Classical Mechanics : : Homework 09

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Problem 01

(a)

For the first particle in position $\theta = \Omega t$.

$$\begin{aligned}\vec{F}_{\text{cor}}(\theta) &= -2m\vec{\omega}\times\vec{v}(t) = -2m\vec{\omega}\times(\vec{\Omega}\times\vec{R}(t)) \\ &= -2m(\omega\hat{z})\times(\Omega\vec{y}\times[R\cos\theta\hat{x} + R\sin\theta\hat{z}]) \\ &= -2m\omega\hat{z}\times(\Omega R\cos\theta(-\hat{z}) + \Omega R\sin\theta(\hat{x})) \\ &= 2m\omega\hat{z}\times(\Omega R\cos\theta\hat{z} - \Omega R\sin\theta\hat{x}) \\ &= 2m\omega\Omega R\sin\theta(\hat{y})\end{aligned}$$

For the second particle is $\vec{F}_{\text{cor}}(\theta+\pi)$. $\vec{F}_{\text{cor}} = 2m\omega\Omega R\sin(\omega)\hat{y}$

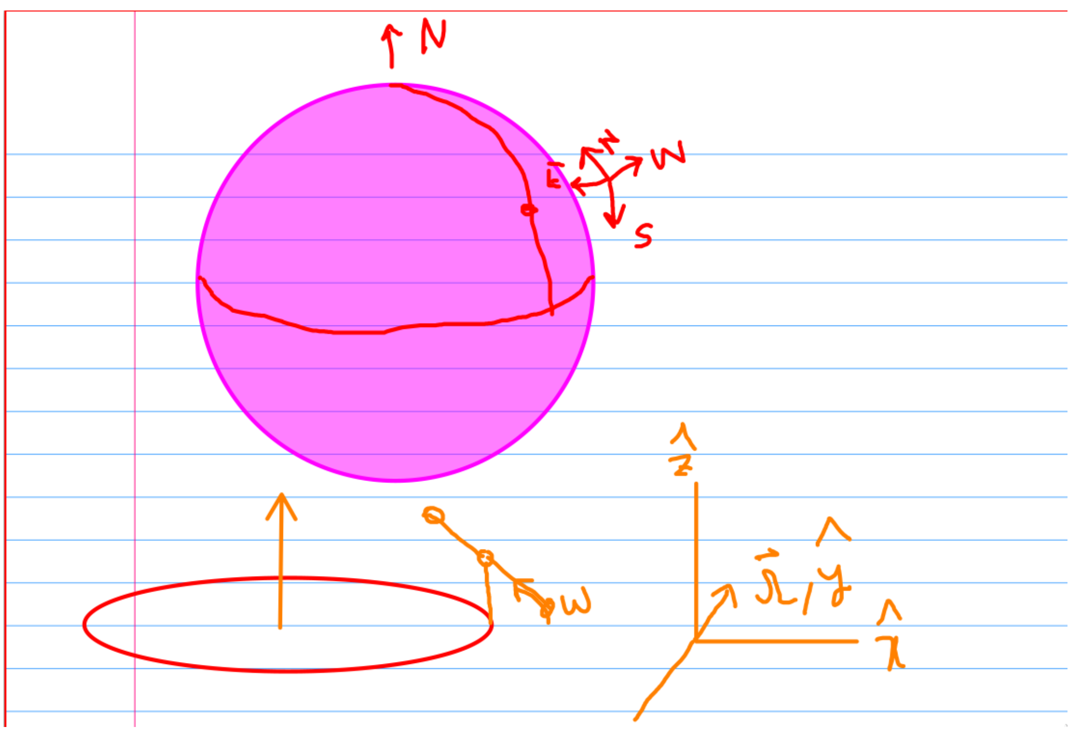

Figure 1: ./ss/9/2.png

(b)

$$
\begin{aligned}
\vec{\tau} &= \vec{R}_1 \times \vec{F}_1 + \vec{R}_2 \times \vec{F}_2 \\
&= (R\cos\theta\hat{x} + R\sin\theta\hat{z}) \times (2m\omega\Omega R\sin\theta\hat{y}) + (-R\cos\theta\hat{x} - R\sin\theta\hat{z}) \times (-2m\omega\Omega R\sin\theta\hat{y}) \\
&= \left[(4m\omega\Omega R^2)\left(\cos\theta\hat{x} \times \sin\theta\hat{y} + \sin\theta\hat{z} \times \sin\theta\hat{y}\right)\right] \\
&= 4m\omega\Omega R^2\left(\cos\theta\sin\theta\hat{z} - \sin^2\theta\hat{x}\right)
\end{aligned}
$$

(c)

$$
\begin{aligned}
\vec{\tau} &= 4m\omega\Omega R^2\left(\cos\theta\sin\theta\hat{z} - \sin^2\theta\hat{x}\right) \\
\langle\vec{\tau}\rangle &= \frac{\int_0^T \mathrm{d}t\, 4m\omega\Omega R^2\left(\cos(\Omega t)\sin(\Omega t)\hat{z} - \sin^2(\Omega t)\hat{x}\right)}{\int_0^T \mathrm{d}t} \\
&= \frac{\int_0^T \mathrm{d}t\, 4m\omega\Omega R^2\left(\cos(\Omega t)\sin(\Omega t)\hat{z} - \sin^2(\Omega t)\hat{x}\right)}{\int_0^T \mathrm{d}t} \qquad (T = 2\pi/\Omega) \\
&= 4m\omega\Omega R^2\frac{\left[\hat{z}\int_0^T \mathrm{d}t\ \cos(\Omega t)\sin(\Omega t) - \hat{x}\int_0^T \mathrm{d}t\ \sin^2(\Omega t)\right]}{2\pi/\Omega} \\
&= 4m\omega\Omega R^2\left(-\frac{1}{2}\hat{x}\right) \\
&= -2m\omega\Omega R^2\hat{x}
\end{aligned}
$$

(d)

New coordinate system where $\hat{y}$ faces the south. And solving for unit mass

$$
\vec{\omega} = \cos(90^\circ - \lambda)\hat{z} - \sin(90^\circ - \lambda)\hat{y} = \sin(\lambda)\hat{z} - \cos(\lambda)\hat{y}
$$

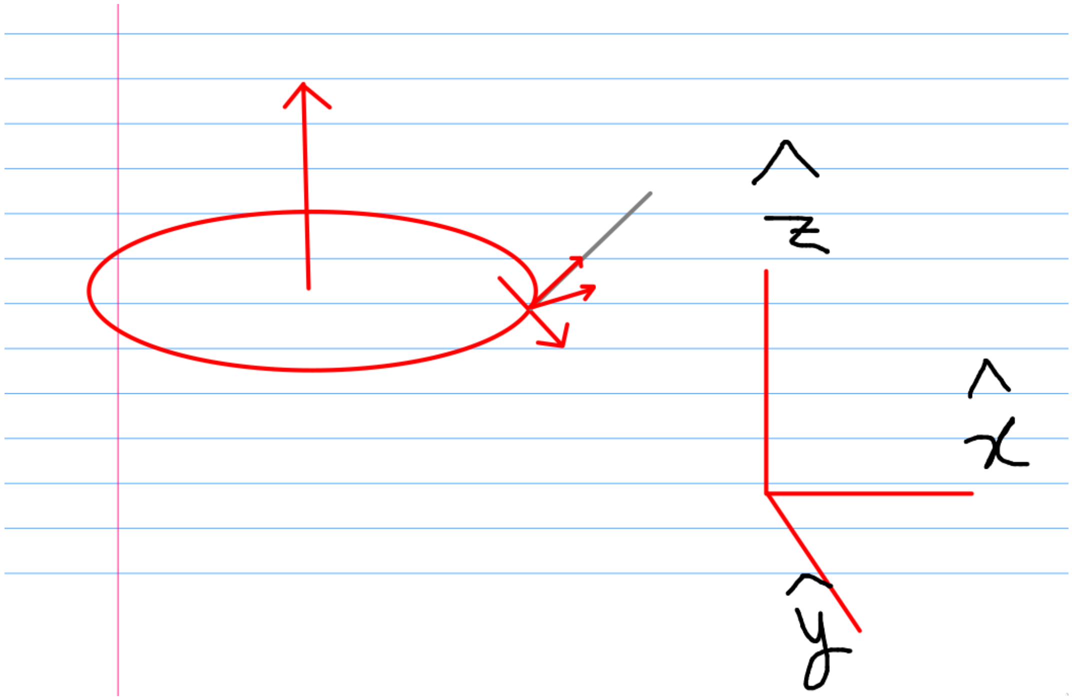

Figure 2: .ss/9/3.png

Coriolis force

$$
\begin{aligned}
\vec{F} &= -2\vec{\omega} \times \vec{v} \\
&= -2\omega\left(\sin(\lambda)\hat{z} - \cos(\lambda)\hat{y}\right) \times (\vec{\Omega} \times \vec{R}) \\
&= -2\omega\left(\sin(\lambda)\hat{z} - \cos(\lambda)\hat{y}\right) \times \left(\Omega\hat{y} \times [R\cos(\theta)\hat{x} + R\sin(\theta)\hat{z}]\right) \\
&= -2\omega(\sin(\lambda)\hat{z} - \cos(\lambda)\hat{y}) \times \Omega R\left(\cos(\theta)\hat{z} - \sin(\theta)\hat{x}\right) \\
&= -2\omega\Omega R(\sin(\lambda)\hat{z} - \cos(\lambda)\hat{y}) \times (\cos(\theta)\hat{z} - \sin(\theta)\hat{x}) \\
&= -2\omega\Omega R\left[(\cos(\lambda)\cos(\theta)\hat{x}) + (-\sin(\lambda)\sin(\theta)(-\hat{y}) + \cos(\lambda)\sin(\theta)(\hat{z}))\right] \\
&= -2\omega\Omega R\left[\cos(\lambda)\cos(\theta)\hat{x} + \sin(\lambda)\sin(\theta)\hat{y} + \cos(\lambda)\sin(\theta)\hat{z}\right]
\end{aligned}
$$

Torque on single object

$$
\begin{aligned}
\vec{\tau} &= \vec{R} \times \vec{F} \\
&= (R\cos(\theta)\hat{x} + R\sin(\theta)\hat{z}) \times \vec{F} \\
&= -2\omega\Omega R^2\left[-\sin\lambda\sin\theta\cos\theta\hat{z} + \cos\lambda\sin\theta\cos\theta\hat{y} - \cos\lambda\sin\theta\cos\theta\hat{y} + \sin\lambda\sin^2\theta\hat{x}\right] \\
&= 2\omega\Omega R^2\left[\sin\lambda\sin\theta\cos\theta\hat{z} - \cos\lambda\sin\theta\cos\theta\hat{y} + \cos\lambda\sin\theta\cos\theta\hat{y} - \sin\lambda\sin^2\theta\hat{x}\right] \\
&= 2\omega\Omega R^2\left[\sin\lambda\sin\theta\cos\theta\hat{z} - \sin\lambda\sin^2\theta\hat{x}\right] \\
\langle\vec{\tau}\rangle &= -\omega\Omega R^2\sin\lambda\hat{x}
\end{aligned}
$$

For two particle, symmetrically we will end up with (considering mass)

$$\langle\vec{\tau}\rangle = -2\omega\Omega R^2\sin\lambda\hat{x}$$

(e)

The average torque can be measured for various orientations of the rotating axis. Giving between $2\omega\Omega R^2$ to $2\omega\Omega R^2 \sin\lambda$. The maximum gives use the idea of where the west-east line lies and minimum gives idea of north-south line.

Then taking care of the direction $-\hat{x}$ we can directly see where the true north is (using similar directions as in the attached figure).

Problem 02

(a)

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{A}\cdot\vec{B}\right)_{\text{fix}} &= \left(\frac{\mathrm{d}\vec{A}}{\mathrm{d}t}\cdot\vec{B}\right)_{\text{fix}} + \left(\vec{A}\cdot\frac{\mathrm{d}\vec{B}}{\mathrm{d}t}\right)_{\text{fix}} \\
\left(\frac{\mathrm{d}\vec{A}}{\mathrm{d}t}\right)_{\text{fix}} &= \left(\frac{\mathrm{d}\vec{A}}{\mathrm{d}t}\right)_{\text{rot}} + \vec{\omega}\times\vec{A} \\
\left(\frac{\mathrm{d}\vec{B}}{\mathrm{d}t}\right)_{\text{fix}} &= \left(\frac{\mathrm{d}\vec{B}}{\mathrm{d}t}\right)_{\text{rot}} + \vec{\omega}\times\vec{B}
\end{aligned}
$$

$$
\begin{aligned}
\implies \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{A}\cdot\vec{B}\right)_{\text{fix}} = \left(\frac{\mathrm{d}\vec{A}}{\mathrm{d}t}\cdot\vec{B}\right)_{\text{fix}} + \left(\vec{A}\cdot\frac{\mathrm{d}\vec{B}}{\mathrm{d}t}\right)_{\text{fix}} &= \left[\left(\frac{\mathrm{d}\vec{A}}{\mathrm{d}t}\right)_{\text{rot}} + \vec{\omega}\times\vec{A}\right]\cdot\vec{B} + \left[\left(\frac{\mathrm{d}\vec{B}}{\mathrm{d}t}\right)_{\text{rot}} + \vec{\omega}\times\vec{B}\right]\cdot\vec{A} \\
&= \left(\frac{\mathrm{d}A}{\mathrm{d}t}\right)_{\text{rot}}\cdot\vec{B} + \left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)_{\text{rot}}\cdot\vec{A} + \left[(\vec{\omega}\times\vec{A})\cdot\vec{B} + (\vec{\omega}\times\vec{B})\cdot\vec{A}\right] \\
&\qquad \text{(check appendix for why third term is zero)} \\
&= \left(\frac{\mathrm{d}A}{\mathrm{d}t}\right)_{\text{rot}}\cdot\vec{B} + \left(\frac{\mathrm{d}B}{\mathrm{d}t}\right)_{\text{rot}}\cdot\vec{A} \\
&= \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{A}\cdot\vec{B}\right)_{\text{rot}}
\end{aligned}
$$

(b)

Recycling what we had above, using $\vec{C} = \vec{A} \times \vec{B}$

$$\begin{aligned}
\left(\frac{d\vec{C}}{dt}\right)_{\text{fixed}} &= \left(\left(\frac{d\vec{A}}{dt}\right)_{\text{rotating}} + \vec{\omega} \times \vec{A}\right) \times \vec{B} + \vec{A} \times \left(\left(\frac{d\vec{B}}{dt}\right)_{\text{rotating}} + \vec{\omega} \times \vec{B}\right). \\
&= \left(\frac{d\vec{A}}{dt}\right)_{\text{rotating}} \times \vec{B} + (\vec{\omega} \times \vec{A}) \times \vec{B} + \vec{A} \times \left(\frac{d\vec{B}}{dt}\right)_{\text{rotating}} + \vec{A} \times (\vec{\omega} \times \vec{B}) \\
&= \left[\left(\frac{d\vec{A}}{dt}\right)_{\text{rotating}} \times \vec{B} + \vec{A} \times \left(\frac{d\vec{B}}{dt}\right)_{\text{rotating}}\right] + \left[(\vec{\omega} \times \vec{A}) \times \vec{B} + \vec{A} \times (\vec{\omega} \times \vec{B})\right] \\
&= \left(\frac{d\vec{A}}{dt}\right)_{\text{rotating}} \times \vec{B} + \vec{A} \times \left(\frac{d\vec{B}}{dt}\right)_{\text{rotating}} + \vec{\omega} \times (\vec{A} \times \vec{B}) \qquad \text{(check appendix for proof)} \\
&= \left(\frac{\mathrm{d}}{\mathrm{dt}}\vec{A} \times \vec{B}\right)_{\text{rotating}} + \vec{\omega} \times (\vec{A} \times \vec{B}) \\
&= \left(\frac{\mathrm{d}}{\mathrm{dt}}\vec{C}\right)_{\text{rotating}} + \vec{\omega} \times \vec{C}
\end{aligned}$$

So vectors abide by the laws of rotation.

Problem 03

In a steady rotational frame, intuitively speaking rough - the position dependent force is *Centrifugal Force* and velocity dependent forces are *Coriolis Force*.

(a)

Consider no force of magnetic field now. Then all the fictitious forces where $\vec{r}$ is measured in rotating frame

$$\vec{F} = m\ddot{\vec{r}} = \vec{F}_{\text{outside}} - 2m\vec{\omega} \times \vec{v} - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Now including magnetic force

$$\vec{v}_{\text{lab}} = \vec{v}_{\text{rot}} + \vec{\omega} \times \vec{r}$$

$$\vec{F}_{\text{B}} = -q\vec{v}_{\text{lab}} \times \vec{B} = -q(\vec{v}_{\text{rot}} + \vec{\omega} \times \vec{r}) \times \vec{B}$$

$$\boxed{\vec{F} = m\ddot{\vec{r}} = -2m\vec{\omega} \times \vec{v}_{\text{rot}} - m\vec{\omega} \times (\vec{\omega} \times \vec{r}) - q(\vec{v}_{\text{rot}} + \vec{\omega} \times \vec{r}) \times \vec{B}}$$

(b)

Substituting $q = 2m\omega/B$ yields

$$\begin{aligned}
\vec{F} &= -2m\omega v\left[\hat{z}\times\hat{v}\right] - m\omega^2 r\left[\hat{z}\times(\hat{z}\times\hat{r})\right] - \left(\frac{2m\omega}{B}\right)\left[(vB)\hat{v}\times\hat{z} + (\omega r B)(\hat{z}\times\hat{r})\times\hat{z}\right] \\
&= -2m\omega v\left[\hat{z}\times\hat{v}\right] - m\omega^2 r\left[\hat{z}\times(\hat{z}\times\hat{r})\right] - 2m\omega v\left[\hat{v}\times\hat{z}\right] + 2m\omega^2 r\left[\hat{z}\times(\hat{z}\times\hat{r})\right] \\
&= m\omega^2 r\left[\hat{z}\times(\hat{z}\times\hat{r})\right] && (\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}) \\
&= -m\omega^2 r\left[(\hat{z}\cdot\hat{z})\hat{r}\right] \\
&= -m\omega^2\vec{r}
\end{aligned}$$

This leaves us with

$$\ddot{\vec{r}} + \omega^2\vec{r} = 0$$

This is a simple harmonic equation, Yippie! Please note that this equation holds in the rotational frame.

Details on shape: Equilibrium is established at $r = 0$ hence establishing the center of the turntable to be the center. Two component solution

$$\begin{aligned}
\ddot{x} + \omega^2 x = 0 &\implies x = x_0\sin(\omega t + \phi_x) \\
\ddot{y} + \omega^2 y = 0 &\implies y = y_0\sin(\omega t + \phi_y)
\end{aligned}$$

This is the very beautiful *Lissayous Curves!*

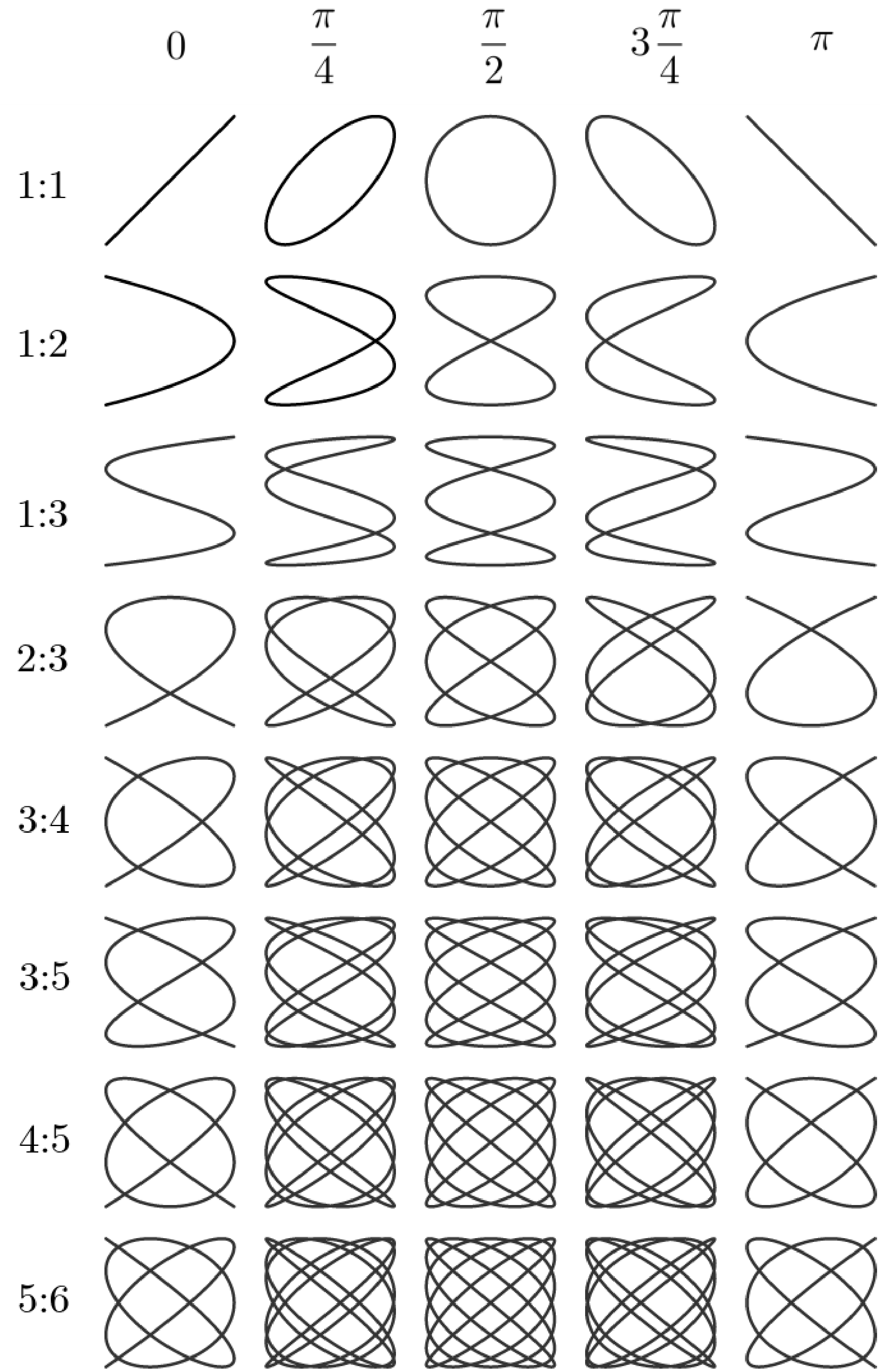

Figure 3: Ratio of $x_0 : y_0$ versus the phase difference $|\phi_x - \phi_y|$

(c)

For half as large q we end up with

$$\begin{aligned}
\vec{F} &= -2m\omega v\left[\hat{z}\times\hat{v}\right] - m\omega^2 r\left[\hat{z}\times(\hat{z}\times\hat{r})\right] - \left(\frac{m\omega}{B}\right)\left[(vB)\hat{v}\times\hat{z} + (\omega rB)(\hat{z}\times\hat{r})\times\hat{z}\right] \\
&= -2m\omega v\left[\hat{z}\times\hat{v}\right] - m\omega^2 r\left[\hat{z}\times(\hat{z}\times\hat{r})\right] - (m\omega v)\left[\hat{v}\times\hat{z}\right] - (m\omega^2 r)\left[(\hat{z}\times\hat{r})\times\hat{z}\right] \\
&= -2m\omega v\left[\hat{z}\times\hat{v}\right] - m\omega^2 r\left[\hat{z}\times(\hat{z}\times\hat{r})\right] + (m\omega v)\left[\hat{z}\times\hat{v}\right] + (m\omega^2 r)\left[\hat{z}\times(\hat{z}\times\hat{r})\right] \\
&= -2m\omega v\left[\hat{z}\times\hat{v}\right] + (m\omega v)\left[\hat{z}\times\hat{v}\right] \\
&= -m\omega v\left[\hat{z}\times\hat{v}\right] \\
&= -m\vec{\omega}\times\vec{v}
\end{aligned}$$

$$\left.\frac{\mathrm{d}\vec{v}}{\mathrm{d}t}\right|_{\text{rot}} = -\vec{\omega}\times\vec{v} = \left(\vec{\Omega}\right)\times\vec{v}$$

So technically in the rotating frame we see the particle itself rotating in $\vec{\Omega} = -\vec{\omega}$

$$\boxed{r_0 = \frac{v_0}{\omega}}$$

Problem 04

Total force on any particle

$$\begin{aligned}
\vec{F}/m &= -2\vec{\omega}\times\vec{v} + \vec{g} \\
&= -2(\omega(-\hat{y})\times v_x\vec{x} + \omega(-\hat{y})\times v_z\hat{z}) - g\hat{z} \\
&= 2(\omega\hat{y}\times v_x\hat{x} + \omega\hat{y}\times v_z\hat{z}) - g\hat{z} \\
&= 2(\omega v_x(-\hat{z}) + \omega v_z\hat{x}) - g\hat{z} \\
&= (-2\omega v_x - g)\,\hat{z} + \omega v_z\hat{x}
\end{aligned}$$

$$\ddot{x} = \omega\dot{z} \qquad \Longrightarrow \quad \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \omega\frac{\mathrm{d}z}{\mathrm{d}t}$$

$$\ddot{z} = -2\omega\dot{x} - g \qquad \Longrightarrow \quad \frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -2\omega\frac{\mathrm{d}x}{\mathrm{d}t} - g$$

Solve the first differential equation

$$\begin{aligned}
&\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \omega\frac{\mathrm{d}z}{\mathrm{d}t} \\
&\text{or, } \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) = \frac{\mathrm{d}}{\mathrm{d}t}(\omega z + c) \\
&\text{or, } \frac{\mathrm{d}x}{\mathrm{d}t} = \omega z + c \\
&\text{now, } \frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -2\omega\frac{\mathrm{d}x}{\mathrm{d}t} - g \\
&\text{or, } \frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -2\omega^2 z - g + d
\end{aligned}$$

So setting $z = A\sin\left(\sqrt{2}\omega t + \phi\right) + (K)$ gives us

$$\frac{\mathrm{d}^2z}{\mathrm{d}t^2} = -2\omega^2\left(A\sin\left(\sqrt{2}\omega t + \phi\right)\right) = -2\omega^2(z-K) = -2\omega^2 z + 2\omega^2 K \implies K = \frac{-g+d}{2\omega^2}$$

For first order considering $\omega^2 \sim 0$.

$$\ddot{x} = \omega\dot{z} \qquad\qquad \ddot{z} = -g$$

So the deviation

$$\begin{aligned}
\Delta x &= v_{0,\mathrm{x}} - \omega v_{0,\mathrm{z}}t^2 - \frac{1}{3}\omega g t^3 \\
\Delta z &= \Delta z_{\mathrm{cor}} + \Delta z_g = v_{0,\mathrm{z}}t - \omega v_{\mathrm{o,x}}t^2 - \frac{1}{2}gt^2 \\
\implies \text{ implying condition } \Delta z = 0 &\implies (v_{0,\mathrm{z}} - \omega v_{0,\mathrm{x}}t - \frac{1}{2}gt) = 0 \implies t = \frac{v_{0,\mathrm{z}}}{\omega v_{0,\mathrm{x}} + g/2} \\
\Delta x &= \frac{v_{0,\mathrm{x}}v_{0,\mathrm{z}}}{\omega v_{0,\,\mathrm{x}} + \frac{1}{2}g} + \frac{\omega v_{0,\mathrm{z}}^3}{(\omega v_{0,\,\mathrm{x}} + g/2)^2} - \frac{\omega g v_{0,\mathrm{z}}^2}{3(\omega v_{0,\mathrm{x}} + g/2)^3}
\end{aligned}$$

Numerically solving

$$\omega = \sqrt{g/r} = \frac{1}{30}\ \text{rad/s}$$

I put the whole thing for Δx in a calculator that gives numerically,

$$\Delta x = 47\,\mathrm{m}$$

(b)

There is no sideways ($\hat{y}$) directional deviation for b. The coriolis force only acts along the vertical and direction of ball's motion.

(c)

There is simply a flip of signs that yields

$$\Delta z = v_{0,\mathrm{z}}t + \omega v_{0,\,\mathrm{x}}t^2 - \frac{1}{2}gt^2 \implies t = v_{0,\mathrm{z}}\frac{1}{g/2 - \omega v_{0,\mathrm{x}}}$$

$$\Delta x = \frac{v_{0,\mathrm{x}}v_{0,\mathrm{z}}}{-\omega v_{0,\,\mathrm{x}} + \frac{1}{2}g} + \frac{\omega v_{0,\mathrm{z}}^3}{(-\omega v_{0,\,\mathrm{x}} + g/2)^2} - \frac{\omega g v_{0,\mathrm{z}}^2}{3(-\omega v_{0,\mathrm{x}} + g/2)^3} = \boxed{57.56\,\mathrm{m}}$$

(d)

Effect on height:

$$a_{\text{field}} = \omega^2 r = \text{ numerically } = 10 \text{ m/s}^2$$

Error for reaching higher altitude $h \sim 30$ m

$$\text{error} = \frac{\omega^2(r - r_0)}{\omega^2 r} = \text{ numerically } = \frac{1}{300}$$

Which is small.
Horizontal Position: The triangle formed by the horizontal deviation

$$z^2 = r_{\text{center}}^2 - r_{\text{end}}^2 \sim \text{ numerically } 8999.86 \text{ m}$$

The subtended angle

$$\cos\theta = \text{ numerically } \implies \theta = 0.32^\circ$$

Also negligible.

$(\vec{\omega}\times\vec{A})\cdot\vec{B} = -\vec{A}\cdot(\vec{\omega}\times\vec{B})$

$$\vec{\omega} = (\omega_x, \omega_y, \omega_z), \quad \vec{A} = (A_x, A_y, A_z), \quad \vec{B} = (B_x, B_y, B_z).$$

$$\vec{\omega}\times\vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ A_x & A_y & A_z \end{vmatrix}.$$

$$\vec{\omega}\times\vec{A} = \big(\omega_y A_z - \omega_z A_y\big)\hat{\imath} - \big(\omega_x A_z - \omega_z A_x\big)\hat{\jmath} + \big(\omega_x A_y - \omega_y A_x\big)\hat{k}.$$

$$(\vec{\omega}\times\vec{A})\cdot\vec{B} = (\omega_y A_z - \omega_z A_y)B_x + (-(\omega_x A_z - \omega_z A_x))B_y + (\omega_x A_y - \omega_y A_x)B_z.$$

$$= \omega_y A_z B_x - \omega_z A_y B_x - \omega_x A_z B_y + \omega_z A_x B_y + \omega_x A_y B_z - \omega_y A_x B_z.$$

$$\vec{\omega}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ B_x & B_y & B_z \end{vmatrix}.$$

$$\vec{\omega}\times\vec{B} = \big(\omega_y B_z - \omega_z B_y\big)\hat{\imath} - \big(\omega_x B_z - \omega_z B_x\big)\hat{\jmath} + \big(\omega_x B_y - \omega_y B_x\big)\hat{k}.$$

$$-\vec{A}\cdot(\vec{\omega}\times\vec{B}) = -\left(A_x(\omega_y B_z - \omega_z B_y) + A_y(-(\omega_x B_z - \omega_z B_x)) + A_z(\omega_x B_y - \omega_y B_x)\right).$$

$$= \omega_y A_z B_x - \omega_z A_y B_x - \omega_x A_z B_y + \omega_z A_x B_y + \omega_x A_y B_z - \omega_y A_x B_z.$$

Comparing the expanded expressions for

$$(\vec{\omega}\times\vec{A})\cdot\vec{B} \quad \text{and} \quad -\vec{A}\cdot(\vec{\omega}\times\vec{B}),$$

we see they are identical. Therefore

$$(\vec{\omega}\times\vec{A})\cdot\vec{B} = -\vec{A}\cdot(\vec{\omega}\times\vec{B}).$$

$(\vec{\omega}\times\vec{A})\times\vec{B} + \vec{A}\times(\vec{\omega}\times\vec{B})$

$$(\vec{\omega}\times\vec{A})\times\vec{B} = \big((\vec{\omega}\times\vec{A})\cdot\vec{B}\big)\vec{A} - \big((\vec{\omega}\times\vec{A})\cdot\vec{A}\big)\vec{B}.$$

$$\vec{A}\times(\vec{\omega}\times\vec{B}) = \big(\vec{A}\cdot\vec{B}\big)\vec{\omega} - \big(\vec{A}\cdot\vec{\omega}\big)\vec{B}.$$

$$(\vec{\omega}\times\vec{A})\times\vec{B} + \vec{A}\times(\vec{\omega}\times\vec{B}) = \vec{\omega}\times(\vec{A}\times\vec{B}).$$