1 Introduction to the Easier type of Algebra

1.1 An example of using Algebra practically

There are a lot of things to do in Algebra. It is the art of taking unknowns and working with them. Later when we have values, then we can do the math practically.

What I mean is, suppose, your age is 5 years. What is your age **after** 3 years ?

As we know, *after* 3 years, our age whatever it is, is going to increase, so after 3 years, your age will be 8 years, simply because $5 + 3 = 8$. Now we can do it for anyone,

- Rafiq's age is 7, so after 3 years his age is $7+3 = 10$ years.
- Srabon's age is 15, so after 3 years his age is $15 + 3 = 18$ years.
- Sokhina's age is 93, so after 3 years her age is $93 + 3 = 96$ years.

We will become tired of continually adding ages with 3 to know what age is would be after 3 years. What we can do is making something simple, so that only knowing the age can yield us the age after 3 years. Let anyones age be $\square$, then after 3 years, his age is,

$$\square + 3 = \text{Age after three years}$$

So if my age is 17 years, use 17 in the box and you'll get my age after 3 years.

$$\square = 17$$

$$\square + 3 = 17 + 3 = 20$$

You can put others age in the box to get their age after 3 years.

But it looks nerd if we use an "x" for the $\square$, then, our special formula to know the age after 3 year will be,

$$x + 3 = \text{Age after three years}$$

So for the ages of Rafiq, Srabon and Sokhina Aunty, we had to take x as 7 for Rafiq, x as 15 for Srabon, x as 93 for Sokhina Aunty. x can take any value and give the right answer if you put the value in the formula.

This is the main point of Algebra. We make a general formula, and put values of various things get answers.

We will discuss more intersting cases. We can find the speed of a ball kicked by Messi, we can find when will an Apple be sweet and many more using algebra.

1.2 Fractions and Multiplication

Let me state some problems. The solution is the idea behind everything.

Problem: You have 3 friends who have 10 Pokemon game card each. How many cards are there in total?

Solution: There are 3 friends with 10 cards. Total number of card is thus,

$$10 + 10 + 10 = 20 + 10 = 30$$

But this also means that,

$$10 + 10 + 10 = 3 \times 10 = 30$$

So there are 30 cards.

Problem: There is a number that can be found if you add 3 four times with itself. What is the number?

Solution: Let's add 3 four times with itself.

$$3 + 3 + 3 + 3 = 6 + 3 + 3 = 9 + 3 = 12$$

But this also means,

$$4 \times 3 = 12$$

Last two problems is the idea of Multiplication. The number is 12.

Problem: There is a 20 meter long stick. What will be the length of the stick if we cut it in 10 equal pieces?

Solution: We have to directly divide this.

$$\frac{20}{10} = 2$$

Thus, the length of each small part will be 2 meter. This makes sense. If you add ten 2 meter small parts, the total length will be 20. You can visualize this by,

$$\frac{20}{10} \times 10 = 2 \times 10$$

This is,

$$20 = 2 \times 10$$

Division is the reverse of multiplication. These come extremely frequently in Physics.

So you can see why we have put the idea of Multiplication. We don't want to waste time by continuously adding and subtracting to do these problems. So, we make the use of the sign $\times$ to do multiplication.

You can also write multiplication in a different form,

$$2 \times 10 = 2(10)$$

In multiplication, you can also reverse position,

$$2 \times 10 = 10 \times 2 = 10(2)$$

1.3 Decimals

We can see,

$$\frac{10}{2} = 5$$

This means that, if we add 5 two times then we get 10. Or, we can add 2 five times to get 10. How?

Like this, multiplying 2 in both the sides,

$$\frac{10}{2} \times 2 = 5 \times 2$$

This yields,

$$10 = 2 \times 5$$

Now,

$$2 \times 5 = 5 + 5 = 10$$

Also,

$$2 \times 5 = 2 + 2 + 2 + 2 + 2 = 10$$

Now let us try to analyze another case.

$$\frac{2}{10}$$

If you do this in a calculator, it means,

$$\frac{2}{10} = 0.2$$

This is still fine. You can add 0.2 ten times to see what you get.

$$\frac{2}{10} \times 10 = 0.2 \times 10$$

Now,

$$0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 + 0.2 = 2$$

2 Algebra: Factoring out

2.1 General idea

For these simple 5 minute algebra, keep in mind that the number that is above the head of any letter [1] is called the *exponent*. The exponent is like writing a lot of things in short, because,

$$x^3 = x \times x \times x$$

$$y^3x^2z = y \times y \times y \times x \times x \times z$$

Like in x^2, 2 is above the head of x so the 2 is our exponent. Now if we have a square root, $\sqrt{}$, then we have to make the exponent to its half. For instance, $\sqrt{x^2}$ should make the exponent 2 in its half, so $2 \times \frac{1}{2} = 1$, henceforth $\sqrt{x^2} = x^1 = x$. Notice whenever we have a one 1, we ignore writing it. It's just for making the math not look buzzed. Another example is that $\sqrt{x^6} = x^{6\times\frac{1}{2}} = x^3$. Just take the exponent to be half.

1. $\frac{2\times3\times4\times x}{3\times2}$
2. $\frac{2\times6\times8\times5}{1\times6\times3}$
3. $\frac{4xyz}{6y}$
4. $\frac{36x^3y^2}{12x^2y}$
5. $\frac{356x^3z^3}{18x^3z^3}$
6. $\frac{6x^8}{5x^7}$
7. $\frac{25x}{5y}$
8. $\frac{16x^{20}}{4x^{10}}$
9. $\frac{2xyzp^3}{9p^6xyz^2}$
10. $\frac{\sqrt{4}yx}{2zx}$
11. $\frac{40\sqrt{x^2y^2z^2}}{20xyz}$
12. $\frac{x^{200}}{x^{100}}$
13. $\frac{x^{1000}}{x^{300}}$
14. $\frac{x^{34}}{y^2x^{14}}$
15. $\frac{ab^4c^9d^3e^3f^8g^8h^2}{g^2d^9b^3f^2c^8e^2h}$
16. $\frac{\sqrt{x^4y^8z^8}}{\sqrt{x^2y^4z^4}}$
17. $\frac{\sqrt{2}}{2}$

[1]we shouldn't call letter, we should say "variable"

I have given the last problem in intentionally.Can you think the matter in this way, taking a square root halves the exponent, so shouldn't a variables exponents be doubled to put it under a square root? I mean that

$$x^2 = \sqrt{x^4}$$

Now you can fairly show that

$$x^2 = \sqrt{x^4} = (\sqrt{x})^4$$

This also suggests that you can exploit the fact in this manner,

$$2 = \sqrt{2^2} = (\sqrt{2})^2 = \sqrt{2} \times \sqrt{2}$$

Use the final suggestion into the last problem and solve it. Not that hard once you catch with the maths and the pattern that it tries to follow. You'll be manipulating this way of using *square root*, and almost everytime given that you pursue physics. It's really very frequent in these sort of maths.

2.2 Advancing a little bit

So you have seen that the math you did previously are like,

$$\frac{x^5}{x^3} = x^2$$

That just means that $x^5 = x \times x \times x \times x \times x$ and $x^3 = x \times x \times x$. In the fraction,

$$\frac{x^5}{x^3} = \frac{x^5 = x \times x \times x \times x \times x}{x \times x \times x} = x \times x = x^2$$

Simple. But have you noticed the fact that in the top of the fraction (called the numerator) the power of x is 5 and in the bottom the power of x is 3. The fraction yields that the result has power of 2. That actually is $5 - 3 = 2$, isn't it?

If we try to define the new rule,

$$\frac{x^5}{x^3} = x^{5-3} = x^2$$

This new method is really much more straight forward and faster.