C11 - 1.1 - Arithmetic Means Notes

Write the first terms 5 of the sequence

$t_1 = 2, d = 3$

2,5,8,11,14

$t_1 = -7, t_3 = 5$

$3 - 1 = 2$

$-7 + 2d = 3$
$2d = 10$
$d = 5$

$-7, -2, 3, 8, 13$

$t_1 = 2, t_4 = -4$

$4 - 1 = 3$

$2 + 3d = -4$
$3d = -6$
$d = -2$

$2, 0, -2, -4, -6$

Find t_1 and r

$t_2 = 2, t_3 = 4$

$3 - 2 = 1$

$2 + d = 4$
$d = 2$

$2, 4, 6, 8, 10$

$t_2 = 2, t_4 = -8$

$4 - 2 = 2$

$2 + 2d = -8$
$2d = -10$
$d = -5$

$2, -3, -8, -13, -18$

$t_2 = 2, t_5 = -13$

$5 - 2 = 3$

$2 + 3d = -13$
$3d = -15$
$d = -5$

$7, 2, -3, -8, -13$

$t_3 = 4, t_{10} = 39$

$10 - 3 = 7$

$4 + 7d = 39$
$7d = 35$
$d = 5$

$-1, 4, 9, 14, 19$

C11 - 1.1 - Arithmetic Sequences Notes

$\underset{\substack{t_1 \\ n=1}}{\underline{\;2\;}}, \; \underset{\substack{t_2 \\ n=2}}{\underline{\;5\;}}, \; \underset{\substack{t_3 \\ n=3}}{\underline{\;8\;}}, \; \underset{t_4}{\underline{\;?\;}}, \; \underset{t_5}{\underline{\;?\;}} \; \ldots\ldots \; \underset{\substack{t_n \\ n=n}}{\underline{\;?\;}}$

+3 +3

$t_1 = first\ term$ (aka: "a")
$d = common\ difference$
$t_n = term\ n$
$n = Term\ \#, or\ number\ of\ terms$

$t_1 = 2$

$d = t_n - t_{n-1}$ $\quad$ $d = t_n - t_{n-1}$ $\quad$ $\boxed{d = t_n - t_{n-1}}$ Difference $\quad$ *A term subtracted by the term before it*

$d = 8 - 5$ $\quad$ $d = 5 - 2$

$d = 3$ $\quad$ $d = 3$ $\quad$ Arithmetic: d must always be the same

1. *Find the General term* $t_n = ?$

$t_n = t_1 + (n-1)d$
$t_n = 2 + (n-1)3$
$t_n = 2 + 3n - 3$
$t_n = 3n - 1$

$\boxed{t_n = t_1 + (n-1)d}$ General term formula

The first term plus 'n − 1' *differences*

2. **What is the tenth term t_{10}?**

$t_n = 3n - 1$
$t_{10} = 3(10) - 1$
$t_{10} = 29$

Check your answer: 2,5,8,11,14,17,20,23,26,29

Or, Start from beginning

$$\boxed{\begin{aligned} t_n &= t_1 + (n-1)d \\ t_{10} &= 2 + (10-1)3 \\ t_{10} &= 2 + 27 \\ t_{10} &= 29 \end{aligned}}$$

Remember: You could have also added the common difference 7 times to Term 3 (t_3)

3. **53 is what term, $t_n = 53, n = ?$**

$t_n = 3n - 1$
$51 = 3n \cancel{- 1}$
$+1 \quad \cancel{+ 1}$
$\frac{52}{3} = \frac{\cancel{3}n}{\cancel{3}}$
$n = 18$

Check your answer: 2,5,8,11,14,17,20,23,26,29,32,35,28,41,44,47,50,53

C11 - 1.2 - Arithmetic Series Notes

$$\underset{\substack{t_1 \\ n=1}}{2} \xrightarrow{+3} \underset{\substack{t_2 \\ n=2}}{5} \xrightarrow{+3} \underset{\substack{t_3 \\ n=3}}{8}, \underset{t_4}{?}, \underset{t_5}{?} \; ..... \; \underset{\substack{t_n \\ n=n}}{?}$$

$t_1 = first\ term$ (aka: "a")
$d = common\ difference$
$t_n = term\ n$
$n = Term\ \#, or\ number\ of\ terms$

$t_1 = 2$

$d = t_n - t_{n-1}$ $d = t_n - t_{n-1}$ $\boxed{d = t_n - t_{n-1}}$ $A\ term\ subtracted\ by\ the\ term\ before\ it$

$d = 8 - 5$ $d = 5 - 2$

$d = 3$ $d = 3$ Arithmetic: d must always be the same

$\mathbf{1.}$ $\boldsymbol{What\ is\ the\ sum\ of\ the\ first\ twelve\ terms\ s_{12}?\ s_{12} = ?, n = 12}$.

$s_n = \frac{n}{2}(t_1 + t_n)$

$s_{12} = \frac{12}{2}(2 + t_{12})$

$s_{12} = 6(2 + 35)$

$s_{12} = 222$

$t_n = 3n - 1$

$t_{12} = 3(12) - 1$

$t_{12} = 35$

$$\boxed{s_n = \frac{n}{2}(t_1 + t_n)}$$

Sum of "n" terms formula: if t_n is known.

$Check\ your\ answer$: 2 + 5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32 + 35 = 222

OR

$s_n = \frac{n}{2}(2t_1 + (n-1)d)$

$s_{12} = \frac{12}{2}(2(2) + (12 - 1)3)$

$s_{12} = 6(4 + (11)3)$

$s_{12} = 6(4 + 33)$

$s_{12} = 6(37)$

$s_{12} = 222$

$$\boxed{s_n = \frac{n}{2}(2t_1 + (n-1)d)}$$

Sum of "n" terms formula: if t_n is not known.

C11 - 1.3 - Geometric Means Notes

Write the first terms 5 of the sequence

$t_1 = 2, r = 3$

2, 6, 18, 54, 162

$t_1 = 4, t_3 = 16$

$3 - 1 = 2$

$4r^2 = 16$
$r^2 = 4$
$r = \pm 2$

2, 4, 8, 16, 32

2, −4, 8, −16, −32

$t_1 = 2, t_5 = 162$

$5 - 1 = 4$

$2r^4 = 162$
$r^4 = 81$
$r = \pm 3$

2, 6, 18, 54, 162

2, −6, 18, −54, 162

$t_2 = 9, t_5 = 243$

$5 - 2 = 3$

$9r^3 = 243$
$r^3 = 27$
$r = 3$

3, 9, 27, 81, 243

Find t_1 *and* r

$t_1 = 3, t_5 = 243$

$5 - 1 = 4$

$3r^4 = 243$
$r^4 = 81$
$r = \pm 3$

3, 9, 27, 81, 243

3, −9, 27, −81, 243

$t_3 = 4, t_{10} = 512$

$10 - 3 = 7$

$4r^7 = 512$
$r^7 = 128$
$r = 2$

1, 2, 4, 8, 16 ...

C11 - 1.3 - Geometric Sequences Notes

$$\underset{\substack{t_1\\ n=1}}{\underline{3}},\ \xrightarrow{\times 2}\ \underset{\substack{t_2\\ n=2}}{\underline{6}},\ \xrightarrow{\times 2}\ \underset{\substack{t_3\\ n=3}}{\underline{12}},\ \ldots\ \underset{t_6}{\underline{?}},\ \ldots\ \underset{\substack{t_n\\ n=n}}{\underline{?}}$$

$t_1 = first\ term\ (aka$: "a")
$d = common\ difference$
$t_n = term\ n$
$n = Term\ \#, or\ number\ of\ terms$

$t_1 = 3$

$r = \frac{t_n}{t_{n-1}}$ $r = \frac{t_n}{t_{n-1}}$ $\boxed{r = \frac{t_n}{t_{n-1}}}$ $A\ term\ divided\ by\ the\ term\ before\ it$

$r = \frac{6}{3}$ $r = \frac{12}{6}$

$r = 2$ $r = 2$ Geometric: r must always be the same

1. $Find\ the\ General\ term\ t_n = ?$

$t_n = t_1 r^{n-1}$
$t_n = 3(2)^{n-1}$

$\boxed{t_n = t_1 r^{n-1}}$ General term formula

2. $\boldsymbol{What\ is\ the\ fifth\ term\ t_5? t_5 = ?, n = 5.}$

$t_n = 3(2)^{n-1}$
$t_5 = 3(2)^{n-1}$
$t_5 = 3(2)^{5-1}$
$t_5 = 3(2)^4$
$t_5 = 48$

$Check\ your\ answer$: 3,6,12,24,48
Remember: You could have also multiplied the common ratio 2 times to t_3

3. $The\ number$ 768 $is\ what\ term?\ t_n = 768, n = ?$

$t_n = 3(2)^{n-1}$
$768 = 3(2)^{n-1}$ $divide\ both\ sides\ by$ 3
$256 = 2^{n-1}$
$2^7 = 2^{n-1}$ $Change\ of\ base$: $256 = 2^8$
$7 = n - 1$ $bases\ ae\ the\ same, exponents\ are\ equal$

$2^x = 2^4$
$x = 4$

C11 - 1.4 - Geometric Series Notes

$$\underset{\substack{t_1 \\ n=1}}{\underline{3}},\ \xrightarrow{\times 2}\ \underset{\substack{t_2 \\ n=2}}{\underline{6}},\ \xrightarrow{\times 2}\ \underset{\substack{t_3 \\ n=3}}{\underline{12}}, \ldots\ \underset{t_6}{\underline{?}}, \ldots\ \underset{\substack{t_n \\ n=n}}{\underline{?}}$$

$t_1 = first\ term$ $(aka$: "a")
$d = common\ difference$
$t_n = term\ n$
$n = Term\ \#, or\ number\ of\ terms$

$t_1 = 3$

$r = \frac{t_n}{t_{n-1}}$ $\quad r = \frac{t_n}{t_{n-1}}$ $\quad \boxed{r = \frac{t_n}{t_{n-1}}}$ $\quad$ $A\ term\ divided\ by\ the\ term\ before\ it$

$r = \frac{6}{3}$ $\quad r = \frac{12}{6}$

$r = 2$ $\quad r = 2$ $\quad$ Geometric: r must always be the same

$\mathbf{1}.\boldsymbol{What\ is\ the\ sum\ of\ the\ first\ eight\ terms\ s_8}?\ \boldsymbol{s_8} = ?, \boldsymbol{n = 8}.$

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$
$$s_8 = \frac{3(1 - 2^8)}{1 - 2}$$
$$s_8 = 765$$

$$\boxed{s_n = \frac{t_1(1 - r^n)}{1 - r}}$$

Sum of "n" terms formula (if number of terms is known)

$Check\ your\ answer$: $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 = 765$

<u>OR</u>

$$s_n = \frac{t_1 - rt_n}{1 - r}$$
$$s_8 = \frac{3 - 2(t_8)}{1 - 2}$$
$$s_8 = \frac{3 - 2(384)}{1 - 2}$$
$$s_8 = 756$$

$$t_n = 3(2)^{n-1}$$
$$t_8 = 3(2)^{8-1}$$
$$t_8 = 3(2)^7$$
$$t_8 = 3(128)$$
$$t_8 = 384$$

$$\boxed{s_n = \frac{t_1 - rt_n}{1 - r}}$$

Sum of "n" terms formula (if last term t_n is known)

$\mathbf{2}.\boldsymbol{What\ is\ the\ sum\ of\ an\ infinite\ number\ of\ terms}?$

$r = 2$ $\qquad r > 1, \therefore\ no\ sum$

C11 - 1.5 - Infinite Geometric Sequences Notes

Convergent, Has sum

What is the sum of the infinite sequence?

$$\underset{t_1}{\underline{8}},\ \underset{t_2}{\underline{4}},\ \underset{t_3}{\underline{2}},\ \underset{t_4}{\underline{1}},\ \underset{t_5}{\underline{\tfrac{1}{2}}},\ \underset{t_6}{\underline{\tfrac{1}{4}}},\ \ldots \qquad \left(\times\tfrac{1}{2},\ \times\tfrac{1}{2}\right)$$

$t_1 = 8$

$$r = \frac{t_n}{t_{n-1}} \qquad r = \frac{4}{8} \qquad r = \frac{1}{2}$$

$$r = \frac{t_n}{t_{n-1}} \qquad r = \frac{2}{4} \qquad r = \frac{1}{2}$$

$-1 < r < 1$

$-1 < \frac{1}{2} < 1$

$\therefore$ Convergent, has sum

$$s_\infty = \frac{t_1}{1-r} \qquad s_\infty = \frac{8}{1-\frac{1}{2}} \qquad s_\infty = \frac{8}{\frac{1}{2}} \qquad s_\infty = 16$$

$$\boxed{s_\infty = \frac{t_1}{1-r}}$$

Sum of "n" terms formula (infinite number of terms)

$Check\ your\ answer$: $8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 15.9375 = 16$

Divergent, No sum

What is the sum of the infinite sequence?

$$\underset{t_1}{\underline{8}},\ \underset{t_2}{\underline{16}},\ \underset{t_3}{\underline{32}},\ \underset{t_4}{\underline{64}},\ \underset{t_5}{\underline{128}},\ \underset{t_6}{\underline{256}},\ \ldots \qquad (\times 2,\ \times 2)$$

$t_1 = 8$

$$r = \frac{t_n}{t_{n-1}} \qquad r = \frac{16}{8} \qquad r = 2$$

$$r = \frac{t_n}{t_{n-1}} \qquad r = \frac{32}{16} \qquad r = 2$$

$r > 1$ $\therefore$ Divergent

$\therefore$ No sum

C11 - 1.1/2/3/4/5 - Sigma Notation - Notes

$\boldsymbol{Take\ the\ sum\ of\ the\ terms\ a_k\ from\ the\ index\ to\ n,\ going\ up\ by\ 1\ each\ time.}$

(Last term of k)

Formula to substitute k into

$\Sigma = Sigma\ (Sum\ terms)$

$$\sum_{k=1}^{n} a_k$$

$\#\ of\ terms = n - k + 1$

Index (first term of k)

Index: What k starts at. Goes up by 1 each time to reach n.

1. Put in k=bottom number
2. Put in k+1
3. Repeat until k=top number

Arithmetic

$\sum_{k=1}^{4} 2k =$ __2____, ___4___, ___6___, ___8___,

k=1 k=2 k=3 k=4

$2 + 4 + 6 + 8 = 20$

$2(1) = 2$

$2(2) = 4$

Geometric

$\sum_{k=2}^{6} 3(2)^{k-1} =$ ___6___, ___12___, ___24___, ___48___, ____96___,

k=2 k=3 k=4 k=5 k=6

$6 + 12 + 24 + 48 + 96 = 186$

$3(2)^{2-1} = 6$

$3(2)^{3-1} = 12$

C11 - 1.1-1.5 - Theory

1.1/1 $Theory: Why\ the\ formulas\ are\ the\ same$

$$s_n = \frac{n}{2}(t_1 + t_n) \qquad t_n = t_1 + (n-1)d$$

$Put\ General\ Term\ Formula\ in\ for\ t_n$

$$s_n = \frac{n}{2}(t_1 + (t_1 + (n-1)d))$$

$$s_n = \frac{n}{2}(2t_1 + (n-1)d) \quad = \quad s_n = \frac{n}{2}(2t_1 + (n-1)d)$$

$Simplify: t_1 + t_1 = 2t_1$

1.3/4 $Theory: Why\ the\ formulas\ are\ the\ same$

$$s_n = \frac{t_1 - rt_n}{1-r} \qquad t_n = t_1 r^{n-1}$$

$Put\ General\ Term\ Formula\ in\ for\ t_n$

$$s_n = \frac{t_1 - r(t_1 r^{n-1})}{1-r}$$

$$s_n = \frac{t_1(1 - r^1 r^{n-1})}{1-r}$$

$$s_n = \frac{t_1(1 - r^n)}{1-r} \quad = \quad s_n = \frac{t_1(1-r^n)}{1-r}$$

$Simplify: Factor\ out\ t_1, r^{1+n-1} = r^1$

1.5 $Theory\ on\ convergence$

$$s_n = \frac{t_1(1-r^n)}{1-r}$$

$$s_\infty = \frac{t_1(1-(\frac{1}{2})^\infty)}{1-r} \qquad \left(\frac{1}{2}\right)^\infty = 0$$

$$s_\infty = \frac{t_1(1-0)}{1-r}$$

$$s_\infty = \frac{t_1}{1-r} \quad = \quad s_\infty = \frac{t_1}{1-r}$$

$-1 < r < 1$ ∴ Convergent ∴ Has sum

1.5 $Theory\ on\ divergence$

$$s_n = \frac{t_1(1-r^n)}{1-r}$$

$$s_\infty = \frac{t_1(1-(2)^\infty)}{1-r} \qquad 2^\infty = \infty$$

$$s_\infty = \frac{t_1(1-\infty)}{1-r}$$

$$s_\infty = \frac{\infty}{1-r}$$

$$s_\infty = \infty$$

$|r| > 1$ ∴ Divergent ∴ No sum

OR

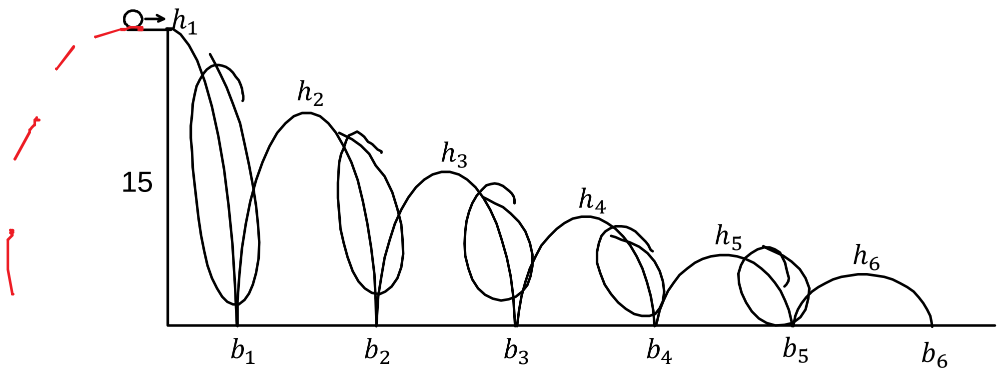
$r > 1$

$r < -1$

C11 - 1.6 - Bouncing Ball Notes (up 60%)

A ball rolls off a building 15 m tall. Each time the ball bounces on the floor, it rises to 60% of the previous height.

Height of Ball vs. Bounces



$r = 60\%$

$r = 0.6$

$\times 0.6 \quad \times 0.6 \quad \times 0.6$

$\underset{h_1}{15}, \quad \underset{h_2}{9}, \quad \underset{h_3}{5.4}, \quad \underset{h_4}{3.24},$

$r = \frac{9}{15}$

$r = .6$

$r = \frac{5.4}{9}$

$r = .6$

How high does the ball bounce after the first bounce? The third bounce?

$t_1 = h_1 = 15$

$h_2 = h_1 \times r$

$h_2 = 15 \times 0.6$

$h_2 = 9\,m$

$h_3 = h_2 \times r$

$h_3 = 9 \times 0.6$

$h_3 = 5.4\,m$

$h_4 = h_3 \times r$

$h_4 = 5.4 \times 0.6$

$h_4 = 3.24\,m$

How high does the ball bounce after the nth bounce? (Find the general formula)

$t_n = t_1 r^{n-1}$

$h_n = h_1 r^{n-1}$

$h_n = 15(0.6)^{n-1}$

How high does the ball bounce after the 4th bounce. $(t_5 = ?)$

$h_n = 15(0.6)^{n-1}$

$h_5 = 15(0.6)^{5-1}$

$h_5 = 15(0.6)^4$

$h_5 = 1.9m$

What is the total vertical distance the ball has travelled when it hits the ground for the 5th bounce? $(s_5 = ? \times \mathbf{2} - \mathbf{15})$

$s_n = \frac{t_1(1 - r^n)}{1 - r}$

$s_5 = \frac{15(1 - (.6)^5)}{1 - .6}$

$s_5 = \frac{15(.92)}{.4}$

$s_5 = 37.6m$

$34.6 \times 2 - 15 = 54.2m$

If it bounces forever, what is the total distance?

$r = 0.6 \quad r < 0$

$s_\infty = \frac{t_1}{1 - r}$

$h_\infty = \frac{h_1}{1 - r}$

$h_\infty = \frac{15}{1 - 0.6}$

$h_\infty = \frac{15}{0.4}$

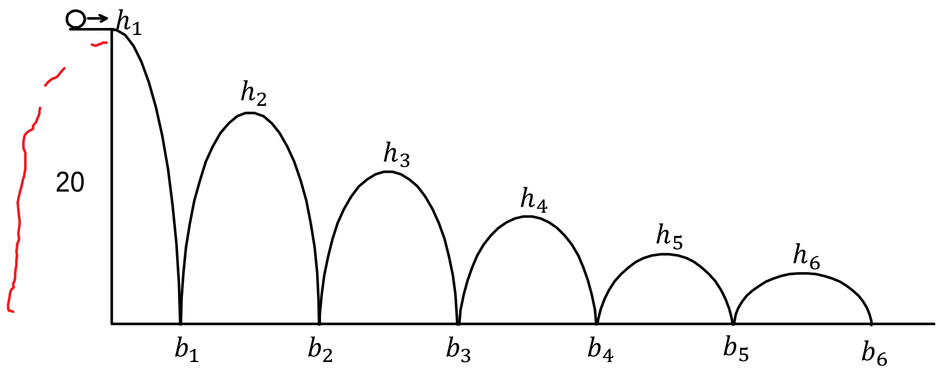
$h_\infty = 37.5\,m$

$\boxed{37.5 \times 2 - 15 = 60m}$

C11 - 1.6 - Bouncing Ball Notes (loses 60%)

A ball rolls off a building 20 m tall. Each time the ball bounces on the floor, it loses 60% of the previous height.

Height of Ball vs. Bounces



$r = 1 - 0.6$
$r = 0.4$

$$\underset{h_1}{\underline{20}} \xrightarrow{\times 0.4} \underset{h_2}{\underline{8}} \xrightarrow{\times 0.4} \underset{h_3}{\underline{3.2}} \xrightarrow{\times 0.4} \underset{h_4}{\underline{1.28}}$$

How high does the ball bounce after the first bounce? The third bounce?

$h_2 = h_1 \times r$
$h_2 = 20 \times 0.4$
$h_2 = 8\,m$

$h_3 = h_2 \times r$
$h_3 = 8 \times 0.4$
$h_3 = 3.2\,m$

$h_4 = h_3 \times r$
$h_4 = 3.2 \times 0.4$
$h_4 = 1.28\,m$

OR

$h_4 = h_1 \times r^3$
$h_4 = 20 \times 0.4^3$
$h_4 = 1.28\,m$

How high does the ball bounce after the nth bounce? (Find the general formula)

$t_n = t_1 r^{n-1}$

$h_n = h_1 \times r^{n-1}$
$h_n = 20 \times 0.4^{n-1}$

What is the total distance the ball has travelled when it hits the ground for the 2nd time? The 4th time?

$d_2 = h_1 + 2h_2$
$d_2 = 20 + 2(8)$
$d_2 = 20 + 16$
$d_2 = 36\,m$

$d_4 = h_1 + 2h_2 + 2h_3 + 2h_4$
$d_4 = 20 + 2(8) + 2(3.2) + 2(1.28)$
$d_4 = 20 + 16 + 6.4 + 1.56$
$d_4 = 43.96\,m$

What is the total distance the ball has travelled when it hits the ground for the 5th bounce.

$$s_n = \frac{t_1(1 - r^n)}{1 - r}$$
$$s_5 = \frac{15(1 - (.4)^5)}{1 - .6}$$
$$s_5 = \frac{15(.99)}{.4}$$
$$s_5 = 37.1m$$

$37.1 \times 2 - 15 = 59.2m$

If it bounces forever, what is the total distance?

$$s_\infty = \frac{t_1}{1 - r}$$
$$h_\infty = \frac{h_1}{1 - r}$$
$$h_\infty = \frac{20}{1 - 0.4}$$
$$h_\infty = \frac{20}{0.6}$$
$$h_\infty = 33.\bar{3}\,m$$

$$\boxed{33.3 \times 2 - 20 = 46.7m}$$