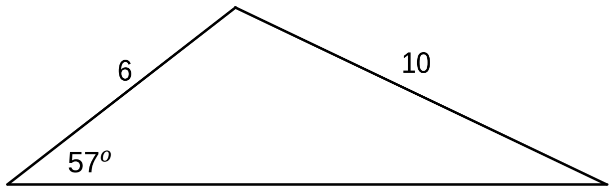
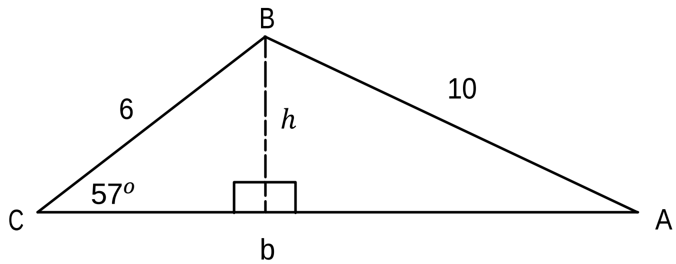
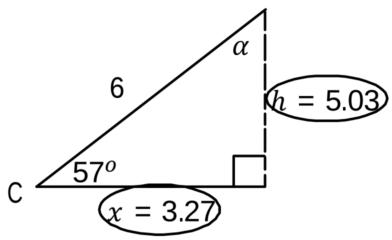
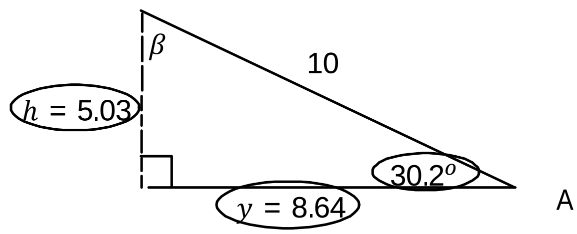
C11 - 2.6 - Solve ASS Triangle Without Sine Law Notes

Solve the triangle with side lengths of 6 m and 10 m, and an angle between the two given sides of 57^o.









$$sin\theta = \frac{O}{H}$$
$$sin57^o = \frac{h}{6}$$
$$6 \times sin57^o = \frac{h}{\cancel{6}} \times \cancel{6}$$
$$6sin57^o = h$$
$$5.03 = h$$
$$h = 5.03$$

$$cos\theta = \frac{A}{H}$$
$$cos57^o = \frac{x}{6}$$
$$6 \times cos57^o = \frac{x}{\cancel{6}} \times \cancel{6}$$
$$6cos57^o = x$$
$$3.27 = x$$
$$x = 3.27$$

$$\alpha = 180^o - (57^o + 90^o)$$
$$\alpha = 180^o - 147^o$$
$$\alpha = 33^o$$

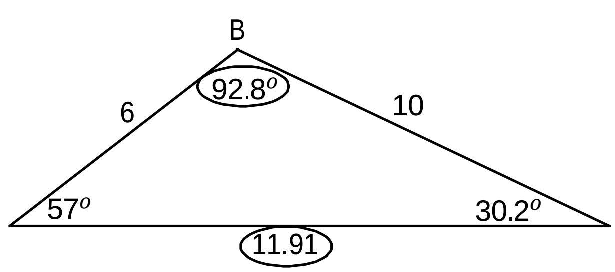
$$sin\theta = \frac{O}{H}$$
$$sin\theta = \frac{5.03}{10}$$
$$sin\theta = 0.503$$
$$\theta = \sin^{-1} 0.503$$
$$\theta = 30.2^o$$

$$cos\theta = \frac{A}{H}$$
$$\cos 30.2^o = \frac{y}{10}$$
$$0.864 = \frac{y}{10}$$
$$10 \times 0.864 = \frac{y}{\cancel{10}} \times \cancel{10}$$
$$8.64 = y$$
$$y = 8.64$$

$$\beta = 180^o - (30.2^o + 90^o)$$
$$\beta = 180^o - 120.2^o$$
$$\beta = 59.8^o$$

$$B = \alpha + \beta$$
$$= 33^o + 59.8^o$$
$$= 92.8^o$$

$$b = x + y$$
$$b = 3.27 + 8.64$$
$$b = 11.91$$

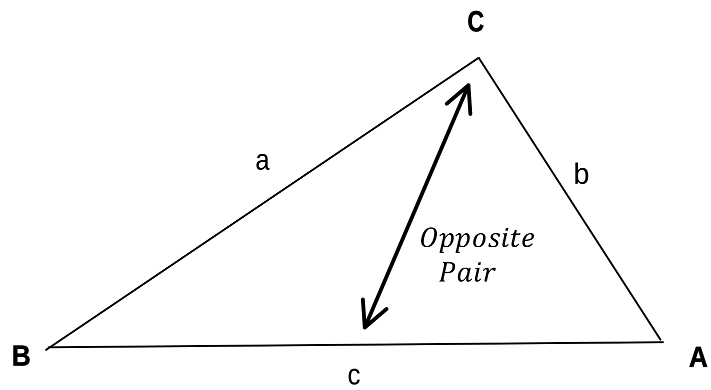


C11 - 2.6 - Sine Law Notes

Sine Law: $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$ or $\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$

$(to\ find\ a\ side)$ $(to\ find\ an\ angle)$

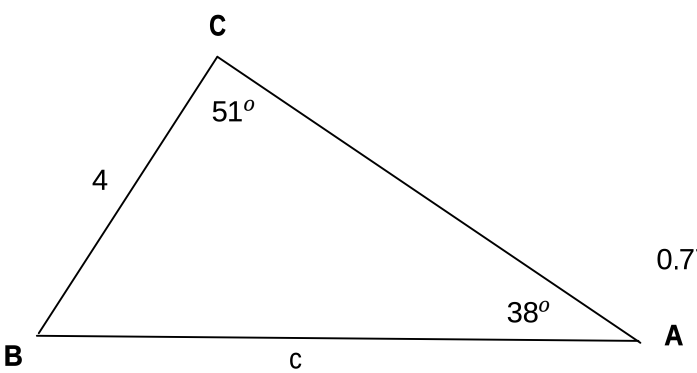
$\boxed{What\ you\ are\ looking\ for\ goes\ on\ top\ but\ algebra\ allows\ you\ to\ do\ either}$



$Notice$: $You\ may\ use\ the\ Sine\ Law\ if\ you\ have$:

$-An\ opposite\ pair$

$-And\ one\ other\ piece\ of\ information$



$$\frac{a}{sinA} = \frac{c}{sinC}$$

$$\frac{4}{sin38^o} = \frac{c}{sin51^o}$$

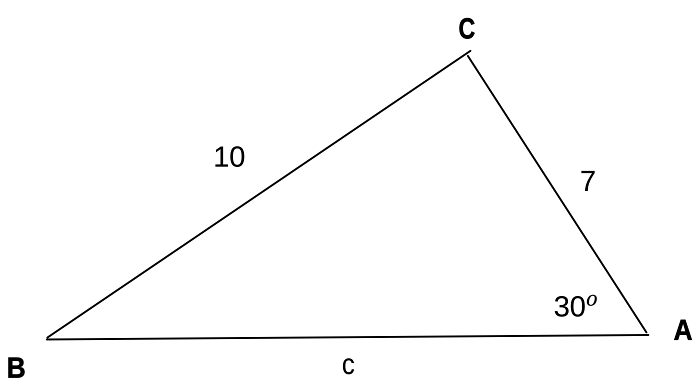
$$6.497 = \frac{c}{0.777}$$

$$0.777 \times 6.497 = \cancel{\frac{c}{0.777} \times 0.777}$$

$$5.048 = c$$

$$c = 5.048$$

$$\boxed{\begin{aligned}\frac{a}{sinA} &= \frac{c}{sinC}\\ \frac{asinC}{sinA} &= c\end{aligned}}$$



$$\frac{sinA}{a} = \frac{sinB}{b}$$

$$\frac{\sin(30)}{10} = \frac{sinB}{7}$$

$$0.05 = \frac{sinB}{7}$$

$$7 \times .05 = \cancel{\frac{sinB}{7} \times 7}$$

$$0.35 = \text{sinB}$$

$$\text{sinB} = 0.35$$

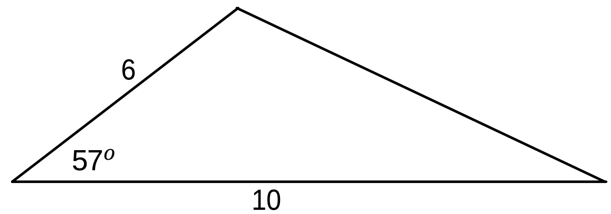
$$\text{B} = \sin^{-1}(0.35)$$

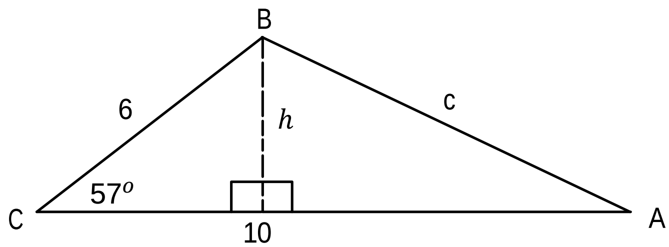
$$B = 20.5^o$$

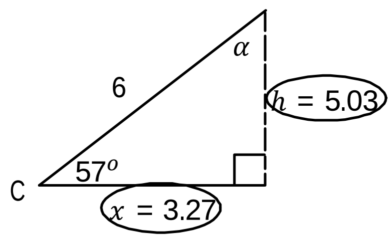
Remember: If you have 2 angles without either opposite side, you may always use 180^o in a triangle.

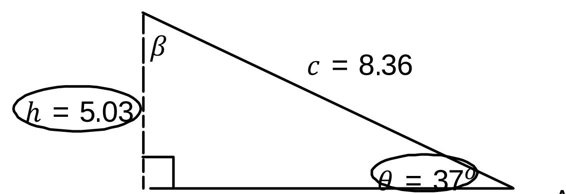
C11 - 2.7 - Solve SAS Triangle Without Cosine Law Notes

Solve the triangle with side lengths of 6 m and 10 m, and an angle between the two given sides of 57^o.









$10 - x =$
$10 - 3.27 = 6.73$

$$\begin{aligned} sin\theta &= \frac{O}{H} \\ sin57^o &= \frac{h}{6} \\ 6 \times sin57^o &= \frac{h}{\cancel{6}} \times \cancel{6} \\ 6sin57^o &= h \\ 5.03 &= h \\ h &= 5.03 \end{aligned}$$

$$\begin{aligned} cos\theta &= \frac{A}{H} \\ cos57^o &= \frac{x}{6} \\ 6 \times cos57^o &= \frac{x}{\cancel{6}} \times \cancel{6} \\ 6cos57^o &= x \\ 3.27 &= x \\ x &= 3.27 \end{aligned}$$

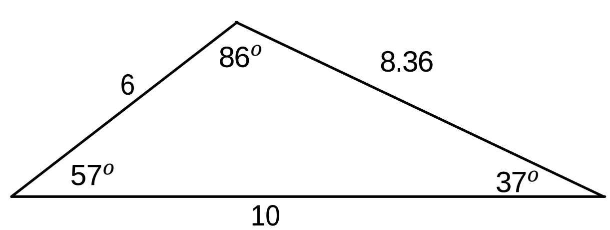
$$\begin{aligned} tan\theta &= \frac{O}{A} \\ tan\theta &= \frac{5.03}{6.73} \\ tan\theta &= 0.7474 \\ \theta &= \tan^{-1}(0.7474) \\ \theta &= 36.77^o \\ \theta &= 37^o \end{aligned}$$

$$\begin{aligned} sin\theta &= \frac{O}{H} \\ sin37 &= \frac{5.03}{c} \\ c \times sin37 &= \frac{5.03}{\cancel{c}} \times \cancel{c} \\ csin37 &= 5.03 \\ \frac{csin\cancel{37}}{\cancel{sin}37} &= \frac{5.03}{sin37} \\ c &= \frac{5.03}{sin37} \\ c &= 8.36 \end{aligned}$$

$$\begin{aligned} 57^o + 90^o + \alpha &= 180^o \\ 147^o + \alpha &= 180^o \\ -147^o \qquad &\ \ -147^o \\ \alpha &= 33^o \end{aligned}$$

$$\begin{aligned} 37^o + 90^o + \beta &= 180^o \\ 127^o + \beta &= 180^O \\ -127^o \qquad &\ \ -127^o \\ \beta &= 53^o \end{aligned}$$

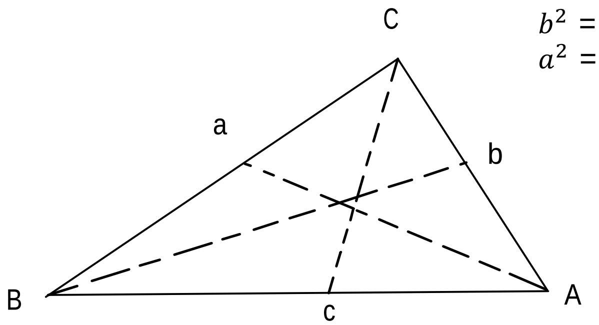
$$\begin{aligned} B &= \alpha + \beta \\ &= 33^o + 53^o \\ &= 86^o \end{aligned}$$



Remember: Find the smallest angle first, and/or 180 minus

C11 - 2.7 - Cosine Law Notes

Cosine Law



$c^2 = b^2 + a^2 - 2abcosC$
$b^2 = c^2 + a^2 - 2cacosB$
$a^2 = b^2 + c^2 - 2cbcosA$

Cosine Law:

$c^2 = b^2 + a^2 - 2abcosC$

Notice: This pattern should occur.

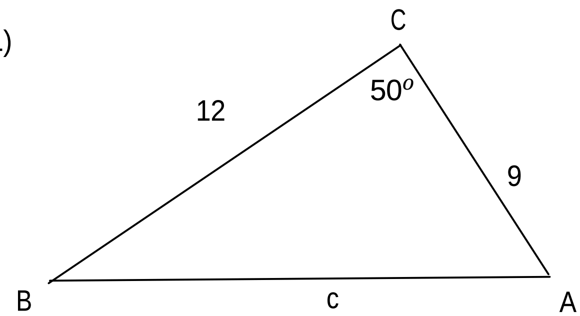
Cosine Law: SSS (hard) and SAS (easy)

Remember: Only one angle in the formula

Remember: We only cos angles.

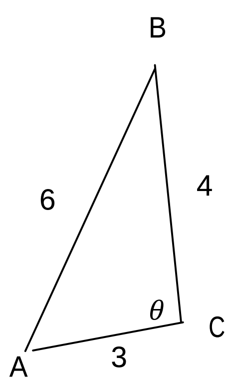
---

1)



$c^2 = b^2 + a^2 - 2abcosC$
$c^2 = 9^2 + 12^2 - 2(12)(9)cos50$ Plug into calculator
$c^2 = 86.2$
$\sqrt{c^2} = \sqrt{86.2}$ Square root both sides
$c = 9.3$

2)



$c^2 = b^2 + a^2 - 2abcosC$
$6^2 = 3^2 + 4^2 - 2(4)(3)cosC$ Substitute values in
$36 = 9 + 16 - 24cosC$ Calculate the squares, multiply
$36 = 25 - 24cosC$ Add

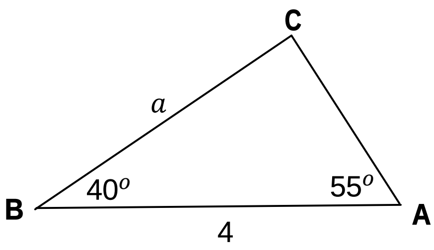
$36 = 25 - 24cosC$
$-25 \quad -25$ Subtract from both sides

$11 = -24cosC$
$\frac{11}{-24} = \frac{-24cosC}{-24}$ Divide both sides
$-\frac{11}{24} = cosC$
$cosC = -\frac{11}{24}$
$C = \cos^{-1}\left(-\frac{11}{24}\right)$ Inverse cos
$C = 117.3^o$

See edit below

C11 - 2.6/7 - Sine/Cosine Law Notes Solve the Triangle

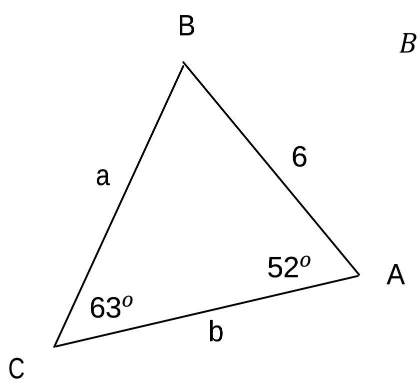
Solve for a.



$C = 180^o - 40^o - 55^o$
$\quad = 85^o$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{a}{sin55^o} = \frac{4}{sin85^o}$$
$$\frac{a}{0.819} = 4.015$$
$$\cancel{0.819} \times \frac{a}{\cancel{0.819}} = 4.015 \times 0.819$$
$$a = 3.289$$

Solve the triangle.

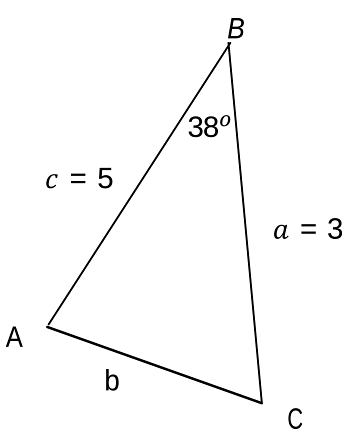


$B = 180^o - 63^o - 52^o$
$\quad = 65^o$

$$\frac{a}{sinA} = \frac{c}{sinC}$$
$$\frac{a}{sin52^o} = \frac{6}{sin63^o}$$
$$\frac{a}{0.788} = 6.734$$
$$\cancel{0.788} \times \frac{a}{\cancel{0.788}} = 6.734 \times 0.788$$
$$a = 6.734 \times 0.788$$
$$a = 5.306$$

$$\frac{b}{sinB} = \frac{c}{sinC}$$
$$\frac{b}{sin65^o} = \frac{6}{sin63^o}$$
$$\frac{b}{0.906} = 6.734$$
$$\cancel{0.906} \times \frac{b}{\cancel{0.906}} = 6.734 \times 0.906$$
$$b = 6.101$$

Solve the triangle *Find the angle opposite of the smaller side 1st.



Cosine Law: Switched b and c

$$c^2 = a^2 + b^2 - 2abcosC$$
$$b^2 = a^2 + c^2 - 2ac \cdot cosB$$
$$b^2 = 3^2 + 5^2 - 2(3)(5) \cdot \cos(38^o)$$
$$b^2 = 9 + 25 - 30\cos(38^o)$$
$$b^2 = 34 - 23.64$$
$$b^2 = 10.36$$
$$\sqrt{b^2} = \sqrt{10.36}$$
$$b = 3.22$$

Sine Law:

$$\frac{sinA}{a} = \frac{sinB}{b}$$
$$\frac{sinA}{3} = \frac{sin38^o}{3.22}$$
$$\frac{sinA}{3} = 0.19$$
$$3 \times \frac{sinA}{3} = 0.19 \times 3$$
$$sinA = 0.57$$
$$A = 35^o$$

180^o in a triangle:

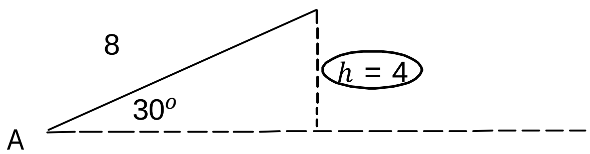
$C = 180^o - 38^o - 35^o$
$\quad = 107^o$

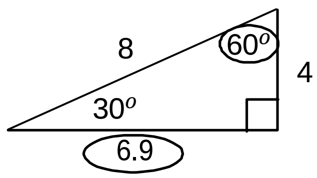
C11 - 2.6 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

Remember: Always find the height first.

$\angle A = 30^o, b = 8, a = 4$





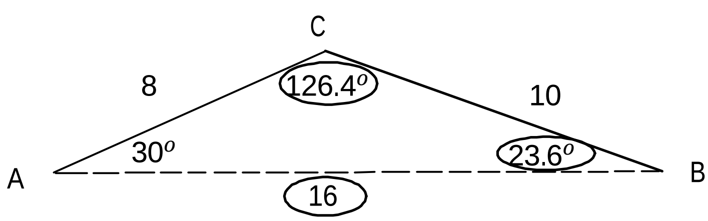
$$\sin\theta = \frac{O}{H}$$
$$\sin 30^o = \frac{h}{8}$$
$$8\sin 30^o = h$$
$$4 = h$$
$$h = 4$$

$$cos\theta = \frac{A}{H}$$
$$cos30^o = \frac{A}{8}$$
$$8\cos 30^o = A$$
$$6.9 = A$$
$$A = 6.9$$

$a = h$
One triangle

$$\theta = 180^o - 30^o - 90^o$$
$$\theta = 60^o$$

$\angle A = 30^o, b = 8, a = 10$



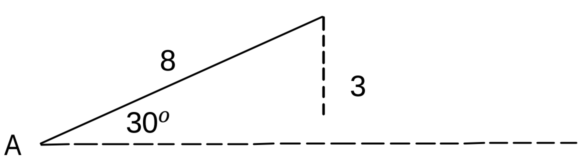
$10 > 8$
$a > b$
One triangle

$$\frac{sinB}{b} = \frac{sinA}{a}$$
$$\frac{sinB}{8} = \frac{\sin 30^O}{10}$$
$$\frac{sinB}{8} = 0.05$$
$$8 \times \frac{sinB}{8} = 0.05 \times 8$$
$$sinB = 0.4$$
$$B = \sin^{-1} 0.4$$
$$B = 23.6^o$$

$$\theta = 180^o - 23.6^o - 30^o$$
$$\theta = 126.4^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 126.4^o} = \frac{10}{\sin 30^o}$$
$$\frac{c}{0.8} = 20$$
$$0.8 \times \frac{c}{0.8} = 20 \times 0.8$$
$$c = 16$$

$\angle A = 30^o, b = 8, a = 3$



$3 < 4$
$a < H$
no triangle

No triangle, can't solve.

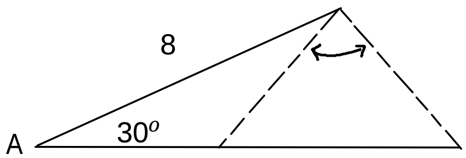
C11 - 2.6 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

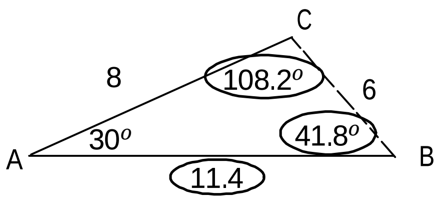
Remember: Always find the height first.

$\angle A = 30^o, b = 8, a = 6$

4 < 6 < 8
$H < a < B$
Two triangles



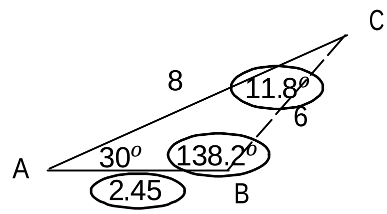
Draw both triangles together and separately.



$$\frac{\sin 30^o}{6} = \frac{\sin B}{8}$$
$$0.08\bar{3} = \frac{sinB}{8}$$
$$8 \times 0.08\bar{3} = \frac{sinB}{8} \times 8$$
$$0.\bar{6} = sinB$$
$$sinB = 0.\bar{6}$$
$$B = \sin^{-1} 0.\bar{6}$$
$$B = 41.8^o$$

$$\theta = 180^o - 30^o - 41.8^o$$
$$\theta = 108.2^o$$

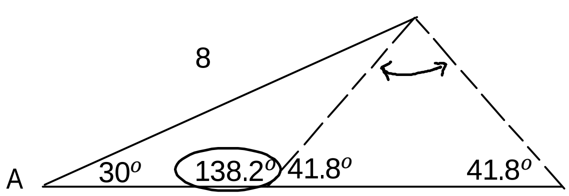
$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 108.2^o} = \frac{6}{\sin 30^o}$$
$$\frac{c}{0.95} = 12$$
$$0.95 \times \frac{c}{0.95} = 12 \times 0.95$$
$$c = 11.4$$



$$\theta = 180^o - 41.8^o$$
$$\theta = 138.2^o$$

$$\theta = 180^o - 30^o - 138.2^o$$
$$\theta = 11.8^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 11.8^o} = \frac{6}{\sin 30^o}$$
$$\frac{c}{0.204} = 12$$
$$0.204 \times \frac{c}{0.204} = 12 \times 0.204$$
$$c = 2.45$$



Notice: Both triangles have an angle of 30^o, a side going up of 8, and a side opposite to 30^o of 6.

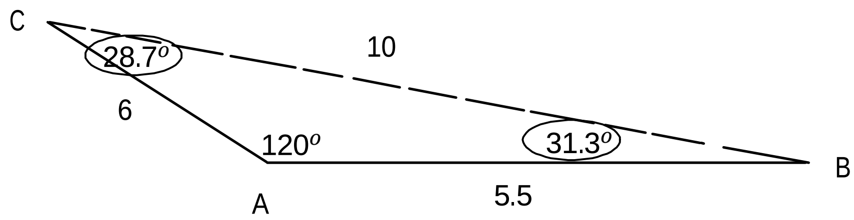
Notice: The isosceles triangle.

C11 - 2.6 - Ambiguous Case of Sine (ASS) Notes

How many triangles? Solve the triangles.

$\angle A = 120^o, b = 6, a = 10$

$10 > 6$
$a > b$
<u>One triangle</u>



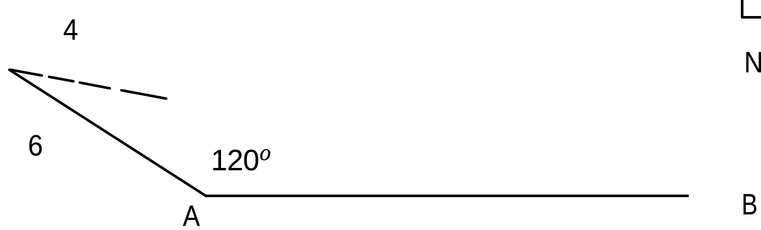
$$\frac{sinB}{b} = \frac{sinA}{a}$$
$$\frac{sinB}{6} = \frac{\sin 120^o}{10}$$
$$\frac{sinB}{6} = 0.0866$$
$$6 \times \frac{sinB}{6} = 0.0866 \times 6$$
$$sinB = 0.52$$
$$B = \sin^{-1} 0.52$$
$$B = 31.3^o$$

$$\theta = 180^o - 31.3^o - 120^o$$
$$\theta = 28.7^o$$

$$\frac{c}{sinC} = \frac{a}{sinA}$$
$$\frac{c}{\sin 28.7^o} = \frac{10}{\sin 120^o}$$
$$\frac{c}{0.48} = 11.55$$
$$0.48 \times \frac{c}{0.48} = 11.55 \times 0.48$$
$$c = 5.5$$

$\angle A = 120^o, b = 6, a = 4$

$4 < 6$
$a < b$
<u>No triangle</u>



No triangle. Can't solve.