# M10 - 5.1 - "FOIL" Notes
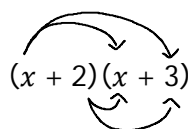
$(x + 2)(x + 3) =$

Method 1: "FOIL" Distribution

**F** - multiply **First** numbers in brackets
**O** - multiply **Outside** numbers in brackets
**I** - multiply **Inside** numbers in brackets
**L** - multiply **Last** numbers in brackets

1)  $(x + 2)(x + 3) =$

$x^2 + 3x + 2x + 6 =$
$x^2 + 5x + 6$

Combine like terms.

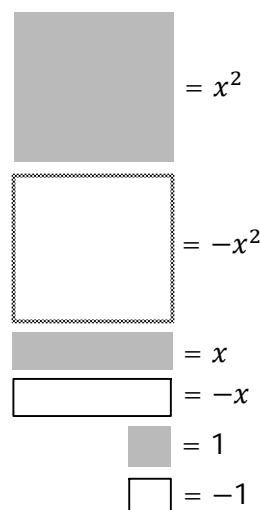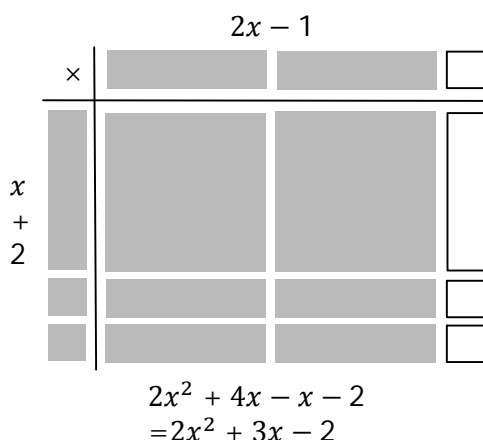| | |
|---|---|
| $(x + 2)(x + 3) =$ <br> $x^2 + 5x + 6$ | Multiply and combine like terms in the same step. |

2)  $(x + 2)(x + 3) =$
$x(x + 3) + 2(x + 3) =$
$x^2 + 3x + 2x + 6 =$
$x^2 + 5x + 6$

Method 2:

3) $(2x - 1)(x + 2)$

Method 3: Algebra Tiles

$2x - 1$



$x + 2$

$= x^2$

$= -x^2$

$= x$

$= -x$

$= 1$

$= -1$

$2x^2 + 4x - x - 2$
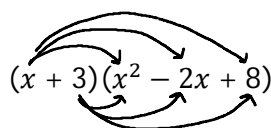$= 2x^2 + 3x - 2$

4)  $(x + 2)^2$
$(x + 2)(x + 2)$
$x^2 + 2x + 2x + 4$
$x^2 + 4x + 4$

5)  $-(x + 2)(x + 4)$
$-(x^2 + 4x + 2x + 8)$
$-(x^2 + 6x + 8)$
$-x^2 - 6x - 8$

"FOIL"
Combine like terms
Distribute the negative

6)  $(x + 3)(x^2 - 2x + 8)$

"FOIL" with more than 2 terms in one bracket

$(x + 3)(x^2 - 2x + 8)$

Multiply both terms in the first set of brackets with each term in the second set of brackets

$x^3 - 2x^2 + 8x + 3x^2 - 6x + 24$
$x^3 + x^2 + 2x + 24$

Combine like terms

# M10 - 5.2 - Factoring GCF Notes

*Remove Greatest Common Factor "GCF."*

1)   $12x + 8$                    $GCF = 4$
   $4(3x + 2)$

   4 times       4 times
   what is 12x   what is 8

<table>
<tr><td>Divide both<br>terms by GCF<br><br>$\dfrac{12x}{4} + \dfrac{8}{2} = 3x + 2$<br><br>Answer goes in<br>brackets</td><td>Check your answer<br>by Distribution<br><br>$4(3x + 2)$<br>$12x + 8$<br><br>The answer should<br>be the same as the<br>original question.</td></tr>
</table>

2)  $2x^2 + 3x =$                $GCF = x$
   $x(2x + 3)$

3)  $12x^2 + 8x =$              $GCF = 4x$
   $4x(3x + 2)$

*Remove Greatest Common Factor "GCF."*

4)  $x^2 + x^3 =$               $GCF = x^2$
   $x^2(1 + x)$

5)  $8x^2y + 4xy$
   $4xy(2x + 1)$              $GCF = 4xy$

6)  $-2x + 8$                   $GCF = -2$
   $-2(x - 4)$

8)      $(2 - x) =$
   $-1(-2 + x) =$
   $-(x - 2)$                $GCF = -1$
                             Rearrange order of the terms

9)  $(-x - 2) =$              $GCF = -1$
   $-1(x + 2) =$
   $-(x + 2)$

10) $x(x + 2) + 4(x + 2) =$    $GCF = (x + 2)$
   $(x + 2)(x - 4)$

11)  $2x - \dfrac{1}{2}$              $2x - 1$

   $2(x - \dfrac{1}{4})$            $2(x - \dfrac{1}{2})$

# M10 - 5.3 - Labelling "$a$", "$b$" and "$c$" in Polynomials Notes

**Identifying "a", "b", and "c" in: $ax^2 + bx + c$**

"a" is the number to the left of the $x^2$ term.
"b" is the number to the left of the $x$ term.
"c" is the number by itself.

## a = 1

1) $y = 1x^2 + 2x + 3$

   $a = 1, b = 2, c = 3$

2) $y = x^2 + 3x + 4$

   $a = 1, b = 3, c = 4$

3) $y = x^2 - 4x + 2$

   $a = 1, b = -4, c = 2$

4) $y = x^2 + 4x - 6$

   $a = 1, b = 4, c = -6$

| $y = kx^2 + mx = 4$ |
| --- |
| $a = k, b = m, c = 4$ |

## a ≠ 1

1) $y = 2x^2 + 3x + 4$

   $a = 2, b = 3, c = 3$

2) $y = 3x^2 + 5x + 7$

   $a = 3, b = 5, c = 7$

3) $y = 2x^2 - 3x - 9$

   $a = 2, b = -3, c = -9$

4) $y = -3x^2 + 7x - 1$

   $a = -3, b = 7, c = -1$

## b = 0

1) $y = 2x^2 + 3$

   $a = 2, b = 0, c = 3$

2) $y = 2x^2 - 5$

   $a = 2, b = 0, c = -5$

## c = 0

1) $y = x^2 + 4x$

   $a = 1, b = 4, c = 0$

2) $y = -3x^2 - 7x$

   $a = -3, b = -7, c = 0$

1)  $x^2 + 5x + 6$

$x^2 + 2x + 3x + 6$
$(x^2 + 2x)(+3x + 6)$
$x(x + 2) + 3(x + 2)$
$(x + 2)(x + 3)$

$a = 1$

*Decompose*
*Group*
*GCF*
*Switch*

_____ X _____ = c

_____ + _____ = b

__2__ X __3__ = ~~c~~ 6        1,2,3,6

__2__ + __3__ = ~~b~~ 5

$x^2 + 5x + 6$
$(x + 2)(x + 3)$

What are two numbers that: multiply to get "c", the last number, and add together to get "b", the middle number. The numbers above on the right go in the brackets on the left.

$(x + 2)(x + 3)$
$x^2 + 3x + 2x + 6$
$x^2 + 5x + 6$

Don't forget to check by multiplying out: FOIL. The answer should be the same as the original question.

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2)  $x^2 + 6x + 8$
    $(x + 2)(x + 4)$

$a = 1$

__2__ X __4__ = ~~c~~ 8

__2__ + __4__ = ~~b~~ 6

3)  $x^2 - 3x - 10$
    $(x - 5)(x + 2)$

$a = 1$

__-5__ X __2__ = c -10

__-5__ + __2__ = b -3

Remember the sign of the numbers you choose goes in the bracket along with the number.

6)  $x^2 + 4x + 15$

Cannot factor

$$x^4 + 5x^2 + 6$$

$$(x^2 + 2)(x^2 + 3)$$

# M10 - 5.3 - Factor by Decomposition $ax^2 + bx + c \ (a \neq 1)$ Notes

**1)**   $2x^2 + 7x + 6$          $a \neq 1$

$2x^2 + 3x + 4x + 6$       *Decompose*   Step 1      $\underline{\ \ 3\ \ }$  X  $\underline{\ \ 4\ \ }$ $= \cancel{ac}\ 12$
$(2x^2 + 3x) \mid (+4x + 6)$   *Group*       Step 2

$x(2x + 3) + 2(2x + 3)$   *GCF*        Step 3      $\underline{\ \ 3\ \ }$  +  $\underline{\ \ 4\ \ }$ $= \cancel{b}\ 7$
$(2x + 3)(x + 2)$         *Switch*      Step 4

$2x + 2y$
$2(x + y)$

$2x^2 + 7x + 6$
$(2x + 3)(x + 2)$

*Step 1 Decompose*: What are two numbers that: multiply to get "a X c" and add to get "b." "b" gets split up into the two numbers above on the right.

*Step 2 Group*: Place brackets around the first two terms, and the second two terms.

*Step 3 GCF*: Remove a GCF from each set of brackets.

*Step 4 Switch*: The GCFs go in a set of brackets together, the identical pair of brackets become one set of brackets below

$(x + 2)(2x + 3)$         Don't forget to check by multiplying out: FOIL. The
$2x^2 + 3x + 4x + 6$       answer should be the same as the original question.
$2x^2 + 7x + 6$

                                          $a \neq 1$

**2)**   $2x^2 - 6x - 8$

                          *Decompose*         $\underline{\ \ 2\ \ }$  X  $\underline{\ -8\ \ }$ $= \cancel{ac}\ -16$
$2x^2 + 2x - 8x - 8$        *Group*
$(2x^2 + 2x) + (-8x - 8)$
                                                $\underline{\ \ 2\ \ }$  +  $\underline{\ -8\ \ }$ $= \cancel{b}\ -6$

$2x(x + 1) - 8(x + 1)$     *GCF*
$(2x - 8)(x + 1)$          *Switch*
$2(x - 4)(x + 1)$          *Factor GCF out each set of brackets*

**3)**   $2x^2 + 4x - 7$           Cannot factor

**4)**   $x^2 + 4x + 3$
$x^2 + 3x + x + 3$          GCF=1
$(x^2 + 3x) + (x + 3)$
$x(x + 3) + 1(x + 3)$
$(x + 1)(x + 3)$

# M10 - 5.4 - Differences of Squares Notes

1)  $x^2 - 9$
    $(x + 3)(x - 3)$

    What squared is $x^2$? That answer goes first in each set of brackets.
    What squared is 9? That number goes second in each set of brackets.
    Put a plus sign in one set of brackets and a minus sign in the other.

    $(x + 3)(x - 3)$
    $x^2 - 3x + 3x - 9$
    $x^2 - 9$

    Don't forget to check by multiplying out: FOIL. (5.1)The answer should be
    the same as the original question.

2)  $4x^2 - 36$
    $(2x)^2 - 6^2$
    $(2x + 6)(2x - 6)$

                                                        36x6=216

    $(2x + 6)(2x - 6)$          Check by "FOIL"
    $4x^2 - 12x + 12x - 36$
    $4x^2 - 36$

3)  $9x^2 - y^2$              Change of base
    $(3x)^2 - y^2$
    $(3x + y)(3x - y)$

    $(3x + y)(3x - y)$          Check by "FOIL"
    $9x^2 - 3xy + 3xy - y^2$
    $9x^2 - y^2$

4)  $-x^2 + 49$
    $49 - x^2$                Rearrange order of terms
    $(7 + x)(7 - x)$

    $(7 + x)(7 - x)$           Check by "FOIL"
    $49 - 7x + 7x - x^2$
    $49 - x^2$

    $x^2 + 4$                 Cannot Factor

# M10 - 5.5 - Factoring Special/Multi-Step Trinomials Notes

1)  $x^2 - 3xy - 10y^2$
    $(x - 5y)(x + 2y)$

$a = 1$

$\underline{\quad -5 \quad}$ X $\underline{\quad 2 \quad}$ = $\cancel{c}$ $-10$

$\underline{\quad -5 \quad}$ + $\underline{\quad 2 \quad}$ = $\cancel{b}$ $-3$

Factor as you would in $a = 1$ and put a $y$ after the last number in both sets of brackets, or do decomposition

$x^2 - 3xy - 10y^2$
$x^2 - 5xy + 2xy - 10y^2$
$(x^2 - 5xy) + (+2xy - 10y^2)$
$x(x - 5y) + 2y(x - 5y)$
$(x + 2y)(x - 5y)$

$(x + 2y)(x - 5y)$ 　　　　　　Check by "FOIL"
$x^2 - 5xy + 2xy - 10y^2$
$x^2 - 3xy - 10y^2$

2)  $2x^2 + 10x + 12$
    $2(x + 5x + 6)$ 　　　　　　Remove GCF
    $2(x + 2)(x + 3)$ 　　　　　　$a = 1$

3)  $-x^2 - 5x - 6$
    $-(x^2 + 5x + 6)$ 　　　　　　Remove GCF
    $-(x + 2)(x + 3)$ 　　　　　　$a = -1$

4)  $3x^2 - 27$
    $3(x^2 - 9)$ 　　　　　　Remove GCF
    $3(x + 3)(x - 3)$ 　　　　　　Differences of squares

5)  $x^4 - 1$
    $(x^2 + 1)(x^2 - 1)$ 　　　　　　Differences of squares
    $(x^2 + 1)(x + 1)(x - 1)$ 　　　　　　Differences of squares

$x^4 + 5x^2 + 6$

$(x^2 + 2)(x^2 + 3)$

# M10 - 5.5 - Factoring Substitution Let x = m+1 Notes

$(m + 1)^2 + 5(m + 1) + 6$

$x^2 + 5x + 6$

$(x + 2)(x + 3)$

$\big((m + 1) + 2\big)\big((m + 1) + 3\big)$

$(m + 3)(m + 4)$

$Let\ x = m + 1$

OR

$(m + 1)^2 + 5(m + 1) + 6$

$m^2 + 2m + 1 + 5m + 5 + 6$

$m^2 + 7m + 12$

$(m + 3)(m + 4)$

$4x^2 - (x + 2)^2$

$a^2 - b^2$

$(a + b)(a - b)$

$(2x + (x + 2))(2x - (x + 2)$

$(3x + 2)(x - 2)$

$let\ a = 2x$

$let\ b = (x + 2)$

Substitute $2x$ and $x + 2$ back in with brackets

$9(x + 2)^2 - 16(x - 1)^2$

$9a^2 - 16b$

$(3a + 4b)(3a - 4b)$

$\big(3(x + 2) + 4(x - 1)\big)\big(3(x + 2) - 4(x - 1)\big)$

$(3x + 6 + 4x - 4)(3x + 6 - 4x + 4)$

$(7x + 2)(-x + 10)$

Let $a = x + 2$

Let $b = x - 1$

Substitute $x + 2$ and $x - 1$ back in with brackets