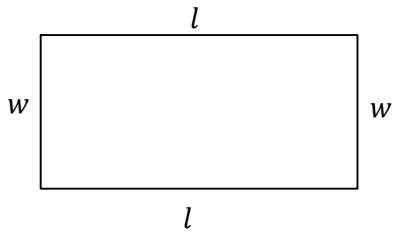
C11 - 4.5 - Rectangular Garden

A rectangular garden has an area of 36 and a perimeter of 30. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$P = 2l + 2w$

$A = l \times w$

Equation 1, equation 2.

$P = 2l + 2w$
$30 = 2l + 2w$
$\frac{30}{2} = \frac{2l}{2} + \frac{2w}{2}$
$15 = l + w$
$-w \quad -w$
$15 - w = l$
$l = 15 - w$

Equation #1
Isolate a variable

$A = l \times w$
$36 = l \times w$
$36 = (15 - w) \times w$
$36 = 15w - w^2$
$+w^2 \quad + w^2$
$36 + w^2 = 15w$
$-15w \quad -15w$
$w^2 - 15w + 36 = 0$
$(w - 12)(w - 3) = 0$

Equation #2
Substitute the isolated variable

Factor

$w - 12 = 0$
$w = 12$

$w - 3 = 0$
$w = 3$

Solve

$l = 15 - w$
$l = 15 - (12)$
$l = 3$

Substitute w into the other equation.

$Length = 12$
$Width = 3$

List the length and width

OR

$l = 15 - w$
$l = 15 - (3)$
$l = 12$

$Length = 3$
$Width = 12$

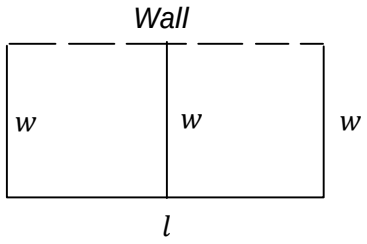
List the length and width

C11 - 4.5 - Fence Split in Two

July 15, 2014 2:37 PM

A rectangular fence that is split in half is against a wall. The total fencing length is 39, and it has a total area of 66. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



Let statements:

$P = l + 3w$

$A = l \times w$

Equation 1, equation 2.

$P = l + 3w$
$39 = l + 3w$
$-3w \quad -3w$
$39 - 3w = l$
$l = 39 - 3w$

Equation #1
Isolate a variable

$A = l \times w$
$66 = (39 - 3w) \times w$
$66 = 39w - 3w^2$
$+3w^2 \quad +3w^2$
$66 + 3w^2 = 39w$
$-39w \quad -39w$
$3w^2 - 39w + 66 = 0$
$3(w^2 - 13w + 22) = 0$
$3(w - 2)(w - 11) = 0$

Equation #2
Substitute the isolated variable

Factor

$w - 2 = 0$
$w = 2$

$w - 11 = 0$
$w = 11$

Solve

$l = 39 - 3w$
$l = 39 - 3(2)$
$l = 39 - 6$
$l = 33$

Substitute w into the other equation.

$Width = 2$
$Length = 33$

List the length and width

or

$l = 39 - 3w$
$l = 39 - 3(11)$
$l = 39 - 33$
$l = 6$

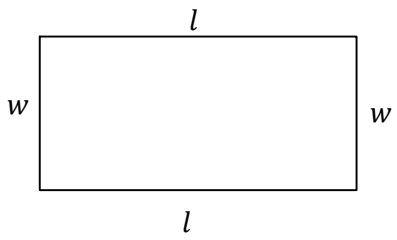
$Width = 11$
$Length = 6$

List the length and width

C11 - 4.5 - Rectangular Garden Quad

A rectangular garden has an area of 61 and a perimeter of 40. What are the lengths and widths?

Let $w = width$
Let $l = length$



Let statements:

$P = 2l + 2w$ $\quad$ $A = l \times w$

Equation 1, equation 2.

$P = 2l + 2w$
$40 = 2l + 2w$
$\frac{40}{2} = \frac{2l}{2} + \frac{2w}{2}$
$20 = l + w$
$-w \quad -w$
$20 - w = l$
$l = 20 - w$

Equation #1
Isolate a variable

$A = l \times w$
$91 = l \times w$
$61 = (20 - w) \times w$
$61 = 20w - w^2$
$+w^2 \quad + w^2$
$61 + w^2 = 20w$
$-20w \quad -20w$
$w^2 - 20w + 61 = 0$

Equation #2
Substitute the isolated variable

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$w = \frac{-(-20) \pm \sqrt{20^2 - 4(1)(61)}}{2(1)}$

Quadratic Formula

$w = \frac{20 - \sqrt{156}}{2(1)}$ $\quad$ $w = \frac{20 + \sqrt{156}}{2(1)}$
$w = 3.755$ $\quad$ $w = 16.245$

Solve

$l = 20 - w$
$l = 20 - (16.245)$
$l = 3.755$

Substitute w into the other equation.

$Length = 16.245$
$Width = 3.755$

List the length and width

OR

$l = 15 - w$
$l = 15 - (3.755)$
$l = 16.245$

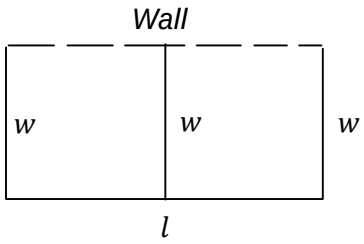
List the length and width

$Length = 3.755$
$Width = 16.245$

C11 - 4.5 - Fence Split in Two Quad

A rectangular fence that is split in half is against a wall. The total fencing length is 61, and it has a total area of 58. What are the dimensions of the fence?

Let $w = width$
Let $l = length$



Let statements:

$P = l + 3w$ $\qquad$ $A = l \times w$

Equation 1, equation 2.

$P = l + 3w$
$61 = l + 3w$
$-3w \quad -3w$
$61 - 3w = l$
$l = 61 - 3w$

Equation #1
Isolate a variable

$A = l \times w$
$58 = (61 - 3w) \times w$
$58 = 61w - 3w^2$
$+3w^2 \qquad +3w^2$
$58 + 3w^2 = 61w$
$-61w \quad -61w$
$3w^2 - 61w + 58 = 0$

Equation #2
Substitute the isolated variable

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$w = \frac{-(-61) \pm \sqrt{61^2 - 4(3)(58)}}{2(3)}$$

Quadratic Formula

$w = \frac{61 + \sqrt{3025}}{6}$ $\qquad$ $w = \frac{61 - \sqrt{3025}}{6}$
$w = 19.\overline{3}$ $\qquad$ $w = 1$
$w = \frac{58}{3}$

Solve

$l = 61 - 3w$
$l = 61 - 3\left(\frac{58}{3}\right)$
$l = 61 - 58$
$l = 3$

Substitute w into the other equation.

$Width = \frac{58}{3}$
$Length = 3$

List the length and width

or

$l = 61 - 3w$
$l = 61 - 3(1)$
$l = 61 - 3$
$l = 58$

$Width = 58$
$Length = 1$

List the length and width