Problem is to solve,

$$I = \int \mathrm{d}\theta \left(\frac{1}{1 + e\cos\theta}\right)^2$$

Solution is to,

$$-e\cos\theta = x$$

This makes it,

$$I = \int \mathrm{d}\theta \left(\frac{1}{1 - x}\right)^2$$

Now the thing is using the taylor series,

$$f(x) = \sum_{i=0}^{\infty} \frac{\partial_x^i f(0)}{i!} x^i$$

Using this,

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$$

That is,

$$\frac{1}{(1-x)^2} = \sum_{i=0}^{\infty} i x^{i-1}$$

Thus the integral is to solve,

$$I = \int \mathrm{d}\theta \sum_{i=0}^{\infty} i x^{i-1}$$

Fixing things and instead of using $-e\cos\theta$, let say $e = -a$ and use $a\cos\theta$ in replacement of x.

$$I = \sum_{i=0}^{\infty} i a^{i-1} \int \mathrm{d}\theta \cos^{i-1}\theta$$

Having this integral doesn't really suck if we know Γ and β functions, one form of β function says,

$$\int_0^{\frac{\pi}{2}} \cos^n \mathrm{d}\theta = \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{n+2}{2}\right)}$$

So the thing that we need to solve is,

$$I = \sum_{i=0}^{\infty} i a^{i-1} \frac{\Gamma\left(\frac{i}{2}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{i+1}{2}\right)}$$

Note this solves the integral from 0 to $\frac{\pi}{2}$.

Now how do I solve this sum?