C11 - 3.5 - Product of Numbers is a Min Notes

The difference between two numbers is 10. Their product is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements: get used to using variables other than x and y

1 $a - b = 10$

2
$a \times b = \cancel{minimum}$
$a \times b = minimum$ y
$y = a \times b$

Equation 1, equation 2.
The minimum or maximum will be y.

$a - b = 10$
$+b \quad + b$
$a = (10 + b)$

Equation #1
Isolate a variable

$y = a \times b$
$y = (10 + b) \times b$
$y = 10b + b^2$
$y = b^2 + 10b$

Equation #2
Substitute the isolated variable

$y = b^2 + 10b$
$y = (b^2 + 10b + 25 - 25)$
$y = (b^2 + 10b + 25) - 25$
$y = (b + 5)^2 - 25$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

Vertex = (−5, −25) b Minimum

$a = 10 + b$
$a = 10 - 5$
$a = 5$

Substitute b into the other equation.

$a = 5$
$b = -5$

List the two numbers and the minimum.

The minimum product is −25.

C11 - 3.5 - Product of Numbers is a Min Notes

Two numbers differ by 10. The product of the larger number and twice the smaller number is a minimum. What are the numbers?

Let statements:

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Equation 1, equation 2.
The minimum or maximum will be y.

1 $a - b = 10$

2 $a \times 2b = minimum$
$a \times 2b = \cancel{minimum}\ y$
$y = a \times 2b$

Equation #1
Isolate a variable

$a - b = 10$
$a = 10 + b$

Equation #2
Substitute the isolated variable

$y = a \times 2b$
$y = (10 + b) \times 2b$
$y = 20b + 2b^2$
$y = 2b^2 + 20b$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

×2

$y = 2b^2 + 20b$
$y = 2(b^2 + 10b + 25 - 25)$
$y = 2(b^2 + 10b + 25) - 50$
$y = 2(b + 5)^2 - 50$

Vertex = (-5, -50)

b Minimum

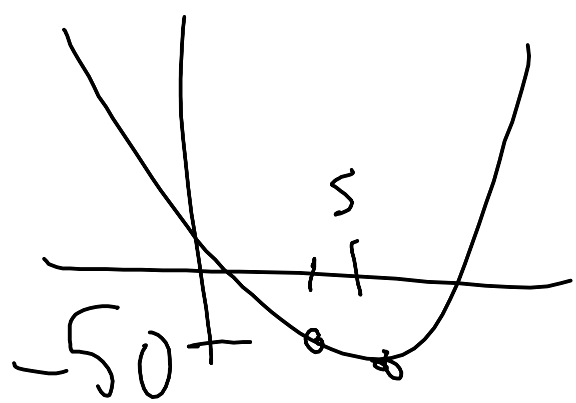
Substitute b into the other equation.

$a = 10 + b$
$a = 10 - 5$
$a = 5$

List the two numbers and the minimum.

$a = 5$
$b = -5$

The minimum product is -50.



C11 - 3.5 - Sum of Squares is a Min Notes

Two numbers sum to 8. The sum of their squares is a minimum.

Let statements:

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Equation 1, equation 2.
The minimum or maximum will be y.

(1) $a + b = 8$

(2) $a^2 + b^2 = minimum$

$a^2 + b^2 = \cancel{minimum}\ y$

$y = a^2 + b^2$

Equation #1
Isolate a variable

$$\begin{aligned} a + b &= 8 \\ -b \quad &\ \ -b \\ a &= 8 - b \\ a &= (8 - b) \end{aligned}$$

Equation #2
Substitute the isolated variable

$$\begin{aligned} y &= a^2 + b^2 \\ y &= (8 - b)^2 + b^2 \\ y &= 64 - 16b + b^2 + b^2 \\ y &= 2b^2 - 16b + 64 \end{aligned}$$

Complete the square.

$\left(\frac{b}{2}\right)^2 = \left(\frac{8}{2}\right)^2 = (4)^2 = 16$

$$\begin{aligned} y &= 2b^2 - 16b + 64 \\ y &= 2(b^2 - 8b) + 64 \\ y &= 2(b^2 - 8b + 16 - 16) + 64 \\ y &= 2(b^2 - 8b + 16) + 64 - 32 \\ y &= 2(b - 4)^2 + 32 \end{aligned}$$

Vertex = (4, 32)

b

Minimum

Substitute b into the other equation.

$$\begin{aligned} a &= 8 - b \\ a &= 8 - (4) \\ a &= 4 \end{aligned}$$

List the two numbers and the maximum.

$a = 4$
$b = 4$

The minimum product is 32.

C11 - 3.5 - Product of Numbers is a Max Notes

The sum of two times one number and six times another is sixty. Find the numbers if their product is a maximum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$

Let statements:

1 $2a + 6b = 60$

2 $a \times b = maximum$
$a \times b = \cancel{maximum}\ y$
$y = a \times b$

Equation 1, equation 2.
The minimum or maximum will be y.

$$\frac{2a}{2} + \frac{6b}{2} = \frac{60}{2}$$
$$a + 3b = 30$$
$$a = 30 - 3b$$

Equation #1
Isolate a variable

$$y = a \times b$$
$$y = (30 - 3b) \times b$$
$$y = 30b - 3b^2$$
$$y = -3b^2 + 30b$$

Equation #2
Substitute the isolated variable

$$y = -3b^2 + 30b$$
$$y = -3(b^2 - 10b + 25 - 25)$$
$$y = -3(b^2 - 10b + 25) + 75$$
$$y = -3(b - 5)^2 + 75$$

Complete the square.
$$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$$

Vertex = (5, 75)

b Maximum

$$a = 30 - 3b$$
$$a = 30 - 3(5)$$
$$a = 15$$

Substitute b into the other equation.

$a = 15$
$b = 5$

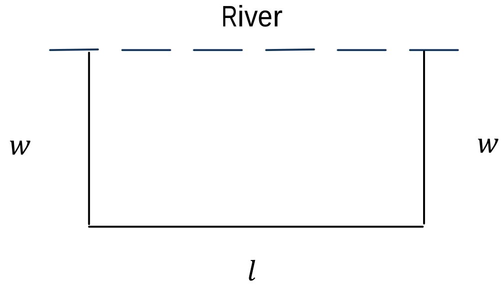
List the two numbers and the maximum.

The maximum product is 75

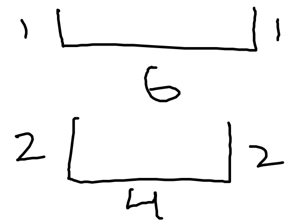
C11 - 3.5 - Fence w/ River Notes (p = 8m)

A rectangular enclosure is bounded on the side of a river. 3 sides total 8m of fencing. Find the dimensions of the largest possible enclosure.

Let $w = width$
Let $l = length$



Let statements:



1 $2w + l = P$
$2w + l = 8$

2 $A = l \times w$

Equation 1, equation 2.
The minimum or maximum will be y.

$$2w + l = 8$$
$$-2w \qquad -2w$$
$$l = 8 - 2w$$

Equation #1
Isolate a variable

$$A = l \times w$$
$$A = (8 - 2w) \times w$$
$$A = 8w - 2w^2$$
$$A = -2w^2 + 8w$$

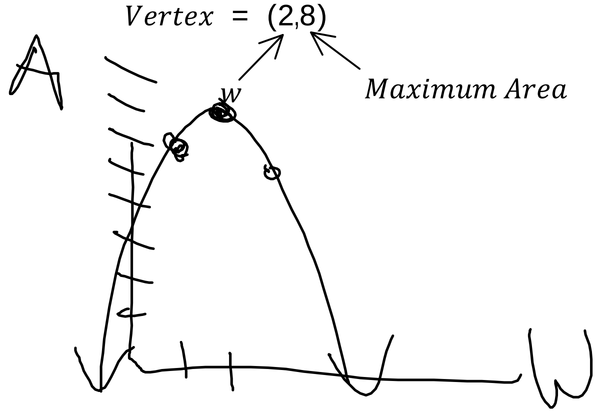
Equation #2
Substitute the isolated variable

$$A = -2w^2 + 8w$$
$$A = -2(w^2 - 4w + 4 - 4)$$
$$A = -2(w^2 - 4w + 4) + 8$$
$$A = -2(w - 2)^2 + 8$$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$Vertex = (2{,}8)$

$Maximum\ Area$



$$l = 8 - 2w$$
$$l = 8 - 2(2)$$
$$l = 4$$

Substitute w into the other equation.

$width = 2$ m
$length = 4$ m

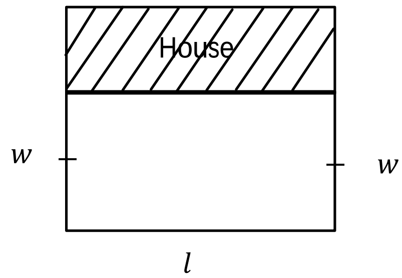
The maximum area is 8 m^2

List the length and width and the maximum area.

C11 - 3.5 - Fence w/ River Notes (p = 60m)

Jack has 60m of fencing to build a three sided fence on the side of his house. Determine the maximum possible area of the fenced area, and the dimensions of the fence.

Let $w = width$
Let $l = length$



Let statements:

1 $P = 2w + \cancel{2}l$
$60 = 2w + l$

2 $\cancel{a} = l \times w$
$\cancel{max} = l \times w$
$y = l \times w$

Equation 1, equation 2.
The minimum or maximum will be y.

$60 = 2w + l$
$-2w \quad -2w$
$60 - 2w = l$
$l = 60 - 2w$

Equation #1
Isolate a variable

$y = l \times w$
$y = (60 - 2w)w$
$y = 60w - 2w^2$
$y = -2w^2 + 60w$

Equation #2
Substitute the isolated variable

$y = -2(w^2 + 30w)$
$y = -2(w^2 + 30 + 225 - 225)$
$y = -2(w^2 + 30 + 225) + 450$
$y = -2(w - 15)^2 + 450$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = (5)^2 = 25$

$Vertex = (15{,}450)$
w $\quad$ $Maximum$

$l = 60 - 2w$
$l = 60 - 2(15)$
$l = 60 - 30$
$l = 30$

Substitute w into the other equation.

$width = 15$m
$length = 30$ m

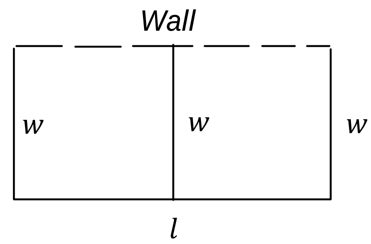
List the length and width and the maximum area.

The maximum area is 450 m^2

C11 - 3.5 - Fence w/ wall Split in Two

A rectangular fence that is split in half is against a wall. The total fencing length is 42 m. What is the max area of the fence?

Let $w = width$
Let $l = length$



Let statements:

$F = l + 3w$

$$\begin{aligned} A &= l \times w \\ max &= l \times w \\ y &= l \times w \end{aligned}$$

Equation 1, equation 2.
The minimum or maximum will be y.

$$\begin{aligned} P &= l + 3w \\ 42 &= l + 3w \\ -3w &\quad - 3w \\ 42 - 3w &= l \\ l &= 42 - 3w \end{aligned}$$

Equation #1
Isolate a variable

$$\begin{aligned} A &= l \times w \\ y &= (42 - 3w) \times w \\ y &= 42w - 3w^2 \\ y &= -3w^2 + 42w \\ y &= -3(w^2 - 14w) \\ y &= -3(w^2 - 14w + 49 - 49) \\ y &= -3(w^2 - 14w + 49) + 147 \\ y &= -3(w - 7)^2 + 147 \end{aligned}$$

Equation #2
Substitute the
isolated variable

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(\frac{-14}{2}\right)^2 = (7)^2 = 49$

$Vertex$: (7,147)

w $\quad$ $Maximum$

The maximum is the y value.

$$\begin{aligned} l &= 42 - 3w \\ l &= 42 - 3(7) \\ l &= 21 \end{aligned}$$

$length = 21m$
$width = 7m$

List the length and width
and the maximum area.

$Max\ area = 147\ m^2$

C11 - 3.5 - Set Up Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. Set up how this question will look.

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ increases}$

$Revenue = price \times quantity$
If $p = 6$, $q = 10$ $r = 6 \times 10$
$r = 60$

$p = 6 + 1x \longrightarrow$ Raising the price by 1 dollar x times.

$q = 10 - 1x \longrightarrow$ Each x times he raises the price, 1 less friend will buy the candy.

$r = p \times q$
$r = (6 + 1x) \times (10 - 1x)$

Price

x	p
-2	4
-1	5
0	6
1	7
2	8

Starting Price and Quantity (zero price increase)

Quantity

x	q
-2	12
-1	11
0	10
1	9
2	8

C11 - 3.5 - Maximize Candy Sales Notes

A student sells candy to all of his friends. Each candy costs 6 dollars, and he has 10 friends who buy the candy each day. Every time he increases the cost by 1 dollar, 1 of his friends decides not to buy the candy. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ increases}$

$Revenue = price \times quantity$
If $p = 6, \quad q = 10$
$r = 60$

$r = p \times q$
$r = 6 \times 10$
$r = \$60$

$p = 6 + 1x \longrightarrow$ If he decides to raise the price by 1 dollar x times.

$q = 10 - 1x \longrightarrow$ One less friend will buy the candy each time he increases the price.

$r = p \times q$
$r = (6 + x)(10 - x)$
$r = 60 - 6x + 10x - x^2$
$r = 60 + 4x - x^2$
$r = -x^2 + 4x + 60$
$r = -(x^2 - 4x) + 60$ $\times(-1)$
$r = -(x^2 - 4x + 4 - 4) + 60$
$r = -(x^2 - 4x + 4) + 60 + 4$
$r = -(x - 2)^2 + 64$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(-\frac{4}{2}\right)^2 = (-2)^2 = 4$

Vertex: $(2, 64)$

$x = 2\ price\ increases$

y = max $revenue = \$64$

$p = 6 + 1x$
$p = 6 + 1(2)$
$p = 6 + 2$
$p = 8$

$\boxed{price = 8}$

$q = 10 - 1x$
$q = 10 - 1(2)$
$q = 10 - 2$
$q = 8$

$\boxed{quantity = 8}$

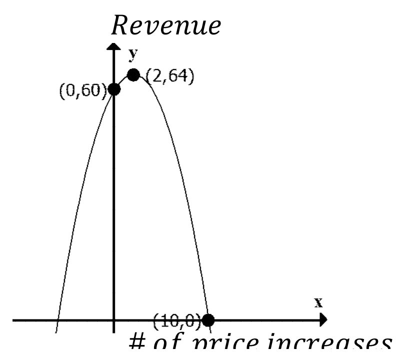
Check with Table of Values

Price	Quantity	(x)	Revenue (y)
6	10	0	60
7	9	1	63
8	8	2	64
9	7	3	63
10	6	4	60
11	5	5	55

1st increase

2nd increase

Max revenue



$\# \, of \, price \, increases$

C11 - 3.5 - Maximize Car Sales Notes

A car salesman sells a car for \$4000, with 20 people buying the car. For every \$200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p =$ \$4000, $q = 20$
$r =$ \$80,000

If they sell 20 cars at \$4000, revenue is \$80,000.

$p = 4000 - 200x$ → If he decides to decrease the price by \$200 x times.

$q = 20 + 2x$ → Two more people will buy the car each time he decreases the price.

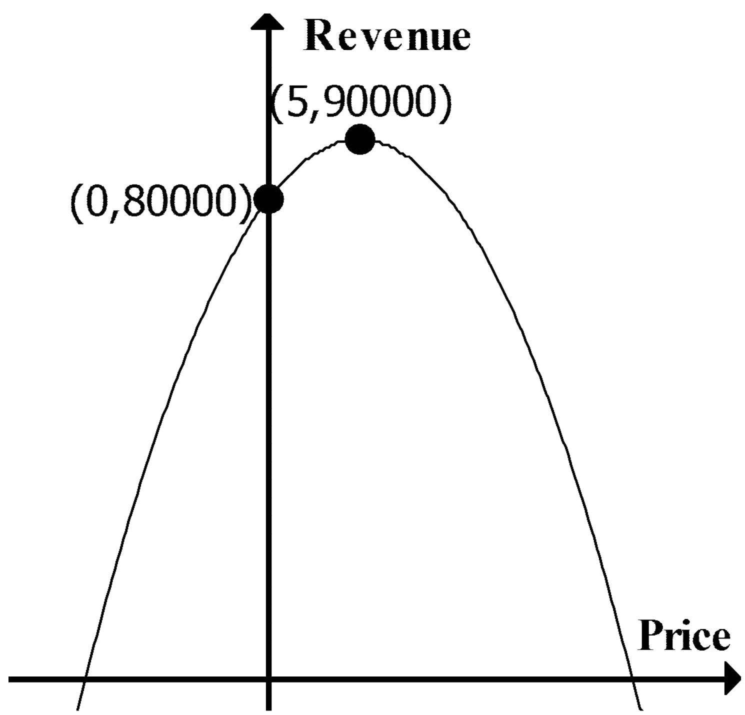
$r = p \times q$
$r = (4000 - 200x)(20 + 2x)$
$r = 80000 + 8000x - 4000x - 400x^2$
$r = -400x^2 + 4000x + 80000$
$r = -400(x^2 - 10x) + 80000$
$r = -400(x^2 - 10x + 25 - 25) + 80000$
$r = -400(x^2 - 10x - 25) + 80000 + 10000$
$r = -400(x - 5)^2 + 90000$

Complete the square.
$\left(\frac{b}{2}\right)^2 = \left(-\frac{10}{2}\right)^2 = (-5)^2 = 25$

Vertex: $(5, 90000)$

$x = 5\ price\ decreases$

y = max $revenue$ = \$90000



$p = 4000 - 200x$
$p = 4000 - 200(5)$
$p = 4000 - 1000$
$p = 3000$

$\boxed{price = \$3000}$

$q = 20 + 2x$
$q = 20 + 2(5)$
$q = 20 + 10$
$q = 30$

$\boxed{quantity = 30\ people}$

C11 - 3.5 - Maximize Car Sales Notes (No Price Increases)

A car salesman sells a car for $2000, with 20 people buying the car. For every $200 he takes off the price, 2 more people buy a car. What is the price that will maximize revenue?

$Let\ p = price$
$Let\ q = quantity$
$Let\ r = revenue$
$\boxed{Let\ x = \#\ of\ price\ decreases}$

$Revenue = price \times quantity$
$If\ p = \$2000,\quad q = 20$
$r = \$40{,}000$
If they sell 20 cars at $8000, revenue is $40,000.

$p = 2000$
$p = 2000 - 200x$ → If he decides to decrease the price by $200 x times.

$q = 20$
$q = 20 + 2x$ → Two more people will buy the car each time he decreases the price.

$r = p \times q$
$r = (2000 - 200x)(20 + 2x)$
$r = 40000 + 4000x - 4000x - 400x^2$
$r = -400x^2 + 40000$
$r = -400(x + 0)^2 + 40000$

Vertex: $(0, 40000)$

$x = 0\ price\ decreases$

y = max $revenue = \$30000$

$p = 2000 - 200x$
$p = 2000 - 200(0)$
$p = 2000$

$\boxed{price = \$2000}$

$q = 20 + 2x$
$q = 20 + 2(0)$
$q = 20 - 0$
$q = 20$

$\boxed{quantity = 20\ people}$