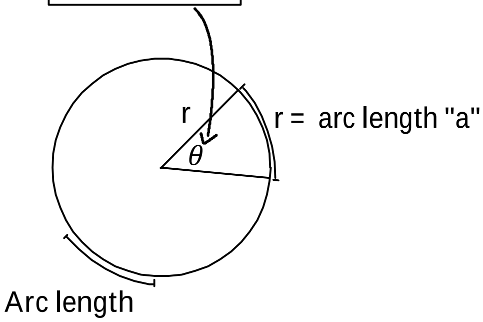
C12 - 4.1 - Radian Notes

"One radian is equal to length of the arc on a circle with radius = 1. With a central angle = 57.3^o OR = $1\,radian$"

$\boxed{1\,rad = 57.3^o}$



1 Radian is the central angle whose arc is equal to the radius

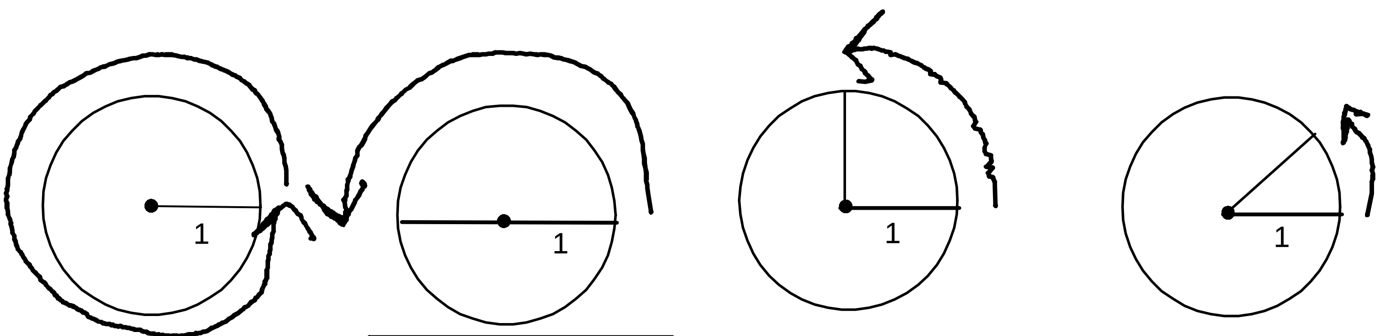
$$\boxed{\begin{array}{l} 1\,rad = 57.3^o \\ 1\,inch = 2.54\,cm \end{array}}$$

$$\boxed{\theta_{rad} = \frac{a}{r}}$$

$$\theta_{rad} = \frac{r}{r}$$

$$\theta_{rad} = 1\,rad$$

One Radian equals 57.3^o



$\theta = 360^o = 2\pi_{rad}$

$C = 2\pi r$
$C = 2\pi(1)$

$C = 2\pi$
$C = 6.28$

$\boxed{\theta = 180^0 = \pi_{rad}}$

$C = 2\pi r$
$C = 2\pi(1)$
$C = 2\pi$

$\frac{C}{2} = \frac{2\pi}{2}$

$C = \pi$

$\theta = 90^0 = \frac{\pi}{2_{rad}}$

$C = 2\pi r$
$C = 2\pi(1)$
$C = 2\pi$

$\frac{C}{4} = \frac{2\pi}{4}$
$C = \frac{\pi}{2}$

$\theta = 45^0 = \frac{\pi}{4_{rad}}$

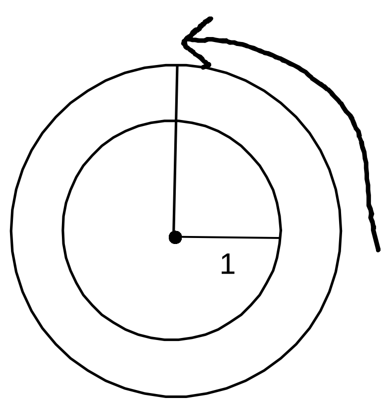
$C = 2\pi r$
$C = 2\pi(1)$
$C = 2\pi$

$\frac{C}{8} = \frac{2\pi}{8}$

$C = \frac{\pi}{4}$

Notice the size of the circle does not matter.

$90^0 = \frac{\pi}{2}$



$\theta = 90^0 = \frac{\pi}{2_{rad}}$

C12 - 4.1 - Degree/Radian Conversion Notes

$\boxed{\frac{180^o}{\pi}}$ $\boxed{\frac{\pi}{180^o}}$

Degrees to Radians: $\times \frac{\pi}{180^o}$

Radians to Degrees: $\times \frac{180^0}{\pi}$

π *and* 180^o *are the same thing, just in different units*

Find θ in radians

$30^0 = ?$ $\quad 30^0 \times \frac{\pi}{180^o} = \frac{30\pi}{180} = \frac{\pi}{6} = 0.52$

$120^0 = ?$ $\quad 120^0 \times \frac{\pi}{180^o} = \frac{120\pi}{180} = \frac{2\pi}{3}$

$99^0 = ?$ $\quad 99^0 \times \frac{\pi}{180^o} = \frac{99\pi}{180} = \frac{11\pi}{20}$

$Degrees$	$Radians$	$Radians$	$Radians$
0^0	0_{rad}	0_{rad}	0_{rad}
15^0	$\frac{\pi}{12}_{rad}$	$\frac{\pi}{12}_{rad}$	0.26_{rad}
30^0	$\frac{2\pi}{12}_{rad}$	$\frac{\pi}{6}_{rad}$	0.52_{rad}
45^0	$\frac{3\pi}{12}_{rad}$	$\frac{\pi}{4}_{rad}$	0.79_{rad}
60^0	$\frac{4\pi}{12}_{rad}$	$\frac{\pi}{3}_{rad}$	1.05_{rad}
75^0	$\frac{5\pi}{12}_{rad}$	$\frac{5\pi}{12}_{rad}$	1.31_{rad}
90^0	$\frac{6\pi}{12}_{rad}$	$\frac{\pi}{2}_{rad}$	1.57_{rad}
180^0	$\frac{12\pi}{12} = \pi_{rad}$	π_{rad}	3.14_{rad}
270^0	$\frac{3\pi}{2}_{rad}$	$\frac{3\pi}{2}_{rad}$	4.71_{rad}
360^0	$2\pi_{rad}$	$2\pi_{rad}$	6.28_{rad}
720^0	$4\pi_{rad}$	$4\pi_{rad}$	12.56_{rad}

Find θ in degrees

$\frac{\pi}{3}_{rad} = ?$ $\quad \frac{\pi}{3}_{rad} \times \frac{180^0}{\pi} = \frac{180\pi}{\pi} = 60^0$

$\frac{2\pi}{5}_{rad} = ?$ $\quad \frac{2\pi}{5}_{rad} \times \frac{180^0}{\pi} = \frac{360\pi}{5\pi} = 72^0$

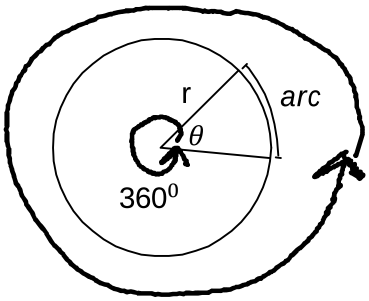
$1.57_{rad} = ?$ $\quad 1.57_{rad} \times \frac{180^0}{\pi} = 90^0$

$3 = ?$ $\quad 3_{rad} \times \frac{180^0}{\pi} = \frac{540}{\pi} = 171.89^0$

If there are no units it is in radians.

C12 - 4.2 - Arc Length, Sector Area Notes

$\theta \text{ in radians}$



$Circumference$

$$\boxed{\frac{arc\ length}{Circumference} = \frac{\theta}{360^O}}$$ Grade 8-11

$$\frac{arc\ length}{Circumference} = \frac{\theta}{2\pi}$$

$\theta \text{ must be in radians}$

$$\frac{a}{2\pi r} = \frac{\theta}{2\pi}$$

$$\cancel{2\pi} \times \frac{a}{\cancel{2\pi} r} = \frac{\theta}{\cancel{2\pi}} \times \cancel{2\pi}$$

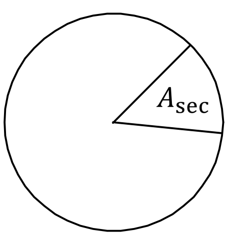
$$\frac{a}{r} = \theta$$

$$\cancel{r} \times \frac{a}{\cancel{r}} = \theta \times r$$

$$a = \theta r$$

$$\boxed{a = \theta r}$$

Sector Area

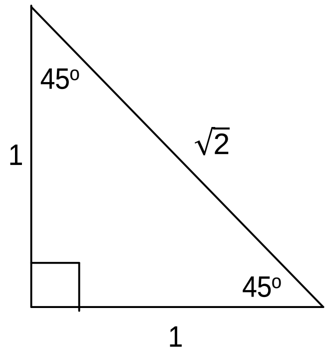


$$\frac{Area_{sector}}{Area_{Total}} = \frac{arc\ length}{Circumference} = \frac{\theta}{2\pi}$$

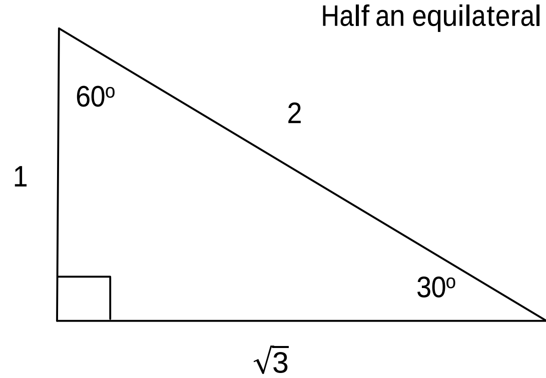
$$\boxed{\frac{A_{\text{sec}}}{\pi r^2} = \frac{a}{2\pi r} = \frac{\theta}{360^o} = \frac{\theta}{2\pi}}$$ They are all equal to each other.

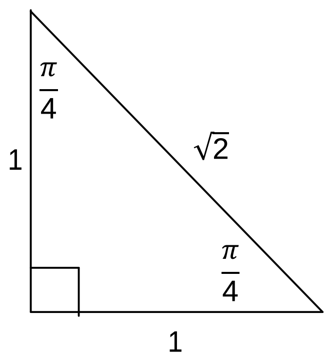
C12 - 4.3 - Special Triangles sin/cos/tan $\frac{\pi}{3},\frac{\pi}{4},\frac{\pi}{6}$ Notes

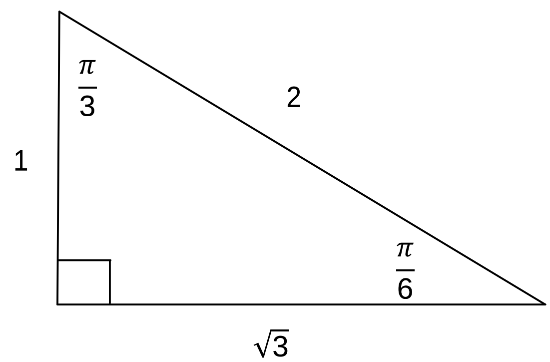
Isoceles, with sides =1



Half an equilateral with sides 2







$\boldsymbol{sin\theta} = \dfrac{\boldsymbol{Opp}}{\boldsymbol{Hyp}}$ $\boldsymbol{cos\theta} = \dfrac{\boldsymbol{Adj}}{\boldsymbol{Hyp}}$ $\boldsymbol{tan\theta} = \dfrac{\boldsymbol{Opp}}{\boldsymbol{Adj}}$

$\sin\frac{\pi}{6} = \frac{1}{2}$ $cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

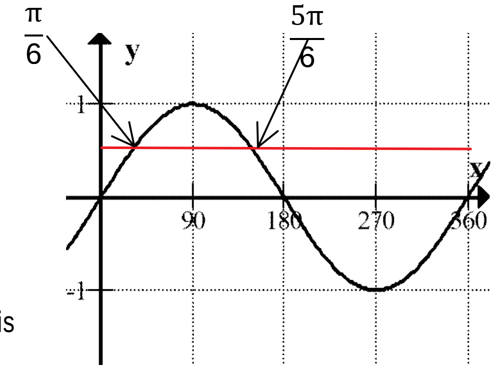
$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $tan\frac{\pi}{4} = 1/1$

$tan\frac{\pi}{4} = 1$

$sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $cos\frac{\pi}{3} = \frac{1}{2}$ $tan\frac{\pi}{3} = \frac{\sqrt{3}}{1}$

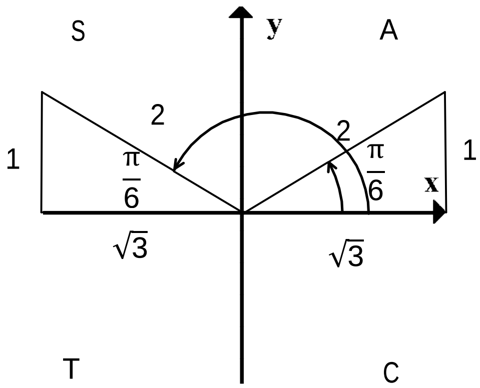
$tan\frac{\pi}{3} = \sqrt{3}$

C12 - 4.3 - $sin\theta = \frac{1}{2}$ Notes



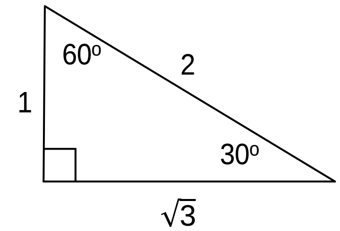
Solve for $\theta, 0^o \le \theta < 2\pi$.

$\sin\theta = \frac{1}{2}$



Draw two triangles where $\sin\theta$ is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles
and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = \frac{\pi}{6}$

$$\theta_{stp} = \pi - \frac{\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$$

Solve for the arrows θ_{stp}

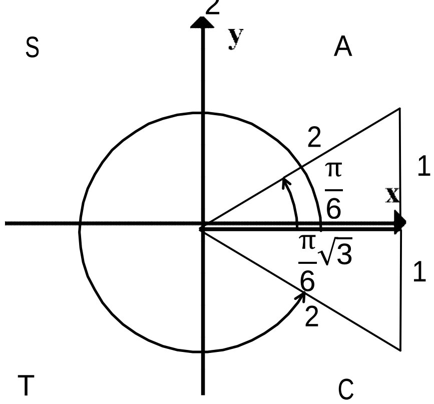
Check your answer: $\sin(\frac{\pi}{6}) = \frac{1}{2}$ $\sin(\frac{5\pi}{6}) = \frac{1}{2}$

$\theta_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$

Calculator must be in radian mode: Mode Radians

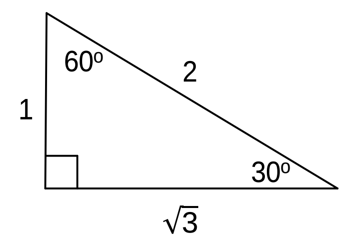
Solve for $\theta, 0^o \le \theta < 2\pi$ *and general solution*.

$cosx = \frac{\sqrt{3}}{2}$



Draw two triangles where $\cos\theta$ is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles
and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = \frac{\pi}{6}$

$$\theta_{stp} = 2\pi - \frac{\pi}{6} = \frac{11\pi}{12}$$

Solve for the arrows θ_{stp}

Check your answer: $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ $\cos(\frac{11\pi}{6}) = \frac{\sqrt{3}}{2}$

$\theta_{stp} = \frac{\pi}{6}, \frac{11\pi}{6}$

General Solution: $\theta = \theta_{stp} \pm pn, n \in I$ $\theta = \theta_{stp} \pm pn, n \in I$

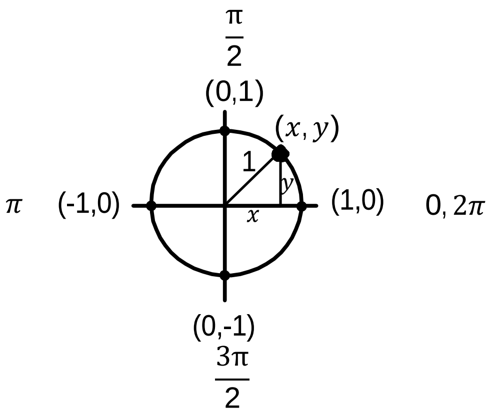
π 11π

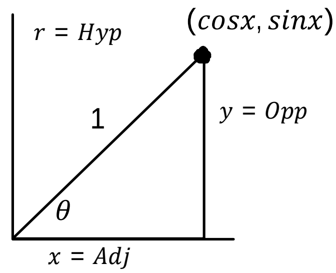
$\theta_{stp} = \frac{\pi}{6}, \frac{11\pi}{6}$

$General\ Solution$: $\theta = \theta_{stp} \pm pn, n \in I$ $\quad$ $\theta = \theta_{stp} \pm pn, n \in I$

$\theta = \frac{\pi}{6} \pm 2\pi n, n \in I$ $\quad$ $\theta = \frac{11\pi}{6} \pm 2\pi n, n \in I$

C11 - 4.4 - Unit Circle sin/cos/tan 90, 180, 270, 360 Notes





$\boxed{sin\theta = y}$

$\boxed{cos\theta = x}$

$\boxed{tan\theta = \frac{y}{x}}$

$Radius\ of\ unit\ circle = 1$

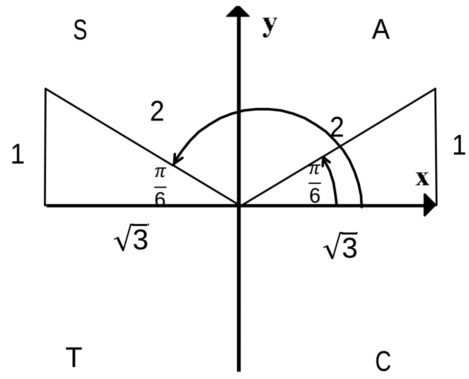
$Hyp = 1$

$\boldsymbol{sin\theta = \frac{Opp}{Hyp}}$ $sin\theta = \frac{y}{1}$ $sin\theta = y$	$\boldsymbol{cos\theta = \frac{Adj}{Hyp}}$ $cos\theta = \frac{x}{1}$ $cos\theta = x$	$\boldsymbol{tan\theta = \frac{Opp}{Adj}}$ $tan\theta = \frac{y}{x}$
$sin0 = \frac{0}{1}$ $sin0 = 0$	$cos0 = \frac{1}{1}$ $cos0 = 1$	$tan0 = \frac{0}{1}$ $tan0 = 0$
$\sin(\frac{\pi}{2}) = \frac{1}{1}$ $\sin\left(\frac{\pi}{2}\right) = 1$	$cos(\frac{\pi}{2}) = \frac{0}{1}$ $cos(\frac{\pi}{2}) = 0$	$tan(\frac{\pi}{2}) = \frac{1}{0}$ $tan(\frac{\pi}{2}) = UND$
$sin\pi = \frac{0}{1}$ $sin\pi = 0$	$\cos\pi = -\frac{1}{1}$ $cos\pi = -1$	$tan\pi = \frac{0}{-1}$ $tan\pi = 0$
$sin(\frac{3\pi}{2}) = \frac{-1}{1}$ $sin(\frac{3\pi}{2}) = -1$	$cos(\frac{3\pi}{2}) = \frac{0}{1}$ $cos(\frac{3\pi}{2}) = 0$	$\tan(\frac{3\pi}{2}) = \frac{-1}{0}$ $tan(\frac{3\pi}{2}) = UND$
$sin2\pi = \frac{0}{1}$ $sin2\pi = 0$	$cos2\pi = \frac{1}{1}$ $cos2\pi = 1$	$tan2\pi = \frac{0}{1}$ $tan2\pi = 0$

C12 - 4.5 - sin 2θ ASTC Special Triangles Notes

Solve for θ $0^o \leq \theta < 2\pi$, and the general solution.

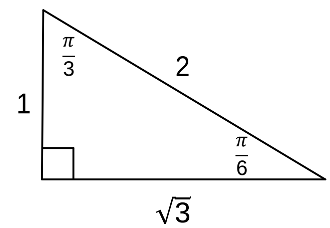
$\sin 2\theta = \frac{1}{2}$



$Let\ m = 2\theta$ $\qquad \sin m = \frac{1}{2}$

Draw two triangles where $\sin m$ is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

Solve for the arrows m_{stp}

$m_{stp} = \frac{\pi}{6}$ $\qquad m_{stp} = \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

$m_{stp} = \frac{\pi}{6}, \frac{5\pi}{6}$

Check your answer: $\qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{5\pi}{6} = \frac{1}{2}$

Substitute 2θ back in for m.

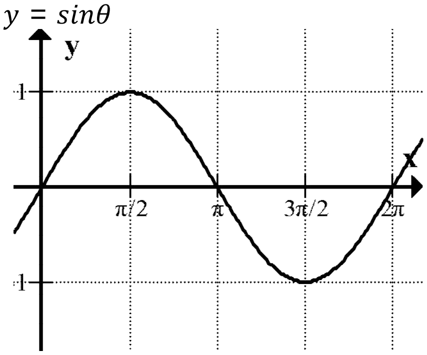
$m = \frac{\pi}{6}$ $\qquad m = \frac{5\pi}{6}$

$2\theta = \frac{\pi}{6}$ $\qquad 2\theta = \frac{5\pi}{6}$

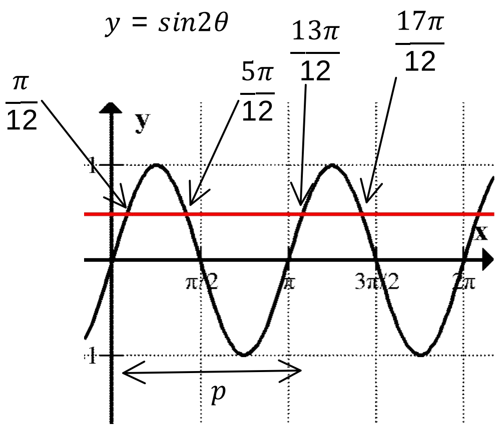
$\theta = \frac{\pi}{6 \times 2}$ $\qquad \theta = \frac{5\pi}{6 \times 2}$

$\theta = \frac{\pi}{12}$ $\qquad \theta = \frac{5\pi}{12}$

$y = sin\theta$



$y = sin2\theta$



$p = \frac{2\pi}{b}$

$p = \frac{2\pi}{2}$

$= \pi$

$\theta = \theta_{stp} \pm p$ $\qquad \theta = \theta_{stp} \pm p$

$\theta = \frac{\pi}{12} + \pi$ $\qquad \theta = \frac{5\pi}{12} + \pi$

$\theta = \frac{13\pi}{12}$ $\qquad \theta = \frac{17\pi}{12}$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ $\qquad 0 \leq \theta \leq 2\pi$

General Solution:	$\theta = \theta_{stp} \pm pn, n \in I$ $\theta = \frac{\pi}{12} \pm \pi n, n \in I$	$\theta = \theta_{stp} \pm pn, n \in I$ $\theta = \frac{5\pi}{12} \pm \pi n, n \in I$