Computational Complex Analysis : : Class 34

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Liouville's Theorem is that if f is holomorphic on $\mathbb{C}$ and $|f(z)| \leq \text{const}$ then, f is constant. If u is harmonic on $\mathbb{C}$ and $|u(z)| \leq \text{ const }$ the u is constant. Proof is use a harmonic conjugate v and $u + iv$ is holomorphic. Take $f = e^{u+iv}$ and $|f| = e^u$.

MVP of Harmonic Functions. If u is harmonic, then $u(z_0) = \text{ average of } u \text{ on circles centered at } z_0$.

Maximum Principle: if u is harmonic on an open connected set, it cannot have local maximum without being constant. Proof is going to be $u \leq M$ where open set and $u = M$ at some point.

Corollary, suppose D is bounded open set and u is continuous D and ∂D and harmonic in D. Then u attains its maximum and min value on D.

A solution, if it exists, is unique. Existence, a given function on a body, we want to find harmonic function on D which takes the given value on the body: "Dirichlet Problem".