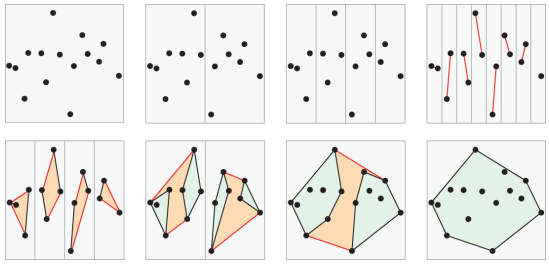
Computational Geometry

Convex Hull

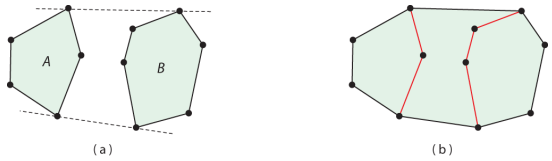
Announcements

- No Lecture on Thursday 2/28
- Midterm will be after Spring Break, around first or second week of April, depending on how we get through the materials

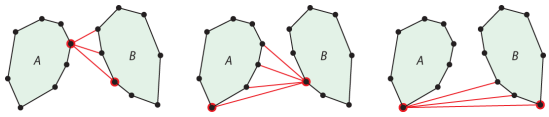
Divide-and-Conquer



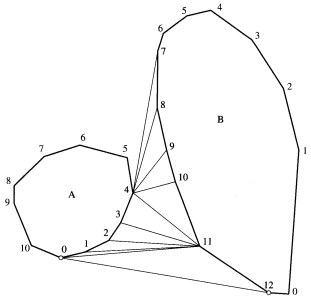
Tangent Lines of Two Hulls



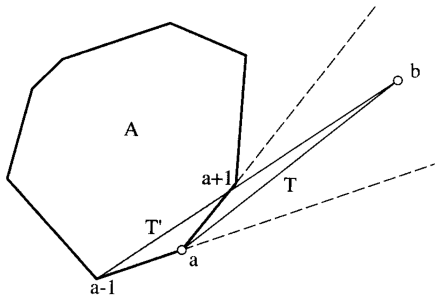
Tangent Line in O(*n*)



Lower Tangent



ab Does Not Cross A

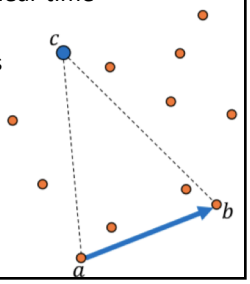


Complexity

- $O(n)$ to find the tangent lines
- $T(n) = 2T\left(\frac{n}{2}\right) + O(n) = 2T\left(\frac{n}{2}\right) + cn$
- $= 2\left(2\left(\frac{T}{4}\right) + \frac{cn}{2}\right) + cn$
- $= 2\left(2\left(2\left(\frac{t}{8}\right) + \frac{cn}{4}\right) + \frac{cn}{2}\right) + cn$
- …
- $= 2^{lgn} + \left(\frac{2^{lgn-1}}{2^{lgn-1}} + \frac{2^{lgn-2}}{2^{lgn-2}} + \cdots + 2^0\right) cn$
- $= O(nlgn)$

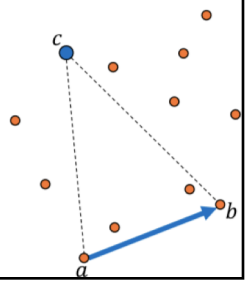
Observation

- Given a hull edge $\overrightarrow{ab}$, we can find the point c furthest away from ab in linear time
  - the point c is on the hull
  - the triangle $\Delta\ abc$ partitions the input into three regions
    - points inside $\Delta\ abc$
    - points to the right of $\overrightarrow{bc}$
    - points to the right of $\overrightarrow{ca}$



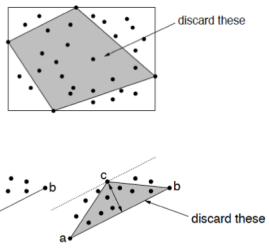
Algorithm

- Recursively
  - Discard points inside $\Delta\ abc$
  - Compute half hull to the right of $\overrightarrow{bc}$
  - Compute half hull to the right of $\overrightarrow{ca}$
  - merge
- $\overrightarrow{ab}$ doesn't need to be a hull edge, a hull diagonal will do (an edge btw two extrema points)



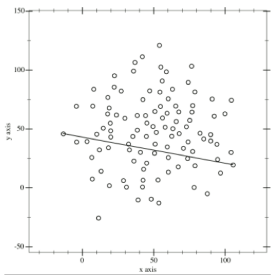
Quick Hull

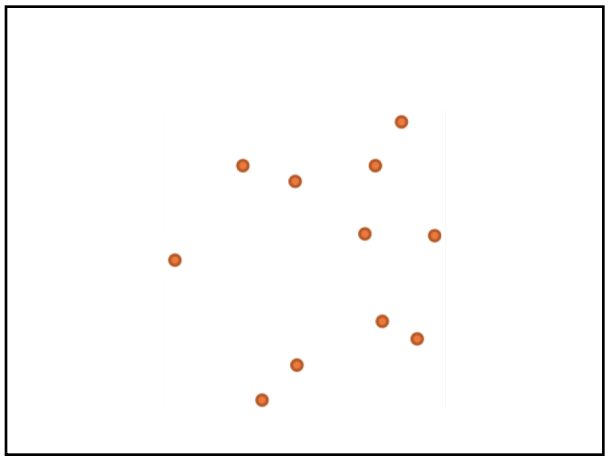
- Compute extreme points in *x* and *y*
- Discard points that lie within the quadrilateral
- Divide the rest into groups and repeat

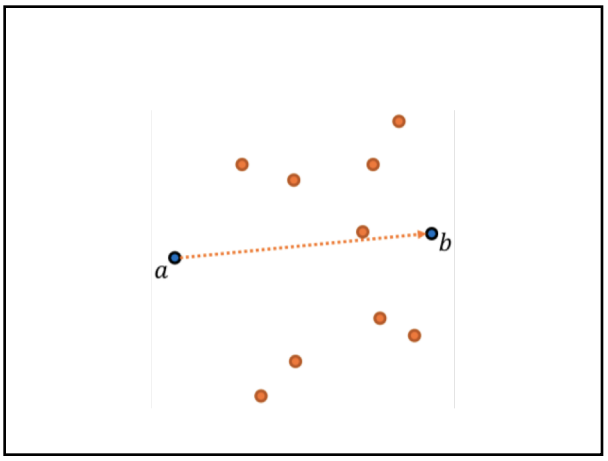


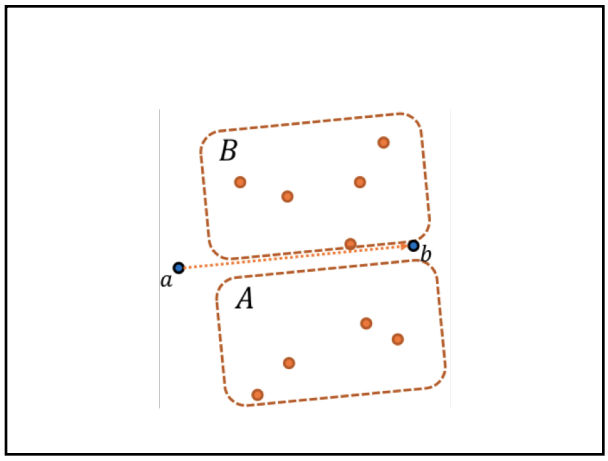
Quick Hull

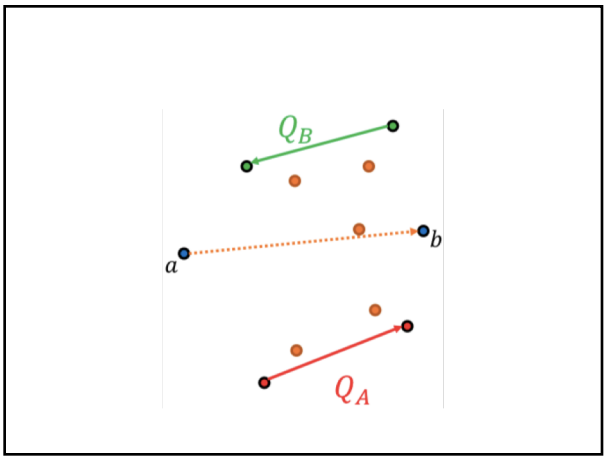
- Find extreme x points
- Use the line segment to divide points into 2 groups
- Recurse on each side:
  - Find point with max distance to the line and form triangle
  - discard interior points

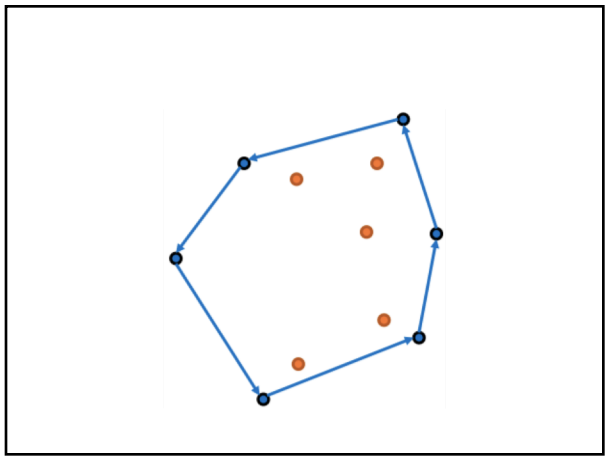


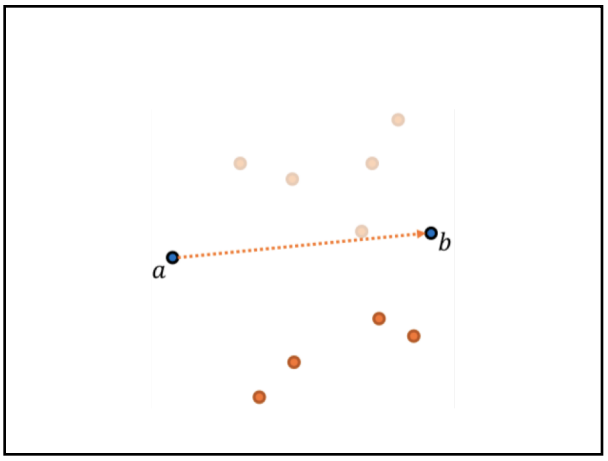



a
b


B
a
b
A


Q_B
a
b
Q_A

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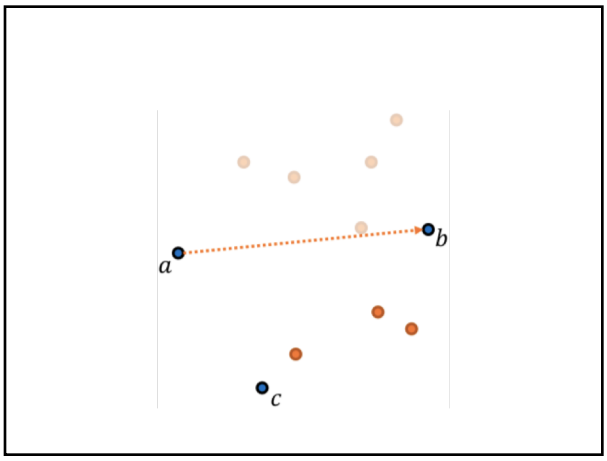
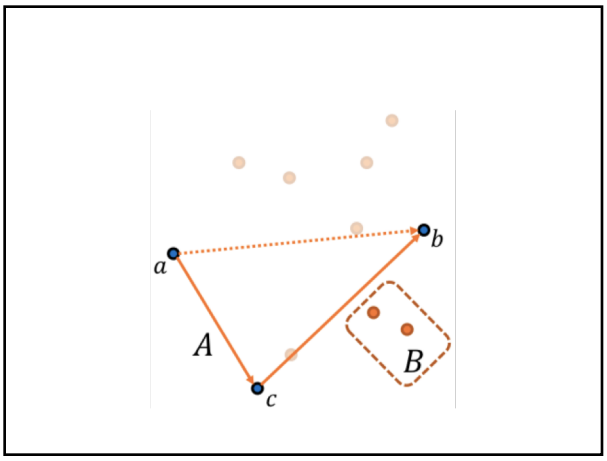
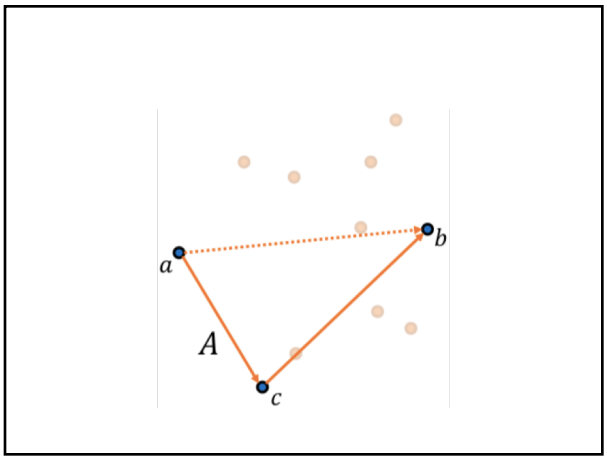

a
b
c


a
b
A
B
c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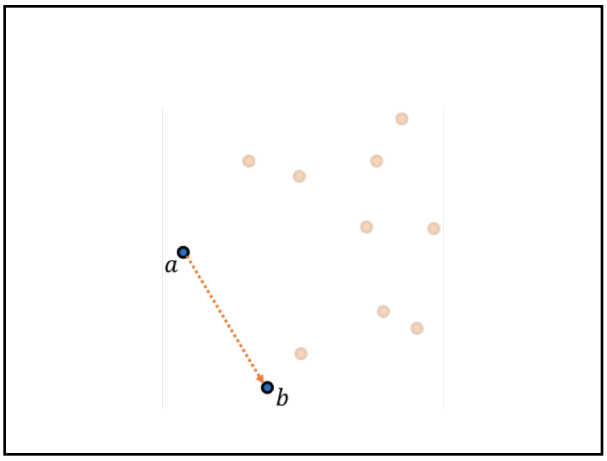
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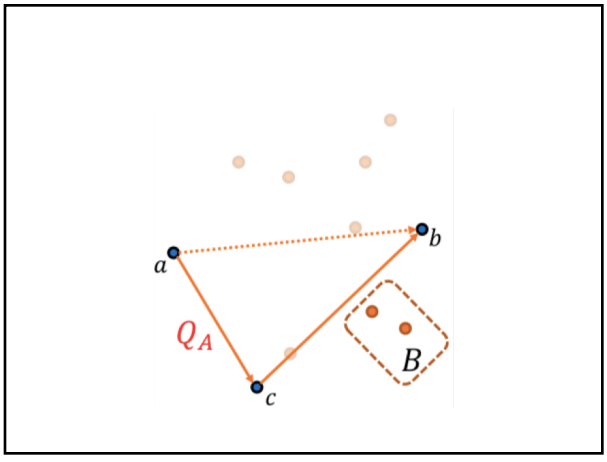
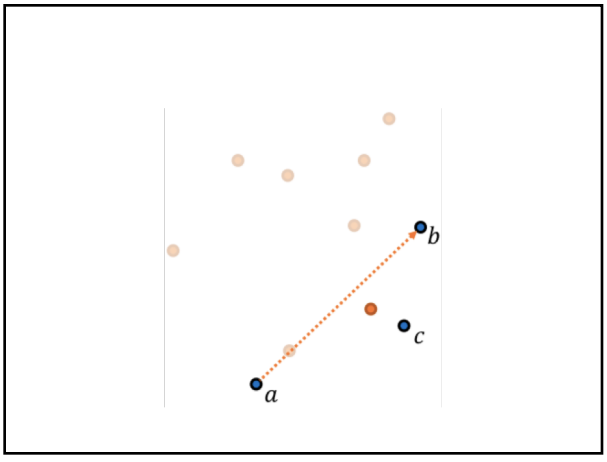

a
b
Q_A
B
c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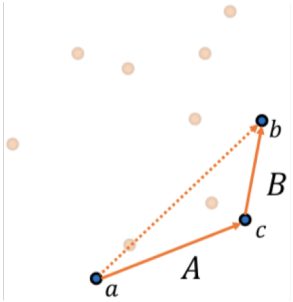
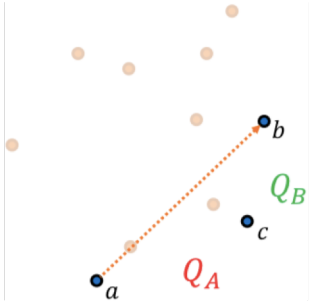
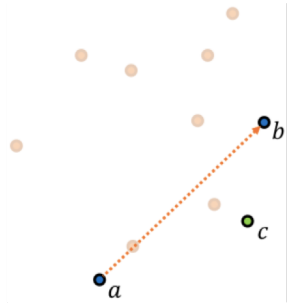
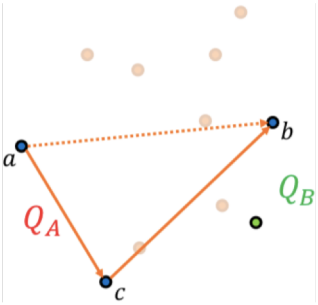
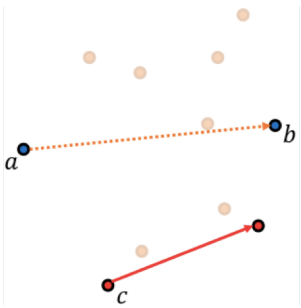
a



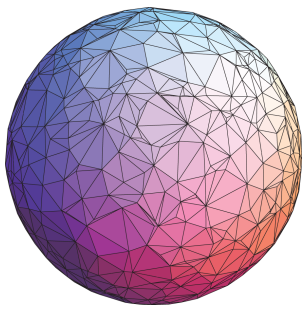




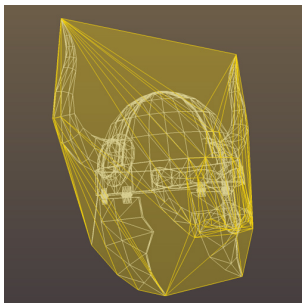




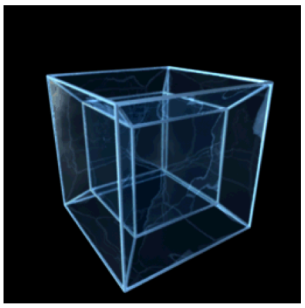
Hull of 758 Random 3D Points



Helmet Hull



Hyper Cube

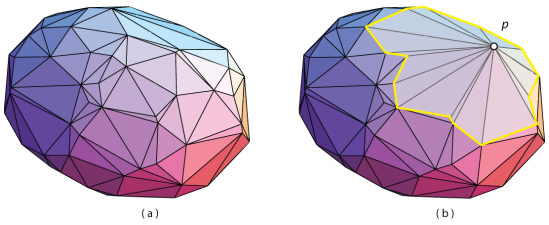


Bounds

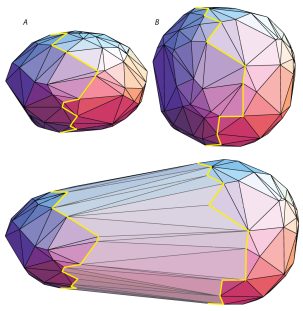
- Convex hull of n points in d dimension has a lower bound of $\Omega(n^{\lfloor d/2 \rfloor})$
-

Algorithm	2D	3D
Incremental	$O(n^2)$	$O(n^2)$
Gift wrapping	$O(nh)$	$O(nf)$
Divide-and-Conquer	$O(n\log n)$	$O(n\log n)$
Graham scan	$O(n\log n)$	
Qhull	$O(n\log h)/O(nf)$	$O(n\log h)/O(nf)$

3D Incremental Hull



3D Divide-and-Conquer



Possible Shadow Boundary Edges