

## Using Force Vectors

Ahmed Saad Sabit

April 15, 2021

## 0.1 Right Angled Triangle - The Triangle with a $90^\circ$ .

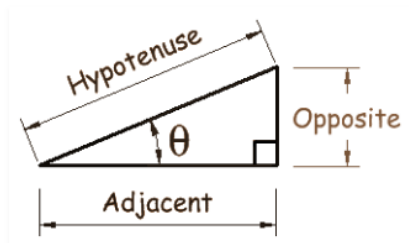


Figure: Right Angled Triangle

Because of some ratio of the right angled triangle we can say that,

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

This also means that,

$$\text{opposite} = \text{hypotenuse} \sin \theta$$

$$\text{adjacent} = \text{hypotenuse} \cos \theta$$

## 0.3 Measuring Angles

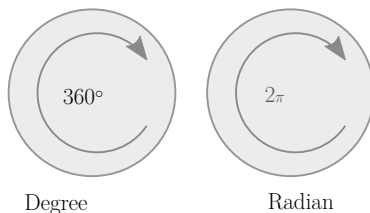


Figure: Comparison

This means that,

$$180^\circ = \pi \text{ radians}$$

The reason we use radian can be shown when needed.

## 0.4 The fact $\alpha + \beta + \gamma = 180^\circ = \pi$

If you add all angles inside a triangle, the sum will always be  $180^\circ$ .

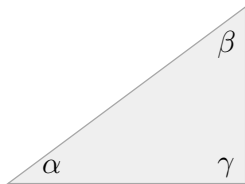


Figure: Summing up angles

$$\alpha + \beta + \gamma = 180^\circ = \pi$$

For a right angle triangle,  $\gamma = 90^\circ = \frac{\pi}{2}$ , hence,

$$\alpha + \beta + 90^\circ = 180^\circ$$

$$\alpha + \beta = 90^\circ$$

Newton's Second Law:  $\vec{F} = m\vec{a}$

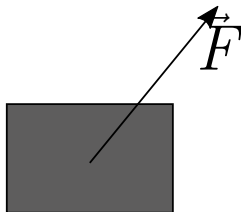


Figure: If you push a block

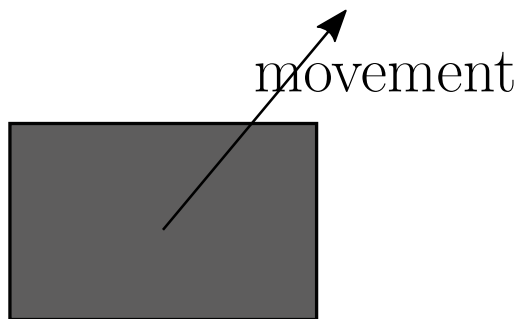


Figure: The movement will be in the same direction

The movement is Acceleration. Acceleration is increase/decrease in speed overtime.

$$\vec{a} = \frac{d\vec{v}}{dt}$$

A required relation of angles  $\alpha$  and  $\beta$ .

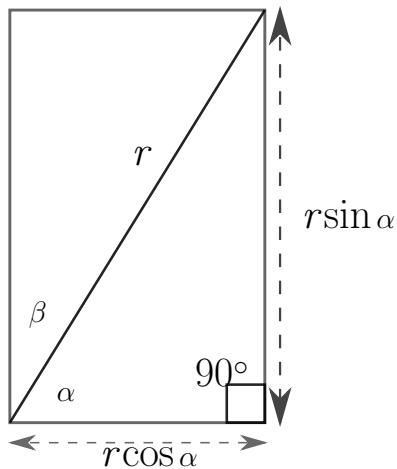


Figure:



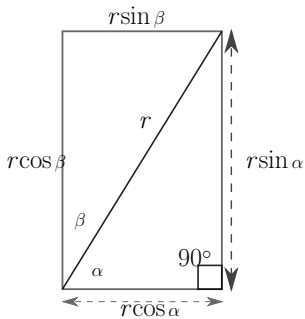


Figure:

This actually shows us that,

$$r \sin \alpha = r \cos \beta$$

$$r \cos \alpha = r \sin \beta$$

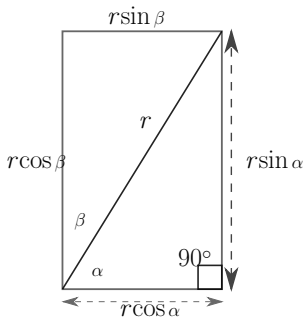


Figure:

This actually shows us that,

$$r \sin \alpha = r \cos \beta$$

$$r \cos \alpha = r \sin \beta$$

Because  $\alpha + \beta = 90^\circ$ ,  $\beta = 90^\circ - \alpha$ .  
Hence,

$$r \sin \alpha = r \cos(90^\circ - \alpha)$$

$$r \cos \alpha = r \sin(90^\circ - \alpha)$$

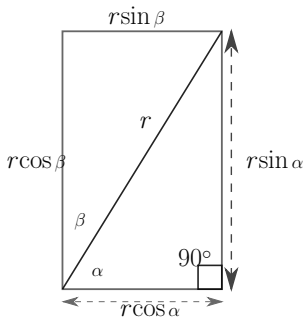


Figure:

This actually shows us that,

$$r \sin \alpha = r \cos \beta$$

$$r \cos \alpha = r \sin \beta$$

Because  $\alpha + \beta = 90^\circ$ ,  $\beta = 90^\circ - \alpha$ .  
Hence,

$$r \sin \alpha = r \cos(90^\circ - \alpha)$$

$$r \cos \alpha = r \sin(90^\circ - \alpha)$$

So, we now know that,

$$\sin(90^\circ - \alpha) = \cos \alpha$$

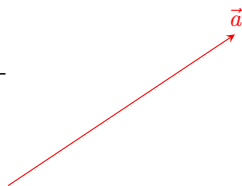
$$\cos(90^\circ - \alpha) = \sin \alpha$$

## Warm Up: Cosine Law — For Vector

We will use vectors very often in physics, so it is better we try it out before starting. At first, vectors can be added in this way,  
Vectors can be added, like,

$$\vec{a} + \vec{b}$$

Which looks like one vector is put in front of another.

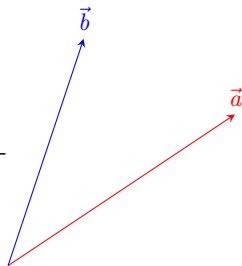


## Warm Up: Cosine Law — For Vector

We will use vectors very often in physics, so it is better we try it out before starting. At first, vectors can be added in this way, Vectors can be added, like,

$$\vec{a} + \vec{b}$$

Which looks like one vector is put in-front of another.

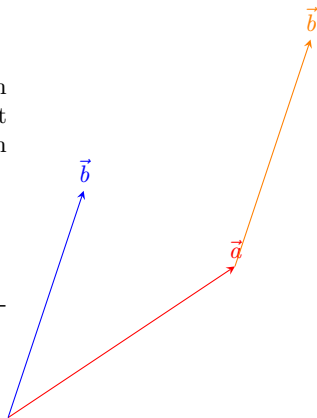


## Warm Up: Cosine Law — For Vector

We will use vectors very often in physics, so it is better we try it out before starting. At first, vectors can be added in this way, Vectors can be added, like,

$$\vec{a} + \vec{b}$$

Which looks like one vector is put in-front of another.

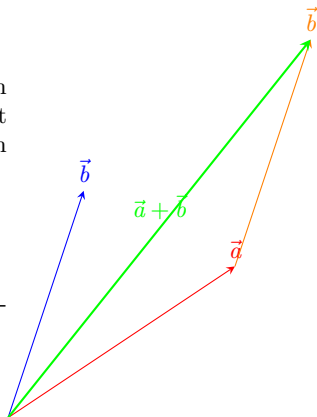


## Warm Up: Cosine Law — For Vector

We will use vectors very often in physics, so it is better we try it out before starting. At first, vectors can be added in this way, Vectors can be added, like,

$$\vec{a} + \vec{b}$$

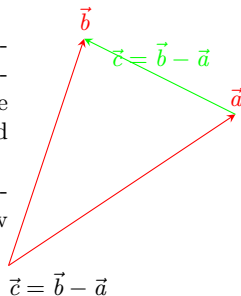
Which looks like one vector is put in front of another.



## Vector Triangle for Cosine Law

If there is a side  $a$  and  $b$ , then assuming the third side is  $c$ , the Law of cosine is to find the length of  $c$  using the lengths of  $a$  and  $b$ , and the angle  $a$  and  $b$  makes.

The easiest solution can be found using  $\vec{a}$  and  $\vec{b}$  vector methods. We know angle  $\theta$  between  $a$  and  $b$



$$\vec{a} + \vec{c} = \vec{b} \rightarrow$$



Now let us have a triangle with side  $\vec{a}$  and  $\vec{b}$ , then the third side is obviously  $\vec{b} - \vec{a}$ . This is same as we saw in last frame. Now, let the third side be  $\vec{c} = \vec{b} - \vec{a}$ , then can we find a scalar solution for the length of vector  $\vec{c}$ ?

$$\begin{aligned}\vec{c} &= \vec{b} - \vec{a} \\ (\vec{c})^2 &= (\vec{b} - \vec{a})^2 \\ \vec{c} \cdot \vec{c} &= \vec{a} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b}\end{aligned}$$

Now,  $\vec{a} \cdot \vec{b}$ , is the dot product. And for a dot product,

$$\vec{a} \cdot \vec{b} = |a| |b| \cos \theta$$

And if,

$$\vec{a} \cdot \vec{a} = |a||a| \cos(0)$$

Because  $\vec{a}$  makes  $0^\circ$  with itself. Hence, we have,

$$c^2 = a^2 + b^2 - 2ab \cos \theta$$

So, we have found length of third side,

$$c = \sqrt{a^2 + b^2 - 2ab \cos \theta}$$

# Sine Law

**The  $\sin \alpha$  Law:** For a triangle, with lowercase denoting the angle and the uppercase the opposite side, the sides and angle follow the relation respect to the circum-circle is that,

$$\frac{a}{\sin A} = \frac{c}{\sin C} = \frac{d}{\sin D} = 2r$$

**Proof:** Let  $\triangle ACD$  be the required triangle, we want to build a relation between the length of the sides and the opposite angles to them motivated from the fact that the opposite angle being large making the side project a larger length.

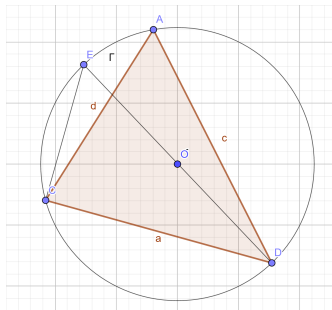
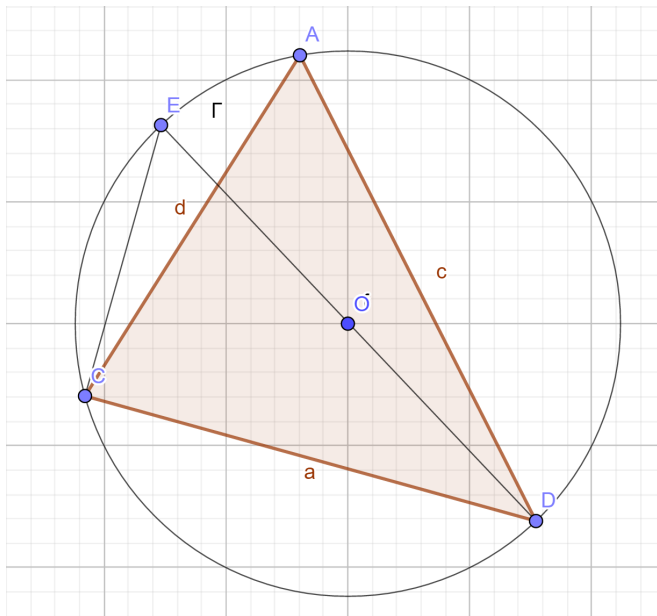


Figure: Sine Law

## We need these two Geometry rules

- If one side a triangle passes through the center of it's circumcircle, then that triangle is a right angled triangle.
- Angle made by two lines by a random point and two end points of the same arc on a circle, is same.



Let  $\Gamma$  be the circumcircle of  $\triangle ACD$  and we draw a diameter  $DE$  that passes through the center of  $\Gamma$  which is point  $O$ .

$\triangle ECD$  is a Right Angled Triangle because it meets the *Semi-circle* and thus,

$$\sin \angle CED = \frac{\overline{CD}}{\overline{ED}}$$

Using the *Arc and Angle Theorem* we can see that

$$\angle CED = \angle CAD = \angle A$$

As we know that  $DE = 2r$  we are left to show that,

$$\begin{aligned} \sin \angle CED &= \frac{\overline{CD}}{\overline{ED}} \\ \sin \angle A &= \frac{a}{2r} \end{aligned} \tag{1}$$

$$\frac{a}{\sin A} = 2r \tag{2}$$

And this can be again and again be proved for all the three remaining sides,

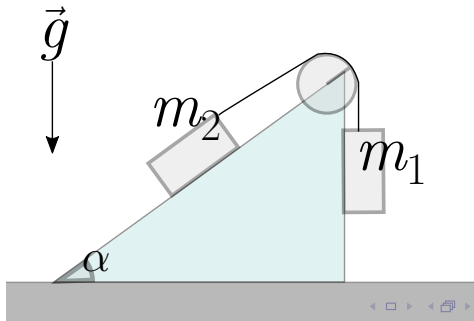
$$\frac{a}{\sin A} = \frac{c}{\sin C} = \frac{d}{\sin D} = 2r \tag{3}$$

And we are done with the proof.

## Problem 1: The General Problem of Force

### Problem.

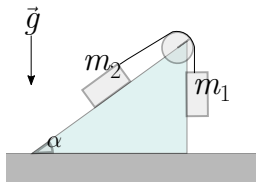
There is a system made of an incline with two masses attached with string as shown. One is  $m_1$  another is  $m_2$ , gravity works  $\vec{g}$ . Find the acceleration of both bodies  $m_1$  and  $m_2$ .



# Problem 1: The General Problem of Force

## Problem.

There is a system made of an incline with two masses attached with string as shown. One is  $m_1$  another is  $m_2$ , gravity works  $\vec{g}$ . Find the acceleration of both bodies  $m_1$  and  $m_2$ .



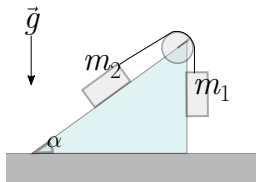
he main force is caused by Gravity

Figure: Problem 1 Diagram

# Problem 1: The General Problem of Force

## Problem.

There is a system made of an incline with two masses attached with string as shown. One is  $m_1$  another is  $m_2$ , gravity works  $\vec{g}$ . Find the acceleration of both bodies  $m_1$  and  $m_2$ .



The main force is caused by Gravity. The rope can pull the  $m_2$  and slow down  $m_1$  when it tries to fall down.

Figure: Problem 1 Diagram



# Problem 1: The General Problem of Force

## Problem.

There is a system made of an incline with two masses attached with string as shown. One is  $m_1$  another is  $m_2$ , gravity works  $\vec{g}$ . Find the acceleration of both bodies  $m_1$  and  $m_2$ .

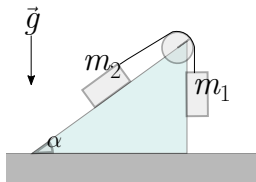


Figure: Problem 1 Diagram

The main force is caused by Gravity. The rope can pull the  $m_2$  and slow down  $m_1$  when it tries to fall down. Here is possibility that too much pull by  $m_2$  can cause the  $m_1$  to lift off the ground, instead of falling down. We have to be careful about all the possibilities.

This is the approach

- Find all the forces, it's the tricky part.

This is the approach

- Find all the forces, it's the tricky part.
- Find the acceleration from the force using  $\vec{F} = m\vec{a}$ . i

## Problem 1: Find All the forces

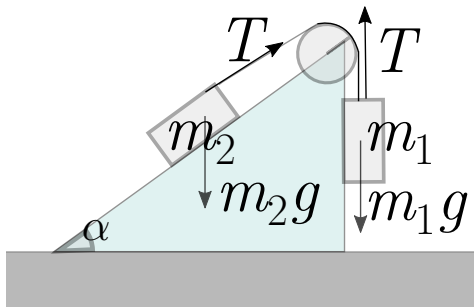


Figure: Sorry for this exploded word size.

We will start isolating the force on the bodies  $m_1$  and  $m_2$ .

Problem 1: Let us find the forces on  $m_2$  first

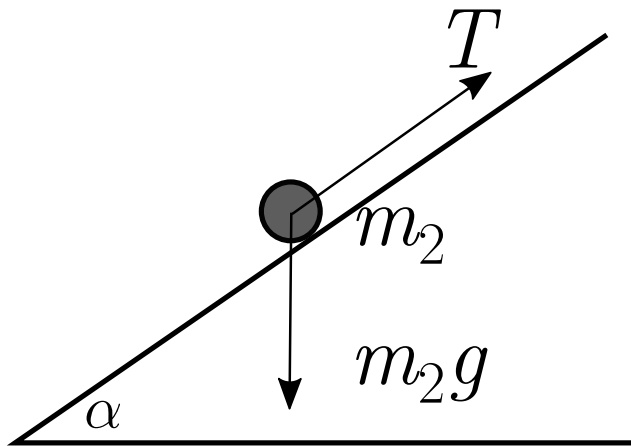


Figure:

Problem 1: Let us find the forces on  $m_1$  then

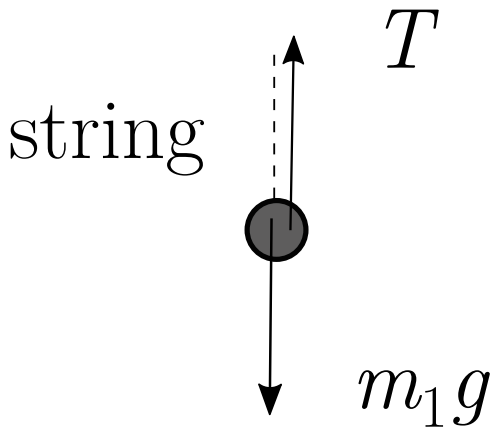


Figure:

# What is $T$ ?

- $T$  is the Tension force, the “Tan” (in Bengali) because of the rope.
- $T$  is same for the whole rope, otherwise a non zero force on the rope would occur.

Let us look the total force on  $m_2$

Note that motion of the body  $m_2$  is constrained (limited) in the incline.

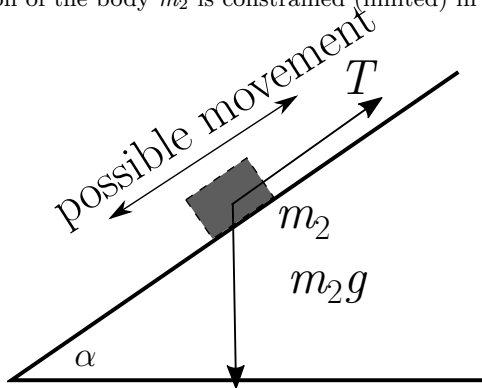


Figure:

Thus, the force on  $m_2$  can only work along the incline. This force is made from 2 components.



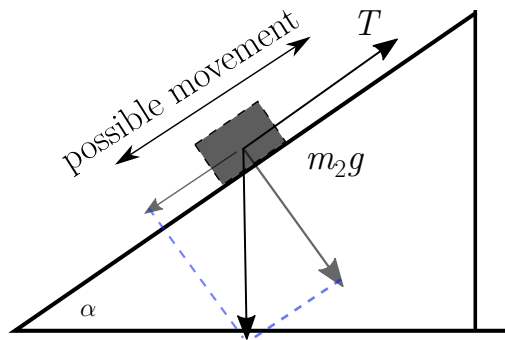


Figure:

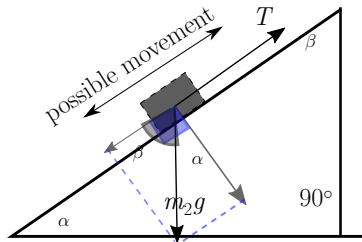


Figure:

$$\alpha + \beta = 90^\circ$$

And you can note that the projection of the  $m_2g$  vector has similar angles.

- We don't need the perpendicular force components.
- Because the block  $m_2$  is constrained to move along the incline.
- So the force perpendicular to the motion are useless.

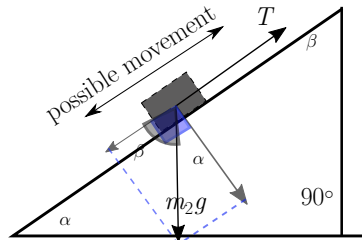


Figure:

# Sign Conventions for Problem

We will consider the sign convention so that,

- If  $m_1$  falls down, then the motion is in positive direction.
- If  $m_1$  lifts up, the the motion is in negative direction.
- $m_1$  falling down means that  $m_2$  will lift up along the ramp. This motion, where  $m_2$  will lift up along the inline/ramp, means the motion is “Positive”.

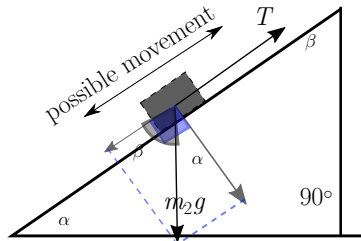


Figure:

The force by gravity, parallel to the direction of possible motion,

$$m_2 g \cos \beta = m_2 g \sin \alpha = F_g$$

This is what we need.

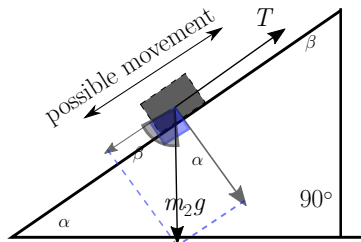


Figure:

Now, the total force on  $m_2$ ,

$$T - F_g$$

This total force can let us know about the acceleration of the  $m_2$  using  $F = ma$ ,

$$m_2 a_2 = T - F_g$$

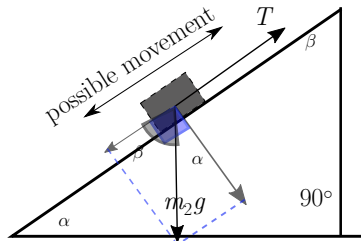


Figure:

Now, we need the total force on  $m_1$ .

The gravity pulls by  $m_1 g$  and the tension pulls against  $T$ , so total force,

$$m_1 a_1 = m_1 g - T$$

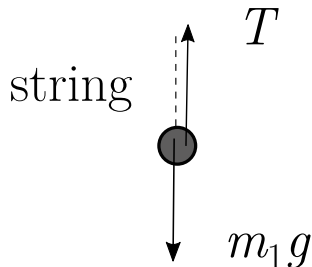


Figure:



We have built up two equations,

$$m_1 a_1 = m_1 g - T$$

$$m_2 a_2 = T - m_2 g \sin \alpha$$

- There are 2 equations
- But there are 3 unknowns. They are,  $a_1$ ,  $a_2$ ,  $T$ .

This system of equation can only be solved if, The number of Unknown **variable** match with the number of **equations**.

This means,

We need *another* equation

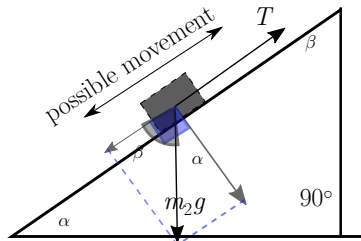


Figure:

Please look at  $a_1$  and  $a_2$ , note that, because of the string, both of the blocks  $m_1$  and  $m_2$  should together, in the same motion. Because, if one is faster than the other, the rope will resist that. So we shall have  $a_1 = a_2$ .

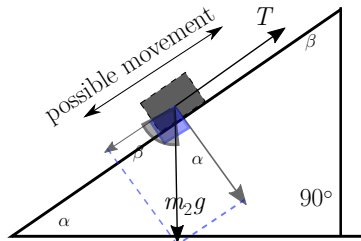


Figure:

Now using the fact  $a_1 = a_2 = a$ ,

$$m_1 a = m_1 g - T$$

$$m_2 a = T - m_2 g \sin \alpha$$

To solve this equations,

- $T = m_2 a + m_2 g \sin \alpha$

Now using the fact  $a_1 = a_2 = a$ ,

$$m_1 a = m_1 g - T$$

$$m_2 a = T - m_2 g \sin \alpha$$

To solve this equations,

- $T = m_2 a + m_2 g \sin \alpha$
- Put this in second equation,  $m_1 a = m_1 g - (m_2 a + m_2 g \sin \alpha)$ .

Now using the fact  $a_1 = a_2 = a$ ,

$$m_1 a = m_1 g - T$$

$$m_2 a = T - m_2 g \sin \alpha$$

To solve this equations,

- $T = m_2 a + m_2 g \sin \alpha$
- Put this in second equation,  $m_1 a = m_1 g - (m_2 a + m_2 g \sin \alpha)$ .
- Solve this for,  $a$ , you will have,  $m_1 g - m_2 g \sin \alpha = (m_1 + m_2) a$
- We have,  $a = \frac{g(m_1 - m_2 \sin \alpha)}{m_1 + m_2}$

Note that, if  $m_1 > m_2 \sin \alpha$ , then  $a > 0$ , which means  $a$  is positive and the acceleration is positive, which follows the convention  $m_1$  falling down is positive. Hence,

$$a = \frac{g(m_1 - m_2 \sin \alpha)}{m_1 + m_2}$$

## Add something to Problem 1

Text **What is the Acceleration if there is friction?**

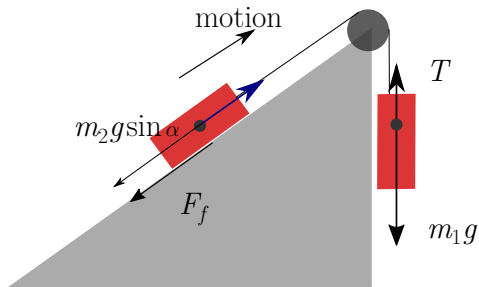


Figure:

The friction is to oppose motion.

- Now, we find the required equations.

- Now, we find the required equations.
- $m_2 a = T - m_2 g \sin \alpha - F_f.$



- Now, we find the required equations.
- $m_2 a = T - m_2 g \sin \alpha - F_f.$
- $m_1 a = m_1 g - T.$

- Now, we find the required equations.
- $m_2 a = T - m_2 g \sin \alpha - F_f$ .
- $m_1 a = m_1 g - T$ .
- Friction is always  $F_f = \mu N$ , where  $N$  is the Normal component of force. Normal force is the force that the incline gives perpendicularly to the block.

- Now, we find the required equations.
- $m_2 a = T - m_2 g \sin \alpha - F_f$ .
- $m_1 a = m_1 g - T$ .
- Friction is always  $F_f = \mu N$ , where  $N$  is the Normal component of force. Normal force is the force that the incline gives perpendicularly to the block.
- Normal force is discussed in next frame.

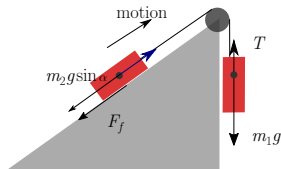


Figure:

We need friction force  $F_f$ .

The normal force is the perpendicular projection of the gravitational force.

- There is no ability to move perpendicular to the inclination.

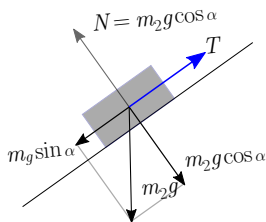


Figure:

$$F_f = m_2 g \mu \cos \alpha$$

We need friction force  $F_f$ .

The normal force is the perpendicular projection of the gravitational force.

- There is no ability to move perpendicular to the inclination.
- So the force is balanced along the perp  $\perp$ .

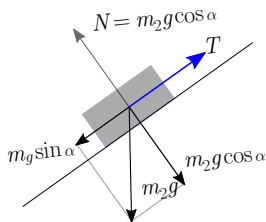


Figure:

$$F_f = m_2 g \mu \cos \alpha$$

We need friction force  $F_f$ .

The normal force is the perpendicular projection of the gravitational force.

- There is no ability to move perpendicular to the inclination.
- So the force is balanced along the perp  $\perp$ .
- Hence, the  $N = m_2 g \cos \alpha$ .
- Friction is,  $F_f = \mu N$ , here,  $\mu$  is the friction coefficient.

$$F_f = m_2 g \mu \cos \alpha$$

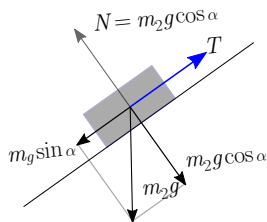


Figure:

We need friction force  $F_f$ .

The normal force is the perpendicular projection of the gravitational force.

- There is no ability to move perpendicular to the inclination.
- So the force is balanced along the perp  $\perp$ .
- Hence, the  $N = m_2 g \cos \alpha$ .
- Friction is,  $F_f = \mu N$ , here,  $\mu$  is the friction coefficient. .

$$F_f = m_2 g \mu \cos \alpha$$

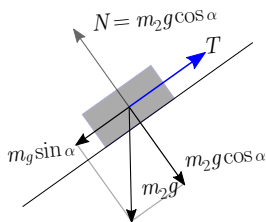


Figure:

The equations that we had,

- $m_2 a = T - m_2 g \sin \alpha - F_f = T - m_2 g \sin \alpha - (m_2 g \mu \cos \alpha).$



The equations that we had,

- $m_2 a = T - m_2 g \sin \alpha - F_f = T - m_2 g \sin \alpha - (m_2 g \mu \cos \alpha).$
- $m_1 a = m_1 g - T.$

The equations that we had,

- $m_2 a = T - m_2 g \sin \alpha - F_f = T - m_2 g \sin \alpha - (m_2 g \mu \cos \alpha).$
- $m_1 a = m_1 g - T.$
- Please note that this equation only hold if there is motion where  $m_2$  lifts up. Otherwise the sign of friction will change.

Similarly solving we get, only for positive motion case,

$$a = \frac{g(m_1 - m_2 \sin \alpha - m_2 \mu \cos \alpha)}{m_1 + m_2}$$

If the motion is reversed,

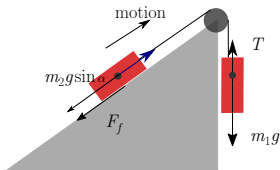


Figure:

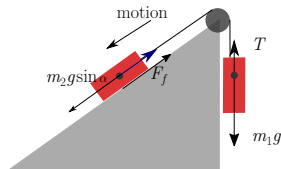


Figure:

We need to write the equation again. Because the direction of friction will change for different motion, where  $m_2$  falls down along the ramp. We have to write,

$$\blacksquare \quad m_2 a = -T + m_2 g \sin \alpha - F_f.$$

This is solved to get,

$$a = \frac{g(m_2 \sin \alpha - m_2 \mu \cos \alpha - m_1)}{m_1 + m_2}$$

If the motion is reversed,

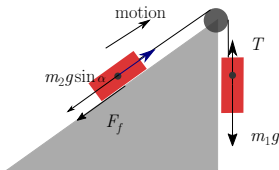


Figure:

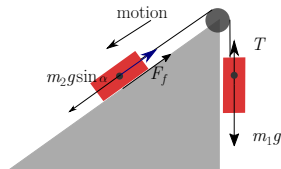


Figure:

We need to write the equation again. Because the direction of friction will change for different motion, where  $m_2$  falls down along the ramp. We have to write,

■  $m_2 a = -T + m_2 g \sin \alpha - F_f.$

■  $m_1 a = T - m_1 g.$

This is solved to get,

$$a = \frac{g(m_2 \sin \alpha - m_2 \mu \cos \alpha - m_1)}{m_1 + m_2}$$

If the motion is reversed,

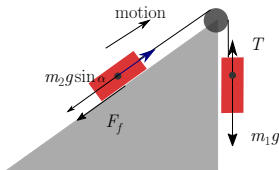


Figure:

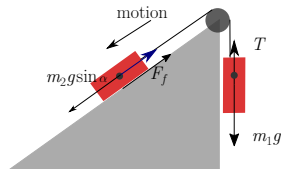


Figure:

We need to write the equation again. Because the direction of friction will change for different motion, where  $m_2$  falls down along the ramp. We have to write,

- $m_2 a = -T + m_2 g \sin \alpha - F_f.$
- $m_1 a = T - m_1 g.$

This is solved to get,

$$a = \frac{g(m_2 \sin \alpha - m_2 \mu \cos \alpha - m_1)}{m_1 + m_2}$$

For 2 cases of friction, we have

$$a = \frac{g(m_2 \sin \alpha - m_2 \mu \cos \alpha - m_1)}{m_1 + m_2}$$

$$a = \frac{g(m_1 - m_2 \sin \alpha - m_2 \mu \cos \alpha)}{m_1 + m_2}$$

These are two cases, first for  $m_2$  getting down, second for  $m_2$  getting up. We can define the condition of getting up, down, or resting in position because of friction.

To  $m_2$  get down, we need,

$$m_2 \sin \alpha - m_2 \mu \cos \alpha - m_1 > 0 \quad \rightarrow \quad m_1 - m_2 \sin \alpha + m_2 \mu \cos \alpha < 0$$

For  $m_2$  get up, we need,

$$m_1 - m_2 \sin \alpha - m_2 \mu \cos \alpha > 0$$

## Problem 2

### Problem.

There is a ring of radius  $R$  and there is a bead that can move along the circle of the ring.

The ring rotates in  $\omega$  angular speed, find a position of the bead in which case it stays in *rest*.

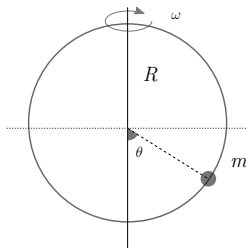


Figure:

This position we are going to define with the angle  $\theta$  respect to rotation axis.

### Idea

If anything is in a constant velocity or rest, then the total force on it is zero

Because  $F = ma$ , if  $a$  is  $a = 0$ , then there is a constant velocity or it is zero. Because,

$$a = \frac{dv}{dt} \rightarrow a = 0 \text{ means } v = \text{const}$$

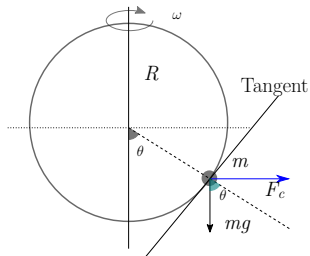


Figure: We will isolate the forces.



## Problem 2: Force

- $\theta^c$  is  $90^\circ - \theta$ .
- $F_C$  is the Centrifugal force  
 $F_C = m\omega^2 r$ , where  $r$  is distance from *axis*.

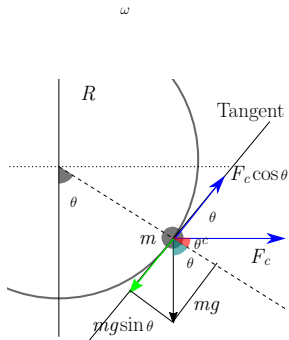


Figure:

## Problem 2: Force

- $\theta^c$  is  $90^\circ - \theta$ .
- $F_C$  is the Centrifugal force  
 $F_C = m\omega^2 r$ , where  $r$  is distance from *axis*.
- The motion of the bead is constrained along the tangent; if there is a component of force along the tangent, the bead will move.

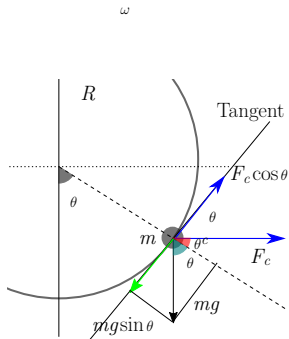


Figure:

## Problem 2: Force

- $\theta^c$  is  $90^\circ - \theta$ .
- $F_C$  is the Centrifugal force  
 $F_C = m\omega^2 r$ , where  $r$  is distance from *axis*.
- The motion of the bead is constrained along the tangent; if there is a component of force along the tangent, the bead will move.
- Projection of gravity on the tangent is  $mg \sin \theta$ .

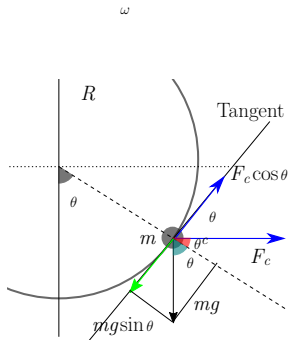


Figure:

## Problem 2: Force

- $\theta^c$  is  $90^\circ - \theta$ .
- $F_C$  is the Centrifugal force  
 $F_C = m\omega^2 r$ , where  $r$  is distance from *axis*.
- The motion of the bead is constrained along the tangent; if there is a component of force along the tangent, the bead will move.
- Projection of gravity on the tangent is  $mg \sin \theta$ .
- Projection of Centrifugal force  $F_C$  on the tangent is  $F_C \cos \theta$ .

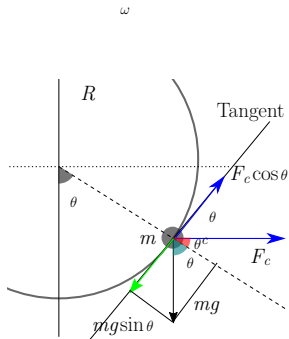


Figure:

## Problem 2: Force

- $\theta^c$  is  $90^\circ - \theta$ .
- $F_C$  is the Centrifugal force  
 $F_C = m\omega^2 r$ , where  $r$  is distance from *axis*.
- The motion of the bead is constrained along the tangent; if there is a component of force along the tangent, the bead will move.
- Projection of gravity on the tangent is  $mg \sin \theta$ .
- Projection of Centrifugal force  $F_C$  on the tangent is  $F_C \cos \theta$ .
- From the diagram evidently two projection are opposite to each other.

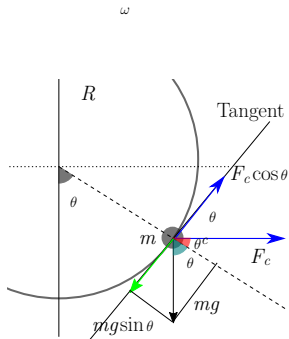


Figure:

## Problem 2: Close look at the forces

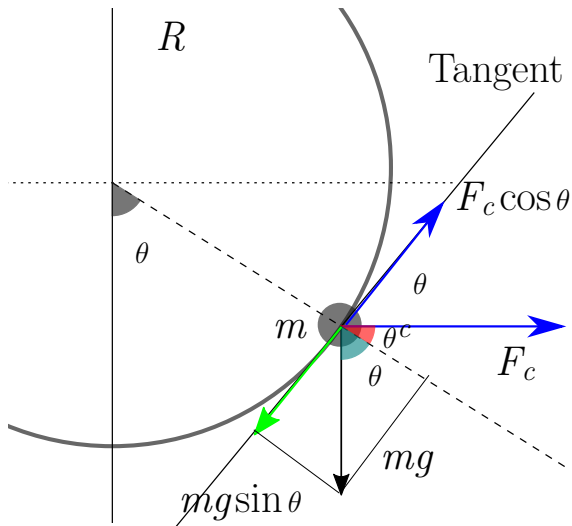


Figure: A closer look

## Problem 2: Centrifugal Force

We need the Radius of Rotation, which is found in the figures, we calculate it,

$$r = R \sin \theta$$

Using these,

$$F_c = m\omega^2 r = m\omega^2 R \sin \theta$$

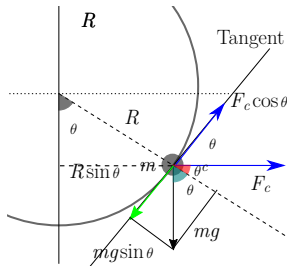


Figure:

## Problem 2: Balance force for Stationary condition

We need total force 0,

$$mg \sin \theta - \left( m \omega^2 R \sin \theta \right) \cos \theta = 0$$

$$g \sin \theta = \omega^2 R \cos \theta \sin \theta$$

And we have,

$$\cos \theta = \frac{g}{\omega^2 R}$$

The angle  $\theta$  is,

$$\theta = \cos^{-1} \left( \frac{g}{\omega^2 R} \right)$$



## Torque

Torque is rotational force.

Like force, Torque causes acceleration of rotation,

$$\tau = I\alpha$$

Where,  $I$  is Moment of Inertia, which is like Mass for rotating bodies.

$I$  depends on the Geometry and Mass of the system.

$$\tau = r \times F$$

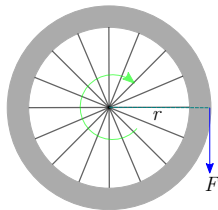


Figure:

## Extending Torque Idea

In case the  $F$  makes an angle with the radius vector  $r$ , we have to add a  $\sin \theta$  factor for the perpendicular projection,

$$\tau = r \times F \sin \theta$$

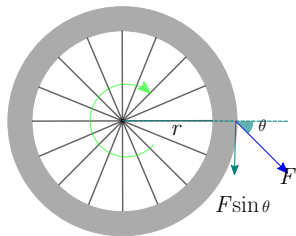


Figure:

### Idea

Most of the rules of Force can be used for Torque too, for example, torque is balanced too.

# Angular Momentum

Like the momentum we learned few minutes ago, angular momentum is also like that,

$$p = mv$$

And similarly

$$L = I\omega$$

Where  $I$  is known, and  $\omega$  is Angular Speed.

Please note that these are vector, but we don't need the vector law for Angular Dynamics right now.

### Problem 3: Some Idea on Torque using last Idea

#### Problem.

Three rods are connected by hinges to each other, the outermost ones are hinged to a ceiling at point  $A$ , and  $B$ , the distance between these are equal of a rod. A weight is hanged at  $C$ , find the minimum force applied at  $D$  to keep the system in Balance.

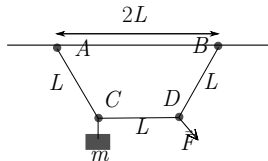


Figure:

If there is no force, then

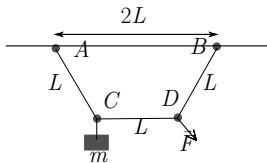


Figure:

Please note, for the pivot  $B$ , if we can stop rotation by axis of  $B$ , then the system can be balanced.

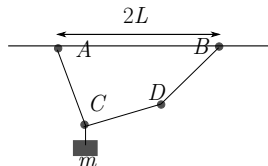


Figure:

We need to know the shapes angles

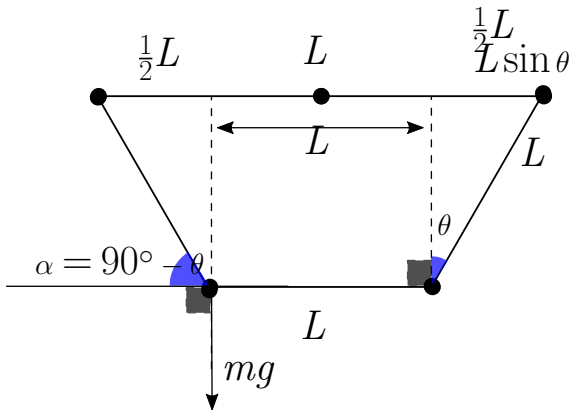


Figure:

## Finding angles for Problem 3

This is sure that,

$$L \sin \theta = \frac{1}{2}L$$

Thus,

$$\theta = 30^\circ$$

From here,

$$\alpha = 90^\circ - \theta = 60^\circ$$

Now, the angle that the  $mg$  vector makes with the left side of the rod is,

$$90^\circ + \alpha = 90^\circ + 60^\circ = 150^\circ$$

So, angle  $m\vec{g}$  makes with the left rod is  $150^\circ$ .

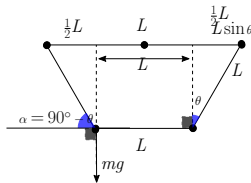


Figure:

## Torque balance rules for Problem 3.

Like force balance, we have to invoke Torque balance, because,

- Force balance is similar to Torque Balance



## Torque balance rules for Problem 3.

Like force balance, we have to invoke Torque balance, because,

- Force balance is similar to Torque Balance
- In this problem, we need to apply force at  $D$ , and if you notice, the motion of  $D$  is like the rotation of right rod pivoting  $B$ .

## Torque balance rules for Problem 3.

Like force balance, we have to invoke Torque balance, because,

- Force balance is similar to Torque Balance
- In this problem, we need to apply force at  $D$ , and if you notice, the motion of  $D$  is like the rotation of right rod pivoting  $B$ .
- To balance the system, we need to balance the *Rotation*.

So, the torque on the left side of the rod,

$$\tau = mgL \sin(150^\circ)$$

And  $\sin 150^\circ = \frac{1}{2}$ , hence,

$$\tau = \frac{mg}{2}L$$

I have something to add here too,

- In the  $mg$  force vector, some force is  $\perp$  perpendicular to the rod (projection)
- Some force is along the rod.

The force components that are along the rod are uimportant, because they pull the left rod and do nothing.

## Final thoughts on Problem 3

Now we add some logic.

## Final thoughts on Problem 3

Now we add some logic.

If left side has torque  $\tau$  where  $\tau = \frac{mg}{2}L$ , then if we apply the same torque on the right side, then we have a torque balance, which will also balance the system, hence, on the right side, we also apply a torque  $\tau'$  which is,

## Final thoughts on Problem 3

Now we add some logic.

If left side has torque  $\tau$  where  $\tau = \frac{mg}{2}L$ , then if we apply the same torque on the right side, then we have a torque balance, which will also balance the system, hence, on the right side, we also apply a torque  $\tau'$  which is,

$$\tau' = \frac{mg}{2}L$$

And  $\tau = FL$ , thus,

$$F = \frac{mg}{2}$$

## Final thoughts on Problem 3

Now we add some logic.

If left side has torque  $\tau$  where  $\tau = \frac{mg}{2}L$ , then if we apply the same torque on the right side, then we have a torque balance, which will also balance the system, hence, on the right side, we also apply a torque  $\tau'$  which is,

$$\tau' = \frac{mg}{2}L$$

And  $\tau = FL$ , thus,

$$F = \frac{mg}{2}$$

This force is perpendicular to the rod, and solely works to balance the system, without any component along the rod, hence, no force is wasted and  $F = \frac{mg}{2}$  is the minimum force possible.

$$F = \frac{mg}{2}$$

Problem 3 Solution  $F = \frac{mg}{2}$

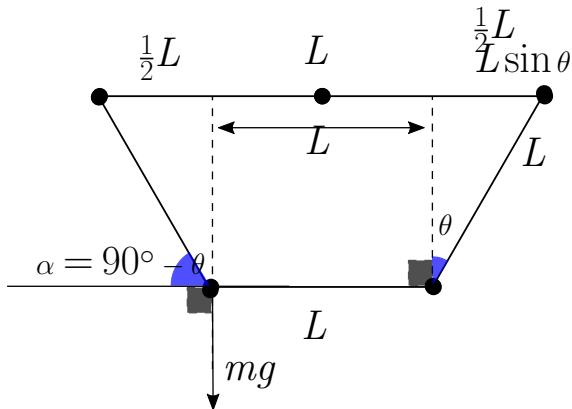


Figure: