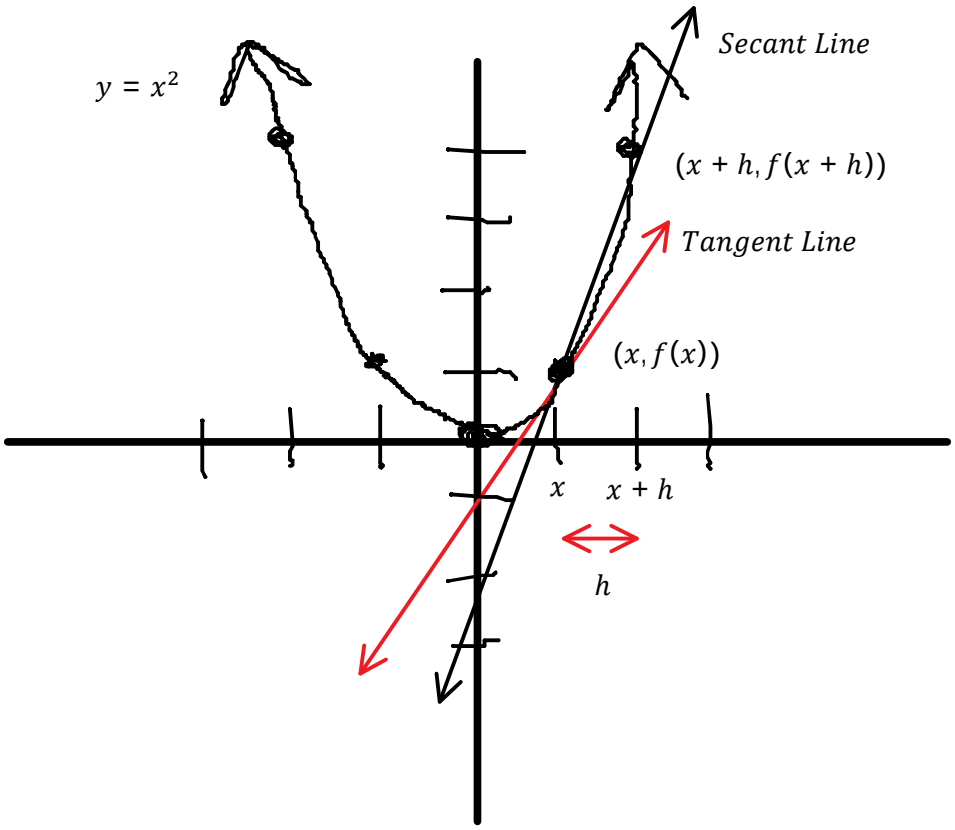
C12 - 2.1 - Definition of Derivative Equation Graph Notes



Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{f(x + h) - f(x)}{x + h - x}$$

$$\boxed{m = \frac{f(x + h) - f(x)}{h}}$$

Definition of the Derivative

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}}$$

$$\lim_{h \to 0} \frac{(x + h)^2 - (x^2)}{h}$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$\lim_{h \to 0} \frac{h(2x + h)}{h}$$

$$\lim_{h \to 0} 2x + h$$

$f'(x) = 2x$ *Slope of Tangent*

(1,1)

$x = 1$

$f'(x) = 2(1)$

$f'(x) = 2$ $\quad m = 2$

$y - y_1 = m(x - x_1)$ (1,1)

$y - 1 = 2(x - 1)$

$y = 2x - 2 + 1$

$y = 2x - 1$ *Tangent Line*

$y = 3x - 2$ *Secant Line*

$f'(x) = 2x$ (2,4)
$f'(x) = 2(2)$
$f'(x) = 4$

$y - y_1 = m(x - x_1)$

$y - 4 = 4(x - 2)$

$y = 4x - 8 + 4$

$y = 4x - 4$ *Tangent Line*

C12 - 2.1234 - Derivative Laws Notes

Basic Rules

$y = 2$
$y' = 0$

$y = 3x$
$y' = 3$

$y = 3x + 2x$
$y' = 3 + 2$
$y' = 5$

$y = 1x$
$y' = 1$

Power Rule

$y = x^n$
$y' = nx^{n-1}$

$y = x^2$
$y' = 2x^{2-1}$
$y' = 2x$

$y = x^3$
$\frac{dy}{dx} = 3x^{3-1}$
$\frac{dy}{dx} = 3x^2$

$y = 2x^3$
$y' = 3 \times 2x^{3-1}$
$y' = 6x^2$

Product Rule

$y = f(x)g(x)$
$y' = f'(x)g(x) + g'(x)f(x)$

$y = uv$
$y' = u'v + v'u$

$y = (2x + 1)(3x - 2)$
$y' = 2(3x - 2) + 3(2x + 1)$
$y' = 6x - 4 + 6x + 3$
$y' = 12x - 1$

Quotient Rule

$y = \frac{f(x)}{g(x)}$
$y' = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$

$y = \frac{u}{v}$
$y' = \frac{u'v - v'u}{v^2}$

$y = \frac{x^2}{2x + 1}$
$y' = \frac{2x(2x + 1) - 2(x^2)}{(2x + 1)^2}$
$y' = \frac{4x^2 + 2x - 2x^2}{(2x + 1)^2}$
$y' = \frac{2x^2 + 2x}{(2x + 1)^2}$

Chain Rule

$y = f(g(x))$
$y' = f'(g(x))(g'(x))$

$y = (2x + 1)^2$
$y' = 2(2x + 1)^{2-1} \times 2$
$y' = (4x + 2) \times 2$
$y' = 8x + 4$

Implicit Differentiation

$x^2 - y^2 = 4$
$2x - 2yy' = 0$
$-2yy' = -2x$
$y' = \frac{x}{y}$

$xy^2 - x = 2$
$1(y^2) + 2yy'(x) - 1 = 0$
$y^2 + 2xyy' - 1 = 0$
$2xyy' = 1 - y^2$
$y' = \frac{1 - y^2}{2x}$

$y^2 + xy = 0$
$2yy' + 1(y) + y'(x) = 0$
$2yy' + y + xy' = 0$
$2yy' + xy' = -y$
$y'(2y + x) = -y$
$y' = -\frac{y}{2y + x}$

GCF = y'

C12 - 2.4 - M8 VS Product VS Quotient VS Chain Rule

$y=(2x)^2$ $\quad y=(2x)^2$ $\quad y=(2x)^2$

$y=4x^2$ $\quad y=2x\cdot 2x$ $\quad y'=2(2x)^1\times 2$

$y'=8x$ $\quad y'=2(2x)+2(2x)$ $\quad y'=8x$

$y'=8x$

$y=x^2$

$y'=2x^1\cdot 1$

$y=\frac{1}{x}$ $\quad y=1(x)^{-1}$ $\quad y=\frac{0x-1}{x^2}$

$y=x^{-1}$

$y'=-x^{-2}$

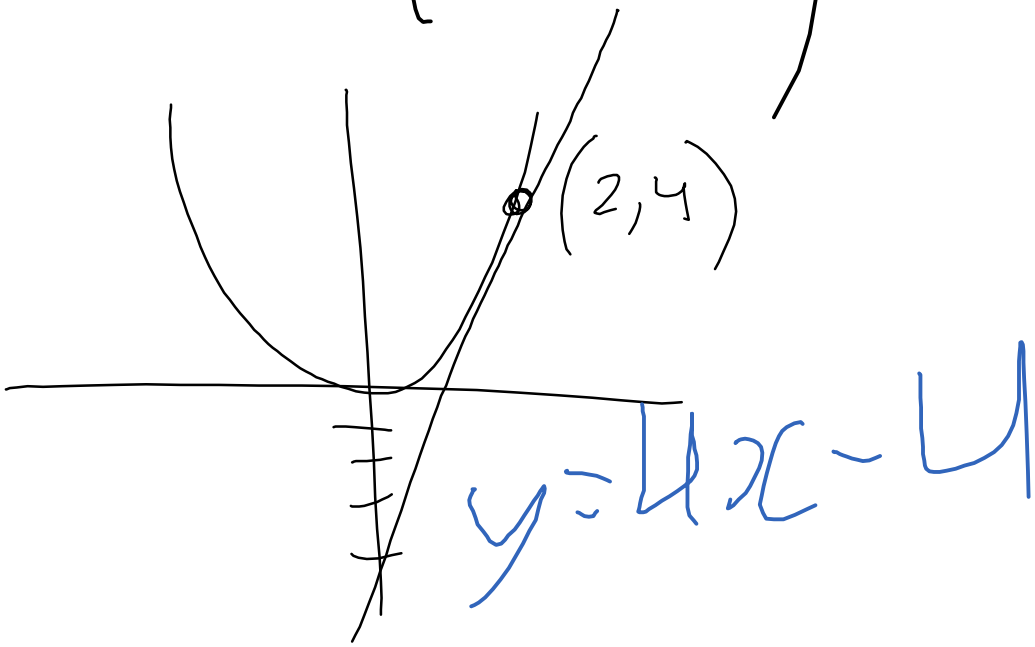
$y = x^2$ $(2, 4)$

$y' = 2x$

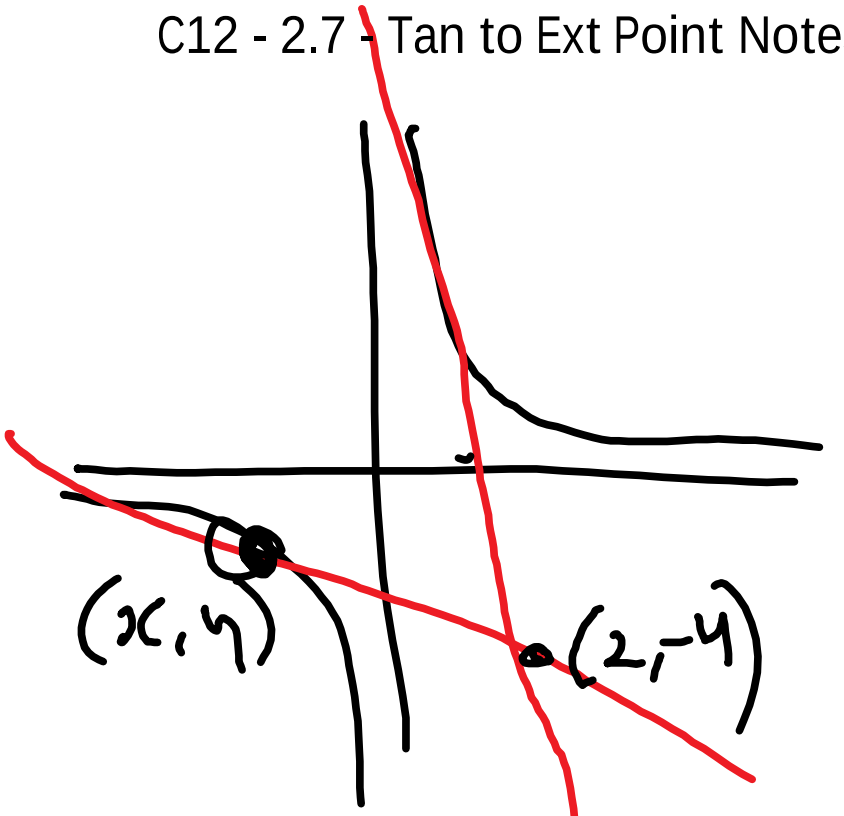
$m = 2(2)$

$m = 4$

$y - 4 = 4(x - 2)$



C12 - 2.7 - Tan to Ext Point Notes



$y = x^{-1}$

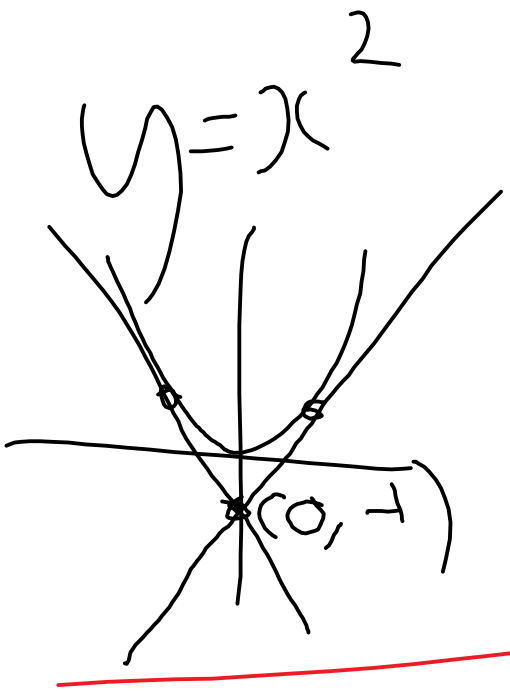
$y' = -x^{-2}$

$y' = \frac{-1}{x^2}$

$$m = \frac{y+4}{x-2} = -\frac{1}{x^2}$$

$$\frac{\frac{1}{x}+4}{x-2} = \frac{-1}{x^2}$$

C12 - 2.7 - Tan to Ext Point Notes



ext pt $(0, -1)$
$x \quad y_1$

$y = x^2$
$\downarrow_2$

$$2x = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2x = \frac{y_2 - (-1)}{x_2 - 0}$$

$$2x = \frac{y+1}{x}$$

$$2x = \frac{x+1}{x}$$

$$2x^2 = x^2 + 1$$

$$x^2 = 1$$

$$x = \pm 1$$

$(1, 1), (-1, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow y - y_1 = m(x - x_1)$$

$x_0 = 2 \qquad f(x) = x^2 - 2x - 5$

$f'(x) = 3x^2 - 2$

$y - y_1 = m(x - x_1)$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$x_1 = 2 - \dfrac{(-1)}{10}$

$x_1 = 2.1$

REPEAT