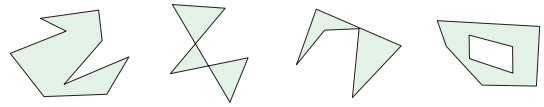


Computational Geometry

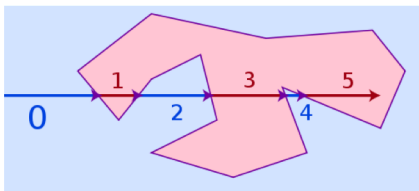
Polygon, Triangulation and Art Gallery

Polygon

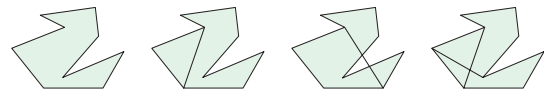


Jordan Curve Theorem

- The boundary δP of a polygon P partitions the plane into two parts.

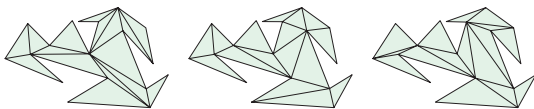


Diagonal



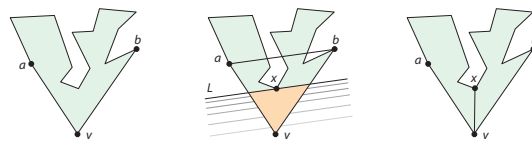
Triangulation

- A triangulation of a polygon is a decomposition into triangles with maximal non-crossing diagonals.



Existence of a Diagonal

- Every polygon with $n > 3$ vertices has a diagonal.

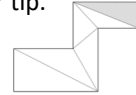


Theorem

- Every polygon admits a triangulation.
- Every triangulation of a polygon P with n vertices has $n-2$ triangles and $n-3$ diagonals.
- Proof by strong induction

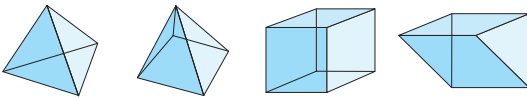
Meister's Two Ears

- Three consecutive vertices a , b and c on the boundary of a polygon form an ear if ac is a diagonal. b is known as an ear tip.

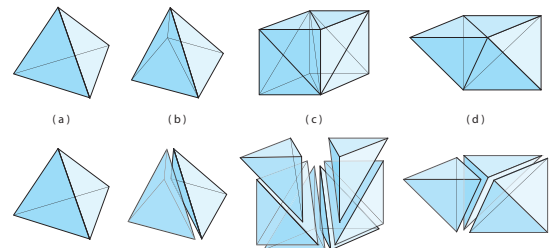


- Every polygon with $n > 3$ vertices has at least two ears.

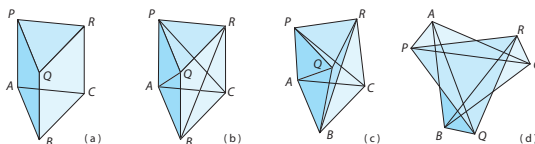
Polyhedra



Tetrahedralization



Schönhardt Polyheron

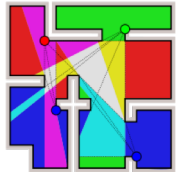


Open Problem

- Determining whether a polyhedron is tetrahedralizable is NP-complete (1992).
- Identifying a large natural class of tetrahedralizable polyhedra?

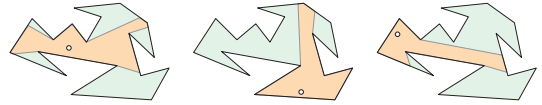
The Art Gallery Problem

- Polygon models the floor plan
- Guards are stationary and have 360° visibility unless blocked by walls
- What is the minimum number of guards needed to cover an arbitrary polygon of n vertices?

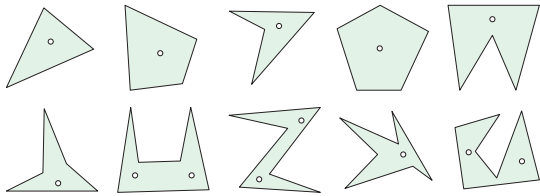


Visibility

- Vertices do not block vision
- $xy \in P \rightarrow x$ sees y

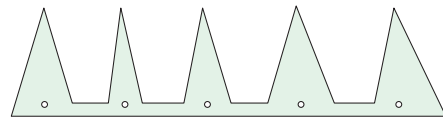


Examples



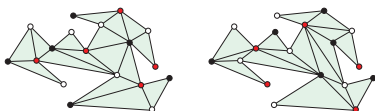
The Necessity of $\lfloor n/3 \rfloor$

- The comb

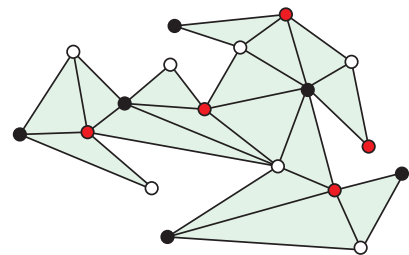


The Sufficiency of $\lfloor n/3 \rfloor$

- Fisk's 1978 proof is based on triangulation and graph coloring
- A coloring of a graph is an assignment of colors to nodes so that no adjacent nodes have the same color

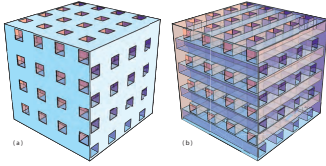


Every Triangulation Can be 3-colored



3D Art Gallery

- Arbitrary polyhedra can not always be tetrahedralized.
- The Seidel polyhedron that requires $>n$ guards



Open Problems

- Edge guards: $\lfloor n/4 \rfloor$?
- Mirror walls