Computational Geometry

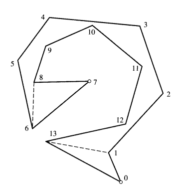
Polygon Triangulation

History of Triangulation Algorithms

Year	Complexity	Reference
1911	O(n^2)	Lennes
1978	O(nlogn)	Garey et al
1983	O(nlogr), r reflex	Hertel & Mehlhorn
1984	O(nlogs), s sinuosity	Chazelle & Incerpi
1988	O(n(1+t_0)), t_0 interior triangles	Toussaint
1988	O(nloglogn)	Tarjan & Van Wyk
1989	O(nlog*n), randomized	Clarkson, Tarjan & Van Wyk
1990	O(nlog*n), simple data structure	Kirkpatrick, Klawe & Tarjan
1991	O(n)	Chazelle
1991	O(nlog*n), randomized	Seidel
2000	O(nlog*n), randomized, simple	Amato, Goodrich, Ramos

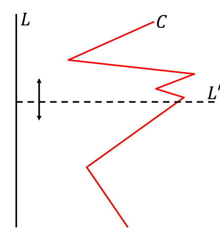
Polygon Triangulation

- By ear removal
  - Search for an ear-diagonal
  - Cut off ear and repeat on the rest of the polygon, which now has *n*-1 vertices
- Complexity?



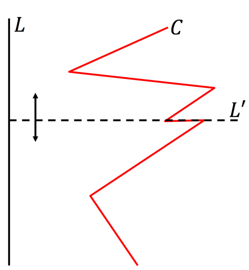
Monotonicity

- Defined with respect to a line
- A polygonal chain C is strictly monotone with respect to L if every line L' orthogonal to L meets C in at most one point



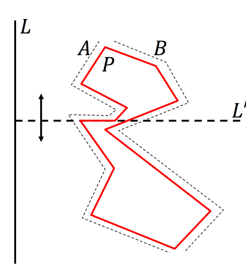
Monotonicity

- A polygonal chain C is monotone with respect to L if every line L' orthogonal to L meets C in at most one connected component



Monotone Polygon

- A polygon P is monotone w.r.t. a line L if its boundary can be split into two polygonal chains such that both are monotone w.r.t. to L

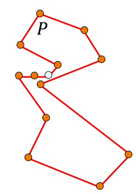


Why do we care?

- Vertices of a y-monotone polygon (w.r.t. the y-axis) can be sorted by y-coordinate in linear time
- $O(n)$: compute the extreme vertices (highest or lowest)
- $O(n)$: merge of two sorted chains

Interior Cusp

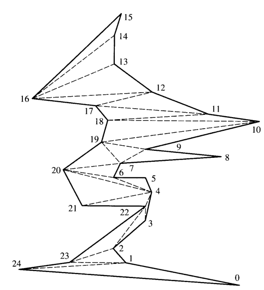
- An interior cusp of a polygon P (w.r.t. the y-axis) is a reflex vertex v whose neighboring vertices are either at or above, or at or below v.
- If P has no interior cusps, then it is monotone
- If it does, it can still be monotone



Monotone Polygon Triangulation

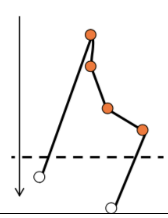
- Sort the vertices top to bottom (linear time)
- For each vertex v, connect v to all vertices above it and visible via a diagonal
- Continue with the next vertex below v

Monotone Polygon



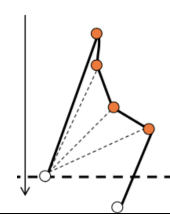
Monotone Polygon Triangulation

- From the top vertex, at any y-value, the un-triangulated vertices above y can be broken up into two chains:
  - single vertex
  - reflex chain



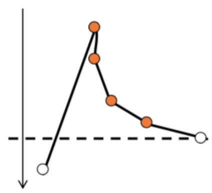
Next Vertex

- One the side with one vertex
- Connect to reflex chain
- pop off triangles



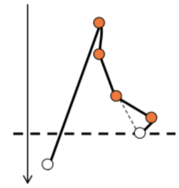
Next Vertex

- On the side of the reflex chain
  - extends the chain

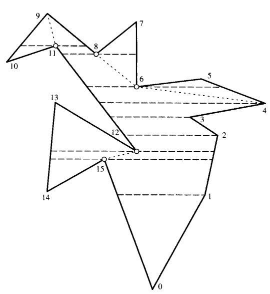


Next Vertex

- On the side of the reflex chain
  - doesn't extend chain
  - connect on the same side and pop

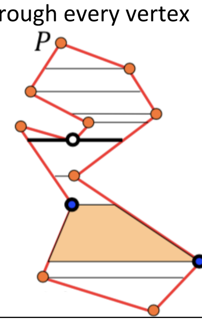


Trapezoidalization



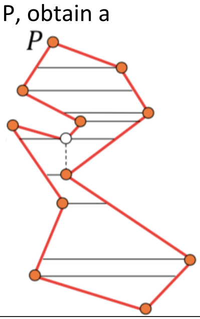
Horizontal Trapezoidalization

- Drawing a horizontal line through every vertex
- supporting vertices
  - those that define the two horizontals
  - cusps are interior to the horizontals



Monotone Partition

- Given a trapezoidalization of P, obtain a partition into monotone polygons
  - upward cusp: connect to support vertex below
  - downward cusp: connect to support vertex above



No Cusp = Monotone