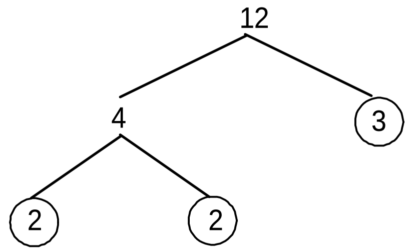
M10 - 4.1 - Entire to Mixed Radicals Notes

What are two numbers that multiply to the number underneath the square root that you know the square root of one of them

$$\sqrt[2]{12} = \sqrt[2]{4 \times 3}$$
$$= \sqrt[2]{4} \times \sqrt[2]{3}$$
$$= 2\sqrt[2]{3}$$

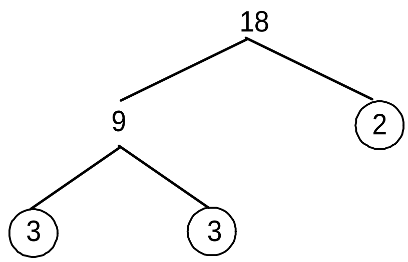
$$\sqrt[2]{12} = \sqrt[2]{2 \times 2 \times 3}$$
$$= 2\sqrt[2]{3}$$



Two identical numbers under a square root: one comes out.

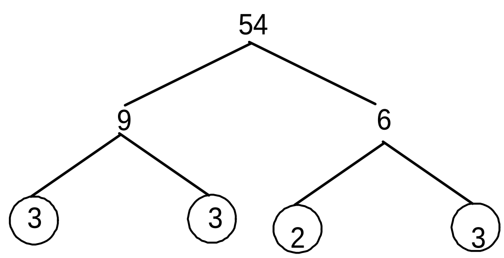
$$\sqrt[2]{2 \times 2} = \sqrt[2]{4} = 2$$

$$\sqrt[2]{18} = \sqrt[2]{3 \times 3 \times 2}$$
$$= 3\sqrt[2]{2}$$



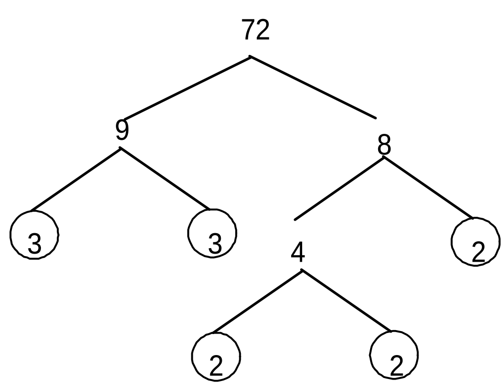
Two identical numbers under a square root: one comes out.

$$\sqrt[2]{54} = \sqrt[2]{3 \times 3 \times 3 \times 2}$$
$$= 3\sqrt[2]{3 \times 2}$$
$$= 3\sqrt[2]{6}$$



Multiply what's left inside.

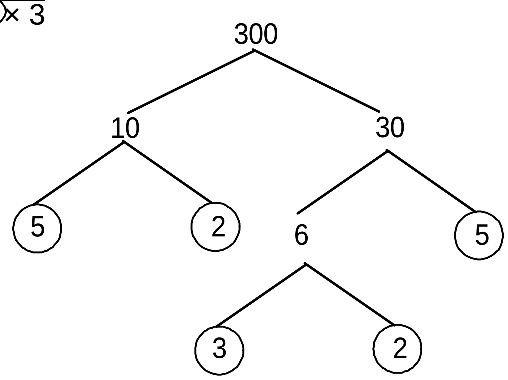
$$\sqrt[2]{72} = \sqrt[2]{3 \times 3 \times 2 \times 2 \times 2}$$
$$= 3 \times 2\sqrt[2]{2}$$
$$= 6\sqrt[2]{2}$$



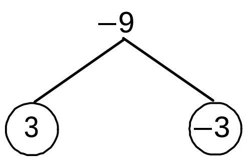
Multiply numbers outside of root.

M10 - 4.1 - Entire to Mixed Radicals Notes

$\sqrt[2]{300} = \sqrt[2]{5 \times 5 \times 2 \times 2 \times 3}$
$= 5 \times 2\sqrt[2]{3}$
$= 10\sqrt[2]{3}$

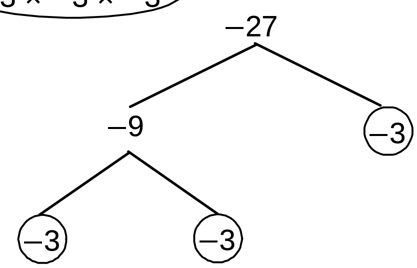


$\sqrt[2]{-9} = \sqrt[2]{3 \times -3}$
Impossible.



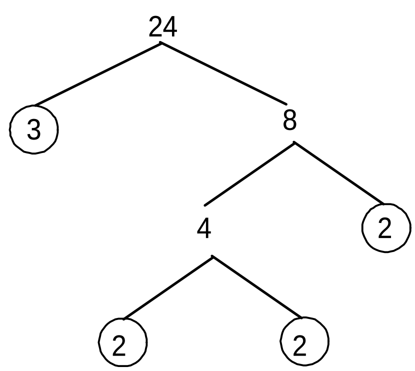
Cannot square root a negative number.

$\sqrt[3]{-27} = \sqrt[3]{-3 \times -3 \times -3}$
$= -3$



You may cube root a negative number.

$\sqrt[3]{24} = \sqrt[3]{2 \times 2 \times 2 \times 3}$
$= \quad 2\sqrt[3]{3}$

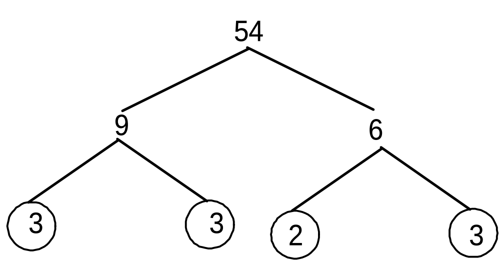


Three identical numbers under a square root: one comes out.

$\sqrt[3]{2 \times 2 \times 2} = \sqrt[3]{8} = 2$

$\sqrt[3]{24} = \sqrt[3]{8 \times 3}$ $= \sqrt[3]{8} \times \sqrt[3]{3}$ $= 2\sqrt[3]{3}$	What are numbers that multiply to the number underneath the cube root that you know the cube root of one of them

$\sqrt[3]{54} = \sqrt[3]{3 \times 3 \times 3 \times 2}$
$= 3\sqrt[3]{2}$



Three identical numbers under a square root: one comes out.

M10 - 4.2 - Mixed to Entire Radicals Notes

$5\sqrt[2]{2} = \sqrt[2]{5 \times 5 \times 2}$
$= \sqrt[2]{25 \times 2}$
$= \sqrt[2]{50}$

One number outside of a square root: two inside.

$-7\sqrt[2]{3} = -\sqrt[2]{7 \times 7 \times 3}$
$= -\sqrt[2]{49 \times 3}$
$= -\sqrt[2]{147}$

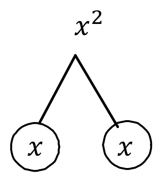
$5\sqrt[3]{2} = \sqrt[3]{5 \times 5 \times 5 \times 2}$
$= \sqrt[3]{125 \times 2}$
$= \sqrt[3]{250}$

One number outside of a cube root: three inside.

$4\sqrt[3]{5} = \sqrt[3]{4 \times 4 \times 4 \times 5}$
$= \sqrt[3]{64 \times 5}$
$= \sqrt[3]{320}$

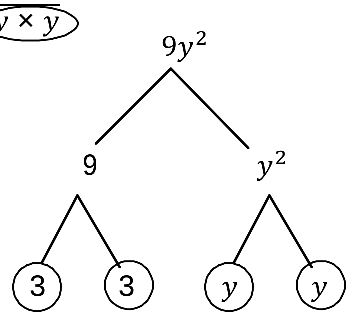
M10 - 4.2 - Simplifying Radicals with Variables Notes

$\sqrt[2]{x^2} = \sqrt[2]{x \times x}$
$= x$

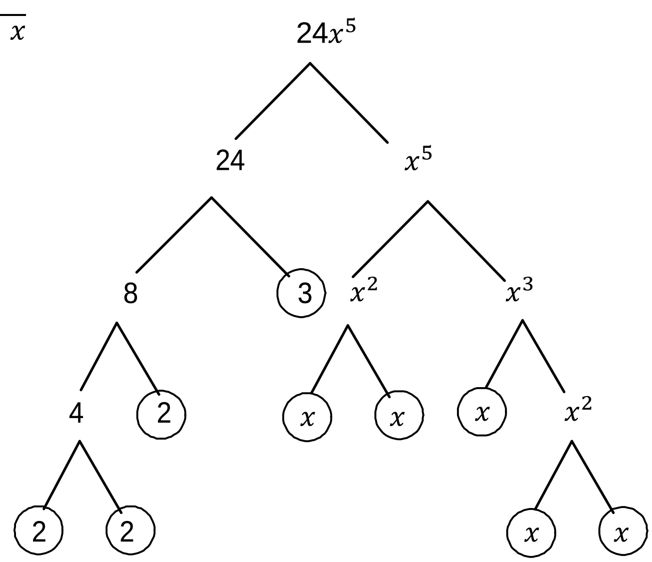


Two identical numbers under a square root: one comes out. Nothing is left.

$\sqrt[2]{9y^2} = \sqrt[2]{3 \times 3 \times y \times y}$
$= 3y$



$\sqrt[3]{24x^5} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times x \times x \times x \times x \times x}$
$= 2x\sqrt[3]{3x^2}$



Remember:
-Must have same base to use laws.

M10 - 4.3 - Add/Sub/Dist Exponents Laws

Exponent

Base $\longrightarrow x^2 \nearrow$

When Multiplying with the same base, Add Exponents.

$x^3 \times x^2 = (x \times x \times x) \times (x \times x) = x^5$

$x^3 \times x^2 = x^{3+2} = x^5$

$\boxed{\text{Add Exponents}}$

$y^2 \times y^4 = (y \times y) \times (y \times y \times y \times y) = y^6$

$y^2 \times y^4 = y^6$

When Dividing with the same base, Subtract Exponents.

$\frac{x^5}{x^2} = \frac{\cancel{x} \times \cancel{x} \times x \times x \times x}{\cancel{x} \times \cancel{x}} = x^3$

$\frac{x^5}{x^2} = x^{5-2} = x^3$

$\boxed{\text{Subtract Exponents}}$

$\frac{z^6}{z^3} = \frac{z \times z \times z \times z \times z \times z}{z \times z \times z} = x^3$

$\frac{z^6}{z^3} = z^{6-3} = z^3$

With Exponents to exponents, Multiply Exponents

$(x^2)^3 = (x \times x)^3 = (x \times x) \times (x \times x) \times (x \times x) = x^6$

$(x^2)^3 = x^{2\times3} = 2^6$

$\boxed{\text{Multiply Exponents}}$

$(y^4)^2 = (y \times y \times y \times y)^2 = (y \times y \times y \times y) \times (y \times y \times y \times y) = y^8$

$(y^4)^2 = y^8$

Ultimately you will either use: Exponent Laws

Or

Repeated Multiplication and Division Theory

M10 - 4.4 - Dist/Neg Exponents Laws

With Product/Quotients to Exponents, Distribute Exponents

$(x \times y)^2 = x^2 \times y^2$

Distribute Exponents

$(2x)^3 = (2x) \times (2x) \times (2x) = 8x^3$

$(2x)^3 = 2^3x^3 = 8x^3$

$$\left(\frac{2x}{y}\right)^2 = \frac{2^2x^2}{y^2} = \frac{4x^2}{y^2}$$

Cannot distribute into a sum!
$(3 + 4)^2 \neq 3^2 + 4^2 = 25$
$(3 + 4)^2 = (3 + 4)(3 + 4) = 7 \times 7 = 7^2 = 49$

Negative Exponents

$x^{-2} = \dfrac{1}{x^2}$ *Bring to the bottom, make exponent positive* $x^{-a} = \dfrac{1}{x^a}$

$\dfrac{1}{x^{-2}} = \dfrac{x^2}{1}$ *Bring to the top, make exponent positive* $\dfrac{1}{x^{-a}} = x^a$

$3a^{-2} = \dfrac{3}{a^2}$ *Bring to the bottom, make exponent positive*

Notice the 3 doesn't come down

$3^{-3}a^{-2} = \dfrac{1}{3^3a^2}$ *Bring to the bottom, make exponent positive*

$(2x)^{-3} = \dfrac{1}{(2x)^3} = \dfrac{1}{8x^3}$ *Bring to the bottom, make exponent positive*

When working with negative exponents always start with a fraction "Over" sign. Put whatever stays. Move whatever needs to be moved. If nothing is left on the top, by division theory, a 1 goes there.

$$\frac{2x^5y^{-2}}{z^{-3}} = \frac{2x^5z^3}{y^2}$$

M10 - 4.4 - Negative Exponents Laws

When you can flip it!

$\left(\frac{x}{y}\right)^{-2} = \frac{x^{-2}}{y^{-2}} = \frac{y^2}{x^2}$

Distribute Exponents
Bring to the bottom, make exponent positive
Bring to the top, make exponent positive

$\left(\frac{x}{y}\right)^{-2} = \left(\frac{y}{x}\right)^{2} = \frac{y^2}{x^2}$ Flip it and make the exponent positive

Alternate Subtraction Method

$\frac{x^2}{x^5} = x^{2-5} = x^{-3} = \frac{1}{x^3}$

Subtract from the top

Theory

$\frac{x^2}{x^5} = \frac{\cancel{x \times x}\ 1}{\cancel{x \times x} \times x \times x \times x} = \frac{1}{x^3}$

$\frac{x^2}{x^5} = \frac{1}{x^{5-2}} = \frac{1}{x^3}$

Subtract from the bottom

$\frac{x^2}{x^{-3}} = x^2x^3 = x^5$

OR

$\frac{x^2}{x^{-3}} = x^{2-(-3)} = x^5$

$\frac{x^{-2}}{x^3} = \frac{1}{x^3x^2} = \frac{1}{x^5}$

OR

$\frac{x^{-2}}{x^3} = \frac{1}{x^{3-(-2)}} = \frac{1}{x^5}$

Theory on "Bring it to the Bottom" and Vice Versa

$3^3 = 27$

$3^2 = 9$ $\div 3$

$3^1 = 3$ $\div 3$

$3^0 = 1$ $\div 3$

$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

If a pattern in Math continues without proving it wrong it is considered Theory for everyone to follow.

The exponents on the left are going down by one,
The numbers on the right are being divided by 3,
This pattern must continue