Day 1

Ahmed Saad Sabit

April 8, 2021

0.1 Right Angled Triangle - The Triangle with a 90°.

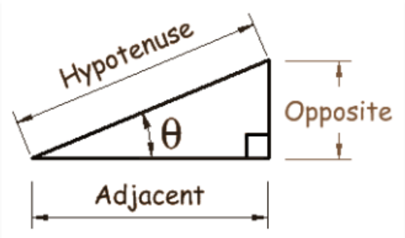


Figure: Right Angled Triangle

Because of some ratio of the right angled triangle we can say that,

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

0.3 Measuring Angles

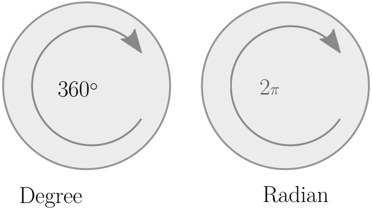


Figure: Comaparison

This means that,

$$180^\circ = \pi \text{ radians}$$

The reason we use radian can be shown when needed.

0.4 The fact $\alpha + \beta + \gamma = 180^\circ = \pi$

If you add all angles inside a triangle, the sum will always be 180°.

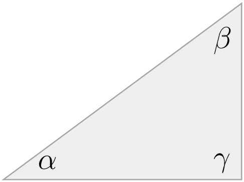


Figure: Summing up angles

$$\alpha + \beta + \gamma = 180^\circ = \pi$$

For a right angle triangle, $\gamma = 90^\circ = \frac{\pi}{2}$, hence,

$$\alpha + \beta + 90^\circ = 180^\circ$$

$$\alpha + \beta = 90^\circ$$

Newton's Second Law: $\vec{F} = m\vec{a}$

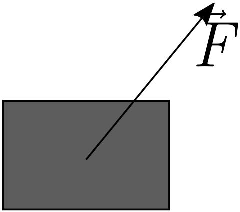


Figure: If you push a block

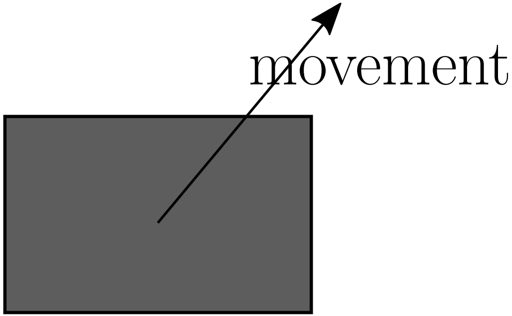


Figure: The movement will be in the same direction

The movement is Acceleration. Acceleration is increase/decrease in speed overtime.

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

A required relation of angles α and β.

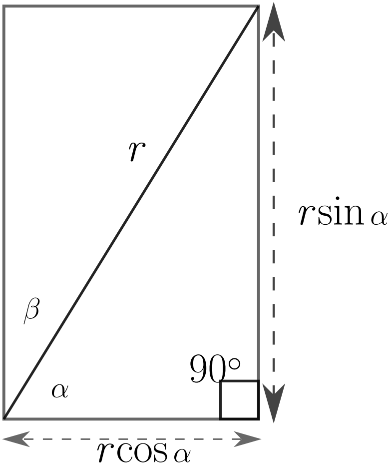


Figure:

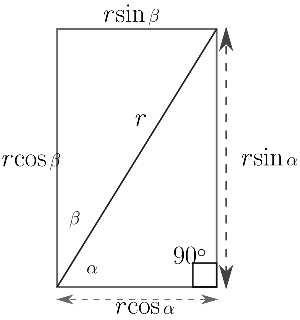
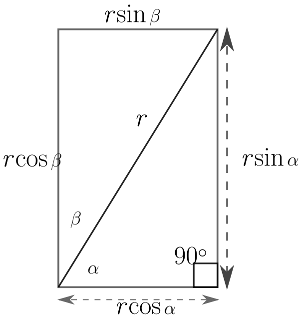


Figure:

This actually shows us that,

$$r \sin \alpha = r \cos \beta$$

$$r \cos \alpha = r \sin \beta$$



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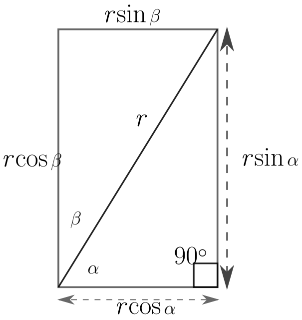
Because $\alpha + \beta = 90°$, $\beta = 90° - \alpha$. Hence,

$$r\sin\alpha = r\cos(90° - \alpha)$$

$$r\cos\alpha = r\sin(90° - \alpha)$$



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$$r\cos\alpha = r\sin(90^\circ - \alpha)$$

So, we now know that,

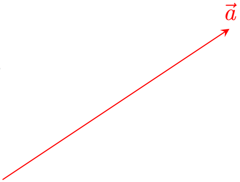
$$\sin(90^\circ - \alpha) = \cos\alpha$$
$$\cos(90^\circ - \alpha) = \sin\alpha$$

Warm Up: Cosine Law — For Vector

We will use vectors very often in physics, so it is better we try it out before starting. At first, vectors can be added in this way,
Vectors can be added, like,

$$\vec{a} + \vec{b}$$

Which looks like one vector is put in-front of another.



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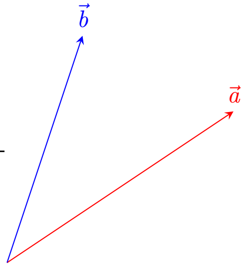

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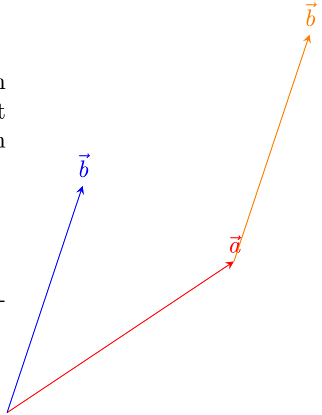

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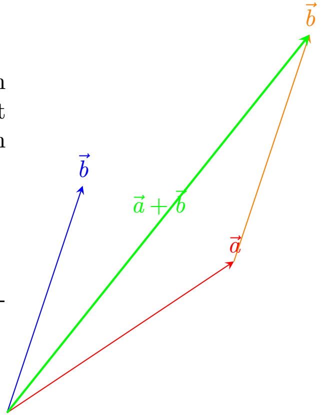
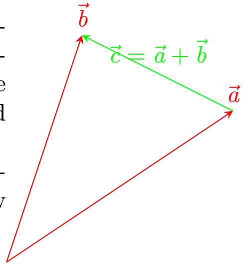


Vector Triangle for Cosine Law

If there is a side a and b, then assuming the third side is c, the Law of cosine is to find the length of c using the lengths of a and b, and the angle a and b makes.
The easiest solution can be found using $\vec{a}$ and $\vec{b}$ vector methods. We know angle θ between a and b



Now let us have a triangle with side $\vec{a}$ and $\vec{b}$, then the third side is obviously $\vec{b}-\vec{a}$. This is same as we saw in last frame. Now, let the third side be $\vec{c}=\vec{b}-\vec{a}$, then can we find a scalar solution for the length of vector $\vec{c}$?

$$\vec{c}=\vec{b}-\vec{a}$$
$$(\vec{c})^2=\left(\vec{b}-\vec{a}\right)^2$$
$$\vec{c}\cdot\vec{c}=\vec{a}\cdot\vec{a}-2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}$$

Now, $\vec{a}\cdot\vec{b}$, is the dot product. And for a dot product,

$$\vec{a}\cdot\vec{b}=|a|\ |b|\ \cos\theta$$

And if,

$$\vec{a}\cdot\vec{a}=|a||a|\cos(0)$$

Because $\vec{a}$ makes 0° with itself. Hence, we have,

$$c^2=a^2+b^2-2ab\cos\theta$$

So, we have found length of third side,

$$c=\sqrt{a^2+b^2-2ab\cos\theta}$$

Now let us have a triangle with side $\vec{a}$ and $\vec{b}$, then the third side is obviously $\vec{b} - \vec{a}$. This is same as we saw in last frame. Now, let the third side be $\vec{c} = \vec{b} - \vec{a}$, then can we find a scalar solution for the length of vector $\vec{c}$?

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$$\begin{aligned}
\vec{c} &= \vec{b}-\vec{a} \\
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The $\sin\alpha$ **Law:** For a triangle, with lowercase denoting the angle and the uppercase the opposite side, the sides and angle follow the relation respect to the circumcircle is that,

$$\frac{a}{\sin A} = \frac{c}{\sin C} = \frac{d}{\sin D} = 2r$$

Proof: Let $\triangle ACD$ be the required triangle, we want to build a relation between the length of the sides and the opposite angles to them motivated from the fact that the opposite angle being large making the side project a larger length.

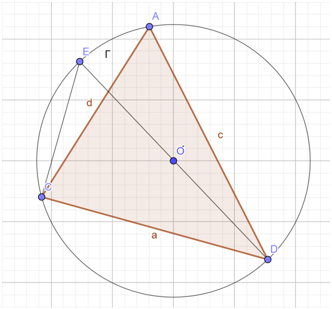
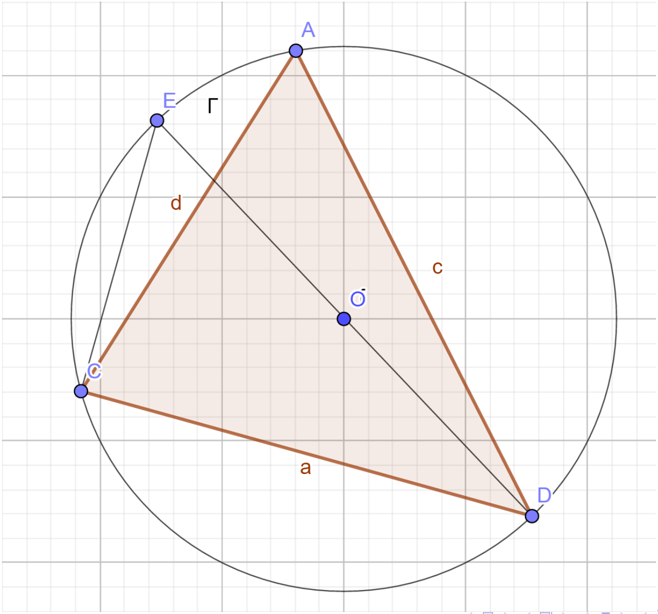


Figure: Sine Law


A
E
Γ
d
c
O
C
a
D

Let Γ be the circumcircle of $\triangle ACD$ and we draw a diameter DE that passes through the center of Γ which is point O.

$\triangle ECD$ is a Right Angled Triangle because it meets the *Semi-circle* and thus,

$$\sin\angle CED = \frac{\bar{CD}}{\bar{ED}}$$

Using the *Arc and Angle Theorem* we can see that

$$\angle CED = \angle CAD = \angle A$$

As we know that $DE = 2r$ we are left to show that,

$$\sin\angle CED = \frac{\bar{CD}}{\bar{ED}}$$

$$\sin\angle A = \frac{a}{2r} \tag{1}$$

$$\frac{a}{\sin A} = 2r \tag{2}$$

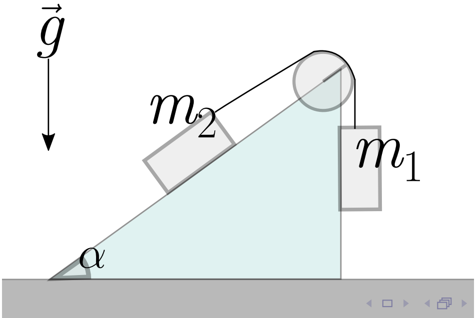
And this can be again and again be proved for all the three remaining sides,

$$\frac{a}{\sin A} = \frac{c}{\sin C} = \frac{d}{\sin D} = 2r \tag{3}$$

And we are done with the proof.

Problem.

There is a system made of an incline with two masses attached with string as shown. One is m_1 another is m_2, gravity works $\vec{g}$. *Find the acceleration of both bodies m_1 and m_2.*



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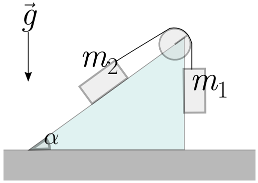

Figure: Problem 1 Diagram

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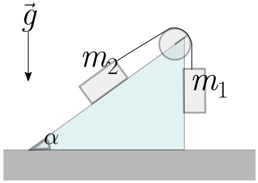

Figure: Problem 1 Diagram

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Problem 1: The General Problem of Force

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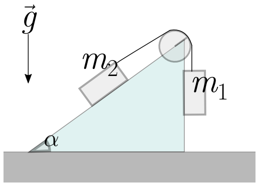

Figure: Problem 1 Diagram

he main force is caused by Gravity The rope can pull the m_2 and slow down m_1 when it tries to fall down. here is possibility that too much pull be m_2 can cause the m_1 to lift off the ground, instead of falling down. We have to be careful about all the possibilities.

This is the approach

- Find all the forces, it's the tricky part.

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- Find all the forces, it's the tricky part.
- Find the acceleration from the force using $\vec{F} = m\vec{a}$. i

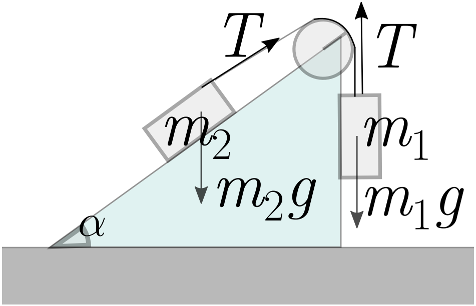


Figure: Sorry for this exploded word size.

We will start isolating the force on the bodies m_1 and m_2.

Problem 1: Let us find the forces on m_2 first

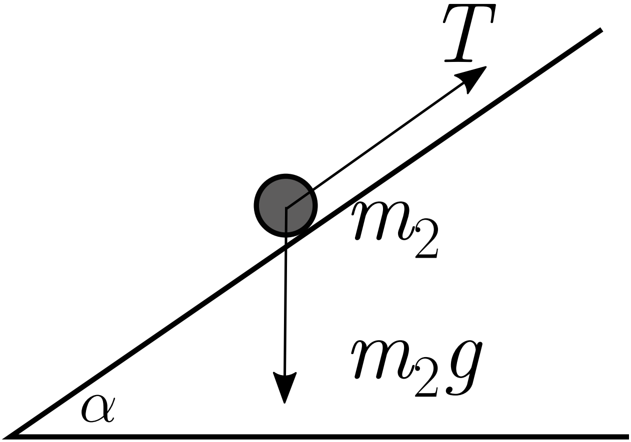


Figure:

Problem 1: Let us find the forces on m_1 then

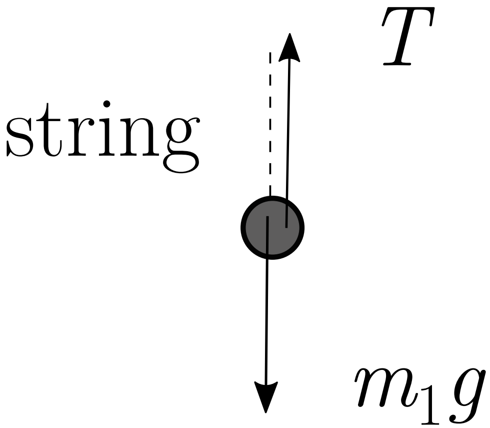


Figure:

What is T?

- T is the Tension force, the "Tan" (in Bengali) because of the rope.
- T is same for the whole rope, otherwise a non zero force on the rope would occur.

Let us look the total force on m_2

Note that motion of the body m_2 is constrained (limited) in the incline.

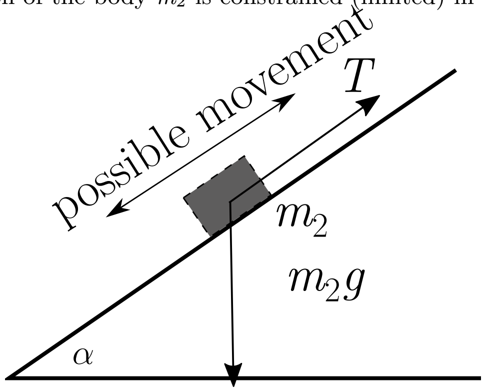
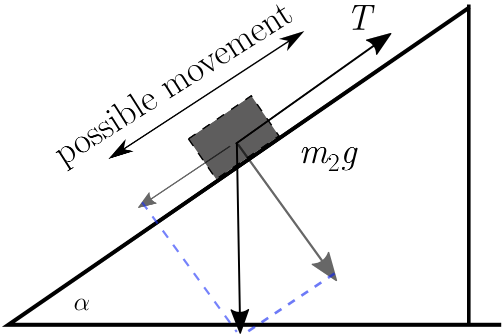


Figure:

Thus, there force on m_2 can only work along the incline. This is force is made from 2 components.



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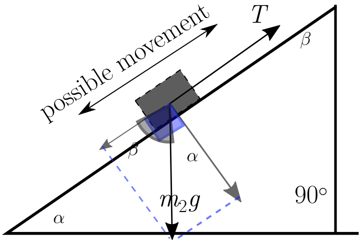

Figure:

$$\alpha + \beta = 90^\circ$$

And you can note that the projection of the m_2g vector has similar angles.

On m_2

- We don't need the perpendicular force components.
- Because the block m_2 is constrained to move along the incline.
- So the force perpendicular to the motion are useless.

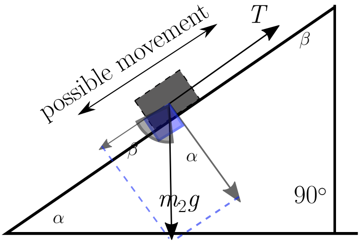

Figure:

We will consider the sign convention so that,

- If m_1 falls down, then the motion is in positive direction.
- If m_1 lifts up, the the motion is in negative direction.
- m_1 falling down means that m_2 will lift up along the ramp. This motion, where m_2 will lift up along the inline/ramp, means the motion is "Positive".

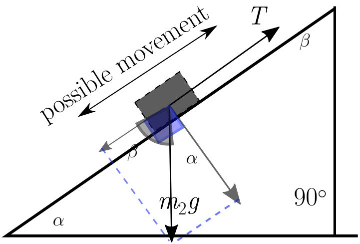

Figure:

The force by gravity, parallel to the direction of possible motion,

$$m_2 g \cos\beta = m_2 g \sin\alpha = F_g$$

This is what we need.

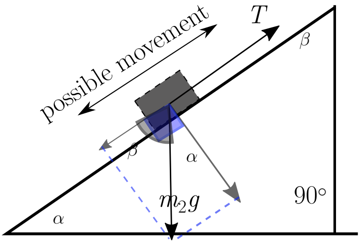

Figure:

Now, the total force on m_2,

$$T - F_g$$

This total force can let us know about the accelearation of the m_2 using $F = ma$,

$$m_2 a_2 = T - F_g$$

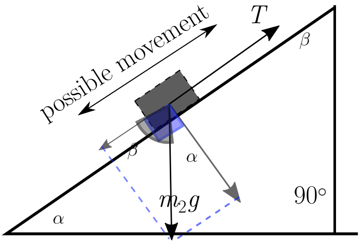

Figure:

Now, we need the total force on m_1.

The gravity pulls by $m_1 g$ and the tension pulls against T, so total force,

$$m_1 a_1 = m_1 g - T$$

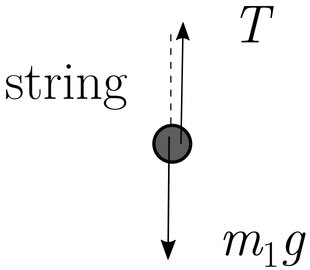

Figure:

We have built up two equations,

$$m_1 a_1 = m_1 g - T$$
$$m_2 a_2 = T - m_2 g \sin\alpha$$

- There are 2 equations
- But there are 3 unknowns. They are, a_1, a_2, T.

This system of equation can only be solved if, The number of Unknown variable match with the number of equations.
This means,
We need *another* equation

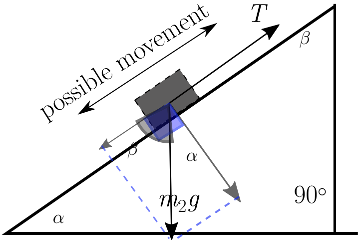

Figure:

Please look at a_1 and a_2, note that, because of the string,
both of the blocks m_1 and m_2 should together, in the same motion.
Because, if one is faster than the other, the rope will resist that.
So we shall have $a_1 = a_2$.

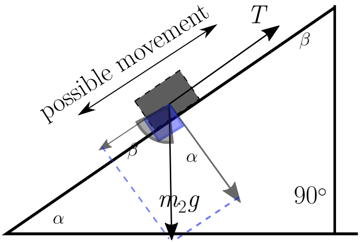

Figure:

Answer final

Now using the fact $a_1 = a_2 = a$,

$$m_1 a = m_1 g - T$$
$$m_2 a = T - m_2 g \sin\alpha$$

To solve this equations,

- $T = m_2 a + m_2 g \sin\alpha$

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To solve this equations,

- $T = m_2 a + m_2 g \sin\alpha$
- Put this in second equation, $m_1 a = m_1 g - (m_2 a + m_2 g \sin\alpha)$.

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To solve this equations,

- $T = m_2 a + m_2 g \sin\alpha$
- Put this in second equation, $m_1 a = m_1 g - (m_2 a + m_2 g \sin\alpha)$.
- Solve this for, a, you will have, $m_1 g - m_2 g \sin\alpha = (m_1 + m_2) a$
- We have, $a = \frac{g(m_1 - m_2 \sin\alpha)}{m_1 + m_2}$

Note that, if $m_1 > m_2 \sin\alpha$, then $a > 0$, which means a is positive and the acceleration is positive, which follows the convention m_1 falling down is positive. Hence,

$$\boxed{a = \frac{g\,(m_1 - m_2 \sin\alpha)}{m_1 + m_2}}$$

Add something to Problem 1

Text **What is the Accelearation if there is friction?**

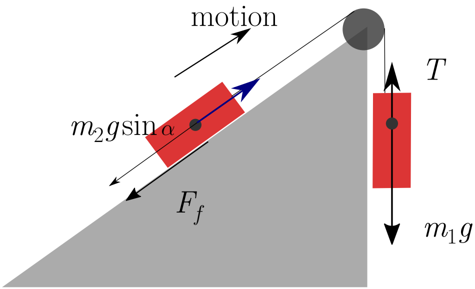

Figure:

The friction is to oppose motion.

- Now, we find the required equations.

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- $m_2 a = T - m_2 g \sin\alpha - F_f$.

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- Normal force is discussed in next frame.

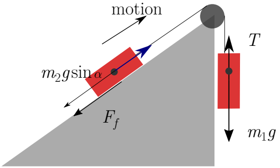

Figure:

The normal force is the perpendicular projection of the gravitational force.

- There is no ability to move perpendicular to the inclination.

$$F_f = m_2 g\mu \cos\alpha$$

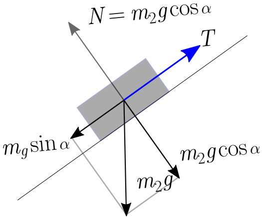

Figure:

We need friction force F_f.

The normal force is the perpendicular projection of the gravitational force.

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- So the force is balanced along the perp $\perp$.

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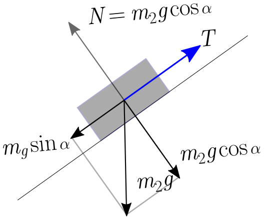

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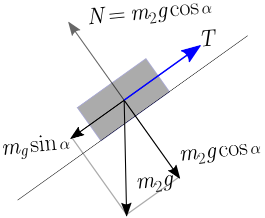

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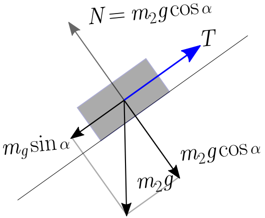

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The equations that we had,

- $m_2 a = T - m_2 g \sin\alpha - F_f = T - m_2 g \sin\alpha - (m_2 g \mu \cos\alpha).$

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- $m_1 a = m_1 g - T$.
- Please note that this equation only hold if there is motion where m_2 lifts up. Otherwise the sign of friction will change.

Similarly solving we get, only for positive motion case,

$$\boxed{a = \frac{g\,(m_1 - m_2 \sin\alpha - m_2 \mu \cos\alpha)}{m_1 + m_2}}$$

If the motion is reversed,

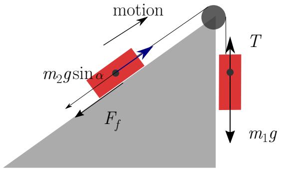

Figure:

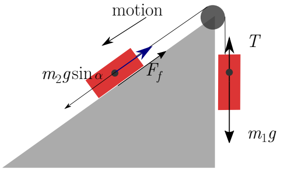

Figure:

We need to write the equation again. Because the direction of friction will change for different motion, where m_2 falls down along the rame. We have to write,

- $m_2 a = -T + m_2 g \sin \alpha - F_f.$

This is solved to get,

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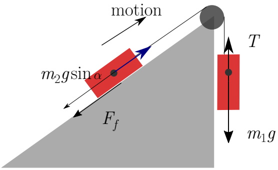

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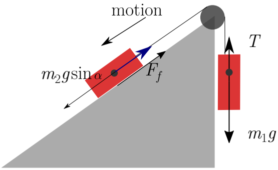

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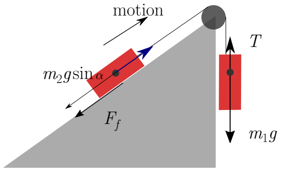

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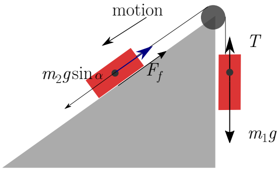

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For 2 cases of friction, we have

$$\boxed{a = \frac{g(m_2 \sin\alpha - m_2\mu\cos\alpha - m_1)}{m_1 + m_2}}$$

$$\boxed{a = \frac{g\left(m_1 - m_2 \sin\alpha - m_2\mu\cos\alpha\right)}{m_1 + m_2}}$$

These are two cases, first for m_2 getting down, second for m_2 getting up. We can define the condition of getting up, down, or resting in position because of friction.
To m_2 get down, we need,

$$m_2 \sin\alpha - m_2\mu\cos\alpha - m_1 > 0 \quad \rightarrow \quad m_1 - m_2 \sin\alpha + m_2\mu\cos\alpha < 0$$

For m_2 get up, we need,

$$m_1 - m_2 \sin\alpha - m_2\mu\cos\alpha > 0$$

Problem 2

Problem.

There is a ring of radius R and there is a bead that can move along the circle of the ring. The ring rotates in ω angular speed, find a position of the bead in which case it stays in *rest*.

This position we are going to define with the angle θ respect to rotation axis.

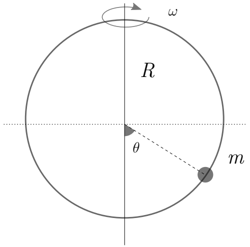


Figure:

Idea

If anything is in a constant velocity or rest, then the total force on it is zero

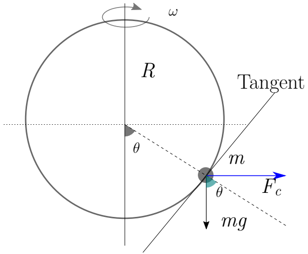


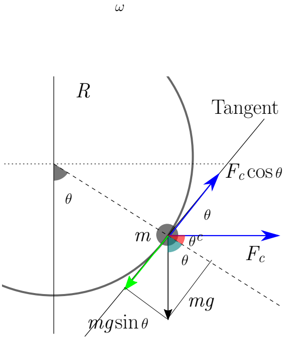
Figure: We will isolate the forces.

Because $F = ma$, if a is $a = 0$, then there is a constant velocity or it is zero. Because,

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad \rightarrow \quad a = 0 \quad \text{means} \quad v = \ \text{const}$$

Problem 2: Force

- θ^c is $90^\circ - \theta$.
- F_C is the Centrifugal force
  $F_C = m\omega^2 r$, where r is distance from *axis*.



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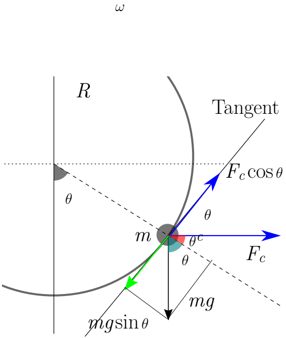

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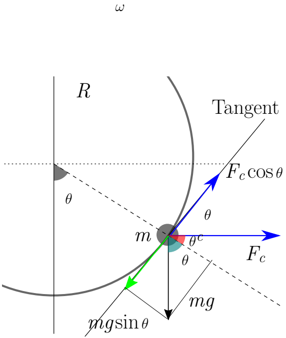

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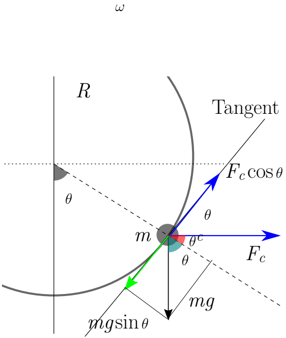

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- Projection of gravity on the tangent is $mg \sin\theta$. .
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- From the diagram evidently two projection are opposite to each other.

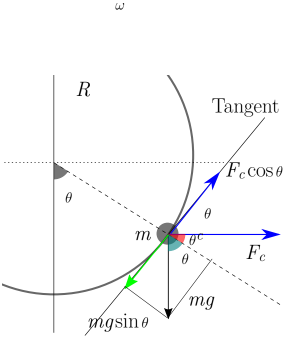

Figure:

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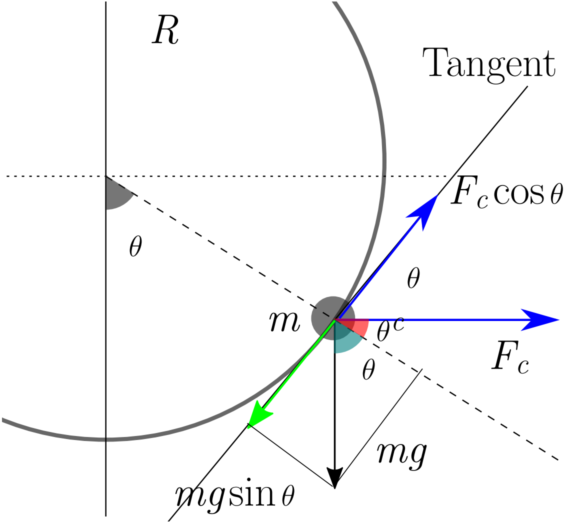


Figure: A closer look

We need the Radius of Rotation, which is found in the figures, we calculate it,

$$r = R\sin\theta$$

Using these,

$$F_c = m\omega^2 r = m\omega^2 R\sin\theta$$

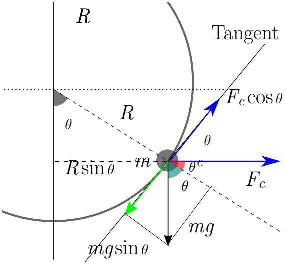


Figure:

Problem 2: Balance force for Stationary condition

We need total force 0,

$$mg\sin\theta - \left(m\omega^2 R\sin\theta\right)\cos\theta = 0$$

$$g\sin\theta = \omega^2 R\cos\theta\sin\theta$$

And we have,

$$\cos\theta = \frac{g}{\omega^2 R}$$

The angle θ is,

$$\boxed{\theta = \cos^{-1}\left(\frac{g}{\omega^2 R}\right)}$$

Torque

Torque is rotational force.
Like force, Torque causes acceleration of rotation,

$$\tau = I\alpha$$

Where, I is Moment of Inertia, which is like Mass for rotating bodies.
I depends on the Geometry and Mass of the system.

$$\tau = r \times F$$

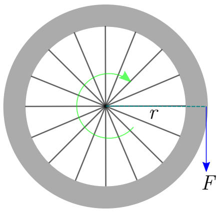


Figure:

Extending Torque Idea

In case the F makes an angle with the radius vector r, we have to add a $\sin\theta$ factor for the perpendicular projection,

$$\tau = r \times F \sin\theta$$

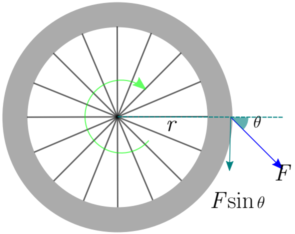


Figure:

Idea

Most of the rules of Force can be used for Torque too, for example, torque is balanced too.

Angular Momentum

Like the momentum we learned few minutes ago, angular momentum is also like that,

$$p = mv$$

And similarly

$$L = I\omega$$

Where I is known, and ω is Angular Speed.
Please note that these are vector, but we don't need the vector law for Angular Dynamcics right now.

Problem 3: Some Idea on Torque using last Idea

Problem.

Three rods are connected by hinges to each other, the outermost ones are hinged to a ceiling at point A, and B, the distance between these are equal of a rod. A weight is hanged at C, find the minimum force applied at D to keep the system in Balance.

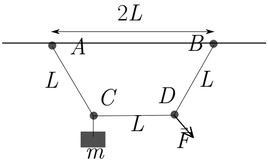
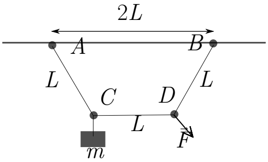


Figure:

If there is no force, then



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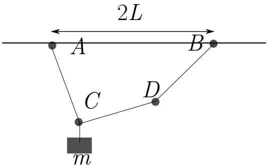


Figure:

Please note, for the pivot B, if we can stop rotation by axis of B, then the system can be balanced.

We need to know the shapes angles

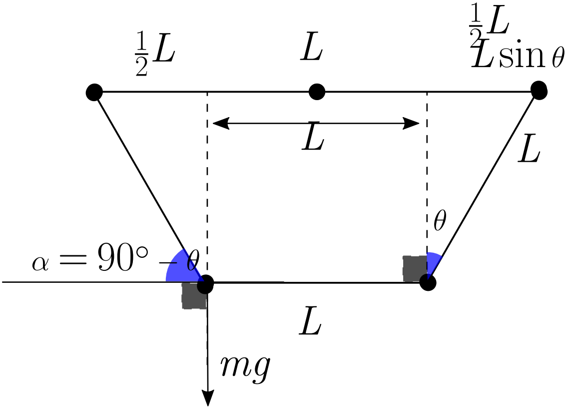


Figure:

Finding angles for Problem 3

This is sure that,

$$L \sin\theta = \frac{1}{2}L$$

Thus,

$$\theta = 30^\circ$$

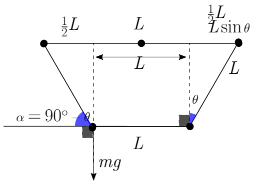
From here,

$$\alpha = 90^\circ - \theta = 60^\circ$$

Now, the angle that the mg vector makes with the left side of the rod is,

$$90^\circ + \alpha = 90^\circ + 60^\circ = 150^\circ$$

So, angle $m\vec{g}$ makes with the left rod is 150°.



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- Force balance is similar to Torque Balance
- In this problem, we need to apply force at D, and if you notice, the motion of D is like the rotation of right rod pivoting B.
- To balance the system, we need to balance the *Rotation*.

So, the torque on the left side of the rod,

$$\tau = mgL\sin(150^{\circ})$$

And $\sin 150^{\circ} = \frac{1}{2}$, hence,

$$\tau = \frac{mg}{2}L$$

I have something to add here too,

- In the mg force vector, some force is $\perp$ perpendicular to the rod (prjection)
- Some force is along the rod.

The force components that are along the rod are uimportant, because they pull the left rod and do nothing.

Final thoughts on Problem 3

Now we add some logic.

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If left side has torque τ where $\tau = \frac{mg}{2}L$, then if we apply the same torque on the right side, then we have a torque balance, which will also balance the system, hence, on the right side, we also apply a torque τ' which is,

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$$\tau' = \frac{mg}{2}L$$

And $\tau = FL$, thus,

$$F = \frac{mg}{2}$$

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$$\tau' = \frac{mg}{2}L$$

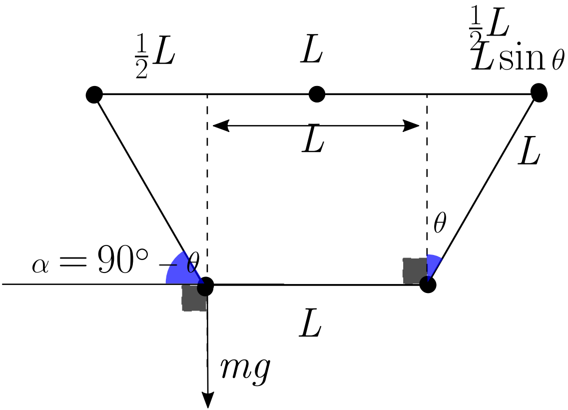
And $\tau = FL$, thus,

$$F = \frac{mg}{2}$$

This force is perpendicular to the rod, and solely works to balance the system, without any component along the rod, hence, no force is wasted and $F = \frac{mg}{2}$ is the minimum force possible.

$$\boxed{F = \frac{mg}{2}}$$

Problem 3 Solution $F = \frac{mg}{2}$



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