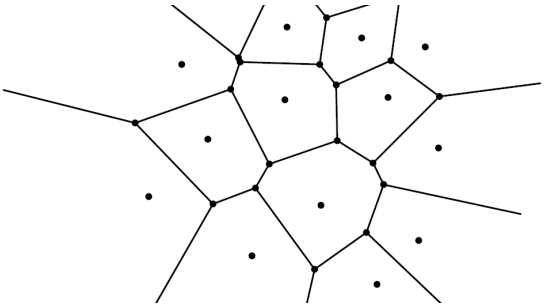
Computational Geometry

Medial Axis

Straight Skeleton

Voronoi



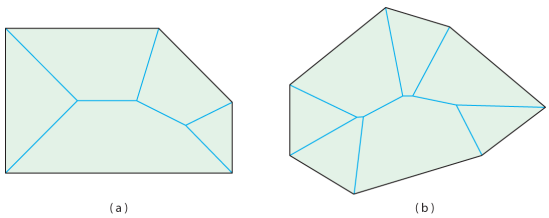
Alternative Voronoi Definitions

- Vor(*S*) is the locus of centers of maximal empty circles – those whose interior contain no site of *S*.
- Vor(*S*) is the locus of points to which there are two or more nearest sites
- Vor(*S*) is the set of "quench points" if the plane is burned uniformly and simultaneously from all sites in *S*.

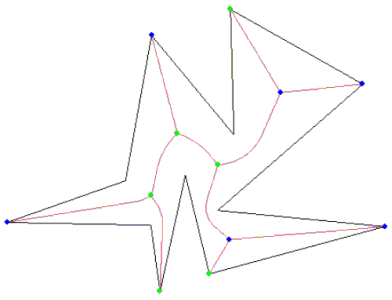
Definition

- The *medial axis* M(*P*) of a polygon *P* is the closure of the set of points in *P* that have two or more closest points among the points on δP.
- This also known as the *cut locus* of δP in Mathematics.

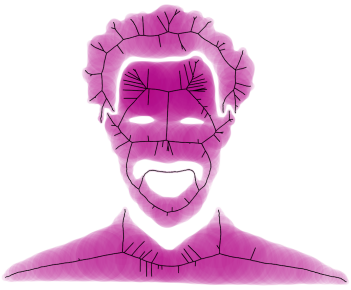
Medial Axes



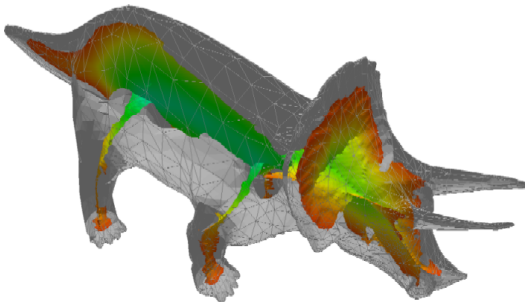
Non-Convex



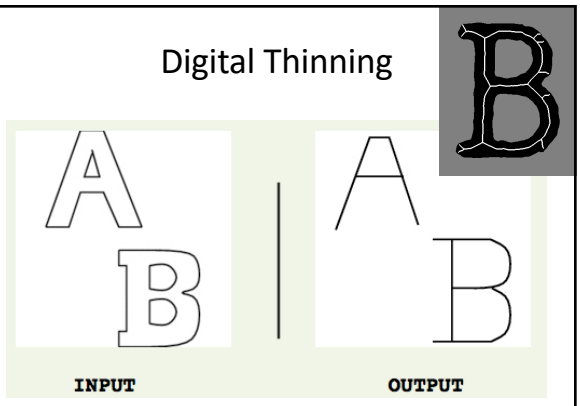
Face



3D Distance Field



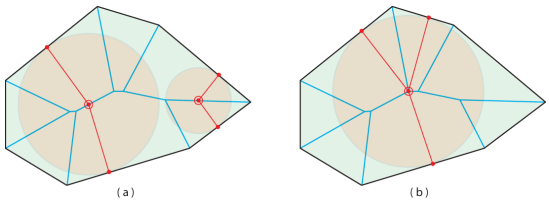
Digital Thinning



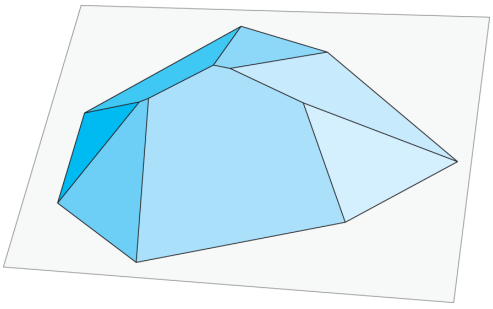
Medial Axis of a Convex Polygon

- A geometric tree of straight segments whose leaves are the vertices of P.
- Points on the medial axis are centers of maximal circles that touch δP in two or more points.
- Starting a fire δP and burning into the interior would again result in the "quench points" being the medial axis.

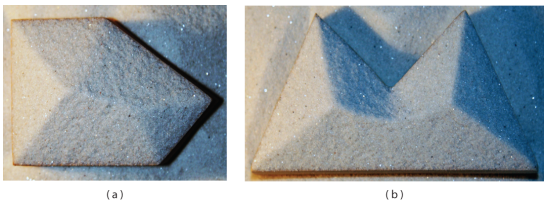
Maximal Circles



Medial Axis Polyhedron

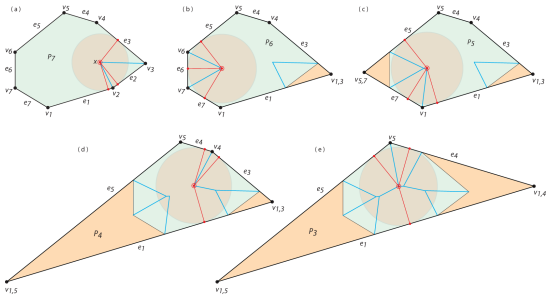


Sand-constructed Physical Models



(a) (b)

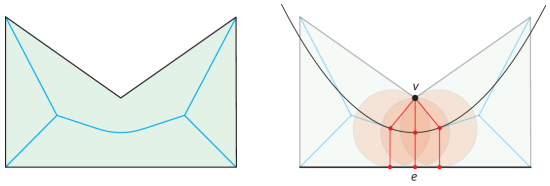
Constructing the Medial Axis



Time Complexity

- Find the first pair of intersecting bisectors by checking all pairs of adjacent vertices – O(n)
- Recursion
- O(n^2)
- O($n$log$n$) is possible with data structure (priority queue)
- O(n) possible with considerably more work and cleverness

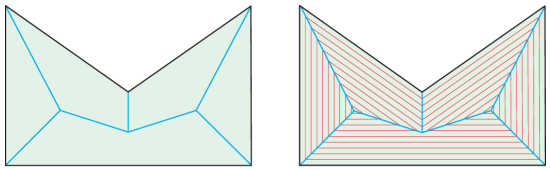
Non-Convexity



Straight Skeleton

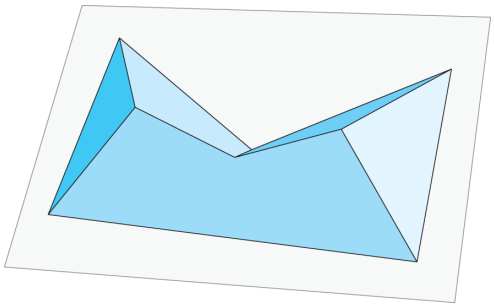
- shrink δP via parallel transformation of all edges inward
- each vertex (including reflex) follows the angle bisector
- stop when
  - an edge is 0 length
  - a reflex vertex collides with an edge – pinch into two polygons and continue

Straight Skeleton



(a) (b)

Straight Skeleton Polyhedron



Notes

- Voronoi definition does not hold
- Best algorithm runs in $O(n^{17/11})$
- Unsolved in 3D (2008)
- Unsolved for higher dimensions