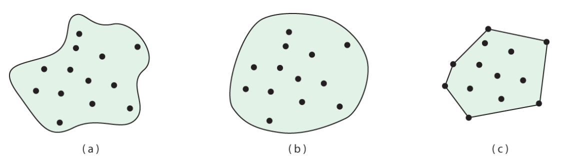
Computational Geometry

Convex Hull

Convexity

- A set S is convex if any two points in S are visible to each other.
- The convex hull of S, denoted by conv(S) or ($\mathcal{H}(S)$), is the intersection of all convex sets that contain S.

Convex Hull

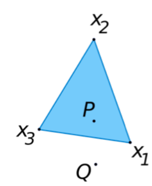


Convex Combinations

- A convex combination of a set of points $S = \{p_1, p_2, \ldots p_n\}$ is of the form:

$$\lambda_1 p_1 + \lambda_2 p_2 + \ldots + \lambda_n p_n,$$

where $\lambda_i \geq 0$ and $\sum \lambda_i = 1$



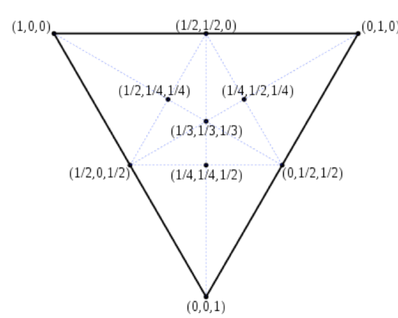
Barycentric Coordinates

- The barycentric coordinates of a point p with respect to a polygon of n vertices are the set of unique real numbers $a_1, a_2, \ldots, a_n$, such that

  $a_1 v_1 + a_2 v_2 + \ldots + a_n v_n = p$,

  where $a_i \geq 0$ and $\sum a_i = 1$
- The point p is known as the Bary center when all the weights a_i are evenly distributed.

Triangular Barycentric Coordiantes



Theorem

- The convex hull of *S* is the set of all convex combinations of *S*.
- Let $M = \{\lambda_1 p_1 + \ldots + \lambda_n p_n \mid \lambda_i \geq 0, \Sigma\ \lambda_i = 1\}$
- conv(S) $\subseteq M$: by proving that *M* is convex
- $M \subseteq$ conv(S) : by induction on *n*