# Notes on Thermodynamics

Covered in Chemistry and Physics Syllabus

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Group 15
Notre Dame College, Dhaka
Bangladesh
4 November, 2021

You can ask for a transcript of transanction/proof to me, sabit.

# Thermal Science Problems

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Included here are a few Thermodynamics problem that would introduce some variants of digestable ideas. Thermodynamics is fascinating and is extremely simple in terms of quality.

Basically I'd throw in some random problem but discuss them in solutions. If you check the solutions and understand them, *you'd at least know what you don't know*, this paper can't solve all your problems, it just has a few example problems for reference.

**Problem 1** (`Reduce pressure by letting it out`)**.** A vessel of volume $V = 30l$ contains ideal gas at temperature $0°$ temperature. After a portion of Gas is let out, the pressure of the vessel decreases by $\Delta p = 0.78\ atm$, the temperature remains constant.

**Find the mass of the gas released.** The density of gas in normal condition is $\rho = 1.3\ \frac{g}{l}$

**Solution.** Start with the $pV = nRT$. Check this, if you change $p$ by, say, $\Delta p$, then given the volume $V$, temperature $T$ is constant, it can be said,

$$\Delta p\ V = \Delta n\ RT$$

Changing $p$ gives a change in $n$, which is the number of moles of gas.
Now see that we let gas flow away, that means the moles of gas reduces in the process. But we need the mass.
Let the mass of 1 mole of the gas be $M$. Then total mass of $n$ mole gas is,

$$m = nM$$

From this we can say,

$$n = \frac{m}{M}$$

From here, if $n$ moles of gas changes by $\Delta n$, then total mass of gas change by $\Delta m$.

$$\Delta n = \frac{\Delta m}{M}$$

Hence we can write,

$$\Delta p V = \frac{\Delta m}{M} RT$$

From this,

$$\Delta p V\ \frac{M}{RT} = \Delta m$$

What is $\frac{M}{RT}$? We can't tell directly. But look, at the end of the problem we are given the density of the gas in normal conditions.
By normal conditions we mean the temperature is $0°$ which is $273\ K$, and pressure is standard $1\ atm = 1 \times 10^5\ Pa$.

Let's find out the equation of Gas density in Standard Condition mentioned above quickly,

$$pV = \frac{m}{M}RT$$
$$p = \frac{RT}{M}\frac{m}{V}$$
$$p = \frac{RT}{M}\rho$$
$$\boxed{\rho = \frac{pM}{RT}}$$

Thus, we can solve for $\frac{M}{RT}$ from here, which is the unkwnown in the equation of $\Delta m$. Thus,

$$\frac{M}{RT} = \frac{\rho}{p} = \frac{\rho}{p_{atm}}$$

Note we have to use $p_{atm}$ in case of the density because it's defined for standard condition, the $p$ in the gas vessel is different.
Let's use the result,

$$\Delta m = \Delta p V \left(\frac{\rho}{p_{atm}}\right)$$

Writing this a little more nicely,

$$\boxed{\Delta m = \rho V \frac{\Delta p}{p_{atm}}}$$

Now we have to put the values, this is a problem, units suck and I hate it, usually you want to use a consistent set of units, thankfully, for this problem the units are arranged nicely.
Check that if we do the Numerical calculation with appropriate units,

$$\Delta m = \left(1.3\frac{g}{\cancel{l}}\right)(30\cancel{l})\left(\frac{0.78\cancel{atm}}{1\cancel{atm}}\right) = 30g$$

You always want the units to cancel out nicely. In this case it did.

This next problem effectively tests Equilibrium conditions of a system.

**Problem 2** (`Two different vessels, connected`)**.** Let's have two gas cylinders, both have same kind of gas. One has a pressure $p_1$ and the other has $p_2$. The one has temperature $T_1$ and the other has temperature $T_2$.

One has $n_1$ moles of gas and the other has $n_2$ moles of gas. Volume of one vessel is $V_1$ and the other is $V_2$. **If the cylinders are connected, then what will be the final pressure of the two cylinders?**

**Solution.** Two vessels are completely different in their states.
But if they are connected, an equilibrium will be established, but what equilibrium?
It will turn out, after leaving the two cylinders connected together for long enough time, both of them will come to some equal states. By that I mean,

- **Mechanical Equilibrium: Pressure equal in both vessels** If the pressure between the two connected vessel isn't equal, then gas will flow from the higher pressure to the lower pressure.

Think about that, a football typically has $3\ atm$ pressure inside it. If you, mischivieously, puncture it, the ball leaks gas from it, why? Because inside the pressure is higher than the atmosphere. Same with anything, same with two cylinders connected.

- **Thermal Equilibrium: Both cylinders will come to a same temperature**Well if one gas is hotter, and one cooler, then heat will flow from the hotter gas to the lower one, coming to a final temperature $T$.

Let's crunch the math, for cylinder 1, before making the connection with cylinder 2,

$$p_1V_1 = n_1RT_1$$

For cylinder 2, before connection,

$$p_2V_2 = n_2RT_2$$

After connection, both the cylinders become basically one large vessel, with volume, $V = V_1 + V_2$,

$$p_{fin}\left(V_1 + V_2\right) = \left(n_1 + n_2\right)RT_{fin}$$

$p_{fin}$ and $T_{fin}$ are final pressure and temperature. We need the final pressure. But we also need to solve the final temperature.
*We have to solve the final temperature* $T_{fin}$. Now, the temperature change of $T_1$ should be $\Delta T_1 = T - T_1$, for $T_2$, the change is $\Delta T_2 = T - T_2$, change in temperature means flow of heat. Let's for sake of doing the math assume that $T_1$ is greater than $T_2$. You can assume anything except them not being equal. The math will hold. Then, final temperautre of both cylinder's gasses would be $T_{fin}$, or just $T$ to save some typing. Hence, heat will flow from $T_1$ cylinder to $T_2$ cylinder, given $T_1$ is greater.
We will assume heat capacity for constant volume process $C$, although it's not mentioned in the problem, because the gas is same, thus from a little experience you'd build up the idea that $C$ will get factored out. No worries. The heat lost by $T_1$,

$$\Delta Q_{\text{lost}} = n_1C\left(T - T_1\right)$$

Heat gained by $T_2$ vessel,

$$\Delta Q_{\text{gained}} = n_2C\left(T - T_2\right)$$

Now the heat flows from $T_1$ to $T_2$. Note that, $Q_{lost}$ is negative in value, because it's flowing out. Similar reasoning, because the $T_1$ temperature vessel is losing heat, it's final temperature $T$ will be lower than $T_1$. Hence, we need the positive value of $Q_{\text{lost}} = n_1C\left(T_1 - T\right)$. The positive value is,

$$\Delta Q_{\text{lost}} = \Delta Q_{\text{gained}}$$

From here,

$$n_1C\left(T_1 - T\right) = n_2C\left(T - T_2\right)$$

Solving this, we can take $T$ to one side of the equation,

$$\frac{n_1}{n_2}\left(T_1 - T\right) = T - T_2$$

Solve for $T$ we get,

$$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

Hence, we can end the problem,

$$\boxed{p_{fin} = \left(\frac{n_1 + n_2}{V_1 + V_2}\right) R \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}}$$

**Problem 3** ( `Metal Piece` ). A thermally insulated piece of metal is being heated under atomospheric conditions, so it receives a constant supply of power $P$. This leads to an increase in the Temperature $T$, so that,

$$T\left(t\right) = T_0[1 + a\left(t - t_0\right)^{\frac{1}{4}}]$$

Here $a$, $t_0$ and $T_0$ are constants. Determine the heat capacity $C_p\left(T\right)$ of the metal.

Note that $T(t)$ means Temperature is a function of time $t$.

**Solution.** It is just taking differentials, because,

$$Q = C\left(T\right)\Delta T \to P = C\frac{\mathrm{d}T}{\mathrm{d}t}$$

So, the solution of the problem is, found from taking the time derivative of the given $T(t)$ equation, and then relating it to the given power.

$$\boxed{C(T)4P\frac{T^3}{aT_0^4}}$$

Chain rule might be needed to be used. Here it's notable that the Heat capacity is the function of the Temperature of the metal.

**Problem 4.** In a system with pressure $p$ and volume $V$, you are given the total system energy to follow,

$$\frac{pV}{E} = 10000$$

Using these information, find the number of particle in the system.

**Solution.** There are some equation that we can directly use, because the degree of freedom not shown we can assume the molecules monoatomic.

$$E = \frac{3}{2}kT$$

and,

$$pV = NkT$$

Equating,

$$N = \frac{3}{2}\frac{pV}{E} = 15,000$$

**Problem 5.** Pressure can be shown by the equation,

$$p = \rho v_x^2$$

Show the equation of pressure in terms of absolute speed $v_0$, here $v_x$ is the speed of the molecule only along $x$ axis.

**Solution.** From the total energy equaiton $E = \frac{1}{2}mv^2$, we can say,

$$v_0^2 = v_x^2 + v_y^2 + v_z^2$$

And because these are all equally probable, speed along any axis is the same. Hence,

$$v_0^2 = 3v_x^2$$

Using this in the rule,

$$\boxed{p = \rho\frac{v_0^2}{3}}$$

**Problem 6.** There is a Tesseract, whose side is made of $l_0$ length sides. Tesseract is a $4D$ cube.The volume of a tesseract with side $l$ is,

$$V_4 = l^4$$

Now, find the 4D volume expansion coefficient in terms of linear thermal expansion coefficient $\alpha$.

**Solution.** Let us call the 4D volume as $D$. Now,

$$\Delta D = \zeta D_0 \Delta T$$

Now, using approximations, we can express this equaiton in terms of $l_0$ which is 1D,

$$D = l^4$$

Thus,

$$\frac{\mathrm{d}D}{\mathrm{d}l} = \frac{\mathrm{d}}{\mathrm{d}l}l^4 = 4l^3$$

We can approximate that this differential change $\mathrm{d}D$ is comparable to a bigger change (it's technically linearity condition)
Bringing the $\mathrm{d}l$ from the left to the right and writing $\mathrm{d}l$ as $\Delta l$,

$$\Delta D = \Delta\left(l^4\right) = 4l^3\Delta l$$

Hence,

$$4l_0^3\Delta l = \zeta l_0^4 \Delta T$$

This leaves us with,

$$\begin{aligned}
4l_0^3\Delta l &= xl_0^4\Delta T \\
4\Delta l &= \zeta l_0 \Delta T \\
4\alpha l_0 \Delta T &= \zeta l_0 \Delta T \\
4\alpha &= \zeta
\end{aligned}$$

We have found the answer,

$$\boxed{4\alpha = \zeta}$$

**Problem 7.** There is a rod that is joined between two mountains. The length is $l_0$ and the linear thermal coefficient of expansion $\alpha$ is small. Because of increase of temperature by $\Delta T$, the rod buckles and breaks at the center. The center rises up by $x$ height above it's natural position. Find the value of $x$ using appropriate approximations.

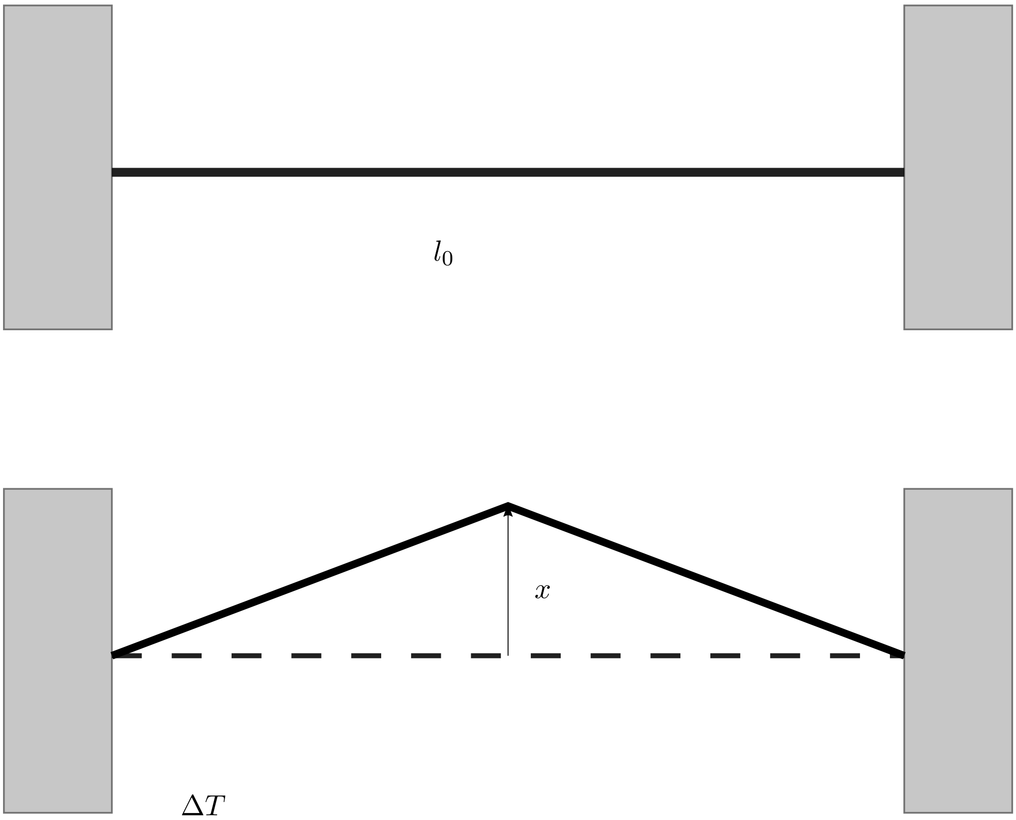


Figure 0.0.1: buckle

**Solution.** We need the length after expansion, and this change in length caused the breaking at the center. So,

$$l = \alpha l_0 \Delta T + l_0$$

Now,

$$x^2 = \left(\frac{l}{2}\right)^2 + \left(\frac{l_0}{2}\right)^2$$

Solving this,

$$\frac{l_0^2}{4}\left((\alpha\Delta T + 1) - 1\right)^2 = \frac{l_0^2}{4}\Big((\alpha\Delta T)^2 + 2(\alpha\Delta T)\Big)$$

We will ignore $\alpha^2$ term, hence,

$$l_0^2 \frac{l_0^2 \Delta T}{2} = x^2$$

Hence,

$$\boxed{x = l_0\sqrt{\frac{\alpha\Delta T}{2}}}$$

When I first tried to solve the next problem, the mistake I did was to try understand what happens when the gas tries to flow when pressure is high across the valve. That mistake taught me a nice lesson, care about the initial and final states when it is suitable, not the middle case when things are not in equilibrium (or moving).

**Problem 8** (`Valve and two Cylinder`). Two identical vessels are connected with a pipe having a valve that lets gas from one side to other if and only if the pressure difference across the valve is $\Delta p \geq 1.1\ atm$. Initially, there is vacuum in one of the vessel and the other vessel is at temperature $T_1 = 27^\circ$ and pressure $p_1 = 1\ atm$. After some time, both of the vessels were heated to the temperature $T_2 = 107^\circ$. What will be the pressure of the vessel now that was in vacuum?

**Solution.** Initially, the state is,

$$p_1 V = nRT_1$$

After the heating is done, and the gasses have moved from one to another accordingly through the valve, there will be two states of the two vessels.
We can feel from now that the difference between the pressure of the two vessel after heating is surely going to be $\Delta p = 1.1\ atm$. Otherwise the gas would flow until this difference is achieved. To be direct,

$$p_2 = p_3 + \Delta p$$

Here $p_2$ is the pressure of the vessel that had gas initially, now heated, and $p_3$ is the pressure after the gas has flowed into it after the heating process. The gas equations,

$$p_2 V = (n - \Delta n)\, RT_2$$

$$p_3 V = (\Delta n)\, RT_2$$

Here the $\Delta n$ is the amount of gas that has flowed from one vessel to another. Now we can start solving equations,

$$\frac{p_3 V}{RT_2} = \Delta n$$

And from initial case,

$$n = \frac{p_1 V}{RT_1}$$

Hence, the $p_2 V = (n - \Delta n)\, RT_2$ equation,

$$p_2 V = \left(\frac{p_1 V}{RT_1} - \frac{p_3 V}{RT_2}\right) RT_2$$

And just taking off the factors,

$$p_2 = p_1 \frac{T_2}{T_1} - p_3$$

And as we know, $p_2 = p_3 + \Delta p$,

$$\Delta p + 2p_3 = p_1 \frac{T_2}{T_1}$$

> That solves,
> $$\boxed{p_3 = \frac{1}{2}\left(p_1\frac{T_2}{T_1} - \Delta p\right) = 0.083\ atm}$$

The first time I was doiing this it took so much time!

## Gas laws and stuffs

Chemistry teacher as usual started with Boyle's law and things, you can cheat and just derive them from $pV = nRT$.

- **Boyle's Law** is, $T = \text{const}$, thus,

  $$pV = nRT = \text{const}$$

  Here amount of gas $n$ or $T$, nothing change, so basically $nRT$ is constant value, thus,

  $$\boxed{pV = \text{const}}$$

  For $T$ constant. Thats' Boyle's Law, derived from Experiment, before we knew $pV = nRT$.

- **Charles Law** is, $p = \text{const}$, thus,

  $$V = \left(\frac{nR}{p}\right)T$$

  Here given $p$ being constant, we can just bring $T$ and $V$ in the same side, and write,

  $$\frac{V}{T} = \left(\frac{nR}{p}\right) = \text{const}$$

  Thus, for constant pressure, Charles Law is,

  $$\boxed{\frac{V}{T} = \text{const}}$$

- **Gay Lussac's Law** is, $V = \text{const}$, thus,

  $$p = \left(\frac{nR}{V}\right)T$$

  Here the $V$ is volume which is constant, hence as usual putting the constant things at one side,

  $$\frac{p}{T} = \left(\frac{nR}{V}\right) = \text{cosnt}$$

  Thus, we can say,

  $$\boxed{\frac{p}{T} = \text{const}}$$

- **Avogradro's Law**, in this case keep $p$ and $T$ constant. Then you can tell,

$$V = n\left(\frac{RT}{p}\right)$$

Here this $\frac{RT}{p}$ is a constant, thus,

$$\boxed{\frac{V}{n} = \text{const}}$$

I don't see any point remembering what's Boyle's law or whats Gay's law, but what do i know.

## Unit converter machine speech

*laugh my fucking ass off*

Let's do a quick problem to tell you what the hell unit converter is quickly as possible.

**Problem 9** (`Unit shit 01`)**.** Find the numerical value of,

$$\frac{p_1}{p_2}\frac{T_2}{T_1}$$

When,

- $p_1$ is $20\ atm$
- $p_2$ is $3251\ mmHg$
- $T_1$ is $342\ K$
- $T_2$ is $200°$

If you ask me these unit conversion problems are pretty fucked up.

**Solution.** When I am confused I take the **safe method**. I'd just directly solve keeping the units as they are,

$$\frac{p_1}{p_2}\frac{T_2}{T_1} = \frac{20\ atm}{3251\ mmHg}\frac{200\ C}{342\ K}$$

Now choose your preffered way of units. I'd definitely go with just simple units we do normally in physics, it's nice and does the job consistenty. We have 1 set of units we like to follow in Theory, it's Meter-Kilogram-Second units, which in my opinion is convinient.
If it's experiment then it's Centimeter-Gram-Second. But some retards have to use Pound-Long Sleeves-Urmom units that pisses me off.
Just be careful about the only quantity $T$ temperature, it follow kinda different rules to work, for example Celcius to Kelvin,

$$K = C + 273$$

Let's do the math normally in normal human units,

$$= \left(\frac{20 \times (200 + 273)}{3251 \times 342}\right)\frac{atm \times K}{mmHg \times K}$$

From a unit converter book or internet page,

$$1\ mmHg = 133.32\ Pa$$

$$1\ atm = 1 \times 10^5\ Pa$$

Now put the pressure unit value's in place,

$$\left(8.5 \times 10^{-3}\right) \frac{10^5\ Pa}{133.2\ Pa}$$

This results in, no units, and value,

$$\frac{p_1}{p_2}\frac{T_2}{T_1} = 6.4$$

**Problem 10** (`Another one 02`). Boyle's Law is basically Isothermal, and you're given,

$$pV = 100\ Pa \cdot dm^3$$

The autistic problem setter wants you to solve this in goddamn ImPeRiAL UnITs.

**Solution.** in this shitty unit pressure is pounds per inches, oh man,

$$1\ Pa = \frac{1}{133.3}\ mmHg$$

$$1\ mmHg = 0.02\ \frac{lb}{inch^2}$$

Thus,

$$1\ Pa = \frac{1}{133.3}\ mmHg = \frac{1}{133.3}\ \left(0.02\ \frac{lb}{inch^2}\right) = 0.00015\ \frac{lb}{inch^2}$$

Imagine how impossibly dumb this math is, it took me over 20 minutes to type the last three lines :facepalm:
And the who the fuck uses decimeter and why and why am I typing this anyway.

$$1dm^3 = 1\,(dm)^3$$

Summon the text straight from Euler,

Kilaiia Hakaiya Dakat Marile Decsher Schanti Milibe

So from that,

$$1dm = 10^{-1}m = 0.1m$$

Hence,

$$1dm^3 = 1\,(0.1\ m)^3 = 1\,(0.1\ \ (39.37\ inch))^3 = 61.02\ inch^3$$

So, after so much circus,

$$pV = 100\,(1\ Pa)\left(1\ dm^3\right) = \left(0.00015\ \frac{lb}{inch^2}\right)\left(61.02\ inch^3\right) \times 100 = 0.9153 \approx 1\ lb\ inch$$

Go ask your Udvasher vaiya if you don't have the head to figure out a quick 2 sec method to change units.

## A nice energy Relation

*Let's check if a gas if monoatomic or not from the speed of sound*

There's a Damn interesting relation between internal energy and $\gamma$ factor in the Adiabatic Process which makes a power relation.

Let me first derive it.

Let's increase the temperature by $\Delta T$, now the increase of Internal Energy is independent how we changed the temperature, all procedures give same answer. Like, we could incrase $\Delta T$ by applying work on the system or just putting heat into it, whatever it would be, the answer would be the same.

The simple way to do it is to use constant volume process $C_v$ molar heat capacity and write the change in Total Internal Energy in terms of $\Delta T$,

$$\Delta U = nC_v \Delta T$$

In class it has been derived that $C_p = C_v + R$, the derivation isn't long, but it's in book so why do I need to do it here lmao.

We were also told,

$$\gamma = \frac{C_p}{C_v}$$

So let's replace the $nC_v\Delta T$ with a $\gamma$ instead of heat capacity,

$$C_p = C_v - R \rightarrow \frac{C_p}{C_v} = 1 - \frac{R}{C_v} = \gamma$$

Thus,

$$C_v = \frac{1}{\gamma - 1} R$$

Hence,

$$\boxed{\Delta U = \frac{1}{\gamma - 1} nR\Delta T}$$

We found this from,

$$\boxed{\Delta U = nC_v \Delta T}$$

From the equipartition theorem thingy, you can also write the thing in Degrees of Freedom, if $\langle i \rangle$ denote Degrees of Freedom, you can write,

$$\boxed{\Delta U = \frac{\langle i \rangle}{2} nR\Delta T}$$

Now time to do a cool problem.

**Problem 11** ( `Speed Sound to Molecular Shape` ). How many degrees of freedom hoave the gas molecules, if under standard conditions, the gas density is $\rho = 1.3 \ \frac{mg}{cm^3}$ and the velocity of the propagation of sound is $v_s = 330 \ \frac{m}{s}$?

**Solution.** Speed of sound is $v_s$,

$$v_s = \sqrt{\gamma \frac{RT}{M}}$$

And from the idea of internal energy of gas, where $n$ is the number of moles,

$$U = \frac{nRT}{\gamma - 1} = \frac{i}{2} nRT$$

And using the ideal gas law in cases,

$$pV = nRT = \frac{m}{M} RT$$

we can find $i$ pretty quickly, which is the degree of freedom. In this case, $i = \dfrac{2}{\dfrac{\rho v_s^2}{p} - 1} = 5$