The Short Vector Calculus proof of Electromagnetic Radiation

AS

November 25, 2019

Time starts in 6:58,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \tag{1}$$
$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3}$$
$$\nabla \times \mathbf{B} = \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \tag{4}$$

For the free space we have the $\rho = 0$ and the field have the ability to exist, we can re write the equation system in free space manner.

$$\nabla \cdot \mathbf{E} = 0 \tag{5}$$
$$\nabla \cdot \mathbf{B} = 0 \tag{6}$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{7}$$
$$\nabla \times \mathbf{B} = \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \tag{8}$$

The time derivative of the equation (8) can yield into a very important symmetry with the equation (7).

$$\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Which is easy to show that,

$$\nabla \times \frac{\partial \mathbf{B}}{\partial t} = -\epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Now this again matches with (7) and we have

$$\nabla \times (\nabla \times \mathbf{E}) = -\epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

The curl of a curl has simple results, as we can use the following corollary,

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) \; - \; \nabla^2 \mathbf{E} \tag{9}$$

The free space equations now we know by (4) that $\nabla \cdot \mathbf{E} = 0$, and find that

$$\nabla^2 \mathbf{E} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{10}$$

Or for assuming the r direction only, we can make that it's possible that

$$\frac{\partial \mathbf{E}}{\partial r} = \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{11}$$

Equation (11) is the wave equation. This shows,

$$c \; = \; \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{12}$$

Which is quite interesting and matches with the real experiments.
The typing ends in 7:34