

## C12 - 6.1 - Multiply Simplify WS

**Simplify**

$$\sin x \sec x$$

$$\cos x \cos x$$

$$\tan x \cot x$$

$$\csc x \csc x$$

$$\sin x \sin x$$

$$\cos x \sec x$$

$$\tan x \csc x$$

$$\sec x \sec x$$

$$\sin x \cos x$$

$$\cos x \csc x$$

$$\tan x \tan x$$

$$\cot x \sec x$$

$$\sin x \cot x$$

$$\cos x \tan x$$

$$\cot x \cot x$$

$$\sin x \csc x$$

$$\cos x \sin x$$

$$\tan x \sin x \cot x$$

**Simplify to  $\sin x$  and  $\cos x$**

$$\sin x \tan x$$

$$\cos x \cot x$$

$$\tan x \sec x$$

$$\csc x \cot x$$

$$\csc x \sec x$$

## C12 - 6.1 - Multiply Simplify WS

**Simplify**

$$\sin^2 x \csc x$$

$$\csc^2 x \sin x$$

$$\csc x \cos^2 x$$

$$\sec^2 x \cos x$$

$$\sin^2 x \csc^2 x$$

$$\sin^2 x \cot^2 x$$

$$\cos^2 x \sec^2 x$$

$$\cos^2 x \tan^2 x$$

$$\csc^2 x \sec^2 x$$

$$\tan^2 x \cot^2 x$$

$$\csc^2 x \cot^2 x$$

$$\sec^2 x \tan^2 x$$

# C12 - 6.1 - Divide Simplify WS

**Simplify**

$$\frac{\sin x}{\sin x}$$

$$\frac{\sin x}{\cos x}$$

$$\frac{1}{\sin x}$$

$$\frac{\csc x}{\csc x}$$

$$\frac{1}{\tan x}$$

$$\frac{\cot x}{\cot x}$$

$$\frac{1}{\cos x}$$

$$\frac{1}{\cot x}$$

$$\frac{\sec x}{\sec x}$$

$$\frac{\cos x}{\cos x}$$

$$\frac{1}{\csc x}$$

$$\frac{\tan x}{\tan x}$$

$$\frac{\cos x}{\sin x}$$

$$\frac{1}{\sec x}$$

$$\frac{\sin x}{\tan x}$$

$$\frac{\cos x}{\cot x}$$

$$\frac{\cos x}{\tan x}$$

$$\frac{\cos x}{\cot x}$$

$$\frac{\sin x}{\cot x}$$

$$\frac{\tan x}{\sin x}$$

$$\frac{\tan x}{\cos x}$$

$$\frac{\sec x}{\cos x}$$

$$\frac{\csc x}{\cos x}$$

$$\frac{\cot x}{\cos x}$$

*Try it in your head!*

$$\frac{\cos x}{\sec x}$$

$$\frac{\cos x}{\csc x}$$

$$\frac{\tan x}{\csc x}$$

$$\frac{\sec x}{\tan x}$$

$$\frac{\cot x}{\sec x}$$

$$\frac{\csc x}{\tan x}$$

$$\frac{\sec x}{\cot x}$$

$$\frac{\csc x}{\sec x}$$

$$\frac{\csc x}{\cot x}$$

## C12 - 6.1 - Multiply Divide Simplify WS

**Simplify**

$$\frac{\sin x \cot x}{\sec x}$$

$$\frac{\cos x \tan x}{\sec x}$$

$$\frac{\csc x \tan x}{\csc x}$$

$$\frac{\cot x \sec^2 x}{\csc^2 x}$$

$$\frac{\tan x \csc^2 x}{\sec^2 x}$$

$$\frac{\cos x \sec^2 x}{\sec^2 x}$$

$$\frac{\sin x \csc^2 x}{\csc^2 x}$$

## C12 - 6.2 - FOIL Factor WS

Distribute/Foil

$$\sin x(1 - \sin x)$$

$$\cos x(\sin x + 1)$$

$$\sin x(1 + \sin x)$$

$$(1 + \cos x)(1 - \cos x)$$

$$(1 + \sin x)(1 - \sin x)$$

$$(\sin x - \cos x)^2$$

$$(\sin x + 2)(\sin x - 1)$$

$$(\cos x + 1)(\cos x - 3)$$

Factor

$$\sin x - \sin^2 x$$

$$\sin x \cos x + \cos x$$

$$\cos x + \cos^2 x$$

$$1 - \sin^2 x$$

$$1 - \cos^2 x$$

$$1 + \sin^2 x$$

$$\cos^2 x + \cos x - 2$$

$$\cos^2 x - 2\cos x - 3$$

$$2\sin^2 x + \sin x - 1$$

$$\csc x^2 - 2\csc x - 3$$

$$2\sin x - \frac{1}{\sin x} + 1$$

## C12 - 6.2 - FOIL Factor WS

Distribute/Foil

$$(\sin x - 2)(\sin x + 1)$$

$$(\sin^2 x - 1)(\sin^2 x + 2)$$

$$(\sin x + 1)(\sin x + 1)$$

$$\cos x(\cos x + 1)(\cos x - 1)$$

$$(\sin x + 1)(\sin x - 1)(\sin^2 x + 2)$$

Factor

$$\sin^2 x + 3\sin x + 2$$

$$\sin^2 x - 2\sin x + 1$$

$$\sin^4 x - 1$$

$$\sin^3 x - \sin x$$

$$\cos^4 x - \cos^2 x - 2$$

$$\cos^3 x + \cos^2 x - 2\cos x$$

## C12 - 6.2 - FOIL Factor Fractions WS

Distribute/Foil

$$(\sec x + 1)(\sec x - 1)$$

$$\frac{(\cos x + 1)(\cos x - 1)}{-\sin^2 x}$$

$$\frac{(\csc x + 1)(\csc x - 1)}{\cos^2 x}$$

$$\frac{(\csc x + \cot x)(\csc x - \cot x)}{\sin^2 x}$$

*Factor*

$$\frac{\sin^2 x - \cos^2 x}{\sin x + \cos x}$$

## C12 - 6.2 - Solve Factor Fractions WS

$$\sin x(1 - \sin x) = 0$$

$$\cos x(\sin x + 1) = 0$$

$$\sin x(1 + \sin x) = 0$$

$$(1 + \cos x)(1 - \cos x) = 0$$

$$(1 + \sin x)(1 - \sin x) = 0$$

$$(\sin x + 2)(\sin x - 1) = 0$$

$$(\cos x + 1)(\cos x - 3) = 0$$

$$\sin x - \sin^2 x = 0$$

$$\sin x \cos x + \cos x = 0$$

$$\cos x + \cos^2 x = 0$$

$$1 - \sin^2 = 0$$

$$1 - \cos^2 x = 0$$

$$1 + \sin^2 x = 0$$

$$\cos^2 x + \cos x - 2 = 0$$

$$\cos x^2 - 2\cos x - 3 = 0$$

$$2\sin x^2 + \sin x - 1 = 0$$

$$2\cos x - \frac{1}{\sec x} + 1 = 0$$

$$2\cos x - \frac{1}{\cos x} + 1 = 0$$



## C12 - 6.2 - Add Subtract Fractions WS

**Simplify**

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$\frac{\cos x}{\sin x} + \frac{1}{\sin x}$$

$$\cot x + \csc x$$

$$\sec x + \tan x$$

$$\sin x + \csc x$$

$$\sin x + \sec x$$

$$\cos x + \sec x$$

$$\cos x + \csc x$$

$$\sin x - \sec x$$

$$\cos x - \cot x$$

$$\cos x + \sin x \tan x$$

$$\sin x + \cos x \cot x$$

## C12 - 6.2 - Add Subtract Fractions Pythag WS

**Simplify**

$$\csc x - \cot x \cos x$$

$$\sec x - \tan x \sin x$$

$$\csc x \cos^2 x + \sin x$$

$$\sec x \sin^2 x + \cos x$$

$$1 + \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{1}{\cos^2 x} - 1$$

$$1 - \frac{1}{\sec^2 x}$$

$$\frac{1}{\sin^2 x} - 1$$

$$\frac{\cos x + \cot x}{1 + \sin x}$$

$$\csc^2 x - \frac{\cot x}{\sin x}$$

## C12 - 6.2 - Add Subtract Fractions Pythag WS

**Simplify**

$$\frac{1}{1 - \sin x} + \frac{1}{1 + \sin x}$$

$$\frac{\cos x}{1 + \cos x} + \frac{\cos x}{1 - \cos x}$$

$$\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x}$$

$$\frac{\cos x}{1 + \cos x} - \frac{\cos x}{1 - \cos x}$$

$$\frac{1}{1 + \cos x} - \frac{1}{1 - \cos x}$$

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$$

## C12 - 6.2 - Add Subtract Complex Fractions WS

**Simplify**

$$\frac{1 + \frac{1}{\sin x}}{\cot x}$$

$$\frac{1 + \frac{1}{\cos x}}{\tan x}$$

$$\frac{\sec x}{1 + \frac{1}{\cos x}}$$

$$\frac{\tan x}{1 + \frac{1}{\cos x}}$$

$$\frac{\sec x}{1 + \frac{\sin x}{\cos x}}$$

$$\frac{1 + \sin x}{1 + \csc x}$$

$$\frac{1 + \csc x}{1 + \sec x}$$

$$\frac{1 + \tan x}{1 + \cot x}$$

$$\frac{\csc x + \sec x}{\cot x + 1}$$

$$\frac{\csc x + \sec x}{\tan x + 1}$$

# C12 - 6.4 - Proofs Reciprocals WS

Prove the left hand side equals the right hand side

$\sin x \sec x$	$\tan x$
<hr/>	

$\cos x \tan x$	$\sin x$
<hr/>	

$\sin x \csc x$	1
<hr/>	

$\cos x \csc x$	$\cot x$
<hr/>	

$\cos x \sec x$	1
<hr/>	

$\tan x \csc x$	$\sec x$
<hr/>	

$\cot x \sec x$	$\csc x$
<hr/>	

$\sin x \cot x$	$\cos x$
<hr/>	

$\cos x \cot x$	$\frac{\cos^2 x}{\sin x}$
<hr/>	

$\cot x \cot x$	$\cot^2 x$
<hr/>	

$\cos x \sin x$	$\sin x \cos x$
<hr/>	

$\sin^2 x$	$\sin x \sin x$
<hr/>	

$\tan x \sec x$	$\frac{\sin x}{\cos^2 x}$
<hr/>	

$\tan x \cot x$	1
<hr/>	

<i>Make one up!</i>	
<hr/>	

## C12 - 6.4 - Proofs Reciprocals WS

Prove the left hand side equals the right hand side

$\frac{\sin x}{\tan x}$	$\cos x$
<hr/>	
<hr/>	

$\frac{\cos x}{\sec x}$	$\cos^2 x$
<hr/>	
<hr/>	

$\frac{1}{\cos x}$	$\sec x$
<hr/>	
<hr/>	

$\frac{\tan x}{\sin x}$	$\sec x$
<hr/>	
<hr/>	

$\frac{\tan x}{\cos x}$	$\frac{\sin x}{\cos^2 x}$
<hr/>	
<hr/>	

$\frac{\sin x}{\sin x}$	1
<hr/>	
<hr/>	

$\frac{\sin x}{\cot x}$	$\frac{\sin^2 x}{\cos x}$
<hr/>	
<hr/>	

$\frac{\sin x}{\cos x}$	$\tan x$
<hr/>	
<hr/>	

$\frac{\cos x}{\cot x}$	$\sin x$
<hr/>	
<hr/>	

$\frac{\sec x}{\tan x}$	$\csc x$
<hr/>	
<hr/>	

$\frac{\tan x}{\csc x}$	$\sec x$
<hr/>	
<hr/>	

$\frac{\csc x}{\cot x}$	$\cos x$
<hr/>	
<hr/>	

## C12 - 6.4 - Proofs Add Subtract Frac WS

Prove the left hand side equals the right hand side

$$\frac{\cot x + \csc x}{\sin x} \quad \frac{\cos x + 1}{\sin x}$$

$$\frac{1 + \sin x}{\cos x} \quad \sec x + \tan x$$

$$\sin x + \csc x \quad \frac{\sin^2 x + 1}{\sin x}$$

$$\sin x + \sec x \quad \frac{\sin x \cos x + 1}{\cos x}$$

$$2\sin x - \frac{1}{\csc x} \quad \sin x$$

$$\sec x - \tan x \sin x \quad \cos x$$

# C12 - 6.4 - Proofs Add Subtract Frac Pythag WS

Prove the left hand side equals the right hand side

$\cos x + \sin x \tan x$	$\sec x$
<hr/>	

$\csc x \cos^2 x + \sin x$	$\csc x$
<hr/>	

$\frac{\cos x + \cot x}{1 + \sin x}$	$\cot x$
<hr/>	

$\csc^2 x - \frac{\cot x}{\sin x}$	$\frac{1}{1 + \cos x}$
<hr/>	

1	$\frac{(1 - \sin^2 x)}{\cos^2 x}$
<hr/>	

$\cot x$	$\frac{\cos^2 x}{1 - \cos^2 x}$
<hr/>	



## C12 - 6.4 - Proofs Add Subtract Frac Pythag WS

Prove the left hand side equals the right hand side

$$\frac{1 - \frac{1}{\sec^2 x}}{\quad} \quad \sin^2 x$$

$$\frac{1 - \frac{1}{\cos^2 x}}{\quad} \quad -\tan^2 x$$

$$\frac{1 + \frac{1}{\tan^2 x}}{\quad} \quad \csc^2 x$$

$$\frac{2 - \frac{1}{\csc^2 x}}{\quad} \quad 1 + \cos^2 x$$

$$\frac{\csc x \cos^2 x + \sin x}{\quad} \quad \csc x$$

$$\frac{\sec x \sin^2 x + \cos x}{\quad} \quad \sec x$$

## C12 - 6.4 - Proofs FOIL Factor Pythag WS

Prove the left hand side equals the right hand side

$$\frac{(\sin x - 2)(\sin x + 1)}{\sin^2 x - \sin x - 2}$$

$$\frac{(1 + \sin x)(1 - \sin x)}{\cos^2 x}$$

$$\frac{(1 + \cos x)(1 - \cos x)}{\sin^2 x}$$

$$\frac{(2\cos x - 1)(\cos x + 2)}{2\cos^2 x + 3\cos x - 2}$$

$$\frac{\phantom{(\sin x - 2)(\sin x + 1)}}{\phantom{\sin^2 x - \sin x - 2}}$$

$$\frac{\phantom{(1 + \sin x)(1 - \sin x)}}{\phantom{\cos^2 x}}$$

## C12 - 6.4 - Proofs Add Subtract Foil Factor Pythag WS

Prove the left hand side equals the right hand side

$$\frac{(cscx + cotx)(cscx - cotx)}{\sin^2 x} \quad \csc^2 x$$

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} \quad 2\sec x$$

$$\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} \quad 2\csc^2 x$$

$$\frac{\cos x}{1 - \cos x} - \frac{\cos x}{1 + \cos x} \quad 2\cot^2 x$$

$$\frac{1}{1 - \cos x} - \frac{1}{1 + \cos x} \quad 2\cot x \csc x$$

$$(\sin x - \cos x)^2 \quad 1 - 2\sin x \cos x$$

# C12 - 6.4 - Proofs Add Subtract Foil Factor Pythag WS

Prove the left hand side equals the right hand side

$$\frac{\cos x - \cot x}{\sin x} = \frac{\sin x(1 + \cos x)}{\sin x}$$

$$\sec x \sin^2 x + \cos x = \sec x$$

$$3 - \sin^2 x = 2 + \cos^2 x$$

$$\sin x - \csc x = \frac{-\cos^2 x}{\sin x}$$

$$1 + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x$$

$$\frac{1 + \sin x}{1 + \csc x} = \frac{\sin x}{1 - \sin x}$$

# C12 - 6.4 - Proofs Add Subtract Comp Frac Pythag WS

Prove the left hand side equals the right hand side

$$\frac{\frac{\cos x + 1}{\sin x}}{\frac{1 + \frac{1}{\cos x}}{\tan x}}$$

$$\frac{1 + \frac{1}{\sin x}}{\cot x} \quad \frac{1 + \sin x}{\cos x}$$

$$\frac{\frac{\csc x}{1 + \frac{1}{\sin x}}}{\frac{1}{1 + \sin x}}$$

$$\frac{\frac{\cos x}{1 + \sin x}}{\frac{\cot x}{1 + \frac{1}{\sin x}}}$$

$$\frac{\frac{\csc x}{1 + \frac{\cos x}{\sin x}}}{\frac{1}{\sin x + \cos x}}$$

$$\frac{1}{1 + \tan x} \quad \frac{\cos x}{\sin x + \cos x}$$

# C12 - 6.4 - Proofs Add Subtract Comp Frac Pythag WS

Prove the left hand side equals the right hand side

$\frac{1 + \sin x}{1 + \csc x}$	$\sin x$
<hr/>	

$\cos x$	$\frac{1 + \cos x}{1 + \sec x}$
<hr/>	

$\frac{1 + \sec x}{1 + \csc x} + 1$	$2$
<hr/>	

$\frac{1 + \cot x}{1 + \tan x}$	$\cot x$
<hr/>	

$\frac{\csc x + \sec x}{\cot x + 1}$	$\sec x$
<hr/>	

$\csc x$	$\frac{\csc x + \sec x}{\tan x + 1}$
<hr/>	

# C12 - 6.4 - Proofs Conjugate HW

$\frac{\sin x}{1 + \cos x}$	$\frac{1 - \cos x}{\sin x}$
<hr/>	
<hr/>	

$\frac{\cos x}{1 - \sin x}$	$\frac{1 + \sin x}{\cos x}$
<hr/>	
<hr/>	

$\frac{\sin x}{1 - \cos x}$	$\frac{1 + \cos x}{\sin x}$
<hr/>	
<hr/>	

$\frac{\cos x}{1 + \sin x}$	$\frac{1 - \sin x}{\cos x}$
<hr/>	
<hr/>	

$\sec x + \tan x$	$\frac{\cos x}{1 - \sin x}$
<hr/>	
<hr/>	

<hr/>	
<hr/>	

## C12 - 6.5 - Expand Sum Difference WS

Expand:

$$\sin\left(x + \frac{\pi}{3}\right)$$

$$\sin(x - \pi)$$

$$\cos\left(x + \frac{\pi}{6}\right)$$

$$\cos\left(x + \frac{\pi}{4}\right)$$

**Find the exact value of the following:**

$$\cos 15^\circ =$$

$$\sin 75^\circ =$$

$$\cos\left(\frac{\pi}{12}\right) =$$

$$\sin -15^\circ =$$

$$\csc 15^\circ$$

$$\cos\left(\frac{7\pi}{12}\right) =$$



## C12 - 6.5 - Simplify Sum Difference WS

**Simplify to a single trigonometric identity:**

$$\cos 2x \cos x + \sin 2x \sin x$$

$$\sin 3x \cos x - \cos 3x \sin x$$

$$\sin A \cos 2A + \cos A \sin 2A$$

$$\cos B \cos 3B - \sin B \sin 3B$$

**Find the exact value of:**

$$\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{6}\right)$$

$$\sin\left(\frac{11\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) - \cos\left(\frac{11\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$$

# C12 - 6.6 - Simplify Double Angle WS

**Simplify the following.**

$$4 \sin 3x \cos 3x =$$

$$6 \sin \frac{x}{2} \cos \frac{x}{2} =$$

$$8 \sin\left(\frac{\pi}{4}\right) \cos \frac{\pi}{4} =$$

$$\cos^2 x - \sin^2 x =$$

$$\cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x =$$

$$2 \cos^2 2x + 2 \sin^2 2x =$$

$$2 \cos^2 \frac{x}{4} - 1 =$$

$$1 - 2 \sin^2 \frac{x}{2} =$$

$$3 - 6 \sin^2 3x =$$

$$2 \cos^2 \frac{\pi}{2} - 1 =$$

$$\sec 10x (\sin^2 5x - \cos^2 5x) =$$

$$2 \sin 4x (\cos^2 2x - \sin^2 2x) =$$

# C12 - 6.6 - Simplify Double Angle WS

**Simplify the following.**

$$1 + \cos 2x =$$

$$1 - \cos 2x =$$

$$\cos 2x + 1 =$$

$$\cos 2x - 1 =$$

$$\frac{1 + \cos 2x}{\sin^2 x} =$$

$$\frac{1 - \cos 2x}{\tan^2 x} =$$

$$\frac{\cos 2x - 1}{\sin^2 x} =$$

$$\frac{\cos 2x - 1}{2\csc^2 x} =$$

## C12 - 6.6 - Solve Double Angle Unit WS

$$\sin x \cos x = 0$$

$$\sin 2x = 0$$

$$\sin 2x = 1$$

$$\cos 2x = 0$$

$$\cos 2x = -1$$

$$\cos 2x = 1$$

$$\sin 4x = 0$$

$$\cos 3x = -1$$

$$\cos\left(\frac{x}{2}\right) = 1$$

$$\sin\left(\frac{1}{3}x\right) = -1$$

## C12 - 6.6 - Solve Double Angle Unit Factor WS

$$\sin 2x + \cos x = 0$$

$$\sin x + \cos 2x = 1$$

$$\sin x - \cos 2x = -1$$

$$\sin 2x = -\sin x$$

$$\sin^2 x + \cos 2x = 0$$

$$\cos^2 x - \cos 2x = 0$$

$$\cos^2 x - \cos 2x = 1$$

$$\sin x - \cos 2x = 0$$

$$\cos x + \cos 2x = 0$$

$$\cos x - \cos 2x = 0$$

$$3\sin x + \cos 2x = -1$$

$$3\cos x + \cos 2x = 1$$

## C12 - 6.6 - Solve Double Angle ASTC WS

$$\sin 2x = \frac{1}{2}$$

$$\cos 2x = -\frac{1}{\sqrt{2}}$$

$$\sin 4x = \frac{1}{\sqrt{2}}$$

$$\cos 3x = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{x}{2}\right) = \frac{1}{2}$$

$$\sin\left(\frac{1}{3}x\right) = \frac{\sqrt{3}}{2}$$

$$\tan 2x = \sqrt{3}$$

## C12 - 6.6 - Solve Double Angle Unit ASTC WS

$$2\cot x \sin^2 x = 1$$

$$2\tan x \sec^2 x = \frac{1}{2}$$

$$2\sin x \cos x + 1 = 0$$

$$4\cos^2 2x - \sqrt{3} = 0$$

$$\cos 2x = 2\sin^2 x$$

$$x^2 = 1/4$$

Determine the non-permissible values of  $x$  in radians, for the following expressions.

$$\frac{1}{\sin x}$$

$$\frac{\sin x}{\cos x}$$

$$\frac{\cos x}{1 - \sin x}$$

$$\frac{\sin x}{\cos x + 1}$$

$$\csc x$$

$$\sec x$$

$$\frac{\cos x}{5}$$

$$\frac{1}{\csc x}$$

$$\tan x$$

$$\frac{1}{\tan x}$$

$$\frac{\tan x}{\sin x}$$

$$\frac{\csc x}{\tan x}$$

$$\frac{\cot x}{\tan x}$$

$$\frac{1}{1 - \sin^2 x}$$

$$\frac{1}{\cos^2 x}$$

$$\frac{1}{\cos^2 x + \cos x - 2}$$

$$\frac{1}{\sin x - \cos x}$$

$$\frac{1}{\sin x - \tan x}$$