C11 - 3.1 - Quadratics Graphing x^2 TOV Notes

Graphing: $y = x^2$

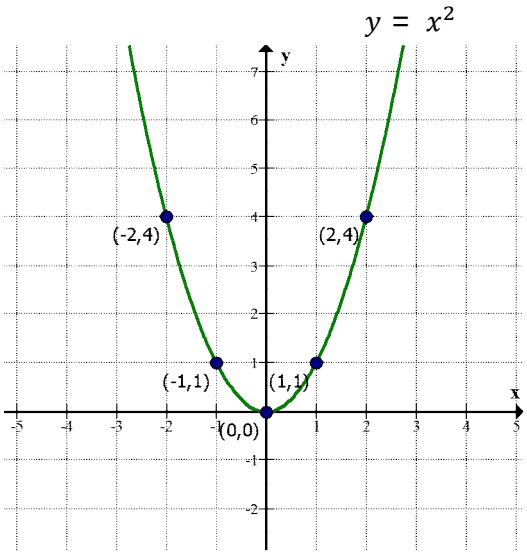


Table of Values

Vertex:

x	y	Pt.
-2	4	(-2,4)
-1	1	(-1,1)
0	0	(0,0)
1	1	(1,1)
2	4	(2,4)

$y = x^2$
$y = (-2)^2$
$y = 4$

$y = x^2$
$y = (-1)^2$
$y = 1$

$y = x^2$
$y = (0)^2$
$y = 0$

$y = x^2$
$y = (1)^2$
$y = 1$

$y = x^2$
$y = (2)^2$
$y = 4$

Notice: the pattern from the vertex (0,0) is **symmetrical** on both sides.

Over 1, 1 squared = 1, up 1. Back to the vertex. Over 2, 2 squared = 4, up 4.

C11 - 3.1 - Quadratic Vertical Translation Notes $y = x^2 + q$

Graphing: $y = x^2 + c$

$y = x^2 + 1$

$y = x^2 + 1$

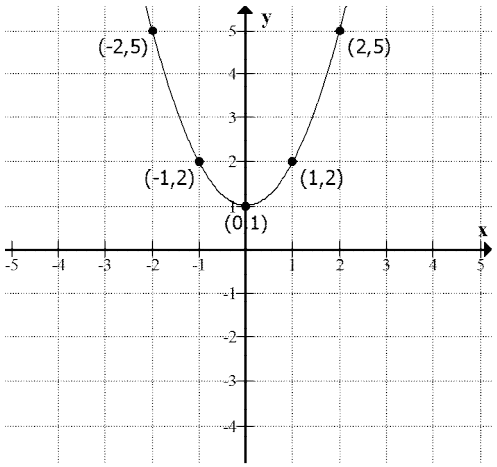


Table of Values

x	y
-2	5
-1	2
0	1
1	2
2	5

Pt.
(-2,5)
(-1,2)
(0,1)
(1,2)
(2,5)

$y = x^2 + 1$
$y = (-2)^2 + 1$
$y = 4 + 1$
$y = 5$

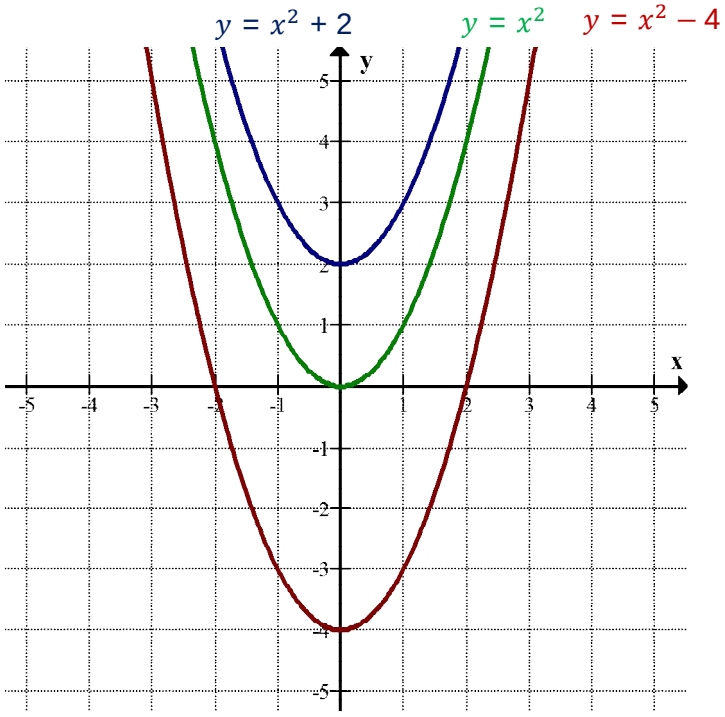
$y = x^2 + 1$
$y = (-1)^2 + 1$
$y = 1 + 1$
$y = 2$

$y = x^2 + 1$
$y = (0)^2 + 1$
$y = 0 + 1$
$y = 5$

$y = x^2 + 1$
$y = (1)^2 + 1$
$y = 1 + 1$
$y = 2$

$y = x^2 + 1$
$y = (2)^2 + 1$
$y = 4 + 1$
$y = 5$

Notice: the graph of $y = x^2 + 1$ is the graph $y = x^2$ shifted up 1. "c" is the y intercept. "c" is only the vertex if there is no "b".



C11 - 3.1 - Quadratics Horizontal Translation Notes $(x-p)^2$

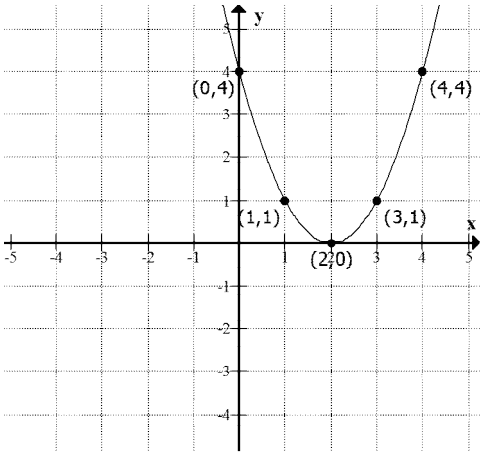
1) Graphing: $y = (x-p)^2$

$y = (x-2)^2$

Table of Values

x	y
0	4
1	1
2	0
3	1
4	4

Pt.
(0,4)
(1,1)
(2,0)
(3,1)
(4,4)



$y = (x-2)^2$
$y = ((0)-2)^2$
$y = (0-2)^2$
$y = (-2)^2$
$y = 4$

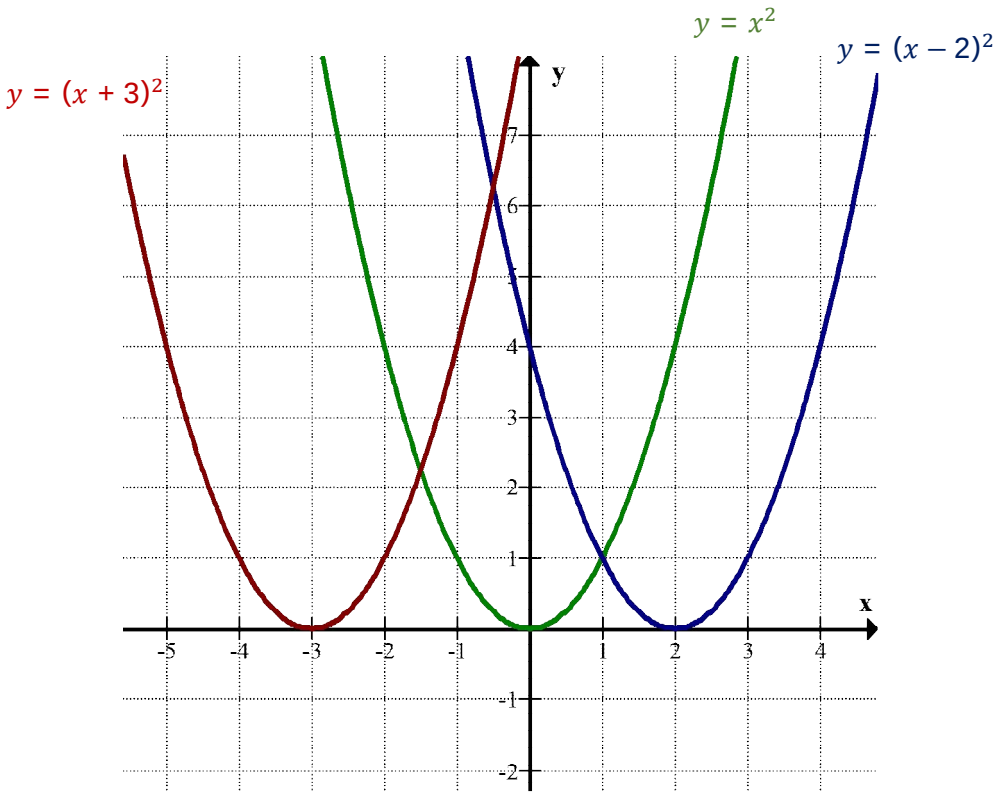
$y = (x-2)^2$
$y = ((1)-2)^2$
$y = (1-2)^2$
$y = (-1)^2$
$y = 1$

$y = (x-2)^2$
$y = ((2)-2)^2$
$y = (2-2)^2$
$y = (0)^2$
$y = 0$

$y = (x-2)^2$
$y = ((3)-2)^2$
$y = (3-2)^2$
$y = (-1)^2$
$y = 1$

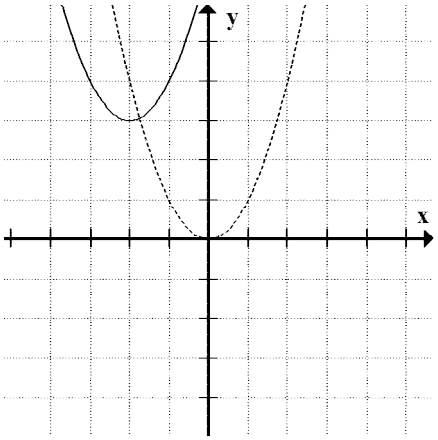
$y = (x-2)^2$
$y = ((4)-2)^2$
$y = (4-2)^2$
$y = (2)^2$
$y = 4$

Notice: the graph of $y = (x-p)^2$ is the graph $y = x^2$ shifted right 2.
Notice we shift the opposite of "p".

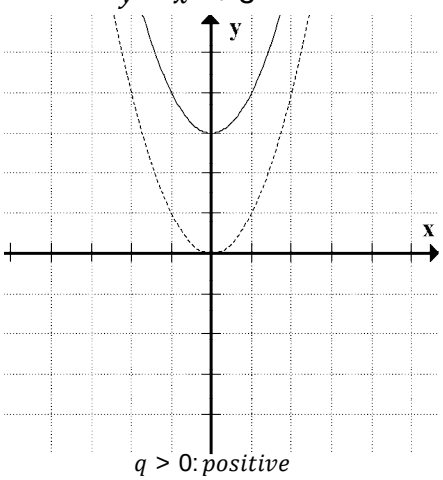


C11 - 3.1 - Quadratics Horizontal/Vertical Summary

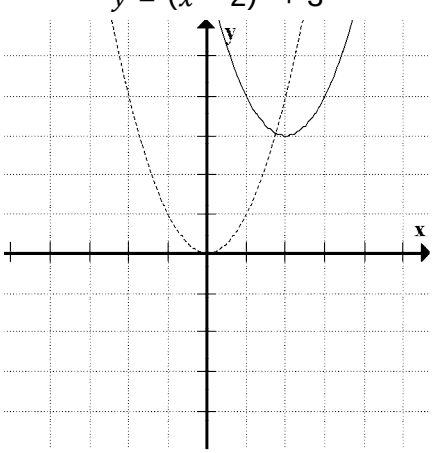
$y = (x + 2)^2 + 3$



$y = x^2 + 3$

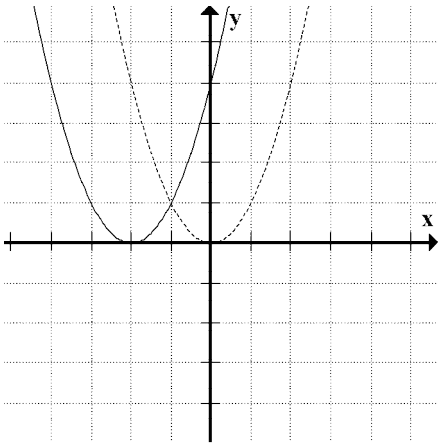


$y = (x - 2)^2 + 3$



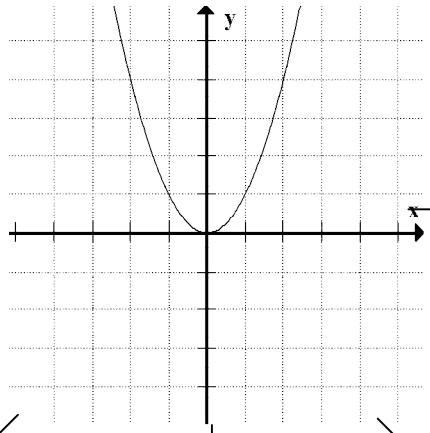
$q > 0$: *positive*
Graph moves up q

$y = (x + 2)^2$

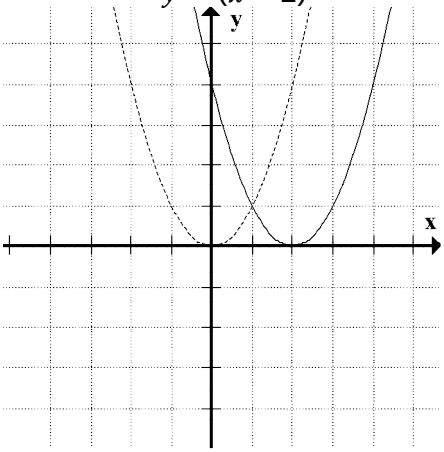


$p > 0$: *negative*
Graph moves left p

$y = x^2$

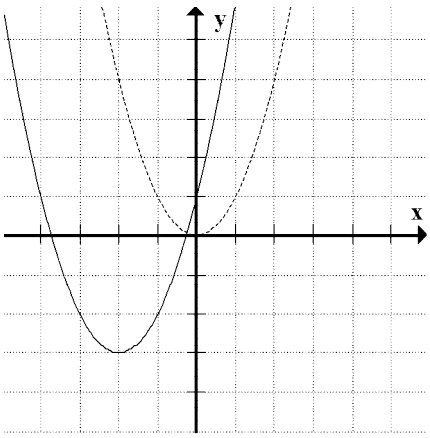


$y = (x - 2)^2$

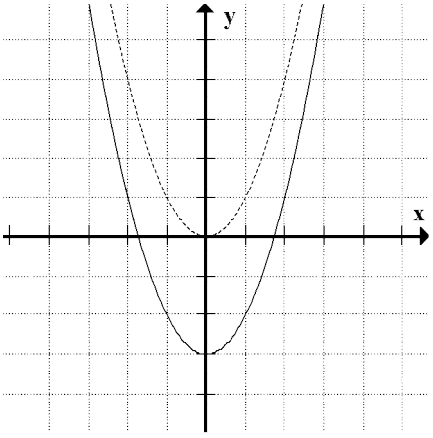


$p < 0$: *negative*
Graph moves right p

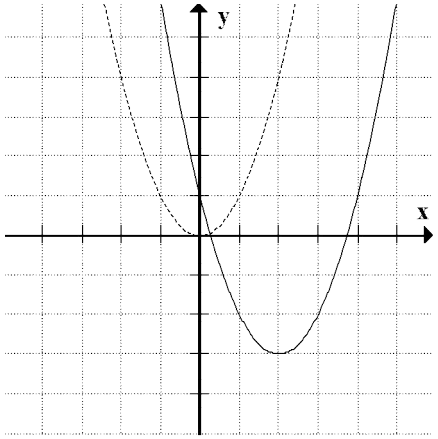
$y = (x + 2)^2 - 3$



$y = x^2 - 3$



$y = (x - 2)^2 - 3$



$q < 0$: *negative*
Graph moves down q

C11 - 3.1 - Quadratics Reflection Notes $-x^2$

1) Graphing: $y = -x^2$

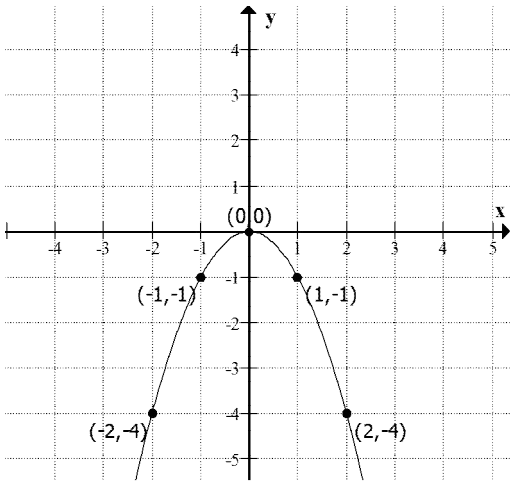
$y = -x^2$

Table of Values

x	y
-2	-4
-1	-1
0	0
1	-1
2	-4

Pt.
(-2,-4)
(-1,-1)
(0,0)
(1,-1)
(2,-4)

$y = -x^2$



$y = -x^2$
$y = -(-2)^2$
$y = -4$

$y = -x^2$
$y = -(-1)^2$
$y = -1$

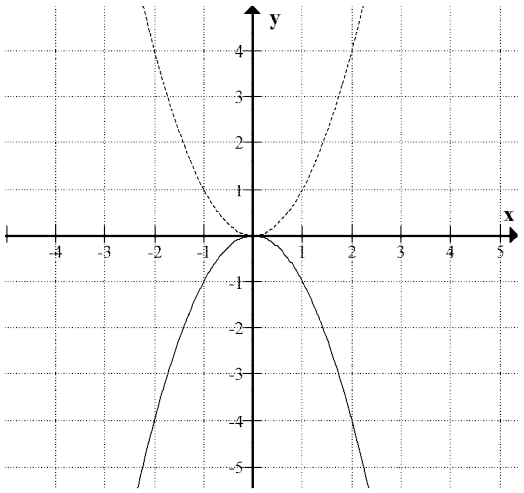
$y = -x^2$
$y = -(0)^2$
$y = -4$

$y = -x^2$
$y = -(1)^2$
$y = -1$

$y = -x^2$
$y = -(2)^2$
$y = -4$

Notice: The graph of $y = -x^2$ is the graph of $y = x^2$ opening downwards. Over 1, 1 squared = 1, down 1. Back to the vertex. Over 2, 2 squared = 4, down 4.

$y = x^2$



$y = -x^2$

C11 - 3.2 - Quadratics Vertical Exp Notes ($2x^2, \frac{1}{2}x^2$)

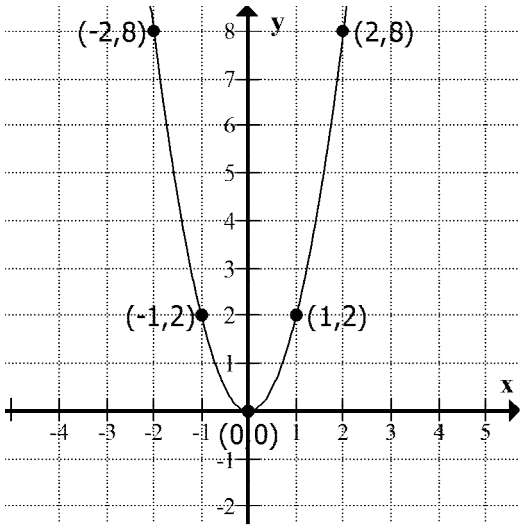
1) Graphing: $y = ax^2$ $y = 2x^2$

$y = 2x^2$

Table of Values

x	y
-2	8
-1	2
0	0
1	2
2	8

Pt.
(−2,8)
(−1,2)
(0,0)
(1,2)
(2,8)



$y = 2x^2$
$y = 2(-2)^2$
$y = 2(4)$
$y = 8$

$y = 2x^2$
$y = 2(-1)^2$
$y = 2(1)$
$y = 2$

$y = 2x^2$
$y = 2(0)^2$
$y = 2(0)$
$y = 0$

$y = 2x^2$
$y = 2(1)^2$
$y = 2(1)$
$y = 2$

$y = 2x^2$
$y = 2(2)^2$
$y = 2(4)$
$y = 8$

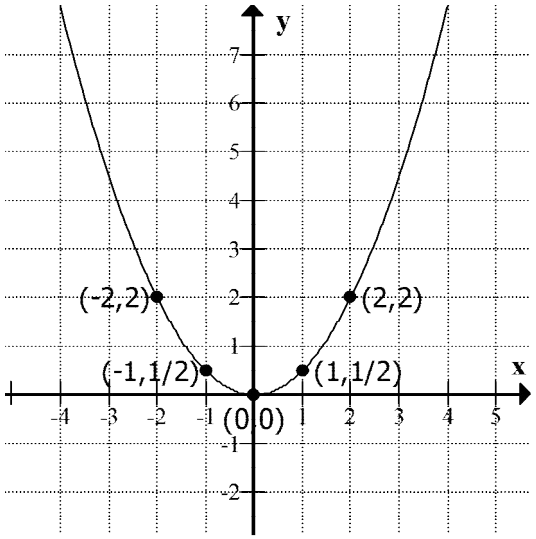
Notice: the pattern from the vertex (0,0) is symmetrical on both sides.
Over 1, 1 squared = 1, 1 times 2 = 2, up 2. Back to the vertex. Over 2, 2 squared = 4, 4 times 2 = 8, up 8.
In the last two steps, we are multiplying by 2 because $a = 2$.

$y = \frac{1}{2}x^2$

Table of Values

x	y
-2	2
-1	$\frac{1}{2}$
0	0
1	$\frac{1}{2}$
2	2

Pt.
(−2,2)
$(-1,\frac{1}{2})$
(0,0)
$(1,\frac{1}{2})$
(2,2)



$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(-2)^2$
$y = \frac{1}{2}(4)$
$y = 2$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(-1)^2$
$y = \frac{1}{2}(1)$
$y = \frac{1}{2}$

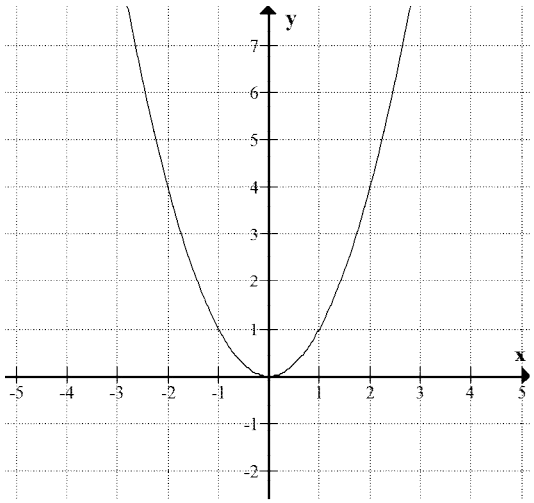
$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(0)^2$
$y = \frac{1}{2}(0)$
$y = 0$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(1)^2$
$y = \frac{1}{2}(1)$
$y = \frac{1}{2}$

$y = \frac{1}{2}x^2$
$y = \frac{1}{2}(2)^2$
$y = \frac{1}{2}(4)$
$y = 2$

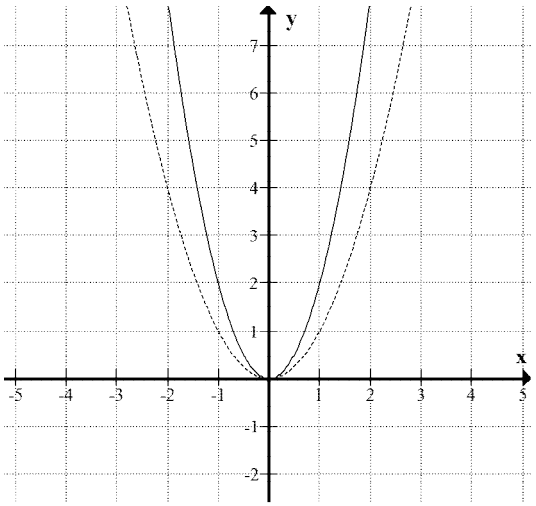
C11 - 3.2 - Quadratics Compression/Expansion Summary

$y = x^2$



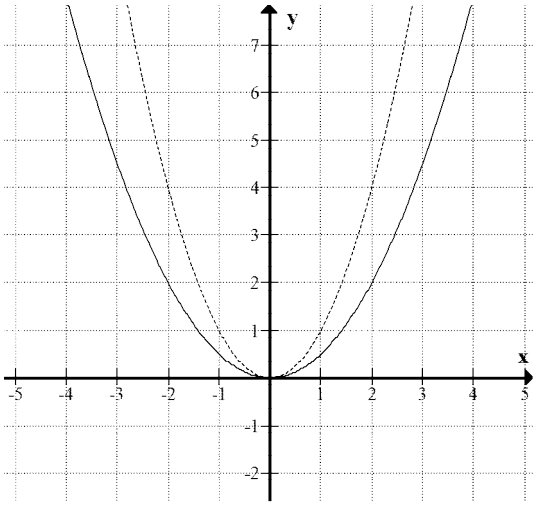
Expand

$y = 2x^2$

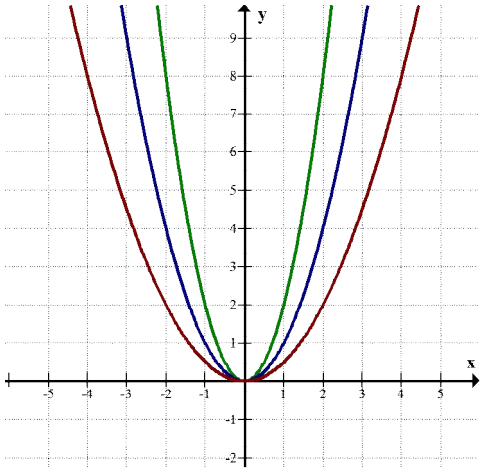


Compress

$y = \frac{1}{2}x^2$



$y = \frac{1}{2}x^2$ $y = 2x^2$ $y = x^2$



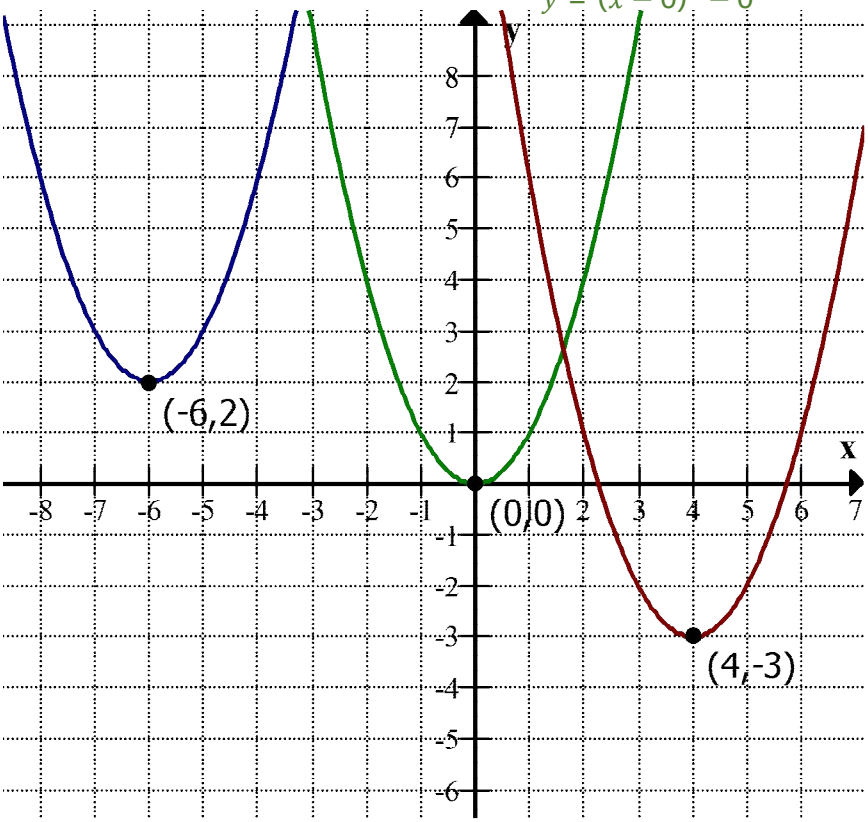
C11 - 3.2 - Quadratics Vertical/Horizontal Combo Notes

$y = (x + 6)^2 + 2$

$y = x^2$

$y = (x - 0)^2 - 0$

$y = (x - 4)^2 - 3$

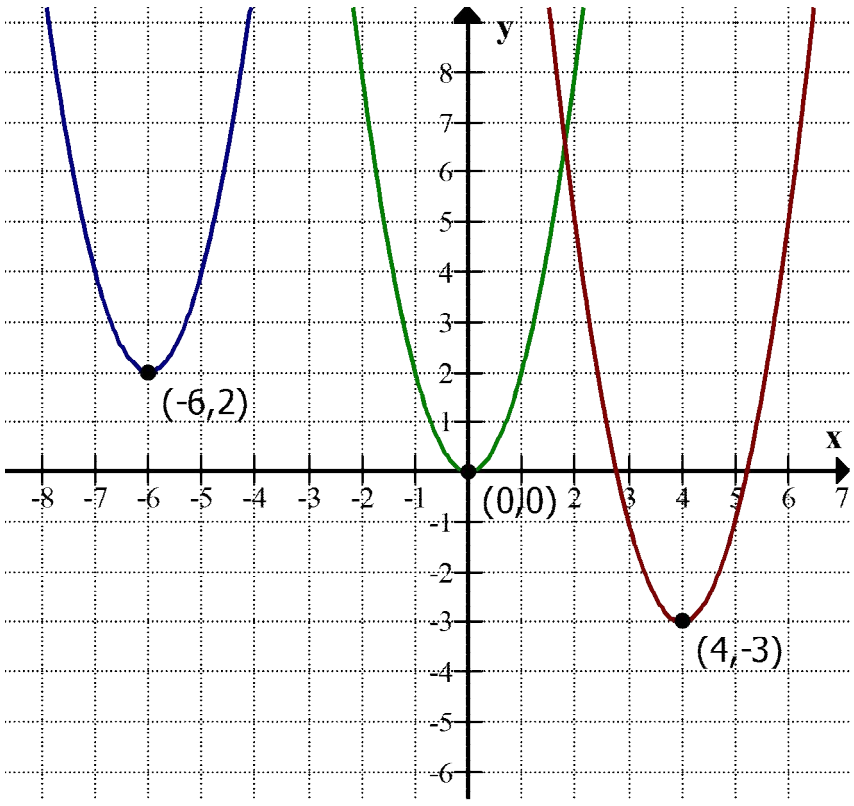


$y = 2(x + 6)^2 + 2$

$y = 2x^2$

$y = 2(x - 0)^2 - 0$

$y = 2(x - 4)^2 - 3$



C11 - 3.3 - Completing the Square Notes

$Standard\ form \longrightarrow Vertex\ form$

$y = ax^2 + bx + c \longrightarrow y = a(x-p)^2 + q \qquad Vertex = (p, q)$

$y = x^2 + 6x + c$

$y = x^2 + 6x + 9$ $\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9}$ "b" divided by 2 all squared: $\left(\frac{b}{2}\right)^2$

$y = (x+3)(x+3)$ Factor

$y = (x+3)^2$

Vertex form: Vertex = $(-3, 0)$

$a = 1$

$y = x^2 - 4x + 3$

$y = (x^2 - 4x) + 3$ $Group\ x\ terms$

$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4}$ "b" divided by 2 all squared: $\left(\frac{b}{2}\right)^2$

$y = (x^2 - 4x + 4 - 4) + 3$ Add and subtract inside brackets

$y = (x^2 - 4x + 4) - 4 + 3$ Remove number not contributing to perfect square (the negative)

$y = (x-2)(x-2) - 1$ Factor brackets, simplify outside

$y = (x-2)^2 - 1$ Vertex form: Vertex = $(2, -1)$

$a \neq 1$

$y = 2x^2 - 8x + 3$

$y = (2x^2 - 8x) + 3$ $Group\ x\ terms$

$y = 2(x^2 - 4x) + 3$ Factor out coefficient of x^2

$\boxed{\left(\frac{b}{2}\right)^2 = \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4}$ New "x" coefficient divided by 2 all squared: $\left(\frac{b}{2}\right)^2$

x2

$y = 2(x^2 - 4x + 4 - 4) + 3$ Add and subtract inside brackets

$y = 2(x^2 - 4x + 4) - 8 + 3$ Remove number not contributing to perfect square (the negative)
Don't forget to multiply by "a"

$y = 2(x-2)(x-2) - 5$ Factor brackets, simplify outside

$y = 2(x-2)^2 - 5$ Vertex form: Vertex = $(2, -5)$

Remember: $\frac{b}{2a}$ or $\frac{"new\ b"}{2}$ is the number that goes inside the brackets with x

C11 - 3.3 - Find Vertex Form Vertex Point Notes

Using the vertex and a point on the parabola, find the equation in Vertex Form.

$\boldsymbol{Vertex}: (\mathbf{5}, \mathbf{4})$ $\boldsymbol{and\ Point}: (\mathbf{7}, \mathbf{8})$

$y = a(x-p)^2 + q$

$y = a(x-p)^2 + q$	Write the general equation.
$y = a(x-(5))^2 + 4$	Put in the values of the Vertex for (p, q)
$y = a(x-5)^2 + 4$	Write in Vertex form. a still unknown.
$8 = a(7-5)^2 + 4$	Put in the values of the point for (x, y)
$8 = a(2)^2 + 4$	
$8 = 4a + 4$	
$-4 \quad -4$	
$\frac{4}{4} = \frac{4a}{4}$	Solve for a.
$1 = a$	
$a = 1$	Put in the value for "a" to find the equation.
$y = 1(x-5)^2 + 4$	

$\boldsymbol{Vertex}: (\mathbf{3}, -\mathbf{2})$ $\boldsymbol{and\ x-intercept = 4}$

$y = a(x-p)^2 + q$
$y = a(x-(3))^2 - 2$
$y = a(x-3)^2 - 2$

$0 = a(4-3)^2 - 2$
$0 = a(1)^2 - 2$
$0 = 1a - 2$
$+2 \quad +2$
$2 = a$
$a = 2$
$y = 2(x-3)^2 - 2$

$\boldsymbol{Vertex}: \left(-\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{1}\right)$ $\boldsymbol{and\ Point}: \left(\frac{\mathbf{1}}{\mathbf{2}}, \mathbf{2}\right)$

$y = a(x-p)^2 + q$
$y = a\left(x-\left(-\frac{1}{2}\right)\right)^2 + 1$
$y = a\left(x+\frac{1}{2}\right)^2 + 1$

$2 = a\left(\frac{1}{2}+\frac{1}{2}\right)^2 + 1$
$2 = a(1)^2 + 1$
$2 = a(1) + 1$
$-1 \quad -1$
$1 = a$
$y = \left(x+\frac{1}{2}\right)^2 + 1$

C11 - 3.3 - Vertex: $(-\frac{b}{2a}, y)$ Quadratics in Standard Form Notes

1) $y = x^2 - 6x + 5$

$Vertex = \left(\frac{-b}{2a}, y\right)$

$Vertex = \left(\frac{-(-6)}{2(1)}, y\right)$

$Vertex = \left(\frac{6}{2}, y\right)$

$Vertex = (3, y)$

$\boxed{Vertex = \left(\frac{-b}{2a}, y\right)}$

$y = x^2 - 6x + 5$
$y = (3)^2 - 6(3) + 5$
$y = 9 - 18 + 5$
$y = -4$

Substitute 3 in for x and solve for y

$Vertex = (3, -4)$

$y = x^2 - 6x + 5$

$Vertex = (3, -4)$

Vertex:

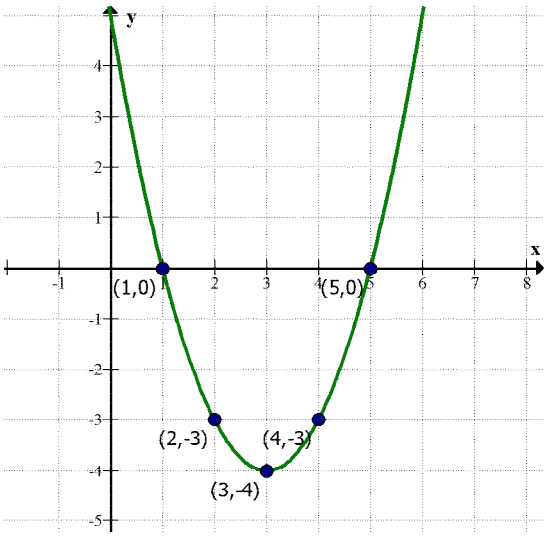
x	y
1	0
2	−3
3	−4
4	−3
5	0

$y = x^2 - 6x + 5$
$y = (1)^2 - 6(1) + 5$
$y = 1 - 6 + 5$
$y = 0$

$y = x^2 - 6x + 5$
$y = (2)^2 - 6(2) + 5$
$y = 4 - 12 + 5$
$y = -3$

$y = x^2 - 6x + 5$
$y = (4)^2 - 6(4) + 5$
$y = 16 - 24 + 5$
$y = -3$

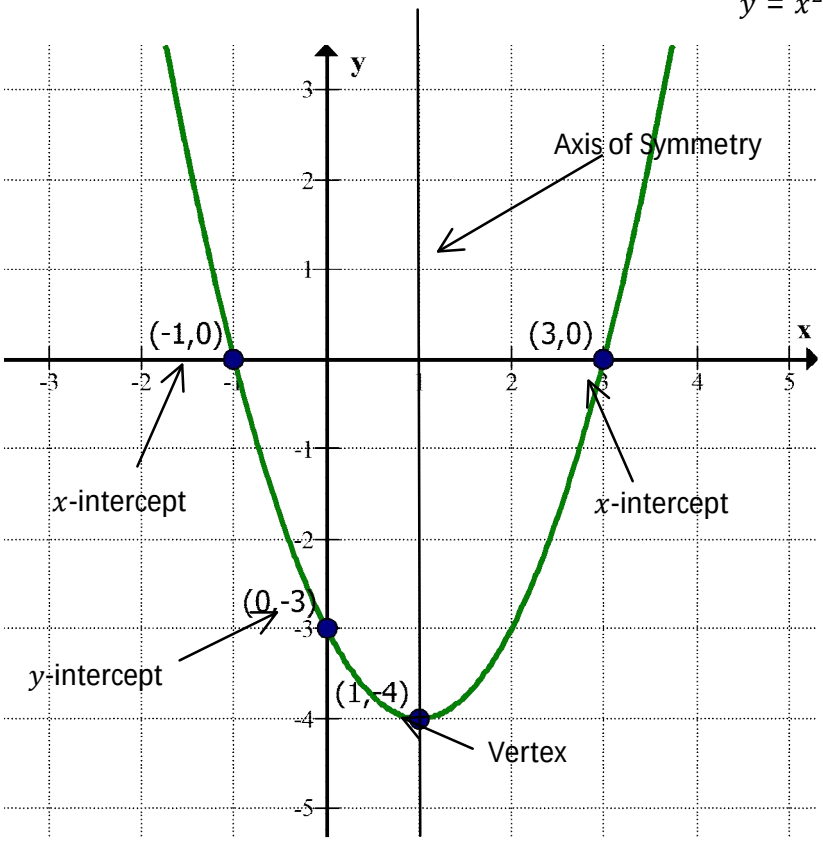
$y = x^2 - 6x + 5$
$y = (5)^2 - 6(5) + 5$
$y = 25 - 30 + 5$
$y = 0$



C11 - 3/4 - Key Points of Quadratic Functions Notes $a = 1$

Important Parts of a Quadratic Function:
- Vertex
- Shape
- x-intercepts
- y-intercept

$y = x^2 - 2x - 3$



Vertex:

x	y
-2	5
-1	0
0	-3
1	-4
2	-3
3	0
4	5

	Vertex Form	Standard Form	Factored Form
Equation	$y = (x - 1)^2 - 4$	$y = x^2 - 2x - 3$	$y = (x + 1)(x - 3)$
Info	Vertex: $(1, -4)$	y-intercept: $(0, -3)$	x-intercepts: $(-1,0), (3,0)$

Standard to Vertex

Standard Form: $y = x^2 - 2x - 3$

↓ Complete the square.

$y = (x^2 - 2x) - 3$

$y = (x^2 - 2x + 1 - 1) - 3$

Vertex Form: $y = (x - 1)^2 - 4$

Standard to Factored

$y = x^2 - 2x - 3$

$y = (x + 1)(x - 3)$

Standard Form: ↓ Factor. Factored Form:

Vertex to Standard

Vertex Form: $y = (x - 1)^2 - 4$

↓ FOIL

$y = (x^2 - 2x + 1) - 4$

$y = x^2 - 2x + 1 - 4$

Standard Form: $y = x^2 - 2x - 3$

Factored to Standard

$y = (x + 1)(x - 3)$

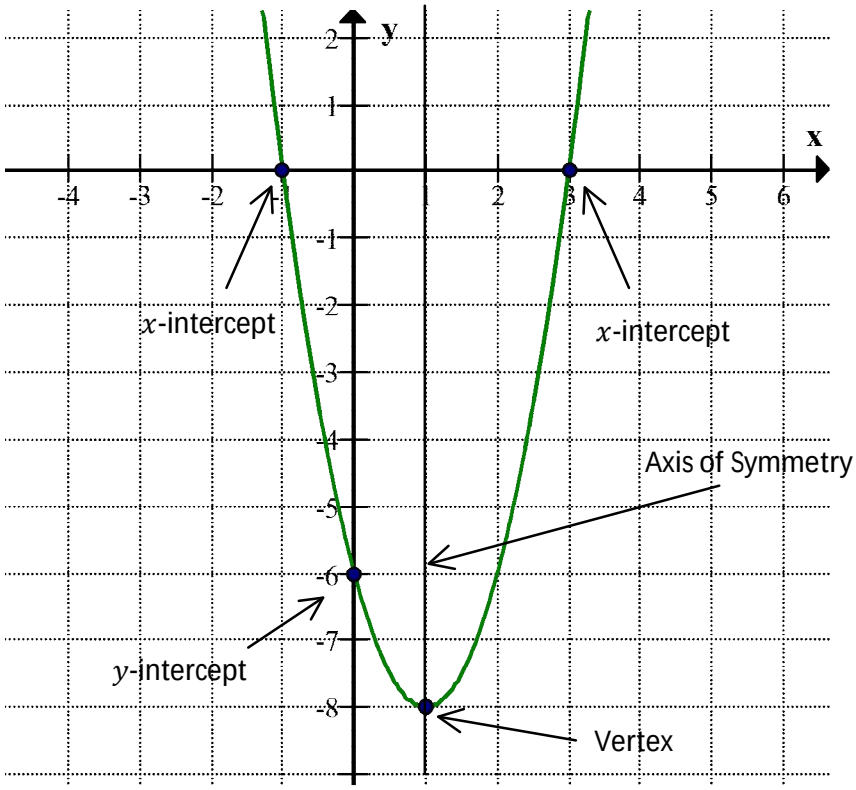
$y = (x^2 - 3x + x - 3)$

$y = x^2 - 2x - 3$

Factored Form: ↓ FOIL Standard Form:

C11 - 3/4 - Key Points of Quadratic Functions Notes $a \neq 1$

Important Parts of a Quadratic Function:
- Vertex
- Shape
- x-intercepts
- y-intercept



Vertex:

x	y
-2	10
-1	0
0	-6
1	-8
2	-6
3	0
4	10

	Vertex Form	Standard Form	Factored Form
Equation	$y = 2(x-1)^2 - 8$	$y = 2x^2 - 4x - 6$	$y = 2(x+1)(x-3)$
Info	Vertex: $(1,-8)$	y-intercept: $(0,-6)$	x-intercepts: $(0,-1), (0,3)$

Standard to Vertex

Standard Form: $y = 2x^2 - 4x - 6$
↓ Complete the square.
$y = 2(x^2 - 2x) - 6$
$y = 2(x^2 - 2x + 1 - 1) - 6$
Vertex Form: $y = 2(x-1)^2 - 6$

Standard to Factored

$y = 2x^2 - 4x - 6$
$y = 2(x^2 - 2x - 3)$
$y = 2(x-3)(x+1)$
Standard Form: ↓ Factor. Factored Form:

Vertex to Standard

Vertex Form: $y = 2(x-1)^2 - 8$
↓ FOIL
$y = 2(x^2 - 2x + 1) - 8$
$y = 2x^2 - 4x + 2 - 8$
Standard Form: $y = 2x^2 - 4x - 6$

Factored to Standard

Factored Form: $y = 2(x+1)(x-3)$
↓ FOIL
$y = 2(x^2 - 4x - 3)$
$y = 2(x^2 - 2x - 3)$
Standard Form: $y = 2x^2 - 4x - 6$