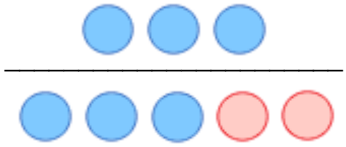
M8 - 11.1 - Blue Marbles out of Bag Notes Visually

You have 3 blue marbles and 2 red marbles in a bag, total 5 marbles.

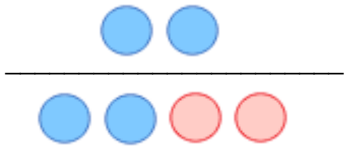
You randomly take a marble out of the bag. What is the probability that it will be a blue marble?



$$\frac{3\ blue\ marbles}{5\ total\ marbles} = 60\%$$

You now take a blue marble out of the bag.

What is now the probability that your next drawn marble will be blue?

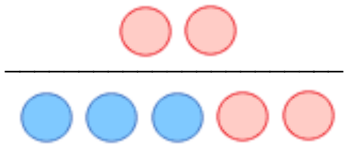


$$\frac{2\ blue\ marbles}{4\ total\ marbles} = 50\%$$

M8 - 11.1 - Taking Red Marbles out of Bag Notes Visually

You have 3 blue marbles and 2 red marbles in a bag, total 5 marbles.

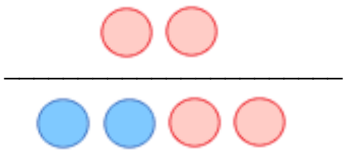
You randomly take a marble out of the bag. What is the probability that it will be a red marble?



$$\frac{2\ red\ marbles}{5\ total\ marbles} = 40\%$$

You now take a blue marble out of the bag.

What is now the probability that your next drawn marble will be red?



$$\frac{2\ red\ marbles}{4\ total\ marbles} = 50\%$$

M8 - 11.1 - Independent Events Notes

$$Probability = \frac{number\ of\ favorable\ outcomes}{total\ outcomes}$$

Example: Flipping a coin.

The probability of flipping a head.

$$P(H) = \frac{1}{2} \quad \begin{matrix} \leftarrow 1\ head \\ \leftarrow 2\ possible\ outcomes \end{matrix}$$

Example: Probability of flipping two heads in a row

What is the probability of flipping two heads in a row?

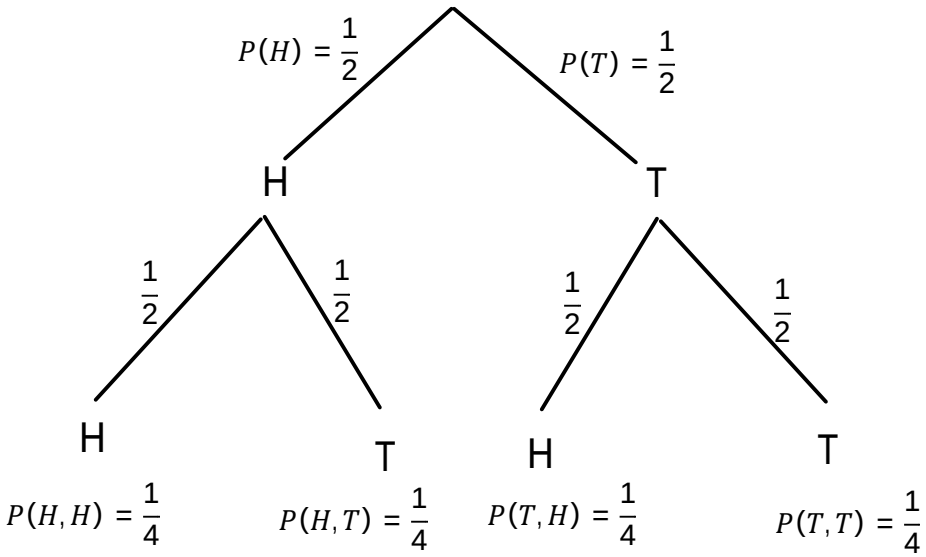
Method 1: Multiply $P(H,H) = P(H) \times P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Method 2: Table

	H	**T**
H	H,H	H,T
T	T,H	T,T

$P(H,H) = \frac{1}{4}$

Method 3: Tree



What is the probability of 1 head and 1 tail (in any order)?

$$\begin{aligned} P(H,T\ or\ T,H) &= P(H,T) + P(T,H) \\ &= \frac{1}{4} + \frac{1}{4} \\ &= \frac{1}{2} \end{aligned}$$

M8 - 11.1 - Independent Events Notes

Example: Rolling a die

The probability of rolling a 6 with a die.

$$P(6) = \frac{1}{6} \quad \begin{matrix} \leftarrow 1\ six \\ \leftarrow 6\ total\ numbers \end{matrix}$$

Example: Rolling two dice.

What is the probability of rolling two 6's?

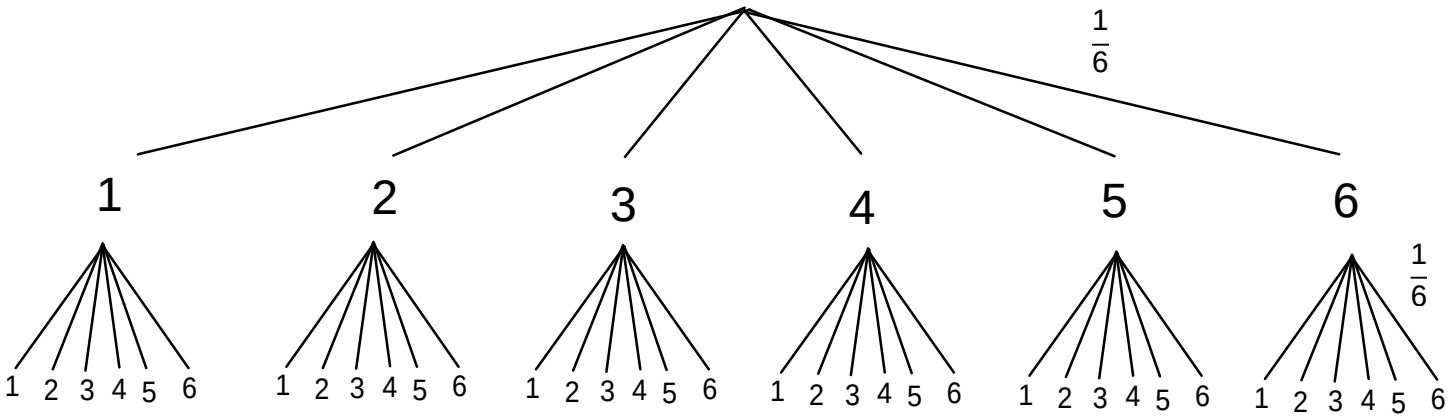
Method 1: $P(6,6) = P(6) \times P(6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Method 2:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$P(6,6) = \frac{1}{36}$

Method 3:



$P(6,6) = \frac{1}{36}$

M8 - 11.1 - Independent Events Notes

Example: Rolling a die and flipping a coin.

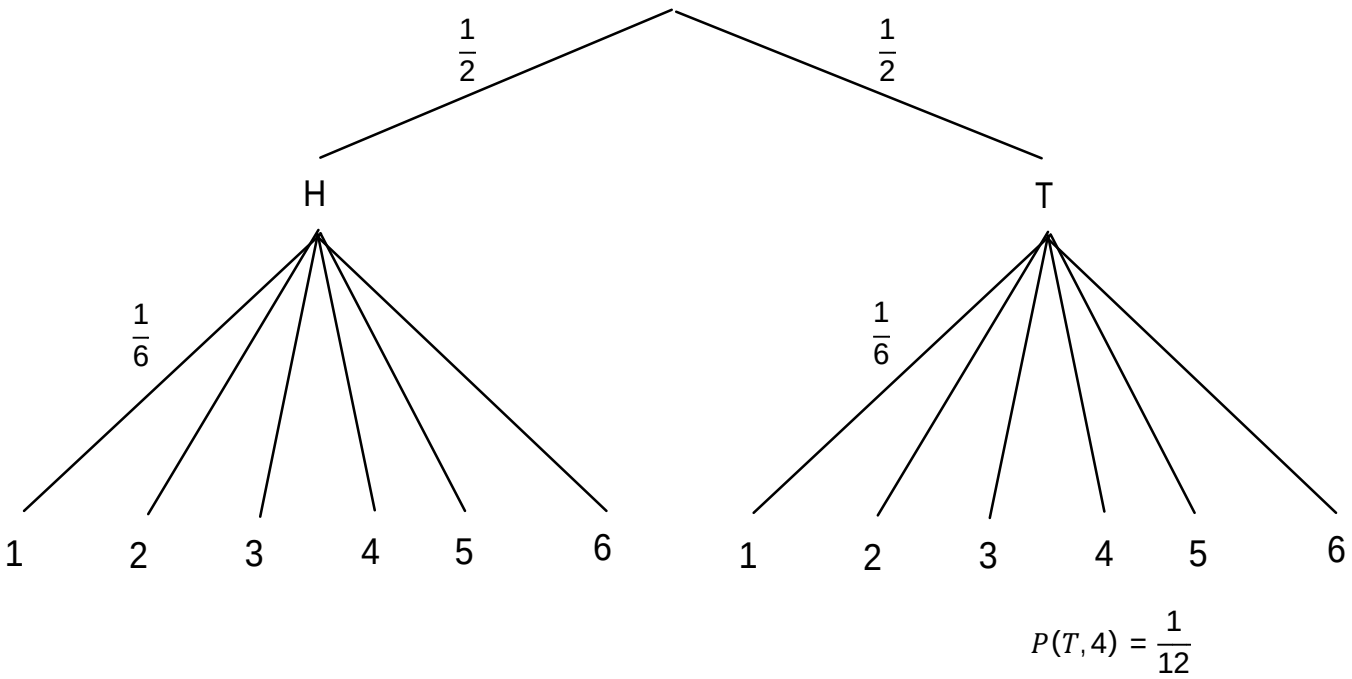
Method 1: $P(T,4) = P(T) \times P(4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Method 2:

	1	**2**	**3**	**4**	**5**	**6**
H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6

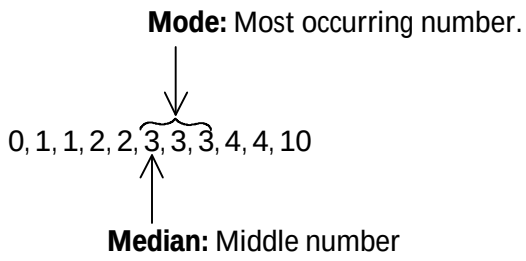
$P(T,4) = \frac{1}{12}$

Method 3:



$P(T,4) = \frac{1}{12}$

M8 - 11.1 - Mean, Median, Mode Notes



$$\underset{\textbf{(Average)}}{\textbf{Mean}} = \frac{\text{All Numbers Added}}{\text{Number of Samples}}$$

$$Mean = \frac{0 + 1 + 1 + 2 + 2 + 3 + 3 + 3 + 4 + 4 + 10}{11}$$

$$Mean = \frac{33}{11}$$

$$Mean = 3$$

M8 - 11.1 - Probability Odds Notes

$$P(x) = \frac{\text{Desired outcomes}}{\text{Total outcomes}}$$ "Probability of x"

What is the probability of drawing an ace from a standard deck?

$$P(Ace) = \frac{4 \leftarrow \text{4 aces}}{52 \leftarrow \text{52 cards}}$$

Odds:

Odds in Favor: Odds Against

4: 48

$$Odds\ Against = Total - Odds\ in\ favour$$

M8 - 11.1 - Probability Rules Notes

Sample Space: List of all possible outcomes.

Event	Sample Space
For a coin toss	heads, tails
Sample space of rolling a six-sided die?	1, 2, 3, 4, 5, 6

Probability Notation

Sample Space: The set of all possible outcomes. For a coin toss, the sample space is $S = \{H, T\}$.

$P(E)$ is the probability of event E taking place.

Probabilities can be expressed as decimals or fractions between 0 and 1 or as percentages between 0 and 100.

1. The probability of an event is between 0 and 1. (i.e. $0 \leq P(E) \leq 1$). The probability can never be less than 0% or greater than 100%.

2. If an event can't happen it has a probability of 0 (i.e. $P(E) = 0$). The probability of rolling a 7 on a standard six-sided die has a probability of 0.

3. If an event will happen with certainty, it has a probability of 1 (i.e. $P(E) = 1$). The probability of getting a head or a tail when flipping a coin is 1.

4. If the probability of an event occurring is $P(E)$, then the probability that it doesn't occur is $P(not\ E) = 1 - P(E)$. For example, the probability of not rolling a 6 is:
   $P(not\ 6) = 1 - P(6) = 1 - \left(\frac{1}{6}\right) = \frac{5}{6}$

5. The sum of probabilities of all outcomes in the sample space must sum to 1. When rolling a dice the sample space is $S = \{1,2,3,4,5,6\}$ and the sum of probabilities of all possible outcomes is

   $P(1,2,3,4,5\ or\ 6) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$

   $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$

M8 - 11.1 - Dependent Events Notes

Example: Choosing marbles from a bag of 3 red and 3 blue marbles.

What is the probability of choosing two blue marbles in a row without replacement?

Probability of choosing a blue marble on the first draw.

$$P(blue_1) = \frac{3\ blue}{6\ total} = \frac{1}{2}$$

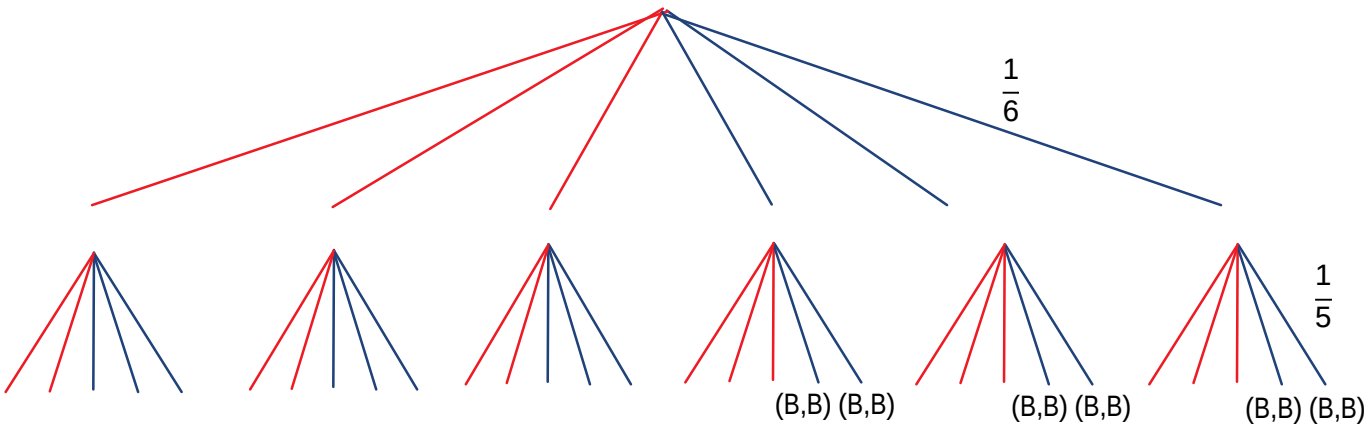
Assuming we chose a blue marble on the first draw, there will only be 2 blue marbles remaining. Therefore the probability of choosing a blue marble on the second draw is.

$$P(blue_2) = \frac{2\ blue}{5\ total} = \frac{2}{5}$$

To calculate the total probability of choosing two blue marbles in a row, we multiply the probabilities of the dependent events.

$$P(blue, blue) = P(blue_1) \times P(blue_2) = \frac{1}{2} \times \frac{2}{5} = \frac{2}{10} = \frac{1}{5}$$

Note: The probability of choosing two blue marbles with replacement is $\frac{1}{4}$.



Using a tree:
There are 6 leaves that correspond to the event described in the question.
Each leaf has a probability of $\frac{1}{30}$
Therefore, $P(blue, blue) = \frac{1}{30} + \frac{1}{30} + \frac{1}{30} + \frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{6}{30} = \frac{1}{5}$

M8 - 11.1 - Complementary Events Notes

Flipping a coin

$\boldsymbol{Event}: Heads$ $\boldsymbol{Complement}: Tails$

The complement of Heads is everything not Heads → Tails

Rolling a die

$\boldsymbol{Event}: Rolling\ a\ 4$ $\boldsymbol{Complement}: Rolling\ a\ 1, 2, 3, 5, 6$

The complement of rolling a 4 is rolling everything not a 4 → 1, 2, 3, 5, 6

Notation:

If the event heads is: H
If the event tails is: T

$The\ complement\ of\ H\ is: \sim H$
$The\ complement\ of\ T\ is: \sim T$