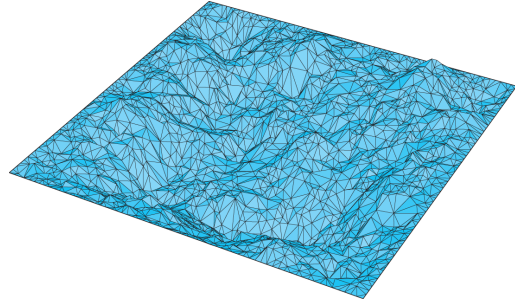


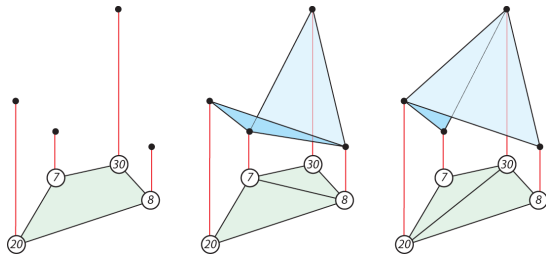
Computational Geometry

Triangulation

Terrain Reconstruction from Sampled Heights

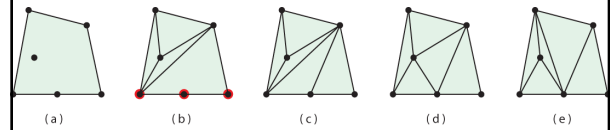


Mountains or Valleys?

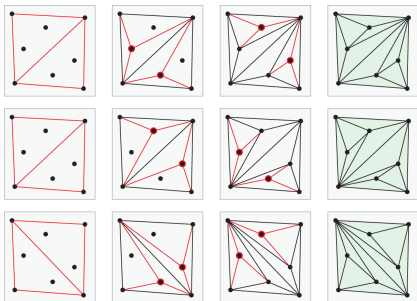


Definition

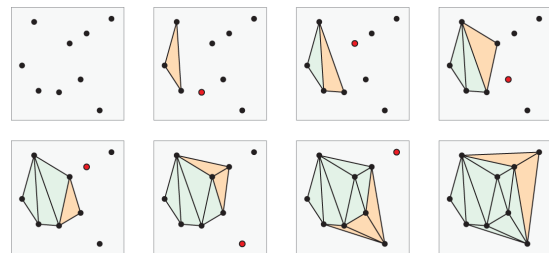
- A triangulation of a planar point set S is a subdivision of the plane determined by a *maximal* set of noncrossing edges whose vertex set is S .



Triangle Splitting



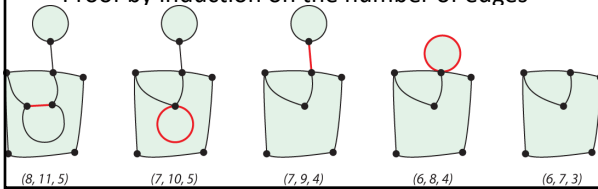
Incremental



Euler's Formula

- Let G be a connected planar graph with V vertices, E edges and F faces, then

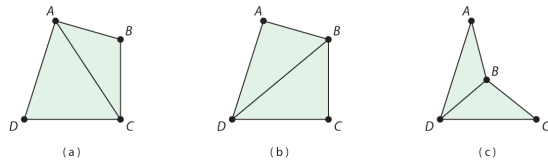
$$V - E + F = 2$$
- The outer face is unbounded
- Proof by induction on the number of edges



Theorem

- Let S be a point set with h points on the hull and k in the interior. If all points are in general position, then any triangulation of S has exactly $2k + h - 2$ triangles and $3k + 2h - 3$ edges.

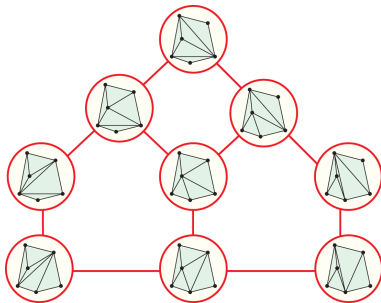
Edge Flip



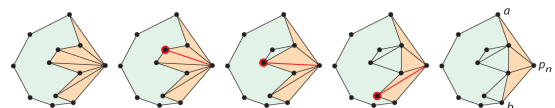
Definition

- For a point set S , a flip graph of S is a graph whose nodes are the set of triangulations of S . Two nodes T_1 and T_2 are connected by an edge if one diagonal of T_1 can be flipped to obtain T_2 .

Flip Graph



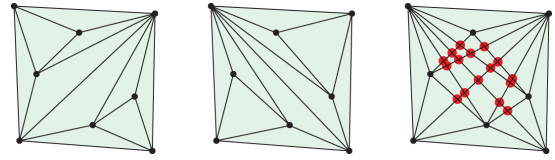
Flipping a Star



Theorem

- The flip graph of any planar point set is connected.
- The *diameter* of a graph is the longest path between any two nodes of the graph, in number of edges.
- For a planar point set S of n points, the diameter of its flip graph is at most $(n - 2)(n - 3)$

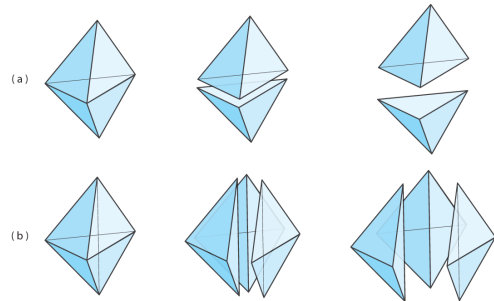
Tighter Bound



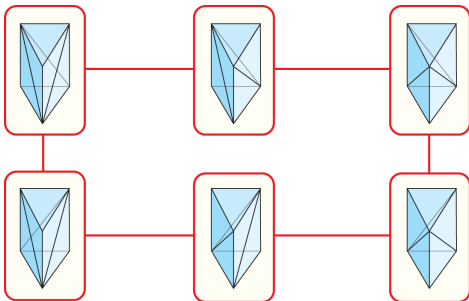
3D

- Let S be a point set in R^3 in general position, with k points in the interior and h on the hull. Then there exists a tetrahedralization of S with at most $3k + 2h - 7$ tetrahedra.

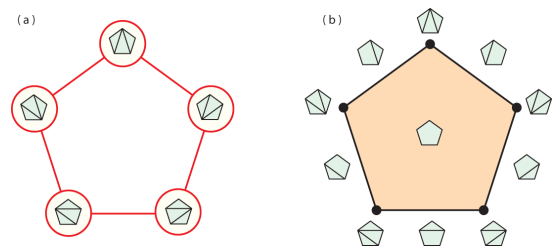
Face Flip



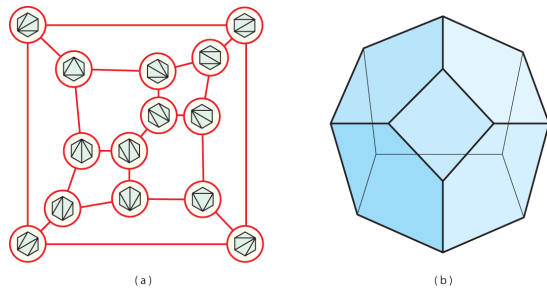
3D Flip Graph



Diagonalization of a Pentagon and the 2D Associahedron



Hexagon Flip Graph and the 3D Associahedron



Associahedra in Higher Dimensions

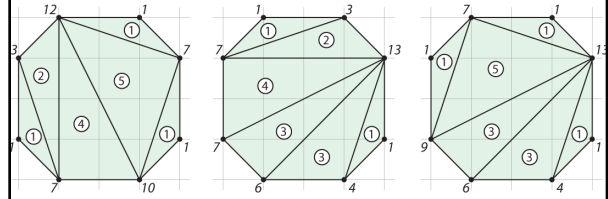
- There exists a convex n -dimensional polytope called the associahedron whose vertices and edges form the flip graph of a convex $(n + 3)$ -sided polygon. The k -dimensional faces of this polytope are in one-to-one correspondence with the diagonalization of the polygon using exactly $n - k$ diagonals.

4D Associahedron Representing the Flip Graph of a Heptagon



Triangulation as a Point in R^n

- Area Vectors
 - Left: (1, 12, 3, 1, 7, 10, 1, 7)
 - Middle: (3, 1, 7, 7, 6, 4, 1, 13)
 - Right: (1, 7, 1, 9, 6, 4, 1, 13)



Theorem

- Given a convex polygon P with n vertices, the convex hull of the area vectors of all triangulations of P is called the secondary polytope of P , denoted $\Sigma(P)$.
- $\Sigma(P)$ is combinatorially equivalent to the associahedron of dimension $n - 3$.
 - The area vectors of P lie in an $(n - 3)$ -dimensional plane of R^n
 - All area vectors lie on the hull