Binding Energy

Ahmed Saad Sabit

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Let there be a sphere of radius r and it is solid with mass m. We want to put a layer of mass $\mathrm{d}m$ over the sphere so that there is increase of radius by $\mathrm{d}r$.

The potential energy of the layer in total,

$$\mathrm{d}V = \frac{Gm}{r}\left(4\rho\pi r^2\ \mathrm{d}r\right)$$

Now, if we are try to find the total potential energy stored from $r = 0$ to $r = R$,

$$\int \mathrm{d}V = \int \frac{G\left(\frac{4}{3}\pi r^3 \rho\right)}{r}\left(4\rho\pi r^2\right)\mathrm{d}r$$

Now if you solve this,

$$V = \frac{16}{3}G\pi^2\rho^2\ \frac{r^5}{5}$$

Now, $M^2 = \frac{16}{9}\rho^2 R^6$, using this in the solution, we get,

$$\boxed{V = \frac{3}{5}\frac{GM^2}{R}}$$

This is the required solution to the problem.