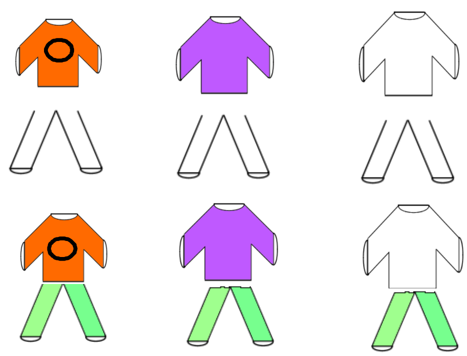
C12 - 11.1 - Fundamental Counting Principle Notes

Step 1: *a* choices
Step 2: *b* choices
Step 3: *c* choices

Total number of choices: $a \times b \times c$

Example: A person has 3 shirts and 2 pairs of pants. How many different outfits can they wear?

$3 \times 2 = 6$



Example: A woman has 4 pairs of shoes, 3 dresses and 5 hats. How many different outfits can she wear?

$4 \times 3 \times 5 = 60$

Example: A fashion designer has 4 different pairs of shoes, 3 different pairs of pants, 2 shirts, 5 necklaces, and 6 hats. How many different outfits can they prepare?

$4 \times 3 \times 2 \times 5 \times 6 = 720$

Example: How many 5 digit numbers are there?

10 $digits\ to\ choose\ from$: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$$\underset{1\text{-}9}{\underline{9}} \times \underset{0\text{-}9}{\underline{10}} \times \underset{0\text{-}9}{\underline{10}} \times \underset{0\text{-}9}{\underline{10}} \times \underset{0\text{-}9}{\underline{10}} = 90{,}000$$

A number can't start with a 0
i.e. 02345 = 2345, which is not a 5 digit number.

C12 - 11.2 - Factorials Notes

Factorial: The product of the consecutive numbers from n to 1.

Examples:

$5! = 5(4)(3)(2)(1) = 120$

$n! = n(n-1)(n-2)\ldots(3)(2)(1)$

$3! = 3 \times 2 \times 1$

$$\frac{7!}{4!} = \frac{7(6)(5)\cancel{(4)(3)(2)(1)}}{\cancel{4(3)(2)(1)}} = \frac{7(6)(5)}{1} = 7(6)(5) = 210$$

Factorials with numbers

$0! = 1$
$1! = 1$
$2! = 2 \times 1 = 2$
$3! = 3 \times 2 \times 1 = 6$
$4! = 4 \times 3 \times 2 \times 1 = 24$
$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$
$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320$

Factorials with variables

$n! = n(n-1)(n-2)(n-3)\ldots \times 3 \times 2 \times 1$

$(n-1)! = (n-1)(n-2)(n-3)\ldots \times 3 \times 2 \times 1$

$(n-2)! = (n-2)(n-3)\ldots \times 3 \times 2 \times 1$

$(n+2)! = (n+2)(n+1)(n)(n-1)\ldots \times 3 \times 2 \times 1$

$(n+1)! = (n+1)(n)(n-1)(n-2)\ldots \times 3 \times 2 \times 1$

You may close the factorial any time you want.

$6! = 6 \times 5 \times 4!$

$10! = 10 \times 9!$

$99! = 99 \times 98 \times 97!$

$$\frac{7!}{4!} = \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{4!}} = 7 \times 6 \times 5 = 210$$

$n! = n(n-1)(n-2)!$

$(n-1)! = (n-1)(n-2)(n-3)!$

$(n-2)! = (n-2)(n-3)(n-4)(n-5)!$

$(n+2)! = (n+2)(n+1)(n)!$

$(n+1)! = (n+1)(n)(n-1)!$

$$\frac{n!}{(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = n(n-1) = n^2 - n$$

Expand the bigger one

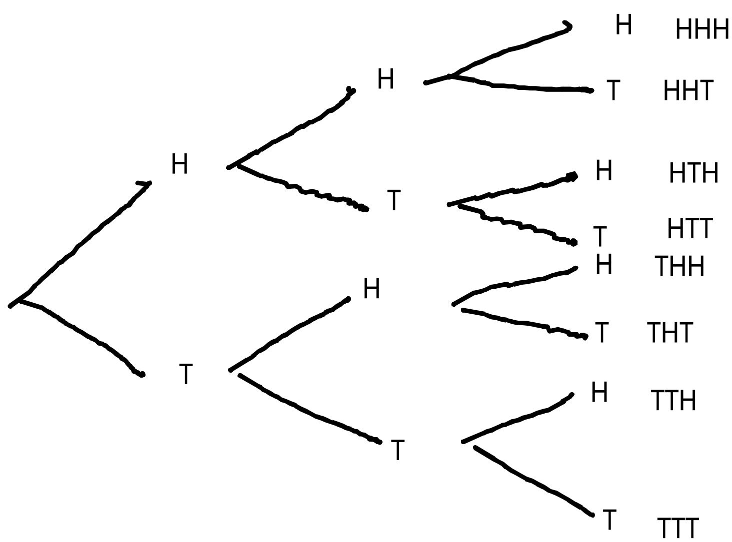
C12 - 11.3 - $(outcomes\ per\ trial)^{\#\ of\ trials}$ $Notes$

If you flip a coin three times what is the total number of outcomes? Draw a tree diagram to confirm.

$$\underset{H,T}{\underline{\quad 2 \quad}} \times \underset{H,T}{\underline{\quad 2 \quad}} \times \underset{H,T}{\underline{\quad 2 \quad}} = 2^3 = 8$$

$(outcomes\ per\ trial)^{\#\ of\ trials}$

$2^3 = 8$



If a test has 10 true and false questions how many answer keys are there possible?

$(outcomes\ per\ trial)^{\#\ of\ trials}$

$2^{10} = 1024$

If a test has A, B, C, D, multiple-choice answers with six questions how many answer keys are there possible?

$(outcomes\ per\ trial)^{\#\ of\ trials}$

$4^6 = 4096$

If a family has 8 children what is the number of combinations of boys and girls?

$(outcomes\ per\ trial)^{\#\ of\ trials}$

$2^8 = 256$

C12 - 11.4 - ABC nPr, n!; nCr Notes

Arranging All the Letters of ABC

No restrictions

$\underset{\text{Eg.(A, B or C)}}{\underline{3}} \times \underset{\text{Eg.(A, B or C)}}{\underline{3}} \times \underset{\text{Eg. (A, B or C)}}{\underline{3}} = 3^3 = 27$

27

AAA	AAB	ABA	BAA	ABC
BBB	AAC	ACA	CAA	ACB
CCC	BBA	BAB	ABB	BAC
	BBC	BCB	CBB	BCA
	CCA	CAC	ACC	CAB
	CCB	CBC	BCC	CBA

No repeats

Permutation *Particular Order matters*

$\underbrace{ABC \quad ACB \quad BAC \quad BCA \quad CBA \quad CAB}_{6}$

$\underset{\text{Eg.(A or B or C)}}{\underline{3}} \times \underset{\text{Eg.(A or C)}}{\underline{2}} \times \underset{\text{Eg. (C)}}{\underline{1}} = 3! = 6$

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 6$$

No repeats

Combination *Order doesn't matter*

$\underbrace{ABC = ACB = BAC = BCA = CBA = CAB}_{1}$

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$

$$_3C_3 = \frac{3!}{3!\,(3-3)!} = \frac{3!}{3! \times 0!} = \frac{3!}{3!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = 1$$

C12 - 11.4 - ABC @β nPr, n!, nCr Notes

Arranging Two of the Letters of ABC

No restrictions

$\underset{(A,\ B\ or\ C)}{\underline{\quad 3 \quad}} \times \underset{(A,\ B\ or\ C)}{\underline{\quad 3 \quad}} = 9$

AA AB AC CB
BB BA CA BC
CC

$\left.\right\}$ 9

No repeats

Permutation *Particular Order matters*

$\underset{(A\ or\ B\ or\ C)}{\underline{\quad 3 \quad}} \times \underset{(B\ or\ C)}{\underline{\quad 2 \quad}} = 6$

~~AA~~ AB AC CB
~~BB~~ BA CA BC
~~CC~~

$\left.\right\}$ 6

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

No repeats

Combination *Order doesn't matter*

$AB = BA$ $AC = CA$ $BC = CB$ $\left.\right\}$ 3

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$

$${}_3C_2 = \frac{3!}{2!\,(3-2)!} = \frac{3!}{2! \times 1!} = \frac{3!}{2!} = \frac{3 \times 2 \times 1}{2 \times 1} = \frac{6}{2} = 3$$

C12 - 11.4 - Combination ABC Cases Notes

Rearranging All the Letters of ABC two at a time

No restrictions

$$\underset{\text{(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{(A, B or C)}}{\underline{\quad 3 \quad}} = 9$$

Case 1: 2 same	Case 2: 2 different		
AA	AB	AC	CB
BB	BA	CA	BC
CC			

} 9

$Case\ 1\text{: } 2\ same$ + $Case\ 2\text{: } 2\ different$

${}_3C_1 + {}_3C_2 \times 2!$
$3 + 3 \times 2!$

$= 3 + 6$
$= 9$

Rearranging All the Letters of ABC

No restrictions

$$\underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(A, B or C)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg. (A, B or C)}}{\underline{\quad 3 \quad}} = 3^3 = 27$$

27

Case 1: 3 same	Case 2: 2 same 1 different			Case 3: 3 different
AAA	AAB	ABA	BAA	ABC
BBB	AAC	ACA	CAA	ACB
CCC	BBA	BAB	ABB	BAC
	BBC	BCB	CBB	BCA
	CCA	CAC	ACC	CAB
	CCB	CBC	BCC	CBA

$Case\ 1\text{: } 3\ same$ + $Case\ 2\text{: } 2\ same, 1\ different$ + $Case\ 3\text{: } 3\ different$

${}_3C_1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_2 \times 2 \times 1 \times 1 + {}_3C_3 \times 3!$
$3 + 3 \times 2 + 3 \times 2 + 3 \times 2 + 1 \times 3!$

$= 3 + 6 + 6 + 6$
$= 27$

C12 - 11.4 - 1,2,3 nPr, n! Notes

How many 3 digit numbers can we make from the numbers 1,2,3 with no repeats?

Permutation

Order matters

6
123
132
231
213
312
321

$$\underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 3)}}{\underline{\quad 2 \quad}} \times \underset{\text{Eg. (3)}}{\underline{\quad 1 \quad}} = 3! = 6$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

Permutation: $${}_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

How many 2 digit numbers can we make from the numbers 1,2,3 with no repeats?

Permutation

Order matters

6
12
21
13
31
23
32

$$\underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 3)}}{\underline{\quad 2 \quad}} = 6$$

$${}_nP_r = \frac{n!}{(n-r)!}$$

Permutation: $${}_3P_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = \frac{3!}{1} = 3! = 3 \times 2 \times 1 = 6$$

How many 3 digit numbers can we make from the numbers 1,2,3 with no restrictions?

27

111	112	221	331	123
222	113	223	332	132
333	121	212	313	231
	131	232	323	213
	122	211	311	321
	133	233	322	312

$$\underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} \times \underset{\text{Eg.(1 or 2 or 3)}}{\underline{\quad 3 \quad}} = 3^3 = 27$$

C12 - 11.4 - Cases 0,1,2,3 nPr, n! Notes

How many four digit numbers can we make from the numbers 0,1,2,3 with no restrictions?

3	×	4	×	4	×	4	$= 3 \times 4^3 = 192$
Eg. (1,2,3) NOT '0'		Eg. (0,1 ,2,3)		Eg. (0,1,2,3)		Eg. (0,1,2,3)	

How many 4 digit numbers can we make from the numbers 0,1,2,3 without repeating numbers?

Permutation: Order matters

18

1230	2130	3210
1203	2103	3201
1320	2310	3120
1302	2301	3102
1023	2013	3021
1032	2031	3012

3	×	3	×	2	×	1	$= 3 \times 3! = 18$
Eg. (1,2,3) NOT '0'		Eg. (0,2,3)		Eg. (0,3)		Eg. (3)	(Not a 4 digit number with the 0 first).

Permutation: ${}_3P_1 \times {}_3P_3 = 18$

How many 4 digit **EVEN** numbers can we make from the numbers 0,1,2,3 with no repeats?

Permutation: Order Matters

1230	+	1302	= 10	
1320		1032		
2130		3012		
2310		3102		
3120				
3210				

An even number must have a 0 or 2 last. If the 0 is last, we can use the 2 first. But, if we use the 2 last, the 0 cannot come first (Not a 4 digit number with the 0 first). Therefore, 2 cases.

Case 1:	3	×	2	×	1	×	1	= 6
	Eg. (1,2,3)		Eg. (2,3)		Eg. (3)		Eg. (0 or ~~2~~)	
Case 2:	2	×	2	×	1	×	1	= 4
	Eg. (1,3)		Eg. (0,3)		Eg. (0)		2	

6 + 4 = 10

Add cases.

If the last number affects the first numbers you can choose from, multiple cases.

C12 - 11.4 - President vs. Committee Notes

How many ways an you organize a committee of 3 people?

$\underset{1,2,3}{\underline{\quad 3 \quad}} \times \underset{1,2}{\underline{\quad 2 \quad}} \times \underset{1}{\underline{\quad 1 \quad}} = 3! = 6$

President Example:

A class is voting on a president, secretary and treasurer out of the 10 people running. How many different choices are there?

A president, secretary, and treasurer are all different positions. Particular Order matters.

$\underset{1\text{-}10}{\underline{\quad 10 \quad}} \times \underset{1\text{-}9}{\underline{\quad 9 \quad}} \times \underset{1\text{-}8}{\underline{\quad 8 \quad}} = 720$

$${}_nP_r = \frac{n!}{(n-r)!}$$
$${}_{10}P_3 = \frac{10!}{(10-3)!}$$
$${}_{10}P_3 = \frac{10!}{7!}$$
$${}_{10}P_3 = \frac{10 \times 9 \times 8 \times 7!}{7!}$$
$${}_{10}P_3 = 10 \times 9 \times 8$$
$${}_{10}P_3 = 720$$

Committee Example:

A class is voting on a committee of 3 people out of the 10 people running. How many different choices are there?

All people on a committee are equal. Order doesn't matter.

$${}_nC_r = \frac{n!}{r!\,(n-r)!}$$
$${}_{10}C_3 = \frac{10!}{3!\,(10-3)!}$$
$${}_{10}C_3 = \frac{10!}{3!\,(7)!}$$
$${}_{10}C_3 = \frac{10!}{3!\,7!}$$
$${}_{10}C_3 = \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1!\,7!}$$
$${}_{10}C_3 = \frac{720}{6}$$
$${}_{10}C_3 = 120$$

$${}_nC_r = \frac{{}_nP_r}{r!}$$
$${}_{10}C_3 = \frac{{}_{10}P_3}{3!}$$
$${}_{10}C_3 = \frac{720}{6}$$
$${}_{10}C_3 = 120$$

The number of ways you can choose a committee is the number of ways you can choose Pres, Vice, and Sec, divided by the number of ways you can organize 3 people.

$${}_nP_r = {}_nC_r \times r!$$
$${}_{10}P_3 = {}_{10}C_3 \times 3!$$
$${}_{10}P_3 = 120 \times 6$$
$${}_{10}P_3 = 720$$

The number of ways you can choose Pres, Vice, and Sec, is the number of ways you can choose 3 people from 10 multiplied by the number of ways you can organize 3 people

C12 - 11.4 - All Minus None Notes

We have three boys and four girls. 3 b's 4 g's

How many different ways can we make a group of three, with no restrictions?

$_7C_3 = 35$ Or $\frac{(3+4)!}{3!\,4!} = 35$

How many different ways can we make a group of three, with exactly two boys and one girl?

$_3C_2 \times {}_4C_1$ Choose two boys from 3 boys, and 1 girl from 4 girls.

How many different ways can we make a group of three, with at least one boy?

<u>Three cases:</u>

Case 1: 1 b, 2 g **Case 2:** 2 b, 1 g **Case 3:** 3 b, 0 g

$_3C_1 \times {}_4C_2 + {}_3C_2 \times {}_4C_1 + {}_3C_3 \times {}_4C_0$

$3 \times 6 + 3 \times 4 + 1 \times 1$

$18 + 12 + 1 = 31$

OR

$All - None$

$_7C_3 - {}_3C_0 \times {}_4C_3$

$35 - 1 \times 4$

$35 - 4 = 31$

(The total number of ways we can choose three people from seven minus a case with no boys)

Note: $_7C_3 = (Case{:}\,0\ boys) + (Case{:}\,1\ boy) + (Case{:}\,2\ boys) + (Case{:}\,3\ boys)$
$35 = 4 + 18 + 12 + 1$
$35 = 35$

We have 10 boys and 11 girls. How many different ways can we make a group of 10 with at least one boy?

$All - None$
$_{21}C_{10} - ({}_{10}C_0 \times {}_{11}C_{10}) = 352705$

We did this instead of adding the cases 1 boys, 2 boys, 3 boys, 4 boys, 5 boys, 6 boys, 7 boys, 8 boys, 9 boys, and 10 boys.

A lot of time it is easier to find out how many times something cant be done, than be done, so it is easier to find the total number of ways you can do something and minus the ways you cant do it.

A family of 5 takes a family photo. How many ways can the parents not sit together? Answer. The total number of ways the family can sit with no restrictions, Minus the number of ways they can sit Together. Think about it! Very Useful!

$$\frac{(\# \; of \; letters)!}{(repeating \; letter)! \, (other \; repeating \; letter)! \ldots}$$

C12 - 11.4 - Identical Objects Notes

How many different words can we make from the letters POLE ?

$4! = 24$

POLE	OLEP	EPOL	LOPE
PELO	OLPE	EPLO	LEPO
PLEO	OPLE	ELPO	LPOE
PLOE	OPEL	ELOP	LPEO
POEL	OELP	EOPL	LPEO
PEOL	OEPL	EOLP	LOEP

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$

How many different words can we make from the letter POLO?

$$\frac{4!}{2!} = \frac{24}{2} = 12 \; combinations$$

POOL	LOOP	OLOP	OPLO
POLO	LOPO	OLPO	OOPL
PLOO	LPOO	OPOL	OOLP

$POOL = POOL$

Because these words are identical, we must divide by the number of ways we are double counting, equal to the number of ways we can organize that many objects.

How many different words can we make from the letters PEEP?

$$\frac{4!}{2!\,2!} = \frac{24}{2 \times 2}$$
$$= \frac{24}{4}$$
$$= 6 \; combinations$$

PEEP	EPPE
PEPE	EPEP
PPEE	EEPP

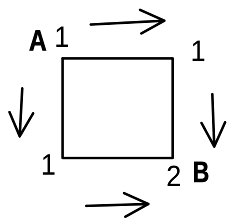
A ten question multiple choice exam has solutions as follows: 5 A's, 3 B's, 1 C, 1 D. In how many different combinations could these answers be ordered?

$$\frac{10!}{5!\,3!} = \frac{10 \times 9 \times 8 \times 7 \times 5!}{5!\,(3 \times 2 \times 1)}$$
$$= \frac{10 \times 9 \times 8 \times 7}{6}$$
$$= 840$$

C12 - 11.4 - Paths in Squares Notes

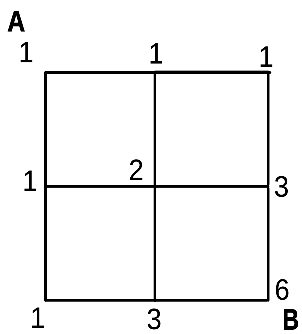
Pick a point, add points coming to it.

Paths in squares formula: $\dfrac{(l+w)!}{l!\,w!}$



Add numbers coming to it.

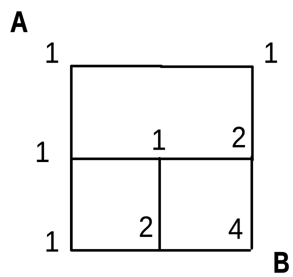
How many different paths can you follow from A to B if you only move down or to the right?



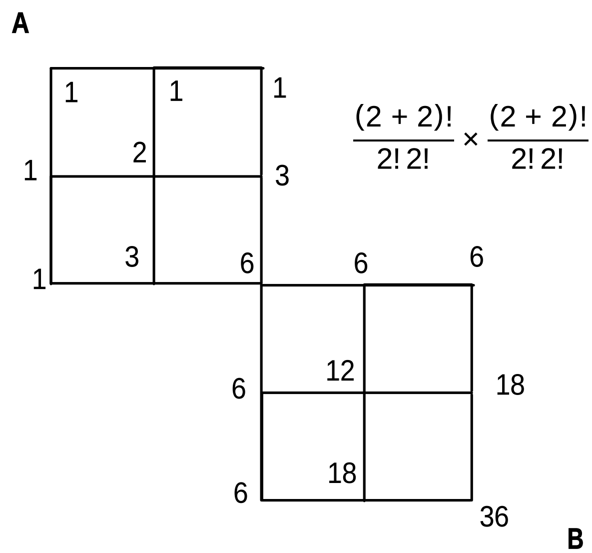
$$\frac{(2+2)!}{2!\,2!} = \frac{4!}{2!\,2!} = \frac{(4\times 3\times 2!)}{2!\,2!} = \frac{4\times 3}{2\times 1} = \frac{12}{2} = 6$$

R,R,D,D

$$\frac{4!}{2!\,2!}$$



You want to ask yourself, how many lines are coming towards that point from the direction they can come and add the numbers.



$$\frac{(2+2)!}{2!\,2!} \times \frac{(2+2)!}{2!\,2!} = 6 \times 6 = 36$$

How many ways can you get from one comer of a 3 sided rubix cube to the opposite comer if you never backtrack.

Paths in rectangular prisms formula: $\dfrac{(l+w+h)!}{l!\,w!\,h!}$ $= \dfrac{(3+3+3)!}{3!\,3!\,3!} = \dfrac{9!}{216} = 1680$

C12 - 11.4 - nPr nCr Algebra Notes

Solve for the missing variable

$_nC_2 = 10$

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$
$$_nC_2 = \frac{n!}{2!\,(n-2)!} = 10$$
$$\frac{n!}{2(n-2)!} = 10$$
$$\frac{n!}{(n-2)!} = 20$$
$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$$
$$n^2 - n = 20$$
$$n^2 - n - 20 = 0$$
$$(n-5)(n+4) = 20$$
$$n = 5 \quad \cancel{n = -4}$$

$_nP_2 = 42$

$$_nP_r = \frac{n!}{(n-r)!}$$
$$_nP_2 = \frac{n!}{(n-2)!} = 42$$
$$\frac{n!}{(n-2)!} = 42$$
$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$$
$$n^2 - n = 42$$
$$n^2 - n - 42 = 0$$
$$(n-7)(n+6) = 20$$
$$n = 7 \quad \cancel{n = -6}$$

$_nC_3 = 4$

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$
$$_nC_3 = \frac{n!}{3!\,(n-3)!} = 4$$
$$\frac{n!}{6(n-3)!} = 4$$
$$\frac{n!}{(n-3)!} = 24$$
$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 24$$
$$n(n-1)(n-2) = 24$$
$$n(n^2 - 3n + 2) = 24$$
$$n^3 - 3n^2 + 2n - 24 = 0$$

See Cubic factoring or guess and check

$_5C_3 = 10$

$_4C_3 = 4 \qquad n = 4$

$_3C_r = 3$

$$_nC_r = \frac{n!}{r!\,(n-r)!}$$
$$_3C_r = \frac{3!}{r!\,(3-r)!} = 3$$
$$\frac{6}{r!\,(3-r)!} = 3$$
$$\frac{6}{3} = r!\,(3-r)!$$
$$2 = r!\,(3-r)!$$

Nope! Guess and check

$_3C_3 = 1$

$_3C_2 = 3 \qquad r = 2$

$_3C_1 = 3 \qquad r = 1$

C12 - 11.5 - Pascal's Triangle

Pascal's triangle with numbers and with ${}_nC_r$'s.

			Sum:
Row 1	1	$n = 0$	$2^0 = 1$
Row 2	1 1	$n = 1$	$2^1 = 2$
Row 3	1 2 1	$n = 2$	$2^2 = 4$
Row 4	1 3 3 1	$n = 3$	$2^3 = 8$
Row 5	1 4 6 4 1	$n = 4$	$2^4 = 16$

The sum of the row is equal to 2^n

$$sum = 2^n$$

${}_4C_3$

$r = 0$ $r = 1$

$\boxed{\text{2nd \# in row is n \# = nC1}}$

$${}_0C_0$$

$${}_1C_0 \quad {}_1C_1$$

$${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$$

$${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$$

$${}_4C_0 \quad {}_4C_1 \quad {}_4C_2 \quad {}_4C_3 \quad {}_4C_4$$

C12 - 11.5 - Binomial Expansion Notes

Binomial Expansion:

$(x + 2)^2 = (x + 2)(x + 2) = x^2 + 4x + 4$
$(x + 2)^3 = (x + 2)(x + 2)(x + 2) = (x + 2)(x^2 + 4x + 4) = x^3 + 4x^2 + 4x + 2x^2 + 8x + 8 = x^3 + 6x^2 + 12x + 8$

$(x + 2)^2 = 1x^2 + 4x + 4$
$(x + 2)^3 = 1x^3 + 6x^2 + 12x + 8$

$(a + b)^n \quad ; n + 1\ terms$

$t_{k+1} = {}_nC_k a^{n-k} b^k$

$k\ is\ always\ one\ less\ than\ the\ term\ number.$

$t_5 = t_{k+1}$
$5 = k + 1$
$4 = k$

$(x - 5)^3$

$n = 3$

$a = x$

$b = -5$

Binomial	"n"	Row #	Expansion	Number of Terms
$(a + b)^0$	0	1	1	1
$(a + b)^1$	1	2	$1a + 1b$	2
$(a + b)^2$	2	3	$1a^2 + 2ab + 1b^2$	3
$(a + c)^3$	3	4	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	4
$(a + b)^4$	4	5	$1a^4 + 4a^3b + 6a^2b^2 + 4xb^3 + 1b^4$	5
$(a + b)^5$	5	6	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	6
$(a + b)^6$	6	7	$1a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + 1b^6$	7

Notice the sum of the exponents is always = n

Pascal's Triangle can aid in the expansion of binomials. Notice the coefficients on each term is equivalent to the numbers in Pascal's Triangle.

General Formula:

$$(a + b)^n = {}_nC_0(a)^n(b)^0 + {}_nC_1(a)^{n-1}(b)^1 + {}_nC_2(a)^{n-2}(b)^2 + \cdots + {}_nC_{n-1}(a)^1(b)^{n-1} + {}_nC_n(a)^0(b)^n$$

What is the 5th term of expansion $(a + b)^6$.

$n = 6$
$a = a$
$b = b$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_5 = {}_6C_4 a^{6-4} b^4$
$t_5 = 15a^2b^4$

$t_{k+1} = t_5$
$k + 1 = 5$
$k = 4$

C12 - 11.5 - Binomial Theorem Middle, x^11, x^0 Notes

FOIL $(x^2+2)^3 = (x^2+2)(x^2+2)(x^2+2) = (x^4+4x^2+4)(x^2+2) = x^6+6x^4+12x^2+8$

Which term in the binomial expansion $(x^2+2)^3$ has x^4? Find the term

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_3C_k (x^2)^{3-k}(2)^k$
$= (x^2)^{3-k}$
$x^{6-2k} = x^4$
$6-2k = 4$
$2 = 2k$
$k = 1$

$(x^2)^{3-k} = x^4$

$(x^2)^{3-k}$; *is the only part that contributes to the exponent of x*

$t_{k+1} =$
$t_{1+1} = t_2$

The second term, t_2, will have an exponent x^4

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_2 = {}_3C_1 (x^2)^{3-1}(2)^1$
$t_2 = 3(x^2)^2 \times 2$
$t_2 = 6x^4$

The second term, $t_2 = 6x^4$

Which term in the binomial expansion $(x^2+2)^3$ is a constant? ($5 = 5x^0$) Find the term.

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_3C_k (x^2)^{3-k}(2)^k$
$= (x^2)^{3-k}$
$x^{6-2k} = x^0$
$6-2k = 0$
$6 = 2k$
$k = 3$

$(x^2)^{3-k} = x^0$

$t_{k+1} =$
$t_{3+1} = t_4$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_4 = {}_3C_3 (x^2)^{3-3}(2)^3$
$t_4 = 1(x^2)^0 \times 8$
$t_4 = 8x^0$
$t_4 = 8$

Which term in the binomial expansion $\left(x^2 - \frac{1}{x}\right)^{10}$ has x^{11}? Find the term. Note: $\left(x^2 - \frac{1}{x}\right)^{10} = (x^2 - x^{-1})^{10}$

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_{10}C_k (x^2)^{10-k}(-x^{-1})^k$
$(x^2)^{10-k}(x^{-1})^k$
$x^{20-2k}x^{-k}$
$x^{20-3k} = x^{11}$

${}_{10}C_k$ and the negative in front of x do not contribute to finding which term it is.

$20-3k = 11$
$9 = 3k$
$3 = k$

$t_{k+1} =$
$t_{3+1} = t_4$

The fourth term will have x^{11}.

$t_{k+1} = {}_nC_k a^{n-k} b^k$
$t_{k+1} = {}_{10}C_k (x^2)^{10-k}(-x^{-1})^k$
$t_4 = {}_{10}C_3 (x^2)^{10-3}(-x^{-1})^3$
$t_4 = {}_{10}C_3 (x^2)^7(-x^{-1})^3$
$t_4 = {}_{10}C_3 x^{14}(-x^{-3})$
$t_4 = {}_{10}C_3 (-x^{11})$
$t_4 = -120x^{11}$

The fourth term, $t_4 = -120x^{11}$