Problem 1. Let's define a line F.

- This line is perpendicular to a line C which is $C(t) = (1, 0, -1) + t(1, -1, 2)$
- This line goes through $(5, 0, 3)$.

Find F.

Solution. What is the vector $\vec{N}$ that points along C line? That vector is simply $(1, -1, 2)$. What are the vectors $\vec{N}_\perp$ that are perpendicular to $\vec{N}$? They are,

$$\vec{N} \cdot \vec{N}_\perp = 0$$

Let's break the vectors down in a way that,

$$\vec{N} = (a, b, c)$$

$$\vec{N}_\perp = (a', b', c')$$

So from the perpendicular condition, we can find (a', b', c').

$$\vec{N} \cdot \vec{N}_\perp = aa' + bb' + cc' = 0$$

For this problem, $(a, b, c) = (1, -1, 2)$ hence,

$$a' - b' + 2c' = 0$$

So any vector (a', b', c') that follow that condition above will be perpendicular to (a, b, c) But the thing is, there are ∞ numbers (a', b', c') that can solve that above equation $a' - b' + 2c' = 0$. We end up getting a whole Plane of points that satisfy that relation, if you imagine that the vector (a', b', c') spins around (a, b, c) vector staying perpendicular. All the direction $\vec{N}_\perp$ can point to builds up a plane. A random plane that satisfies above condition can be

$$x - y + 2z = D$$

Where D is some random number.
There are infinite planes. But we are interested on the one that contains one of the point $(5, 0, 3)$ as shown in the problem. To find the plane that specifically contains the point $(5, 0, 3)$ we have to use,

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

The derivation is avoided here, but you can find it out online. Putting the values we get the plane,

$$x - y + 2z = -1$$

This plane includes $(5, 0, 3)$.
This plane is perpendicular to the C line (because it's perpendicular to it's direction as we have noted). There is a point where C intersects $x - y + 2z = -1$. This intersection point, if connected to $(5, 0, 3)$ will give us the final answer which is the line that is perpendicular to C and goes through that point.
Turns out for the C line, the intersection is simply $1, 0, -1$. So the line that connects

$$(1, 0, -1) \to (5, 0, 3)$$

is the solution. $\square$