Mock Physics Olympiad Shortlist

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Contents

1 Decisions

- 3:00 – 4:30 PM
- 25 April
- 1 hour 30 mins
- Although there are many problems, we don't want the pupil to solve all of it, rather the max. The contest would be still open after it is done, so there is still ways to check after the contest ends.

2 Example Sheet

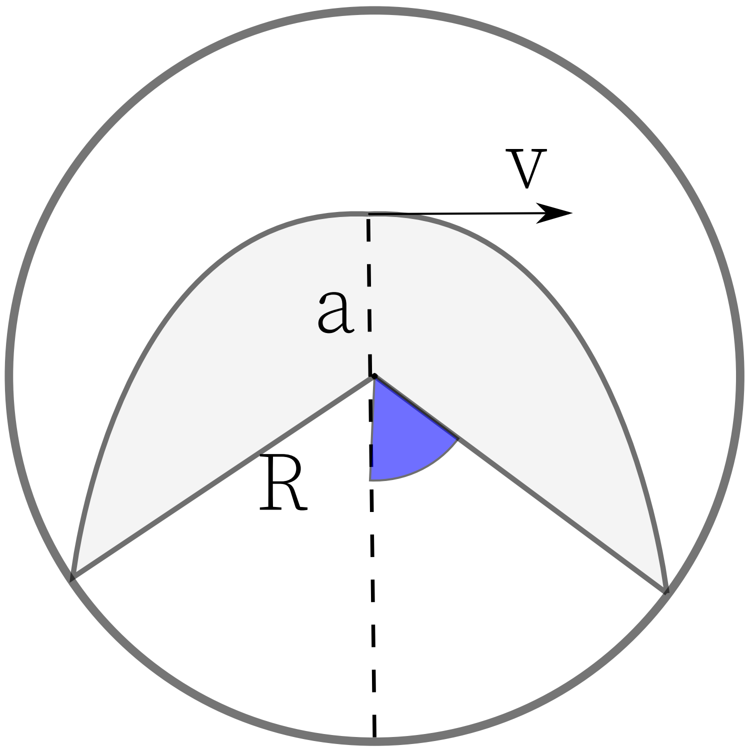

Figure 1: This area sweeped is marked in gray

Problem 1. Consider a trajcetory that is *Parabolic* and the mass is a comet that is in the Earth Sun orbital plane. Now, if they have the mass M_s and m, then what will be the maximum time for the comet, possible to stay in the orbit?

Sabit

- **score** selected problem (example)
- score 20 points
- time est for this 20 mins (all are example)

Idea 1. We can use Energy and Angular Momentum conservation laws to find some required equations. From our intuition we know that the *Area sweeped must be proportional to time*, hence what we are going to do is use,

$$L = mr^2 \frac{d\theta}{dt}$$

Solution 1. Let the comet pass through the perigee. The distance of focus (Sun) and perigee be a. Now we need to look for a. We shall have the total energy $E = 0$ and L as the angular momemtum.

$$\frac{1}{2}v^2 - \frac{GM}{a} = 0$$

$$\left(\frac{L^2}{m^2a^2}\right) = \frac{2GM}{a}$$

With the above two equation we can find,

$$a = \frac{L^2}{2GMm^2}$$

Or, converse is also true,

$$L = m\sqrt{2GMa}$$

We shall now concentrate on the time taken. From the definition of angular momemtum,

$$L = mr^2 \frac{\mathrm{d}\theta}{\mathrm{d}t}$$

So, the integral,

$$\int_0^t \mathrm{d}t = \left(\frac{2m}{L}\right)\left(\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, \mathrm{d}\theta\right)$$

Above, the term $\int_{\theta_1}^{\theta_2} r^2 \, \mathrm{d}\theta$ is merely the area of the region of orbit, bounded by the points where the comet orbit cut Earth orbit, and focus. We call that, because, if we assume a line r emerging from the focus to any point in the trajectory, the height gained for an infinitesimal movement is $rdm\theta$, so the area is half times base times hight, follow, $\frac{1}{2}r(rdm\theta)$, taking the integral around theta determines the full region. But, finally, if we call the area A, then the time period is,

$$t = \frac{2mA}{L}$$

Solving for the area, we find that,

$$A = \frac{2}{3}\sqrt{a(R-a)}(2a + R)$$

Plugging onto the equations,

$$t = \frac{2}{3}\sqrt{\frac{2}{GM}}\sqrt{R-a}\left(\frac{a}{R} + \frac{1}{2}\right)$$

Maximizing this function is just maximizing the function,

$$f(x) = (1-x)(x + \frac{1}{2})^2$$

We can see that maximum occurs at $x = \frac{1}{2}$. So the maximum time,

$$\boxed{T_{max} = \frac{4}{3}\sqrt{\frac{R^3}{GM}}}$$

$\square$

3 Mechanics

A simple warm up problem.

Problem 2. There is a charge q that is a distance away from another stationary q charge. The initial speed of the charge is shown as u and it start moving at time $t = 0$. After some time, the speed is v and the distance from the q charge is r. Find the angle made by the velocity vector $\vec{v}$ with the position vector (respect to stationary q charge) $\vec{r}$. Initially, $\vec{u}$ and $\vec{a}$ are perpendicular (make $90°$ between them).

Sabit

- Selected
- Score 2
  - Vector notation: 1
  - Angular Momentum: 1

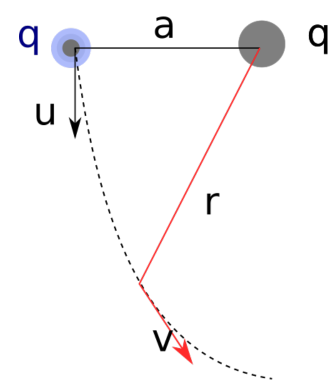

Figure 2: The charge problem.

Idea 2. Angular Momentum is conserved for $1/r^2$ form and any type central force. This includes classical gravitation and coulomb's law. Because if the force has another component than radial force, that will contribute to give a torque, hence causing change to angular momentum.

Solution 2. Just from the adjoined idea, we have,

$$L = mua = mvr \sin\theta$$

Hence,

$$\theta = \arcsin\left(\frac{ua}{vr}\right)$$

□

Problem 3. Coefficient of restitution is $C = 0.9$, for a system where a ball falls on the ground and hops off. The C means that if the ball falls with v speed, it will bounce up with $0.9v$ speed. So, at first the ball is vertically thrown upward with speed $50\ m/s$, at $t = 0$ time. After what time the ball comes to rest and does not bounce anymore?

Sabit

- selected
- Point 4

Idea 3. Some problems have the feature of being recursive and changing after every step in a linear or geometric form. Those forms can be solved either by reduction through through any Series like Taylor or McLauren, or, they can even be Geometric Series additions.

Solution 3. Every time the ball hops off, then there is a time it spends on air. That time is,

$$T = \frac{2v}{g}$$

Given the ball hops off with the speed v. Initially speed is $v_0 = 50\ m/s$, but when it bounces next time, it will have speed $v = 0.9 \times 50\ m/s$, or we can say, $v = v_0 C$. After it returns to ground again, it will bounce of with $v = C(v_0 C)$ that is, $v = C^2 v_0$. And this keeps going. For every bounce n, there will be bounce speed $v_0 C^n$. We have to sum up all the time taken by the ball in air.

$$T = \sum T_i = \sum_{i=0}^{\infty} 2\frac{v_0}{g} C^i$$

There will be alot of small bounces made, so we sum up until infinity, but that does not mean that time will also be infinite, because with every bounce the speed decreases, so as the time the ball spends on air.

$$T = \frac{2v_0}{g} \sum_{i=0}^{\infty} C^i$$

We have $C < 1$ that helps us to take this sum, it will be,

$$\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$$

Our sum would be,

$$\boxed{T = \frac{2v_0}{g} \frac{1}{1-C}}$$

That gives the numerical value,

$$T = 100\ s$$

Not infinite. □

Problem 4. Suppose there is a block of mass $m = 2\ kg$ resting on another block of mass $M = 4\ kg$ which is resting on a surface. The bottom block is given a force $F = 80\ N$ to the right. What are the blocks' acceleration to the right? All surface surface interface has coefficient of static and kinetic friction $\mu = 0.2$ and free-fall acceleration, $g = 10\ m/s^2$

Abrar

- selected
- Point 5

Solution 4. There are 4 variables, namely a_m, a_M, N_m, N_M and there are 4 force equations. Considering positive x-axis to the right and positive y-axis to the up, we get

$$F - \mu N_M - \mu N_m = Ma_M$$

$$\mu N_m = ma_m$$

$$N_M - Mg - mg = 0$$

$$N_m - mg = 0$$

Solving the third and fourth equations, we can find that $N_m = 20\ N$ and $N_M = 80\ N$. Plugging them in the first and second equations, we can find that $\boxed{a_m = 2\ /s^2 \text{ and } a_M = 4/ms^2}$

□

Problem 5. A pendulum of mass m and length l while hanging in vaccum gives a period of T. if it is submerged in a liquid which accounts for an effective weight loss of $\frac{mg}{4}$, the period becomes T_{liquid} considering no viscous friction. if the ratio $T : T_{liquid}$ can be expressed in $\sqrt{\frac{a}{b}}$, then what is the values of a and b?

Abrar

- backup
- Point 4

Solution 5.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

When submerged in the liquid, the equation of motion will become using small angle approximation

$$ml^2\theta'' = \frac{3m}{4}gl\theta \Rightarrow \theta'' = \frac{3g}{4l}\theta$$

therefore,

$$T_{liquid} = 2\pi\sqrt{\frac{4l}{3g}}$$

so $\boxed{\text{a} = 3,\ \text{b} = 4}$ □

Problem 6. A uniform string of mass m and length l is hung vertically with a mass M attached to its free end. what is the speed of a wave transmitted across the string at a distance y from the top of the string, $V(y)$. What is the time required for a wave to go from the top to the bottom?

Abrar

- Hard backup (difficult)
- Point 10

Solution 6. Tension at a distance y from the top is

$$T(y) = \frac{mg(l-y)}{l} + Mg$$

Therefore, the speed will be

$$v(y) = \sqrt{\frac{T(y)}{\frac{m}{l}}} \Rightarrow v(y) = \sqrt{\frac{\frac{mg(l-y)}{l} + Mg}{\frac{m}{l}}}$$

$$\boxed{\Rightarrow v(y) = \sqrt{g(l-y) + \frac{Mgl}{m}}}$$

Then, the required time will be

$$\int_0^t dt = \int_0^l \frac{1}{v(y)}\, dy$$

$$\Rightarrow t = \int_0^l \frac{1}{\sqrt{g(l-y) + \frac{Mgl}{m}}}\, dy$$

Substituting $v = g(l-y) + \frac{Mgl}{m}$, we find that $dv = -gdy$ and the upper and lower bound are respectively $gl + \frac{Mgl}{m}$ and $\frac{Mgl}{m}$

$$\int_0^l \frac{1}{\sqrt{g(l-y) + \frac{Mgl}{m}}}\, dy \ = -\frac{1}{g} \int_{gl+\frac{Mgl}{m}}^{\frac{Mgl}{m}} \frac{1}{\sqrt{v}}\, dv$$

$$\Rightarrow t = -\frac{2}{g}\left(\sqrt{\frac{Mgl}{m}} - \sqrt{gl + \frac{Mgl}{m}}\right)$$

$$\boxed{\Rightarrow t = 2\sqrt{\frac{l}{mg}}\left(\sqrt{M+m} - \sqrt{M}\right)}$$

□

Problem 7. A solid cube with side length **a** is placed as the figure shows. A surface stays upon $x - y$ plane, while an edge is fixed along z axis. It rotates around the z axis with angular speed $\boldsymbol{\omega} = \omega\hat{\mathbf{k}}$. Find it's angular momentum in this format: $\mathbf{L} = L_x\hat{\mathbf{i}} + L_y\hat{\mathbf{j}} + L_z\hat{\mathbf{k}}$

Meshkat

- selected
- Point 3

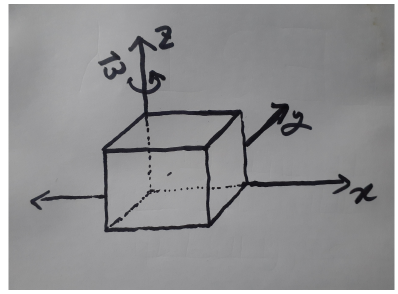

Figure 3: Problem 10

Solution 7. □

Problem 8. There is a cylinder with non-uniform mass distribution which is rolling on the ground. The center of mass is located r from the axis of rotation. What is the maximum speed the cylinder can roll without hopping off the ground?

Sabit

- Selected
- Point 1

Solution 8. Hopping off the ground will occur if,

$$\frac{v^2}{r} = g$$

Hence,

$$\boxed{v = \sqrt{gr}}$$

□

4 Thermodynamics

Problem 9. Suppose a empty tank of volume $10V_0$ is connected to a cylinder of volume V_0 which contains n moles of ideal gas with pressure P_0. when equilibrium is reached, the tank is disconnected and another full cylinder is connected to it. How many cylinders are needed to make the Tank's pressure $\frac{P_0}{2}$?

Abrar

- Selected
- Point 7

Solution 9. When pressure equilibrium is reached, then P, n, R will be constant.As a result, the ideal gas law $PV = nRT$ will take the form of $V \propto n$. Now according to that equation, to reach the desired pressure, $5n$ mole of ideal gas is needed in the tank. If we denote n_i as the number of mole of gas in the tank after disconnecting from the ith cylinder, then

$$n_0 = 0 \ and \ n_i = (n + n_{i-1}) \times \frac{10}{11}, \ so$$

$$n_1 = \frac{10}{11} \times n$$

$$n_2 = \frac{10}{11}\left(1 + \frac{10}{11}\right) \times n$$

$$n_3 = \frac{10}{11}\left(1 + \frac{10}{11} + \left(\frac{10}{11}\right)^2\right) \times n$$

We can see a pattern.

$$n_i = \sum_{j=1}^{i} \left(\frac{10}{11}\right)^j \times n$$

$$\Rightarrow n_i = \frac{1 - \left(\frac{10}{11}\right)^i}{\frac{1}{11}} \times n$$

solving for i, we find $i = 7.3$. So, 8 cylinders are needed. □

Problem 10. At troposphere, the lowest level of Earth's atmosphere, air near the ground rises slowly because of density gradient and in this process, heat exchange with the surrounding air can be neglected, making the process adiabatic. Assume air molecules are diatomic with the mass of $m = 1.38 \times 10^{-25}\ kg$. Since air rises slowly, it can be assumed in a equilibrium at all time. Derive the value of $\frac{dP}{dz}$ at $z = 500\ m$ in terms of, $n(z)$, the number density of air molecules at height z and other necessary terms in Pascal per meter. Also by deriving $\frac{dT}{dz}$, find T at $350\ m$ in terms of z and other necessary terms. What is the sum of the numbers? Temperature at surface is $T_0 = 300\ K$, $n(500) = 10^{23}$, $g = 10ms^{-2}$, $K_b = 1.38 \times 10^{-23} m^2 kg s^{-2} K^{-1}$

Abrar

- tell that the process is adiabatic
- Selected
- Point 8

Solution 10. As for the first part,

$$\frac{dP}{dz} = -n(z)\, mg$$

, as gravity is the only net acting force. By nature, the process of rising up is adiabatic.

Therefore, using the ideal gas law, we find this alternative of our adiabatic gas law

$$P^{\gamma-1}T^{-\gamma} = constant \Rightarrow -\gamma T^{-\gamma-1}P^{\gamma-1} + (\gamma-1)\,T^{-\gamma}P^{\gamma-2}\frac{dP}{dT} = 0$$

$$\Rightarrow dP = \frac{\gamma}{\gamma-1}\frac{P}{T}dT \Rightarrow -n\,(z)\,mg = \frac{\gamma}{\gamma-1}\frac{P}{T}\frac{dT}{dz}$$

Now by the ideal gas law,

$$PV = Nk_bT \Rightarrow P = n\,(z)\,k_bT$$

Substituting P, we find that

$$\frac{dT}{dz} = -\left(\frac{\gamma-1}{\gamma}\frac{mg}{K_b}\right) \Rightarrow dT = -\frac{\gamma-1}{\gamma}\frac{mg}{K_b}dz$$

Integrating and plugging in the initial value, we find that

$$T = T_0 - \frac{\gamma-1}{\gamma}\frac{mg}{K_b}z$$

□

Problem 11. 1 mole of ideal gas with internal energy $U = \frac{3}{2}RT$, expands from initial volume $V_i = \frac{1}{10}V_0$ following the equation

$$p = \frac{p_0}{V_0}V + p_0$$

- What is the highest temperature reached by the gas during the expansion?
- What is the maximum amount of heat taken in by the gas?

Ishtiak

- Selected
- 2 parts, 1st part: 3
- 2 parts, 2nd part: 4

Solution 11. □

Problem 12. A sealed container with cross section area $S = 0.1m^2$ is divided into two parts by a movable valve of mass $m = 0.1kg$. The friction between the valve and the container is negligible. When the valve at equilibrium, the air pressure $P = 101325Pa$ on the two sides of the valve are equal with volumes $V_1 = 0.3m^3$ and $V_2 = 0.5m^3$. When the valve is slightly displaced from equilibrium and then relaxed, it oscillates around its equilibrium position. The system can be considered as isothermal. What is period of the oscillation T? Your answer should be in seconds rounded up to the nearest hundredth.

Ishtiak

- In case of lacking of oscillaitons problems, this can be considered.
- Backup for oscillation
- Point 5

Solution 12. □

Problem 13. An air bubble of volume $20cm^3$ is at the bottom of a lake $40m$ deep, where the temperature is $4°C$.The bubble rises to the surface, which is at a temperature of $20°C$.Take the temperature of the bubble's air to be the same as that of the surrounding water. Just as the bubble reaches the surface, what is its volume? Assume that the pressure inside the bubble is same as the pressure outside. Your answer should be in cm^3, rounded to the nearest integer.

Ishtiak

- We can talk about added mass
- this problem is in 3 parts, normal, surface tension, added mass.
- Selected
- Points 3, 3 2 (to be known) , 2

Solution 13. □

5 Electrodynamics

Problem 14. There is a loop in which case the loop material is taken into superconducting state, in superconducting state, all you need to assume is that the loop has a very very low resistance, in the order of micro to lower order ohms. If radius of it is r and if we run a current I initially and after a moment take away the current source, it takes t time for the current to reduce by ΔI. Find the small resistance of R.

Sabit

- Selected
- Point 4

Solution 14. The current along the loop is going to cause a flux into itself, it will cause an EMF. This EMF can be calculated using the Faraday's Law and using it we can solve for R. □

Problem 15. A beam of protons is accelerated from rest through a potential difference of 2000 V and enters a uniform magnetic field which is perpendicular to the direction of the proton beam. If the magnetic field is 0.2 T, calculate the radius of the path of the beam. Answer in milimeters to the nearest tenth.

Ishtiak

- Selected as a hello world type.
- Point 2

Solution 15. □

Problem 16. An isosceles triangle has one of it's vertex located at $(x, y) = (0, 0)$ and one of it's side on x axis. Each side of the triangle is of length 1 meter. On each of it's vertex, there is a 1C charge. (x_i, y_i) are points on the triangle where electric field is 0. Find the sum of all the x_is. Your answer should be in meter, rounded to the nearest tenth.

Ishtiak

- Selected after modification.
- Point (to be known)

Solution 16. □

6 DC Circuits

Problem 17. Referring to the DC circuit below: Find the current **I**. Find **I** if $n = \infty$

- For discussion
- Point 7

Meshkat

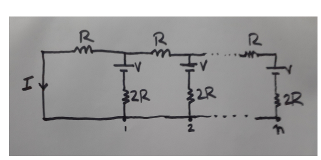

Figure 4: Problem 15

Solution 17. ☐

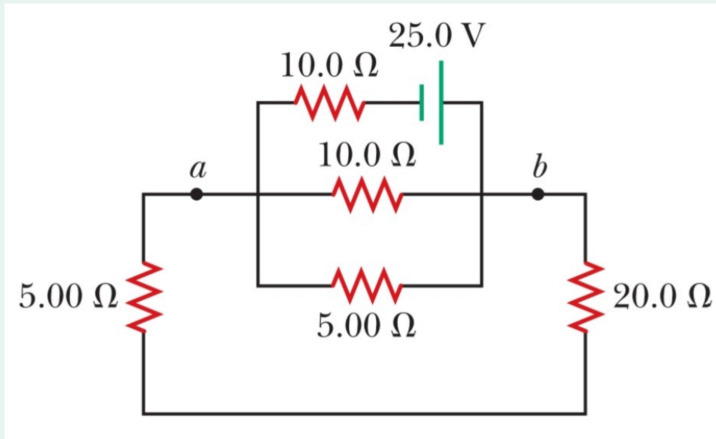

Figure 5: Problem 16

Problem 18. Find the current through the 20Ω resistor. Hint: Think about the resistors connected to a and b.

Abrar

- For discussion
- Point 4

Solution 18. All resistors connected to a and b are in parallel. So their equivalent resistance is

$$R = \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{20+5}\right)^{-1} \ \Omega = \frac{50}{17} \ \Omega$$

. Then, R and the 10 Ω resistor with the battery are in series. Therefore, the total current of the circuit is

$$I = \frac{25}{10 + \frac{50}{17}} \ A = \frac{85}{44} \ A$$

This is the current through the given resistor. ☐

7 Optics

Problem 19. There is an object between a concave mirror to the right and a convex lens to the left with a separation of 40 cm. Two real image with $m = 1.50$ magnification can be seen to the right of the lens, one upright and one inverted. The focal length of the lens is $f_l = 10\ cm$. What is the focal length of the mirror, f_m?

Abrar

- Selected
- Point 6

Solution 19. The upright image is the image of the real image of the real image from the mirror of the given object.we know that

$$m = \frac{v}{u}\ so\ v = 1.50u$$

As for both images, v is the same, so u is too. Now,

$$\frac{1}{f_l} = \frac{1}{u} + \frac{1}{1.50u}\ or\ u = \frac{50}{3}\ cm$$

For the image from the concave mirror to be equal in length to the object, The distance between itself and the object must be twice its focal lnegth.

$$f_m = \frac{40 - \frac{50}{3}}{2}\ cm = \frac{35}{3}\ cm$$

□

Problem 20. Sabit wants to determine the refractive index of a liquid by using diffraction grating. He submerged a diffraction grating setup in the liquid. The light has wavelength of $\lambda = 500\ nm$, the separation the slits are $d = 10\ micrometer$, the distance between the slit and the screen is $s = 10\ cm$. In air of refractive index $n = 1$, the second maxima was 5 mm apart from the central maxima. In the liquid that distance becomes 3.75 mm. What is the refractive index of the liquid? (Note: The angles of the maximas are small.)

Abrar

- Selected
- Point 4

Solution 20. Any transparent medium affects the wavelength of any incoming light. If the maxima separation is y and the angle of the maxima is θ, then using small angle approximation and formula of diffraction grating,

$$d \sin\theta \approx d \tan\theta = d\frac{y}{s} = \frac{\lambda}{n}$$

, where n is the refractive index of the liquid.

$$\rightarrow n = \frac{s\lambda}{dy}$$

Plugging the values, we find that

$$n = \frac{4}{3}$$

□

8 Relativity

Problem 21. The motion of a transparent medium influences the speed of light in it. Suppose, a block of water is moving at 0.5c speed having a refractive index of $\frac{4}{3}$. Then the speed of light in that water will be $\frac{ac}{b}$. What is the value of a and b if they are co-prime?

Abrar

- Selected
- Point 6

Solution 21. Using the relativistic addition of velocities for $\frac{c}{n}$ and v, the speed of water in non-moving water and the speed of water respectively, we can find the the speed of light in the moving water is

$$u = \frac{v + \frac{c}{n}}{1 + \frac{v\left(\frac{c}{n}\right)}{c^2}} = \frac{c}{n}\left(\frac{1 + nv/c}{1 + v/nc}\right)$$

Plugging values, we can find that

$$u = \frac{10c}{11}$$

Therefore, $\boxed{\text{a=10, b=11}}$ □

Problem 22. A particle of 0.0001 kg is undergoing uniform circular motion with radius r = 100 km and velocity of $\frac{\sqrt{3}c}{2}$. How much relativistic centripetal force is the particle having? $c = 3 \cdot 10^8 m/s$

Abrar

- Selected
- 5

Solution 22. If P is the relativistic momentum of the particle, then

$$F_{centripetal} = \frac{dP}{dt} = \gamma m_0 v \; = \; \gamma m_0 \frac{dv}{dt}$$

here the Lorentz factor and mass is constant.Using classical argument, we can find that

$$\frac{dv}{dt} = \frac{v^2}{r}$$

Plugging values, we can find that

$$\boxed{F = 135000000 = 1.35 \times 10^8 \ N}$$

$\square$