C12 - Table of Contents

Duotang/Notes/Homweork

C12 - Methods

Limits

Extreme Table of Values
Substitution
Factoring
Conjugate; Top, bottom or both. Foil only Conjugates
Multiply by "1"
Graphing
Separate Fractions

LCD

Add/Subtract Fractions top and bottom, flip and multiply or Multiply by LCD Complex Fractions)

Squeeze Theorem $f(x) \leq g(x) \leq h(x)$

L'Hopital's Rule $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ IF $\frac{f(a)}{g(a)} = \frac{0}{0}$ OR $\frac{\pm\infty}{\pm\infty}$

Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 1,2,3 \ldots$$

Horizontal Asymptotes

Limit as x approaches positive or negative infinity
Divide top and bottom by x to the highest exponent of x in denominator

Vertical Asymptotes

Set denominator = 0 and solve

$Definition\ of\ the\ Derivative$

$$m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Derivative Laws

Integration Formulas

FUNDAMENTAL THEOREM OF CALCULUS

$$A = \int_a^b f(x)dx = F(b) - F(a)$$

$F(x)$ *is the antiderivative of* $f(x)$

Volume

$$V = \int_a^b A(x)dx$$

C12 - Remember

$$y' = \frac{dy}{dx} = f'(x)$$

$Limit: What\ y\ is\ approaching.$

$Limit\ Exists\ if\ and\ only\ if:$ $Left\ hand\ Limit = Right\ Hand\ Limit$

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

$\lim_{x \to a^-} f(x)$ Left hand limit

$\lim_{x \to a^+} f(x)$ Right hand limit

$Limit\ Does\ Not\ Exists$ $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$ DNE

$Continuous$ $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a} f(x)$

$\lim_{x \to a} f(x) = f(a)$ $Limit\ exists\ and\ equals\ the\ value\ of\ the\ function.\ Obviously!$

$a^2 - b^2 = (a - b)(a + b)$	Difference of Squares	$x^2 - 4 = (x - 2)(x + 2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	Difference of Cubes	$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	Sum of cubes	$x^3 + 1 = (x + 1)(x^2 - 1x + 1)$

SOAP

$Chain\ Rule$ $-2xy$

$-2xy$
$(-2x)(y)$
$-2y + (-2x)y'$
$-2y - 2xy'$

$-2(x)(y)$
$-2(1y + xy')$
$-2y - 2xy'$

$Trig$

$$\lim_{x \to 0} \frac{sinx}{x} = 1 \qquad \lim_{x \to 0} \frac{tanx}{x} = 1 \qquad \lim_{x \to 0} \frac{1 - cosx}{x} = 0 \qquad \lim_{x \to 0} \frac{cosx - 1}{x} = 0$$

m

y	$\updownarrow$	$positiin$
y'		$velocity$
y''		$accaleration$

y

Critical points: the x location where the derivative is equal to zero.

C12 - Properties of Limits

$\lim_{x \to c} f(x) = L$ $\quad$ $\lim_{x \to c} g(x) = M$ $\quad$ *Given*

$\lim_{x \to c} (f(x) + g(x)) = L + M$ $\quad$ *Sum Rule*

$\lim_{x \to c} (f(x) - g(x)) = L - M$ $\quad$ *Difference Rule*

$\lim_{x \to c} (k \times g(x)) = k \times M$ $\quad$ *Constant Rule*

$\lim_{x \to c} (f(x) \times g(x)) = L \times M$ $\quad$ *Product Rule*

$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}$ $\quad$ *Quotient Rule*

$\lim_{x \to c} \left(f(x)\right)^{\frac{m}{n}} = L^{\frac{m}{n}}$ $\quad$ *Power Rule*

Duotang

C12 - Derivative Poetry

Power

Bring the exponent down in front
Subtract one from the exponent

Not very Poetic!

Product Rule

Derivative of the first, times the second, Plus
Derivative of the second times the first

Product Rule

Derivative of the first, times the second, Plus
The first, times the derivative of the second

Quotient Rule

Derivative of the top, times the bottom, Minus
Derivative of the bottom, times the top,

All over bottom squared

Power/Chain Rule

Bring the exponent down in front
Write what we are doing <u>power rule on</u>
Subtract one from the exponent
Multiply by the derivative of what you did the <u>power rule on</u>
<u>Possibly do Chain Rule again</u>

C12 - English Sentences

Slope of Tangent Line

We always take the derivative of the equation
We always substitute the X value of the point into the derivative to find the slope value
We sometimes substitute the X value back into the original equation to figure out the Y value
We now write down the equation in slope point form or y=mx+b or general form

Implicit Differentiation Don’t forget y'

Take derivative
Combine primes on one side
Everything else on the other side
Factor out y prime
Divide both sides
Sometimes sub y back in
Possibly sub (x,y) in first

Max/Min

Diagram
Equation
Derivative=0
Solve
Number Line Check
Answer the Question

Related Rates

We always write down info and draw a diagram
We always write a formula relating the information
We always take the derivative
(We substitute constants into the formula)
We sometimes use that formula to figure out other information we need
We sometimes find a relationship between the variables so when we take the derivative we don't need to do a product rule and we can do the derivative with respect to one variable due to limited information
We choose the formula based on information that's given
*Negative Derivatives
Answer the Question

If the derivative is a product rule which we can't solve we need to find a relationship between the two letters

Integration by Substitution

Choose a "u" who's derivative is present

C12 - Derivative Laws y^1 =

$$\boxed{y' = f'(x) = \frac{dy}{dx}}$$

$\boxed{\boxed{\text{CHAIN RULE}}}$

Basic Rules

$$y = c \qquad y' = 0$$

$$y = f(x) \qquad y' = f'(x)$$

$$y = cf(x) \qquad y' = cf'(x)$$

$$y = f(x) \pm g(x) \qquad y' = f'(x) \pm g'(x)$$

$$y = x \qquad y' = 1$$

Power rule

$$y = x^n$$
$$y' = nx^{n-1}$$

Product rule

$$y = f(x)g(x)$$
$$y' = f'(x)g(x) + g'(x)f(x)$$

$$\boxed{\begin{aligned} y &= uv \\ y' &= u'v + v'u \end{aligned}}$$

Chain rule

$$\boxed{\begin{aligned} y &= f\big(g(x)\big) \\ y' &= f'\big(g(x)\big)\big(g'(x)\big) \end{aligned}}$$

Quotient rule

$$y = \frac{f(x)}{g(x)}$$
$$y' = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$$

$$\boxed{\begin{aligned} y &= \frac{u}{v} \\ y' &= \frac{u'v - v'u}{v^2} \end{aligned}}$$

Logarithmic rules

$$y = \log_a x$$
$$y' = \frac{1}{xlna}$$

$$y = lnx$$
$$y' = \frac{1}{x} \times \frac{1}{\cancel{lne}}$$
$$y' = \frac{1}{x}$$

$$\boxed{Note: \ln e = 1}$$

Exponential rules

$$y = e^x$$
$$y' = e^x \cancel{lne}$$
$$y' = e^x$$

$$y = a^x$$
$$y' = a^x lna$$

Trigonometric rules

$$y = sinx \qquad y' = cosx$$

$$y = secx \qquad y' = secxtanx$$

$$y = tanx \qquad y' = \sec^2 x$$

$$y = cosx \qquad y' = -sinx$$

$$y = cscx \qquad y' = -cscxcotx$$

$$y = cotx \qquad y' = -\csc^2 x$$

Inverse trigonometric rules

$$y = sin^{-1}x \qquad y' = \frac{1}{\sqrt{1 - x^2}}$$

$$y = \sec^{-1} x \qquad y' = \frac{1}{|x|\sqrt{x^2 - 1}}$$

$$y = \tan^{-1} x \qquad y' = \frac{1}{1 + x^2}$$

$$y = cos^{-1}x \qquad y' = -\frac{1}{\sqrt{1 - x^2}}$$

$$y = \csc^{-1} x \qquad y' = -\frac{1}{|x|\sqrt{x^2 - 1}}$$

$$y = \cot^{-1} x \qquad y' = -\frac{1}{1 + x^2}$$

C12 - Derivative Laws $\frac{d}{dx} =$

$y' = f'(x) = \frac{dy}{dx}$

CHAIN RULE

Basic Derivatives

$\frac{d}{dx} c = 0$

$\frac{d}{dx}[cf(x)] = cf'(x)$

$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

Power Rule

$\frac{d}{dx} x^n = nx^{n-1}$

Product Rule

$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$

Chain Rule

$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

Quotient Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

Exponential and Logarithmic Functions

$\frac{d}{dx} a^x = a^x \ln a$

$\frac{d}{dx} \log_a x = \frac{1}{x} \times \frac{1}{\ln a}$

$\frac{d}{dx} e^x = e^x \cancel{\ln e} = e^x$

$\frac{d}{dx} \ln|x| = \frac{1}{x} \times \frac{1}{\cancel{\ln e}} = \frac{1}{x}$

$Note: \ln e = 1$

Trigonometric Functions

$\frac{d}{dx} sinx = cosx$

$\frac{d}{dx} secx = secxtanx$

$\frac{d}{dx} tanx = \sec^2 x$

$\frac{d}{dx} cosx = -sinx$

$\frac{d}{dx} cscx = -cscx\, cotx$

$\frac{d}{dx} cotx = -\csc^2 x$

Inverse Trigonometric Functions

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2 - 1}}$

$\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$

$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2 - 1}}$

$\frac{d}{dx} \cot^{-1} x = -\frac{1}{1 + x^2}$

Bring the exponent down in front, write down what you did the power rule on. Subtract one from the exponent. Multiply by the derivative of what you did the power rule on. Possibly do chain rule again...

Get all y primes on one side, get things without y prime on the other, factor out a y prime, divide both sides by what you factored y prime out of. Possibly substitute for y from original so all in terms of x.

C12 - Integration Formulas

$$f(x) = F'(x)$$

Basic Formulas

$\int kdx = kx + C$ $\quad (k: \ a\ constant)$

$\int kf(x)dx = k\int f(x)dx$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $\quad (n \neq -1)$

$\int \frac{1}{x}dx = \ln|x| + C$

Exponential and Logarithmic Functions

$\int e^x dx = \frac{e^x}{lne} + c = e^x + C$

$\int lnx\, dx = x\ln x - x + C$

$\int a^x\, dx = \frac{a^x}{lna} + C \ (a \neq -1)$

Trigonometric Functions

$\int sinx\, dx = -cosx + C$

$\int secx\, tanx\, dx = secx + C$

$\int \sec^2 x\, dx = tanx + C$

$\int cosx\, dx = sinx + C$

$\int cscx\, cotx\, dx = -cscx + C$

$\int \csc^2 x\, dx = -cotx + C$

$\int tanx\, dx = \ln|secx| + C$

$\int secxdx = \ln|secx + tanx| + C$

$\int cscx\, dx = \ln|cscx - cotx| + C$

Inverse Trigonometric Functions

$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C$

$\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}|x| + C$

$\int \frac{1}{x^2+1}dx = \tan^{-1}x + C$

$\int -\frac{1}{\sqrt{1-x^2}}dx = \cos^{-1}|x| + C$

$\int -\frac{1}{x^2+1}dx = \cot^{-1}x + C$

Advanced Integrals

$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{\text{a}} \bullet \sec^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{1}{x^2+a^2}dx = \frac{1}{a} \bullet \tan^{-1}\left(\frac{x}{a}\right) + C$

Reverse Chain Rule

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{n+1} \times \frac{1}{a} + c$

$\int \frac{1}{bx}dx = \frac{lnbx}{b} + C$

$\int \frac{f'(x)}{f(x)}\, dx = ln|f(x)| + C$

$\int e^{kx}dx = \frac{e^{kx}}{k} + C$

$\int a^{kx}\, dx = \frac{1}{k} \bullet \frac{a^{kx}}{lna} + C$

$\int sinkx\, dx = -\frac{coskx}{k} + C$

$\int coskx\, dx = \frac{sinkx}{k} + C$

Integration by Substitution

$\int f\big(g(x)\big)g'(x)dx = \int f(u)du = F(x) + c$

Integration by Parts

$\int uv'dx = uv - \int u'vdx$