Heat Light and Waves : : Homework 03

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Problem 01

(a)

$$e_1 = 1 - \frac{T_1}{T_h}$$
$$e_2 = 1 - \frac{T_2}{T_1}$$
$$e_3 = 1 - \frac{T_c}{T_2}$$

Re-writing,

$$T_1 = T_h(1 - e_1)$$
$$T_2 = T_1(1 - e_2) = T_h(1 - e_1)(1 - e_2)$$
$$T_c = T_c(1 - e_3) = T_h(1 - e_1)(1 - e_2)(1 - e_3)$$
$$e_{\text{net}} = 1 - \frac{T_c}{T_h} = 1 - (1 - e_1)(1 - e_2)(1 - e_3)$$
$$\boxed{e_{\text{net}} = 1 - (1 - e_1)(1 - e_2)(1 - e_3)}$$

b

With the input of heat Q_0

$$W_1 = Q_0 - Q_0\frac{T_1}{T_h}$$
$$W_2 = Q_0\frac{T_1}{T_h} - Q_0\frac{T_2}{T_h}$$
$$W_3 = Q_0\frac{T_2}{T_h} - Q_0\frac{T_c}{T_h}$$

Now using

$$W_1 = W_2 = W_3$$
$$1 - \frac{T_1}{T_h} = \frac{T_1}{T_h} - \frac{T_2}{T_h} = \frac{T_2}{T_h} - \frac{T_c}{T_h}$$

I took pen and paper and computed the results for T_1 and T_2 with solving the linear equations (we could also use matrices)

$$\boxed{T_1 = \frac{2T_h + T_c}{3}} \quad \boxed{T_2 = \frac{T_h + 2T_c}{3}}$$

Problem 02

(a)

$$T_a = \frac{p_a V_a}{R} = \frac{2 \cdot 10^3}{R}$$
$$T_b = \frac{p_b V_b}{R} = \frac{2 \cdot 10^3}{R}$$

This process is Isothermal because of equal temperature.

(b)

- $a \to b$ is expelling heat. As we've seen the gas is compressing isothermally.
- $b \to c$ absorbs heat as it is expanding.
- $c \to a$ expelling heat since temperature drops.

(c)

- $T_a = \frac{2 \cdot 10^5 \times 0.01}{R} = 240.5\,K$
- $T_b = 240.5\,K$ [isothermal]
- $T_c = \frac{4 \cdot 10^5 \times 0.01}{R} = 481.1\,K$

(d)

- $a \to b$ then $Q = -nRT\ln(V_1/V_f) = -240.5R\ln 2 = -1385\,J$
- $b \to c$ then $Q = nC_p\Delta T = \frac{7}{2}R(240.5) = 6998\,J$
- $c \to a$ then $Q = nC_v\Delta T = \frac{5}{2}R(-240.5) = -4998\,J$

Computing Q_{net}

$$Q_{\text{net}} = 615\,J$$

(e)

$$W = -Q_{\text{net}} = -615\,J$$

(f)

$$e = 1 - \frac{Q_c}{Q_h} = 1 - \frac{6383}{6998} = 0.087$$

Problem 03

(a)

- $B \to C$ and $D \to A$ is adiabatic hence $Q = 0$
- $A \to B$ hence $Q = nC_v\Delta T = n\left(\frac{5R}{2}\right)(T_B - T_A) > 0$ which is Q_h.

(b)

- $C \to D$ hence $Q = nC_v\Delta T = n\frac{5R}{2}(T_D - T_C) < 0$ which is Q_C.

(c)

- $D \to A$ adiabatic so $P_D V_B^\gamma = P_A V_A^\gamma$
- $B \to C$ adiabatic so $P_C V_B^\gamma = P_B V_A^\gamma$

$$(P_D - P_C)V_B^\gamma = (P_A - P_B)V_A^\gamma \to P_D - P_C = (P_A - P_B)\left(\frac{V_A}{V_B}\right)^\gamma$$

For the idea gas we have

$$P_B V_A = nRT_B,\ P_A V_A = nRT_A$$

$$P_C V_B = nRT_C,\ P_D V_B = nRT_D$$

We can do a rewrite

$$|T_D - T_C| = \left|\frac{1}{nR}(P_D - P_C)V_B\right|$$

$$|T_B - T_A| = \left|\frac{1}{nR}(P_B - P_A)V_A\right|$$

$$e = 1 - \frac{|Q_C|}{|Q_h|} = 1 - \frac{|\ln\frac{5}{2}R(T_D - T_C)|}{|\ln\frac{5}{2}R(T_B - T_A)|} = 1 - \frac{|T_D - T_C|}{|T_B - T_A|}$$

$$e = 1 - \frac{|(1/nR)(P_D - P_C)V_B|}{|(1/nR)(P_B - P_A)V_A|} = 1 - \frac{|(V_A/V_B)^\gamma V_B|}{|V_A|}$$

$$e = 1 - \frac{|(V_A/V_B)^\gamma\, V_B|}{|V_A|} = 1 - \left|\left(\frac{V_A}{V_B}\right)^\gamma \frac{V_B}{V_A}\right|$$

Now

$$c_v = \frac{5}{2}R \to c_p = c_v + R = \frac{7}{2}R$$

$$\gamma = \frac{c_p}{c_v} = \frac{7}{5}$$

$$e = 1 - \left|\left(\frac{V_A}{V_B}\right)^{\frac{7}{5}} \frac{V_B}{V_A}\right| = 1 - \left(\frac{V_A}{V_B}\right)^{\frac{2}{5}}$$

Hence,

$$\boxed{e = 1 - \left(\frac{V_A}{V_B}\right)^{2/5}}$$

Problem 04

(a)

$$\Delta S_{\text{tea}} = \int_i^f \frac{\text{d}Q}{T} = \int_{T_i}^{T_f} \frac{mc\text{d}T}{T} = mc\ln\left(\frac{T_f}{T_i}\right) = -229.18\,\frac{J}{K}$$

(b)

$$\Delta S_{\text{air}} = \frac{\Delta Q_{\text{air}}}{T} = \frac{-mc\Delta T_{\text{tea}}}{T_{\text{air}}} = 256.23\,\frac{J}{K}$$

(c)

$$\Delta S_{\text{net}} = -229.18 + 256.23 = 27.05\,\frac{J}{K}$$

Problem 05

(a)

$$\Delta S_{\text{ice}} = \frac{mc_i}{T} = 302.19\,\frac{J}{K}$$
$$\Delta S_{\text{water}} = mc\ln\left(\frac{T_f}{T_i}\right) + \Delta S_{\text{ice}} = 393.84\,\frac{J}{K}$$

(b)

$$\Delta S_{\text{air}} = \frac{-\Delta Q_{\text{ice}}}{T_{\text{air}}} = 364.59\,\frac{J}{K}$$

(c)

$$\Delta S_{\text{net}} = 393.84 - 364.59 = 29.25\,\frac{J}{K}$$

Problem 06

(a)

The forces are

$$F_1 = -k_1 x$$
$$F_2 = -k_2 x$$
$$F_{\text{net}} = -k_1 x - k_2 x = -(k_1 + k_2)x$$

$$k_{\text{eff}} = k_1 + k_2$$

(b)

The forces are

$$F_1 = -k_1 x$$
$$F_2 = -k_2 x$$
$$F_{\text{net}} = -k_1 x - k_2 x = -\left(k_1 + k_2\right) x$$
$$k_{\text{eff}} = k_1 + k_2$$

(c)

$$F_1 = -k_1 x_1$$
$$F_2 = -k_2 x_2$$
$$x_1 = -\frac{F_1}{k_1}$$
$$x_2 = -\frac{F_2}{k_2}$$
$$F_1 = F_2 = F_{\text{net}}$$
$$x_1 + x_2 = -\left(\frac{F}{k_1} + \frac{F}{k_2}\right)$$
$$F_{\text{net}} = -k_{\text{eff}} x = -k_{\text{eff}}(x_1 + x_2) = k_{\text{eff}}\left(\frac{F}{k_1} + \frac{F}{k_2}\right)$$
$$F = F k_{\text{eff}}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$
$$k_{\text{eff}} = \frac{1}{\left(\frac{1}{k_1} + \frac{1}{k_2}\right)} = \frac{1}{\frac{k_1 + k_2}{k_1 k_2}} = \frac{k_2 k_1}{k_1 + k_2}$$

(d)

Treat original spring as chain of $10x$ new spring. From part (c),

$$\frac{1}{k_{\text{old}}} = \frac{10}{k_{\text{new}}} \to k_{\text{new}} = 10 k_{\text{old}}$$
$$T_{\text{new}} = 2\pi\sqrt{\frac{m}{10 k_{\text{old}}}}$$
$$T_{\text{old}} = 2\pi\sqrt{\frac{m}{k_{\text{old}}}}$$
$$\frac{T_{\text{new}}}{T_{\text{old}}} = \frac{2\pi\sqrt{\frac{m}{10 k_{\text{old}}}}}{2\pi\sqrt{\frac{m}{k_{\text{old}}}}} = \frac{1}{\sqrt{10}}$$