C11 - 7.1 - Absolute Value: $|x|$ Notes

$|2| =$
$|2| = 2$

$|-3| = 3$

$|2-4| =$
$|-2| = 2$

$|3| - |-5| =$
$3 - 5 = -2$

Do whatever is inside the absolute value, then make it positive.

$-|3| = -3$

$-|-5| =$
$-(5) = -5$

Solve algebraically.

$|x| = 4$

"+" case:

$+(x) = 4$
$x = 4$

"−" case:

$-(x) = 4$
$x = -4$

Distribute a positive into the absolute value

Distribute a negative into the absolute value

Check your answer. (Left Hand Side LHS = RHS Right Hand Side)

$|x| = 4$
$|4| = 4$
$4 = 4$ ✓

$|x| = 4$
$|-4| = 4$
$4 = 4$ ✓

$|x-2| = 2$

"+" case:

$+(x-2) = 2$
$x - 2 = 2$
$x = 4$

"−" case:

$-(x-2) = 2$
$-x + 2 = 2$
$-x = 0$
$x = 0$

Check your answer. (LHS =RHS)

$|x-2| =$
$|4-2| =$
$|2| = 2$ ✓

$|x-2| =$
$|0-2| =$
$|-2| = 2$ ✓

C11 - 7.1 - Absolute Value: $|x|$ Notes

$2|x-2| = 6$

"+" case:

$+2(x-2) = 6$
$2x-4 = 6$
$2x = 10$
$x = 5$

"−" case:

$-2(x-2) = 6$
$-2x+4 = 6$
$-2x = 2$
$x = -1$

Check your answer.(LHS =RHS)

$2|x-2| =$
$2|5-2| =$
$2|3| = 6$ ✓

$2|x-2| =$
$2|-1-2| =$
$2|-3| = 6$ ✓

$|x| = -6$
Impossible.

C11 - 7.1 - Absolute Value Inequalities: $|x|$ Notes

5. $|x| \geq 2$

"+" case: "−" case:

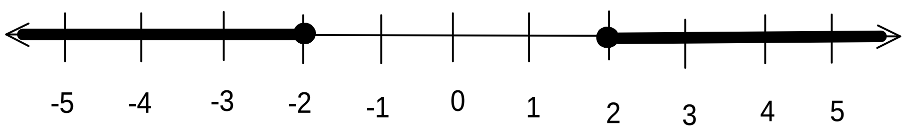
$+(x) \geq 2$

$x \geq 2$

$-(x) \geq 2$

$x \leq -2$

Divide by a negative, change direction of sign.



$\geq, \leq =$ ●

Shade greater than two, and less than negative two.

Check your answer. Test values in shaded region.

$|3| \geq$
$|3| \geq 3$
$3 \geq 2$

$|-3| \geq$
$|-3| \geq$
$3 \geq 2$

6. $|x - 3| < 2$

"+" case: "−" case:

$+(x - 3) < 2$

$x - 3 < 2$

$x < 5$

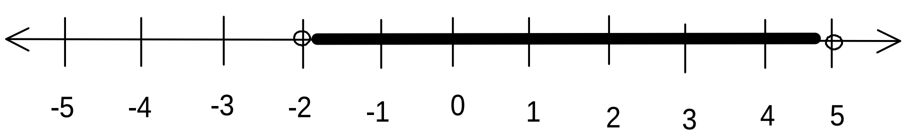
$-(x - 3) < 2$

$-x + 3 < 2$

$-x < 2$

$x > -2$

Divide by a negative, change direction of sign.



$>, < =$ ○

Shade less than five, and greater than negative two.

Check your answer. Test values in shaded region.

$|3| \geq$
$|3| \geq 3$
$3 \geq 2$

$|-3| \geq$
$|-3| \geq$
$3 \geq 2$

C11 - 7.2 - Linear Absolute Value: $y = |x + c|$ Notes

Graphing Absolute Values

$y = |x + 2|$

"+" case: Distribute a positive into the absolute value

$y_1 = +(x + 2)$

$y_1 = x + 2$

"−" case: Distribute a negative into the absolute value

$y_2 = -(x + 2)$

$y_2 = -x - 2$

$y = |x + 2|$

Table of Values

x	y
-5	-3
-3	-1
-2	0
-1	1
0	2

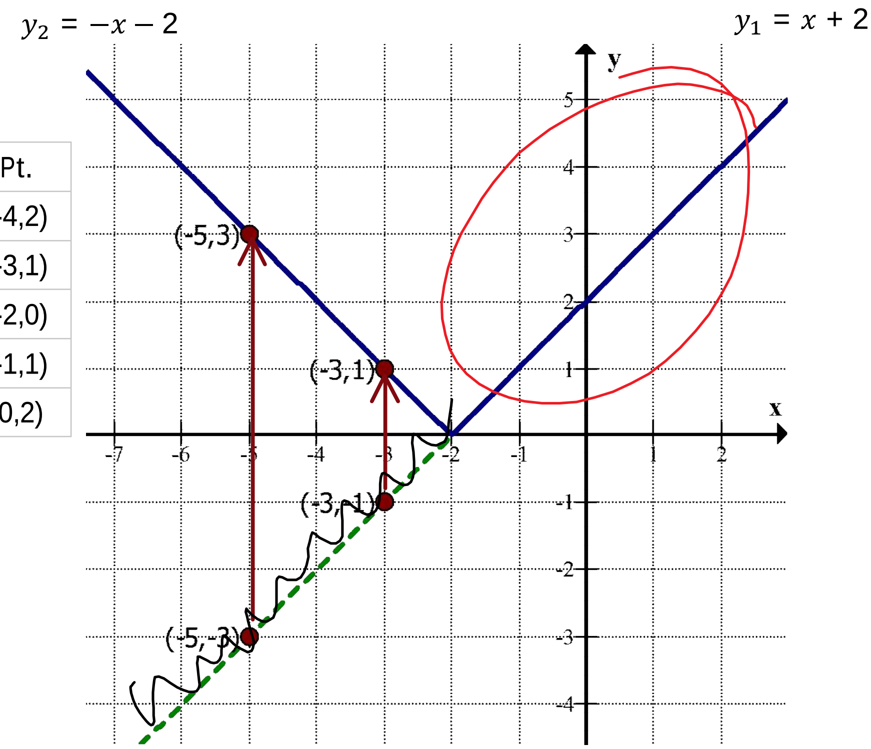
$y = x + 2$

→

x	y
-5	3
-3	1
-2	0
-1	1
0	2

$y = |x + 2|$

Pt.
(-4,2)
(-3,1)
(-2,0)
(-1,1)
(0,2)



Notice the graph of $y = |x + 2|$ is the graph of $y = x + 2$ and $y = -x - 2$ without any negative y values. Transfer any negative y value to a positive y value.

Piecewise function: $y = \begin{cases} x + 2, if\ x \geq -2 \\ -x - 2, if\ \ x < -2 \end{cases}$

$y = \begin{cases} \text{"+" case, Domain of "+" case} \\ \text{"−" case, Domain of "−" case} \end{cases}$

Notice: The domain of the negative case is not equal to.

Domain of positive case:

$x + 2 \geq 0$
$-2 \quad\quad -2$
$x \geq -2$

Set what is inside the absolute value greater than or equal to zero.

Domain of negative case:

$x + 2 < 0$
$-2 \quad\quad -2$
$x < -2$

Set what is inside the absolute value less than zero.

C11 - 7.2 - Linear Absolute Value Equations $|x| = c$ Notes

Solve algebraically

$|x+2| = 4$

"+" case:

$$+(x+2) = 4$$
$$x+2 = 4$$
$$x = 2$$

"−" case:

$$-(x+2) = 4$$
$$-x-2 = 4$$
$$-x = 6$$
$$x = -6$$

Check your answer.

$$|x+2| =$$
$$|2+2| =$$
$$|4| = 4$$

$$|-6+2| =$$
$$|-4| =$$
$$|-4| = 4$$

Solve graphically.

$|x+2| = 4$

Left hand side (LHS)

"+" case:

$$y_1 = +(x+2)$$
$$y_1 = x+2$$

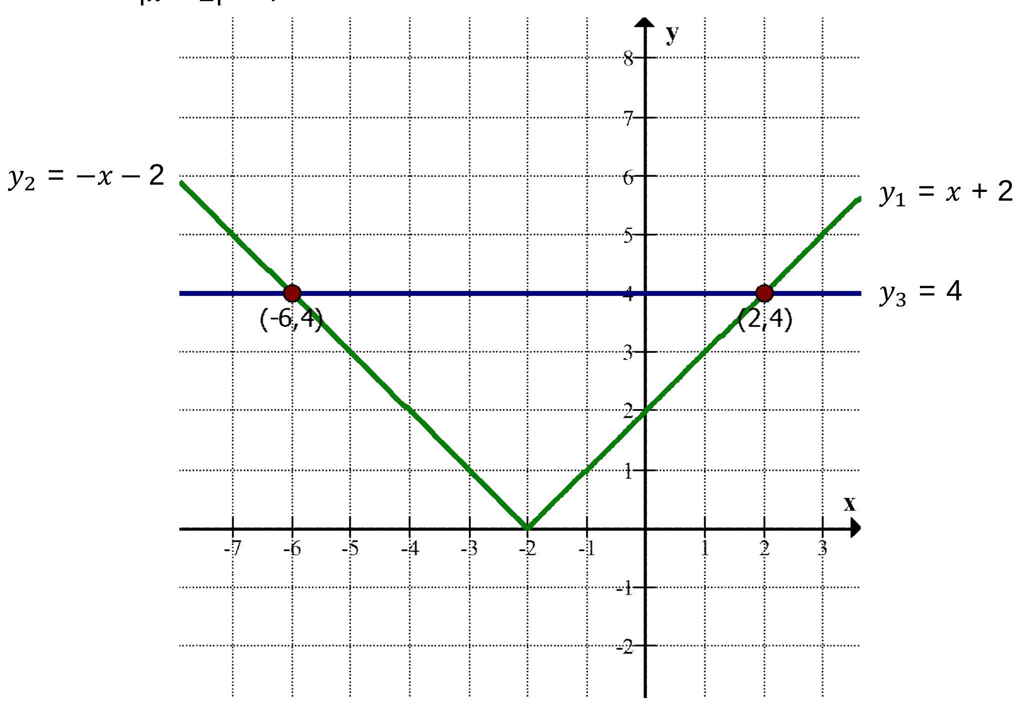
"−" case:

$$y_2 = -(x+2)$$
$$y_2 = -x-2$$

Right hand side (RHS)

$$y_3 = 4$$

$|x+2| = 4$



C11 - 7.3 - Quadratic Absolute Value Notes

2. $y = |x^2 - 4|$

"+" case:

$y_1 = +(x^2 - 4)$

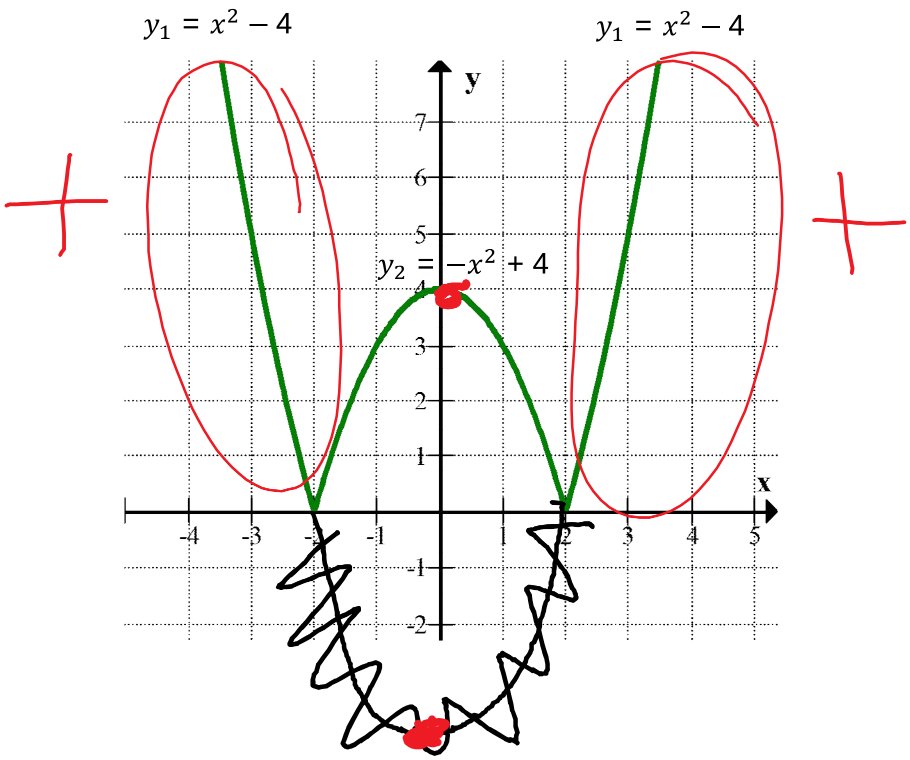
$y_1 = x^2 - 4$

"−" case:

$y_2 = -(x^2 - 4)$

$y_2 = -x^2 + 4$

$y = |x^2 - 4|$



Notice the graph of $y = |x^2 - 4|$ is the graph of $y_1 = x^2 - 4$ less than two and greater than two and is the graph of $y_2 = -x^2 + 4$ less than two and greater than negative two.

Piecewise function:

$$y = \begin{cases} x^2 - 4, & if\ x \geq 2, x \leq -2 \\ -x^2 + 4, & if -2 < x < 2 \end{cases}$$

C11 - 7.3 - Quadratic Absolute Value Equations Notes

Solve algebraically.

$|x^2 - 4| = x + 2$

"+" case:

$$+(x^2 - 4) = x + 2$$
$$x^2 - 4 = x + 2$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x = 3, -2$$

"−" case:

$$-(x^2 - 4) = x + 2$$
$$-x^2 + 4 = x + 2$$
$$0 = x^2 + x - 2$$
$$0 = (x + 2)(x - 1)$$
$$x = -2, 1$$

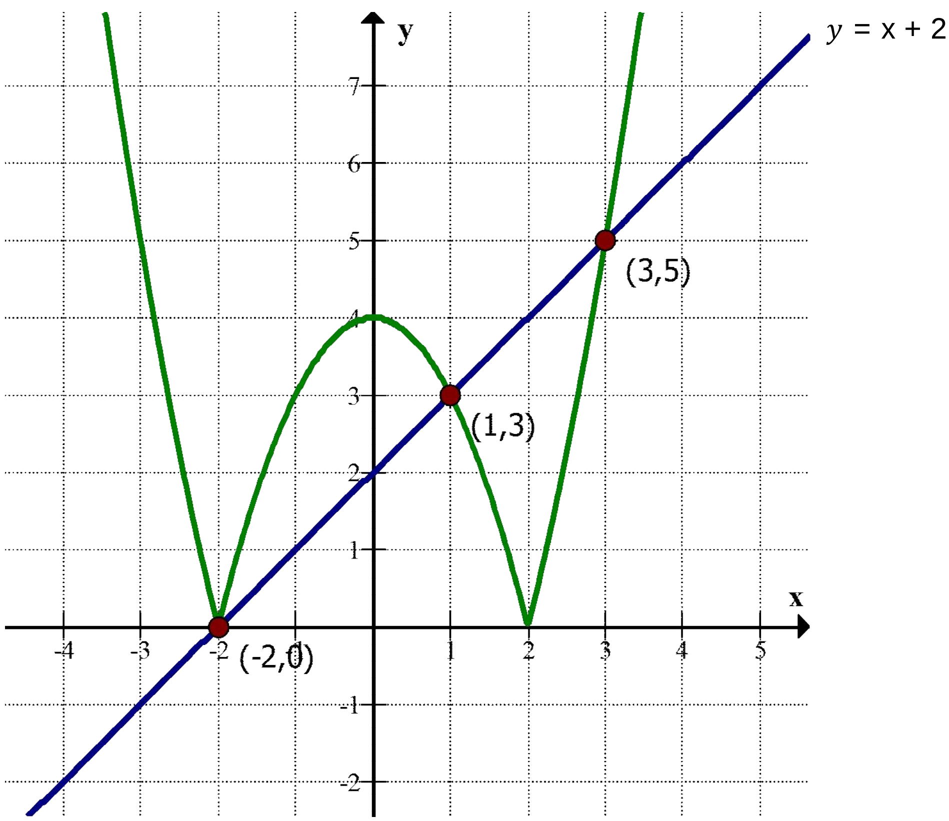
Check Answers!

$x = 3, -2$

$x = -2, 1$

Solve Graphically

$y = |x^2 - 4|$



C11 - 7.4 - Linear Reciprocals Notes

$y = x + 4$ Line

$y = \frac{1}{x+4}$ Reciprocal line

Solve algebraically: set denominator = 0, 1, -1.

Vertical asymptote (VA):
Denominator = 0

$x + 4 = 0$
$x = -4$

VA: $x = -4$

Invariant points (IP):
Denominator = 1

$x + 4 = 1$
$x = -3$

$(-3, 1)$

Invariant points (IP):
Denominator = -1

$x + 4 = -1$
$x = -5$

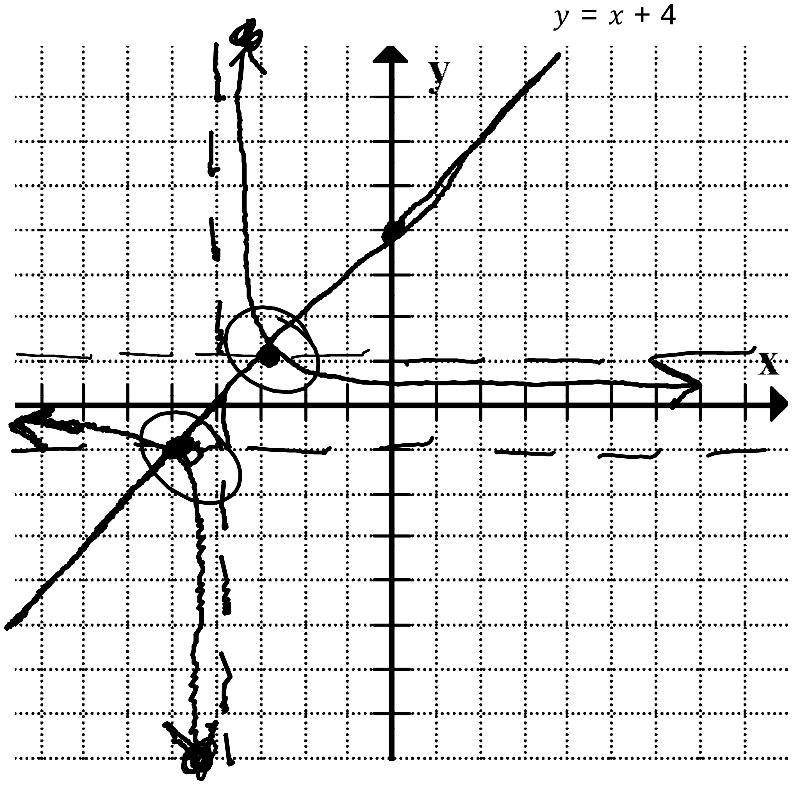
$(-5, -1)$

1. Graph original
2. Graph VA: Dotted line
3. Graph IP's
4. Graph reciprocal

x	y
-100	$-.01$
-5	-1
-4.1	-10
-4.01	-100
-4	UND
-3.99	100
-3.9	10
-3	1
100	.01

$y = x + 4$

$y = \frac{1}{x+4}$



Notice: The invariant points are the intersection of the original and the lines $y = 1, y = -1$

Notice: The vertical asymptote(s) of the reciprocal is the X intercept of the original

C11 - 7.4 - Quadratic Reciprocals Notes

$y = x^2 - 4$ Parabola

$y = \dfrac{1}{x^2 - 4}$ Reciprocal Parabola

Solve algebraically: set denominator = 0, 1, -1.

Vertical asymptote (VA):
Denominator = 0

$$x^2 - 4 = 0$$
$$(x + 2)(x - 2) = 0$$
$$x = 2, -2$$

VA's: $x = 2$
$x = -2$

Invariant points (IP):
Denominator = 1

$$x^2 - 4 = 1$$
$$x^2 = 5$$
$$x = \sqrt{5}, -\sqrt{5}$$

$(\sqrt{5}, 1)$
$(-\sqrt{5}, 1)$

Invariant points (IP):
Denominator = -1

$$x^2 - 4 = -1$$
$$x^2 = 3$$
$$x = \sqrt{3}, -\sqrt{3}$$

$(\sqrt{3}, -1)$
$(-\sqrt{3}, -1)$

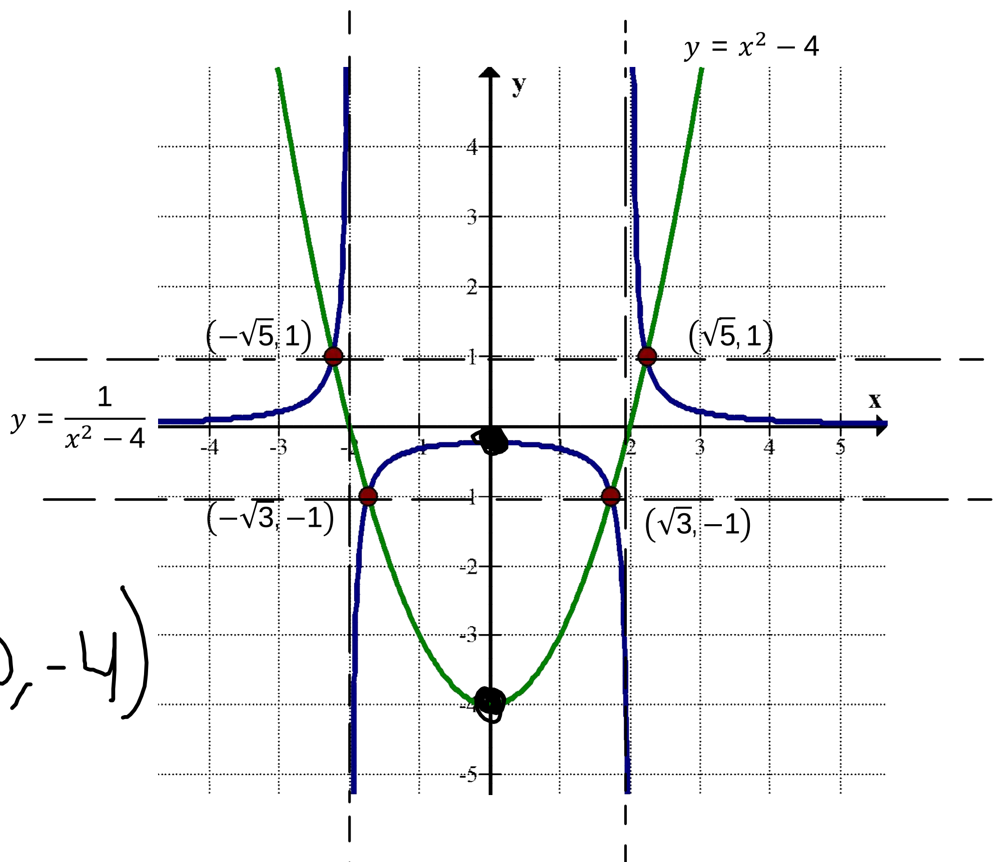
Solve graphically.

$y = x^2 - 4$
$y = \dfrac{1}{x^2 - 4}$

1. Graph original
2. Graph VA's: Dotted lines
3. Graph IP's
4. Graph reciprocal

$\left(0, -\frac{1}{4}\right)$

$(0, -4)$



Notice: The inflection points are the intersection of the original and the lines $y = 1, y = -1$