C12 - 9.0 - Definition of Rational Notes

Rational Expression:

$$\frac{1st\ polynomial}{2nd\ polynomial} \qquad \frac{g(x)}{h(x)}$$ ratio of two polynomials

Example: $\frac{x+2}{x-1}$ $;\ Grade\ 11$

Rational Function:

$$1st\ polynomial = \frac{2nd\ polynomial}{3rd\ polynomial} \qquad f(x) = \frac{g(x)}{h(x)}$$

Example: $f(x) = \frac{x+2}{x-1}$

Example: $f(x) = \frac{x^2 + 2x + 1}{1}$ *1 is a polynomial*

Example: $f(x) = 3x^2 + 5x$ *1 is always the denominator. 1 is a polynomial.*

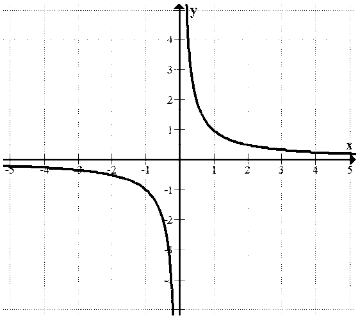
NOT a Rational Function:

$f(x) = \frac{x+3}{\sqrt{x-2}}$ $\sqrt{x-2}\ \ is\ \ not\ a\ polynomial$

C12 - 9.1 - Vertical Asymptotes Notes

To find Vertical Asymptotes: Cannot have a denominator of 0. $\boldsymbol{VA: Vertical\ Asymptote}$

$$f(x) = \frac{1}{x}$$

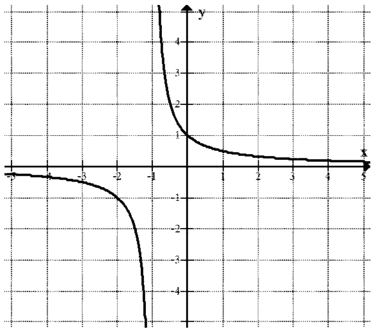


$VA: x = 0$ $\quad Set\ denominator = 0, and\ solve.$

NPVs, Restrictions:
$x = 0$

Domain:
$x \neq 0$

$$f(x) = \frac{1}{x+1}$$



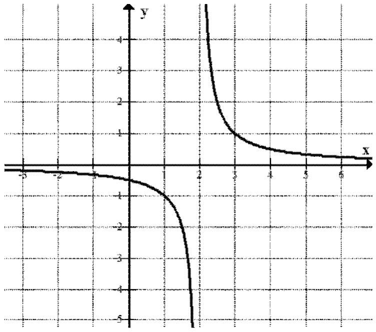
$VA: x + 1 = 0$
$x = -1$ $\quad Set\ denominator = 0, and\ solve.$

NPVs, Restrictions:
$x = -1$

Domain:
$x \neq -1$

$Notice: The\ vertical\ asymptote\ has\ shifted\ 1\ to\ the\ left\ from\ \frac{1}{x}$

$$f(x) = \frac{1}{x-2}$$



$VA: x - 2 = 0$
$x = 2$ $\quad Set\ denominator = 0, and\ solve.$

NPVs, Restrictions:
$x = 2$

Domain:
$x \neq 2$

$Notice: The\ vertical\ asymptote\ has\ shifted\ 2\ to\ the\ right\ from\ \frac{1}{x}.$

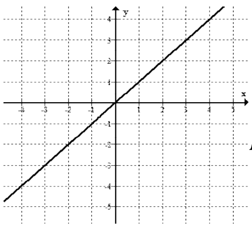
A vertical asymptote by definition is the limit as x approaches $\pm x$ value of vertical asymptote. Substitute ± x values close to the vertical assympotoe into a table of values. If y equals $+infinity$ on one side and $-infinity$ on the other it is a vertical asymptote.

C12 - 9.2 - Horizontal Asymptotes Cases Notes

Case 1: (No Horizontal asymptote)

You may cross a horizontal asymptote

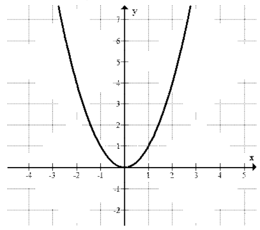
$\boldsymbol{y = x}$ $\quad y = \frac{x^2}{x}$



HA: None

$Range: y \in R$

$\boldsymbol{y = x^2}$



HA: None

$Range: y \geq 0$

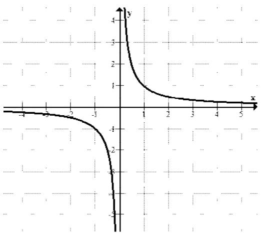
If the exponent of x is higher on the top than the bottom, no horizontal asymptote.

Unless there is a slant.

Case 2: (Horizontal Asymptote at $y = 0$)

$\boldsymbol{y = \frac{1}{x}}$

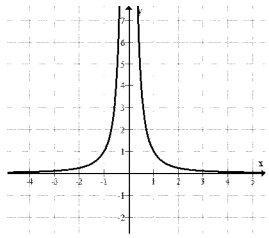
$y = \frac{x}{x^2}$



HA: $y = 0$

$Range: y \neq 0$

$\boldsymbol{y = \frac{1}{x^2}}$



HA: $y = 0$

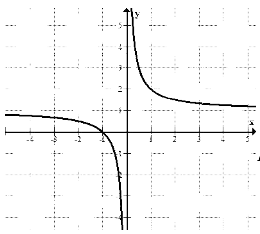
$Range: y > 0$

If the exponent of x is higher on the bottom, HA: $y = 0$

Modified Case 2: (Horizontal Asymptote at $y = c$)

$\boldsymbol{y = \frac{1}{x} + c}$

$y = \frac{1}{x} + 1$

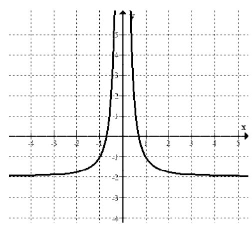


HA: $y = 1$

$Range: y \neq 1$

$\boldsymbol{y = \frac{1}{x^2} + c}$

$y = \frac{1}{x^2} - 2$



HA: $y = -2$

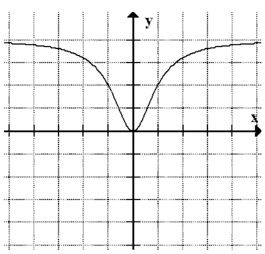
$Range: y > -2$

If case 2 is shifted up or down = $c, HA: \ y = c$

Case 3: (Horizontal Asymptote at $y = \frac{a}{b}$)

$y = \frac{ax^n}{bx^n}$

$y = \frac{4x^2}{(x^2 + 1)}$



$HA: \ y = \frac{4}{1}$

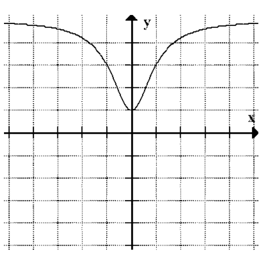
$Range: 0 \leq y < 4$

If the exponent of x is the same on the top as the bottom, HA: y = fraction of coefficients

Modified Case 3: (Horizontal Asymptote at $y = \frac{a}{b} + c$)

$y = \frac{ax^n}{bx^n} + c$

$y = \frac{4x^2}{(x^2 + 1)} + 1$



$HA: \ y = \frac{4}{1} + 1 = 5$

$Range: 1 \leq y < 5$

If case 3 is shifted up or down = c, $HA: \ y$ = fraction of coefficients + c

A horizontal asymptote by definition is the limit as x approaches ±infinity. Substitute ±infinity for x into a table of values.

C12 - 9.3 - x,y Intercepts Notes

To find x-intercept: $Set\ y = 0$ **To find y-intercept:** $Set\ x = 0$

Find the x and y intercepts of the following.

$$\boldsymbol{y = \frac{x-1}{x+1}}$$

$y - interceptes: Set\ x = 0$

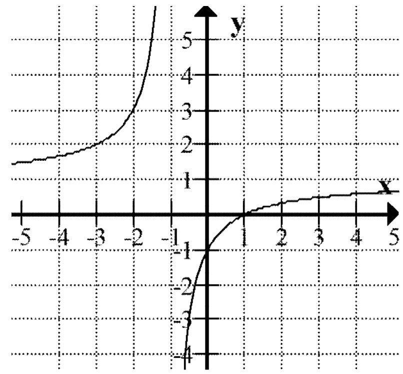
$$y = \frac{0-1}{0+1}$$
$$y = -\frac{1}{1}$$
$$y = -1$$

$y\ \boldsymbol{in}t: (0, -1)$

$x - intercept: Set\ y = 0$

$$0 = \frac{(x-1)}{x+1}$$
$$0(x+1) = x-1$$
$$0 = x-1$$
$$1 = x$$

$x\ int: (1{,}0)$



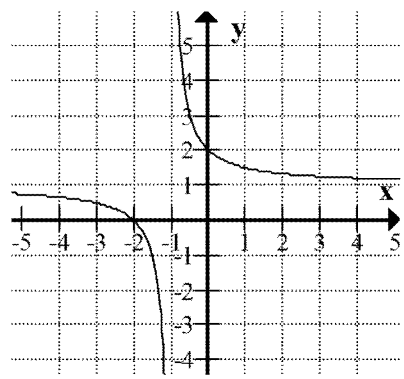
$$\boldsymbol{y = \frac{1}{x+1} + 1}$$

$$y = \frac{1}{0+1} + 1$$
$$y = \frac{1}{1} + 1$$
$$y = 1 + 1$$
$$y = 2$$

$y\ intercept: (-2{,}0)$

$$0 = \frac{1}{x+1} + 1$$
$$-1 = \frac{1}{x+1}$$
$$-1(x+1) = 1$$
$$-x - 1 = 1$$
$$-x = 2$$
$$x = -2$$

$x\ intercept: (0{,}2)$



C12 - 9.3 - x,y Intercepts Notes

Find intercepts

$$\boldsymbol{y = \frac{x^2 + 6x + 8}{x - 2}}$$

To find y-intercept: $Set\ x = 0$

Factor

$$y = \frac{(x+2)(x+4)}{x-2}$$
$$y = \frac{(0+2)(0+4)}{0-2}$$
$$y = \frac{(2)(4)}{-2}$$
$$y = -4$$

$y\ intercept$: $(0, -4)$

To find x-intercept: $Set\ y = 0$

$$y = \frac{(x+2)(x+4)}{x-2}$$
$$0 = \frac{(x+2)(x+4)}{x-2}$$
$$0(x-2) = (x+2)(x+4)$$
$$0 = (x+2)(x+4)$$

$x = -2$ $\quad$ $x = -4$

$x\ intercepts$: $(-4, 0), (-2, 0)$

$$\boldsymbol{y = \frac{x^2 - 1}{x^2 + x}}$$

Factor

$$y = \frac{(x+1)(x-1)}{x(x+1)}$$
$$y = \frac{(0+1)(0-1)}{0(0+1)}$$
$$y = \frac{-1}{0}$$

Undefined. No y-intercept.

$$y = \frac{x^2 - 1}{x^2 + x}$$
$$0 = \frac{x^2 - 1}{x^2 + x}$$
$$0 = x^2 - 1$$
$$1 = x^2$$
$$\pm\sqrt{1} = x$$
$$\pm 1 = x$$

$x = 1, -1$

or

$$0 = (x-1)(x+1)$$

$x = 1, -1$

$x\ intercepts$: $(1, 0), (-1, 0)$

$$y = \frac{1}{x+1} - 2$$

$$y = \frac{1}{0+1} - 2$$
$$y = 1 - 2$$
$$y = -1$$

$y\ intercept$: $(0, -1)$

$$y = \frac{1}{x+1} - 2$$

$$0 = \frac{6}{x+1} - 2$$
$$2 = \frac{6}{x+1}$$
$$2(x+1) = 1$$
$$2x + 2 = 6$$
$$2x = 4$$
$$x = 2$$

$x\ intercept$: $(2, 0)$

C12 - 9.4 - Holes Notes

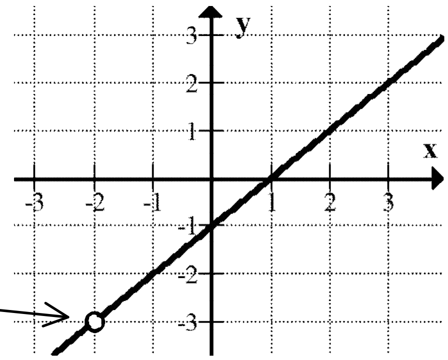
$f(x) = \frac{(x-1)\cancel{(x+2)}}{\cancel{x+2}}$

If you can cross out a piece of the denominator, that piece is a hole.
Where there is a hole, there is not a vertical asymptote.

$f(x) = (x-1)$

Hole: $x + 2 \neq 0$
$x \neq -2$

Set what you've crossed off can't equal to zero and solve.



To find coordinate, plug back in.

$f(x) = (x-1)$
$f(-2) = (-2-1)$
$f(-2) = -3$

Hole at: $(-2, -3)$

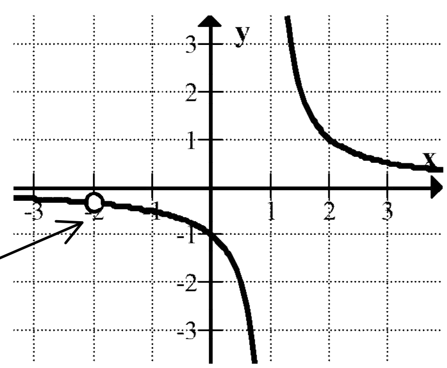
$f(x) = \frac{\cancel{x+2}}{(x-1)\cancel{(x+2)}}$

$f(x) = \frac{1}{x-1}$

Hole: $x + 2 \neq 0$
$x \neq -2$

To find coordinate, plug back in.

$f(x) = \frac{1}{x-1}$
$f(-2) = \frac{1}{(-2)-1}$
$f(-2) = \frac{1}{-3}$



Hole at: $\left(-2, -\frac{1}{3}\right)$

C12 - 9.5 - Slant Notes

$$y = \frac{3x^2 + 4x + 2}{x + 1}$$

quotient

$$\begin{array}{r|l} & 3x + 1 \\ \hline x + 1 & 3x^2 + 4x + 2 \\ - & 3x^2 + 3x \\ \hline & \quad x + 2 \\ - & \quad x + 1 \\ \hline & \qquad 1 \end{array}$$

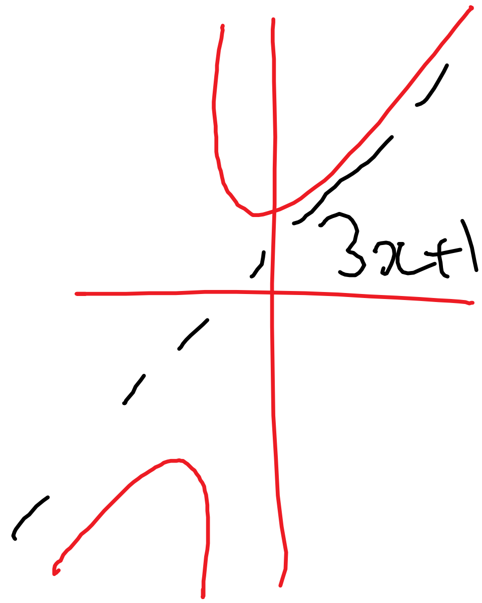
dividend

divisor

remainder

So slant asymptote is quotient.

$Slant\ asymptote = 3x + 1$



$$\begin{array}{c|ccc} -1 & 3 & 4 & 2 \\ + & \downarrow & -3 & -1 \\ \hline & 3 & 1 & 1 \leftarrow R \end{array}$$

$3x + 1$