The Physics Intro to Sidereal and Solar Time

Ahmed Saad Sabit

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I am motivated from the fact, how vague the way the general astronomy text seem to be. We shall make everything clear on the difference of the two time keeping systems.

Definition of a Sidereal Day: The Time taken by Earth to make a full rotation of $2\pi\, rad$ about its own axis because of Diurnal Motion is a $1\, SiderealDay$.

Note to avoid confusion, many texts say the formal definition; that the Sidereal Day is the time interval between two successive passage of the γ (Vernal Equinox) through the observers Meridian[1].Now this is okay, but in reality if the Earth diurnal motion was absent then γ would be static, it can only make a full apparent rotation if we rotate! Thus I prefer the First Definition.

Definition of a Solar Day: The Time taken by Earth by its Diurnal Motion so that Sun returns to its previous position after a full cycle is a Day. Hence, $2\pi\, rad$ full rotation of Sun relative to us yields $1\, day$. You should say $1\, SolarDay$.

Keep in mind that the Annual Motion of Earth around Sun is also responsible for an effect on Solar Day!

The fact here is to be convinced to make the sense that, Sidereal and Solar are completely different things. Because we have adopted the Solar Day measure, it is wise to denote Sidereal Day in terms of Solar Day.

Relating the Solar and Sidereal Day:

We point out the General Effects.

- Earth rotates some angle to make Sun apparently return to its same position as previous day.
- But as the Earth Rotates in its own Diurnal Axis, it also makes considerable motion along its trajectory around Sun; simply, it also moves around the Sun by some angle (that our Astronomers have measured Perfectly).
- So, overall we break 1 Solar Day in a few parts, so that we can relate it with a Sidereal Day.
  - Earth completes a full Sidereal Day.
  - Earth has Sun not yet in its previous position, because it (Earth) is displaced from its previous position, because of Annual Motion. So, Earth has to rotate an extra α angle to ensure a day.
- We notice that 1 Solar Day > 1 Sidereal Day.

Using Degree measures, we denote the Angular Velocities. Knowing the 1 year is 365.25 Solar Day. Earths Annual Motion angular velocity respect to Sun as axis,

$$A = \frac{360^\circ}{365.25\, Solar\ Day} = 0.986^\circ / Solar\ Day \tag{1}$$

So in 1 day, Earth advances (the rotation excess too)

$$\alpha = At = A \times 1\, Solar\ Day = 0.986^\circ \tag{2}$$

We can note that the time taken for Earth to make $360^\circ + \alpha$ angle rotation is 1 Solar day. And time making 360° only is a Sidereal Day. Hence, using good old Unitary method,

Making $360^\circ + \alpha$ rotation takes 1 Solar Day.
Making 1° rotation takes $\frac{1}{360^\circ + \alpha}$ Solar Day.

[1]Meridian: The 90° curve joining the Zenith and South point of Horizon.

Making $360°$ rotation takes $\frac{1}{360°+\alpha} \times 360°$ Solar Day.

So it takes 0.9976 Solar Days to make a bare $360°$ full rotation = 1 Sidereal Day. You can use Hours and Minutes to show that Sidereal Day is just 4 minutes less than Solar Day.