

## Computational Geometry

### Introduction

### Resources

- [www.cs.brynmawr.edu/cs310](http://www.cs.brynmawr.edu/cs310)
- `~dxu/handouts/cs310`

### Prerequisites

- Discrete Math (CS/MATH 231)
- Mathematical maturity
- Programming proficiency

### Requirements

- Class participation (10%)
- 6-7 assignments (40%)
  - individual or group
- Midterm (25%)
- Final project (25%)

### Polygons

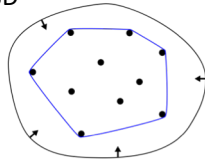
- Every polygon of  $n$  vertices may be guarded by  $n/3$  vertex guards.
- Not every polyhedron of  $n$  vertices may be guarded with one guard at every vertex
- Any pair of polygons of the same area have a common dissection
- Not every pair of polyhedra of the same volume have a common dissection

### Lindgren's Dissection of a Greek Cross to an Equilateral Triangle



## Convex Hulls

- The convex hull of  $n$  points in 2D can be constructed as quickly as those  $n$  points can be sorted.
- The same complexity can be achieved for the convex hull of  $n$  points in 3D



## Merging two 3D Hulls

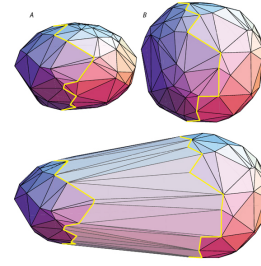


Figure 2.15. Two hulls A and B along with the hull of A ∪ B. The shadow boundaries are marked.

## Triangulations

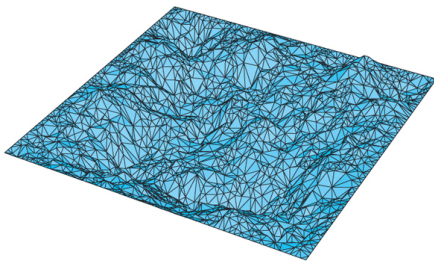
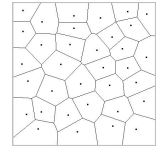


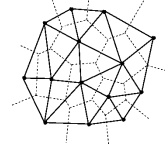
Figure 3.17. A piecewise-linear terrain reconstruction.

## Voronoi Diagram

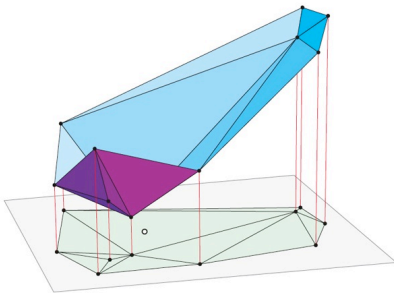
- Proximity diagram



- Delaunay Triangulation

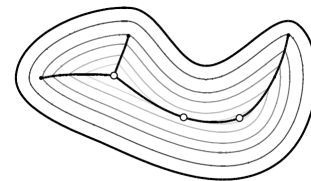


The Delaunay Triangulation is the projection of convex hulls in 3D



## Curves

- Medial axis – a complete shape descriptor
- Generalization of the Voronoi diagram



## Curve Shortening

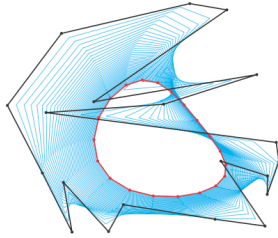


Figure 5.23. A discrete flow of a simple 20-gon (in black) with 40 iterations using  $\delta = 1/10$ .

## Polyhedra

- Euler's formula
- Gauss-Bonnet Theorem: The total curvature on any polyhedron is a constant:  $4\pi$ .
- Can any convex polyhedron be unfolded to a planar set?

