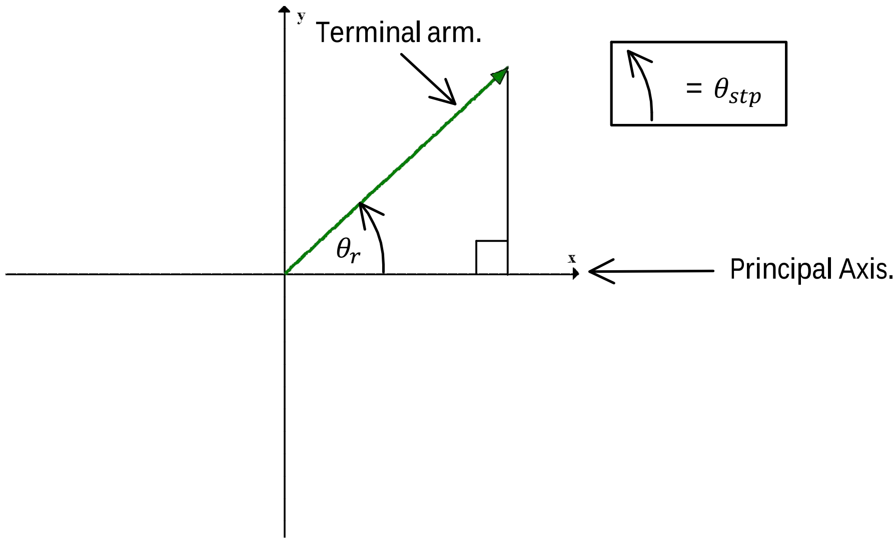
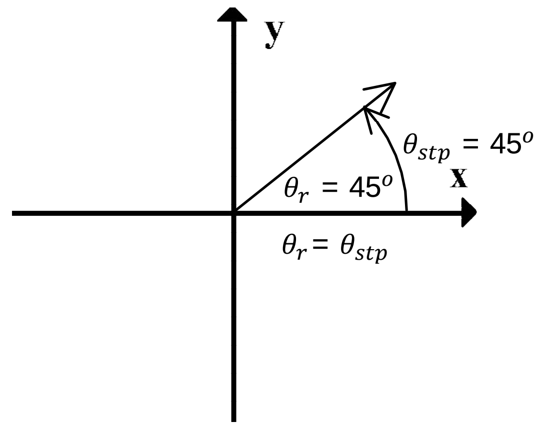
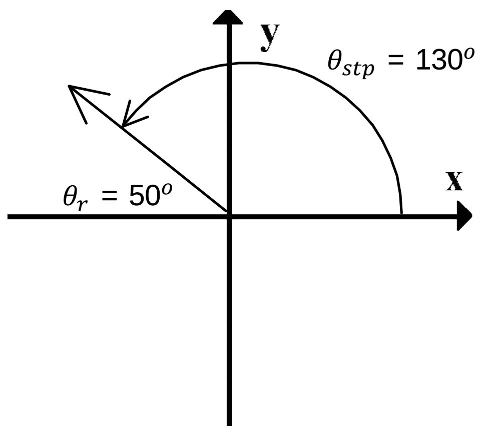
C11 - 2.1 - θ_r, θ_{stp} Notes

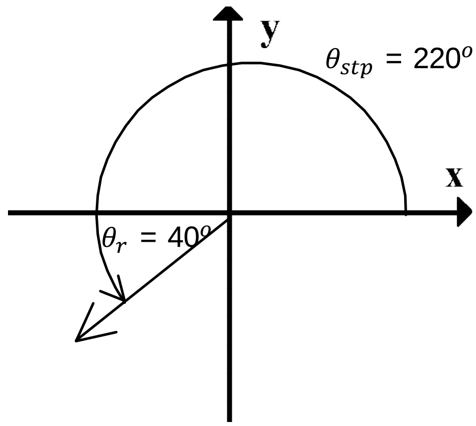
θ_r: the "reference angle"is the angle between the teminal arm and the x-axis (always positive, between 0^o and 90^o).

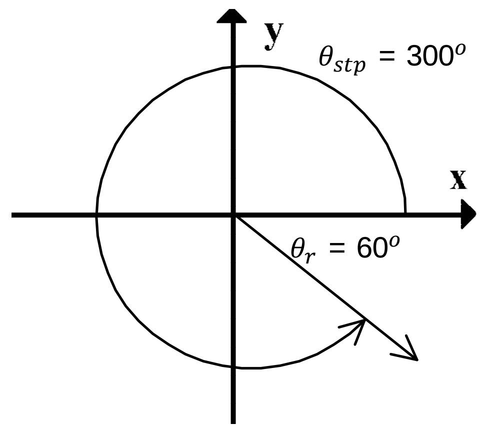
θ_{stp}: the "angle in standard position" from the principal axis to the terminal arm.







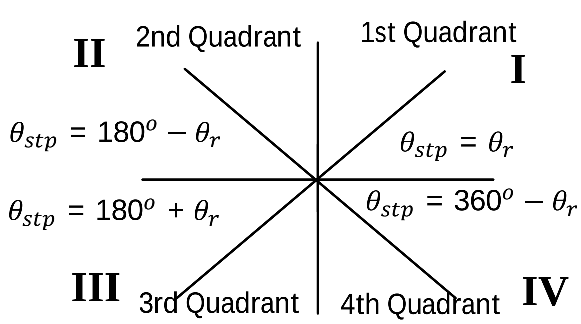


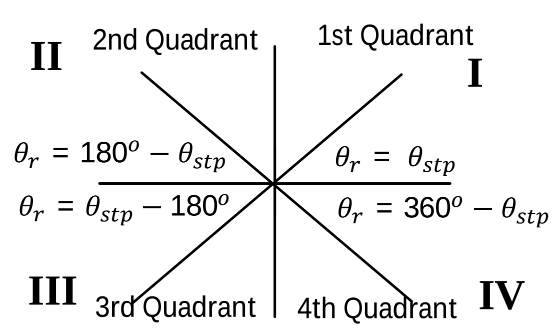


θ_{stp} = $180^O - 50^o$
θ_{stp} = 130^O

θ_{stp} = $180^O + 40^o$
θ_{stp} = 220^O

θ_{stp} = $360^O - 50^o$
θ_{stp} = 310^O



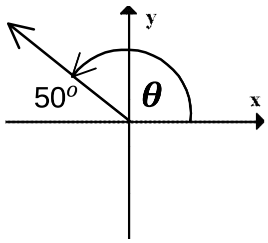


Basic logic will calculate θ_{stp} and θ_r much more easily than using these formulas.

C11 - 2.1 - $\pm\, \theta_{stp}, \theta_{cot}, \theta_{pri}$ Notes

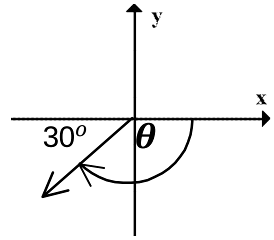
$$\boxed{\theta_{cot} = \theta_{stp} \pm 360^{o} n,\ nEI}$$

Counter-clockwise rotation is a positive $\boldsymbol{\theta_{stp}}$



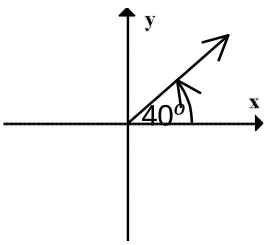
$\theta_{stp} = 180^{o} - 50^{o}$
$\theta_{stp} = 130^{o}$

Clockwise rotation is a negative $\boldsymbol{\theta_{stp}}$

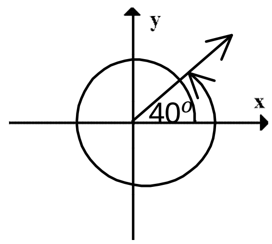


$\theta_{stp} = -(180^{o} - 30^{o})$
$\theta_{stp} = -150^{o}$

Positive Co-terminal Angles (θ_{cot})



$\theta_{r} = 40^{o}, \theta_{stp} = 40^{o}$

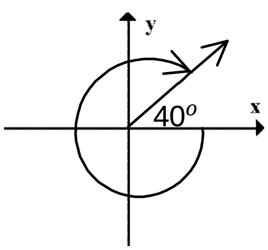


$\theta_{cot} = 360^{o} \pm \theta_{stp}$
$\theta_{cot} = 360^{o} + 40^{o}$
$\theta_{cot} = 400^{o}$

$\theta_{r} = 40^{o}, \theta_{stp} = 40^{o}, \theta_{stp} = 400^{o}$

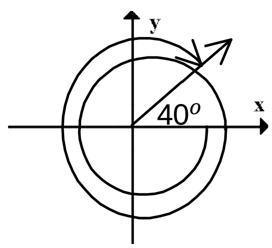
$\theta_{\mathrm{cot}} = 40^{o}, 400^{o}, 760^{o}, 1120^{o}, 1480^{o}, \ldots$

Negative Co-terminal Angles (θ_{cot})



$\theta_{cot} = 360^{o} \pm \theta_{stp}$
$\theta_{cot} = -(360^{o} - 40^{o})$
$\theta_{cot} = -(320^{o})$
$\theta_{cot} = -320^{o}$

$\theta_{r} = 40^{o}, \theta_{stp} = -320^{o\,o}$



$\theta_{cot} = 360^{o} \pm \theta_{stp}$
$\theta_{cot} = -(360^{o} + (360^{o} - 40)^{o})$
$\theta_{cot} = -(360^{o} + 320^{o})$
$\theta_{cot} = -680^{o}$

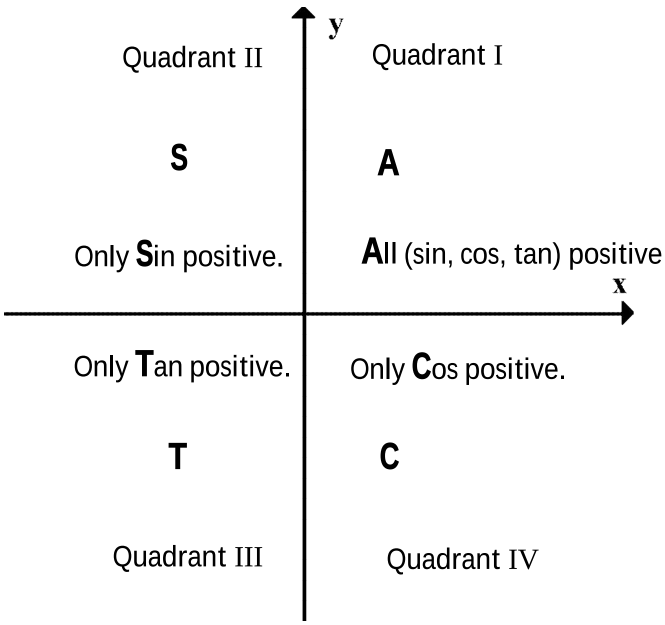
$\theta_{r} = 40^{o}, \theta_{stp} = 40^{o}, \theta_{stp} = -680^{o}$

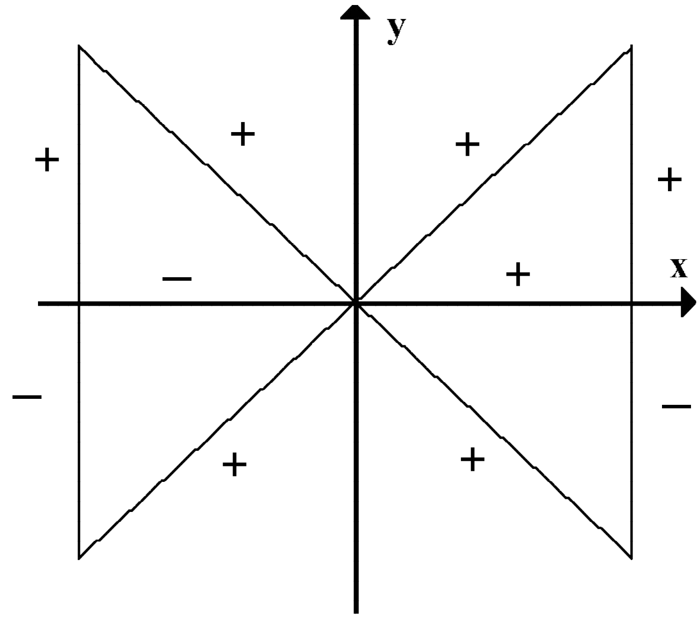
$\theta_{\mathrm{cot}} = 40^{o}, -320^{o}, -680^{o}, -1040^{o}, -1400^{o}, \ldots$

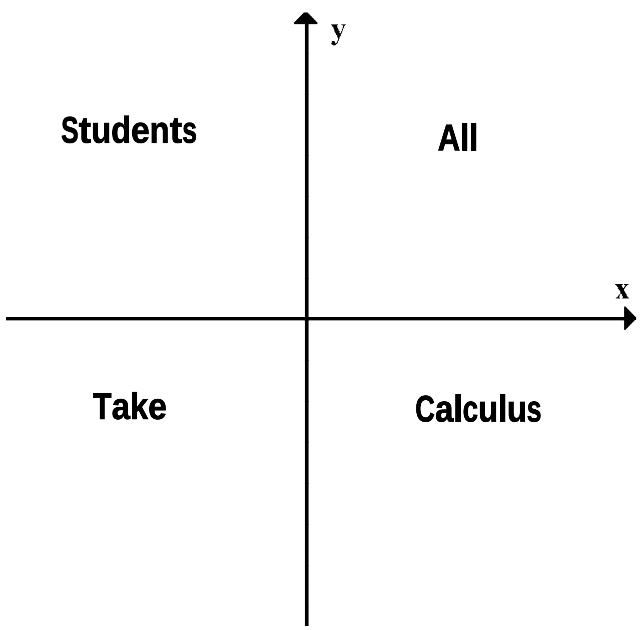
Basic logic will calculate θ_{cot} much more easily than using these formulas.

$$\boxed{\theta_{principle} = smallest\ positive\ \theta_{stp}\ coterminal.}$$

C11 - 2.2 -ASTC Notes







Remember: the hypotenuse is always positive.

Quadrant II:

$$sin\theta = \frac{+}{+} = +$$
$$cos\theta = \frac{-}{+} = -$$
$$tan\theta = \frac{+}{-} = -$$

Quadrant I:

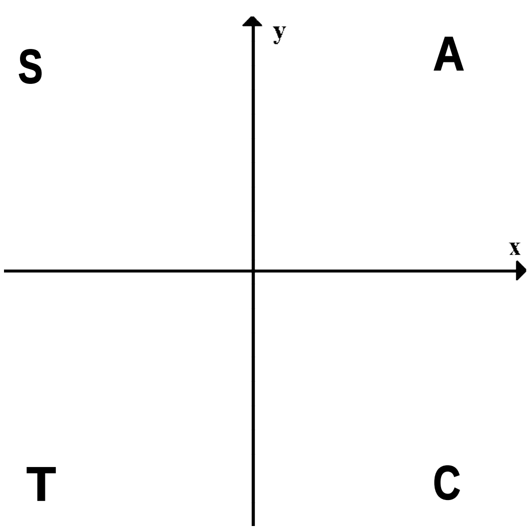
$$sin\theta = \frac{+}{+} = +$$
$$cos\theta = \frac{+}{+} = +$$
$$tan\theta = \frac{+}{+} = +$$

Quadrant III:

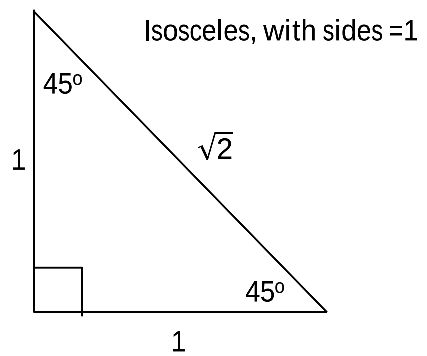
$$sin\theta = \frac{-}{+} = -$$
$$cos\theta = \frac{-}{+} = -$$
$$tan\theta = \frac{-}{-} = +$$

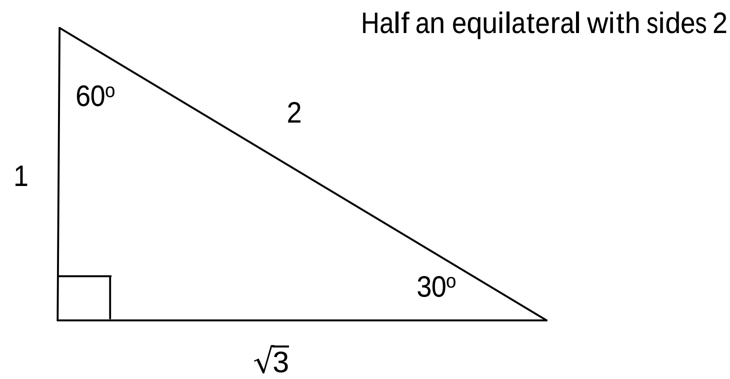
Quadrant IV:

$$sin\theta = \frac{-}{+} = -$$
$$cos\theta = \frac{+}{+} = +$$
$$tan\theta = \frac{-}{+} = -$$



C11 - 2.3 - Special Triangles 30,45,60 sin/cos/tan Notes





$\boldsymbol{sin\theta} = \frac{\boldsymbol{Opp}}{\boldsymbol{Hyp}}$ $\boldsymbol{cos\theta} = \frac{\boldsymbol{Adj}}{\boldsymbol{Hyp}}$ $\boldsymbol{tan\theta} = \frac{\boldsymbol{Opp}}{\boldsymbol{Adj}}$

$sin30^o = \frac{1}{2}$ $cos30^o = \frac{\sqrt{3}}{2}$ $tan30^o = \frac{1}{\sqrt{3}}$

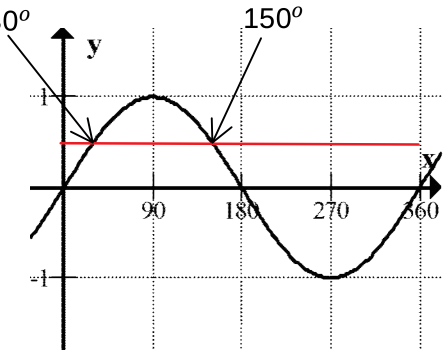
$sin45^o = \frac{1}{\sqrt{2}}$ $cos45^o = \frac{1}{\sqrt{2}}$ $tan45^o = \frac{1}{1}$

$tan45^o = 1$

$sin60^o = \frac{\sqrt{3}}{2}$ $cos60^o = \frac{1}{2}$ $tan60^o = \frac{\sqrt{3}}{1}$

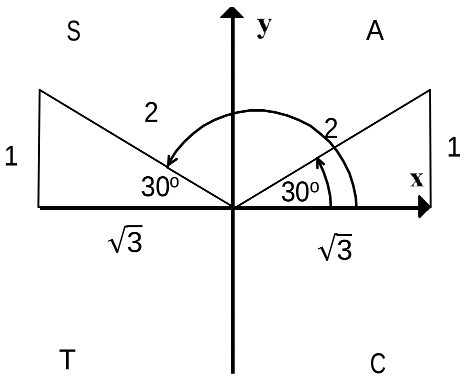
$tan60^o = \sqrt{3}$

C11 - 2.3 - $sin\theta = \frac{1}{2}$ Notes

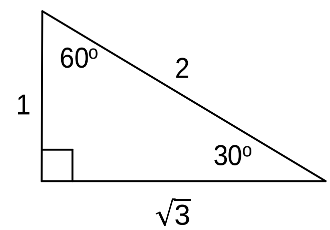


Solve for $\theta, 0^o \leq \theta < 360^o$.

sin$\theta = \frac{1}{2}$



Draw two triangles where sin θ is positive:
ASTC Quadrant I, II



Label the triangles according to special triangles
and SOH CAH TOA

Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = 30^o$ $\quad$ $\theta_{stp} = 180^o - 30^o$
$= 150^o$

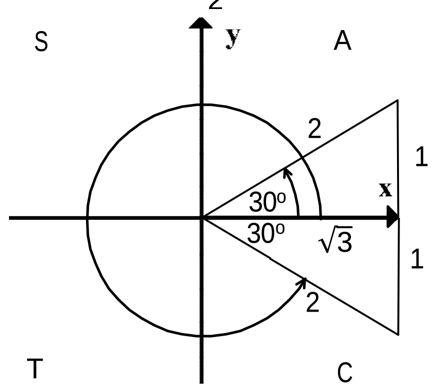
$\theta_{stp} = 30^o, 150^o$

Solve for the arrows θ_{stp}

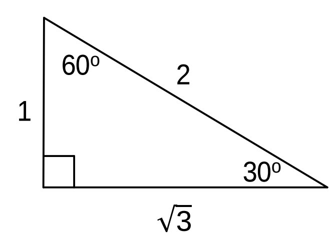
Check your answer: $\quad sin30^o = \frac{1}{2} \quad sin150^o = \frac{1}{2}$

Solve for $\theta, 0^o \leq \theta < 360^o$ *and general solution*.

$cosx = \frac{\sqrt{3}}{2}$



Draw two triangles where cos θ is positive:
ASTC Quadrant I, II



Label the triangles according to special triangles
and SOH CAH TOA

Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = 30^o$ $\quad$ $\theta_{stp} = 360^o - 30^o$
$= 330^o$

$\theta_{stp} = 30^o, 330^o$

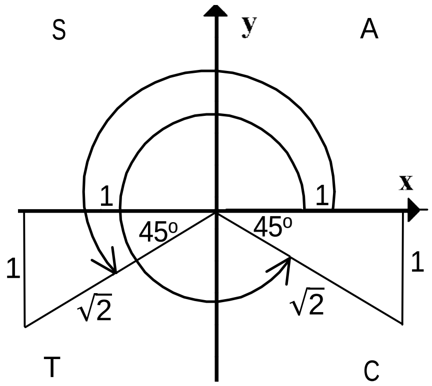
Solve for the arrows θ_{stp}

Check your answer: $\quad cos30^o = \frac{\sqrt{3}}{2} \quad cos330^o = \frac{\sqrt{3}}{2}$

$General\ Solution: \theta = \theta_{stp} \pm pn, n \in I$ $\qquad$ $\theta = \theta_{stp} \pm pn, n \in I$
$\theta = 30^o \pm 360^o n, n \in I$ $\qquad$ $\theta = 330^o \pm 360^o n, n \in I$

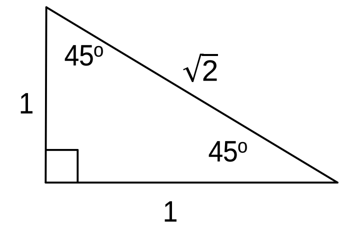
C11 - 2.3 - $sin\theta = -\frac{1}{\sqrt{2}}$ Notes

$\mathbf{sin}\,\boldsymbol{x} = -\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$



Draw two triangles where $\sin\theta$ is negative:
ASTC Quadrant III, IV

Label the triangles according to special triangles and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = 180^o + 45^o$
$= 225^o$

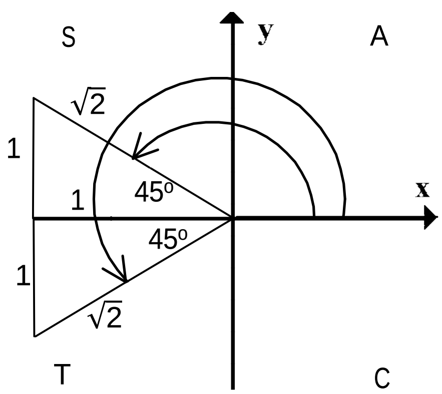
$\theta_{stp} = 360^o - 45^o$
$= 315^o$

Solve for the arrows θ_{stp}

$\theta_{stp} = 225^o, 315^o$

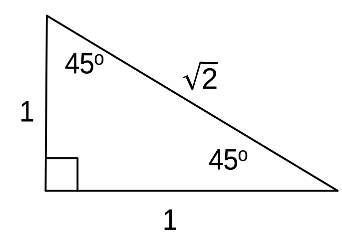
Check your answer: $sin225^o = -\frac{1}{\sqrt{2}}$ $sin315^o = \frac{1}{\sqrt{2}}$

$\boldsymbol{cos\theta} = -\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$



Draw two triangles where $\cos\theta$ is negative:
ASTC Quadrant II, III

Label the triangles according to special triangles and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = 180^o + 45^o$
$= 225^o$

$\theta_{stp} = 180^o - 45^o$
$= 135^o$

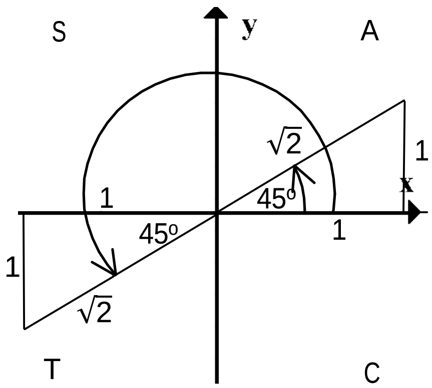
Solve for the arrows θ_{stp}

$\theta_{stp} = 225^o, 135^o$

Check your answer: $cos225^o = -\frac{1}{\sqrt{2}}$ $cos135^o = -\frac{1}{\sqrt{2}}$

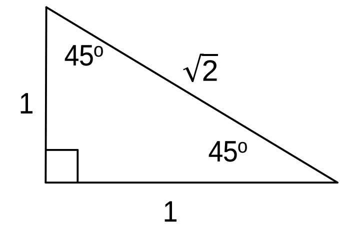
C11 - 2.3 - $tan\theta = 1$ Notes

$\tan x = 1$ $\quad\quad$ $\tan x = \frac{1}{1}$



Draw two triangles where tan θ is positive:
ASTC Quadrant I, III

Label the triangles according to special triangles and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = 45^o$
$= 45^o$

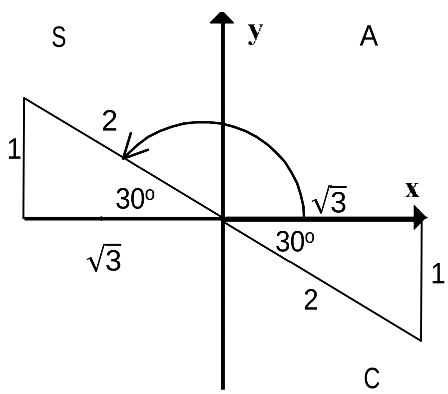
$\theta_{stp} = 180^o + 45^o$
$= 225^o$

Solve for the arrows θ_{stp}

$\theta_{stp} = 45^o, 225^o$

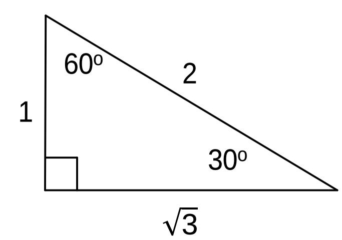
Check your answer: $\quad tan45^o = \frac{1}{1} \quad tan225^o = \frac{1}{1}$

$tan\theta = -\frac{1}{\sqrt{3}}$



Draw two triangles where tan θ is negative:
ASTC Quadrant II, IV

Label the triangles according to special triangles and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

$\theta_{stp} = 180^o - 30^o$
$= 150^o$

$\theta_{stp} = 360^o - 30^o$
$= 330^o$

Solve for the arrows θ_{stp}

$\theta_{stp} = 150^o, 330^o$

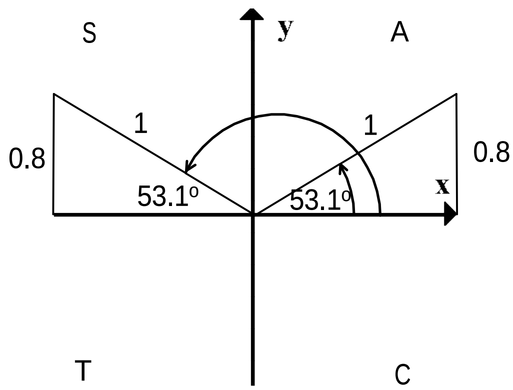
Check your answer: $\quad tan150^o = -\frac{1}{\sqrt{3}} \quad tan330^o = -\frac{1}{\sqrt{3}}$

C11 - 2.3 - $sin\theta = .8$ Notes

Solve for $\theta, 0^o \leq \theta < 360^o$ and general solution

$\sin\theta = 0.8$

$sin\theta = \frac{0.8}{1} = \frac{8}{10}$



Draw two triangles where $\sin\theta$ is positive:
ASTC Quadrant I, II

Label the triangles according to SOH CAH TOA

$Solve\ for\ \theta_r:\ \theta_r = sin^{-1}\left(\frac{0.8}{1}\right)$

Draw an arrow from the principal axis to the first terminal arm,
draw an arrow from the principal axis to the second terminal arm.

Solve for the arrows θ_{stp}

$\theta_{stp} = 53.1^o$ $\quad \theta_{stp} = 180^o - 53.1^o$
$= 126.9^o$

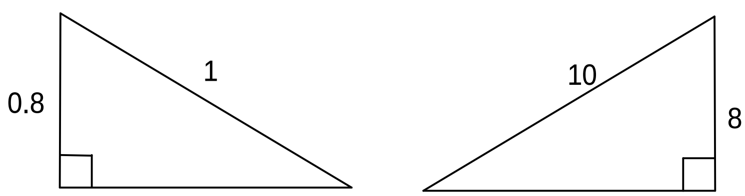
$\theta_{stp} = 53.1^o, 126.9^o$

Check your answer: $sin 53.1^o = 0.8$ $\quad sin 126.9^o = 0.8$

$sin\theta = \frac{0.8}{1}$

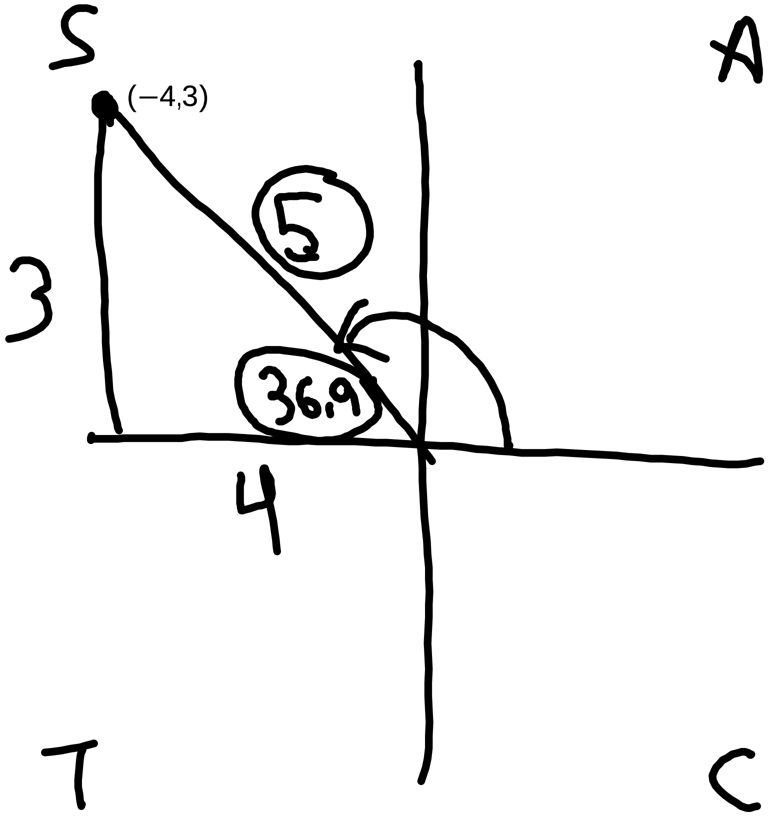
$\theta = \sin^{-1}\left(\frac{0.8}{1}\right)$

$\theta = 53.1^o$



$General\ Solution: \theta = \theta_{stp} \pm pn, n \in I$	$\theta = \theta_{stp} \pm pn, n \in I$
$\theta = 53.1^o \pm 360^o\ n, n \in I$	$\theta = 126.9^o \pm 360^o n, n \in I$

*C11 -·2.3 - Trig Point on Graph Notes



$a^2+b^2=c^2$

$c=5$

$\tan\theta=\frac{o}{a}$

$\tan\theta=\frac{3}{4}$

$\theta=\tan^{-1}\left(\frac{3}{4}\right)$

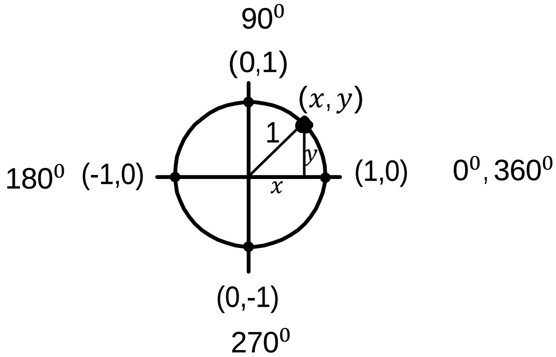
$\theta=36.9^\circ$ $\quad\theta=180-36.9$

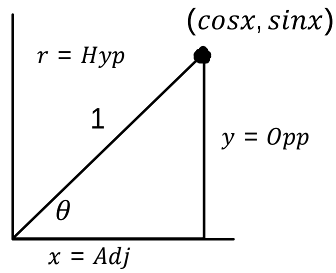
$\theta=143.1$

$\boxed{*+4}$

$\tan 143.1=.75$ ✓

C11 - 2.4 - Unit Circle sin/cos/tan 90, 180, 270, 360 Notes





$\boxed{sin\theta = y}$

$\boxed{cos\theta = x}$

$\boxed{tan\theta = \frac{y}{x}}$

$Radius\ of\ unit\ circle = 1$

$Hyp = 1$

$\boldsymbol{sin\theta = \frac{Opp}{Hyp}}$ $sin\theta = \frac{y}{1}$ $sin\theta = y$	$\boldsymbol{cos\theta = \frac{Adj}{Hyp}}$ $cos\theta = \frac{x}{1}$ $cos\theta = x$	$\boldsymbol{tan\theta = \frac{Opp}{Adj}}$ $tan\theta = \frac{y}{x}$
$sin0^o = \frac{0}{1}$ $sin0^o = 0$	$cos0^o = \frac{1}{1}$ $cos0^o = 1$	$tan0^o = \frac{0}{1}$ $tan0^o = 0$
$sin90^o = \frac{1}{1}$ $sin90^o = 1$	$cos90^o = \frac{0}{1}$ $cos90^o = 0$	$tan90^o = \frac{1}{0}$ $tan90^o = UND$
$sin180^o = \frac{0}{1}$ $sin180^o = 0$	$\cos 180^o = -\frac{1}{1}$ $cos180^o = -1$	$tan180^o = \frac{0}{-1}$ $tan180^o = 0$
$sin270^o = \frac{-1}{1}$ $sin270^o = -1$	$cos270^o = \frac{0}{1}$ $cos270^o = 0$	$tan270^o = \frac{-1}{0}$ $tan270^o = UND$
$sin360^o = \frac{0}{1}$ $sin360^o = 0$	$cos360^o = \frac{1}{1}$ $cos360^o = 1$	$tan360^o = \frac{0}{1}$ $tan360^o = 0$

$$(x, y)$$

$$(sin\theta, cos\theta)$$

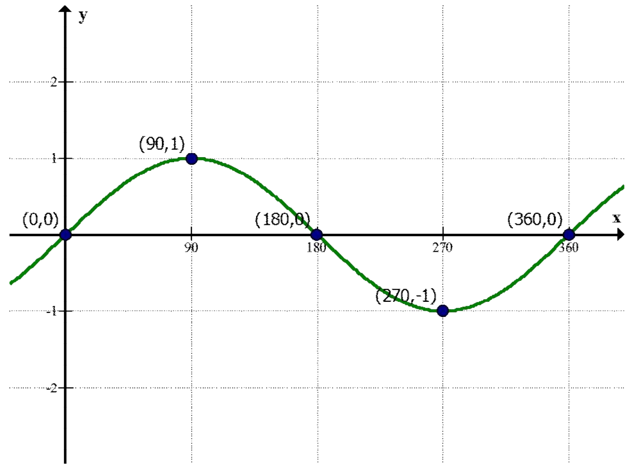
C11 - 2.8 - TOV^0 sinx,cosx,tanx Graphs Notes

$y = \sin x$

Table of Values

x	y
0º	0
90º	1
180º	0
270º	-1
360º	0

Pt.
(0,0)
(90,1)
(180,0)
(270,-1)
(360,0)

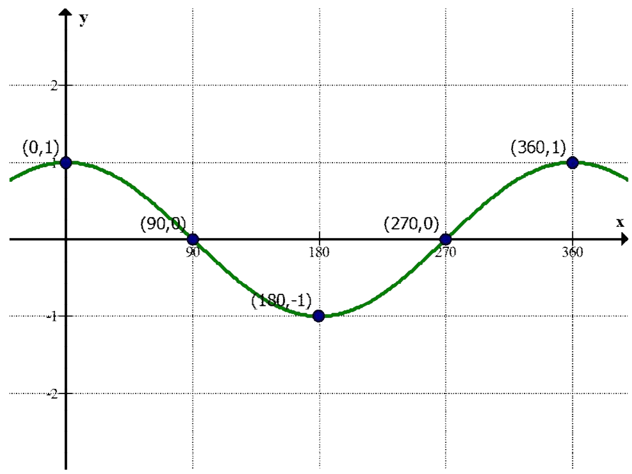


$y = \cos x$

Table of Values

x	y
0º	1
90º	0
180º	-1
270º	0
360º	1

Pt.
(0,1)
(90,0)
(180,-1)
(270,0)
(360,1)

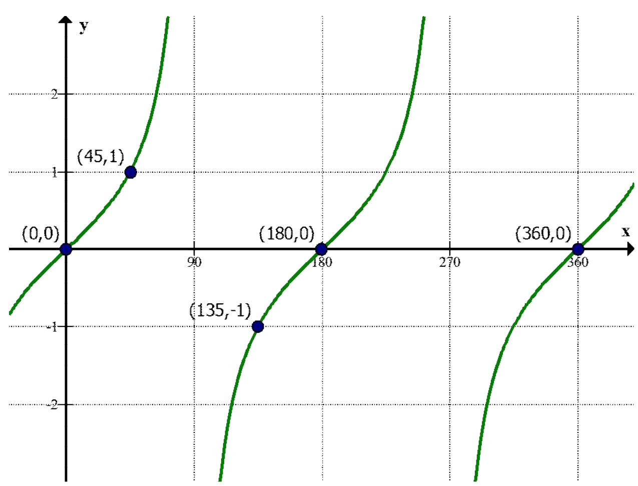


$y = \tan x$

Table of Values

x	y
0º	0
45º	1
90º	und
135º	-1
180º	0

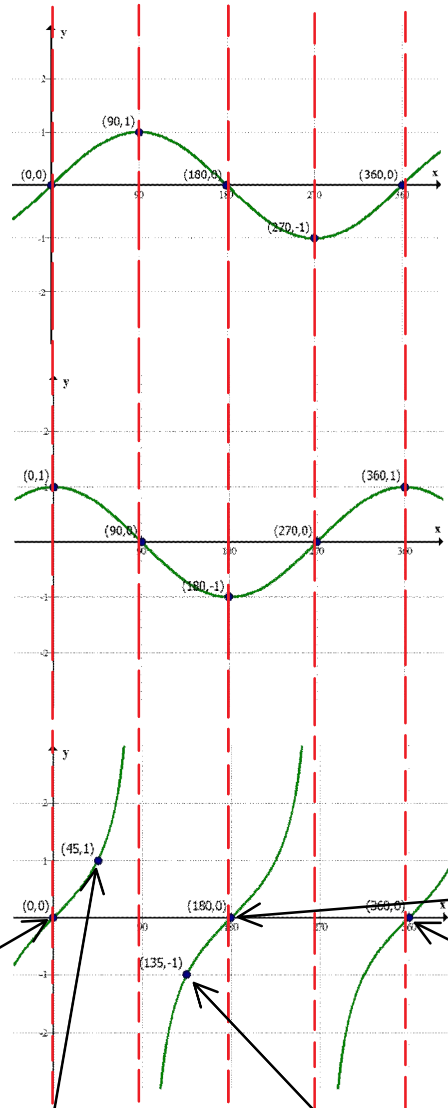
Pt.
(0,0)
(45,1)
(90,und)
(135,-1)
(180,0)



$$tanx = \frac{sinx}{cosx}$$

C11 - 2.8 - Theory Graph $tanx = \frac{sinx}{cosx}$ Notes

$$tanx = \frac{sinx}{cosx}$$



$sinx$

$cosx$

$tanx = \frac{sinx}{cosx}$

$At\ x = 180^o$:

$\sin x = 0$
$cosx = -1$

$\frac{sinx}{cosx} = \frac{0}{-1} = 0$

$At\ x = 0^o$:

$\sin x = 0$
$cosx = 1$

$\frac{sinx}{cosx} = \frac{0}{1} = 0$

$At\ x = 360^o$:

$\sin x = 0$
$cosx = 1$

$\frac{sinx}{cosx} = \frac{0}{1} = 0$

$At\ x = 45^o$:

$\sin x = \frac{1}{\sqrt{2}}$

$cosx = \frac{1}{\sqrt{2}}$

$\frac{sinx}{cosx} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$At\ x = 135^o$:

$\sin x = \frac{1}{\sqrt{2}}$

$cosx = -\frac{1}{\sqrt{2}}$

$\frac{sinx}{cosx} = \frac{\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = -1$

$At\ x = 90^o$:

$\sin x = 1$
$cosx = 0$

$\frac{sinx}{cosx} = \frac{1}{0} = DNE$

$Vertical\ Asymptote$

$At\ x = 270^o$:

$\sin x = -1$
$cosx = 0$

$\frac{sinx}{cosx} = \frac{-1}{0} = DNE$

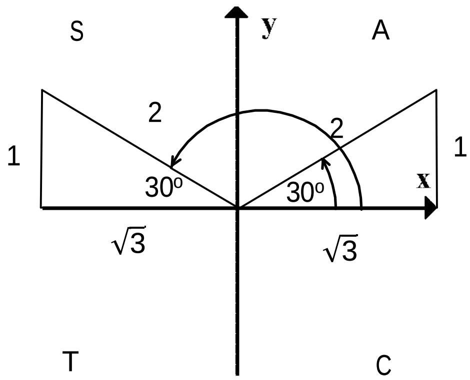
$Vertical\ Asymptote$

C11 - 2.8 - sin 2θ Notes

Solve for θ $0^o \le \theta < 360^o$, and the general solution.

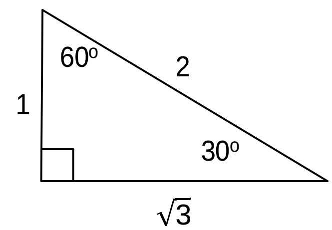
$\sin 2\theta = \frac{1}{2}$

$Let\ m = 2\theta$ $\sin m = \frac{1}{2}$



Draw two triangles where sin m is positive:
ASTC Quadrant I, II

Label the triangles according to special triangles and SOH CAH TOA



Label the reference angle according to special triangles.

Draw an arrow from the principal axis to the first terminal arm, draw an arrow from the principal axis to the second terminal arm.

Solve for the arrows m_{stp}

$m_{stp} = 30^o$ $m_{stp} = 180^o - 30^o = 150^o$

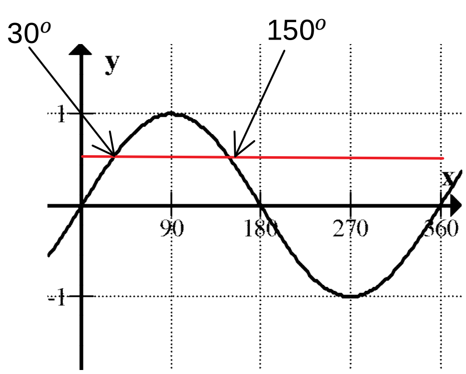
$m_{stp} = 30^o, 150^o$

Substitute 2θ back in for m.

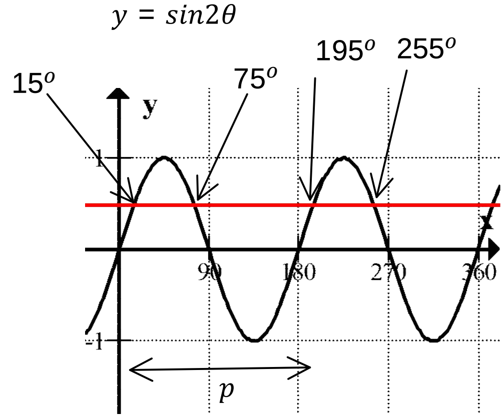
$m = 30^o$
$2\theta = 30^o$
$\frac{2\theta}{2} = \frac{30^o}{2}$
$\theta = 15^o$

$m = 150^o$
$2\theta = 150^o$
$\frac{2\theta}{2} = \frac{150^o}{2}$
$\theta = 75^o$

$y = sin\theta$



$y = sin2\theta$



$p = \frac{360^0}{b}$

$p = \frac{360^o}{2} = 180^o$

$\theta = \theta_{stp} \pm p$
$\theta = 15^o + 180^o$
$\theta = 195^o$

$\cancel{\theta = \theta_{stp} \pm p}$
$\cancel{\theta = 195^o + 180^o}$
$\cancel{\theta = 375^o}$

$\theta = \theta_{stp} \pm p$
$\theta = 75^o + 180^o$
$\theta = 255^o$

$0 \le \theta \le 360^o$
$\theta = 15^o, 75^o, 195^o, 225^o$

$General\ Solution$: $\theta = \theta_{stp} \pm pn, n \in I$	$\theta = \theta_{stp} \pm pn, n \in I$
$\theta = 15^o \pm 180^o\, n, n \in I$	$\theta = 75^o \pm 180^o n, n \in I$