Honors Multivariable Calculus : : Homework 1x

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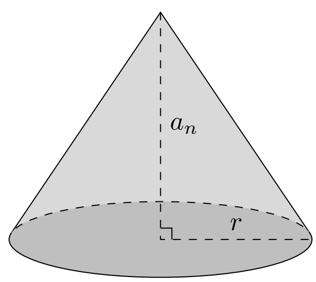
Problem 01

Aid for my brain: Let the compact region be $R \in \mathbb{R}^{n-1}$ where $n = 3$. Common sense tells us R is basically a disk in $\mathbb{R}^2$ or for now $x - y$ plane if we want what is in the figure. The tip of the cone is $\vec{a}$.
I think from the question our R doesn't really need to be necessarily a disk. Now the region is all the lines that join from $\vec{a}$ to R. If $\vec{x} \in R$ then considering a linear map $\gamma_{\vec{x}} : [0,1] \to \mathbb{R}^n$ such that $\gamma_{\vec{x}}(0) = \vec{a}$ and $\gamma_{\vec{x}}(1) = \vec{x} \in R$



The line segment is set of all points such that,

$$\Gamma = \{s \in \mathbb{R}^n : s = \vec{a}t + (1-t)\vec{x} \text{ where } t \in [0,1], \vec{x} \in R, \vec{a} \in \mathbb{R}^n\}$$

Here $x \in R$. Every point $\vec{p} \in \Gamma$ is a memeber of the cone.

The volume of the region Γ (which is the defined cone) is going to be,

$$\int_{\Gamma} 1 = \text{Volume}$$

Now the burden is to find a region Γ.