Quantum Mechanics : : Homework 0X

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Problem 4

(a)

$$
\begin{aligned}
\sigma_B &= \vec{P} \cdot \vec{n} \\
&= (k\vec{r}) \cdot \vec{n} \\
&= kr
\end{aligned}
$$

At the surface the charge is $\sigma_B = kR$. Total charge at the surface,

$$Q_{\text{surface}} = 4\pi R^3$$

$$
\begin{aligned}
\rho_B &= -\nabla \cdot \vec{P} \\
&= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr) \\
&= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^3 k) \\
&= -\frac{k}{r^2}(3r^2) \\
&= -3k
\end{aligned}
$$

Total charge in the volume is

$$Q_{\text{volume}} = \frac{4}{3}\pi R^3(-3k) = -4\pi R^3$$

Answers: Surface charge density

$$\boxed{\sigma_B = kr}$$

Volume charge density

$$\boxed{\rho_B = -3k}$$

(b)

For the homogeneous charge distribution inside the surface, hence using Gauss's law we know that the Electric field is just going to be

$$E_r(4\pi r^2) = (-3k)(4\pi r^3/3\epsilon_0) \implies E_r = -\frac{k}{\epsilon_0} \qquad (r < R)$$

Outside, using Gauss's law

$$E_r(4\pi \mathrm{r}) = \frac{Q_{\text{surface}} + Q_{\text{volume}}}{\epsilon_0} = 0 \qquad (\mathrm{r} > R)$$

The field outside the ball is zero.

Answers: Electric Field

$$\boxed{E_r = \begin{cases} -k/\epsilon_0 & r < R \\ 0 & r > R \end{cases}}$$

Problem 5

The uniform polarization along $\vec{P} = P\hat{z}$. Bound charge

$$\rho_B = -\nabla \cdot \vec{P} = \frac{\mathrm{d}P}{\mathrm{d}z} = 0$$

Surface charge (on flat ends) because curved surface would be zero. Top face

$$\sigma_B^{\text{top}} = \vec{P} \cdot \hat{n} = P$$

Bottom face

$$\sigma_B^{\text{lower}} = \vec{P} \cdot \hat{n} = -P$$

Problem 6

Compute the total charge

$$Q = \int \mathrm{d}q_{\text{volume}} + \int \mathrm{d}q_{\text{surface}}$$

$$\begin{aligned} Q &= \int \mathrm{d}V(\rho_b) + \int \mathrm{d}A(\sigma_b) \\ &= -\int \mathrm{d}V(\nabla \cdot \vec{P}) + \int \mathrm{d}A(\vec{P} \cdot \vec{n}) \\ &= -\int \mathrm{d}\vec{S} \cdot \vec{P} + \int \mathrm{d}A(\vec{P} \cdot \vec{n}) \\ &= -\int \mathrm{d}A\vec{P} \cdot \hat{n} + \int \mathrm{d}A(\vec{P} \cdot \vec{n}) \\ &= 0 \end{aligned}$$