Problem 1.1

Lauren Prohovich

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1 Problem

Let (X,d) be a pseudometric space, and suppose d has the property that $d(a,b) > 0$ whenever a $\neq$ b. Prove that every finite subset of X is closed.

2 Solution

Let S be a finite subset of X. For the metric space (X,d), consider the elements p$\in$X such that p$\notin$S. For every element s$\in$S, take d(p,s). Let min(d(p,s))=A, and let A/2= ε.

We have the definition of a limit point to be that if (X,d) is a pseudometric space and S is a subset of X, a point p in X is a limit point of S if for every $\varepsilon > 0$, there is a point S$\neg$p such that d(p,s)< ε.

However, if we have that ε=min(d(p,s))/2, d(p,s) will always be greater than ε. Therefore, every point p$\in$X such that p$\notin$S is not a limit point of S. Since a limit point cannot be a point in the subset S, the limits points are not in S. Since the limit point is not in S or in X/S, the limit points do not exist. Thus, cl(S)=S, so S is closed.