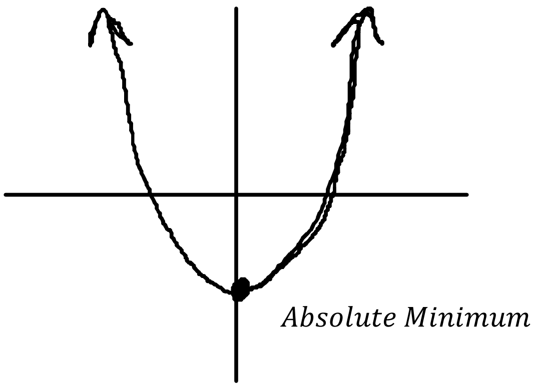
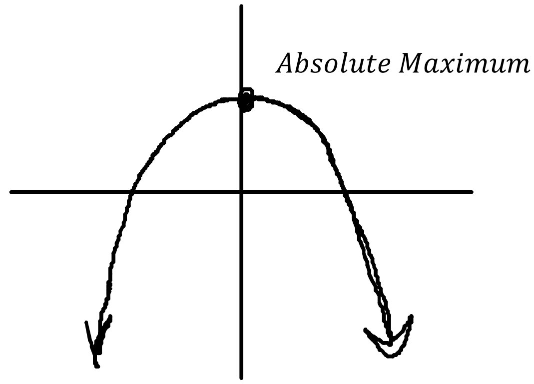
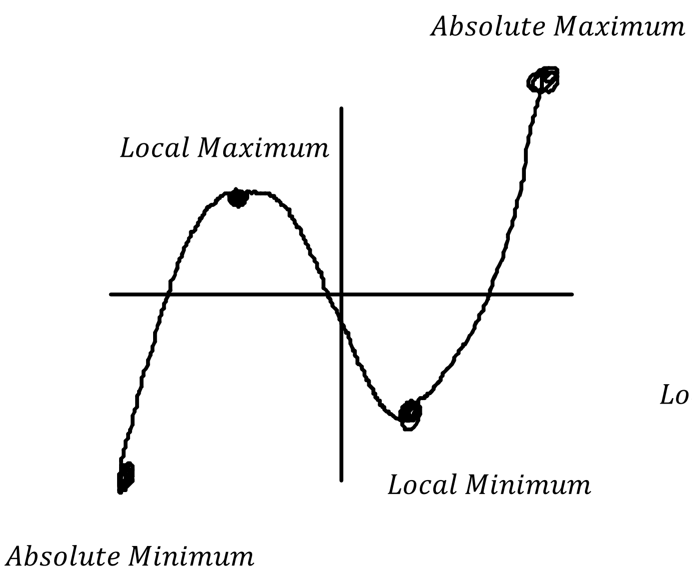
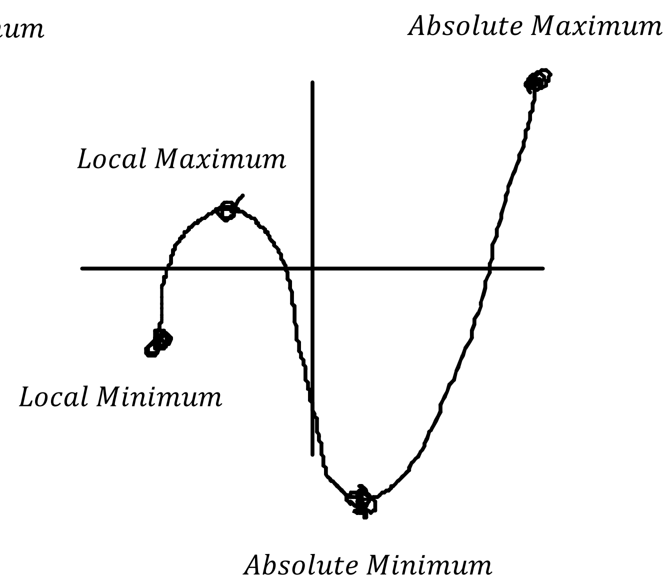
C12 - 3.1 - Absolute/Local Max/Min Notes





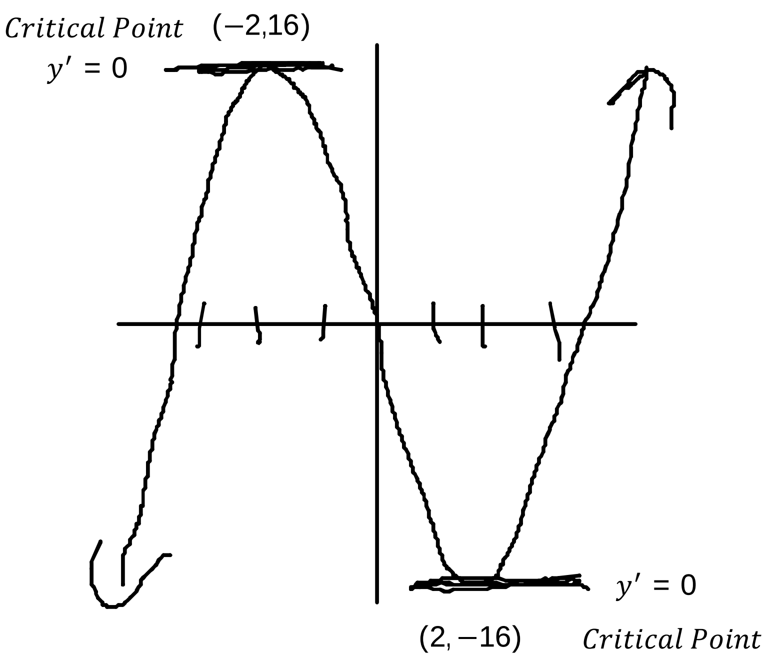




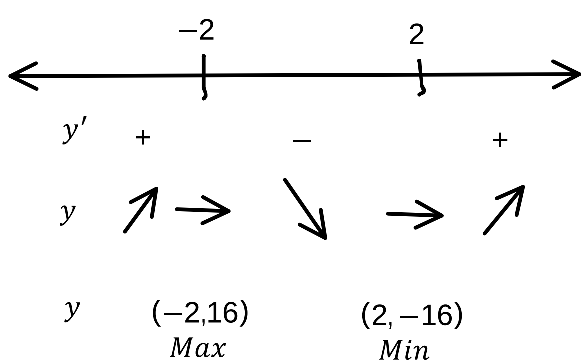
C12 - 3.1 - Critical Points Notes

Find the critical points. Find the derivative and set it equal to zero. Draw a graph and show the location of the horizontal slopes.

$y = x^3 - 12x$	
$y' = 3x^2 - 12$	Find the derivative
$0 = 3x^2 - 12$	Set the derivative equal to zero
$3x^2 = 12$	
$x^2 = 4$	
$x = \pm 2$	Solve



Prove derivative is positive to the left of -2. Negative between -2 and 2. And positive to the right of 2.



C12 - 3.2 - Curve Sketching Notes

$$\boxed{y = x^3 + 12x^2 + 36x}$$

Domain:

Vertical Asymptotes (VA): None $x \in R$

Range:

Horizontal Asymptotes (HA): None $y \in R$

$$\lim_{x\to\infty} x^3 + 12x^2 + 36x = \infty$$

$$\lim_{x\to-\infty} x^3 + 12x^2 + 36x = -\infty$$

$$y' = 3x^2 + 24x + 36 = 0$$
$$x^2 + 8x + 12 = 0$$
$$(x+6)(x+2) = 0$$
$$x = -6, -2$$

	-10	-6	-4	-2	0
y'	$+$	0	$-$	0	$+$
y	↗	→	↘	→	↗

Sign Analysis

$$y' = (x+6)(x+2)$$

$$y'(-10) = (-)(-) = +$$
$$y'(-4) = (+)(-) = -$$
$$y'(0) = (+)(+) = +$$

Critical Points (CP): y $(-6, 0)$ *Max:* $(-2, -32)$ *Min:*

$$y = x^3 + 12x^2 + 36x$$
$$y = (-6)^3 + 12(-6)^2 + 36(-6)$$
$$y = 0$$

$$y = x^3 + 12x^2 + 36x$$
$$y = (-2)^3 + 12(-2)^2 + 36(-2)$$
$$y = -32$$

Intervals of Increase/Decrease

Increasing: $(-\infty, -6)$, $(-2, \infty)$
Decreasing: $(-6, -2)$

$$y'' = 6x + 24 = 0$$
$$6(x+4) = 0$$
$$x = -4$$

	-10	-4	0
y''	$-$	0	$+$
y	∩		∪

Sign Analysis

$$y'' = 6(x+4)$$

$$y''(-10) = +(-) = -$$
$$y''(0) = +(+) = +$$

Inflection Point (IP): y $(-4, -16)$ *(IP)*

$$y = x^3 + 12x^2 + 36x$$
$$y = (-4)^3 + 12(-4)^2 + 36(-4)$$
$$y = -16$$

Intervals of Concavity

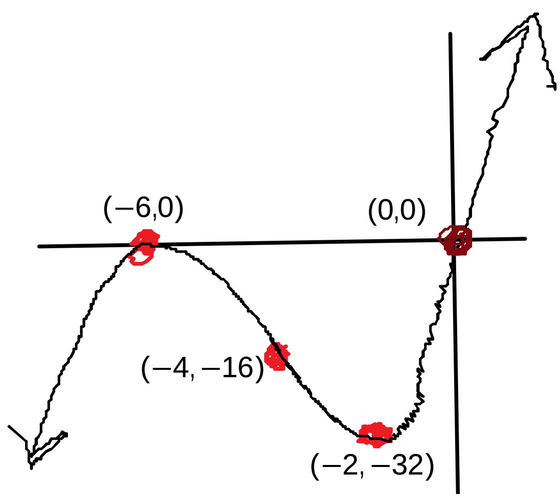
Concave Up: $(-4, \infty)$
Concave Down: $(-\infty, -4)$

x − intercepts

$$y = x^3 + 12x^2 + 36x$$
$$0 = x^3 + 12x^2 + 36x$$
$$0 = x(x^2 + 12x + 36)$$
$$0 = x(x+6)(x+6)$$

$$x = 0, -6$$

$(0,0)$ $(-6,0)$



y − intercepts

$$y = x^3 + 12x^2 + 36x$$
$$y = (0)^3 + 12(0)^2 + 36(0)$$
$$y = 0$$

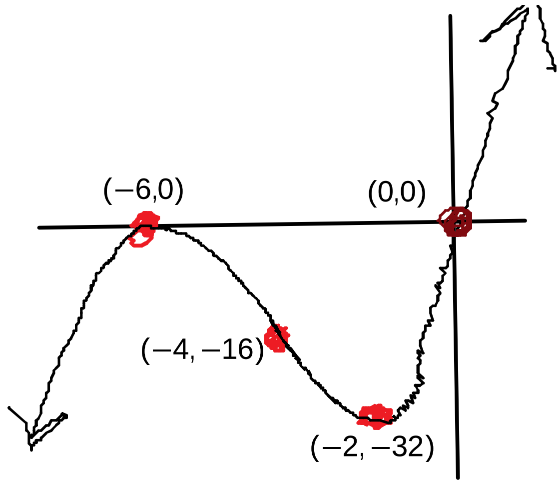
$(0,0)$

$x - intercepts$

$y = x^3 + 12x^2 + 36x$
$0 = x^3 + 12x^2 + 36x$
$0 = x(x^2 + 12x + 36)$
$0 = x(x + 6)(x + 6)$

$x = 0, -6$

(0,0) (−6,0)



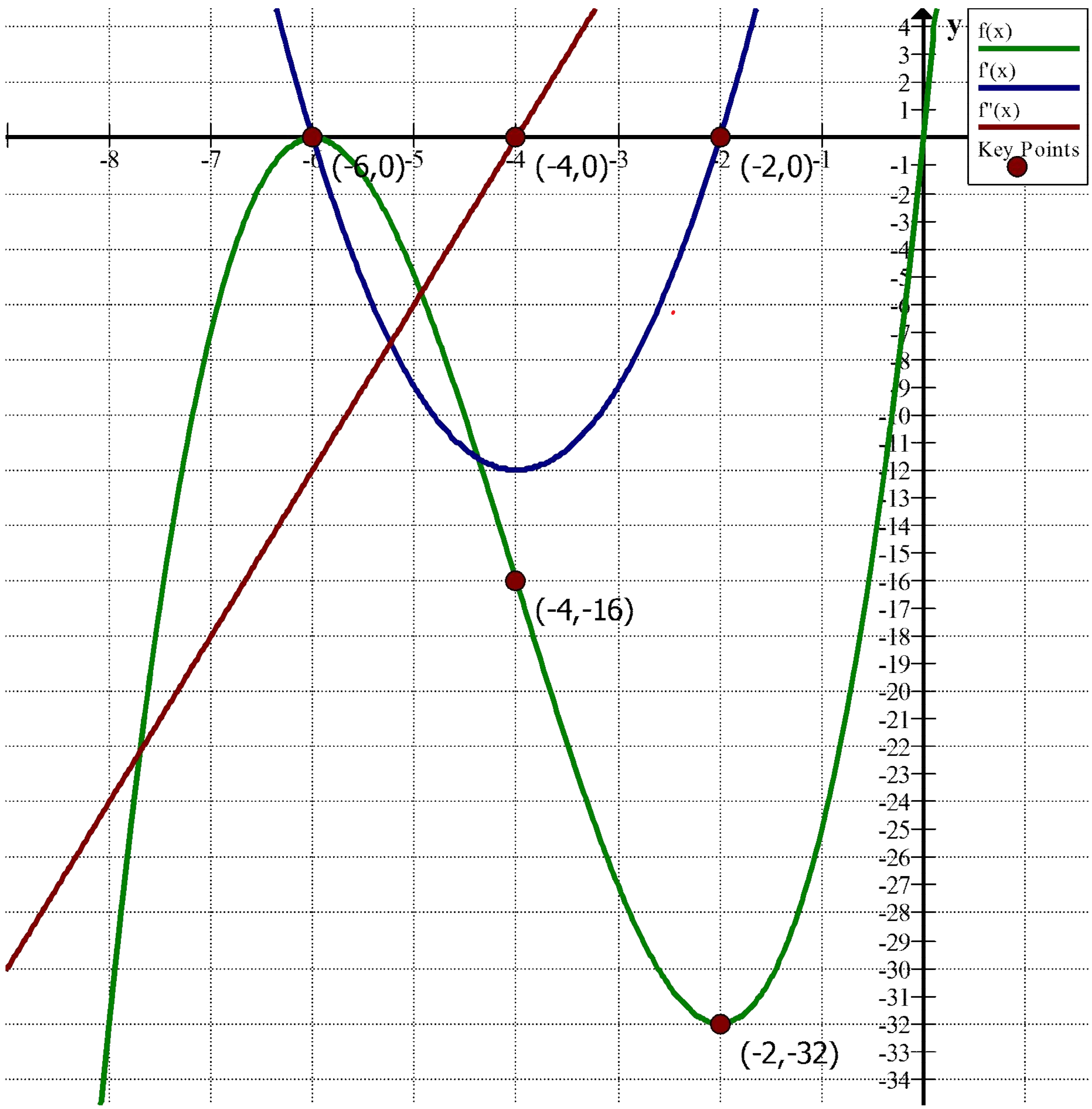
$y - intercepts$

$y = x^3 + 12x^2 + 36x$
$y = (0)^3 + 12(0)^2 + 36(0)$
$y = 0$

(0,0)

C12 - 3.2 - Curve Sketching Notes

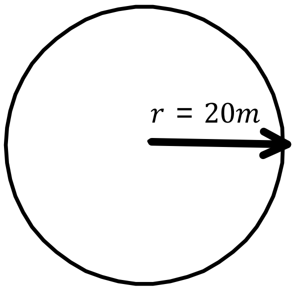
Sketch the function: $y = x^3 + 12x^2 + 36x$



C12 - 3.3 - Circle/Sphere Related Rates Notes

Find the rate of change.

The radius of a circle is growing at a rate of 4 m/s. What is the rate at which the area within the circle changing when the radius is 20m?



$\frac{dr}{dt} = 4$

$\frac{dA}{dt}\Big|_{r=20} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi r \cdot (4)$

$\frac{dA}{dt} = 8\pi r$

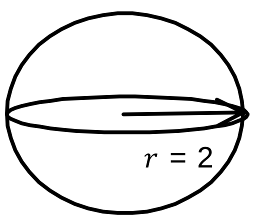
$= 8\pi(20)$

$= 160\pi$

$\frac{dA}{dt} = 160\pi \frac{m}{s^2}$

Therefore the area is changing at a rate of $160\pi \frac{m^2}{s}$ when the radius is $20m$.

The volume of a balloon is increasing at 4 meters cubed per second. How fast is the radius increasing when the radius is two meters?



$\frac{dV}{dt} = 256 \frac{m}{s^3}$

$\frac{dr}{dt}\Big|_{r=2} = ?$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 3 \times \frac{4}{3}\pi r^{3-1} \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$256 = 4\pi(2)^2 \frac{dr}{dt}$

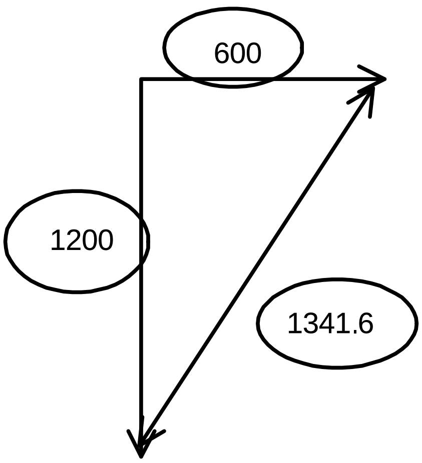
$\frac{dr}{dt} = \frac{16}{\pi}\frac{m}{s}$

Therefore the radius is changing at $\frac{16}{\pi}\frac{m}{s}$ when the radius is 2 m.

C12 - 3.3 - Train Pythag Related Rates Notes

Find the rate of change.

Train 'a' leaves Vancouver heading South at 10 m/s and train 'b' leaves heading East at 5 m/s? How far are they a part after two minutes? What is the speed at which the trains are moving apart at that time?



$\frac{da}{dt} = 10$ $\frac{db}{dt} = 5$ $\left.\frac{dc}{dt}\right|_{t=2} = ?$

$$a^2 + b^2 = c^2$$
$$1200^2 + 600^2 = c^2$$
$$c = 1341.6$$

$$a^2 + b^2 = c^2$$
$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$
$$2(1200)(10) + 2(600)(5) = 2(1341.6)\frac{dc}{dt}$$
$$30000 = 2683.2\frac{dc}{dt}$$
$$\frac{dc}{dt} = 11.1\frac{m}{s}$$

$2\,minutes = 120\,seconds$

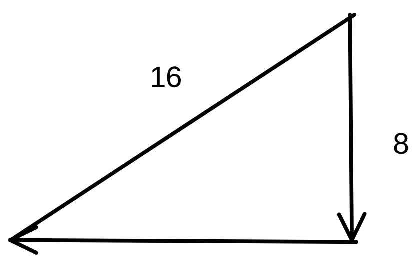
$$a = vt$$
$$a = 10 \times 120$$
$$a = 1200$$

$$b = vt$$
$$b = 5 \times 120$$
$$b = 600$$

C12 - 3.3 - Ladder Trig Related Rates Notes

Find the rate of change.

The top of a 16 ft ladder slides down a wall at a rate of 3 ft/s. At what rate is the base of the ladder sliding away from the wall when the latter is at a height of 8 ft on the wall.



$$\frac{dy}{dt} = -3\frac{ft}{s}$$

*Length is shrinking: Derivative is Negative.

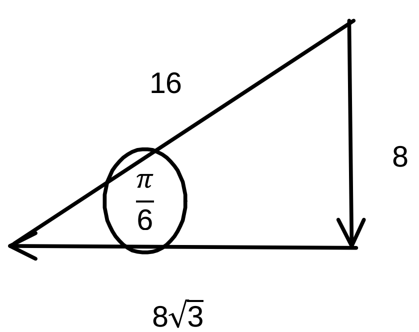
$$\left.\frac{dx}{dt}\right|_{y=8} = ?$$

$$x^2 + y^2 = c^2$$
$$x^2 + 8^2 = 16^2$$
$$x = \sqrt{16^2 - 8^2}$$
$$x = \sqrt{192}$$
$$x = 8\sqrt{3}$$

$$x^2 + y^2 = c^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 16^2$$
$$2(8\sqrt{3})\frac{dx}{dt} + 2(8)(-3) = 0$$
$$\frac{dx}{dt} = \frac{3}{\sqrt{3}}$$
$$\frac{dx}{dt} = \sqrt{3}\frac{ft}{s}$$

*We can substitue constants into the formula

What is the rate the angle at the bottom of the ladder changing?



$$sin\theta = \frac{8}{16}$$
$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$
$$\theta = \frac{\pi}{6}$$

*Real life is in Radians. Degrees are for wimps.

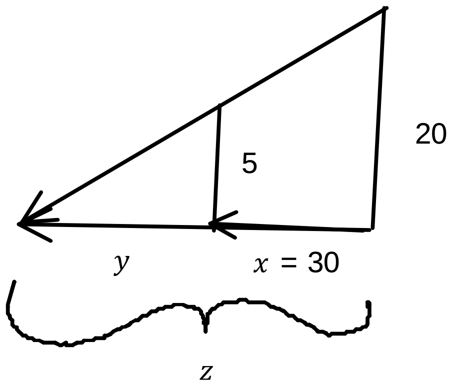
$$cos\theta = \frac{x}{r}$$
$$cos\theta = \frac{x}{16}$$
$$-sin\theta\frac{d\theta}{dt} = \frac{1}{16}\frac{dx}{dt}$$
$$-\frac{8}{16}\frac{d\theta}{dt} = \frac{1}{16}\sqrt{3}$$
$$\frac{d\theta}{dt} = -\frac{\sqrt{3}}{8}\frac{rad}{s}$$

*I used cos because it used the rate I already solved on the top. Using sin and tan is possible but much more difficult based on the information and previously solved.

C12 - 3.3 - Similar Triangles/Cos Law Related Rates Notes

Find the rate of change.

At 5 foot tall woman is walking away from a 20 foot lamp post at 3 m/s. What is the rate of her shadow when she is 30 feet from the lamp post; and is her shadow getting bigger or smaller.

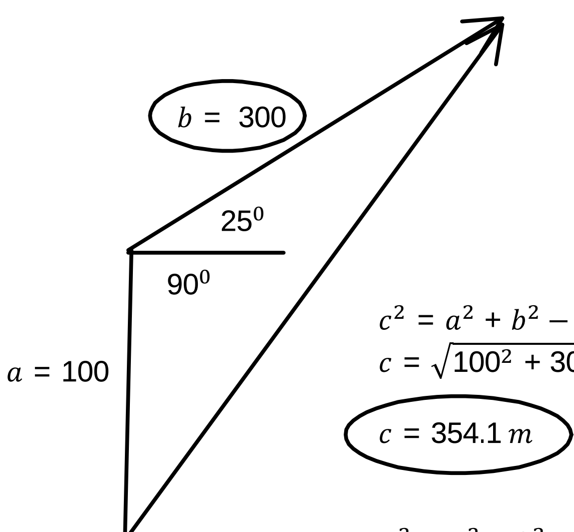


$\frac{dx}{dt} = 3\frac{m}{s}$ $\qquad$ $\left.\frac{dz}{dt}\right|_{x=30} = ?$

$$\frac{5}{20} = \frac{y}{x+y}$$
$$5x + 5y = 20y$$
$$5x = 15y$$
$$x = 3y$$
$$\frac{dx}{dt} = 3\frac{dy}{dt}$$
$$3 = 3\frac{dy}{dt}$$
$$\frac{dy}{dt} = 1\frac{ft}{s}$$

$$x + y = z$$
$$\frac{dx}{dt} + \frac{dy}{dt} = \frac{dz}{dt}$$
$$3 + 1 = \frac{dz}{dt}$$
$$\frac{dz}{dt} = 4\frac{ft}{s}$$

A float plane rising at 25° above the horizontal flies over a boat at an altitude of 100 m at 60 m/s. How fast is the distance between the boat and the plane increasing after five seconds?



$\frac{db}{dt} = 60$ $\qquad$ $\left.\frac{dc}{dt}\right|_{t=5} = ?$

$$v = \frac{d}{t}$$
$$d = vt$$
$$d = 60 \times 5$$
$$d = 300\,m$$

$$c^2 = a^2 + b^2 - 2abcosC$$
$$c = \sqrt{100^2 + 300^2 - 2(100)(300)cos115}$$
$$c = 354.1\,m$$

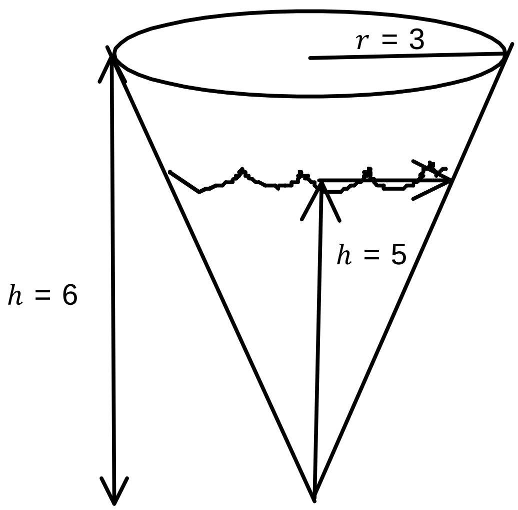
$$c^2 = a^2 + b^2 - 2abcosC$$
$$2c\frac{dc}{dt} = 0 + 2b\frac{db}{dt} - 2acosC\frac{db}{dt}$$
$$2(354.1)\frac{dc}{dt} = 0 + 2(300)(60) - 2(100)(-0.4226)(60)$$
$$\frac{dc}{dt} = 57.99\frac{m}{s}$$

*That would have been a tough product rule if more things were changing

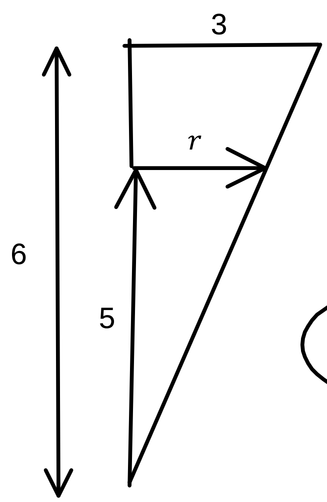
C12 - 3.3 - Cone/Sim Tri/Cos Law Related Rates Notes

Find the rate of change.

A cone with a radius of 3 cm and height of 6 cm is filling with water with the height of the water level is increasing at a rate of 0.2 cm/s. What is the rate the volume is increasing when the height of the water is level 5 cm.



$$\frac{dh}{dt} = 0.2$$



$$\frac{h}{6} = \frac{r}{5}$$

$$r = \frac{5h}{6}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{5h}{6}\right)^2 h$$

$$V = \frac{25}{108}\pi h^3$$

$$\frac{dV}{dt} = 3 \times \frac{25}{108}\pi h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 3\pi \times \frac{25}{108}\pi(5)^2(0.2)$$

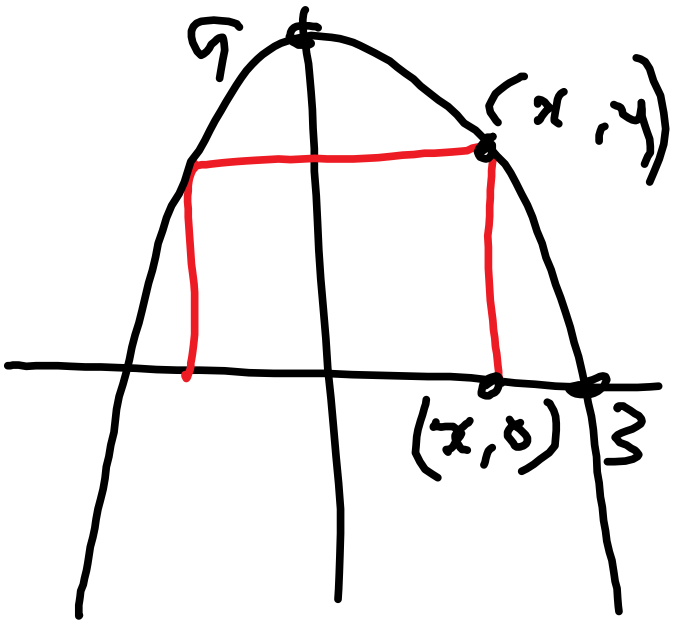
$$\frac{dV}{dt} = \frac{125\pi}{36}\frac{cm}{s}$$

*We can't take this product so we must use similar triangles

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + \frac{dh}{dt}r^2\right)$$

C12 - 3.4 - Max Area Rectangle under Parabola

$y = 9 - x^2$



$A = lw$

$A = 2xy$

$A = 2x(9 - x^2)$

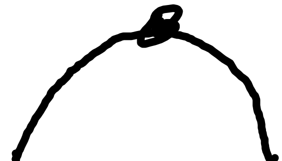
$\boxed{A = 12\sqrt{3}}$

$A' = 18 - 6x^2$

$0 = 3 - x^2$

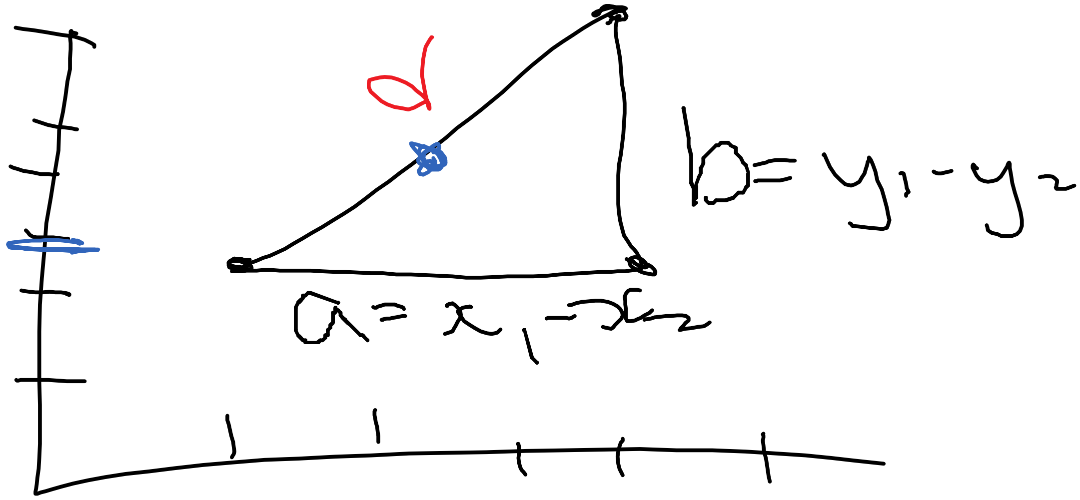
$\boxed{x = \pm\sqrt{3}}$

1		2
+	$\sqrt{3}$	−



max ✓

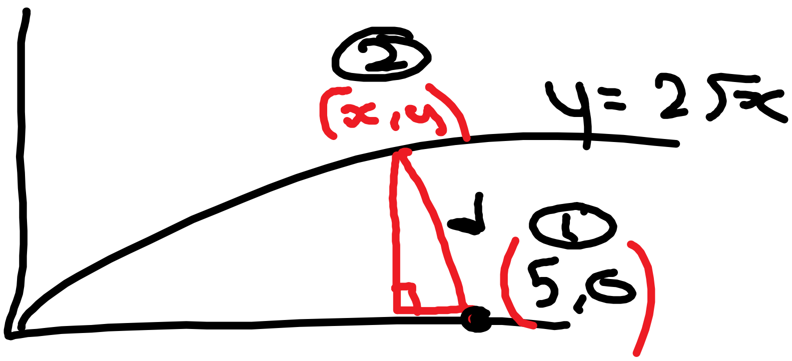
C12 - 3.4 - Distance Formula Notes



$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$(x - 5$ $\quad$ $\cancel{y - 0}$ $\quad$ $(2\sqrt{x})^2$

$$d = (x^2 - 10x + 25 + 4x)^{1/2}$$

$$d = (x^2 - 6x + 25)^{1/2}$$

$$d' = \frac{1}{\cancel{2}}\left(\_\_\_\_\_\_\right)^{-1/2} \cdot (\cancel{2}x - \cancel{6}^{\,3})$$

$$0 = \cancel{d'} = \frac{x - 3}{\cancel{\sqrt{x^2 - 6x + 25}}}$$

$x = 3$

$(3, 2\sqrt{3})$

	1	3	4
d'	$-$		$+$
d	↘	min	↗

√