Finding out several solution to the same Problem -Irodov Mechanics

AS

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TAKE A MOMENT DOING THE PROBLEM YOURSELF, TRY A WORTHWHILE HOUR. IF YOU GET STUCK, THEN TRY TO CHANGE THE APPROACH, THE DAY YOU BECOME PRO IS THE DAY YOU REALIZE HOW UNHELPFUL SEEING A SOLUTION IS

Problem 1.138 A horizontal plane supports a stationary vertical cylinder of Radius R and a disk with a thread of length l_0 (refer to the figure adjoined; top view). An initial vertical velocity of v_0 has been imparted on the disk and it follows a trajectory as it winds around the cylinder.
How long will it take until it strikes the cylinder? Of course assume there is no friction any where. Hint is the figure.

Answer $\frac{l_0^2}{2Rv_0}$

Fundamental Realization The important realization here plays part all about the different ways of doing this same problem. It has to understood that the *Energy has to be conserved.* It is right that the direction of the velocity vector is going to change, but *that will not cause any change in the energy, that is no Work would be done.* And to worry about the angular momentum of the system will just be an unnecessary worry. In my solutions, that I had to work for a few hours before sitting to type, the energy conservation gives an important *Kinetical Hack.*

The hack is that, if we assume the mass of the disk to be m, then the total energy being conserved tells that later when the direction of the v_0 has changed,x, y component of the velocity then add the energy like the following,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2$$

That implies that,

$$v_0^2 = v_x^2 + v_y^2 \tag{1}$$

Drawing a nice diagram it's apparent that the v_x component vectorally added with v_y becomes v_0, meaning the speed will go unchanged.

Solution 1 This is the solution where I put one of my previous 'Idea'. It is noted in my other Latex doc, I copy-paste it here,

Idea 1 The solution of that Irodov problem (1.12) of three particles each at the vertex of an Equilateral Triangle is an idea itself. Whenever you face that the length changing in this and this way, then note that it is as sign that there is a $\frac{dl}{dt} = v_0$ in the situation.

Example: *Three particles are in the vertex of an equilateral triangle with length l. Each starts to move towards the one in front in a cyclic order, as particle 1 move to 2; and 2 to 3 and 3 to 1 at a velocity v. Find when the triangle shall converge to a point.* I tried to lamely trace the triangle to infinity, that was useless. Note that the particles are causing the length of the triangle in a way that

$$\frac{dl}{dt} = -v - v\cos\frac{\pi}{3}$$

Making an integration to t time till the $l \to 0$ shall do it.

Simply what I mean is that when we have, say a line, on whom its end has a velocity vector, then the vector projection of the velocity vector on the line is the rate of change (the 'derivative') of the line. The line here is chosen by another of my previous idea.

Idea 2 Turn all the problem into a system that you are familiar with. No need to stress over for finding a parameter that are you are not sure exists. Use an outlook that dissolves the problem into something learned.

Example: *Find the velocity of a particle that rotates around a circle, whose radius is R and the position vector $\vec{r}$ from the point of the trajectory path to particle rotates at ω.* Using the idea that the ω_0 from the center of the circle is the angular velocity, and using the isosceles identity of the circle, a relation can be built with the point with the center of the circle that efficiently solves the problem.

SO, my target is building relations of *Velocity Projections* and make equations that make sense with the *Center of Cylinder*. But it turns out that the same idea can be used in two different ways, so we will see the both. For now, the relatively easy one, the way I solved it very first (partially).

There is the velocity vector v_0, it is perpendicular to the l_0 line initially and throughout, but there is projection of v_0 on the x line,it's the line that joins the disk and the center of the cylinder. And *This would cause the x line to reduce with time, and thus the work now is finding the time for reduction from $x = \sqrt{l^2 + R^2}$ length to R.*

$$
\begin{aligned}
&\rightarrow \frac{dx}{dt} = -v_0 \cos\theta \\
&or, \frac{dx}{dt} = v_0 \frac{R}{x} && (2) \\
&or, x\,dx = v_0 R\,dt \\
&or, \int_{\sqrt{R^2+l_0^2}}^{R} x\,dx = v_0 R \int_0^t dt \\
&or, \left[\frac{x^2}{2}\right]_{\sqrt{R^2+l_0^2}}^{R} = v_0 R\,(t-0) \\
&or, \frac{R^2}{2} - \frac{R^2+l_0^2}{2} = v_0 R t \\
&or, \frac{R^2}{2} - \frac{R^2}{2} + \frac{l_0^2}{2} = v_0 R t \\
&or, \frac{l_0^2}{2Rv_0} = t && (3)
\end{aligned}
$$

Above is our required solution. The next solution makes a little different way, that is more convincing.

□

Solution 2 We more directly use the properties of a circle.

Referring to the figure, we can see that

$$x^2 = R^2 + l_0^2 \tag{4}$$

After some small amount of time, dt, there would be little decrement about the x and l_0, which is

$$
\begin{aligned}
(x-dx)^2 =& R^2 + (l_0 - Rd\theta)^2 \\
or,\ x^2 - 2x\,dx + (dx)^2 =& R^2 + l_0^2 - 2Rl_0\,d\theta + (d\theta)^2 \\
or,\ x^2 - 2x\,dx =& R^2 + l_0^2 - 2Rl_0\,d\theta
\end{aligned}
$$

We ignore when the small values have any exponents. Now, the change (the fundamental idea of Derivatives),

$$
\begin{aligned}
(x-dx)^2 - x^2 =& \\
or,\ x^2 - 2x\,dx - x^2 =& R^2 + l_0^2 - 2Rl_0\,d\theta - R^2 + l_0^2 \\
or, x\,dx =& R\,l_0\,d\theta
\end{aligned}
$$

l_0 can be any l at given time following the winding relation

$$l = l_0 - R\theta$$

Yes, I know, we can also directly come into the same using derivative directly. Prefer that if you know that.

Now divide the both sides with dt to get a dx/dt.

$$\begin{aligned}
&or, x\,\frac{dx}{dt} = R\,l\,\frac{d\theta}{dt}\\
&or, x\,v_0\frac{R}{x} = R\,l\,\frac{d\theta}{dt}\\
&or, v_0\,dt = l\,d\theta
\end{aligned}$$

The θ has a meaning. When the disk would eventually hit the cylinder when the l_0 string winds up around the cylinder, as it winds up, the hitting point shall make an angle θ. We have to know this θ for making the upper-limit of the integration, the lower is of course 0, because initially, there is no winding.

The number of windings that could be made with the string of length l_0,

$$n = \frac{l_0}{2\pi R}$$

When we wind the string around the cylinder, let it make remainder portion.[1] So, for n winding, length taken around the cylinder $2\pi Rn$. Let this create θ from starting point of starting the winding. So,

$$or,\ R\theta = 2\pi Rn$$

$$or,\ \theta = 2\pi\frac{l_0}{2\pi R}$$

$$or,\ \theta = \frac{l_0}{R}$$

This is quite non convincing in first site. Better derive yourself. So θ is $\frac{l_0}{R}$, upperlimit for integration. The θ upper limit is thus the point when string completes winding.

[1]We remember that if n is some integer like 1,2,3,4 or whatever, then wind ends where it starts. 1.5, 2.5, 3.5, like this, then the end of winding with complete just opposite side, 180 degree other side.

$$
\begin{aligned}
&\rightarrow v_0 \int_0^t dt = \int_0^{\frac{l_0}{R}} l \, d\theta \\
&or, v_0 \int_0^t dt = \int_0^{\frac{l_0}{R}} (l_0 - R\theta) \, d\theta \\
&or, v_0 \int_0^t dt = \int_0^{\frac{l_0}{R}} l_0 \, d\theta - \int_0^{\frac{l_0}{R}} R\theta \, d\theta \\
&or, v_0 t = l_0 \left[\theta\right]_0^{l_0/R} - R \left[\frac{\theta^2}{2}\right]_0^{l_0/R} \\
&or, v_0 t = \frac{l_0^2}{R} - \frac{l_0^2}{2R} \\
&or, v_0 t = \left(1 - \frac{1}{2}\right) \frac{l^2}{R} \\
&or, t = \frac{l_0^2}{2Rv_0}
\end{aligned}
$$

This is the end of the first part of the document that took the time from 7:30 to 10:45 in the **Christmas Day**.