C12 - Table of Contents

Duotang/Notes/Homework

C12 - Methods

Transformations

General Formula
Graphing
Translations
Expansions/Compressions
Reflections
Inverse
Invariant Points
Domain/Range

Radicals

Transformations

Polynomials

Factoring Cubics

Long Division –
Synthetic Division +

Factor theorem
Remainder theorem

Potential Factors
Solve by Inspection
x-intercepts $(x, 0)$

Graphing

End behavior
Multiplicity
y-intercept (0,y)

TOV

Trigonometry

Radians/Conversions
Arc Length/Sector Area
Solving Equations
ASTC/Unit Circle
Special Triangles
STP/Reference Angles
Substitution
$m = 2x$
$m = sinx$
$\theta_r = \sin^{-1}(+)$

$0 \leq x < 2\pi$
Gen Sol: $\theta = \theta + pn, nei$

Linear/Angular Velocity

Trig Functions

Box Model
DACB
TOV

$$p = \frac{2\pi}{b}$$

Trig Identities

Identities
Fractions/LCD
Factoring/FOIL
Conjugates

Exponentials

TOV
Change of Base
Take both sides to reciprocal exponent
Exponent/Radical laws

Logarithms

Laws
TOV
Domain: Set the thing you are logging to greater than or equal to zero, then solve.

Rational's

Factor
Holes
VA's
HA's
SA's

Function operations

$f(x) + g(x)$
$f(x) - g(x)$
$f(x) \times g(x)$
$f(x) \div g(x)$
$f\big(g(x)\big)$
Inverse

Combinatorics

FCP
Factorials
Tree Diagram
nPr, nCr
Cases
All minus none
Identical Objects
Paths in Squares
Pascal 'Triangle
Binomial Theorem

C12 - Remember

Transformations

Remember: Factor out "b" so "x" has a coefficient of "1"

$(2x)^2 = 4x^2$

Inverse Check: $f(g(x)) = x$
$g(f(x)) = x$

$\sqrt{x^2} = |x|$

Radicals

$\sqrt{4x} = 2\sqrt{x}$

Polynomials

Missing Terms "Insert 0"
Store x, 2nd Calc Zero

Trigonometry

Radian mode only matters if
you press sin cos or tan

Trig Functions

$sin\theta = 0 \text{ @ } 0, \pi, 2\pi$
$cos\theta = 0 \text{ @} \frac{\pi}{2}, \frac{3\pi}{2}$
$tan\theta = \frac{sin\theta}{cos\theta}$

Sin starts in the middle and goes up

Cos starts from the top and goes down

$\boldsymbol{x}$ **−Increments =** $\frac{\pi}{\boldsymbol{lcd\ c,p}}$

Trig Identities

$\sin^2 x = sinx \times sinx = (\sin(x))^2 \neq \sin x^2$

$\cos(x + \pi) \neq cosx + cos\pi$

Exponentials

$2(2)^x \neq 4^x$

Variable Exponents

Logarithms

The base of the log is the base of the exponent
The exponent is the Answer
$\log(x + 3) \neq logx + log3$

Rational's

Holes before VA's
A graph can cross a horizontal asymptote
but not a vertical asymptote

Function operations

$f(x)^3$ *We dont write this*

$f(x^3) \neq= (f(x))^3 = f^3(x)$

Combinatorics

; $n + 1$ *terms*
k is always one less than the term number.

C12 - Formula Sheet

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities

$sec\theta = \dfrac{1}{cos\theta}$ $\qquad csc\theta = \dfrac{1}{sin\theta}$ $\qquad cot\theta = \dfrac{1}{tan\theta}$

$$\boxed{\begin{aligned} tan\theta &= \frac{sin\theta}{cos\theta} \\ cot\theta &= \frac{cos\theta}{sin\theta} \end{aligned}}$$

Addition Identities

$\sin(\alpha + \beta) = sin\alpha\, cos\beta + cos\alpha\, sin\beta$

$\sin(\alpha - \beta) = sin\alpha\, cos\beta - cos\alpha sin\beta$

$\cos(\alpha + \beta) = cos\alpha\, cos\beta\, - sin\alpha\, sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\, cos\beta + sin\alpha\, sin\beta$

Double Angle Identities

$$\begin{aligned} cos2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \end{aligned}$$

$sin2\theta = 2sin\theta cos\theta$

Permutations and Combinations

${}_nP_r = \dfrac{n!}{(n-r)!}$ $\qquad {}_nC_r = \dfrac{n!}{r!\,(n-r)!}$ $\qquad t_{k+1} = {}_nC_k a^{n-k} b^k$

$Note\colon\ {}_nC_r = \dfrac{{}_nP_r}{r!}$

Arc Length

$a = \theta r$

$\tan(\alpha + \beta) = \dfrac{tan\alpha + tan\beta}{1 - tan\alpha\, tan\beta}$ $\qquad \tan(\alpha - \beta) = \dfrac{tan\alpha - tan\beta}{1 + tan\alpha\, tan\beta}$ $\qquad \tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

C12 - 1.0 - Transformations Review

$$y = af\big(b(x-h)\big) + k$$

$+k$: Vertical Translation $VT = k$ (up)
$-k$: Vertical Translation $VT = k$ (down)

$(x-h)$: Horizontal Translation $HT = h$ (right) (h is positive)
$(x+h)$: Horizontal Translation $HT = h$ (left) (h is negative)

$a < 0$: Vertical Reflection (VR) over the x-axis
$|a| > 1$: Vertical Expansion $VE = a$
$0 < |a| < 1$: Vertical Compression $VC = a$

$b < 0$: Horizontal Reflection (HR) over the y-axis
$|b| > 1$: Horizontal Compression $HC = \frac{1}{b}$
$0 < |b| < 1$: Horizontal Expansion $HE = \frac{1}{b}$

Methods: Equation Substitution

Vertical Expansion of 2:	Put $\frac{1}{2}y$ in for 'y'.
Vertical Compression of $\frac{1}{2}$:	Put $2y$ in for 'y'.
Horizontal Expansion of 2:	Put $\frac{1}{2}x$ in for 'x'.
Horizontal Compression of $\frac{1}{2}$:	Put $2x$ in for 'x'.
Vertical Reflection:	Put $-y$ in for y.
Horizontal Reflection:	Put $-x$ in for x.
Vertical Translation up 3:	Put $(y-3)$ in for 'y'.
Vertical Translation down 2:	Put $(y+2)$ in for 'y'.
Horizontal Translation left 2:	Put $(x+2)$ in for 'x'
Horizontal Translation right 4:	Put $(x-4)$ in for 'x'

Mapping

(x, y) ; $old\ point$ $\longrightarrow$ $\left(\frac{x}{b} + h, ay + k\right)$; $new\ point$

(2,3) $\rightarrow$ (2,6)	multiply y-value by two	$(x, 2y)$	(x, ay)
(3,4) $\rightarrow$ (3,2)	multiply y-value by a half	$(x, \frac{1}{2}y)$	
(2,3) $\rightarrow$ (4,3)	multiply x-value by two	$(2x, y)$	$(\frac{1}{b}x, y)$
(4,1) $\rightarrow$ (2,1)	multiply x-value by a half	$(\frac{1}{2}x, y)$	
(3,5) $\rightarrow$ (3,−5)	Multiply y-value by -1	$(x, -y)$	
(−4,3) $\rightarrow$ (4,3)	Multiply x-value by -1	$(-x, y)$	
(2,1) $\rightarrow$ (2,4)	add 3 to y-value	$(x, y+3)$	$(x, y \pm k)$
(1,4) $\rightarrow$ (1,2)	Subtract 2 from y-value	$(x, y-2)$	
(4,1) $\rightarrow$ (2,1)	Subtract 2 from x-value	$(x-2, y)$	$(x \pm h, y)$
(2,3) $\rightarrow$ (6,3)	Add 4 to x-value	$(x+4, y)$	

Remember: Horizontal Translations are the Opposite of what you see inside the brackets.

Remember: "k" may be on the left hand side of the equation. $g(x) - k = f(x-h)$. So add or subtract "k" to both sides.

Remember: Horizontal Compressions and Expansions are the Reciprocal of "b"

Remember: "a" may be on the left side of the equation: $ag(x) = f(x-h)$. So multiply or divide by ato both sides.

Remember: Factor the brackets so x has a coefficient of 1

Inverse ($f^{-1}(x)$): switch x and y
$y = f(x) \rightarrow x = f(y)$
A reflection over the xy axis ($y = x$)
$f\big(g(x)\big) = x \qquad g\big(f(x)\big) = x$

Invariant Points:
Horizontal Reflections: y-intercepts (0,y)
Vertical Reflections: x-intercepts (x,0)
Inverses: (a,a) (2,2) (any points on the line y=x)
Roots: (x,0) (x,1)

*We work functions stuff 1st. Bedmas

C12 - 3.0 - Polynomials Review

Synthetic

Ascending Order

Steps:
Bring Down
Multiply
Add
Repeat last two steps

$a \,|\, Synthetic = f(a) = x - int\ (a, 0)$

$+$

$Factor = (x - a)$

The x intercept, the thing you put into synthetic division and $f(\ )$ is all the same.

The factor is the only opposite

Insert 0 if you are missing a degree term $x^3 + 0x^2 - 2x + 4$

If you are going to put the opposite sign of the x intercept into synthetic division, you must subtract

Potential Factors

$= \frac{\pm factors\ of\ d}{\pm factors\ of\ a}$

Solve by Inspection

$f(a) = 0$

$(x - a);\ is\ a\ factor$

$f(a) = R$

$(x - a);\ is\ not\ a\ factor$

x-intercepts

$x - 2 = 0$
$x = 2$

$x - int$: $(2,0)$

y-intercept

$(0, y)$

End behavior

$\pm x^{even}, \pm x^{odd}$

TOV

Multiplicity

$(x-2)^1(x-1)^2(x+3)^3$

Degree

	xeR x^{even}	xeR x^{odd}
Max number of x-intercepts	Degree	Degree
Min number of x-intercepts	0	1
Max number of turns	Degree −1	Degree −1
Min number of turns	1	0*
	$y \geq min$ $y \leq max$	yeR

Min Degree = # turns +1

Long Division

Steps:
Goes Into
Multiply
Subtract
Bring Down
Repeat

Coefficient Binomials, Quadratic divisors - Use Long Division

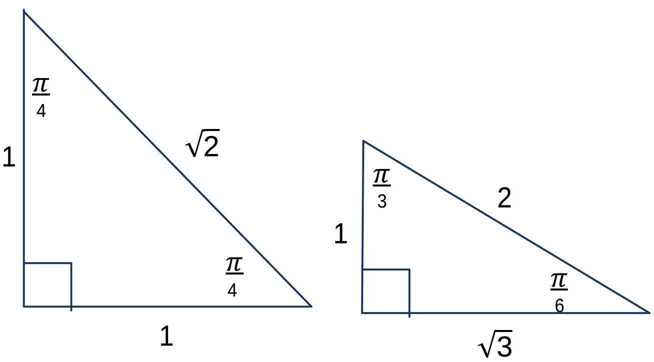
Factored Form

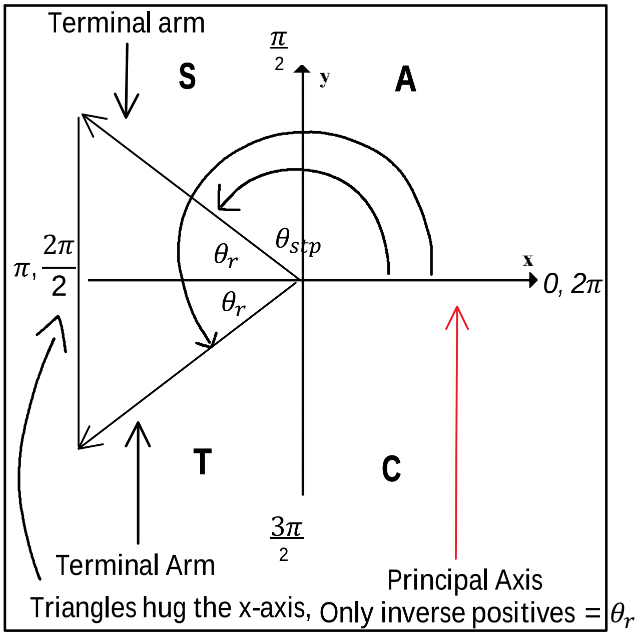
$y = a(x-z)^1(x-r)^2(x-s)^3 \ldots$

$y = 5;\quad Constant$
$y = 2x;\quad Linear$
$y = -x^2;\quad Quadratic$
$y = 5x^3;\quad Cubic$
$y = -2x^4;\quad Quartic$
$y = 2x^5;\quad Quintic$

C12 - 4.0 - Trigonometry Review

Special Triangles.





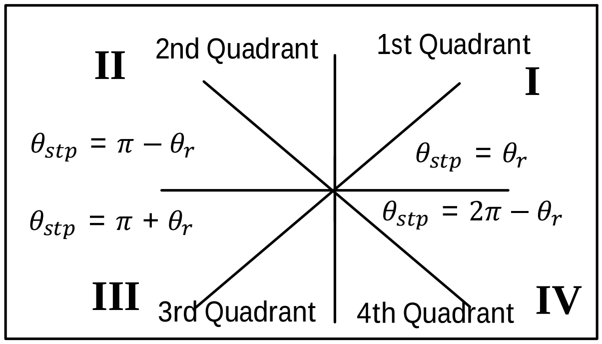
SOH - CAH - TOA

$sin\theta = \frac{O}{H}$ | $cos\theta = \frac{A}{H}$ | $tan\theta = \frac{O}{A}$

$csc\theta = \frac{1}{sin\theta} = \frac{H}{O}$ | $sec\theta = \frac{1}{cos\theta} = \frac{H}{A}$ | $cot\theta = \frac{1}{tan\theta} = \frac{A}{O}$

$\theta = \sin^{-1}(\frac{O}{H})$

$tan\theta = \frac{sin\theta}{cos\theta}$

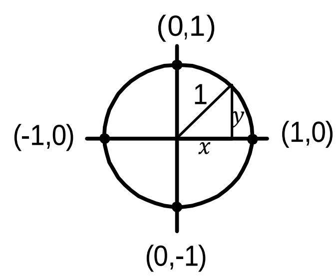


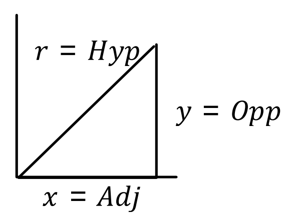
$\theta_{cot} = \theta_{stp} \pm 2\pi n$

$p = \frac{2\pi}{b}$ (sin, cos) $\quad p = \frac{360^o}{b}$ (sin, cos)

General Solution:
$\theta = \theta_{stp} \pm pn, n \in I$

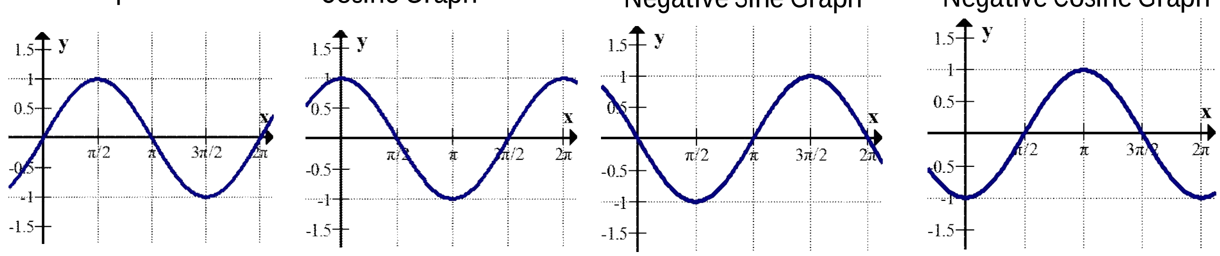
$p = \frac{\pi}{b}$ (tan) $\quad p = \frac{180^o}{b}$ (tan)





$sin\theta = y$ $\quad cos\theta = x$ $\quad tan\theta = \frac{y}{x}$

Sine Graph | Cosine Graph | Negative Sine Graph | Negative Cosine Graph



$y = sinx$ $\quad y = cosx$ $\quad y = -sinx$ $\quad y = -cosx$

$$\boxed{y = a\sin(b(x - c)) + d}$$

GRAPH tan(x)

Tan is Zero when sin is zero
Tan is und when cos is zero

C12 - 5.0 - Trigonometric Functions Review *(h,k)(c,d)

$y = \text{a}\sin(b(x-h)) + k \qquad y = \text{a}\cos(b(x-h)) + k$

$$y = a\sin(b(x-c)) + d$$

$Amplitude\colon\ |a|$

$Period\colon p = \dfrac{2\pi}{b}$

$Phase\ Shift\colon c$

$Horizontal\ center\ line\colon d$

Remember: Factor the brackets so x has a coefficient of 1

$$y = \text{a}\tan(b(x-c)) + d$$

$Period\ of\ tan\colon \dfrac{\pi}{b}$

Tan is Zero when sin is zero
Tan is und when cos is zero

x-intercepts / Domain Restrictions

x-intercepts:

$sinx\colon b(x-c) = \pi n, nEI$

$cosx\colon b(x-c) = \dfrac{\pi}{2} + \pi n, nEI$

$tanx\colon b(x-c) = \pi n, nEI$

Domain:

$\dfrac{\square}{sinx}\colon b(x-c) \neq \pi n, nEI$

$\dfrac{\square}{cosx}\colon b(x-c) \neq \dfrac{\pi}{2} + \pi n, nEI$

$\dfrac{\square}{tanx}\colon b(x-c) \neq \pi n, \dfrac{\pi}{2} + \pi n, nEI$

$\boxed{\dfrac{\square}{\frac{sinx}{cosx}}\colon sinx \neq 0, cosx \neq 0}$

Transformations

$sinx = \cos(x-90)$

$cosx = sin(x+90)$

$\sin(-x) = -sinx$

$\cos(-x) = cosx$

$\tan(-x) = -tanx$

$sinx = \cos(90-x)$

$cosx = \sin(90-x)$

Rearranged Formula

$y = \text{a}\sin\left(\dfrac{2\pi}{p}(x-c)\right) + d \qquad b = \dfrac{2\pi}{p} \qquad y = a\sin\left(\dfrac{2\pi(x-c)}{p}\right) + d$

C12 - 6.0 - Steps Trig Identities Review

Trig Identities: Steps

Get into sin and cos
Identities

Fractions
Simplify
Adding and subtracting fractions
LCD: Do to the top do to the bottom
Multiplying by 1
Flip and multiply
Separate fractions

Multiply by LCD (Complex Fractions)

OR

Add/Subtract Fractions top and bottom, flip and multiply

Rearrange terms

Factoring
GCF
Trinomials
Differences of Squares

Conjugates

Distribution and FOIL (where necessary)

Choose a $cos2\theta$ to cross off the '1' or combine with the #

$Add\ and\ subtract\ 1$

C12 - 6.0 - Trig Proofs

Pythagorean Identity

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{o}{h}\right)^2 + \left(\frac{a}{h}\right)^2 = 1$

$\frac{o^2}{h^2} + \frac{a^2}{h^2} = 1$

$o^2 + a^2 = h^2$

$1 + tan^2\theta = sec^2\theta$

$1 + \left(\frac{o}{a}\right)^2 = \left(\frac{h}{a}\right)^2$

$1 + \frac{o^2}{a^2} = \frac{h^2}{a^2}$

$o^2 + a^2 = h^2$

$1 + cot^2\,\theta = csc^2\,\theta$

$1 + \left(\frac{a}{o}\right)^2 = \left(\frac{h}{o}\right)^2$

$1 + \frac{a^2}{o^2} = \frac{h^2}{o^2}$

$o^2 + a^2 = h^2$

$Pythagerous\ Theorm$

Reciprocal Identities

$\frac{sinx}{cosx} = \frac{o}{h} \div \frac{a}{h} = \frac{o}{h} \times \frac{h}{a} = \frac{o}{a} = tanx$

$sec\theta = \frac{1}{cos\theta} = \frac{1}{(\frac{a}{h})} = 1 \times \frac{h}{a} = \frac{h}{a}$

$sec\theta = \frac{h}{a}$

$csc\theta = \frac{1}{sin\theta} = \frac{1}{\left(\frac{o}{h}\right)} = 1 \times \frac{h}{o} = \frac{h}{o}$

$csc\theta = \frac{h}{o}$

$cot\theta = \frac{1}{tan\theta} = \frac{1}{\left(\frac{o}{a}\right)} = 1 \times \frac{a}{o} = \frac{a}{o}$

$cot\theta = \frac{a}{o}$

Double Angle Identities

$sin2x = 2sinxcosx$

$\sin(x + x) =$

$sinxcosx + cosxsinx = 2sinxcosx$

$cos2x = cos^2\,x - sin^2 x$

$\cos(x + x) =$

$cosxcosx - sinxsinx = \cos^2 x - \sin^2 x$

$cos2x = 1 - 2\sin^2 x$

$cos2x = 1 - 2(1 - \cos^2 x)$

$cos2x = 1 - 2 + 2\cos^2 x$

$cos2x = -1 + 2\cos^2 x$

$cos2x = 2\cos^2 x - 1$

$cos2x = \cos^2 x - \sin^2 x$

$cos2x = (1 - sin^2x) - \sin^2 x$

$cos2x = 1 - 2\sin^2 x$

C12 - 6.0 - Fractions Algebra Fact/FOIL Theory

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$a \times \frac{b}{c} = \frac{ab}{c}$

$\frac{a}{b} \times c = \frac{ac}{b}$

$\frac{a}{b} \div \frac{c}{d} =$

$\frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} =$

$\frac{a}{b} \div \frac{c}{d} =$

$\frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$

$a \div \frac{b}{c} =$

$a \times \frac{c}{b} = \frac{ac}{b}$

$\frac{a}{\left(\frac{b}{c}\right)} =$

$a \div \frac{b}{c} =$

$a \times \frac{c}{b} = \frac{ac}{b}$

$\frac{a}{b} \div c =$

$\frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$

$\frac{\left(\frac{a}{b}\right)}{c} =$

$\frac{a}{b} \div c =$

$\frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad+cb}{bd}$

$\frac{x+1}{x} = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x}$

$x \times x = x^2$

$x \times x^2 = x^3$

$\frac{x^2}{x} = x$

$\frac{x^3}{x^2} = x$

$\frac{x^3}{x} = x^2$

$\frac{x}{x} = 1$

$x^2 + x$
$x(x+1)$

$x^2 - x - 2$
$(x-2)(x+1)$

$x^2 - 1$
$(x+1)(x-1)$

FOIL Conjugates: FL

$a + b + c = a + c + b$

$ab = ba$

$\frac{1}{a} \times a = 1$

C12 - 7.0 - Exponential Review

Interest

$\boxed{F = P(1 \pm r)^t}$

KEY

F : *Future Amount*
P : *Present Amount*
r : *Interest rate as decimal*
t : *time*

$F = 100(.87)^t$

$;+ = Growth$
$- = Decay$

$\boxed{F = P\left(1 \pm \frac{r}{n}\right)^{tn}}$; with Compounding

n : *# of compounding periods per year*
$\frac{r}{n}$: *Rate per period*
tn: *number of periods*

Yearly; $n = 1$
Monthly; $n = 12$
Weekly; $n = 52$

Growth & Decay

$\boxed{F = P(r)^{\frac{t}{T}}}$; Growth with "T"

T: *Time / Amount for Rate to* ***OCCUR***

$F = 100\left(\frac{1}{2}\right)^{\frac{t}{5}}$

$\boxed{F = Pe^{kt}}$; Continuous Growth

e: *constant* ≈ 2.718
k: *proportional constant*

Growth	10% = .1	15% = .15	40% = .4	50% = .5	60% = .6	100% = 1.00	*Double*
$(1+r)$	(1 + .1)	(1 + .15)	(1 + .4)	(1 + .5)	(1 + .5)	(1 + 1.00)	
(.....)	(1.1)	(1.15)	(1.4)	(1.5)	(1.5)	(2)	(2)

Decay	10% = .1	15% = .15	40% = .4	50% = .5	60% = .6	95% = .95	*Half − Life*
$(1-r)$	(1 − .1)	(1 − .15)	(1 − .4)	(1 − .5)	(1 − .5)	(1 − .95)	
(.....)	(.9)	(.85)	(.6)	(.5)	(.4)	(.05)	$\left(\frac{1}{2}\right)$

Method: *Arbitrarily set* $P = 100\%$ *or* 100 *or* 1

Remember: *The exponent is the time or the number of time periods.*

Intensity

$\boxed{I = 10^{b-s}}$; Earthquakes, pH

$I = 10^{\frac{b-s}{10}}$; Sound

KEY

$I = \frac{I_b}{I_s}$: *Intensity*
b − *Larger Richter, Debibel, pH etc*
s − *Smaller Richter, Decibel, pH etc*

$pH = -\log(H^+)$
H^+- Concentration of Hydrogen

C12 - 8.0 - Laws of Logarithms Review

Math Alpha Math

Laws of logarithms

'a' is "the thing you are logging"

1. $\log_b a = c \longrightarrow a = b^c$ — What power must you raise "b" to, to equal "a"? Slide "b" across.
   $a > 0$
   $b > 0, b \neq 1$

2. $\log_b a^m = m\log_b a$ — The exponent of "a" can come down in front. Vice versa.

3. $\log_b a = \dfrac{\log a}{\log b}$ — Change of base. $\boxed{\log_b a = \dfrac{\log_c a}{\log_c b} \quad \text{:"c" is arbitrary}}$

4. $\log_b m + \log_b n = \log_b mn$ — Positives go on top, negatives go on bottom and vice versa.

5. $\log_b m - \log_b n = \log_b \dfrac{m}{n}$ — Rule 3, 4 and 5 Must have coefficient of 1.

6. $\log_b a = \log_{b^n} a^n$ — "a" and "b" can both be taken to the same exponent, $n \neq 0$

7. $b^{\log_b a} = a$ — Same base of exponent as logarithm, answer is "a"

8. $\log_a a = 1$

9. $\log_a 1 = 0$

10. $\log x = \log_{10} x$

Methods: "log" both sides
"de-log" both sides
"inverse": switch x and y
Set Log arbitrarily $= x$

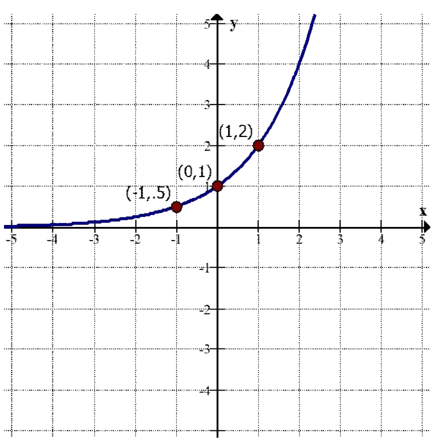
Natural Logarithms: (same rules as logs)

$\ln e = 1$

$\boxed{\ln x = \log_e x \qquad \ln x \text{ is } \log_e x}$

Methods: "ln" both sides.
"de-ln" both sides.

$f(x) = 2^x \qquad y = 2^x$

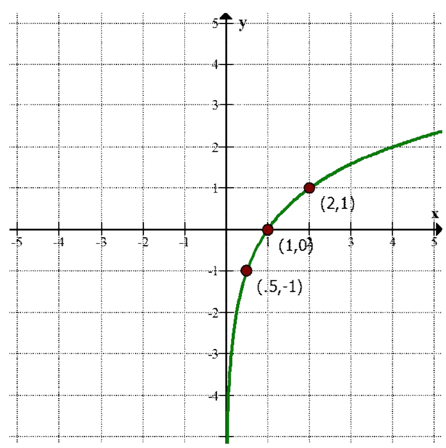


x	y
-1	$\frac{1}{2}$
0	1
1	2

Domain: $x \epsilon R$
Range: $y > k \quad (a > 0)$
$y < k \quad (a < 0)$
$HA: y = k$

$y = a(C)^{b(x-h)} + k$

$f^{-1}(x) = \log_2 x \qquad y = \log_2 x$



x	y
$\frac{1}{2}$	-1
1	0
2	1

Domain: $b(x-h) > 0$ $\boxed{\&\ base > 0, base \neq 1}$
Range: $y \epsilon R$
$VA: b(x-h) = 0$

$y = a\log(b(x-h)) + k$

C12 - 9.0 - Rationals Review

$$y = \frac{a}{(b(x-h))} + k$$

Remember: Check for holes first because they might cancel out a Vertical Asymptote.

Holes:

Factor the top, Factor the bottom
If a term cancels, there is a hole when the term=0

$$y = \frac{x-3}{(x-3)(x+2)} = \frac{1}{x+2} \qquad y = \frac{1}{(3)+2} = \frac{1}{5}$$

$(x-3)$ cancels so,

Hole at $x - 3 = 0$
$x = 3$

Hole at $\left(3, \frac{1}{5}\right)$

Vertical Asymptote:

$$y = \frac{1}{x+1} \qquad x + 1 = 0 \qquad x = -1$$

$Set\ denominator\ equal\ to\ zero\ and\ solve.$ $\qquad denominator = 0$

VAs, NPVs:

$x = -1$

Domain Restriction:

$x \neq -1$

Horizontal Asymptote:

Case 1: $x^2, \frac{x^2}{x}$ (No HA)

Case 2: $\frac{1}{x}, \frac{1}{x^2}$ (Asymptote at $y = 0$)

Case 2: $\frac{1}{x} + c, \frac{1}{x^2} + c$ (Asymptote at $y = c$)

Case 3: $\frac{3x^2}{2x^2}$ (Asymptote at $y = \frac{3}{2}$)

Case 3: $\frac{3x^2}{2x^2} + c$ (Asymptote at $y = \frac{3}{2} + c$)

Range Restrictions: $y \neq HA\ or\ Holes$

A graph can cross a horizontal asymptote but not a vertical asymptote

x	y
99999	?

<u>Horizontal Asymptotes</u>

Divide top and bottom by highest exponent of x in denominator

$Intercepts$

$x - intercepts: Set\ y = 0\ and\ Solve$

$y - interceptes: Set\ x = 0\ and\ Solve$

Slant Asymptote:

$Do\ Synthetic\ or\ Long\ Division\ and\ if\ the\ Quotient, the\ Answer, is\ a\ linear\ function\ that\ is\ the\ equation\ of\ the\ slant\ asymptote.$ $(Case\ \#1)$

C12 - 10.0 - Function Operations Review

Operations

$f(x) + g(x) = (f + g)(x)$ — *Add y − values*

$f(x) - g(x) = (f - g)(x)$ — *Subtract y − values*

Pick an x-value to talk about. We aren't talking about another x-value till were done talking about that x-value.

$f(x) \cdot g(x) = f(x) \times g(x)$ — *Multiply y − values*

$\frac{f(x)}{g(x)} = \frac{f}{g}(x)$ — *Divide y − values*

$(fg)(x) = f(x) \cdot g(x)$ AKA

Composite Functions

$f \circ g(x) = f(g(x))$ — *Put g(x) into f's x*

$g \circ f(x) = g(f(x))$ — *Put f(x) into g's x*

Inverse

$$
\begin{aligned}
y &= 2x + 4 && \textit{Switch x and y}\\
x &= 2y + 4 \\
x - 4 &= 2y \\
\frac{x}{2} - 2 &= y && \textit{Algebra}\\
y &= \frac{1}{2}x - 2 && \textit{Solve for } y =
\end{aligned}
$$

$f(g(x)) = x$

$g(f(x)) = x$

Remember: If you put g(x) f's x, and if you put f(x) into g's x, both should solve to x.

C12 - 11.0 - Combinatorics Review

$\boxed{\text{Logic}}$

Fundamental Counting Principle

___ , ___ , ___ Blanks

$\frac{\#\ options}{options}$

Repeats?

$\boxed{\times 2}$

$(outcomes\ per\ trial)^{number\ of\ trials}$

Tree Diagrams

Cases: Multiply within cases, add separate cases.

Cases!

$All - None$

Factorial Notation!

$$\frac{(\#\ of\ letters)!}{(repeating\ letter)!\,(other\ repeating\ letter)!\ldots}$$

Identical Objects

Guess and Check

$Paths\ in\ Squares\colon \frac{(l+w)!}{l!\,w!}$

$Paths\ in\ Cubes\colon \frac{(l+w+h)!}{l!\,w!\,h!}$

$Circle\colon\ (n-1)!$

Combinatorics Formulas

Order Matters/Doesn't Matter

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$

$$\boxed{{}_nC_r = \frac{{}_nP_r}{r!}}$$

Binomial Theorem

$t_{k+1} = {}_nC_k a^{n-k} b^k$; $(a+b)^n$; $n+1\ terms$

Probability

$\frac{\#\text{ of}}{\text{Options}}$

C12 - 11.0 - nCr, nPr Tables

$_{n}P_{n} = n!$	$_{n}P_{1} = n$	$_{n}P_{0} = 1$	$_{n}P_{n-1} = n!$	$_{n}P_{n-2} = n(n-1)!$
$_{3}P_{3} = 3!$ $_{5}P_{5} = 5!$	$_{3}P_{1} = 3$ $_{5}P_{1} = 5$	$_{6}P_{0} = 1$ $_{2}P_{0} = 1$	$_{6}P_{5} = 720$ $_{2}P_{1} = 2$	$_{6}P_{5} = 720$ $_{2}P_{1} = 2$

$_{0}P_{0} = 1$	$_{1}P_{0} = 1$	$_{2}P_{0} = 1$	$_{3}P_{0} = 1$	$_{4}P_{0} = 1$	$_{5}P_{0} = 1$
	$_{1}P_{1} = 1$	$_{2}P_{1} = 2$	$_{3}P_{1} = 3$	$_{4}P_{1} = 4$	$_{5}P_{1} = 5$
		$_{2}P_{2} = 1$	$_{3}P_{2} = 6$	$_{4}P_{2} = 12$	$_{5}P_{2} = 20$
			$_{3}P_{3} = 6$	$_{4}P_{3} = 24$	$_{5}P_{3} = 60$
				$_{4}P_{4} = 24$	$_{5}P_{4} = 120$
					$_{5}P_{5} = 120$

$_{n}C_{n} = 1$	$_{n}C_{1} = n$	$_{n}C_{0} = 1$	$_{n}C_{n-1} = n$	$_{n}C_{r} = {}_{n}C_{n-r}$
$_{3}C_{3} = 1$ $_{5}C_{5} = 1$	$_{3}C_{1} = 3$ $_{5}C_{1} = 5$	$_{6}C_{0} = 1$ $_{2}C_{0} = 1$	$_{5}C_{4} = 5$ $_{7}C_{6} = 7$	

$_{0}C_{0} = 1$	$_{1}C_{0} = 1$	$_{2}C_{0} = 1$	$_{3}C_{0} = 1$	$_{4}C_{0} = 1$	$_{5}C_{0} = 1$
	$_{1}C_{1} = 1$	$_{2}C_{1} = 2$	$_{3}C_{1} = 3$	$_{4}C_{1} = 4$	$_{5}C_{1} = 5$
		$_{2}C_{2} = 1$	$_{3}C_{2} = 3$	$_{4}C_{2} = 6$	$_{5}C_{2} = 10$
			$_{3}C_{3} = 1$	$_{4}C_{3} = 4$	$_{5}C_{3} = 10$
				$_{4}C_{4} = 1$	$_{5}C_{4} = 5$
					$_{5}C_{5} = 1$

$_{n}P_{r}\,,\ _{n}C_{r} \to r \le n$

We can only choose objects from
a larger number of objects.

C12 - 11.0 - Table of Cards

Hearts ♥	**Diamonds ♦**	**Spades ♠**	**Clubs ♣**
Ace ♥	Ace ♦	Ace ♠	Ace ♣
2 ♥	2 ♦	2 ♠	2 ♣
3 ♥	3 ♦	3 ♠	3 ♣
4 ♥	4 ♦	4 ♠	4 ♣
5 ♥	5 ♦	5 ♠	5 ♣
6 ♥	6 ♦	6 ♠	6 ♣
7 ♥	7 ♦	7 ♠	7 ♣
8 ♥	8 ♦	8 ♠	8 ♣
9 ♥	9 ♦	9 ♠	9 ♣
10 ♥	10 ♦	10 ♠	10 ♣
Jack ♥	Jack ♦	Jack ♠	Jack ♣
Queen ♥	Queen ♦	Queen ♠	Queen ♣
King ♥	King ♦	King ♠	King ♣

Ace is both high and low

52 card deck
4 suits
13 cards in each suit
4 of each rank

5 card poker hands $_{52}C_5 = 2598960\ Hands$ $P(hand) = \#\frac{of}{_{52}C_5}$

Hand							$_nC_r$	# of
Royal Flush	Ace ♥	King ♥	Queen ♥	Jack ♥	10 ♥	10-Ace same suit	$_4C_1 \times 1 = 4$	4
Straight Flush	5 ♠	6 ♠	7 ♠	8 ♠	10 ♠	5 card run same suit	$_4C_1 \times 10 - 4 = 36$	36
4 of a Kind	7 ♥	7 ♦	7 ♠	7 ♣	3 ♦	4 same rank, 1 other	$_{13}C_1\ _4C_4\ _{48}C_1$	624
Full House	2 ♥	2 ♦	2 ♠	4 ♦	4 ♣	3 same rank, 1 pair	$_{13}C_1\ _4C_3\ _{12}C_1\ _4C_2$	3744
Flush	4 ♠	8 ♠	Jack ♠	2 ♠	6 ♠	All same suit, no straight	$_4C_1 \times\ _{13}C_5 - 40$	5108
Straight	3 ♥	4 ♣	5 ♦	6 ♠	7 ♣	5 card run, not all same suit	$(\ _4C_1)^5 \times 10 - 40$	10200
3 of a kind	9 ♥	9 ♦	9 ♠	2 ♦	5 ♣	3 kind, 2 others not a pair	$_{13}C_1\ _4C_3\ _{12}C_2\ (_4C_1)^2$	54912
2 pair	4 ♥	4 ♠	5 ♦	5 ♠	Queen ♠	2 different pairs, 1 other	$_{13}C_2(\ _4C_2)^2\ _{44}C_1$	*123552*
Pair	King ♦	King ♠	6 ♥	9 ♠	2 ♦	1 pair 3 others	$_{13}C_1\ _4C_2\ _{12}C_3(\ _4C_1)^3$	1098240
High Card	Jack ♠	8 ♦	4 ♠	2 ♦	7 ♠	None of the above	$_{52}C_5 - above\ sum$	1302540

We choose 2,3 etc when we don't want them to be the same