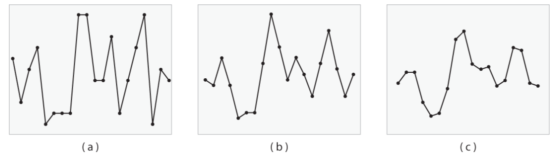
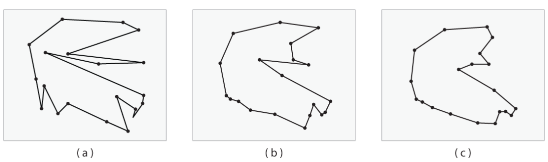
Computational Geometry

Curve Shortening

Midpoint Transformation



On a Closed Curve



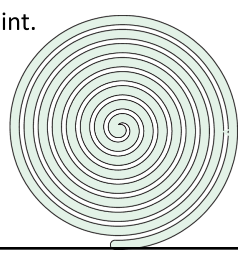
Curve Evolution

- Let C(s) = (x(s), y(s)) be a smooth closed curve parameterized by arc length s.
- Add a time variable t, defining a curve C(s, t). The curve evolves with t according to the differential equation:

  $\partial C/\partial t = \partial^2 C/\partial s^2 = K\eta$
- Each point p of the curve move along the normal at p with speed proportional to the curvature

Curve Shortening Theorem

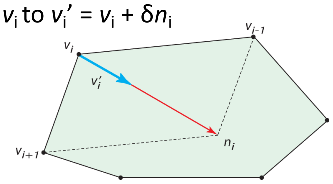
- Every smooth, simple closed curve C evolves under the flow defined by the equation so that it remains simple for all time and converges to a round point.
- Convexifies without self-intersection
- [twisted curve]
- [spiral]
- [flower]



Discrete Flow

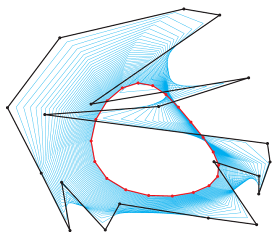
- Replace n-sided polygon P with its midpoint transformation
- Approximate normal at v_i as

  $n_i = (v_{i+1} - v_i) + (v_{i-1} - v_i)$
- Move vertex v_i to $v_i' = v_i + \delta n_i$



Discrete Curve Shortening

- Every simple polygon evolves under the flow so that it converges to a point whose shape is asymptotically an affine transformation of a regular polygon.



64-gon with 100 Iterations