Heat Light and Waves : : Homework 08

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Problem 01

(a)

$$v_1 = \frac{c}{n_1}$$
$$v_2 = \frac{c}{n_2}$$

$$d_1 = \sqrt{h_1^2 + x^2}$$
$$d_2 = \sqrt{h_2^2 + (L-x)^2}$$

$$\boxed{t = n_1 \frac{\sqrt{h_1^2 + x^2}}{c} + n_2 \frac{\sqrt{h_1^2 + (L-x)^2}}{c}}$$

(b)

$$\begin{aligned}
\frac{\mathrm{d}t}{\mathrm{d}x} = 0 &= \frac{n_1}{c} \frac{x}{\sqrt{h_1^2 + x^2}} - \frac{n_2}{c} \frac{L-x}{\sqrt{h_1^2 + (L-x)^2}} \\
&\implies n_1 \left(\frac{x}{\sqrt{h_1^2 + x^2}} \right) = n_2 \left(\frac{L-x}{\sqrt{h_1^2 + (L-x)^2}} \right) \\
&\implies n_1 \sin\theta_1 = n_2 \sin\theta_2
\end{aligned}$$

Problem 02

(a)

Look at critical angles first.

$$\sin\theta_C = \frac{n_2}{n_3}$$

Now using that related to θ_m

$$
\begin{aligned}
n_1 \sin\theta_m &= n_3 \sin\theta_2 \\
\sin\theta_m &= n_3 \cos\theta_c && (90^\circ - \theta_{2,m} = \theta_c) \\
\theta_m &= \arcsin\left(n_3\sqrt{1-\left(\frac{n_2}{n_3}\right)^2}\right)
\end{aligned}
$$

Hence what we get is

$$\boxed{\theta_m = \arcsin\left(\sqrt{n_3^2 - n_2^2}\right)}$$

Similar problem : Asian Physics Olympiad 2024 Theory Examination.

(b)

Writing the obvious equations and finding them using the calculator

$$
\begin{aligned}
n_3 \sin\theta_{\text{core}} = n_2 &\implies \text{numerically, } \theta_{\text{core}} = 59.97^\circ \\
n_2 \sin\theta_{\text{clad}} = n_1 &\implies \text{numerically, } \theta_{\text{clad}} = 50.27^\circ
\end{aligned}
$$

$$
\begin{aligned}
&n_3 \sin\theta_3 = n_2 \sin\theta_{\text{clad}} \\
&\qquad \implies \text{numerically, } \theta_3 = \arcsin\left(\frac{n_2}{n_3}\sin\theta_{\text{clad}}\right) = 41.81^\circ
\end{aligned}
$$

which satisfies $\theta_3 < 59.97^\circ$

(c)

$$\cos\theta_2 = \cos\left(90^\circ - \theta_{\text{core}}\right) = \sin\theta_{\text{core}} = \frac{n_2}{n_3}$$

$$t_1 = \frac{L}{c/n_3} = \frac{Ln_3}{c}$$

$$t_2 = \left(L/\cos\theta_2\right)/\left(c/n_3\right) = \frac{Ln_3^2}{cn_2}$$

$$\Delta t = t_2 - t_1 = \frac{Ln_3^2}{cn_2} - L\frac{n_3}{c} = \frac{Ln_3}{c}\left(\frac{n_3}{n_2} - 1\right)$$

$$\boxed{\Delta t = n_3\left(\frac{n_3}{n_2} - 1\right)\frac{L}{c}}$$

Problem 03

(a)

$$\frac{1}{f_2} = \frac{1}{d} + \frac{1}{x}$$
$$\frac{2}{R_2} - \frac{1}{d} = \frac{1}{x}$$
$$\frac{1}{\frac{2}{R_2} - \frac{1}{1-0.75}} = x \implies x = -0.136\,\mathrm{m}$$

$$\boxed{D = x - d = 0.886\,\mathrm{m}}$$

(b)

$$R_1 = 1\,m$$
$$f_1 = 0.5\,m$$

Leaving R_2 as unknown

$$d = 0.5\,m - 0.75\,m$$
$$\frac{1}{f_2} = \frac{1}{-0.25} + \frac{1}{-0.136} \implies f_2 = -0.088\,m$$
$$R_2 = 2f_2 = -0.176\,m$$

The secondary mirror should be a convex mirror with radius $-0.176\,m$

Problem 04

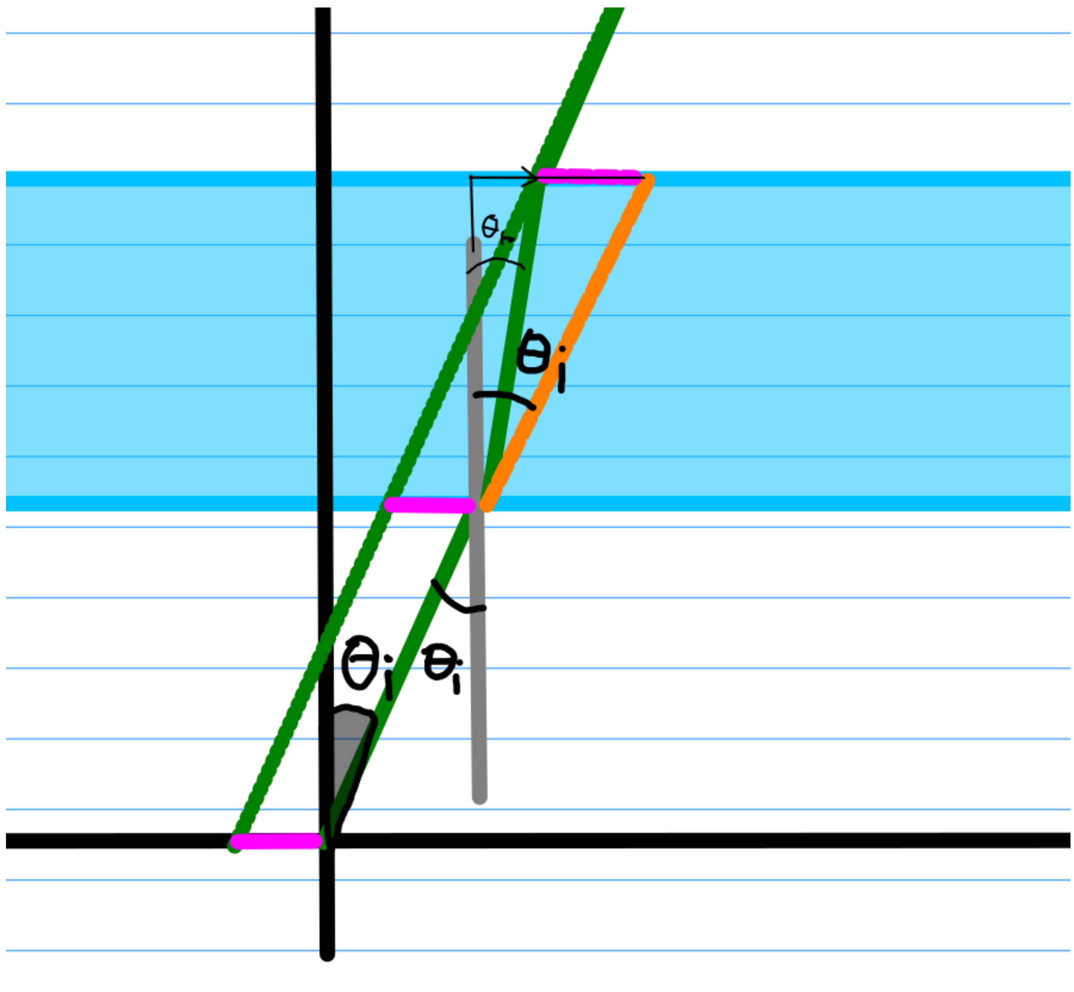

Figure 1: ./ss/8/1.png

We just need to care about the Purple colored line which is the shift of the image. With small angle,

$$\sin\Theta_i = n\sin\Theta_r \implies \Theta_r \approx \frac{\Theta_i}{n}$$

$$\begin{aligned}
\Delta x &= (4)(\tan(\Theta_i) - \tan(\Theta_r)) \\
&= 4\left(\frac{\sin\Theta_i}{\sqrt{1-\sin^2\Theta_i}} - \frac{\sin\Theta_r}{\sqrt{1-\sin^2\Theta_r}}\right) \\
&\approx 4\left(\frac{\theta_i}{\sqrt{1-\theta_i^2}} - \frac{\theta_i/n}{\sqrt{1-\frac{\theta_i^2}{n^2}}}\right) \\
&\approx 4\theta_i\left(1-\frac{1}{n}\right) = 1.42\theta_i
\end{aligned}$$

The shift in the position is a function of the small incident angle θ_i

NOTE: Geometrically it's obvious the shift in the image has NO dependence on the distance between the glass from the ground. It's only the angle that matters here.

Problem 05

(a)

$$F = (n-1)\left(\frac{1}{r_1} - \frac{1}{\infty}\right)$$
$$\implies \frac{F}{n-1} = \frac{1}{r_1} \implies r_1 = \frac{n-1}{F} = 1.1\,\mathrm{m}$$

(b)

$$n_{\text{lens-water}} = \frac{n_{\text{lens}}}{n_{\text{water}}}$$
$$r_1 = \frac{\frac{n_{\text{lens}}}{n_{\text{water}}} - 1}{F} \implies r_1 = 0.33\,\mathrm{m} \text{ convex when facing it}$$

Problem 06

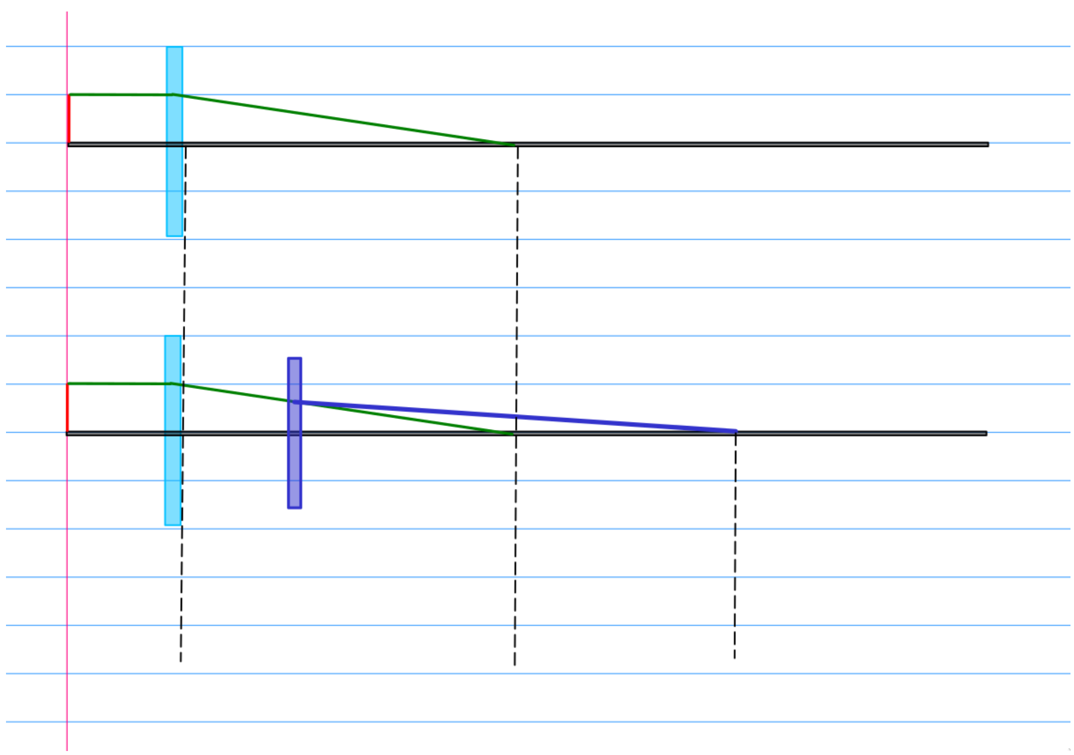

Figure 2: ./ss/8/2.png

$$\frac{1}{f_2} = \frac{1}{34.2} - \frac{1}{-15} \implies f_2 = 10.43\,\mathrm{cm}$$
$$\boxed{f_2 = -10.43\,cm}$$

Problem 07

(a)

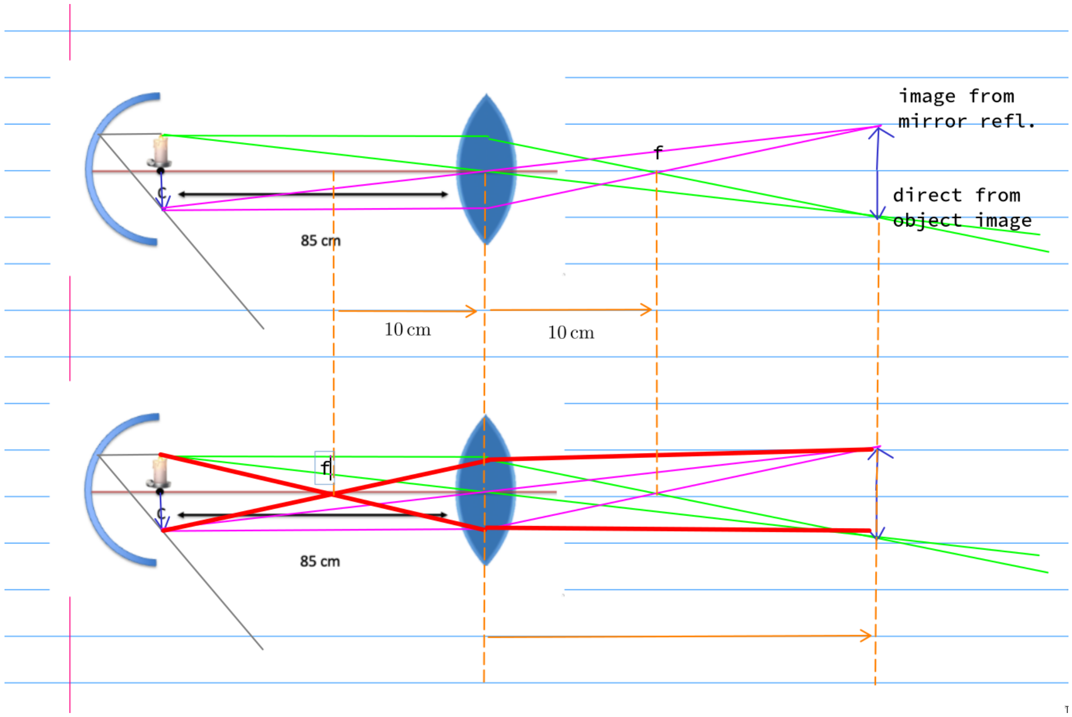

Figure 3: ./ss/8/3.png

(b)

For the direct image

$$\frac{1}{f} = \frac{1}{x} - \frac{1}{-85} \implies x = 8.95\,\text{cm}$$

The image is a real image.

$$m = -\frac{8.95}{85} = 0.105$$

$$h' = mh = 0.21\,\text{cm, upright}$$

For the mirror,

$$\frac{1}{10} = \frac{1}{20} + \frac{1}{x} \implies x = 20\,\text{cm}$$

$$m_{\text{mirror}} = -\frac{20}{20} = -1$$

$$h' = -2\,\text{cm}$$

For lens

$$\frac{1}{10} = \frac{1}{d} + \frac{1}{-85} \implies d = 8.95\,\text{cm}$$

$$m_{\text{lens}} = \frac{8.95}{-85} = 0.105$$

$$h' = -0.21\,\text{cm}$$

$$m_{\text{total}} = -0.1053\,\text{inverted}$$