C11 - 4.1 - Solving $x-intercepts$ Notes

Solve for $x-$ intercepts.

$y = x^2 + 6x + 5$

$\underline{1} \times \underline{5} = 5$
$\underline{1} + \underline{5} = 6$

$y = (x + 1)(x + 5)$

Factor

$0 = (x + 1)(x + 5)$

Set y equal to zero, $(y = 0)$

$x + 1 = 0$ $\quad$ $x + 5 = 0$

Set the brackets equal to zero seperately

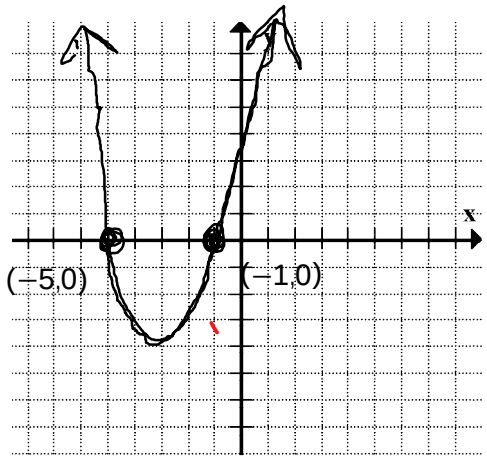
$-1 \quad -1$ $\quad$ $-5 \quad -5$

Solve

$x = -1$ $\quad$ $x = -5$

$x-int$: $(-1,0)$ $\quad$ $(-5,0)$

State $x-$ intercepts $(x, 0)$



Draw a graph and label $x-$ intercepts.

Nicholas Cragg at 2015-08-24 1:18 PM

$y = 2x^2 - 3x - 2$
$y = 2x^2 - 4x + 1x - 2$
$y = (2x^2 - 4x)(+1x - 2)$
$y = 2x(x - 2) + 1(x - 2)$
$y = (x - 2)(2x + 1)$
$0 = (x - 2)(2x + 1) = 0$

Factor
Decompose
Group
GCF
Switch

$x - 2 = 0$ $\quad$ $2x + 1 = 0$

Set the brackets equal to zero seperately

$+2 \quad +2$ $\quad$ $-1 \quad -1$

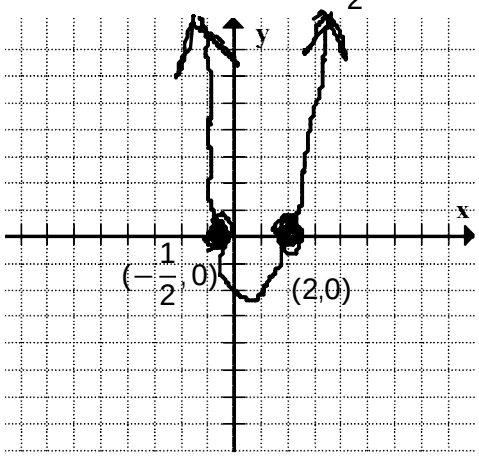
$x = 2$ $\quad$ $\frac{2x}{2} = -\frac{1}{2}$

Solve

$x = -\frac{1}{2}$

State $x-$ intercepts $(x, 0)$

$x-int$: $(2,0)$ $\quad$ $(-\frac{1}{2}, 0)$



Draw a graph and label $x-$ intercepts.

C11 - 4.1 - Solving $x-intercepts$ Notes

Set $y = 0$ *and factor to find* $x-intercepts.$ $(x, 0)$

$y = x^2 - 6x + 5$
$0 = x^2 - 6x + 5$
$0 = (x-5)(x-1)$

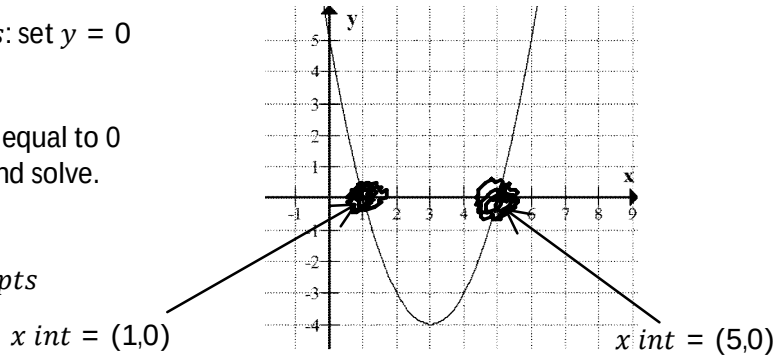
x *intercepts*: set $y = 0$
Factor.

$x - 5 = 0$ $\quad$ $x - 1 = 0$
$+5 \quad +5$ $\quad$ $+1 \quad +1$
$x = 5$ $\quad$ $x = 1$

Set brackets equal to 0 separately and solve.

(5,0) $\quad$ (1,0)

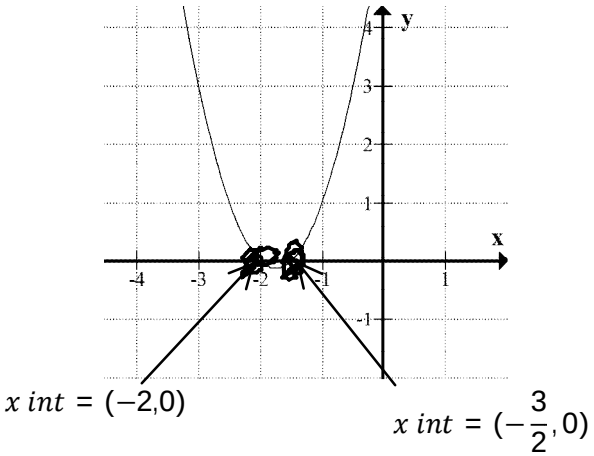
$x-intercepts$



$y = 2x^2 + 7x + 6$
$0 = 2x^2 + 7x + 6$
$0 = 2x^2 + 4x + 3x + 6$
$0 = 2x(x+2) + 3(x+2)$
$0 = (2x+3)(x+2)$

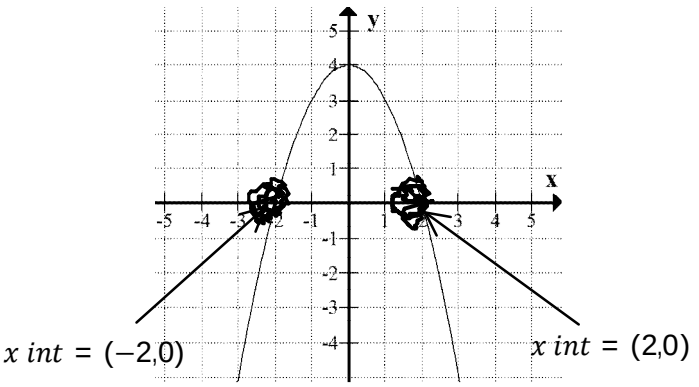
$2x + 3 = 0$
$-3 \quad -3$
$2x = -3$
$\frac{2x}{2} = \frac{-3}{2}$
$x = -\frac{3}{2}$

$x + 2 = 0$
$-2 \quad -2$
$x = -2$



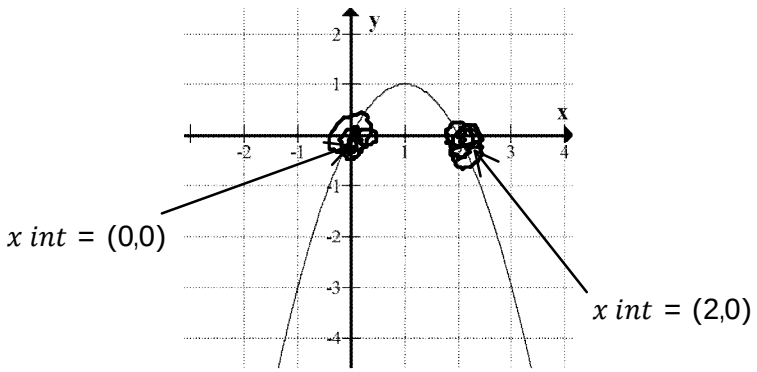
$y = -x^2 + 4$
$0 = -x^2 + 4$
$0 = -(x^2 - 4)$ GCF: -1
$0 = -(x+2)(x-2)$ Factor.

$x + 2 = 0$ $\quad$ $x - 2 = 0$
$-2 \quad -2$ $\quad$ $+2 \quad +2$
$x = -2$ $\quad$ $x = 2$



$y = -x^2 + 2x$
$0 = -x^2 + 2x$
$0 = -x(x-2)$

$x = 0$ $\quad$ $x - 2 = 0$
$+2 \quad +2$
$x = 2$



C11 - 4.2 - $x - int$/Standard Form Notes

$x\ int$ = (2,0) , (6,0)

$x = 2$ $\quad$ $x = 6$

$-2 \quad -2$ $\quad$ $-6 \quad -6$

$x - 2 = 0$ $\quad$ $x - 6 = 0$

$y = (x - 2)(x - 6)$

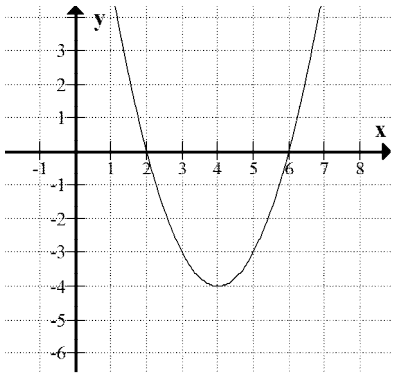
$y = x^2 - 8x + 12$

Write down the x values.

Add or subtract to both sides to make = 0

Factored Form

Standard Form



$x\ int = \left(\frac{1}{2}, 0\right), (4,0)$

$x = \frac{1}{2}$ $\quad$ $x = 4$

$2 \times x = \frac{1}{2} \times 2$ $\quad$ $-4 \quad -4$

$2x = 1$ $\quad$ $x - 4 = 0$

$-1 \quad -1$

$2x - 1 = 0$

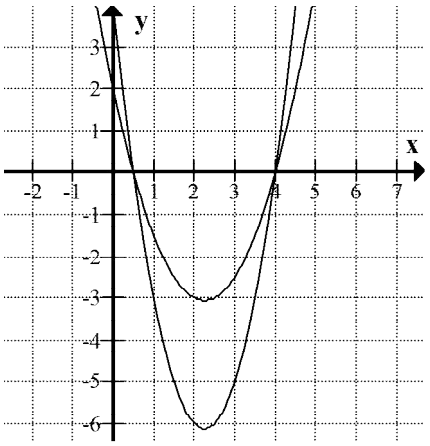
$y = (2x - 1)(x - 4)$

$y = 2x^2 - 9x + 4$

Multiply and Add or subtract to both sides to make = 0

$y = x^2 - \frac{9}{2}x + 2$

$0 = x^2 - \frac{9}{2}x + 2$



$y = 2x^2 - 9x + 4$

$0 = 2x^2 - 9x + 4$

$x\ int = \left(\frac{1}{2}, 0\right), (4,0)$

$x = \frac{1}{2}$ $\quad$ $x = 4$

$-\frac{1}{2} \quad -\frac{1}{2}$ $\quad$ $-4 \quad -4$

$x - \frac{1}{2} = 0$ $\quad$ $x - 4 = 0$

$y = \left(x - \frac{1}{2}\right)(x - 4)$

$y = x^2 - 4x - \frac{1}{2}x + 2$

$y = x^2 - \frac{9}{2}x + 2$

Notice: two different graphs in standard form can have the same x-intercepts.

C11 - 4.2 - Find Standard Form x-int "a" and a Point Notes

Find equation in Standard Form using $x-$ intercepts and "a"

$y = a(x + \#)(x + \#)$

$x-int = 2$ *and* 6
$a = 1$

$x = 2$
$-2 \quad -2$
$x - 2 = 0$

$x = 6$
$-6 \quad -6$
$x - 6 = 0$

Set $x-int = \#$ and make equal to zero

$y = a(x + \#)(x + \#)$
$y = 1(x - 2)(x - 6)$
$y = (x - 2)(x - 6)$
$y = x^2 - 8x + 12$

Write Factored Form
Substitute Factors

Foil

$x-int = 2$ *and* -2
$a = 2$

$x = 2$
$-2 \quad -2$
$x - 2 = 0$

$x = -2$
$+2 \quad +2$
$x + 2 = 0$

$y = a(x + \#)(x + \#)$
$y = 2(x - 2)(x + 2)$
$y = 2(x^2 + 2x - 2x - 4)$
$y = 2(x^2 - 4)$
$y = 2x^2 - 8$

$x-int = \frac{3}{2}$ *and* $-\frac{7}{2}$

$x = \frac{3}{2}$
$2 \times x = \frac{3}{2} \times 2$
$2x = 3$
$-3 \quad -3$
$2x - 3 = 0$

$x = -\frac{7}{2}$
$2 \times x = \frac{3}{2} \times 2$
$2x = -7$
$+7 \quad +7$
$2x + 7 = 0$

$y = a(x + \#)(x + \#)$
$y = (2x - 3)(2x + 7)$
$y = 4x^2 + 14x - 6x - 21$
$y = 4x^2 + 8x - 21$

$x-int = -1$ *and* 3
$(2, -6)$

$y = a(x + 1)(x - 3)$
$-6 = a(2 + 1)(2 - 3)$
$-6 = a(3)(-1)$
$-6 = -3a$
$a = 2$

$y = 2(x + 1)(x - 3)$

C11 - 4.3 - x –Intercepts/Vertex Form

$y = x^2 - 2x - 8$
$y = (x - 2)(x + 4)$

$x - 2 = 0$ $\quad$ $x + 4 = 0$
$+2 \quad +2$ $\quad$ $-4 \quad -4$
$x = 2$ $\quad$ $x = -4$ $\quad$ $x - int$: $\quad$ (2,0) $\quad$ (−4,0)

The x coordinate of the vertex is always halfway between the two x-intercepts.

$x = \frac{(2) + (-4)}{2} = \frac{-2}{2}$
$x = -1$ Find the average between the two x-intercept values.

Vertex: $(-1, y)$

$y = (x - 2)(x + 4)$
$y = ((-1) - 2)((-1) + 4)$ Find the y value of the vertex by putting in the x value of the vertex
$y = (-3)(3)$
$y = -9$

Vertex: $(-1, -9)$

Vertex:

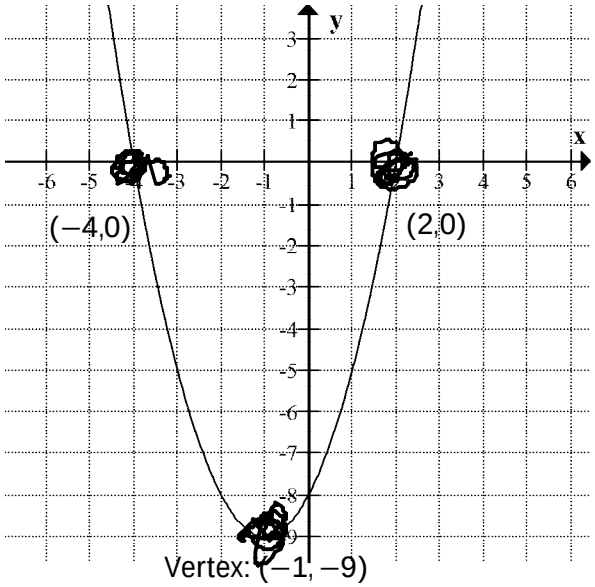
x	y
−3	−5
−2	−8
−1	−9
0	−8
1	−5

$y = (x - 2)(x + 4)$
$y = \big((-3) - 2\big)\big((-3) + 4\big)$
$y = (-5)(1)$
$y = -5$

$y = (x - 2)(x + 4)$
$y = \big((-2) - 2\big)\big((-2) + 4\big)$
$y = (-4)(2)$
$y = -8$

$y = (x - 2)(x + 4)$
$y = \big((0) - 2\big)\big((0) + 4\big)$
$y = (-2)(4)$
$y = -8$

$y = (x - 2)(x + 4)$
$y = \big((1) - 2\big)\big((1) + 4\big)$
$y = (-1)(5)$
$y = -5$



Vertex: $(-1, -9)$

Notice: the pattern from the vertex $(-1, -9)$ is symmetrical on both sides.
Over 1, 1 squared = 1, up 1. Back to the vertex. Over 2, 2 squared = 4, up 4.

Divide the with in half and square it to find how far down the vertex is

C11 - 4.3 - Solving by Square Root Method Notes

$x^2 - 4 = 0$
$+4 \quad +4$
$x^2 = 4$
$\sqrt{x^2} = \pm\sqrt{4}$
$x = \pm 2$

Need a ± because the 2 could have been positive or negative before it was squared.

$x = 2 \qquad x = -2$

$(x-2)^2 - 9 = 0$
$+9 \quad +9$
$(x-2)^2 = 9$
$\sqrt{(x-2)^2} = \pm\sqrt{9}$
$x - 2 = \pm 3$

$x - 2 = 3$
$+2 \quad +2$
$x = 5$

$x - 2 = -3$
$+2 \quad +2$
$x = -1$

$2(x+1)^2 - 8 = 0$
$+8 \quad +8$
$2(x+1)^2 = 8$
$\frac{2(x+1)^2}{2} = \frac{8}{2}$
$(x+1)^2 = 4$
$\sqrt{(x+1)^2} = \pm\sqrt{4}$
$x + 1 = \pm 2$

$x + 1 = 2$
$-1 \quad -1$
$x = 1$

$x + 1 = -2$
$-1 \quad -1$
$x = -3$

$x^2 + 16 = 0$
$-16 \quad -16$
$x^2 = -16$
$\sqrt{x^2} = \pm\sqrt{-16}$

Can't square root a negative. DNE

$(x+2)^2 + 2 = 0$
$-2 \quad -2$
$(x+2)^2 = -2$
$\sqrt{(x+2)^2} = \sqrt{-2}$

Can't square root a negative. DNE

C11 - 4.3 - Solve by Completing the Square Notes

Steps to solve by completing the square:

$x^2 - 2x - 1 = 0$

$Group\ x\ terms.$

$(x^2 - 2x) - 1 = 0$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$ "b" divided by 2 all squared: $\left(\frac{b}{2}\right)^2$

$(x^2 - 2x - 1 + 1) - 1 = 0$

$(x^2 - 2x + 1) - 1 - 1 = 0$

Add and subtract inside brackets.
Remove number not contributing to perfect square.

$(x^2 - 2x + 1) - 2 = 0$

$(x - 1)(x - 1) - 2 = 0$

$(x - 1)^2 - 2 = 0$

Factor brackets, simplify outside

$+2 \quad +2$

Add or subtract that number on both sides of the equation.

$(x - 1)^2 = 2$

$\sqrt{(x - 1)^2} = \pm\sqrt{2}$

Square root to eliminate the square.

$x - 1 = \pm\sqrt{2}$

$+1 \quad +1$

$x = \pm\sqrt{2} + 1$

Add or subtract constant to isolate x.

$x = \sqrt{2} + 1 \quad x = -\sqrt{2} + 1$

Make two equations equal to x.

$x^2 - 2x - 1 = 0$

$+1 \quad +1$

Keep all x terms on one side. Add or subtract constant to both sides.

$x^2 - 2x = 1$

$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$ "b" divided by 2 all squared: $\left(\frac{b}{2}\right)^2$

$+1 \quad +1$

Add or subtract that number on both sides of the equation.

$x^2 - 2x + 1 = 2$

$(x - 1)(x - 1) = 2$

Factor the left hand side.

$(x - 1)^2 = 2$

Write the perfect square on the left.

$\sqrt{(x - 1)^2} = \pm\sqrt{2}$

Square root both sides.

$x - 1 = \pm\sqrt{2}$

$+1 \quad +1$

$x = \pm\sqrt{2} + 1$

Isolate for x

$x = -\sqrt{2} + 1 \quad x = \sqrt{2} + 1$

Split into two equations.

C11 - 4.4 - Quadratic Equation Notes

Solve for $x-$ intercepts

$y = \overset{a}{1}x^2 + \overset{b}{8}x + \overset{c}{15}$

$a = 1$
$b = 8$
$c = 15$

Quadratic Equation

$$x_{int} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{int} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{int} = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{64 - 60}}{2}$$

$$= \frac{-8 \pm \sqrt{4}}{2}$$

$$= \frac{-8 \pm 2}{2}$$

$\frac{-8-2}{2} = \frac{-10}{2} = -5$

$\frac{-8+2}{2} = \frac{-6}{2} = -3$

2 rational roots.

$x\ int: -5$

$x\ int: -3$

$2x^2 - 6x - 7 = 0$

$a = 2$
$b = -6$
$c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{36 + 56}}{4}$$

$$x = \frac{6 \pm \sqrt{92}}{4}$$

$$x = \frac{6 \pm 2\sqrt{23}}{4}$$

$$x = \frac{3 \pm \sqrt{23}}{2}$$

Simplify $\div$ *top and bottom by* 2

$x = \frac{3+\sqrt{23}}{2}$

$x = \frac{3-\sqrt{23}}{2}$

2 irrational roots.

C11 - 4.4 - Quadratic Equation Notes Part 2

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$2x^2 - 5x + 1 = 0$

$a = 2$
$b = -5$
$c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x == \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - 8}}{4}$$

$$x = \frac{5 - \sqrt{17}}{4}$$

$$x_1 = \frac{5 + \sqrt{17}}{4}$$

$$x_2 = \frac{5 - \sqrt{17}}{4}$$

Discriminant > 0
2 Real Roots.

2 Irrational Roots.

$x + 6x + 11 = 0$

$a = 1$
$b = 6$
$c = 11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(11)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 44}}{2}$$

$$x = \frac{-6 \pm \sqrt{-8}}{2}$$

Discriminant < 0
No Real Roots.

$3v^2 - 6v + 3 = 0$

$a = 3$
$b = -6$
$c = 3$

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(3)}}{3}$$

$$v = \frac{6 \pm \sqrt{36 - 36}}{3}$$

$$v = \frac{6 \pm 0}{3}$$

$$v = \frac{6}{3}$$

$$v = 2$$

$v_1 = 2$

Discriminant = 0
No Real Roots.

1 rational root.

C11 - 4.4 - Discriminant Notes

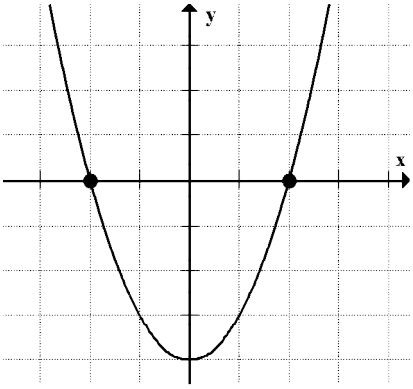
Quadratic Formula: $x_{int} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

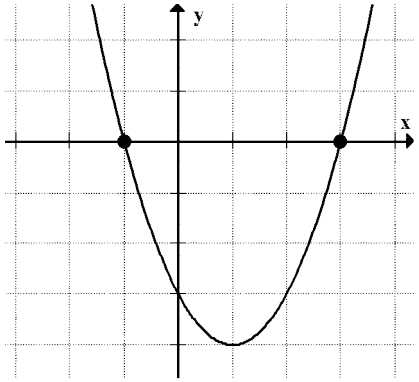
$$x_{int} = \frac{-b \pm \sqrt{DISCRIMINANT}}{2a}$$

Discriminant: $b^2 - 4ac$

Inside the root is positive

Case 1: $b^2 - 4ac > 0$

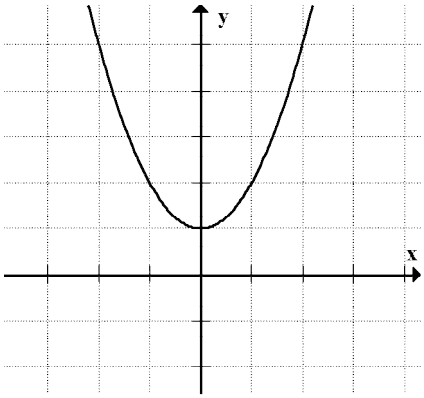


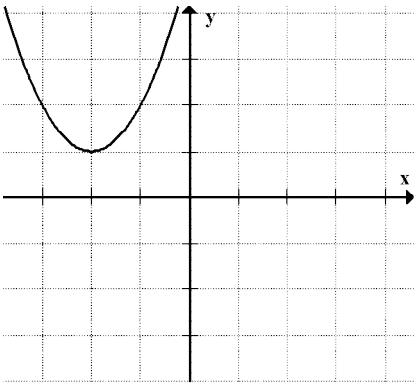


Two x-intercepts
Two Real Roots
Two Solutions

Inside the root is negative

Case 2: $b^2 - 4ac < 0$

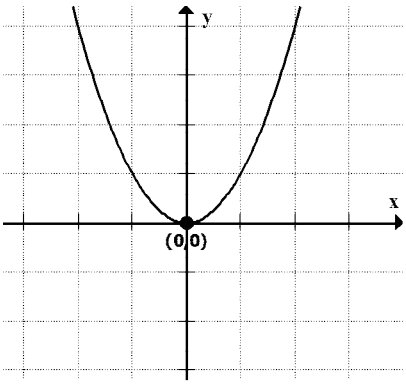


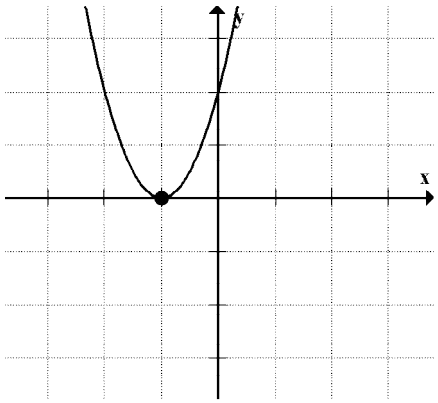


Zero x-intercepts
No Real Roots
No Solutions
Imaginary Roots

Inside the root is zero

Case 3: $b^2 - 4ac = 0$





One x-intercepts
Two equal/real roots
One Solution

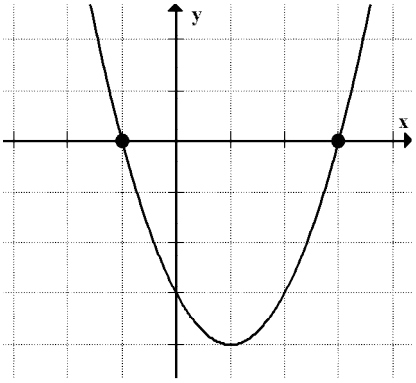
C11 - 4.4 - Discriminant Notes w/ Eqns

Quadratic Formula: $x_{int} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x_{int} = \frac{-b \pm \sqrt{DISCRIMINANT}}{2a}$$

Discriminant: $b^2 - 4ac$

Case 1: $b^2 - 4ac > 0$



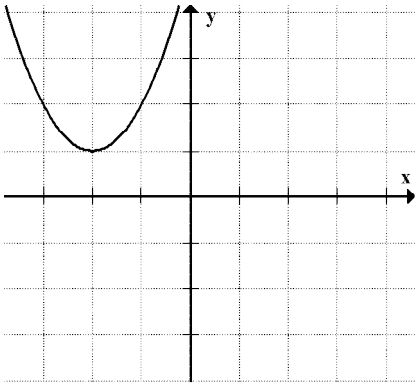
$x^2 - 2x - 3$

$b^2 - 4ac$
$(-2)^2 - 4(1)(-3)$
$4 + 12$
16

Inside the root is positive

Two x-intercepts
Two Real Roots
Two Solutions

Case 2: $b^2 - 4ac < 0$



$x^2 + 4x + 5$

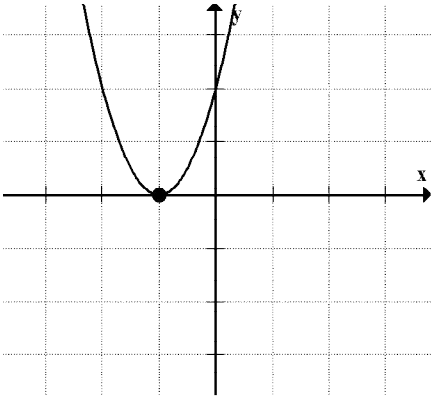
$b^2 - 4ac$
$(4)^2 - 4(1)(5)$
$16 - 20$
-4

Inside the root is negative

Cant Square Root Negatives

Zero x-intercepts
No Real Roots
No Solutions
Imaginary Roots

Case 3: $b^2 - 4ac = 0$



$2x^2 + 4x + 2$

$b^2 - 4ac$
$(4)^2 - 4(2)(2)$
$16 - 16$
0

Inside the root is zero

One x-intercepts
Two equal/real roots
One Solution