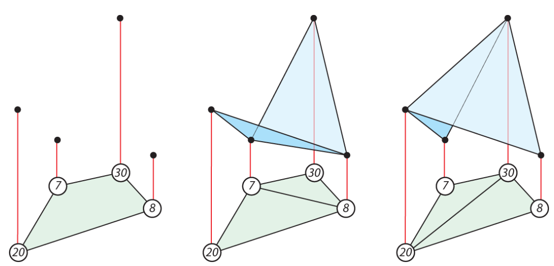
Announcements

- Lab today will be office hours instead
- No lecture on Thursday 2/28
- Switching lab from Tuesday 3/5 to Thursday 3/7 – next week only

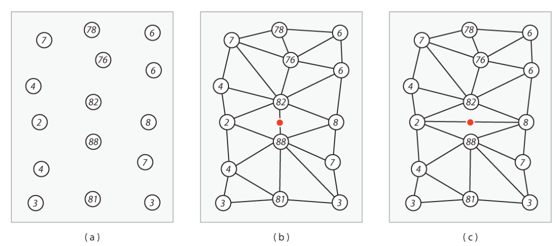
Computational Geometry

Delaunay and Other Special Triangulations

Lifting the Triangles



Skinny is Bad

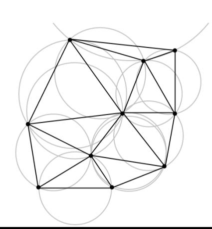


Angle Sequence

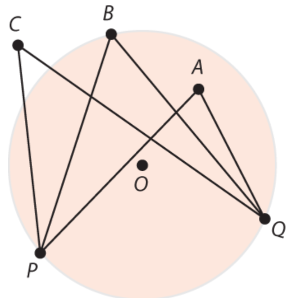
- Let T be a triangulation of a point set S, and suppose T has n triangles. The angle sequence $\{a_1, a_2, \ldots, an\}$ lists all $3n$ angles of T in sorted order.
- A triangulation T_1 is fatter than T_2 ($T_1 > T_2$) if the angle sequence of T_1 is lexicographically greater than T_2's.
  - $\{30°, 45°, 65°, 120°\} > \{30°, 45°, 60°, 120°\}$

Delaunay Triangulation

- For each convex quad in a triangulation T_1 with diagonal e, if a diagonal flip results in a triangulation T_2, s.t. $T_1 \geq T_2$, then e is legal.
- A Delaunay triangulation is a triangulation with all legal edges.

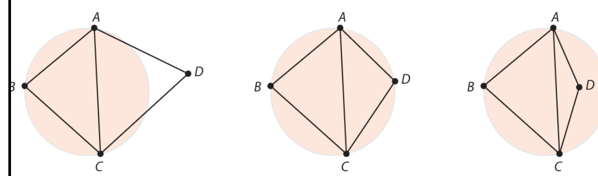


Thales' Theorem

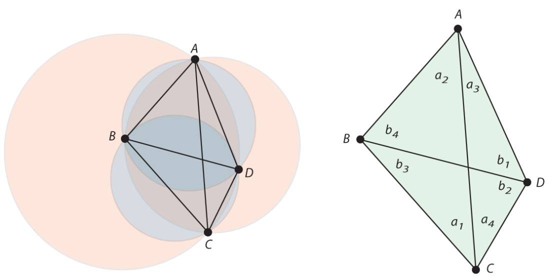


Thales' and Illegal Edges

- Let *e* be an edge of a triangulation, where *e* = *AC* belongs to the two triangles *ABC* and *ACD*. Then *e* is legal if *D* is outside of the circumcircle of *ABC* and illegal if *D* is inside.

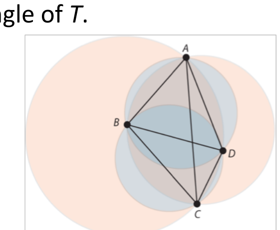


Proof

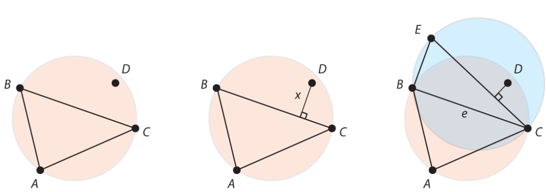


Empty Circle Property

- Let *S* be a point set in general position. A triangulation *T* is Delaunay if and only if no point from *S* is in the interior of any circumcircle of any triangle of *T*.



Proof



(a) (b) (c)

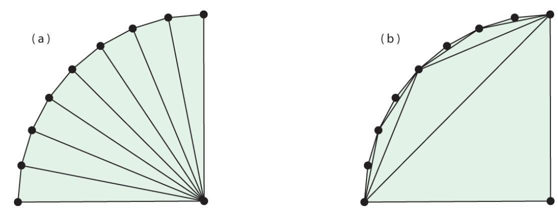
Naïve Delaunay

```
Delaunay({p₁,…,pₙ})
  for i in [1, n]
    for j in [1, i)
      for k in [1, j)
        (c, r) = Circumcircle(pᵢ, pⱼ, pₖ)
         isValid = true
         for l in [1, k)
           if (|pₗ-c| < r)
             isValid = false
         if (isValid) Output(pᵢ, pⱼ, pₖ)
```

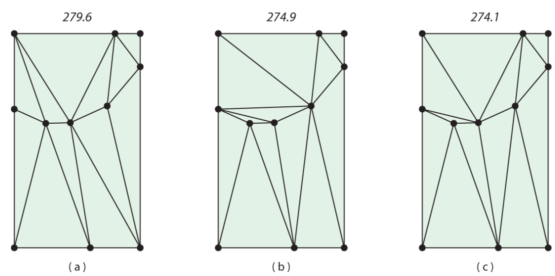
Delaunay via Edge Flipping

- Create any triangulation (incremental, say)
- Flip every illegal edge until done
- Complexity?

Delaunay is not MWT



Delaunay vs. Greedy vs. MWT



Theorem

- For point set *S*, a minimum spanning tree of *S* is a subset of the Delaunay triangulation of *S*.
- Proof by contradiction.

Compatible Triangulations