Quantification Theory

→ Symbolization 5-6 marks
→ Proofs → Valid 5-6 "
→ Invalid 4 "

* All Alsatians are Dogs.
All Dogs are Mammals.
∴ All Alsatians are Mammals.

P
Q
∴ R

Predicate Logic
We talk in terms of Predication

What we learnt earlier was Propositional Logic.

→ How Do we Symbolize?

u { All S is P, No S is P } Universal Quantifier — Represented by (x) [For all x]

Exists { Some S is P, Some S is not P } Existential Quantifier $(\exists x)$ [There exist x]

1. All S is P

$$(x)\left[S_x \supset P_x\right] \rightarrow A.$$

For all x, If x is S then x is P.

x

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(2) No S is P

$$(x)\,[S_x \supset \sim P_x] \quad E$$

For all x, If x is S then x is not P.

(3) Some S is P.

$$(\exists x)\,[S_x \cdot P_x]$$

There exists (at least one) x such that x is S and x is P.

(4) Some S is not P

$$(\exists x)\,[S_x \cdot \sim P_x]$$

There exists at least one x such that x is S and x is not P

→ What if it is not in Standard Form?

Understand → Intention
→ Usage.

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Eg
- Only citizens are voters.
- Arbitrary ← An elephant is a mammal.
- Existence ← There are white elephants.
- Existence ← Children are present.
- Not all snakes are poisonous.

1) All voters are citizens.

$\longrightarrow (x) [V_x \supset C_x]$.

2) All elephants are mammals.

$(x) [E_x \supset M_x]$.

3) Some elephants are white.

$(\exists x) [E_x \cdot W_x]$

4) Some children are present.

$(\exists x) [C_x \cdot P_x]$

5) Some snakes are not poisonous.

$(\exists x) [S_x \cdot \sim P_x]$

↓

Square of opposition

(F) A E

I O (T)

||

$\sim (x) [S_x \supset P_x]$

It is not the case that, For all x, If x is S then x is P.

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Q) All that glitters is not gold.
(F)

Some that glitters is not gold. O

$\sim A \equiv O$
$\sim E \equiv I$
$\sim I \equiv E$
$\sim O \equiv A$

$$(\exists x)\,[G_x \cdot \sim D_x]$$
$$\equiv \sim (x)\,[G_x \supset D_x]$$

There exists at least one x such that x glitters and x is not gold.

symbolizes V, OR

Q) Apples and oranges are nourishing and delicious.

$$(x)\,[(A_x \vee O_x) \supset (N_x \cdot D_x)]$$

~~Ther~~ For all x, if x is an apple or an orange, then x is nourishing and x is delicious.

For all x, if x is an apple or an orange, x is nourishing & x is delicious

Or, $(x)\,[A_x \supset (N_x \cdot D_x)] \cdot (x)\,[O_x \supset (N_x \cdot D_x)]$

Q) Pickles are edible unless they are rotten.

$$(x)\,[P_x \supset (E_x \vee R_x)]$$

For all x, if x is a Pickle, then x is edible or x is rotten.

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$\equiv \quad (x)\left[P_x \supset (\sim E_x \supset R_x)\right]$

VVIP

Q) Among snakes only Rattlers and Copperheads are Poisonous & Fatal.

A) $\cancel{(x)\{S_x \supset [(R_x \vee C_x) \supset (P_x \cdot F_x)]\}}$.

$\cancel{(\exists x)\left[S_x \cdot \sim P_x \cdot \sim F_x\right]}$

Among snakes $\boxed{\text{only}}$ R & C are P&F.

S only. R&C are P&F.

$(x)\{S_x \supset [(P_x \cdot F_x) \supset (C_x \vee R_x)]\}$.

$\equiv (x)\{(S_x \cdot P_x \cdot F_x) \supset (C_x \vee R_x)\}$.

Only S is P.

~~If P is~~ If x is P then x is S

Q) Cats and dogs bite if they are frightened or harassed.

$(x)\{(C_x \vee D_x) \supset [(F_x \vee H_x) \supset B_x]\}$.

* Provided that ≡ if ≡ when

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Q) Irene is either a doctor or a lawyer.

$$\cancel{(x)\left[I_x \supset (D_x \vee L_x)\right]}$$

→ $D \vee L$ — Propositional logic

→ $D_i \vee L_i$ — Predicate logic

Q) The Times of India is a newspaper.

$$N_t$$

Q) If Paris is beautiful, then Andre told the truth.

$$B_p \supset T_a$$

Q) Everything is ~~modern~~ mortal

A

$$(x) M_x$$

Q) Something is mortal

I

$$(\exists x) M_x.$$

$\sim(\exists x) G_x$ $\qquad \exists_x A_x$

Q) Animals exist $\qquad (x)\left[A_x \supset E_x\right]$

Ghosts do not exits $\qquad (x)\left[G_x \supset \sim E_x\right]$

Anything is conceivable $\qquad (x)\, C_x$

$\exists_x A_x \quad ; \quad \sim(\exists x) G_x \quad ; \quad (x)\, C_x.$

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Q) There are happy marriages

$$(\exists x)\,[Hx \cdot Mx].$$

Q) Not a ~~p~~ single psychologist attended the conference.

$$(x)\,[Px \supset \sim Cx]$$

Q) Something is not mortal. O

Nothing is mortal. E

A E
I O

✓ $(\exists x)(\sim Mx) \equiv \sim (x)(Mx)$ ∝

✓ $(x)(\sim Mx) \equiv \sim (\exists x)(Mx)$ ∝

* Always go for LITERAL meaning.

Q) Some medicines are dangerous only if taken in excessive amount.

P only if Q
P ⊃ Q.

$(Dx \supset Ex)$

$$(\exists x)\,[Mx \cdot \cancel{(Ex \supset Dx)}]$$

Particular ⟹ main connective (·)

Universal ⟹ main connective (⊃, ≡)

Q) None but the brave deserve the fair.

$$(x)\,[Dx \supset Bx]$$

None But : Only

* Only if
* Only

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Q) Not every visitor stayed for dinner.

$$\sim (x) \cancel{\exists_x} [V_x \supset S_x]$$

$$\equiv \exists_x [V_x \cdot \sim S_x]$$

VIP

Q) Not any visitor stayed for dinner.

$$(x) [V_x \supset \sim S_x]$$

* Not any : No (E)
* Not every : ($\sim$ A) $\sim$ All

VIP

Q) Everything enjoyable is either immoral, illegal or Fattening.

A) $\cancel{(x) [E_x \supset (I_x \vee L_x \vee F_x)]}$

$$(x) [E_x \supset (\sim M_x \vee \sim L_x \vee F_x)]$$

* We cannot eat or drop-off negation but we should develop negation.

✓ Immoral → Not Moral ✓

× Not Mortal → Immortal ×

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Q) Not every actor is talented who is famous.

$$\cancel{\sim (x) [F_x \supset T_x]}$$

$$\sim (x) \{A_x \supset [F_x \supset T_x]\}$$

$$\equiv \sim (x) \{(A_x \cdot F_x) \supset T_x\}.$$

$$\equiv \exists_x [(A_x \cdot F_x) \cdot \sim T_x].$$

* $\sim (x) [P_x \supset Q_x] \equiv \cancel{\sim} (\exists x) \sim [P_x \supset Q_x]$

$$\equiv (\exists x) \sim [\sim P_x \vee Q_x]$$

$$\equiv (\exists x) [P_x \cdot \sim Q_x].$$

Existential Negation Theorem.

* Singular & General proposition.

Subject is individual; Predicate is class.

Singular → Mohan is a student

General → (A E I O), All S is P, No S is P, Some S is P, Some S is not P

S_P is a proposition ; S_x is not a proposition

* S_x is called propositional function because its truth value depends on what x is. It is a function of x.

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(x) → Universal quantifier
($\exists x$) → Existential quantifier

P_d d : Donald Trump
P : President of USA.

Q) All politicians & business persons are honest.

$$(x)\left[(P_x \vee B_x) \supset H_x\right].$$

For all x, if x is a Politician or x is a B.P., then, x is honest.

Q) All politicians who are business persons are honest.

$$(x)\left[(P_x \cdot B_x) \supset H_x\right]$$

Q) No policeman who has below 6 ft height is a Black Commando.

$$(x)\left[(P_x \cdot H_x) \supset \sim B_x\right]$$

VVIP

Q) If BJP wins in 2019 election, then some politicians will be disappointed.

$$\cancel{(\exists x)\left[(E_b \cdot P_x) \cdot D_x\right]}.$$

$$\times \quad E_b \supset (\exists x)\left[P_x \cdot D_x\right].$$

Disappointed → Lose hope → $\sim H_x$.

$$\checkmark \quad \underbrace{E_b}_{\text{Antecedent : singular}} \supset \underbrace{(\exists x)\left[P_x \cdot \sim H_x\right]}_{\text{Consequent : general}}$$

chart

GOD ⇄ DOG

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VVVIP

Q) If all bereaucrats and vendors conspire the price of (all) public goods rise.

A) $(x)[(B_x \vee V_x) \supset C_x] \supset \cancel{R_p}\ (x)[P_x \supset R_x]$

VIP

Q) God exist $\longrightarrow$ $(\exists_x)\ G_x$ } Not singular $\boxed{\text{General}}$

$\cancel{E_g \rightarrow X}$

In quanti theory, existence is not considered as a predicate (quality)

General props

Dogs exists $\rightarrow$ $(\exists_x)\ D_x$. $\cancel{E_d}$

Trees exit $\rightarrow$ $(\exists_x)\ T_x$ $\cancel{E_t}$.

V.V.

4 Rules of Quantification

1. Universal instantiation
2. Universal generalization
3. Existential instantiation
4. Existential generalization.

Q) All politicians & govt. employees are either drawing their salary from public account or from charitable trust.

A) $(x)[(P_x \vee G_x) \supset (A_x \vee C_x)]$

Q) No horse that is well trained fails to be gentle.

$(x)[(H_x \cdot W_x) \supset \sim\sim G_x]$

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① Universal Instantiation (UI)

↓

Singular.

All students are studious

$(x)\,[\,S_x \supset T_x\,]$

∀

• Removing the quantifier in order to apply M.I, Trans. etc → Universal Instantiation

$$\frac{x\ \phi(x)}{\phi(a)} \longrightarrow \text{an example of } x$$

→ No condition.

$(x)\,[S_x \supset T_x] \longrightarrow S_a \supset T_a$.

② Universal Generalization (UG)

y → individual constant

→ Retrieve the quantifier.

$$\frac{\phi_y}{\cancel{x\,\phi(x)}\ (x)\phi_x}$$

Conditions
① y is a randomly selected constant
② y is not under any assumption.

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$1. Ga \longrightarrow$ assumption

$2. (x) Gx$ $\quad$ 1, UG $\quad$ X.

$$\frac{Sa \supset Ta}{(x)(Sx \supset Tx)}$$

(4) Existential Instantiation (E.I.)

(3) Existential Generalization (E.G.)

$$\frac{\phi_\nu}{(\exists x)\phi x}$$

no condition

(4) Existential Instantiation (E I)

$$\frac{(\exists x)\phi x}{\phi_\nu}$$

Condition

(1) ν is an individual constant which doesnot have a prior occurrence.

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Q) 1. $(x)\ (A_x \supset B_x)$

2. $\sim B_t$ $\quad / \therefore \sim A_t$

3. $A_t \supset B_t$ $\quad$ 1, U.I.

4. $\sim B_t \supset \sim A_t$ $\quad$ 3, Trans.

5. $\sim A_t$ $\quad$ 4, 2 M.P.

Q) 1. $(x)\ (C_x \supset D_x)$

2. $(x)\ (E_x \supset \sim D_x)$ $\quad / \therefore (x)\ (E_x \supset \sim C_x)$

3. $C_t \supset D_t$ $\quad$ 1, UI

4. $E_t \supset \sim D_t$ $\quad$ 2, UI

5. $D_t \supset \sim E_t$ $\quad$ 4, Trans.

6. $C_t \supset \sim E_t$ $\quad$ 3, 5 H.S.

7. $E_t \supset \sim C_t$ $\quad$ 6, Trans.

8. $(x)\ (E_x \supset \sim C_x)$ $\quad$ 7, UG.

Q) ~~F.~~ All members are both officers & gentlemen. All officers are fighters. Only a pacifist is either a gentleman or not a fighter. No pacifists are gentlemen if they are fighters. Some members are fighters if and only if they are officers. Therefore not all members are fighters.

A) 1. $(x)\ [M_x \supset (O_x \cdot G_x)]$

2. $(x)\ [O_x \supset F_x]$ $\quad$ ~~$(x)\ [P_x \supset (G_x \vee \sim F_x)]$~~

3. ~~$(x)\ [(G_x \vee \sim F_x) \supset P_x]$~~

$(x)\ [(G_x \vee \sim F_x) \supset P_x]$

4.

$P \vee Q$

Q

$M_P \rightarrow O_P \cdot G_P \rightarrow O_P \rightarrow F_P \rightarrow G_P \rightarrow P_P \cdot F_P.$

$\sim G_P \cdot F_P$ $\quad G_P$ $\quad \sim G_1$ $\quad \sim P_P \vee \sim F_P$ $\quad F_P \cdot \sim F_P$

$\sim P_P.$

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4. $(x)[(P_x \cdot F_x) \supset \sim G_x]$

5. ~~$(\exists_x)\{[(M_x \cdot F_x)] \equiv O_x\}$~~ $/ \therefore (\exists_x)[M_x \cdot \sim F_x]$.

$(\exists_x)\{M_x \cdot (F_x \equiv O_x)\}$.

* Start with instantiation of existential proposition. → 5

✓ Begin with EI <u>not</u> with UI.

6. $M_p \cdot (F_p \equiv O_p)$
7. $M_p \supset (O_p \cdot G_p)$
8. $O_p \supset F_p$
9. $P_p \supset (G_p \vee \sim F_p)$
10.

$(x)[(G_x \vee \sim F_x) \supset P_x]$

*

$$P \supset Q$$

Sufficient condition ↙ ↘ Necessary condition.

36 Q) Only well-trained horses are gentle.

~~$(x)[(G_x \cdot W_x) \supset$~~

$(x)[G_x \supset (W_x \cdot H_x)]$.

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37) Only horses are gentle if ~~they~~ they are well-trained

$$\cancel{(x)\left[H_x \cdot (W_x \supset G_x)\right]}$$

$$\cancel{(x)\left[(H_x \cdot W_x) \supset G_x\right]}$$

$$(x)\left[W_x \supset (G_x \supset H_x)\right]$$

$$(x)\left[(W_x \cdot G_x) \supset H_x\right]$$

Q) Not every person who talks a great deal has a great deal to say.

A) $$(\exists x)\left[P_x \cdot T_x \cdot \sim S_x\right]$$

Q) None but the brave deserve the fair
only

A) $$(x)\left[D_x \supset B_x\right]$$

Q) All that glitters is not gold.

A) $$\exists x\left[G_x \cdot \sim A_x\right].$$

27 Q) Some horses are gentle only if they are well-trained.

if P then Q
$P \supset Q$

Q if P
$P \supset Q$

Q only if P
$Q \supset P$

$$\cancel{\exists x\left[(H_x \cdot G_x) \supset W_x\right]}$$

$$(\exists x)\left[H_x \cdot (G_x \supset W_x)\right]$$

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28 Q) Some horses are gentle if they have been well trained.

$$(\exists x)\,[Hx \cdot (Wx \supset Gx)]$$

29) It is not true that {every watch will [keep good time if and only if (it is wound regularly and not abused)]}.

A) ✗ $\sim (x)\,[(Wx \supset Gx) \equiv (Rx \cdot \sim Ax)]$ ✗

✓ $\sim (x)\,\{Wx \supset [Gx \equiv (Rx \cdot \sim Ax)]\}$

39) ~~40)~~ Some horses are gentle even though they have not been well-trained.

A) $$(\exists x)\,[Hx \cdot \sim Tx \cdot Gx]$$

$$(\exists x)\,[Hx \cdot (Gx \cdot \sim Tx)]$$

→ Main connective

Even Though : But : Conjunction

Bracket $(Gx \cdot \sim Tx)$ is necessary to make Semantic significance. The first $(\cdot)$ is the main connective.

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Q) Bees and wasps sting if they are either angry or frightened. Therefore any bee stings if it is angry.

A) ~~$(x)\left[(B_x \vee W_x)\right.$~~

A) 1. $(x)\left\{\left[(B_x \vee W_x) \cdot (A_x \vee F_x)\right] \supset S_x\right\}$.

2/∴ $(x)\left[(B_x \cdot A_x) \supset S_x\right]$

2. $\left\{(B_p \vee W_p) \cdot (A_p \vee F_p)\right\} \supset S_p$.

3. $\sim\left\{(B_p \vee W_p) \cdot (A_p \vee F_p)\right\} \vee S_p$

4. $\left\{\sim(B_p \vee W_p) \vee \sim(A_p \vee F_p)\right\} \vee S_p$.

5. $\left[(\sim B_p \cdot \sim W_p) \vee (\sim A_p \cdot \sim F_p)\right] \vee S_p$.

6. $\left\{\left[\sim B_p \vee (\sim A_p \cdot \sim F_p)\right] \cdot \left[\sim W_p \vee (\sim A_p \cdot \sim F_p)\right]\right\} \vee S_p$

7. $\left\{(\sim B_p \vee \sim A_p) \cdot (\sim F_p \cdot \sim B_p)\right\} \vee S_p$

8. $(\sim B_p \vee \sim A_p) \vee S_p$

9. ~~$\sim B_p \vee +$~~ $\sim (B_p \cdot A_p) \vee S$

9. $(x)\left[(B_x \cdot A_x) \supset S_x\right]$

Easy
*

✓ By SCP

- Neat
- Simple
- Easy

$\to B_x$.

$(B_x \vee W_x)$

$(A_x \vee F_x) \supset S_x$

$\to A_x$

$A_x \vee F_x$

S_x

$(A_x \supset S_x)$

$(B_x) \supset (A_x \supset S_x)$

$(B_x \cdot A_x) \supset S_x$

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Indian Logic

Logic is Universal. → To help us to use the words correctly

Definition → To avoid confusion / Vagueness

- → Aristotle (Horos = Definition Horismos)
- → ~~Nyanya~~ Nyāya (Laksana = Definition).

Definiendum → What is being defined?
Phusis → encapsulating essence.

Genus and Difference are provided to Encapsulate the essence of Definiendum

Human defined as featherless biped.
(Chicken whose feathers were plucked = Critic).

Avyapti → Definition should not be too wide
Too wide = Ativyapti

Revised definition of 'Man'

Man is a rational animal.

Nyaya Sutra → Nyanya Theory of Laksha...

Nyaya Bhashya → Vatsyayana

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$\frac{2}{3}$ rd of Indo-European languages based in Sanskrit.

Atatva vyavachchedaka Dharma
= Definition according to Nyaya theory.

Difference b/w Nyaya & Aristotle's ~~the~~ theory

- In Nyaya theory, we need not state the essence.
- In Nyaya theory, we can have more than 1 defn. But only 1 defn in Aristotle's theory

Economy → ~~Lat~~ Laghava.

1) Economy in $\boxed{\text{Sharira}}$ (constitution)
   - We prefer one with lesser constituents. (definition)
2) Economy in $\boxed{\text{Sambandha}}$ (relation to definiendum)
   - we prefer sakshat (direct) reln as compared to parampara (indirect).

$\boxed{\text{Upasthiti}}$

3) Comprehensiveness (links to other concepts)
   - We prefer one defn which is more comprehensive

Skepticism → There is no true knowledge. All present knowledge is full of doubts

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Induction.

Induction is needed for the premise of a deduction.

Principle of uniformity of nature → nature is uniform. This assumption needs to be made to prove induction. But this itself is an induction!

Circularity: To use the definition of something to define that thing itself.

"Either accept induction or reject all science"
— Bertrand Russell.

① There are uniform causal lines in nature
② There is spatio-temporal continuity in nature.

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U.I

$(x)\ (P_x \supset Q_x)$

$P_y \supset Q_y$

R2 E.I.

$(\exists x)\ (P_x \cdot Q_x)$

$P_a \cdot Q_a$

where a is arb. chosen const

R1 U.G.

$P_y \supset Q_y$

$(x)\ (P_x \supset Q_x)$

E.G.

$P_a . Q_a$

$(\exists x)(P_x \cdot Q_x)$

Restriction.

R1 : You cannot apply U.G. on a line which is a consequence of E.I.
⇒ (any line which is coming out of E.I.)

R2 : E.I. cannot be performed with a constant which is having a free occurrence in any of the above lines.

free : not bounded by a quantifier

Remarks.

- If you have to instantiate 2 particular proposition in the same question, then, you may not require to instantiate both of them.

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- In case, you have to universally & existentially instantiate ;
  (i) Instantiate the existential proposition first.
  (ii) Use constants.

venn ↓ U first

Q) 1. $(x)(F_x \supset \sim G_x)$
2. $(\exists x)(H_x \cdot G_x)$ $/\therefore (\exists x)(H_x \cdot \sim F_x)$
~~3. $F_y \supset \sim G_x$~~ ($\because R_2$ is violated)
3. $Ha \cdot Ga$ — 2, E.I.
4. $Fa \supset \sim Ga$ — 1, U.I.
5. Ga — 3, Comm., Simp.
6. $\sim\sim Ga$ — 5, D.N.
7. $\sim Fa$ — 4, 6, M.T.
8. Ha — 3, Simp
9. $Ha \cdot \sim Fa$ — 8, 7, Conj
10. $(\exists x)(H_x \cdot \sim F_x)$ — 9, E.G.

Q-4 VIP

Q) A 1. $(x)[I_x \supset J_x]$
O 2. $(\exists x)[I_x \cdot \sim J_x]$ $/\therefore (x)[J_x \supset I_x]$
3. $Ia \cdot \sim Ja$ — 2, E.I
4. $Ia \supset Ja$ — 1, U.I.
5. Ia — 3, Simp.
6. Ja — 4, 5 M.P.
7. $\sim Ja$ — 3, Comm., Simp
→ * 8. $Ja \vee (x)[J_x \supset I_x]$ — 6, ~~7~~, Add
9. $(x)[J_x \supset I_x]$ — 8, 7, D.S.

* $\because$ premisses are A & O, contradictions will arise

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Q) 1. $(x)\ [(Bx \supset Cx) \cdot (Dx \supset Ex)]$

2. $(x)\ [(Cx \vee Ex) \supset \{[Fx \supset (Gx \supset Fx)] \supset (Bx \cdot Dx)\}]$

$/\therefore (x)\ [Bx \equiv Dx]$

$a \rightarrow y$

3. $(B\cancel{y} \supset C\cancel{y}) \cdot (D\cancel{y} \supset E\cancel{y})$ 1, U-I.

4. $(C\cancel{y} \vee E\cancel{y}) \supset \{[F\cancel{y} \supset (G\cancel{y} \supset F\cancel{y})] \supset (B\cancel{y} \cdot D\cancel{a})\}]$ 2, U-I.

5. Ba

6. $Ba \supset Ca$.

7. Ca

8. $Ca \vee Ea$

9. $[Fa \supset (Ga \supset Fa)] \supset (Ba \cdot Da)$

10. $Fa \cdot Ga$

11. $\cancel{Ga}\ Fa$

12. $(Fa \cdot Ga) \supset Fa$

13. $Fa \supset (Ga \supset Fa)$

14. $Ba \cdot Da$

15. Da

16. $Ba \supset Da$

OR.

Fy

$Fy \vee \sim Gy$

$\sim Gy \vee Fy$

$Gy \supset Fy$

$Fy \supset (Gy \supset Fy)$

17. Da

18. $Da \supset Ea$

19. Ea

20. $Ea \vee Ca$

21. $Ca \vee Ea$

22. $[Fa \supset (Ga \supset Fa)] \supset (Ba \cdot Da)$

23. $Fa \cdot Ga$

24. Fa

25. $(Fa \cdot Ga) \supset Fa$

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26. $Fa \supset (Ga \supset Fa)$
27. $Ba \cdot Da$
28. Ba
29. $Da \supset Ba$

30. $(Ba \supset Da) \cdot (Da \supset Ba)$
31. $(By \equiv Dy)$
32. $(x)(Bx \equiv Dx)$.

19 Q) Doctors and Lawyers are college graduates. Any altruist is an idealist. Some Lawyers are not idealists. Some doctors are altruists. Therefore, some college graduates are idealists.

A) 1. $(x)\,[(Dx \vee Lx) \supset Gx]$
2. $(x)\,[Ax \supset Ix]$
3. $(\exists x)\,[Lx \cdot \sim Ix]$
4. $(\exists x)\,[Dx \cdot Ax]$ $\quad / \therefore (\exists x)\,[Gx \cdot Ix]$
5. $\cancel{La \cdot \sim Ia}$ $Da \cdot Aa$ — 4, E.I.
6. $Aa \supset Ia$ — 2, U.I.
7. $(Da \vee La) \supset Ga$ — 1, U.I.
8. $\cancel{La}$ Da — 5, Simp.
~~9. $La \vee Da$~~
10. $(Da \vee La)$ — 8, Add.
11. Ga — 7, 10, M.P.
12. Ia — 5, Com, Simp, 6, M.P.
13. $Ga \cdot Ia$ — 11, 12, Conj.
14. $(\exists x)(Gx \cdot Ix)$ — 13, E.G.

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23. Q) All members are both officers and gentlemen. All officers are fighters. Only a pacifist is either a gentleman ~~and~~ or not a fighter. No pacifist are gentlemen if they are fighters. Some members are fighters if and only if they are officers. Therefore not all members are fighters.

1. $(x)\,[Mx \supset (Ox \cdot Gx)]$
2. $(x)\,[Ox \supset Fx]$
3. $(x)\,[(Gx \vee \sim Fx) \supset Px]$
4. $(x)\,[Px \supset (Fx \supset \sim Gx)]$
5. $(\exists x)\,[Mx \cdot (Fx \equiv Ox)]$

$/\therefore (\exists x)\,[Mx \cdot \sim Fx]$

~Pv~F.

6.	$Ma \cdot (Fa \equiv Oa)$	5, E.I.
7.	$Ma \supset (Oa \cdot Ga)$	1, U.I.
8.	Ma	6, Simp.
9.	$Oa \cdot Ga$	7, 8, M.P.
10.	$Oa \supset Fa$	2, U.I.
11.	Oa	9, Simp.
12.	Fa	10, 11 M.P.
13.	$Pa \supset (Fa \supset \sim Ga)$	4, U.I.
14.	$(Pa \cdot Fa) \supset \sim Ga$	13, Exp.
15.	$Ga \supset \sim (Pa \cdot Fa)$	14, Trans.
16.	Ga	9, Comm., Simp.
17.	$\sim (Pa \cdot Fa)$	15, 16 M.P.
18.	$\sim Pa \vee \sim Fa$	17, DeM.

Informal fallacy ?

Date ___ / ___ / ___

19. $(Ga \vee \sim Fa) \supset Pa$ — 3, U.I.
20. $Ga \vee \sim Fa$ — 16, Add.
21. Pa — 19, 20. M.P.
22. $\sim\sim Pa$ — 21, D.N.
23. $\sim Fa$ — 18, 22 D.S.
24. $Ma \cdot \sim Fa$ — 8, 23 Conj.
25. $(\exists x)(Mx \cdot \sim Fx)$ — 24, E.G.

Proving Invalidity.

3 variables.

$(x)[Px \supset Qx]$

$$(Pa \supset Qa) \cdot (Pb \supset Qb) \cdot (Pc \supset Qc)$$

$(\exists x)[Px \cdot Qx]$

$$(Pa \cdot Qa) \vee (Pb \cdot Qb) \vee (Pc \cdot Qc)$$

Q) 1. $(\exists x)(Xx \cdot Yx)$
2. $(x)(Xx \supset Zx)$
3. $(\exists x)(Zx \cdot \sim Xx)$ $\quad /\therefore (\exists x)(Zx \cdot \sim Yx)$
4.

Xa	
Xb	
Xc	

Ya	
Yb	
Yc	

Za	
Zb	
Zc	

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Easy $(X_a \cdot Y_a) \vee (X_b \cdot Y_b) \vee (X_c \cdot Y_c)$ T

Diff $(X_a \supset Z_a) \cdot (X_b \supset Z_b) \cdot (X_c \supset Z_c)$

Easy $(Z_a \cdot \sim X_a) \vee (Z_b \cdot \sim X_b) \vee (Z_c \cdot \sim X_c)$

$/ \therefore (Z_a \cdot \sim Y_a) \vee (Z_b \cdot \sim Y_b) \vee (Z_c \cdot \sim Y_c)$

$Z_a\ Z_b\ Z_c \to F$ or $\sim Y_a, \sim Y_b, \sim Y_c \to F$

X_a	~~F~~	T
X_b	~~F~~	~~T~~ F
X_c	~~F~~	~~T~~ F

Y_a	T
Y_b	T
Y_c	T

Z_a	~~F~~	T
Z_b	~~F~~	T
Z_c	~~F~~	T

Start with difficult first $(\cdot)$

Put any one as T or F.

Start with Conclusion False

Q) Abbots and Bishops are churchmen. No churchmen are either dowdy or elegant. Some bishops are elegant and fastidious. Some Abbots are not fastidious. Therefore, some abbots are dowdy.

A) 1. $(x)\, [(A_x \vee B_x) \supset C_x]$

E) 2. $(x)\, [C_x \supset \sim (D_x \vee E_x)]$

I) 3. $(\exists x)\, [B_x \cdot E_x \cdot F_x]$

4. $(\exists x)\, [A_x \cdot \sim F_x]$ $\quad / \therefore (\exists x)\, [A_x \cdot D_x]$

E ⟷ I Contradiction

Date ___/___/____

5.	$Ba \cdot (Ea \cdot Fa)$	3, E.I.
6.	Ba	5, Simp
7.	$Ea \cdot Fa$	5, Comm; Simp.
8.	Ea	7, Simp.
9.	$Ba \vee Aa$	6, Add.
10.	$Aa \vee Ba$	9, Comm.
11.	$(Aa \vee Ba) \supset Ca$	1, U.I.
12.	Ca	11, 10, Conj.
13.	$Ca \supset \sim(Da \vee Ea)$	2, U.I.
14.	$(Da \vee Ea) \supset \sim Ca$	13, Trans
15.	$Ea \vee Da$	8, Add.
16.	$Da \vee Ea$	15, Comm
17.	$\sim Ca$	14, 16, M.P.
18.	$Ca \vee (\exists x)[Ax \cdot Dx]$	12, Add.
19.	$(\exists x) \cdot [Ax \cdot Dx]$	18, 17, D.S