The General Matrix problem of Oscillation

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Problem 1. There are two masses m_1 and m_2 that are positioned between two walls in a coupled system. Find the modes of oscillations *using Matrices*.

Solution. For a displacement, x_1 nad x_2, For the first mass,

$$-kx_1 + k(x_2 - x_1) = -k(2x_1 - x_2)$$

For the Second mass,

$$-k\left(x_2 - x_1\right) - kx_2 = -k(2x_2 - x_1)$$

The equation of motion,

$$\ddot{x_1} = -\frac{k}{m}\left(2x_1 - x_2\right)$$

$$\ddot{x_2} = -\frac{k}{m}\left(2x_2 - x_1\right)$$

Assume that the solution of these are,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^{i\alpha t}\begin{pmatrix} A \\ B \end{pmatrix}$$

Thus,

$$\ddot{x_1} = -\alpha^2 A e^{i\alpha t} = -\alpha^2 x_1 = -\omega^2\left(2A - B\right)e^{i\alpha t}$$

$$\ddot{x_2} = -\alpha^2 B e^{i\alpha t} = -\alpha^2 x_2 = -\omega^2\left(2B - A\right)e^{i\alpha t}$$

From there, we can write,

$$-\alpha^2 A + 2\omega^2 A - \omega^2 B = 0$$

$$-\alpha^2 B + 2\omega^2 B - \omega^2 A = 0$$

In matrix form,

$$\begin{pmatrix} -\alpha^2 + 2\omega^2 & -\omega^2 \\ -\omega^2 & -\alpha^2 + 2\omega^2 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0$$

This requires the $\det m$ to be zero,

$$\left(-\alpha^2 + 2\omega^2\right)^2 - \left(\omega^2\right)^2$$

$$= \alpha^4 + 4\omega^4 - \omega^4 - 2\alpha^2 \cdot 2\omega^2$$

$$\alpha^4 - 4\alpha^2\omega^2 + 3\omega^4$$

This is equal to zero,

$$\alpha^4 - 4\alpha^2\omega^2 + 3\omega^4 = 0$$

This solves,

$$\alpha^2 = \frac{4\omega^2 \pm \sqrt{16\omega^4 - 12\omega^4}}{2}$$

The solutions are,

$$\alpha = \pm\omega$$