C12 - 6.1 - Ratios $tanx$ $secx$ Notes

$$\boxed{\frac{sinx}{cosx} = tanx \qquad secx = \frac{1}{cosx}}$$

$\frac{sinx}{sinx} = 1$ $\frac{\sin^2 x}{sinx} = sinx$ $\frac{\sin^3 x}{sinx} = \sin^2 x$ $\frac{\sin^3 x}{\sin^2 x} = \sin x$

$\frac{\cos x}{cosx} = 1$ $\frac{\cos^2 x}{cosx} = cosx$ $\frac{\cos^3 x}{cosx} = \cos^2 x$ $\frac{\cos^3 x}{\cos^2 x} = \cos x$

$$\boxed{\begin{array}{l} \sin^2 x = (sinx)(sinx) \neq sinx^2 \\ \cos^2 x = (cosx)(cosx) \neq cosx^2 \end{array}}$$

$\frac{sinx}{1} \times \frac{1}{cosx}$
$\frac{sinx}{cosx}$
$= tanx$

$sinxtanx$
$sinx \times \frac{sinx}{cosx}$
$= \frac{\sin^2 x}{cosx}$

$cosxtanx$
$cosx \times \frac{sinx}{cosx}$
$= sinx$

$\frac{sinxcosx}{sinx} = cosx$

$\frac{cosxsinx}{cosx} = sinx$

$\frac{sinx}{tanx}$
$\frac{sinx}{\left(\frac{sinx}{cosx}\right)}$
$sinx \div \frac{sinx}{cosx}$
$sinx \times \frac{cosx}{sinx}$
$= cosx$

$\frac{cosx}{tanx}$
$\frac{cosx}{\left(\frac{sinx}{cosx}\right)}$
$cosx \div \frac{sinx}{cosx}$
$cosx \times \frac{cosx}{sinx}$
$= \frac{\cos^2 x}{sinx}$

$\frac{tanx}{cosx}$
$\frac{\left(\frac{sinx}{cosx}\right)}{\frac{cosx}{1}}$
$\frac{sinx}{cosx} \div \frac{cosx}{1}$
$\frac{sinx}{cosx} \times \frac{1}{cosx}$
$= \frac{sinx}{\cos^2 x}$

$\frac{tanx}{sinx}$
$\frac{\left(\frac{sinx}{cosx}\right)}{\frac{sinx}{1}}$
$\frac{sinx}{cosx} \div \frac{sinx}{1}$
$\frac{sinx}{cosx} \times \frac{1}{sinx}$
$\frac{1}{cosx}$
$= secx$

Flip and Multiply

C12 - 6.1 - Ratios $cscx\ cotx$ Notes

$$secxcosx =$$
$$\frac{1}{cosx} \times cosx =$$
$$\frac{cosx}{cosx} = 1$$

$$\boxed{secx = \frac{1}{cosx}}$$

$$secxsinx =$$
$$\frac{1}{cosx} \times sinx =$$
$$\frac{sinx}{cosx} = tanx$$

$$\boxed{\frac{sinx}{cosx} = tanx}$$

$$secxtanx =$$
$$\frac{1}{cosx} \times \frac{sinx}{cosx} = \frac{sinx}{\cos^2 x}$$

$$cscxsinx = \frac{1}{sinx} \times sinx = \frac{sinx}{sinx} = 1$$

$$\boxed{cscx = \frac{1}{sinx}}$$

$$csxcosx =$$
$$\frac{1}{sinx} \times cosx =$$
$$\frac{cosx}{sinx} = cotx$$

$$\boxed{\frac{cosx}{sinx} = cotx}$$

$$cscxtanx =$$
$$\frac{1}{sinx} \times \frac{sinx}{cosx} =$$
$$\frac{1}{\cos x} = secx$$

C12 - 6.2 - Factoring Distribution Notes

$\sin^2 x - \cos^2 x$
$(sinx + cosx)(sinx - cosx)$

Factoring/Distribution

$\sin x - \sin^2 x$
$sinx(1 - sinx)$

$sinxcosx + sinx$
$sinx(cosx + 1)$

$sinx + \sin^2 x$
$sinx(1 + sinx)$

$GCF\ - Distribution = sinx$

$\cos x - \cos^2 x$
$cosx(1 - cosx)$

$sinxcosx + cosx$
$cosx(sinx + 1)$

$cosx + \cos^2 x$
$cosx(1 + cosx)$

$GCF = cosx$

$(1 + cosx)(1 - cosx)$
$1 - \cancel{cosx} + \cancel{cosx} - \cos^2 x$
$1 - \cos^2 x$
$\sin^2 x$

$(1 + sinx)(1 - sinx)$
$1 - \cancel{sinx} + \cancel{sinx} - \sin^2 x$
$1 - \sin^2 x$
$\cos^2 x$

$FOIL\ - Differences\ of\ squares$

$(sinx + 1) = (1 + sinx)$

$-\cos^2 x + 1 = 1 - \cos^2 x$

$Rearange\ order\ of\ Terms$

$2 + sinx + \sin^2 x = \sin^2 x + sinx - 2$
$= (sinx + 2)(sinx - 1)$

$\boxed{let\ m = sinx}$

$Factor$

$m^2 + m - 2$
$(m + 2)(m - 1)$

$sinx - \cos^2 x - 1 = 0$
$sinx - (1 - \sin^2 x) - 1 = 0$
$sinx - 1 + \sin^2 x - 1 = 0$
$\sin^2 x + sinx - 2 = 0$

$(sinx - 1)(sinx + 2) = 0$

$cosx = 2\sin^2 x - 1$
$cosx = 2(1 - \cos^2 x) - 1$
$cosx = 2 - 2\cos^2 x - 1$
$2\cos^2 x + cosx - 1 = 0$

$(2cosx + 1)(cosx - 1) = 0$

C12 - 6.2 - Fractions Notes

$\frac{1}{cosx} - \frac{\sin x}{cosx} = \frac{1 - sinx}{cosx}$ $\quad Add\ and\ Subtract\ Fractions: LCD$

$\frac{1}{sinx} - sinx$

$\frac{1}{sinx} - sinx \times \frac{sinx}{sinx}$ $\quad Add\ and\ Subtract\ Fractions: LCD$

$\frac{1}{sinx} - \frac{\sin^2 x}{sinx}$

$\frac{1 - \sin^2 x}{sinx}$

$\frac{\cos^2 x}{sinx}$ $\quad \cos^2 x = 1 - \sin^2 x$

$\frac{secx}{sinx} - tanx$

$\frac{\frac{1}{cosx}}{\frac{sinx}{1}} - \frac{sinx}{cosx}$

$\frac{1}{cosx} \times \frac{1}{sinx} - \frac{sinx}{cosx}$

$\frac{1}{sinxcosx} - \frac{sinx}{cosx}$

$\frac{1}{sinxcosx} - \frac{sinx}{cosx} \times \frac{sinx}{sinx}$

$\frac{1}{sinxcosx} - \frac{\sin^2 x}{sinxcosx}$

$\frac{1 - \sin^2 x}{sinxcosx}$

$\frac{\cos^2 x}{sinxcosx}$

$\frac{cosx}{sinx}$

$cotx$

$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}$

$\left(\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}\right) \times \frac{cosx}{cosx}$ $\quad LDC = cosx$

$\frac{\frac{cosx}{cosx} - \cos^2 x}{cosx - \frac{sinxcosx}{cosx}}$

$\frac{1 - \cos^2 x}{cosx - sinx}$

$\frac{\sin^2 x}{cosx - sinx}$

$\frac{sinx + cosx}{cosx}$

$\frac{sinx}{cosx} + \frac{cosx}{cosx}$ $\quad Separate\ Fractions$

$tanx + 1$

C12 - 6.4 - Proofs Pythag Reciprocal Frac Notes

$\tan x \csc x$	=	$\sec x$
$\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}\right)$		
$\frac{1}{\cos x}$		
$\sec x$		

$\frac{\cot x}{\csc x}$	=	$\cos x$
$\frac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\sin x}\right)}$		
$\frac{\cos x}{\sin x} \times \frac{\sin x}{1}$		
$\cos x$		

$1 + \tan^2 x$	=	$\sec^2 x$
$1 + \frac{\sin^2 x}{\cos^2 x}$		$\frac{1}{\cos^2 x}$
$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$		
$\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$		
$\frac{1}{\cos^2 x}$		

$\csc x \cos^2 x + \sin x$	=	$\csc x$
$\frac{1}{\sin x} \times \cos^2 x + \sin x$		$\frac{1}{\sin x}$
$\frac{\cos^2 x}{\sin x} + \sin x \times \frac{\sin x}{\sin x}$		
$\frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$		
$\frac{\cos^2 x + \sin^2 x}{\sin x}$		
$\frac{1}{\sin x}$		

$\cot x + \tan x$	=	$\csc x \sec x$
$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$		$\csc x \sec x$
$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$		
$\frac{1}{\sin x \cos x}$		
$\left(\frac{1}{\sin x}\right)\left(\frac{1}{\cos x}\right)$		
$\csc x \sec x$		

$\frac{1 + \cos x}{1 + \sec x}$	=	$\cos x$
$\left(\frac{1 + \cos x}{1 + \frac{1}{\cos x}}\right)$		$\cos x$
$\frac{(1 + \cos x)}{\left(\frac{\cos x + 1}{\cos x}\right)}$		
$(1 + \cos x) \times \frac{\cos x}{\cos x + 1}$		
$\frac{\cos x(1 + \cos x)}{\cos x + 1}$		
$\cos x$		

C12 - 6.4 - Proofs Conjugate Notes

Conjugate:

$a + b \longleftrightarrow a - b$

$a - b \longleftrightarrow a + b$

Conjugate:

$1 - sinx \longleftrightarrow 1 + sinx$

$1 + sinx \longleftrightarrow 1 - sinx$

Conjugate:

$1 + cosx \longleftrightarrow 1 - cosx$

$1 - cosx \longleftrightarrow 1 + cosx$

$$\frac{\square}{1 + cosx} \times \frac{1 - cosx}{1 - cosx} \qquad \frac{\square}{1 + sinx} \times \frac{1 - sinx}{1 - sinx}$$

$$\frac{\square}{1 - cosx} \times \frac{1 + cosx}{1 + cosx} \qquad \frac{\square}{1 - sinx} \times \frac{1 + sinx}{1 + sinx}$$

Prove that the two sides are equal.

$$\frac{sinx}{1 + cosx} = \frac{1 - cosx}{sinx}$$

The conjugate $\quad \dfrac{sinx}{1 + cosx} \times \dfrac{1 - cosx}{1 - cosx}$	$\dfrac{(1 - cosx)}{\sin x}$
$\dfrac{sinx\,(1 - cosx)}{(1 + cosx)(1 - cosx)}$	
FOIL (FL) $\quad \dfrac{sinx(1 - cosx)}{1 - \cancel{cosx} + \cancel{cosx} - \cos^2 x}$	
Now we have the Pythagorean identity $\quad \dfrac{sinx(1 - cosx)}{1 - \cos^2 x}$	

1) Multiply the top and bottom by the conjugate of the denominator $\quad \times \dfrac{1 - cosx}{1 - cosx}$
2) FOIL the bottom
3) Pythagorean Identity
4) Simplify

$$(1 + cosx)(1 - cosx)$$
$$1 - \cancel{cosx} + \cancel{cosx} - \cos^2 x$$
$$1 - \cos^2 x$$

$$(a + b)(a - b)$$
$$a^2 - \cancel{ab} + \cancel{ab} + b^2$$
$$a^2 - b^2$$

$$\sin^2 x - \cancel{\cos^2 x} = 1$$
$$\sin^2 x = 1 - \cos^2 x$$

$$\frac{sinx(1 - cosx)}{\sin^2 x}$$

$$\frac{(1 - cosx)}{\sin x}$$

RHS ✓

C12 - 6.4 - Proofs Foil Conjugate Fact Frac Notes

Foil

$$(secx - 1)(secx + 1) = \frac{1}{\cot^2 x}$$

$\sec^2 x - 1$	$\frac{1}{\cot^2 x}$
$\tan^2 x$	$\tan^2 x$

Conjugate!

$$\frac{sinx}{1 + cosx} = \frac{1 - cosx}{sinx}$$

$\frac{sinx}{1 + cosx} \times \frac{1 - cosx}{1 - cosx}$	$\frac{(1 - cosx)}{\sin x}$
$\frac{sinx(1 - cosx)}{1 - \cos^2 x}$	
$\frac{sinx(1 - cosx)}{\sin^2 x}$	
$\frac{(1 - cosx)}{\sin x}$	

Factor

$$\frac{1 + cosx}{\sin^2 x} = \frac{1}{1 - \cos x}$$

$\frac{1 + cosx}{1 - \cos^2 x}$	$\frac{1}{1 - \cos x}$
$\frac{1 + cosx}{(1 - \cos x)(1 + cosx)}$	
$\frac{1}{1 - \cos x}$	

Add and Subtract Fractions

$$\frac{1}{1 + cosx} + \frac{1}{1 - cosx} = 2\csc^2 x$$

$\frac{1}{1 + cosx} \times \frac{1 - cosx}{1 - cosx} + \frac{1}{1 - cosx} \times \frac{1 + cosx}{1 + cosx}$	$2 \times \frac{1}{\sin^2 x}$
$\frac{(1 - cosx) + (1 + cosx)}{(1 - cosx)(1 + cosx)}$	$\frac{2}{\sin^2 x}$
$\frac{1 - cosx + 1 + cosx}{1 - \cos^2 x}$	
$\frac{2}{\sin^2 x}$	

C12 - 6.5 - Sum Difference Notes

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$285 = 225 + 60$$

$$\sin(a+b) = sinacosb + sinbcosa$$
$$\sin(a-b) = sinacosb - sinbcosa$$
$$\cos(a+b) = cosacosb - sinasinb$$
$$\cos(a-b) = cosacosb + sinasinb$$

$$\begin{aligned}\sin(x+\pi) &= sinxcos\pi + sin\pi cosx \\ &= sinx \times -1 + 0 \times cosx \\ &= -sinx\end{aligned}$$

$$\begin{aligned}cos45cos30 + sin45sin30 &= \cos(45^o + 30^o) \\ &= cos75\end{aligned}$$

$$\begin{aligned}\cos\left(x+\frac{\pi}{4}\right) &= cosxcos\left(\frac{\pi}{4}\right) - \sin xsin\left(\frac{\pi}{4}\right) \\ &= cosx \times \frac{1}{\sqrt{2}} - sinx \times \frac{1}{\sqrt{2}} \\ &= \frac{cosx}{\sqrt{2}} - \frac{sinx}{\sqrt{2}} \\ &= \frac{cosx - sinx}{\sqrt{2}}\end{aligned}$$

Or

$$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, 2\pi$$

$$\frac{2\pi}{12}, \frac{3\pi}{12}, \frac{4\pi}{12}, \frac{6\pi}{12}, \frac{12\pi}{12}, \frac{24\pi}{12}$$

$$\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{4} - \frac{\pi}{3}$$

$$\begin{aligned}\sin\left(\frac{\pi}{12}\right) &= \\ sin15^0 &= \\ \sin(45^0 - 30^0) &= sin45cos30 - sin30cos45 \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}}\end{aligned}$$

$15 = 45 - 30$ $\quad special - quadrantal - combo\ angles$

$$\begin{aligned}sec15^o &= \\ \frac{1}{cos15^o} &= \\ \frac{1}{\cos(45^o - 30^o)} &= \frac{1}{(cos45cos30 + sin45sin30)} \\ &= \frac{1}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}} \\ &= \frac{1}{\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)} \\ &= \frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \\ &= 1 \times \frac{2\sqrt{2}}{\sqrt{3}+1} \\ &= \frac{2\sqrt{2}}{\sqrt{3}+1}\end{aligned}$$

$$\begin{aligned}cos75^o &= \\ \cos(45^o + 30^o) &= cos45cos30 + sin45sin30 \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}}\end{aligned}$$

$$\cos(-75) = \cos(-45 - 30)$$

C12 - 6.6 - Double Angle Notes

$$sin2x = 2sinxcosx$$

$4\sin 6x = 8\sin 3x\cos 3x$ Double the number in front. Half the angle.

$2\sin x = 4\sin\frac{1}{2}x\cos\frac{1}{2}x$

$\frac{1}{2}\sin 4x = 1\sin 2x\cos 2x$

$2sin\pi = 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) = 0$

$$2sinxcosx = sin2x$$

$8\sin 3x\cos 3x = 4\sin 6x$ Half the number in front. Double the angle.

$4\sin\frac{1}{2}x\cos\frac{1}{2}x = 2\sin x$

$4\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$$\begin{aligned} cos2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$\cos^2 2x - \sin^2 2x = cos4x$ Double the angle

$2\cos^2 3x - 2\sin^2 3x =$
$2\left(\cos^2 3x - \sin^2 3x\right) = 2cos6x$ GCF, Double the angle

$\boxed{GCF: -1}$

$2\cos^2 2x - 1 = cos4x$ Double the angle

$4\cos^2 x - 2 =$
$2(2\cos^2 x - 1) = 2cos2x$ GCF, Double the angle

$1 - 2\sin^2 \pi = cos2\pi = 1$

$1 - 2\sin^2\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$