C12 - 8.1 - $\log_b a$ = ? Definition Notes

Log Form	Exponential Form		
$\log_2 16 = ?$ $\log_2 16 = 4$	$2^4 = 16$	$2^x = 16$ $2^x = 2^4$ $x = 4$	Think: What power do you have to raise 2 to, to equal 16.
$\log_5 125 = ?$ $\log_5 125 = 3$	$5^3 = 125$	$5^x = 125$ $5^x = 5^3$ $x = 3$	Remember: The base of the log is the base of the exponent

$log_5 625 = ?$
$\log_5 5^4 = ?$
$4\log_5 5 = ?$
$4 \times 1 = 4$

Change of Base
Bring Exponent down in front
Log Rules
Solve

$\log_5 5 = 1$

$\log_{\frac{1}{2}} 16 = ?$
$\log_{\frac{1}{2}} 16 = -4$

$\left(\frac{1}{2}\right)^{-4} = 16$

$\left(\frac{1}{2}\right)^{x} = 16$
$(2^{-1})^x = 2^4$
$2^{-x} = 2^4$
$-x = 4$
$x = -4$

The answer of the log is the exponent

$\log_3\left(\frac{1}{27}\right) = ?$
$\log_3\left(\frac{1}{27}\right) = -3$

$3^{-3} = \frac{1}{27}$

$3^x = \frac{1}{27}$
$3^x = \frac{1}{3^3}$
$3^x = 3^{-3}$
$x = -3$

$\log_{2x} 16x^4 = ?$
$\log_{2x} 16x^4 = 4$

$(2x)^4 = 16x^4$

$(2x)^m = 16x^4$
$(2x)^m = (2x)^4$
$m = 4$

$\log_{10} 10^{\frac{1}{2}} = ?$
$\log_{10} 10^{\frac{1}{2}} = \frac{1}{2}$

$10^{\left(\frac{1}{2}\right)} = 10^x$
$x = \frac{1}{2}$

$\log_{10} 1000 = ?$
$\log_{10} 10^3 = ?$
$\log_{10} 10^3 = 3$

$10^3 = 10^x$
$x = 3$

$\log_0 1 = und$	$a > 0$
$\log_2 0 = und$	$b > 0$
$\log_2 -3 = und$	
	$b \neq 1$
$\log_1 11 = und$	

C12 - 8.2 - $\log_b x = c, \log_x a = c, \log_b a = x$ *Notes*

Find x

$\log_4 x = 2$

$x = 4^2$

$x = 16$

Exponential Form

Remember: The base of the log is the base of the exponent
Remember: The Exponent is the answer

$\log_5 x = -2$

$x = 5^{-2}$

$x = \frac{1}{5^2}$

$x = \frac{1}{25}$

$\log_2 16 = x$

$16 = 2^x$

$2^4 = 2^x$

$x = 4$

$\log_x 64 = 3$

$64 = x^3$

$4^3 = x^3$

$x = 4$

$\log_x 27 = \frac{3}{2}$

$27 = x^{\frac{3}{2}}$

$27^{\frac{2}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}$

$27^{\frac{2}{3}} = x^1$

$\sqrt[3]{27^2} = x$

$x = 9$

$\log_2(x-5) = 2$

$x - 5 = 2^2$

$x = 4 + 5$

$x = 9$

$\log_{36}(x^2 + 5x) = \frac{1}{2}$

$x^2 + 5x = 36^{\frac{1}{2}}$

$x^2 + 5x = 6$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6 \quad x = 1$

$log_{x-3} 2 = 2$

$2 = (x-3)^2$

$2 = (x-3)(x-3)$

$2 = x^2 - 6x + 9$

$0 = x^2 - 6x + 7$

$0 = (x-7)(x+1)$

$x = 7 \quad \cancel{x = -1}$

Reject -1

Domain Restriction: Set inside $log > 0$ *and solve.*

$x - 3 > 0$

$x > 3$

C12 - 8.3 - Change of Base Notes

Exponential Form

$\frac{log16}{log2} =$

$\log_2 16 = 4$

Change of Base

$16 = 2^4$

$\frac{\log_2 16}{log_2 4} =$

$\log_4 16 = 2$

Change of Base

$16 = 4^2$

$\log_2 4 =$

$\frac{\log_2 4}{\log_2 2} = \frac{2}{1} = 2$

Change of Base

$4 = 2^2$

$2 = 2^1$

Choose the Base you want!
Think about it.

$\log_3 27 =$

$\frac{log27}{log3} =$

Change of Base

$27 = 3^3$

$\frac{log3^3}{log3} =$

Exponent down in front

$\frac{3log3}{log3} = 3$

$\log_8 16 =$

$\frac{\log_2 16}{\log_2 8} = \frac{4}{3}$

Change of Base

$16 = 2^4$
$8 = 2^3$

$\frac{1}{\log_8 2} =$

$\frac{1}{\left(\frac{log2}{log8}\right)} =$

Change of Base

$1 \times \frac{log8}{log2} =$

$\frac{log8}{log2} =$

$\log_2 8 = 3$

$8 = 2^3$

C12 - 8.4 - $\log_b m + \log_b n = \log_b mn \quad log_b m - log_b n = log_b \frac{m}{n}$

Exponential Form

$\log_2 4 + \log_2 8 = \log_2 4 \times 8 = \log_2 32 = 5$ Add-Multiply $32 = 2^5$

$\boxed{2 + 3 = 5}$

$\log_3 3 + \log_3 9 = log_3 3 \times 9 = \log_3 27 = 3$ Add-Multiply $27 = 3^3$

$\boxed{1 + 2 = 3}$

$log1 + log5 + log7 = log1 \times 5 \times 7 = \log 35$ Add-Multiply

$\log_3 27 - \log_3 3 = \log_3 \frac{27}{3} = \log_3 9 = 2$ Subtract-Divide

$\boxed{3 - 1 = 2}$

$log4 + log20 - log10 = log \frac{4 \times 20}{10} = log\, 8$ Positives on top, Negatives on Bottom

$log5 - log2 - log10 = log \frac{5}{2 \times 10} = \log \frac{1}{4}$

$log5 - log2 + log10 = log \frac{5 \times 10}{2} = log25$

$log64 = log4 \times 16 = log\, 4 + log16$ Separate into an addition of logs

$log5 = \log\left(\frac{10}{2}\right) = log10 - log2$ Separate into a subtraction of logs

C12 - 8.4 - $\log_b m + \log_b n = \log_b mn$ $log_b m - log_b n = log_b \frac{m}{n}$

$\log 3 + \log(x+1) = log3(x+1) = \log(3x+3)$ Add-Multiply

$\log(x-2) + \log(x+1) = \log(x-2)(x+1) = \log(x^2-x-2)$ Add-Multiply

$logx + \log x = \log x \times x = \log x^2$ Add-Multiply

$\log x^3 - \log x^2 = \log \frac{x^3}{x^2} = logx$ Subtract-Divide-Simplify

$\log(x^2-1) - \log(x+1) = \log \frac{x^2-1}{x+1} = \log \frac{(x+1)(x-1)}{(x+1)} = \log(x-1)$ Subtract-Divide-Factor-Simplify

C12 - 8.5 - Log Operation Notes

$log8 = 0.9031$ $\quad$ $log_4 7 = 1.4037$ $\quad$ Calculator $\quad$ Math, Alpha, Math

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$log6^2 =$
$2log6 = 1.5563$

Bring your exponent down in front. Bahm!

$2log5^3 =$		If there is a number in front multiply
$3 \times 2log5 =$		
$6log5 = 4.1938$		

$log25 = 1.3979$
$log5^2 = 1.3979$
$2log5 = 1.3979$

Change of Base $\quad$ $25 = 5^2$

If for example you know:	$log5 = a$	$2log5 = 2a$

$log5^{x+2} =$
$(x+2)log5 =$
$xlog5 + 2log5$

Distribute

$3xlog7 - xlog2 =$
$x(3log7 - log2)$

$GCF = x$

$logxy^2 =$
$logx + logy^2 =$
$logx + 2logy$

The exponent only applies to the y value

$logx^2y^2 =$
$logx^2 + logy^2 =$
$2logx + 2logy$

$\log(xy)^2 =$
$2logxy =$
$2(logx + logy) =$
$2logx + 2logy$

You can bring this exponent down in front. Remember: If you separate into an addition you must distribute the 2.

$(logx)^2 = logx \times logx$ $\quad$ Cannot bring exponent down in front

$log5 \times log2 = log5 \times \log 2 = 0.2104$ $\quad$ Cannot multiply 2 logs

$logx \times logx = logx \times logx$

$\frac{log2x}{logx} = \frac{log2x}{logx}$ $\quad$ Cannot Divide 2 logs

$2log5 = 2log5$ $\quad$ Cannot distribute into a log

$\log(x+2) = \log(x+2)$ $\quad$ Cannot distribute

C12 - 8.5 - Operation $\log_{b^n} a^n$ *Notes*

Exponential Form

$\log_2 8 = \log_{2^2} 8^2 = \log_4 64 = 3$

Take the base and the log to any exponent you like!

$8 = 2^3$ $64 = 4^3$

$\log_{\frac{1}{2}} 4 = \log_{(\frac{1}{2})^{-1}} 4^{-1} = \log_2 4^{-1} = -1 \log_2 4 = -2$

$\frac{1}{4} = 2^{-2}$

$\left(\frac{1}{2}\right)^{-1} = 2$

Bring your exponent down in front

$\log_2 4 = 2$

$4 = 2^2$

$-1 \times 2 = -2$

Take the base and the thing you are logging to an exponent to get like bases to use log *laws*

C12 - 8.6 - Log/Delog Both Sides Notes

$$4 = 2^x$$
$$log4 = log2^x$$
$$log4 = xlog2$$
$$\frac{log4}{log2} = x$$
$$\log_2 4 = x$$
$$4 = 2^x$$
$$2^2 = 2^x$$
$$x = 2$$

Log Both Sides
Bring Exponents Down In Front
Divide

Change of base
Exponential Form
Change of base
Solve

$$\boxed{\begin{aligned} 4 &= 2^x \\ \log_2 4 &= x \\ x &= 2 \end{aligned}}$$

Quick Method
Change to log *form*

$$3 = 5^x$$
$$log3 = log5^x$$
$$log3 = xlog5$$
$$\frac{log3}{log5} = x$$
$$\log_5 3 = x$$
$$x = 0.6826$$

Algebraic answer

$$\boxed{\begin{aligned} 3 &= 5^x \\ \log_5 3 &= x \\ x &= 0.6826 \end{aligned}}$$

Check Answer:

$$5^{0.6828} = 3$$

Before you log both sides!

$$3 = 2^x - 1$$
$$4 = 2^x$$

Add/Subtract First

$$8 = 2 \times 2^x$$
$$4 = 2^x$$

Divide First

Or
$$\boxed{\begin{aligned} 8 &= 2 \times 2^x \\ log8 &= \log(2 \times 2^x) \\ log8 &= log2 + log2^x \end{aligned}}$$

$$4 = 7^{x+1}$$
$$log4 = log7^{x+1}$$
$$log4 = (x+1)log7$$
$$log4 = xlog7 + \log 7$$
$$log4 - log7 = xlog7$$
$$\frac{log4 - log7}{log7} = x$$
$$x = \frac{log4 - log7}{log7}$$
$$x = 0.29$$

Distribute
Combine Like terms
Divide

Or divide log7
and minus 1

$$8 = 3^{2x}$$
$$log8 = log3^{2x}$$
$$log8 = 2xlog3$$
$$\frac{log8}{log3} = 2x$$
$$\frac{\log_3 8}{2} = x$$
$$\frac{1}{2}\log_3 8 = x$$
$$x = \log_3 8^{\frac{1}{2}}$$

Bring Fraction In Front.
Bring Coefficient Up to
Exponent of log

$$2^{2x-5} = 9^{x+2}$$
$$log2^{2x-5} = log9^{x+2}$$
$$(2x-5)log2 = (x+2)log9$$
$$2xlog2 - 5log2 = xlog9 + 2log9$$
$$2xlog2 - xlog9 = 2log9 + 5log2$$
$$x(2log2 - log9) = 2log9 + 5log2$$
$$x = \frac{2log9 + 5log2}{2log2 - log9}$$

GCF = x
Divide

De-log Both sides

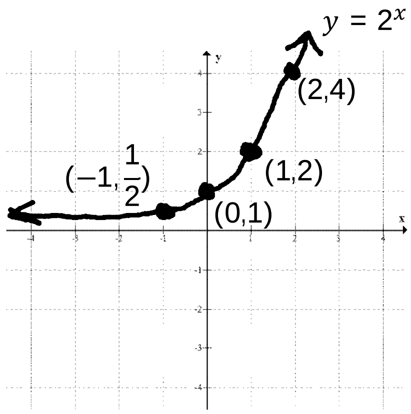
$$\log_2 4 = \log_2 x$$
$$4 = x$$

Rule 7 Proof

$$b^{\log_b a} = a$$
$$logb^{\log_b a} = loga$$
$$\log_b a \times logb = \log a$$
$$\frac{loga}{logb} \times \log b = loga$$

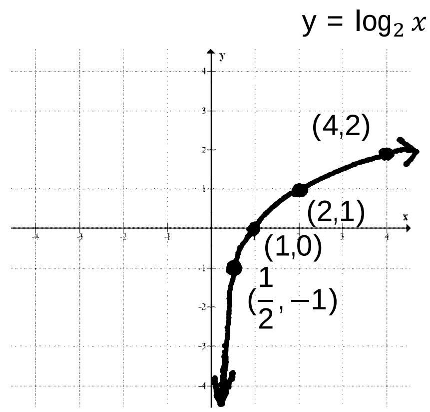
Remember: You may only log both sides if SAMD is complete. Bedmas backwards.
Remember: If you do log a product you must separate into an addition of logs.
Remember: if you log a sum you must use brackets
Remember: You may only de-log both sides if one log equals one log.

C12 - 8.7 - Inverse Log Graphs Notes



x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

x	y
0	*und*
$\frac{1}{2}$	-1
1	0
2	1
2	4

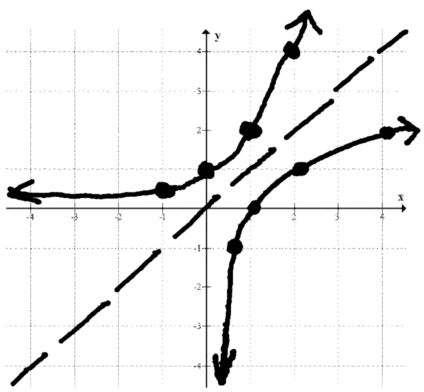


$$
\begin{aligned}
y &= 2^x \\
x &= 2^y && \textit{Switch x and y} \\
logx &= log2^y && \text{Log Both Sides} \\
logx &= ylog2 && \text{Bring Exponents Down In Front} \\
\frac{logx}{log2} &= y && \text{Divide} \\
\log_2 x &= y && \text{Change of base} \\
y &= \log_2 x && \text{Mirror} \\
f^{-1}(x) &= \log_2 x && \text{Inverse Function notation}
\end{aligned}
$$

$$
\begin{aligned}
y &= 2^x \\
x &= 2^y && \textit{Switch x and y} \\
y &= \log_2 x && \text{Exponential to } log \text{ Form} \\
f^{-1}(x) &= \log_2 x
\end{aligned}
$$

Back the Other Way!

$$
\begin{aligned}
y &= \log_2 x \\
x &= \log_2 y \\
2^x &= y \\
y &= 2^x \\
f^{-1}(x) &= 2^x
\end{aligned}
$$



$$
\begin{aligned}
y &= 2^{x+1} - 3 \\
x &= 2^{y+1} - 3 \\
x + 3 &= 2^{y+1} \\
\log(x+3) &= (y+1)log2 \\
\frac{\log(x+3)}{log2} &= y + 1 \\
\log_2(x+3) &= y + 1 \\
\log_2(x+3) - 1 &= y \\
y &= \log_2(x+3) - 1 \\
f^{-1}(x) &= \log_2(x+3)
\end{aligned}
$$

$\boxed{\textit{Inverse Proof}}$

$$
\begin{aligned}
y &= \log_2(x+3) - 1 \\
x &= \log_2(y+3) - 1 \\
x + 1 &= \log_2(y+3) \\
2^{x+1} &= y + 3 \\
2^{x+1} - 3 &= y \\
y &= 2^{x+1} - 3 \\
f^{-1}(x) &= 2^{x+1} - 3
\end{aligned}
$$

Remember: Inverse: Switch x and y
Remember: A diagonal reflection over the line $y = x$