general solution to time independent schrodinger equation

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$$

is non trivial if $\hat{H}$ depend on time but trivial if independent of time

$$\hat{H}(t) = |\gamma| \hat{\vec{S}}\, \vec{B}(t) =$$

$$\left(\hat{S}_x B_x(t) + \hat{S}_y B_y(t) + \hat{S}_z B_z(t)\right)|\gamma|$$

solving for magnetic field,

$$\vec{B}(t) = \begin{cases} B_y \vec{n}_y & 0 \le t \le t_0 \\ B_z \vec{n}_z & t_0 < t < \infty \end{cases}$$

so,

$$\vec{H}(t) = \begin{cases} |\gamma| \hat{S}_y B_y & \text{before } t_0 \\ \hat{S}_z B_z |\gamma| & \text{after} \end{cases}$$

$\hat{H}_1(t)$ and then it becomes $\hat{H}_2(t)$

writing small like this is quite easy.

The initial state is $t=0$

$|\psi_0\rangle = |m=1\rangle$ inter connected

example $\hat{S}_z = \pm\hbar = \hbar$, we are required to find the final state for spin $t > t_0$.

(a) evolution,

$$|\psi_1\rangle = e^{-\frac{i}{\hbar}\hat{H}_1 t}|\psi_0\rangle$$

after this

$$|\psi_2\rangle = e^{-\frac{i}{\hbar}\hat{H}_2(t-t_0)}\, e^{-\frac{i}{\hbar}\hat{H}_1 t_0}|\psi_0\rangle$$

is there a cleaner way to write this?

$$|\psi_1\rangle = \exp\left(-\frac{i}{\hbar}\hat{H}_1 t\right)|\psi_0\rangle \quad t < t_0$$

$$|\psi_2\rangle = \exp\left(-\frac{i}{\hbar}\hat{H}_2(t-t_0)\right)\exp\left(-\frac{i}{\hbar}\hat{H}_1(t_0)\right)|\psi_0\rangle \quad t > t_0$$

and then,

$$|\psi_2\rangle = \exp\left(-\frac{i}{\hbar}|\gamma|\hat{S}_z B_z\left(t - \frac{\pi}{2|\gamma| B_y}\right)\right)\exp\left(-\frac{i}{\hbar}\hat{S}_y B_y \frac{|\gamma|\pi}{2|\gamma| B_y}\right)|\psi_0\rangle$$

$$= \exp\left(-\frac{i}{\hbar}|\gamma|\hat{S}_z B_z\left(t - \frac{\pi}{2|\gamma| B_y}\right)\right)\exp\left(-\frac{i}{\hbar}\frac{\hat{S}_y}{2}\pi\right)|\psi_0\rangle$$

$$\sim \exp\left(-\frac{i}{\hbar}\hat{S}_y\left(\frac{\pi}{2}\right)\right)|\psi_0\rangle$$

$$\sim \hat{R}_y(\pi/2)|\psi_0\rangle$$

and now, (b), probability of getting $S_z = \hbar$ means $\langle\psi_2|S_z|\psi_s\rangle^2 = \hbar$?

the state with $S_z = \hbar$ is $|\psi_0\rangle$ and hence,

$$\langle\psi_0|\psi_2\rangle^2 = \text{probability}$$

what we get is, that thing is pretty long lol. I don't quite know what to really do here lol.

$$\hat{J}_z = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \hat{J}_z|m\rangle = m|m\rangle$$

$$\det(\hat{J}_z - mI) = 0$$

we get $m = -1, 0, 1$

$m=0$ means, state with 0 eigenvalue

$$e^{-i\theta\hat{J}_z}|m=0\rangle = e^0|m=0\rangle = |m=0\rangle$$

eigenvector

$m = \pm 1$,

$$e^{-i\theta\hat{J}_z}|m=\pm1\rangle = \underbrace{e^{\pm i\theta}}_{\text{basically a complex phase}}|m=\pm1\rangle$$

what is a complex phase for state?

now, $|n_z\rangle = \begin{bmatrix}0\\0\\1\end{bmatrix}$ state (our basis)

$|n_x\rangle = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $|n_y\rangle = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$|n_z\rangle, |n_x\rangle, |n_y\rangle = \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$

eigenstate $|m=0\rangle = \begin{bmatrix}0\\0\\1\end{bmatrix}$

eigenstate $|m=+1\rangle = \begin{bmatrix}1\\i\\0\end{bmatrix}$ $\quad |0\rangle, |+1\rangle, |-1\rangle = \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\i\\0\end{bmatrix}, \begin{bmatrix}1\\-i\\0\end{bmatrix}$

eigenstate $|m=-1\rangle = \begin{bmatrix}1\\-i\\0\end{bmatrix}$

$\hbar$

? -1 0 ?

$$R_y(0) \to \begin{bmatrix} \frac{1+\cos(\pi/2)}{2} \\ \sin(\pi/2) \\ \frac{1-\cos(\pi/2)}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{bmatrix}$$

problems 1

1 The basis vectors are $|\uparrow\rangle_z, |\downarrow\rangle_z$

ein $\hat{\sigma}^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\hat{\sigma}^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\hat{\sigma}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

find the eigenvector $\hat{S}_x$ and $\hat{S}_y$. Express the eigenvectors

$|\uparrow\rangle_x, |\downarrow\rangle_x \quad |\uparrow\rangle_y \quad |\downarrow\rangle_y$

$(\hat{S}_x - \hat{I}m)|\uparrow\rangle_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \lambda\hat{I} = \cancel{\frac{\hbar}{2}\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix}} \rightarrow$

$\begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix} - \lambda I = \begin{bmatrix} -\lambda & a \\ a & -\lambda \end{bmatrix} \rightarrow \lambda^2 - a^2 = 0$

$(\lambda + a)(\lambda - a) = 0$

$\therefore \lambda = a, -a$

$= \frac{\hbar}{2}, -\frac{\hbar}{2}$

eigenvalue $\frac{\hbar}{2}, -\frac{\hbar}{2}$ now from there

$\frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \frac{\hbar}{2}(1)\hat{I} \rightarrow \frac{\hbar}{2}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} -x_1 + x_2 \\ x_1 - x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad x_1 = x_2 = 1$

$\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ✓ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

✓ $\begin{bmatrix} 1 \\ -1 \end{bmatrix} = (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $x_1 = -x_2$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -1$

$$\boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

Smei $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -\lambda & -i \\ i & -\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \cancel{\begin{bmatrix} -\lambda x_1 - i x_2 \\ i x_1 - \lambda x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}} \rightarrow$

$\lambda^2 - [-i \cdot i] = \lambda^2 - [-i^2] = \lambda^2 - [1] = (\lambda+1)(\lambda-1) \quad \lambda = 1, -1$

$\begin{bmatrix} -1 & -i \\ i & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 - x_2 i \\ x_1 i - x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{matrix} x_1 = -x_2 i \\ x_1 i = x_2 \end{matrix} \quad \begin{matrix} x_1 = 1 \\ x_2 = i \end{matrix} \quad \boxed{\begin{bmatrix} 1 \\ i \end{bmatrix}}$

$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{matrix} 0 + 1 \\ i + 0 \end{matrix} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ ✓

$$\begin{bmatrix} 1 & -i \\ i & +1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 i \\ x_1 i + x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{matrix} x_1 = x_2 i \\ x_1 i = -x_2 \end{matrix} \quad \begin{matrix} x_1 = i \\ x_2 = 1 \end{matrix}$$

$$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} i \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ -1 \end{bmatrix} = -1\begin{bmatrix} i \\ 1 \end{bmatrix}$$

$$\boxed{\underset{1}{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}}, \underset{-1}{\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix}}}$$

(b) $|\psi\rangle = \alpha|\uparrow\rangle_z + \beta|\downarrow\rangle_z = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$\Delta S_x^2 = \langle\psi|\hat{S}_x\hat{S}_x|\psi\rangle + \left(\langle\psi|\hat{S}_x|\psi\rangle\right)^2$

compute $\langle\psi|\hat{S}_x|\psi\rangle = \sum\langle\psi|\hat{S}_x|\lambda_i\rangle\langle\lambda_i|\psi\rangle$

$= \langle\psi|\hat{S}_x|\uparrow\rangle_z\alpha + \langle\psi|\hat{S}_x|\downarrow\rangle_z\beta$

$\uparrow\uparrow \quad \uparrow\downarrow$

$\downarrow\uparrow \quad \downarrow\downarrow$

$= {}_z\langle\uparrow|\hat{S}_x|\uparrow\rangle_z\alpha\alpha^* + {}_z\langle\downarrow|\hat{S}_y|\uparrow\rangle_z\alpha\beta^* +$

${}_z\langle\uparrow|\hat{S}_x|\downarrow\rangle_z\beta\alpha^* + {}_z\langle\downarrow|\hat{S}_x|\downarrow\rangle\beta\beta^*$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad = (\alpha\beta^* + \alpha^*\beta)^2$

$\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \qquad \hat{S}_x\hat{S}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 0+1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \hat{I}$

$\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \langle\psi|\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}|\psi\rangle = \alpha\alpha^* + \beta\beta^*$

$\begin{bmatrix} 1, 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \Rightarrow \sqrt{\alpha\alpha^* + \beta\beta^* - (\alpha\beta^* + \alpha^*\beta)^2}$

ψ eigen state $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\cancel{\sqrt{2}}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \sqrt{1 + 1 - (1+1)^2}$

$\sqrt{2 - 4} =$

$\langle \hat{S}_x^2 \rangle - \langle S_x \rangle^2$

$2 - 4$

$1 - \left(\frac{1}{2} + \frac{1}{2}\right)^1 = 0$ ✓

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. $|\psi\rangle = \alpha|\uparrow\rangle_z + \beta|\downarrow\rangle_z$

$(\vec{n})_x = \langle\psi|\sigma'|\psi\rangle$

$= \langle\psi|\lambda_i\rangle\langle\lambda_i|\sigma'|\lambda_j\rangle\langle\lambda_j|\psi\rangle$

$= \langle\psi|\uparrow\rangle\langle\uparrow|\sigma|\uparrow\rangle\langle\uparrow|\psi\rangle +$
$\langle\psi|\uparrow\rangle\langle\uparrow|\sigma|\downarrow\rangle\langle\downarrow|\psi\rangle +$
$\langle\psi|\downarrow\rangle\langle\downarrow|\sigma|\uparrow\rangle\langle\uparrow|\psi\rangle +$
$\langle\psi|\downarrow\rangle\langle\downarrow|\sigma|\downarrow\rangle\langle\downarrow|\psi\rangle$

$= \alpha^*\alpha\,(\sigma'_{11})$
$+ \alpha^*\beta\,(\sigma'_{12})$
$+ \beta^*\alpha\,(\sigma'_{21})$
$+ \beta^*\beta\,(\sigma'_{22})$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$\sigma_1 \rightarrow (\vec{n}_x) = \alpha^*\beta + \beta^*\alpha = \cancel{(\alpha^*\beta)^2 + (\beta^*\alpha)^2} + 2(\alpha^*\alpha\,\beta\beta^*)$

$(\vec{n}_y) = -i\alpha^*\beta + i\,\beta^*\alpha = i^2\left[-\alpha^*\beta + \beta^*\alpha\right]^2 = (-1)\cancel{[(\alpha\beta^*)^2 + (\alpha^*\beta)^2}$
$- 2(\alpha^*\alpha\beta\beta^*)$

$(\vec{n}_z) = \alpha^*\alpha - \beta^*\beta = (\alpha^*\alpha)^2 + (\beta\beta^*)^2 - 2(\alpha^*\alpha \times \beta^*\beta)$

$\alpha^2 + \beta^2 = 1$

$= 2(\alpha\alpha^*\beta\beta^*) = \cancel{2\left(\frac{1}{2}\cdot\frac{1}{2}\right)} =$

$\alpha^2 - 2\alpha\beta + \beta^2 =$

$\alpha^2 + \beta^2 + 2(\alpha^*\alpha\,\beta^*\beta) = 1$ $\langle\psi|\psi\rangle$ ohie

$\vec{n}\cdot\hat{\vec{S}}|\psi\rangle = \left[(\hat{n}_x)\hat{x} + (\hat{n}_y)\hat{y} + (\hat{n}_z)\hat{z}\right]\{S\,\hat{x} + S$

$\vec{n}\cdot\hat{\vec{S}} = (n_x)S_1 + (n_2)S_2 + (n_3)S_3$

$= \frac{\hbar}{2}n_x\sigma_1 + \frac{\hbar 2}{2}n_x\sigma_2$

$\langle\psi|\vec{n}\cdot\vec{S}|\psi\rangle = \langle\psi|$

$\hat{\vec{S}} = \hat{S}_x \hat{x} + \cdots$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\langle\psi| \frac{\hbar}{2}(\vec{n}_x)\hat{\sigma}'|\psi\rangle$

$\hat{\vec{n}} = (\vec{n})_x \hat{x} + \cdots$ $\frac{\hbar}{2}(\vec{n}_x)\langle\psi|\hat{\sigma}'|\psi\rangle$

$(\vec{n})_x \hat{S}_x + \cdots |\psi\rangle$ $\frac{\hbar}{2}(\vec{n}_x)(\vec{n}_x) + \cdots$

$\frac{\hbar}{2}(\vec{n})_x \hat{\sigma}' + \cdots |\psi\rangle$ $\frac{\hbar}{2} \cdots (1)$

$\frac{\hbar}{2}(\vec{n}_x)\hat{\sigma}'|\psi\rangle + \cdots$

$\frac{\hbar}{2}(\vec{n}_x)\begin{bmatrix} \quad \end{bmatrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$

$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} -\beta \\ \alpha \end{bmatrix} i$

$\frac{\hbar}{2}\left(\begin{bmatrix} \alpha' & \beta' \end{bmatrix}\begin{bmatrix} \beta \\ \alpha \end{bmatrix}\right)\begin{bmatrix} \beta \\ \alpha \end{bmatrix}$

$\frac{\hbar}{2}\left(\begin{bmatrix} \alpha' & \beta' \end{bmatrix}\begin{bmatrix} -\beta \\ \alpha \end{bmatrix}\right)\begin{bmatrix} -\beta \\ \alpha \end{bmatrix}$ $\begin{bmatrix} n_x & n_y & n_z \end{bmatrix}\begin{bmatrix} \hat{S}_x \\ \hat{S}_y \\ \hat{S}_z \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} \boxed{\times}$

$\frac{\hbar}{2}\left(\begin{bmatrix} \alpha' & \beta' \end{bmatrix}\begin{bmatrix} \alpha \\ -\beta \end{bmatrix}\right)\begin{bmatrix} \alpha \\ -\beta \end{bmatrix}$

$\cancel{(\alpha^*\beta + \beta^*\alpha)\beta +}$

$\alpha^* \beta^* \quad (\alpha^*\beta + \beta^*\alpha)\beta + (-\alpha^*\beta + \beta^*\alpha)(-\beta) + (\alpha^*\alpha - \beta^*\beta^*)\alpha$

$(\alpha^*\beta + \beta^*\alpha)\alpha + (-\alpha^*\beta + \beta^*\alpha)(\alpha) + (\alpha^*\alpha - \beta^*\beta^*)(-\beta)$

$\sigma|\psi\rangle\langle\psi|T\rangle = \sigma|T\rangle$? $\alpha^*\beta\beta + \alpha\beta\beta^* + \alpha^*\beta\beta - \alpha\beta\beta^* + \alpha^*\alpha\alpha - \alpha\beta^*\beta^*$

$\cancel{\alpha^*\alpha\beta} + \alpha\alpha\beta^* - \alpha\alpha^*\beta + \alpha\alpha\beta^* - \cancel{\alpha^*\alpha\beta} + \beta^*\beta^*\beta$

$\alpha\alpha\beta^* - \alpha\alpha^*\beta + \beta^*\beta^*\beta$

$\sigma|T\rangle$

$\langle\psi|\Pi\rangle = \langle\lambda|\Pi\rangle \Rightarrow |\psi\rangle = |\lambda\rangle$ $= \sigma I|T\rangle$

$\langle\psi|\Pi\rangle - \langle\lambda|\Pi\rangle = 0$ $= \sigma \sum |P_i\rangle\langle P_i|T\rangle$

or, $(\langle\psi| - \langle\lambda|)|\Pi\rangle = 0$

$|\Pi\rangle \neq 0$ then $\langle\psi| - \langle\lambda| = 0$

3. products of rotations

$[\hat{\sigma}^a, \hat{\sigma}^b] = 2\delta^{a,b}\hat{I}$ (Clifford Algebra) $\quad [\hat{\sigma}^a, \hat{\sigma}^b] = 2i\varepsilon^{abc}\hat{\sigma}^c$

$\hat{\sigma}^a\hat{\sigma}^b = \delta^{a,b}\hat{I} + i\varepsilon^{abc}\hat{\sigma}^c$ $\quad$ {PHYS 311}

(a) $\hat{U} = e^{S_y\theta_3} \; S_z\theta_2 \; S_y\theta_1$

$$e^{-i\frac{\hat{S}_y}{\hbar}\theta_1} = e^{-i\frac{\hat{\sigma}^2}{2}\theta_1} = \cos\left(\frac{\theta_1}{2}\right)\hat{I} - i\hat{\sigma}^2\sin\left(\frac{\theta_1}{2}\right)$$

$$\hat{U} = \cos\left(\frac{|\vec{\phi}|}{2}\right)\hat{I} - i\vec{n}_\phi\cdot\hat{\vec{\sigma}}\sin\left(\frac{|\vec{\phi}|}{2}\right)$$

$c^{x+y} = cxcy - sxsy$

$cxcy = c^{x-y} = cxcy + s$

$$e^{-i\frac{S_y}{\hbar}\theta_1} =$$

$$\left[\cos\left(\frac{\theta_3}{2}\right)\hat{I} - i\hat{\sigma}^2\sin\left(\frac{\theta_3}{2}\right)\right]\left[\cos\left(\frac{\theta_2}{2}\right)\hat{I} - i\hat{\sigma}^3\sin\left(\frac{\theta_2}{2}\right)\right]$$
$$\left[\cos\left(\frac{\theta_1}{2}\right)\hat{I} - i\hat{\sigma}^2\sin\left(\frac{\theta_1}{2}\right)\right]$$

$$\cos\frac{\theta_3}{2}\cos\frac{\theta_2}{2}\hat{I}\cdot\hat{I} - \left[i\sin\left(\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_3}{2}\right)\right]\hat{I}\cdot\hat{\sigma}^3$$
$$- \left[i\sin\left(\frac{\theta_3}{2}\right)\cos\left(\frac{\theta_2}{2}\right)\right]\hat{\sigma}^2\cdot\hat{I}$$
$$+ i^2\hat{\sigma}^2\hat{\sigma}^3\sin\left(\frac{\theta_3}{2}\right)\sin\left(\frac{\theta_2}{2}\right)$$

$\sigma^2\sigma^3 =$

2 3 1

$\cancel{\cos\left(\frac{\theta_3}{2}\right)\cos}$

$$\cos\frac{\theta_3}{2}\cos\frac{\theta_2}{2}\hat{I} - i\left[\hat{\sigma}^3\sin\left(\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_3}{2}\right) + \hat{\sigma}^2\sin\left(\frac{\theta_3}{2}\right)\cos\left(\frac{\theta_2}{2}\right)\right]$$
$$+ (-i)\left[\hat{\sigma}_1\sin\left(\frac{\theta_3}{2}\right)\sin\left(\frac{\theta_2}{2}\right)\right]$$

$$\cos\frac{\theta_3}{2}\cos\frac{\theta_2}{2}\hat{I} - i\left[\hat{\sigma}_1\sin\frac{\theta_3}{2}\sin\frac{\theta_2}{2} + \hat{\sigma}_2\frac{\sin\theta_3}{2}\cos\frac{\theta_2}{2} + \hat{\sigma}^3\sin\frac{\theta_2}{2}\cos\frac{\theta_3}{2}\right.$$

Scanned with CamScanner

$$\cos\frac{\theta_2}{2}\cos\frac{\theta_3}{3}\hat{I} - i\left[\hat{\sigma}_1 \sin\frac{\theta_2}{2}\sin\frac{\theta_3}{2} + \hat{\sigma}_2\cos\frac{\theta_2}{2}\frac{\sin\theta_3}{2} + \hat{\sigma}^3\sin\frac{\theta_2}{2}\cos\frac{\theta_3}{2}\right]$$

$$\left[\cos\frac{\theta_1}{2}\hat{I} - i\hat{\sigma}^2\sin\frac{\theta_1}{2}\right]$$

$$\cos\frac{\theta_1}{2}\cos\frac{\theta_3}{3}\cos\frac{\theta_2}{2}\hat{I} - i\left[\hat{\sigma}_1\sin\frac{\theta_2}{2}\sin\frac{\theta_3}{2}\cos\frac{\theta_1}{2} + \hat{\sigma}_2\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\sin\frac{\theta_3}{2} + \hat{\sigma}_3\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\cos\frac{\theta_3}{3}\right] +$$

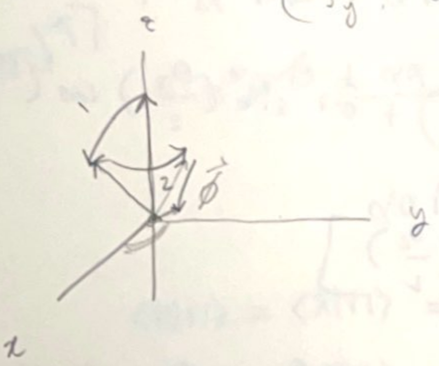
$$(-i\sigma^2)\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\cos\frac{\theta_3}{2}$$

$$\cos\frac{\theta_2}{2}\cos\frac{\theta_3}{3} = \frac{1}{2}\left(\cos\left(\frac{\theta_2}{2}+\frac{\theta_3}{3}\right) + \cos\left(\frac{\theta_2}{2}-\frac{\theta_3}{2}\right)\right)$$

$$e^{-i/\hbar(\hat{S}_y\theta_3 + \hat{S}_z\theta_2 + \hat{S}_y\theta_1)}$$

commutator does not hold for rotation

$$(\hat{S}_y, \hat{S}_z, \hat{S}_y)\begin{pmatrix}\theta_1\\ \theta_2\\ \theta_3\end{pmatrix} \;?$$



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?

$$\vec{\phi} = \theta_1 \hat{S}_y n_y$$

$$|\vec{\phi}| = \phi_1$$

$$\hat{U} = \cos\left(\frac{|\vec{\phi}|}{2}\right)\hat{I} - i\,\vec{n}_\phi\cdot\vec{\sigma}\,\sin\left(\frac{|\vec{\phi}|}{2}\right)$$

$$e^{-i\frac{S_y}{\hbar}\theta_1} = e^{-i\frac{\sigma^2}{2}\theta_1} = e^{-i\sigma^2\left(\frac{\theta_1}{2}\right)}$$

$$= \cos\frac{\theta_1}{2}\hat{I} - i\sin\left(\frac{\theta_1}{2}\right)\underline{\hat{\sigma}^2}$$

$$|\vec{\phi}| = \theta_1 = \sqrt{\phi_1^2+\phi_2^2+\phi_3^2}$$

$$\vec{n}_\phi\cdot\hat{\vec{\sigma}} = n_x\sigma^1 + \underbrace{n_2\sigma^2}_{\sigma^2} + \sigma^3 n_3$$

$$\vec{n}_\phi = \frac{\vec{\phi}}{|\vec{\phi}|} = \frac{a\hat{x} + b\hat{y} + c\hat{z}}{|\vec{\phi}|}$$

$$n_2 = 1 = \frac{\theta}{\theta}\sigma^2$$

$$\hat{U} = \cos\frac{\theta_2}{2}\cos\frac{\theta_3}{2}I - i\left[\hat{\sigma}_1\frac{\sin\theta_2}{2}\sin\frac{\theta_3}{2} + \hat{\sigma}_2\cos\frac{\theta_2}{2}\frac{\sin\theta_3}{2} + \hat{\sigma}_3\frac{\sin\theta_2}{2}\cos\frac{\theta_3}{2}\right]$$

$\hat{U} =$

$$e^{-i[\,]} = \sum \frac{-i\sigma\theta}{2} \qquad e^x = \sum\frac{x^n}{n!}$$

$$= \sum \frac{i\theta/2}{n!}\sigma^{n!}$$

$$\cos\left(\frac{|\vec{\phi}|}{2}\right) = \cos\left(\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_3}{2}\right)$$

$$\cos\left(\frac{a+b}{2}\right) = c(A)c(B) - \cancel{\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)}$$

has to zero

$$\sum \frac{\left(\frac{-i\theta_1}{2}\right)^n}{n!}\sigma^n$$

$$\sum_j\sum_n \frac{\left(\frac{-i\theta_1}{2}\right)^n\left(\frac{-i\theta_2}{2}\right)^j}{n!\,j!}\sigma_y^n\sigma_z^j$$

$$c\frac{\theta_2}{2}c\frac{\theta_3}{2}I - i\sigma_1\frac{\sin\theta_2}{2}\sin\frac{\theta_3}{2}$$

$$c\frac{\theta_2}{2}c\frac{\theta_3}{2}I - s\frac{\theta_2}{2}s\frac{\theta_3}{2} + s\frac{\theta_2}{2}s\frac{\theta_3}{2} - i$$

$$(\sigma_1\sigma_1 - i\sigma_1)\,s\frac{\theta_2}{2}s\frac{\theta_3}{2}$$

$$\sigma_1\sigma_1 - \sigma_2\sigma_3$$

1 2 3 1 2 3

$\sigma^1\sigma^1 = \delta I$

$\sigma^1\sigma^2 = i\sigma^3 \qquad \sigma^3\sigma^2 = -i\sigma^1$

$\sigma^2\sigma^3 = i\sigma^1 \qquad \sigma^2\sigma^1 = -i\sigma^3$

$\sigma^3\sigma^1 = i\sigma^2 \qquad \sigma^1\sigma^3 = -i\sigma^2$

$c\frac{\theta_2}{2} c\frac{\theta_3}{3}\hat{I} - \left[i\sigma_1 \sin\frac{\theta_2}{2}\sin\frac{\theta_3}{2} + i\hat{\sigma}_2 \cos\frac{\theta_2}{2}\sin\frac{\theta_3}{2} + i\hat{\sigma}_3 \sin\frac{\theta_2}{2}\cos\frac{\theta_3}{2}\right.$

$cx_2\, cx_3\left(\frac{\sigma^1\sigma^1 + \sigma^2\sigma^2 + \sigma^3\sigma^3}{2}\right)$

$e^{-i\frac{\sigma^2}{2}\theta_1}\, e^{i\frac{\sigma^2}{2}\theta_1} = \cos\left(\frac{\theta_1}{2}\right)$

$\underbrace{c\frac{\theta_2}{2} c\frac{\theta_3}{3}\hat{I} - s\frac{\theta_2}{2}s\frac{\theta_3}{2}I} + \underbrace{s\frac{\theta_2}{2}s\frac{\theta_3}{2}I} - i\sigma_1 \sin\frac{\theta_2}{2}\sin\frac{\theta_3}{2}$

$s\frac{\theta_2}{2}s\frac{\theta_3}{2}\sigma_1\sigma_1 - i\sigma_1 \sin \sin$

$\sigma_1\sigma_1 + \sigma_3\sigma_2$

$\sigma_1\sigma_1 - \sigma_2\sigma_3$

$\cos\left(\frac{\theta_2}{2} + \frac{\theta_3}{2}\right)\hat{I} + (\sigma_1\sigma_1 - \sigma_2\sigma_3)\sin\frac{\theta_2}{2}\sin\frac{\theta_3}{2}$

$-\sigma^3\sigma^1 + \cancel{\sigma^1\sigma^3}\cos\frac{\theta_2}{2}\sin\frac{\theta_3}{2}$

$-\sigma^1\sigma^2 + \cancel{\sigma^2\sigma^1}\sin\frac{\theta_2}{2}\cos\frac{\theta_3}{2}$

$\sigma^3\sigma^1$

$$\left(\cos\left(\frac{\theta_2}{2}\right)\hat{I} - i\hat{\sigma}^3 \sin\left(\frac{\theta_2}{2}\right)\right)\left(\cos\left(\frac{\theta_1}{2}\right)\hat{I} - i\hat{\sigma}^2 \cancel{\xi}\sin\frac{\theta_1}{2}\right)$$

$$\cos\left(\frac{\theta_2}{2}\right)\hat{I}\cos\left(\frac{\theta_1}{2}\right)\hat{I} - i\hat{\sigma}^2 \sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}$$

$$- i\hat{\sigma}^3 \sin\left(\frac{\theta_2}{2}\right)\cos\left(\frac{\theta_1}{2}\right) + \underbrace{\hat{\sigma}^3\hat{\sigma}^2}_{-i\hat{\sigma}^1}\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}$$

$$c\frac{\theta_2}{2}\,c\frac{\theta_1}{2} - i\hat{\sigma}^2\frac{\sin\theta_1}{2}\cos\frac{\theta_2}{2} - i\sin\theta_2\cos - a$$

$$c\left(\frac{\theta_2}{2}+\frac{\theta_1}{2}\right)\quad\left(I - i\hat{\sigma}^1\right)\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}$$

$$\boxed{e^{ia(\hat{n}\cdot\vec{\sigma})} = I\cos a + i(\hat{n}\cdot\vec{\sigma})\sin a}$$

$$-i\sigma^2 - i\sigma^3$$

$$-i\sigma^2 + i\sigma^2$$

$$-i\sigma^2\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2} - i\sigma^3\sin\frac{\theta_2}{2}\cos\frac{\theta_1}{2}$$

$$\cancel{-i\sigma^2\big(\sin}$$

$$-i\sigma^2\left(\sin\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right) - \sin\frac{\theta_2}{2}\cos\frac{\theta_1}{2}\right)$$

$$-i\sigma^3\left(\sin\left(\frac{\theta_2}{2}+\frac{\theta_1}{2}\right) - \sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right)$$

$$\left(-i\sigma^2 - i\sigma^3\right)\left(\sin\left(\frac{\theta_1}{2}+\frac{\theta_2}{2}\right)\right)$$

$$-i\sigma^2\left(\sin\frac{\theta_2}{2}\cos\frac{\theta_1}{2}\right) +$$

$$i\sigma^3\big($$

4. $\hat{H}(t) = |\gamma| \hat{\vec{S}} \cdot \vec{B}(t)$

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$\hat{J}_z |m\rangle = m|m\rangle \qquad m \in \{1, 0, -1\}$

$|m = \pm 1\rangle = \frac{1}{\sqrt{2}} \left(|\vec{n}_x\rangle + i|\vec{n}_y\rangle\right) \qquad |m=0\rangle = |\vec{n}_z\rangle$

$\hat{R}_z(\theta)|m\rangle = e^{-i\theta \hat{J}_z}|m\rangle = e^{-im\theta}|m\rangle$

the states $|m = \pm 1\rangle$ acquires phases under rotation

i did this yesterday.

$$\vec{B}(t) = \begin{cases} B_y \vec{y}_y & 0 \le t \le t_y \quad H_1 \\ B_z \vec{n}_z & t \ge t_y \quad H_2 \end{cases}$$

---

init state $t = 0 \quad |\psi_0\rangle = |m = 1\rangle \qquad \hat{S}_z = +\hbar$

evolution until $t_y = \frac{\pi}{2|\gamma| B_y}$ compute final state $t > t_y$

---

so, propagator,

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle = e^{\frac{-i\hat{H}(t)t}{\hbar}}|\psi_0\rangle$$

$0 \le t \le t_y$

$$= e^{-\frac{i\hat{H}_1 t}{\hbar}}|\psi_0\rangle$$

$$= e^{\left(-i\frac{H_1 t}{\hbar}\right)}|\psi_0\rangle$$

---

after,

$$|\psi(t)\rangle = U(t)|\psi(t_B)\rangle$$

$$= e^{\frac{-iH_2(t-t_B)}{\hbar}} e^{\frac{iH t_B}{\hbar}} |\psi_0\rangle$$

$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times [B_b C_c \varepsilon_{abc}]$

I FORGOT HOW THEY COME INTO BEING

$\left(e^{-i\frac{\sigma^3}{2}\theta_2}\right)\left(e^{-i\frac{\sigma^2}{2}\theta_1}\right)$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$= \left(\hat{I} + \left(-i\frac{\sigma^3}{2}\theta_2\right) + \frac{1}{2}\left(-i\frac{\sigma^3}{2}\theta_2\right)^2 + \frac{1}{3!}\left(-i\frac{\sigma^3}{2}\theta_2\right)^4 + \cdots\right)$

$= ($

$\cancel{= \hat{I} + \left(-i\frac{\sigma^3\sigma^2}{2}\theta_2\right.}$ $= \hat{I}\,(\text{series}) +$

$\left(\hat{I} + \left(-i\frac{\sigma^3}{2}\theta_2\right) + \frac{1}{2}\left(-i\frac{\sigma^3}{2}\theta_2\right)^2 + \cdots\right)\left(\hat{I} + \left(-i\frac{\sigma^2}{2}\theta_1\right) + \frac{1}{2}\left(-i\frac{\sigma^2}{2}\theta_1\right) + \cdots\right)$

$\hat{I} + \left(-i\frac{\sigma^2}{2}\theta_1\right) + \frac{1}{2}\left(-i\frac{\sigma^2}{2}\theta_1\right) +$

$-\sigma \quad = e^{x+y} \rightarrow \quad e^{-\frac{i}{\hbar}(\hat{S}_y\theta_1 + S_z\hat{\theta}_2 = \hat{S}_y\theta_1)}$