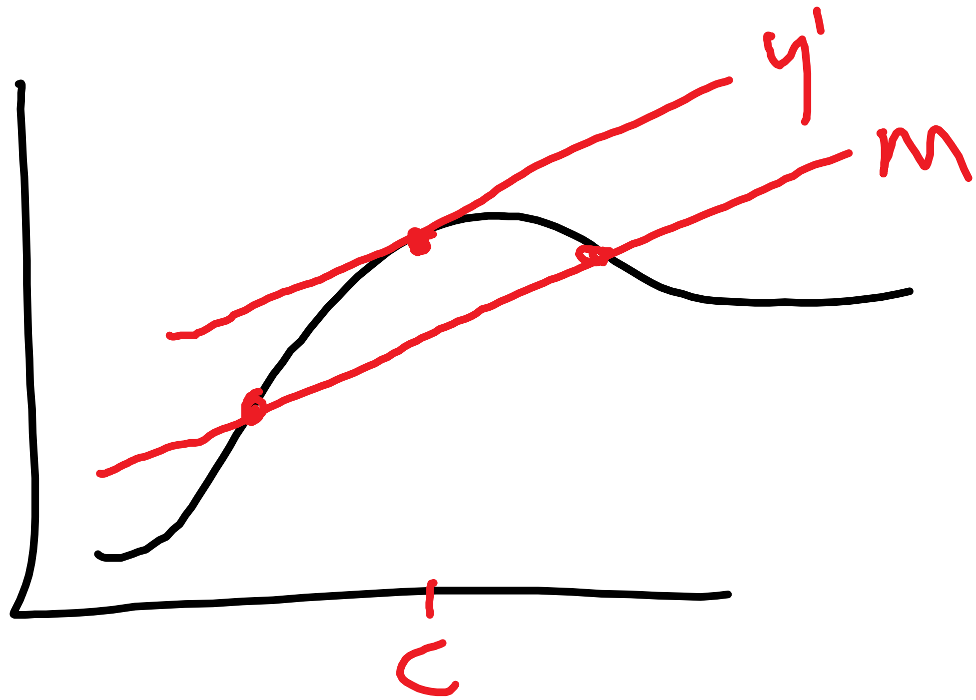








C12 - 1.9 - Mean Value Theorem MVT

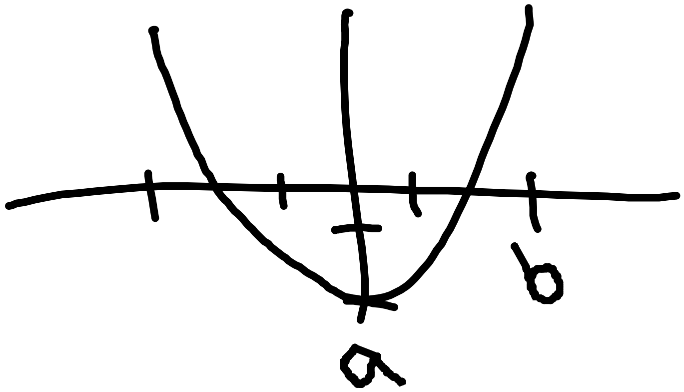


$$\boxed{y' = \frac{y_2 - y_1}{x_2 - x_1}}$$

C12 - 1.9 - Intermediate Value Theorem IVT

$$x^2 = 2$$

$$x^2 - 2 = 0$$



$$\boxed{f(a) \le f(c) \le f(b)}$$

$$f(0) \le f(c) \le f(2)$$

$$-2 \le 0 \le 2$$

& CONT

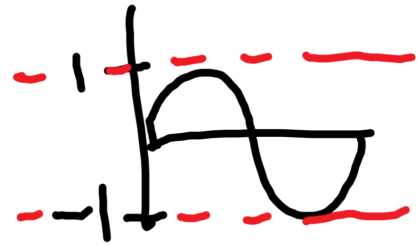
$$\therefore\ 0 < c < 2$$

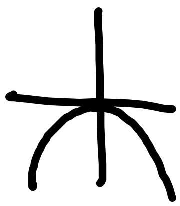
✓

C12 - 1.9 - $x^2 sinx$ Squeeze/Sandwich Theorem

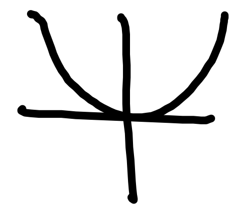
$$\lim_{x \to 0} x^2 \sin x = 0$$

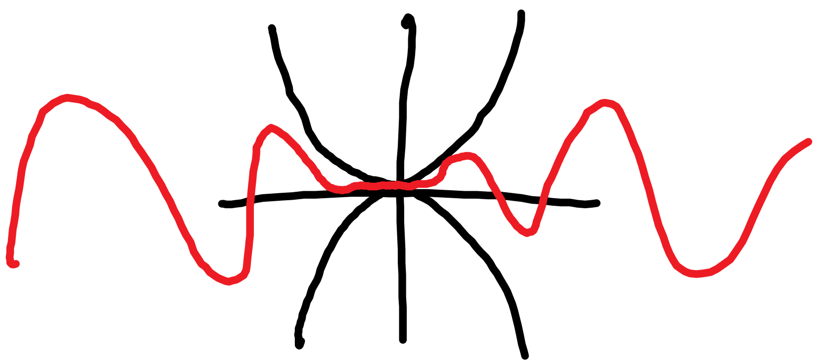
$$\boxed{-1 \leq \sin x \leq 1}$$





$$-1x^2 \leq x^2 \sin x \leq 1x^2$$

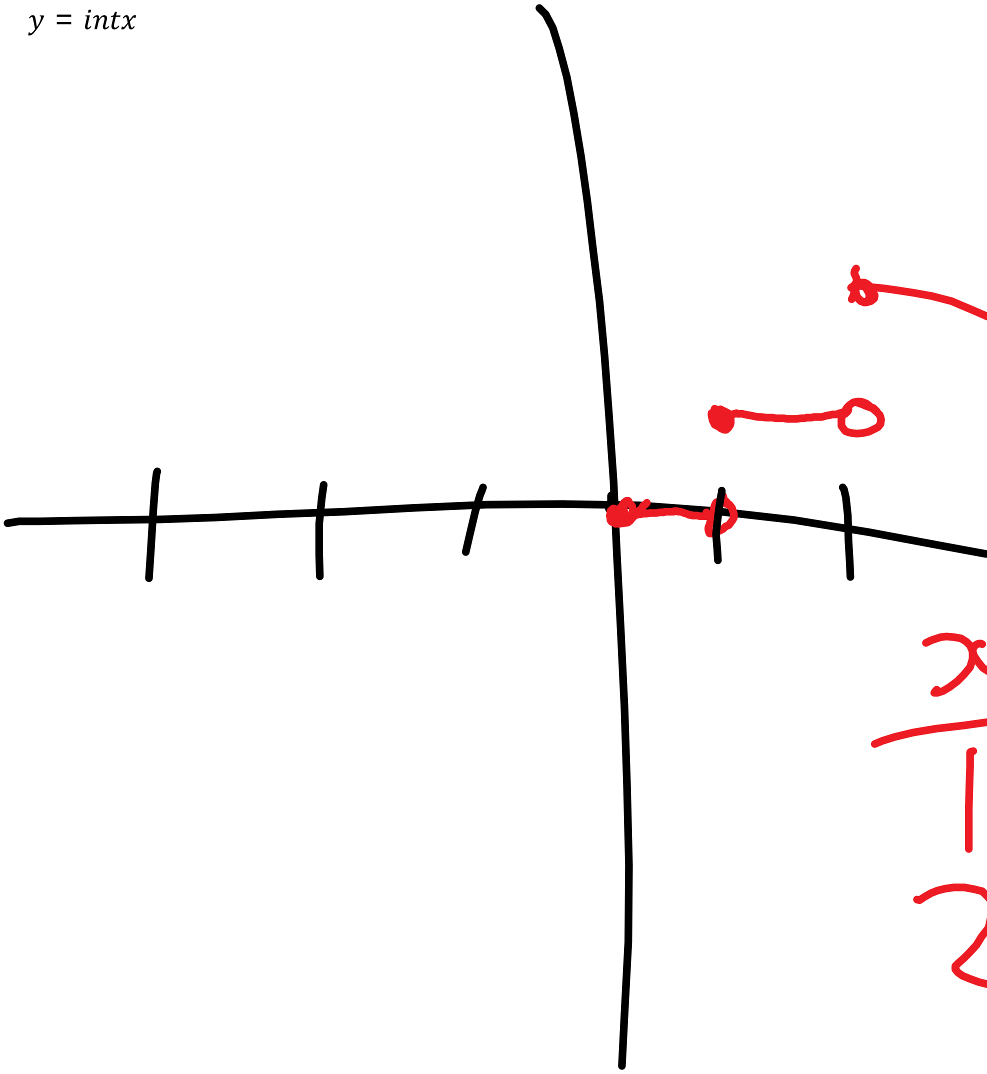




$$0 \leq 0 \leq 0$$

C12 - 1.9 - Int Function Limits

$y = intx$



x	y
1	1
2	2

C12 - 1.9 - Difference of cubes

$$a^3 - b^3$$

$$(x+1)^3 - 2^3$$

$$\therefore \left((x+1) - 2\right)\left((x+1)^2 + 2(x+1) + 2^2\right)$$

$$x - 1$$

$$\lim_{x \to \pm\infty}\left(1+\frac{1}{x}\right)^{x} = \lim_{x \to 0}\left(1+x\right)^{\frac{1}{x}} = e$$

Must be reciprocals of each other. Make them!