THIS WORK IS STILL UNDER DEVELOPMENT
Conjugate Gradients Method by Example in Qt/C++

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Abstract

Work under active development.

Contents

1 Introduction

Practical implementation of CGM.

2 The Realistic Example

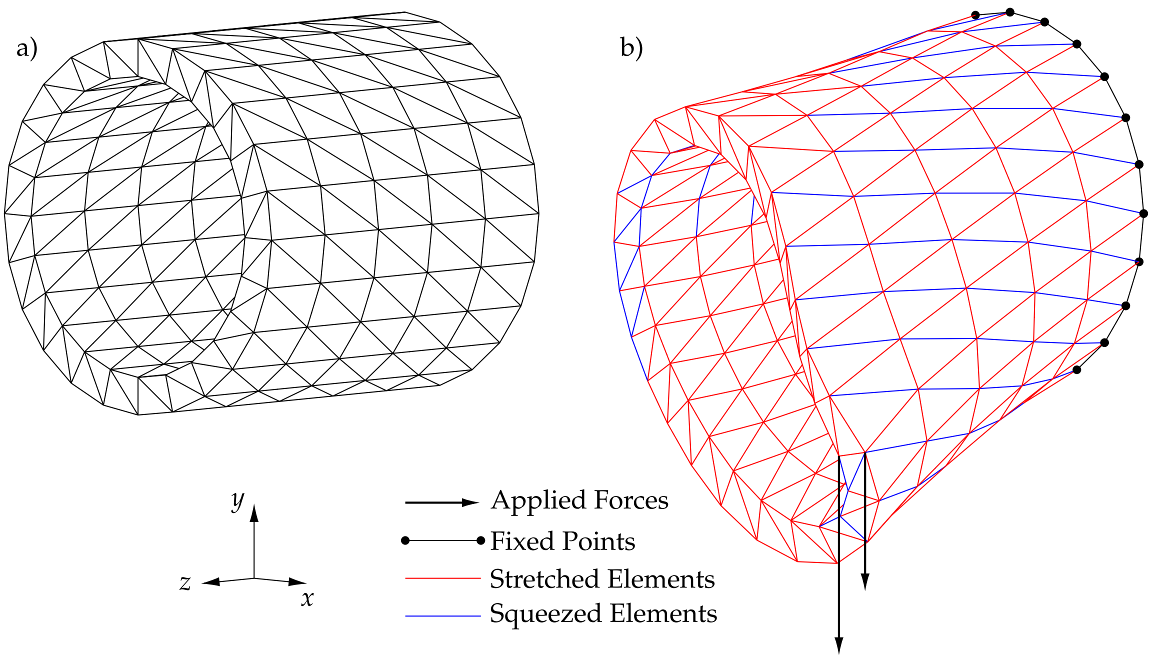

Figure 1: Pipe

3 The Simplest Geometry

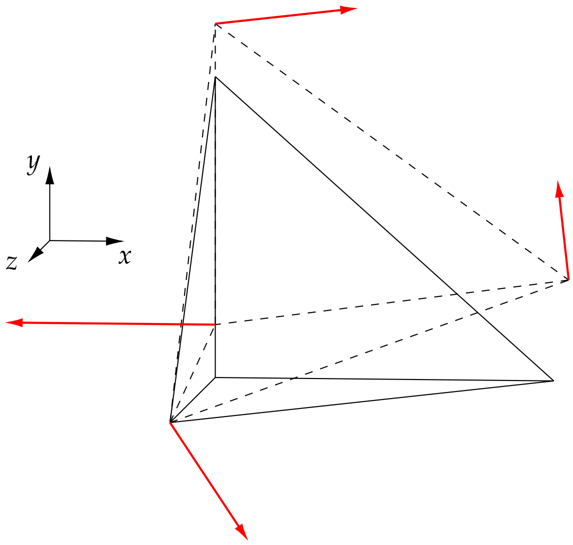

Figure 2: Tetrahedron

4 The Steepest Descent Method

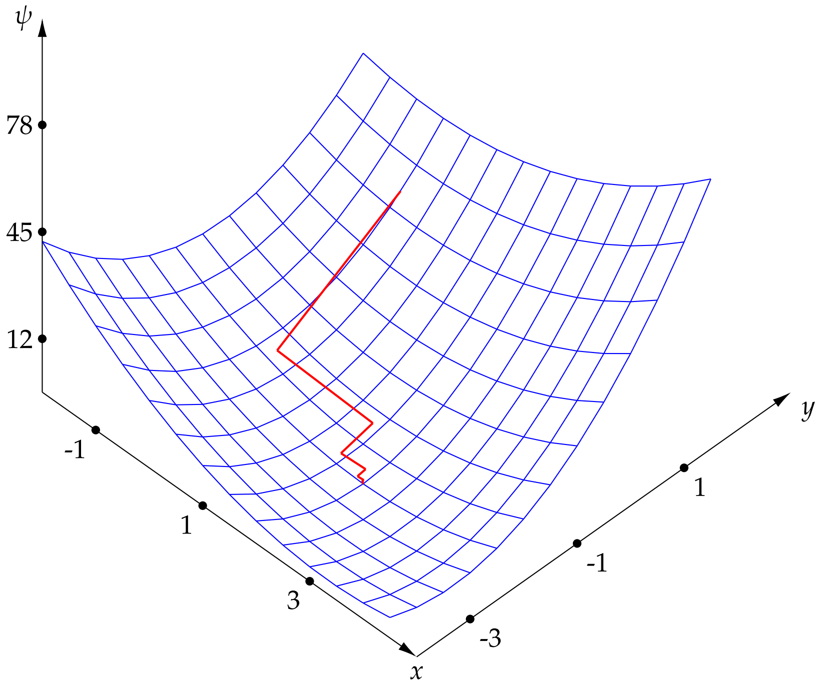

Figure 3: Quadratic Form

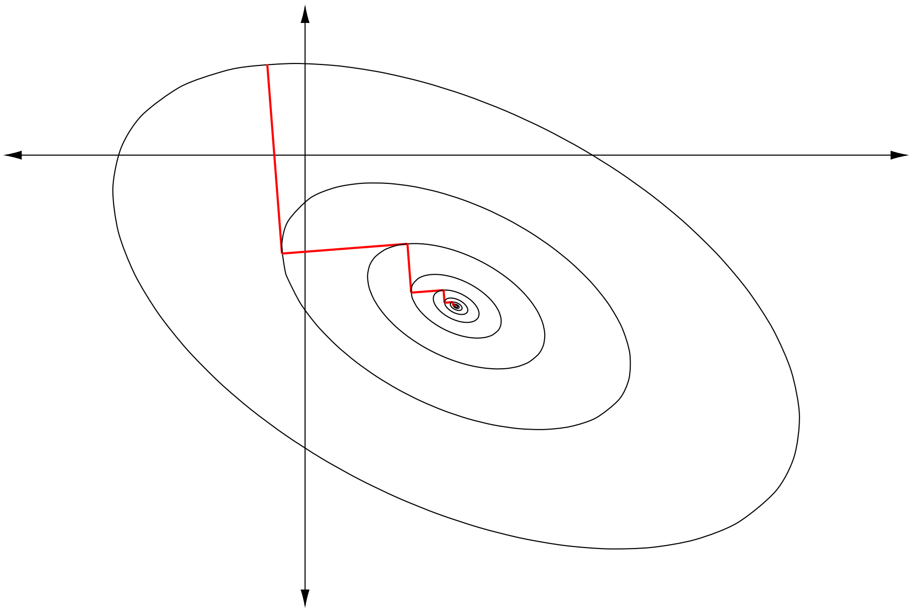

Figure 4: Intersection for $\psi = 0$

5 Conjugate Gradients Method

$$Ax = f \tag{1}$$

$$\Phi = \frac{1}{2}x^T Ax + x^T f \tag{2}$$

$$\nabla\Phi = \frac{1}{2}IAx + \frac{1}{2}x^T AI - If \tag{3}$$

$$\nabla\Phi = \begin{bmatrix} \frac{\partial\Phi}{\partial x_1} \\ \frac{\partial\Phi}{\partial x_2} \\ \vdots \\ \frac{\partial\Phi}{\partial x_n} \end{bmatrix} \tag{4}$$

$$\nabla\Phi = Ax - f \tag{5}$$

$$x = x_0 + Ps \tag{6}$$

$$p_i^T Ap_j = 0 \ \text{ for } \ i \neq j \tag{7}$$

$$P^T AP = D \tag{8}$$

$$d_i = p_i^T Ap_i \tag{9}$$

$$\nabla\Phi = Ax - f = Ax_0 + APs - f \tag{10}$$

$$\nabla\Phi = APs + r_0 \tag{11}$$

$$r_0 = f - Ax_0 \tag{12}$$

$$P^T APs + P^T r_0 = 0 \tag{13}$$

$$Ds + P^T r_0 = 0 \tag{14}$$

$$s_i = \frac{p_i^T r_0}{p_i^T Ap_i} \tag{15}$$

$$x = x_0 + Ps \tag{16}$$

6 Calculations for Tetrahedron

$$\begin{bmatrix} 250 & -250 & . & . & . \\ -250 & 366 & 82.47 & -73.31 & . \\ . & 82.47 & 363 & -82.47 & -64 \\ . & -73.31 & -82.47 & 73.31 & . \\ . & . & -64 & . & 85.33 \end{bmatrix} \begin{bmatrix} 0.7031 \\ 0.7031 \\ 0.2531 \\ 1.397 \\ 0.5414 \end{bmatrix} = \begin{bmatrix} . \\ . \\ . \\ 30 \\ 30 \end{bmatrix} \tag{17}$$

Figure 5: Equation K for Tetrahedron

$$\begin{bmatrix} . & 64 & 64 & . & -85.33 \\ . & -42.67 & . & . & . \\ . & . & -48 & . & 64 \\ . & -64 & . & . & . \\ . & -82.47 & -92.78 & 82.47 & . \\ . & . & . & . & . \\ . & . & -222.2 & . & . \end{bmatrix} \begin{bmatrix} 0.7031 \\ 0.7031 \\ 0.2531 \\ 1.397 \\ 0.5414 \end{bmatrix} = \begin{bmatrix} 15 \\ -30 \\ 22.5 \\ -45 \\ 33.75 \\ . \\ -56.25 \end{bmatrix} \tag{18}$$

Figure 6: Equation T for Tetrahedron