Time Value of Money (TVM)

One of the central concepts of finance is that a dollar today is worth more than a dollar in the future.

In order to be compensated for delaying a dollar today for a dollar in the future one must receive an additional benefit-like interest.

Intuitively, you are already quite aware of the TVM concepts........

Time Value of Money (TVM) con't

For example, if I asked "how much money would you have in 1 year if you invested $100 at a rate of 10%?"

Usually, responses are a very quick... $110.00!

Well, if I asked "how much money would you have in another year if you invested the $110.00 at 10%?"

A response (just a bit slower) is a quick... $121.00

Time Value of Money (TVM) con't

Notice that in going from $100 to $110, and then from $110 to $121, we made these leaps without dealing with a great deal of terminology.

Let's introduce some terms::::
- Present Value (PV)-value in today's terms. ($100)
- Future Value (FV)-value of some amount in the future.($110)
- Interest rate (*i*)-rate of return (10%)
- Time (*n*) - measure of time in years.

Time Value of Money (TVM) con't

Also, we made the leaps without too heavy of use of algebra.
If I asked how (exactly) you did the calculations....?

Time Value of Money (TVM) con't

Most people would respond that they took 10% of $100 and added it back to $100. For the second part of the problem they took 10% of $110 and added it back to $110.

Or.....
(.10)100 + 100 = 110

And.....
(.10)110 + 110 = 121

Time Value of Money (TVM) con't

If we break this down a little we can get to the point where we can solve larger problems,,,,,like how much would the $100 grow in 10 years @ 10%.

(.10)100 + 100 =
(a) $\underline{100(1+.10) = 110}$
PV *i* FV

And.....
(b) (.10)110 + 110 = 110(1+.10) = 121
but from (a) above we know that...
110 = 100(1+.10) & if we substitute into
(b) 100(1+.10)(1+.10) = 121

Time Value of Money (TVM) con't

Following the lines........

(.10)100 + 100 =

(a) 100(1+.10) = 110

PV i FV

And....

(b) (.10)110 + 110 = 110(1+.10) = 121

but from (a) above we know that...

110 = 100(1+.10) & if we substitute into

(b) 100(1+.10)(1+.10) = 121

or 121 = 100 (1+ .10)^2

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Time Value of Money (TVM) con't

Following the lines........

121 = 100 (1+ .10)^2

Or

in a more general format..........

REMEMBER THIS---

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Time Value of Money (TVM) con't

Following the lines........

121 = 100 (1+ .10)^2

in a more general format..........

$$FV = PV(1+i)^n$$

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

Now, to the question asked earlier........

How much would the

$100 grow in 10 years @ 10%????????

The $100 is the PV, we are looking for the FV in 10 years.

$$FV = PV(1+i)^n$$

$$FV = 100(1+.10)^{10}$$

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

Continuing........

How much would the

$100 grow in 10 years @ 10%????????

$$FV = PV(1+i)^n$$

$$FV = 100(1+.10)^{10}$$

$$FV = 100(1.10)^{10}$$

$$FV = 100(2.5937424601) = 259.37$$

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

Continuing........

How much would the

$100 grow in 10 years @ 10%????????

$$FV = PV(1+i)^n$$

$$FV = 100(1+.10)^{10}$$

$$FV = 100(1.10)^{10}$$

$$FV = 100(2.5937424601) = 259.37$$

now would be a good time to make sure you can use your calculator & come up w/ the same numbers......

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

$FV = PV(1+i)^n$

$FV = 100(1+.10)^{10}$

$FV = 100(1.10)^{10}$

$FV = 100(2.5937424601) = 259.37$

The number in parenthesis is called the Future Value factor. In the 'good old days' before calculators, look-up tables were computed. If you look at Table A-3 in the Appendix of your your text you can find the intersection of 10% and 10 periods and find the future value factor.........The book also provides financial calculator keystrokes...

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

$FV = PV(1+i)^n$

$FV = 100(1.10)^{10}$

$FV = 100(2.5937424601) = 259.37$

A new question---how much would you pay today for a promise of $259.37 in 10 years, given a suitable discount rate of 10%?????

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

$FV = PV(1+i)^n$

$FV = 100(1.10)^{10}$

$FV = 100(2.5937424601) = 259.37$

A new question---how much would you pay today for a promise of $259.37 in 10 years, given a suitable discount rate of 10%?????

$FV = PV(1+i)^n$ or $PV = FV/(1+i)^n$

$259.37 = PV(1.10)^{10}$

$PV = 259.37 \frac{1}{(1.10)^{10}}$

$PV = 259.37(.38554328943) = \$100.00$

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

$FV = PV(1+i)^n$

$FV = 100(1.10)^{10}$

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A new question---how much would you pay today for a promise of $259.37 in 10 years, given a suitable discount rate of 10%?????

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$259.37 = PV(1.10)^{10}$

$PV = 259.37 \frac{1}{(1.10)^{10}}$

$PV = 259.37(.38554328943) = \$100.00$

This number is the Present Value Factor found in Table A-1

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

Notice that when dealing with single amounts (*lumps*, in my terminology) the PV factor is just the reciprocal of the FV factor.

One of my complaints regarding texts is that they will present four formulas----one solving for FV, one for PV, one for *i* and another for *n*. All we really need is the general formula presented above.........then we can do the math from there......

A couple example problems................

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

1. If I deposit $2,000 in an account that earns a annually compounded rate of 8%, how much will my account be worth in 15 years???

2. A zero coupon bond pays $1,000 at maturity in 20 years. If the appropriate required return for this level of risky asset is 12%, how much would I be willing to pay for this bond???(assume annual cmpd/discounting)

3. What interest rate (annual basis) is required for a deposit to double in 10 years???

4. At a rate of 12%, how long would it take for an amount to double???

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Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

1. If I deposit $2,000 in an account that earns a annually compounded rate of 8%, how much will my account be worth in 15 years???

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3. What interest rate (annual basis) is required for a deposit to double in 10 years???

4. At a rate of 12%, how long would it take for an amount to double???

Remember to attempt these before moving on....

Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

1. If I deposit $2,000 in an account that earns a annually compounded rate of 8%, how much will my account be worth in 15 years???

$$FV = 2000(1.08)^{15} = 2000(3.172169) = \$6,344.34$$

2. A zero coupon bond pays $1,000 at maturity in 20 years. If the appropriate required return for this level of risky asset is 12%, how much would I be willing to pay for this bond???(assume annual cmpd/discounting)

3. What interest rate (annual basis) is required for a deposit to double in 10 years???

4. At a rate of 12%, how long would it take for an amount to double???

Did you get the same answer????....

Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

1. If I deposit $2,000 in an account that earns a annually compounded rate of 8%, how much will my account be worth in 15 years???

$$FV = 2000(1.08)^{15} = 2000(3.172169) = \$6344.34$$

2. A zero coupon bond pays $1,000 at maturity in 20 years. If the appropriate required return for this level of risky asset is 12%, how much would I be willing to pay for this bond???(assume annual cmpd/discounting)

$$1000 = PV(1.12)^{20} \rightarrow PV = 1000/9.64629309 = \$103.67$$

3. What interest rate (annual basis) is required for a deposit to double in 10 years???

4. At a rate of 12%, how long would it take for an amount to double???

Did you get the same answer????....

Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

1. If I deposit $2,000 in an account that earns a annually compounded rate of 8%, how much will my account be worth in 15 years???

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2. A zero coupon bond pays $1,000 at maturity in 20 years. If the appropriate required return for this level of risky asset is 12%, how much would I be willing to pay for this bond???(assume annual cmpd/discounting)

$$1000 = PV(1.12)^{20} \rightarrow PV = 1000/9.64629309 = \$103.67$$

3. What interest rate (annual basis) is required for a deposit to double in 10 years???

$$2 = 1(1+i)^{10} \rightarrow \sqrt[10]{(1+i)^{10}} = \sqrt[10]{2} \rightarrow i = 1.07177 - 1 = .07177 \text{ or } 7.177\%$$

Note: on your calculator the tenth root of a number is equivalent to taking number to the power of 1/10 or .10. Also, this can easily be done on the financial calculator using the keys shown in the appendix and solving for *i*

4. At a rate of 12%, how long would it take for an amount to double???

Did you get the same answer????

Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

1. If I deposit $2,000 in an account that earns a annually compounded rate of 8%, how much will my account be worth in 15 years???

$$FV = 2000(1.08)^{15} = 2000(3.172169) = \$6344.34$$

2. A zero coupon bond pays $1,000 at maturity in 20 years. If the appropriate required return for this level of risky asset is 12%, how much would I be willing to pay for this bond???(assume annual cmpd/discounting)

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$$2 = 1(1+i)^{10} \rightarrow \sqrt[10]{(1+i)^{10}} = \sqrt[10]{2} \rightarrow i = 1.07177 - 1 = .07177 \text{ or } 7.177\%$$

4. At a rate of 12%, how long would it take for an amount to double???

$$2 = 1(1.12)^n \rightarrow \ln 2 = n(\ln 1.12) \rightarrow .693147 = n(.1133287) \rightarrow n = 6.116 \text{ years}$$

Did you get the same answers???

Time Value of Money (TVM) con't

$$FV = PV(1+i)^n$$

This general format is used throughout the text, although it is sometimes disguised by use of different variables.

If you feel comfortable with this presentation, please move on to the next section.

TVM 2

Time Value of Money 2 (TVM2)

$$FV = PV(1+i)^n$$

In TVM we made an assumption right in the beginning of the slide show.........remember.....

For example, if I asked "how much money would you have in 1 year if you invested $100 at a rate of 10%?"

Usually, responses are a very quick...
$110.00!
Well, if I asked "how much money would you have in another year if you invested the $110.00 at 10%?"
A response (just a bit slower) is a quick...
$121.00

TVM2 con't

$$FV = PV(1+i)^n$$

The assumption had to do with compound interest. We assumed that we were dealing only with annual compounding of interest (interest only earns interest after it has been added into the principal--in those problems only once per year). In many practical applications, interest is compounded more often than once per year.

For example, some institutions compound interest semi-annually or quarterly.

A note on interest rates----when rates are provided they are almost always stated in terms of an annual rate---even if the term of the security is only days--as in the case of U.S. Treasury bills.

TVM2 con't

$$FV = PV(1+i)^n$$

Interest rates are stated in annual terms.

NOMINAL RATE=STATED RATE

But let's go back to our earlier example--What if I deposited $100 at a nominal rate of 10% for 1 year, & interest is compounded semi-annually. How much money would I have at the end of 1 year?

TVM2 con't

$$FV = PV(1+i)^n$$

Interest rates are stated in annual terms.

NOMINAL RATE=STATED RATE

But let's go back to our earlier example--What if I deposited $100 at a nominal rate of 10% for 1 year, & interest is compounded semi-annually. How much money would I have at the end of 1 year?

Half of 10% would be credited to the account at the end of 6 months and the balance at the end of 1 year,..... or,..........

TVM2 con't

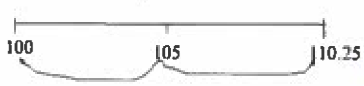
$$FV = PV(1+i)^n$$

Interest rates are stated in annual terms

NOMINAL RATE=STATED RATE

But let's go back to our earlier example--What if I deposited $100 at a nominal rate of 10% for 1 year, & interest is compounded semi-annually. How much money would I have at the end of 1 year?

Half of 10% would be credited to the account at the end of 6 months and the balance at the end of 1 year,..... or,..........



TVM2 con't

$$FV = PV(1+i)^n$$

Half of 10% would be credited to the account at the end of 6 months and the balance at the end of 1 year,..... or,..........

$100 would grow to $105, & then the $5 would start earning interest for 1/2 year at a rate of 10% (or 5%) and earn $0.25,..... & the $100 would earn $5 interest.......
...........to give a grand total of $110.25!!!!!!!!

We can use the general format above & just add another variable to the equation for problems like this one.....

TVM2 con't

$$FV = PV(1+i)^n$$

We can use the general format above & just add another variable to the equation for problems like this one.....Notice we had received half the interest but rec'd it twice. Our new variable is <u>m,</u> which stands for the <u>number of compounding periods per year</u>.

$$FV = PV\left(1+\frac{i}{m}\right)^{n \cdot m}$$

TVM2 con't

$$FV = PV(1+i)^n$$

Always remember,,,,when dealing with compounding periods more often than once per year,,,,,always divide i by m and multiply n times m. <u>m=#compounding periods/yr.</u>

$$FV = 100\left(1+\frac{.10}{2}\right)^{1 \cdot 2} = 100(1.05)^2 = 110.25$$

$$FV = PV\left(1+\frac{i}{m}\right)^{n \cdot m}$$

Please note: This is <u>not</u> another formula, its just adapting the familiar one by adding the compounding feature.

TVM2 con't

$$FV = PV(1+i)^n \quad FV = PV\left(1+\frac{i}{m}\right)^{n \cdot m}$$

Using this knowledge, we can answer the previous problem-- How much would $100 grow in 10 years at a nominal rate of 10%...........with interest compounded.......

a) annually?

b) semi-annually?

c) quarterly?

d) monthly?

e) daily?

Please work these before continuing!!!!

TVM2 con't

$$FV = PV(1+i)^n \quad FV = PV\left(1+\frac{i}{m}\right)^{n \cdot m}$$

Using this knowledge, we can answer the previous problem-- How much would $100 grow in 10 years at a nominal rate of 10%........... with interest compounded ......

a) annually?

$$FV = 100(1+.10)^{10} = 100(1.10)^{10} = 259.37$$

b) semi-annually?

$$FV = 100\left(1+\frac{.10}{2}\right)^{10 \cdot 2} = 100(1.05)^{20} = 265.33$$

c) quarterly?

$$FV = 100\left(1+\frac{.10}{4}\right)^{10 \cdot 4} = 100(1.025)^{40} = 268.51$$

d) monthly?

$$FV = 100\left(1+\frac{.10}{12}\right)^{10 \cdot 12} = 100(1.008\overline{33})^{120} = 270.70$$

e) daily?

$$FV = 100\left(1+\frac{.10}{365}\right)^{10 \cdot 365} = 100(1.00027397)^{3650} = 271.79$$

TVM2 con't

As you can tell, the same 10% nominal rate can provide quite different answers based on the compounding frequency.

When Individual Retirement Accounts were first allowed and gaining popularity I remember that the bank I worked for had advertised a 5-year Certificate of Deposit IRA at a rate of 10% with interest compounded quarterly. The competitor offered a 10.25% IRA w/ interest compounded annually. Which was the best deal????

TVM2 con't

10% with interest compounded quarterly or the 10.25% IRA w/ interest compounded annually. Which was the best deal????

Of course the 10% w/ quarterly compounding.

Truth in advertising rules have been established to help clear up consumer confusion. Banks must advertise the <u>effective annual rate (EAR)</u> {this is also called the annual percentage yield in some texts and advertisements}

The EAR is the equivalent rate of an annually compounded security to that of a security w/ multiple compounding periods.

TVM2 con't

Another way of dealing w/ EAR::::: I should come up with the same FV of a multiple compounding instrument at a nominal rate & the FV of an annual compounding instrument at the EAR.

Looking at our earlier example:

What if I deposited $100 at a nominal rate of 10% for 1 year, & interest is compounded semi-annually.

How much money would I have at the end of 1 year?

Find the EAR of this security.

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TVM2 con't

Another way of dealing w/ EAR::::: I should come up with the same FV of a multiple compounding instrument at a nominal rate & the FV of an annual compounding instrument at the EAR.

Looking at our earlier example:

What if I deposited $100 at a nominal rate of 10% for 1 year, & interest is compounded semi-annually.

How much money would I have at the end of 1 year?

The FVs should be the same......

$$FV = 100(1+.10/2)^{1\cdot 2}\ and$$

$$FV = 100(1+EAR)^{1} are\_equal$$

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TVM2 con't

Or.....

$$FV = 100(1+.10/2)^{1\cdot 2}\ and$$

$$FV = 100(1+EAR)^{1} are\_equal$$

$$or \rightarrow 100(1.05)^{2} = 100(1+EAR)$$

$$or \rightarrow (1.05)^{2} = (1+EAR)$$

$$\& \rightarrow EAR = (1.05)^{2} - 1 = 10.25\%$$

$$generally \rightarrow EAR = \left(1+\frac{i_{nom}}{m}\right)^{m} - 1$$

I can never remember this formual, however, if one just remembers this is just a restatement of our general formula, you don't have to remember it, it can always be derived.......

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TVM2 con't

Now with quarterly compounding.....

$$FV = 100(1+.10/4)^{1\cdot 4}\ and$$

$$FV = 100(1+EAR)^{1} are\_equal$$

$$or \rightarrow 100(1.025)^{4} = 100(1+EAR)$$

$$or \rightarrow (1.025)^{4} = (1+EAR)$$

$$\& \rightarrow EAR = (1.025)^{4} - 1 = 10.3813\%$$

So, the 10% compounded quarterly was a better deal.

When dealing w/ interest rates make sure you are comparing apples w/ apples, nominals w/ nominals, or EARs w/ EARs.

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TVM2 con't

Generally, in finance courses you are provided the nominal rate. If you feel good about this section please go forward to TVM part 3.

TVM 3

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