Corrigendum to "Strong Normalization Proof with CPS-Translation for Second Order Classical Natural Deduction"

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Our paper [1] contains a serious error. Proposition 4.6 of [1] is actually false and hence our strong normalization proof does not work for the Curry-style $\lambda\mu$-calculus. However, our method still can show that (1) the correction of Proposition 5.4 of [2], and (2) the correction of the proof of strong normalization of Church-style $\lambda\mu$-calculus by CPS-translation.

Firstly, our method is still effective for the correction of Proposition 5.4 of [2]. The proposition claims that for any Curry-style $\lambda\mu$-term u, which is not necessarily typable, if u^* is strongly normalizable, then u is strongly normalizable too. But its proof does not work, since Proposition 5.1 (i) of [2] is false because of erasing-continuation. Our method proves the similar result for the Curry-style $\lambda\mu$-calculus by Propositions 4.3 and 4.12 of [1].

Proposition. *For any Curry-style $\lambda\mu$-term u, if there exists an augmentation u^+ of u such that u^{+*} is strongly normalizable, then u is strongly normalizable.*

Secondly, as mentioned in the concluding remarks of [1], our method is effective for the strong normalization proof of the Church-style $\lambda\mu$-calculus, which is called the second-order typed $\lambda\mu$-calculus in [2]. The strong normalization of the typed $\lambda\mu$-calculus is proved in [2], but its proof with CPS-translation does not work since Proposition 5.5 of [2] is false because of erasing-continuation.

For the Church-style system, the CPS-translation preserves typability of terms, and the strong normalization is proved by our method in [1]. Definition 4.7 in [1] is naturally changed for Church-style terms as follows:

$$\mathrm{Aug}(\mu\alpha^A.t) = \{\mu\alpha^A.(\lambda z^\perp.t^+)([\alpha^A]c^{\forall X.X}\vec{a});$$
$$t^+ \in \mathrm{Aug}(t),\ z^\perp \text{ is a fresh } \lambda\text{-variable and } \vec{a} \text{ is a finite sequence of terms and types}\}.$$

Then, similarly to the case of the Curry-style, we can prove the following facts, where $\rhd_\lambda$, $\rhd_\mu$ and $\rhd_\forall$ are defined as in [2].

Lemmas. (1) *If $t : \Gamma \vdash A, \Delta$ is provable in the typed $\lambda\mu$-calculus, then there is an augmentation t^+ of t such that $t^+ : \Gamma, (\forall X.X)^c \vdash A, \Delta$.*

(2) *If $t \rhd^1_\lambda u$ and t^+ is an augmentation of t, then there exists an augmentation u^+ of u such that $t^{+*} \rhd^+ u^{+*}$.*

(3) *If $t \rhd^1_\mu u$ or $t \rhd^1_\forall u$, and t^+ is an augmentation of t, then there exists an augmentation u^+ of u such that $t^{+*} \rhd u^{+*}$.*

Using these lemmas, the strong normalization of the typed $\lambda\mu$-calculus is proved as follows.

Theorem. *Any typed term of the typed $\lambda\mu$-calculus is strongly normalizable.*

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Proof. Suppose that there exists an infinite sequence of typed $\lambda\mu$-terms $\{t_i\}_{i<\omega}$ such that $t_i \rhd^1 t_{i+1}$ for all $i < \omega$. Then there are infinitely many i such that $t_i \rhd^1_\lambda t_{i+1}$ as proved in [2]. We can find an augmentation t_0^+ of t_0 which is typed, then there is an infinite sequence of λ-terms $\{t_i^{+*}\}$ such that $t_i^{+*} \rhd t_{i+1}^{+*}$ by (2) and (3) of the above lemmas. Then there are infinitely many i such that $t_i^{+*} \rhd^+ t_{i+1}^{+*}$ by (2) of the above lemmas, but it contradicts the strong normalization of the second-order λ-calculus. $\Box$

References

[1] K. Nakazawa, M. Tatsuta, Strong Normalization Proof with CPS-Translation for Second Order Classical Natural Deduction, *Journal of Symbolic Logic* **68** (3) (2003) 851-859.

[2] M. Parigot, Proofs of Strong Normalization for Second Order Classical Natural Deduction, *Journal of Symbolic Logic* **62** (4) (1997) 1461-1479.