Time Value of Money 3 (TVM 3)

With the FV & PV of lumps formula we can solve most all problems. However, when there are numerous cash flows and amounts involved in a problem the number of separate calculations can become extremely time consuming.
For example, if I asked, " what is the future value of $1000 deposited in an account for ten years at a rate of 10%." This is a simple problem, but if I added on, "and the following year I deposit another $1000,,,,,and the following year I deposit another $1000,,,,and the following.."

Obviously, this becomes a very long problem using $FV=PV(1+i)^{10}$, then $FV=PV(1+i)^{9}$, and on & on.

TVM3 con't

There are a couple of short-cut formulas and concepts worth introducing at this point.
•Perpetuity-equal-sized cash flows that extend infinitely into the future.

There are examples of perpetuities. Some preferred stocks pay a fixed dividend (like 6% of a par value of $100, or $6.00/year) indefinitely.

•Annuity--equal-sized cash flows over equally-spaced & specified time periods.

TVM3 con't

Perpetuity-equal-sized cash flows that extend infinitely into the future.
•In finance, we are usually concerned with the present value (or PV) of a perpetuity. "How much would I pay for a cash flow?", or "How much would I charge someone today for a promise to pay them a certain amount indefinitely?"
•There is a calculation for the FV of a perpetuity, however, I can think of no application nor useful reason to know it. Therefore, don't worry about it.
•The present value of a perpetuity can be calculated:::

$$PV_{perp} = \frac{CF}{i}$$

TVM3 con't

Perpetuity-equal-sized cash flows that extend infinitely into the future

$$PV_{perp} = \frac{CF}{i}$$

where *i* is the appropriate discounting interest rate for this level of risky asset.

Of more practical use is the annuity......

TVM3 con't $PV_{perp} = \frac{CF}{i}$

•Annuity--equal-sized cash flows over equally-spaced & specified time periods

Annuities are ubiquitous in our society. Examples include questions such as....

- •How much money will I have in 20 years if I save $x.xx per year at a rate of x%?
- •How much money must I deposit each year/month in order to save $1,000,000 in 20 years at a nominal rate of 8%? 12%?
- •Would I pay $10,000 for an annuity which returned $1,500/year, given a required return of 10%?
- •If I buy a car and finance $10,000 at a rate of 9% for 48 months, what is my monthly payment?

TVM3 con't $PV_{perp} = \frac{CF}{i}$

•Annuity--equal-sized cash flows over equally-spaced & specified time periods.

There are two kinds of annuities:

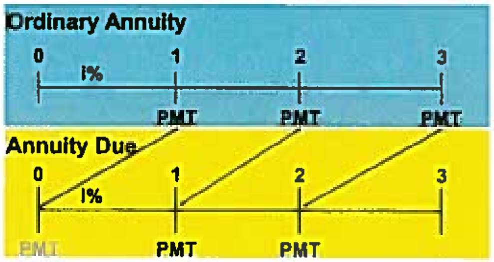
1. Regular or ordinary annuities-the cash flow is at the end of each period. (Always assume a regular annuity even when only annuity is mentioned)

2. Annuity Dues-the cash flow is at the beginning of each period.

See next Slide..........

TVM3 con't

What's the difference between an ordinary annuity and an annuity due?

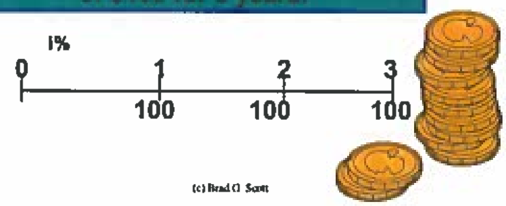


Believe it or not, most applications are regular annuities, so let's look at those first & annuity dues last.

TVM3 con't $PV_{perp} = \frac{CF}{i}$

•Annuity--equal-sized cash flows over equally-spaced & specified time periods.
Regular or ordinary annuities-the cash flow is at the end of each period (Always assume a regular annuity)

Time line for an ordinary annuity of $100 for 3 years.

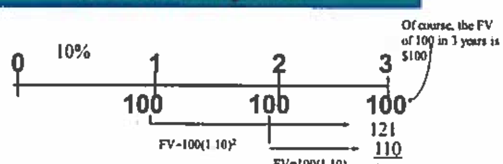


(c) Brad G. Scott

$FV = PV(1+i)^n$ TVM3 con't $PV_{perp} = \frac{CF}{i}$

•To get Future Value, we could just use the PV of a lump formula.

Time line for an ordinary annuity of $100 for 3 years.



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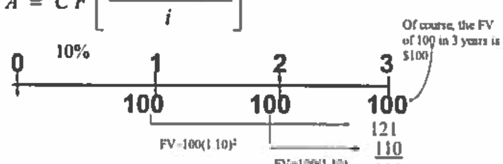
$FV = PV(1+i)^n$ TVM3 con't $PV_{perp} = \frac{CF}{i}$

•To get Future Value, we could just use the PV of a lump formula.

OR.......we could learn the Future Value of an Annuity (FVA) formula: (sidenote: this can be derived from the FV formula above, however, I will not go through that exercise here.....trust me, it can!)

$$FVA = CF\left[\frac{(1+i)^n - 1}{i}\right]$$



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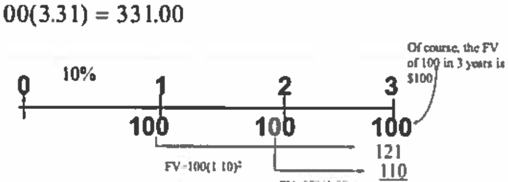
$FV = PV(1+i)^n$ TVM3 con't $PV_{perp} = \frac{CF}{i}$

Future Value of an Annuity (FVA) formula: working on the below problem:

$$FVA = CF\left[\frac{(1+i)^n - 1}{i}\right] = 100\left[\frac{(1+.10)^3 - 1}{.10}\right] =$$

$$= 100(3.31) = 331.00$$



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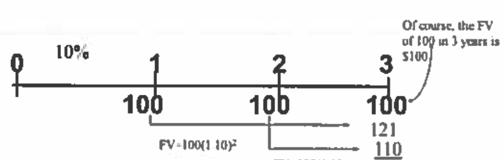
$FV = PV(1+i)^n$ TVM3 con't $PV_{perp} = \frac{CF}{i}$

$$FVA = CF\left[\frac{(1+i)^n - 1}{i}\right] = 100\left[\frac{(1+.10)^3 - 1}{.10}\right] =$$

$$= 100(3.31) = 331.00$$

Now, let's look at the financial calculator solution:



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TVM3 con't

Financial Calculator Solution

INPUTS	3	10	0	-100	
	N	I/YR	PV	PMT	FV
OUTPUT					331.00

Have payments but no lump sum PV, so enter 0 for present value.

Note: make sure financial calculator is on END mode. (one of the common mistakes during tests)

$FV = PV(1+i)^n$ TVM3 con't $PV_{perp} = \frac{CF}{i}$

Points on the FVA formula
–the # in the [brackets] is the FVA factor, Appendix Table A-4.

$$FVA = CF\left[\frac{(1+i)^n - 1}{i}\right]$$

TVM3 con't

If we were to look at an annuity due, we can observe that we had already found a FVA of \$331.00,,,,,,BUT now it's at t=2,,,,instead of t=3 where we want it.

Find the FV and PV if the annuity were an annuity due.

0	1	2	3
10%			
100	100	100	

$FV = PV(1+i)^n$ TVM3 con't

If we were to look at an annuity due, we can observe that we had already found a FVA of \$331.00,,,,,,BUT now it's at t=2,,,,instead of t=3 where we want it.

Find the FV and PV if the annuity were an annuity due.

0	1	2	3
10%			
100	100	100	
		121	
		110	
		331 $(1.10)^1$=	\$364.10

all we need to do is grow it by one period using *FV=PV(1+i)^n*

$FV = PV(1+i)^n$ TVM3 con't

NOTE: we did not need another formula for FVA of annuity dues,,,,,we just need to recognize the end period for our regular annuity formula, & tweak it—NOT ANOTHER FORMULA

Find the FV and PV if the annuity were an annuity due.

0	1	2	3
10%			
100	100	100	
		121	
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all we need to do is grow it by one period using *FV=PV(1+i)^n*

$FV = PV(1+i)^n$ TVM3 con't $PV_{perp} = \frac{CF}{i}$

Points (AGAIN) on the FVA formula
–the # in the [brackets] is the FVA factor, Appendix Table A-4.
–when dealing with BOTH CFs and compounding periods more often than annually----the formula is adjusted the same way as the lump formula---divide each *i* by *m* and multiply each *n* by *m*. (remember *m* is the # of compounding periods/year) or…

$$FVA = CF\left[\frac{(1+\frac{i}{m})^{n \cdot m} - 1}{\frac{i}{m}}\right]$$

$FV = PV(1+i)^n$ **TVM3 con't** $PV_{perp} = \frac{CF}{i}$

Points (AGAIN) on the FVA formula

—when dealing with BOTH CFs and compounding periods more often than annually—the formula is adjusted the same way as the lump formula—divide each i by m and multiply each n by m (remember m is the # of compounding periods/year)

Now a couple of problems: (Assume ordinary annuitites--end of period cash flows)

1) If I save $1200 per year in an account earning 8% (annually compounded) for 20 years, how much money should the account be worth?

2) If I save $100 per month in an account earning a nominal rate of 8%/yr., for 20 years, how much money whould the account be worth

$$FVA = CF\left[\frac{(1+\frac{i}{m})^{n\cdot m}-1}{\frac{i}{m}}\right]$$

Do problems before proceeding.

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$FV = PV(1+i)^n$ **TVM3 con't** $PV_{perp} = \frac{CF}{i}$

Assume ordinary annuitites--end of period cash flows

1) If I save $1200 per year in an account earning 8% (annually compounded) for 20 years, how much money should the account be worth?

$$FVA = 1200\left[\frac{(1+.08)^{20}-1}{.08}\right] = 1200[45.76196] = \$54,914.36$$

2) If I save $100 per month in an account earning a nominal rate of 8%/yr., for 20 years, how much money whould the account be worth?

$$FVA = 100\left[\frac{(1+\frac{.08}{12})^{20\cdot 12}-1}{\frac{.08}{12}}\right] = 100[589.02] = \$58,902.04$$

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$FV = PV(1+i)^n$ **TVM3 con't** $PV_{perp} = \frac{CF}{i}$

Assume ordinary annuitites--end of period cash flows

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$$FVA = 100\left[\frac{(1+\frac{.08}{12})^{20\cdot 12}-1}{\frac{.08}{12}}\right] = 100[589.02] = \$58,902.04$$

Does it make sense that problem 2 would yield a higher future value (even though you're still only contributing $1200/yr)????

What would be the answer to 1) if it were an annuity due?????

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$FV = PV(1+i)^n$ **TVM3 con't** $PV_{perp} = \frac{CF}{i}$

What would be the answer to 1) if it were an annuity due?????

1) If I save $1200 per year in an account earning 8% (annually compounded) for 20 years, how much money should the account be worth?

$$FVA = 1200\left[\frac{(1+.08)^{20}-1}{.08}\right] = 1200[45.76196] =$$

$$\$54,914.36 \cdot (1.08) = \$59,307.51$$

Did you get the same answer? With a financial calculator one would just change from 'end' to 'begin' mode of payment.

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$FV = PV(1+i)^n$ **TVM3 con't** $PV_{perp} = \frac{CF}{i}$

If you are comfortable with FVAs, then move on to the next section TVM 4……(which will work with present value of annuities PVA)…….

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$FV = PV(1+i)^n$ Time Value of Money 4 (TVM4)

The time value of money concept of Present Value (PV) is used in many applications. As we saw before, we can use the formula above to solve for any problems in which we have future values, but are looking for present values. (Questions like, "how much would I pay today for an amount in the future.")

As a review let's look at the PV calculation before we go on to PVA (present values of annuities).

$FV = PV(1+i)^n$ TVM 4 con't

What's the PV of $100 due in 3 years if i = 10%?

Finding PVs is discounting, and it's the reverse of compounding.

0 — 10% — 1 — 2 — 3

PV = ?- - - - - - - - - - - - - - - - 100

$FV = PV(1+i)^n$ TVM 4 con't

What's the PV of $100 due in 3 years if i = 10%?

We can use the above formula to solve for this problem.

$$FV = PV(1+i)^n \rightarrow PV = FV/(1+i)^n$$

0 — 10% — 1 — 2 — 3

PV = ?- - - - - - - - - - - - - - - - - 100

$$100 = PV(1+.10)^3 \rightarrow PV = 100/(1.1)^3 = 75.13148$$

$FV = PV(1+i)^n$ TVM 4 con't

Financial Calculator Solution

INPUTS	3	10		0	100
	N	I/YR	PV	PMT	FV
OUTPUT			-75.13		

Either PV or FV must be negative. Here PV = -75.13. Put in $75.13 today, take out $100 after 3 years.

We can use the general formula to solve for annuities as well.........

$FV = PV(1+i)^n$ TVM 4 con't

What's the PV of this ordinary annuity?

$100/year for 3 years at 10%...........

0 — 10% — 1 — 2 — 3

100 100 100

90.91 ← 100/(1.1)

82.64 ← $100/(1.1)^2$

75.13 ← $100/(1.1)^3$ =

248.69 = PV

This calc can be used for each CF

$FV = PV(1+i)^n$ TVM 4 con't

What's the PV of this ordinary annuity?

$100/year for 3 years at 10% ....

We also have a shortcut formula for the present value of an annuity......(also derivable from the above general format!!!!) Remember, an annuity must be equal-sized cashflows at regular intervals. The formula is.......

$$PVA = CF\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]$$

$FV = PV(1+i)^n$ TVM 4 con't

What's the PV of this ordinary annuity?

$100/year for 3 years at 10%..........

Using the formula to solve for the prior problem.....

$$PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right] \rightarrow 100\left[\frac{1-\frac{1}{(1.10)^3}}{.10}\right] \rightarrow 100\left[\frac{1-(1.10)^{-3}}{.1}\right]$$

$$\rightarrow 100\left[\frac{.248685199}{.10}\right] \rightarrow 100(2.48685199) = \$248.6852$$

Note: $1/(1.10)^3 = (1.10)^{-3}$

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$FV = PV(1+i)^n$ TVM 4 con't

Usually at this point, I require everyone to do this calculation on their calculator. Start w/ the 1.1 to the negative third power and work outwards.........Can everyone do this problem?

$$PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right] \rightarrow 100\left[\frac{1-\frac{1}{(1.10)^3}}{.10}\right] \rightarrow 100\left[\frac{1-(1.10)^{-3}}{.1}\right]$$

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(c) Brad G Scott

$FV = PV(1+i)^n$ TVM 4 con't

Usually at this point, I require everyone to do this calculation on their calculator. Start w/ the 1.1 to the negative third power and work outwards.........Can everyone do this problem?

Note that the number in the parentheses is the PVA factor found in the Appendix of the book.

$$PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right] \rightarrow 100\left[\frac{1-\frac{1}{(1.10)^3}}{.10}\right] \rightarrow 100\left[\frac{1-(1.10)^{-3}}{.1}\right]$$

$$\rightarrow 100\left[\frac{.248685199}{.10}\right] \rightarrow 100(2.48685199) = \$248.6852$$

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Appendix Table A-2

$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

Using a financial calculator....

INPUTS	3	10		100	0
	N	I/YR	PV	PMT	FV
OUTPUT			-248.69		

Have payments but no lump sum FV, so enter 0 for future value.

So now you have four methods by which a PVA can be calculated

- calc each lump sum *separately*
- use the PVA formula
- use the appendix
- use the financial calculator

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$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

Using a financial calculator....

So now you have four methods by which a PVA can be calculated

- calc each lump sum separately
- use the PVA formula
- use the appendix
- use the financial calculator

One should be able to use both the formulas and the financial calculator. I find the formulas very useful when one is working w/ a spreadsheet application and for applying finance TVM concepts to other ideas. The financial calculator is extremely useful when solving for i and uneven cash flows. I would suggest one work the homework both ways & see if you get the same answer.

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$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

Another feature of the PVA formula is that I can tweak it just like the others if I'm dealing with cash flows and compounding periods more often than annually. I do the exact same thing......divide every i by m and multiply every n by m. m is the # of compounding periods per year.

$$PVA = CF\left[\frac{1-\frac{1}{(1+\frac{i}{m})^{n \cdot m}}}{\frac{i}{m}}\right]$$

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TVM 4 con't

$FV = PV(1+i)^n \qquad PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

Example are really very plentiful.
The PVA is used in many facets of our life.

Think about a monthly car payment. It is an equal-sized CF (your payment) that happens at regular intervals. The present value of the annuity is the amount borrowed.

$$PVA = CF\left[\frac{1-\frac{1}{(1+\frac{i}{m})^{n \cdot m}}}{\frac{i}{m}}\right]$$

TVM 4 con't

$FV = PV(1+i)^n \qquad PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

I will borrow $20,000 to finance a car purchase. The bank offers a rate of 9% (stated nominal rate) for a 48 month note. What will be my monthly car payment?

$$PVA = CF\left[\frac{1-\frac{1}{(1+\frac{i}{m})^{n \cdot m}}}{\frac{i}{m}}\right]$$

TVM 4 con't

$FV = PV(1+i)^n \qquad PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

I will borrow $20,000 to finance a car purchase. The bank offers a rate of 9% (stated nominal rate) for a 48 month note. What will be my monthly car payment?
The present value is $20,000. The n is 4 years. The m is 12 (for 12 compounding periods per year). The interest rate is .09.

$$PVA = CF\left[\frac{1-\frac{1}{(1+\frac{i}{m})^{n \cdot m}}}{\frac{i}{m}}\right] \qquad 20{,}000 = CF\left[\frac{1-\frac{1}{(1+\frac{.09}{12})^{4 \cdot 12}}}{\frac{.09}{12}}\right]$$

TVM 4 con't

$FV = PV(1+i)^n \qquad PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

I will borrow $20,000 to finance a car purchase. The bank offers a rate of 9% (stated nominal rate) for a 48 month note. What will be my monthly car payment?
The present value is $20,000. The n is 4 years. The m is 12 (for 12 compounding periods per year). The interest rate is .09.

$$20{,}000 = CF\left[\frac{1-\frac{1}{(1+\frac{.09}{12})^{4 \cdot 12}}}{\frac{.09}{12}}\right]$$

$$20{,}000 = CF(40.184781885)$$
$$CF = 20{,}000/(40.184781885)$$
$$CF = \$497.70$$

TVM 4 con't

$FV = PV(1+i)^n \qquad PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

$$20{,}000 = CF\left[\frac{1-\frac{1}{(1+\frac{.09}{12})^{4 \cdot 12}}}{\frac{.09}{12}}\right]$$

$$20{,}000 = CF(40.184781885)$$
$$CF = 20{,}000/(40.184781885)$$
$$CF = \$497.70$$

Let's go through the keystrokes to solve using the calculator & then we'll look at the financial calculator solution. [When I use the ^ symbol that is the same as the y^x button on some calculators. The <-> means change the sign by using the +/- button on your calculator.] Notice that when you don't stop your calculator there aren't rounding errors.
START: .09 divided by 12 = {save this number}, +1, ^ 48 <-> = <->, + 1 = [now we have the numerator of the parentheses], divided by .0075 [saved from previous calc] = ...[you now have the PVA factor]...hit your 1/x button. X 20000 = [now you should have the answer $497.70008475!!!!!!!!!]

TVM 4 con't

$FV = PV(1+i)^n \qquad PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

$$20{,}000 = CF\left[\frac{1-\frac{1}{(1+\frac{.09}{12})^{4 \cdot 12}}}{\frac{.09}{12}}\right]$$

$$20{,}000 = CF(40.184781885)$$
$$CF = 20{,}000/(40.184781885)$$
$$CF = \$497.70$$

	P/YR	N	I/Y	PV	PMT	FV
INPUTS	12	48	9	20000		0
OUTPUT					-497.70	

R

Now you can see where the numbers for the financial calculator come from.

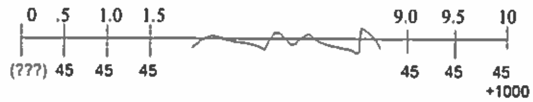
$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

When using the PVA formula, or the financial calculator, it is important that the compounding period and cash flows match in timing (like monthly cash flows with semiannual compounding would not work).

Let's look at a combination problem with semi-annual compounding.
A bond pays $45 every six month and also pays its maturity value of $1000 at the maturity in 10 years.

$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

A bond pays $45 every six month and also pays its maturity value of $1000 at the maturity in 10 years. If the required return of investors purchasing this bond is 8%, what is the price (present value) of this bond?
First, let's look at a time-line of the cash flows of this instrument. (time on top, CFs on bottom of bad line)



This is really a combination problem, we have both an annuity portion and a lump portion.......

$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

A bond pays $45 every six month and also pays its maturity value of $1000 at the maturity in 10 years. If the required return of investors purchasing this bond is 8%, what is the price (present value) of this bond?

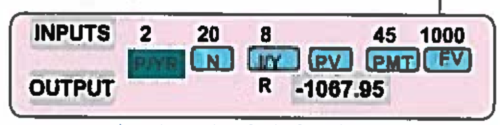
$$PVA = 45\left[\frac{1-\frac{1}{(1+\frac{.08}{2})^{10\cdot 2}}}{\frac{.08}{2}}\right] = 611.5646855$$

$$+PV_{lump} = 1000/(1+\frac{.08}{2})^{10\cdot 2} = \underline{456.386946}$$

$1,067.95

$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

And on a calculator,....



Now you can see where the numbers for the financial calculator come from.

The steps are always the same:

- Is this an annuity, lump, or combination?
- Do I have the interest rate, number of years, compounding and CF frequency, CFs, PV amount, or FV amount?
- Solve for un-known.

$FV = PV(1+i)^n$ TVM 4 con't $PVA = CF\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$

To avoid the risk of overwhelming, I'm stopping here.
I advise using the forum as you go, & let me know what is missing or unclear. This section is very important!!!