Elastic and viscous properties of Silly Putty

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Abstract

We consider in this paper the elastic and viscous properties of silly putty and confirm the well known fact that the properties depend on the rate at which the material is deformed. Rapid deformations were studied by dropping masses onto one end of a silly putty cylinder, and slow deformations were studied by compressing the cylinder in a materials testing machine. The results were compared with a simple engineering model of viscoelastic materials to estimate the stiffness and the viscosity of the silly putty cylinder. It was found that stress induced in silly putty relaxes with a time constant of about 0.1 s, Young's modulus for a rapid deformation is about 1.7×10^6 N/m^2, and the viscosity for a slow compression is about 8×10^4 Pa-s. When subject to a short impact, silly putty vibrates as a result of compressional wave propagation through the material.

I. INTRODUCTION

Silly putty is probably the best known example of all viscoelastic materials since it is so obviously elastic on a small time scale and so obviously viscous on a long time scale. A heavy mass dropped onto a piece of silly putty will bounce off the putty, but the putty can be squashed flat in a few seconds with a gentle press of one finger. Baseballs[1] and other sports balls[2] are also viscoelastic, as are a wide variety of other materials including food, nylon, human tissue, wood, rubber, clay and lava.[3] Silly putty owes its fame to the fact that it is more highly elastic and has a lower viscosity than most other viscoelastic solids. Materials similar to silly putty are now being used in sports wear and foot wear to minimise impact shock.[4]

Silly putty is a silicon–based polymer, consisting of a mixture of silicone (polydimethylsiloxane), silica and boric acid. A home-made version can be manufactured from white glue and borax.[5] The long polymer molecules are entangled, like strands of spaghetti, and are linked by weak hydrogen bonds. Under stress, the bonds can be broken over a relatively long time scale and the strands can be untangled. Hydrogen bonds can reform over the same time scale. If the stress is applied rapidly then relatively few bonds are broken and the material behaves as an elastic solid.[3,5–7]

Despite the fact that the properties of silly putty are well known in a qualitative sense, it is difficult to find quantitative measurements of these properties. The object of the present work was to measure the properties, using relatively simple apparatus. The experiment could form the basis of an interesting undergraduate project to measure the properties of other materials as well. Viscoelastic behavior has previously been investigated in an undergraduate laboratory by measuring the vibration frequency of a stretched nylon string[8] or of a mass attached to a rubber band.[9] However, the latter techniques are not suited to measuring the properties of silly putty.

In the present paper, a more direct approach was taken in order to measure the physical properties of a small cylinder of silly putty. The elastic properties were investigated by bouncing various masses off the cylinder to measure its stiffness and energy loss. The viscous properties were measured by compressing the cylinder slowly in a materials testing machine. The results were interpreted in terms of a standard viscoelastic engineering model consisting of two springs and a dashpot.[8,9] The model has previously been used to describe other viscoelastic materials, although many other models have also been described in the literature since not all viscoelastic materials behave in the same manner.[3,7] The model was successful in describing the compression of silly putty but failed to account for the permanent deformation observed when the applied force was reduced to zero.

II. THEORETICAL MODEL

The standard model used to describe viscoelastic materials is shown in Fig. 1. It consists of a spring in parallel with a so–called Maxwell element which contains a spring in series with a dashpot. Each spring obeys Hooke's law, with $F = kx$, where k is the spring constant and x is the compression of each spring. The dashpot obeys the relation $\sigma = \eta\, d\epsilon/dt$ where σ is the stress, ϵ is the strain and η is the viscosity of the dashpot. A dashpot has the property that it does not spring back to its original position after it is compressed or extended since there is no restoring force while the strain remains constant. The dashpot therefore represents the viscous nature of the material while the two springs represent its elastic properties.

The restoring force provided by spring k_2 in Fig. 1 allows silly putty to relax back to its original uncompressed length if it is compressed and then the compression force is removed. In fact, silly putty remains permanently compressed if it is compressed for a time interval of more than about 0.1 seconds. A simple solution to this problem is to add a second dashpot, in series with spring k_2, so that the model consists of two Maxwell elements in parallel. Such a model allows stress relaxation to occur on two different time scales, and it results in permanent deformation when the stress is reduced to zero. If the second Maxwell element has a relatively long time constant, then it does not alter the behaviour of the model on a short time scale. The equation describing this model is second order in both F and x and does not provide the insights afforded by the simpler model in Fig. 1. Some insights into the behaviour of two or more parallel Maxwell elements can be found in a related, more technical paper on the properties of silly putty.[10]

For a cylinder of cross–sectional area A and length L, $\sigma = F/A$ and $\epsilon = x/L$ where x is the compression of the cylinder. The force, F_1, acting on the dashpot is therefore given by

$$F_1 = k_1 x_S = \left(\frac{A\,\eta}{L}\right)\frac{dx_D}{dt} \tag{1}$$

where x_S is the compression of the Maxwell element spring and x_D is the compression of the dashpot. The net force $F = F_1 + k_2 x$ and the overall compression $x = x_S + x_D$. From these relations it is easy to show that

$$\frac{dF}{dt} = (k_1 + k_2)\,\frac{dx}{dt} + \frac{(k_2 x - F)}{\tau} \tag{2}$$

where $\tau = A\eta/(k_1 L)$ is the time constant or the relaxation time of the Maxwell element. The relaxation time is a property of the material rather than a function of the material dimensions. For a cylinder of length L and area A, $E = \sigma/\epsilon = kL/A$, where E is Young's modulus, so $\tau = \eta/E$. If the time constant is relatively long or if the force, F, is applied for a time much shorter than τ, then $F = (k_1 + k_2)x$ to a good approximation, and the model describes an elastic solid with spring constant $k_1 + k_2$.

Suppose that a force is applied in such a way that it increases rapidly from zero to a value F_o and then remains constant for a time much longer than the relaxation time. That process would simulate the effect of adding a weight on top of a cylinder of silly putty. If the time for the rapid increase is much shorter than the relaxation time then the model will compress (or extend under a tensile force) initially by an amount $x = F_o/(k_1 + k_2)$. Subsequently, we find from Eq. (2) that

$$x = \frac{F_o}{k_2}\left(1 - \frac{k_1}{k_1 + k_2}\,e^{-At}\right) \tag{3}$$

where $A = k_2/[(k_1 + k_2)\tau]$. After a long time, x will approach the value F_o/k_2, in which case dF/dt and dx/dt are both zero in Eq. (2).

The result described in the previous paragraph seems to conflict with the known fact that a short cylinder of silly putty will stretch many times its original length when pulled slowly with a constant force. In the latter case the cross–sectional area decreases and the length of the cylinder increases, in which case the stiffness of the cylinder decreases well below its initial value. On the other hand, a cylinder shortens and fattens when it is compressed, resulting in an increase in the initial stiffness. As a result, a cylinder of silly putty compresses by a finite amount after a long time under a constant load. For that reason, a ball or cylinder of silly putty does not deform under its own weight into a thin, flat disk when left on a flat surface, even after several weeks.

A common procedure when testing materials is to compress or extend the material at a constant rate. Suppose that $V = dx/dt$ remains constant, then the solution of Eq. (2) is

$$F = V[k_2 t + k_1\,\tau\,(1 - e^{-t/\tau})] \tag{4}$$

While $t \ll \tau$, $F = V(k_1 + k_2)t$. If the compression is continued well beyond the relaxation time, then $F = V(k_2 t + k_1 \tau)$. The three parameters of the model can be determined simply by compressing the material at a constant rate and fitting the resulting $F(t)$ curve using Eq. (4).

An alternative method of measuring the three parameters is to compress the cylinder rapidly, to a value x_o, and then maintain that compression for a relatively long time. If the cylinder is compressed in a time much less than τ then the force will increase to a value $F_o = (k_1 + k_2)x_0$. Subsequently, while x remains constant, F is given by

$$F = (k_2 + k_1 e^{-t/\tau})x_0 \,, \tag{5}$$

and will therefore decrease exponentially to a value $F = k_2 x_o$ after a sufficiently long time. For silly putty, a few seconds is more than sufficient since the relaxation time is about 0.1 s. Other viscoelastic materials can take hours or years or centuries to relax.[3]

III. EXPERIMENTAL PROCEDURE

Room temperature impact properties of silly putty were measured using the technique shown in Fig. 2. The objective was to impact one end of a 12.8 g, 35 mm long, 20 mm diameter cylinder of silly putty using steel balls of different mass in order to vary the impact duration from a few milliseconds up to about 40 ms. The balls were dropped onto the top end of the cylinder from a height of about 5 or 10 cm to impact the cylinder at a speed of around 1 m/s. The impact duration was measured from the output voltage of a piezoelectric disk, the output being directly proportional to the force on the disk.[11] The silly putty cylinder was mounted directly on the top face of the disk. The impact was filmed with a Casio EX-F1 video camera at 300 frames/s in order to measure (a) the incident and rebound speed of the ball, and (b) the compression of the silly putty cylinder. After each impact, the silly putty was re–formed into a new cylinder of length 35 ± 1 mm, and diameter 20 ± 1 mm.

The net force on the ball, including the gravitational force, was integrated over time to obtain the velocity of the ball as a function of time during the impact. The velocity of the ball just before and just after the impact was obtained from the video data. The velocity was then integrated over time to calculate the displacement of the ball as a function of time. In this experiment, the putty was much softer than the ball so the displacement of the ball was equal to the compression of the putty, the compression of the ball itself being negligible. The stiffness of the cylinder for a rapid compression could therefore be obtained by plotting the force on the cylinder as a function of its compression. For relatively long impacts, the compression and the "restitution" of the cylinder could also be observed and measured as a function of time, at 3.3 ms intervals, from the video data. Many experiments have previously been described on the coefficient of restitution of a bouncing ball,[2,11] but the actual compression and restitution of a ball is difficult to measure or observe due to the short time scale of the collision. It was easy to observe both phases for the silly putty cylinder since the compression was both large and slow when using heavy balls to squash the cylinder.

The response of the silly putty cylinder to a slow compression was measured in a small materials testing machine, as indicated in Fig. 3. The machine was computer controlled and was programmed to increase the compression to a maximum of about 10 mm, at a fixed rate, which was varied from 200 mm/min to 1000 mm/min. Once the compression reached a value of about 10 mm, the compression was maintained at a constant value for 1.2 s before separating the plates to remove the cylinder. During the constant compression phase, the force on the cylinder decreased rapidly to zero as a result of stress relaxation. If a metal spring was compressed in this manner, the force on the spring would remain constant while the compression was maintained at a constant value.

Given that materials testing machines are not generally available in undergraduate physics laboratories, readers may be interested in the simpler and much cheaper substitute described by Lewis et al.[2] Furthermore, force can be measured with a variety of other devices, including a piezoelectric disk or a strain gauge or small weighing scales.

An additional measurement was made, using the apparatus shown in Fig. 2, by compressing the cylinder rapidly by 10 mm and then holding the compression constant. This technique was therefore similar to the test conducted with the materials testing machine, but the compression rate was increased to about 35,000 mm/min. In order to increase the compression rate, a flat wood block was pushed onto the top end of the silly putty cylinder by hand, at a speed of 0.59 m/s. Stops either side of the 8.3 kg cylinder shown in Fig. 2 were used to limit the travel of the block and to hold the compression constant at 10 mm for several seconds.

An independent measure of the viscosity of silly putty was attempted by measuring the time taken for a 12 mm diameter, 8.3 g steel bearing ball to sink to the bottom of a 30 mm diameter, 15 mm tall cylinder of silly putty. Normally, this experiment is undertaken in a student laboratory by allowing a ball to fall through a viscous fluid such as glycerine. Since silly putty is sticky, it was felt that confining the putty in a container would interfere with normal laminar flow around the ball, so the putty was placed at rest on a flat surface, unconfined. Adhesion of the putty to the side of the container would not matter if the container was say 20 cm wide, but only a small sample of silly putty was available. In theory, the viscosity can be estimated from the terminal velocity of the ball using Stokes' law. However, the ball submerged below the upper surface by only half its diameter after 24 hours, so the experiment was abandoned. The observed speed was much lower than expected, presumably because the putty below the ball compressed against the flat surface and was unable to flow unimpeded around the ball.

IV. IMPACT RESULTS

Figure 4 shows the results of dropping two steel balls on the silly putty cylinder, one of mass 28.2 g, and one of mass 2.0 kg. Both balls bounced up off the cylinder but the rebound speed was less than the incident speed in each case. The ratio of the two speeds, defined as the coefficient of restitution for the collision (COR), was 0.73 for the lighter ball and 0.45 for the heavier ball. The impact duration was 4.4 ms for the light ball and 38 ms for the heavy ball.

Both balls generated a compressional wave in the cylinder, causing the cylinder to vibrate at a frequency that was inversely proportional to the height of the cylinder. For a 35 mm tall cylinder, the vibration frequency was about 360 Hz, as indicated in Fig. 4(a). In the latter case, the vibration persisted well after the light ball bounced clear. In both cases, the cylinder remained attached to the piezo so the pressure wave in the cylinder was recorded by the piezo even after the ball bounced clear. In some cases, but not those in Fig. 4, the cylinder jumped vertically off the piezo as a result of the collision since elastic energy remained stored in the cylinder after the ball bounced clear. In other cases, the cylinder did not jump, presumably because it adhered more firmly to the piezo. The cylinder was sticky but not as sticky as adhesive tape.

The area enclosed by the hysteresis curves in Figs. 4(b) and 4(d) represents the energy loss during the collision. The COR is a measure of the fraction of the stored elastic energy that is lost during the collision. A greater fraction was lost in the heavy ball collision, and the COR was therefore lower, as evidenced by the fact that the cylinder remained visibly squashed after the collision. As indicated in Fig. 4(d) the maximum compression of the cylinder was 9.6 mm and the compression was still 4.5 mm at the end of the collision. The cylinder ended up being 4.5 mm shorter for several seconds after the collision, as indicated on the video film, although it recovered slightly over the next few minutes.

The impact result with the 2 kg ball contrasted sharply with a simple experiment where the 2 kg ball was placed gently on top of the 35 mm tall cylinder. Despite the fact that the force on the cylinder was then about seven times smaller than the impact force shown in Fig. 4(c), the cylinder squashed to a thickness of about 5 mm in about four seconds and it remained squashed when the ball was lifted up.

The longitudinal stiffness, k, of the silly putty cylinder can be estimated from the slope of the F vs x curves in Figs. 4(b) and (d). In both cases k is about 15 N/mm during both the compression and expansion phases of the collision. By comparison, the stiffness of a tennis ball is typically about 20 N/mm. An alternative measure of the stiffness can be obtained in terms of the impact time, T. For a mass m incident on a spring of spring constant k, it is easy to show that $T = \pi\sqrt{m/k}$. For the 28.2 g mass, $k = 14.2$ N/mm. For the 2 kg mass, $k = 13.7$ N/mm, consistent with the values of k obtained from the results in Fig. 4. From the known dimensions of the cylinder, it can be concluded that Young's modulus for silly putty, when subject to a rapid deformation, is about 1.7×10^6 N/m^2, in good agreement with room temperature results obtained previously.[10]

V. SLOW COMPRESSION RESULTS

Results obtained by compressing the silly putty cylinder in a materials testing machine are shown in Fig. 5. It was found that the stiffness and viscosity of silly putty increase by a factor of about two when left in a refrigerator for half an hour, but all results presented in this paper were obtained at a temperature of 24 C. The maximum compression speed was 1000 mm/min (0.0167 m/s), about sixty times slower than the compression speed in the impact experiments. The force on the cylinder increased with the compression speed, indicating viscous behaviour of the silly putty. If the viscosity remained constant, and if the cylinder behaved as a simple dashpot, then the force would be directly proportional to the compression speed. In fact, the force increased at a lower rate. For example, there was a factor of about three increase in the force, at any given x, when the compression rate increased by a factor of five, from 200 to 1000 mm/min.

When the compression reached a value of about 10 mm it was maintained at that value for 1.2 s in order to measure the rate of stress relaxation. In each case, the force decreased rapidly with a time constant of about 0.12 s, at least for the first 0.2 s. After the first 0.2 s, the time constant increased to about 0.4 s, as indicated in Fig. 5(b).

The dot points shown in Fig. 5(a) are best fit solutions given by Eq. (4). The solutions were obtained by varying the three parameters k_1, k_2 and τ to fit each of the three experimental curves separately. The parameters for each of the best fit solutions are listed in Table 1. In terms of the model solutions, the linear slope of the curves in Fig. 5(a) at large x depends only on k_2 while the intercept of the straight section of the curves (extrapolated back to $x = 0$) depends on both the compression rate, V, and the viscosity, according to the relation $F = k_2 x + VA\eta/L$.

Table 1. Parameters used to model each of the three curves in Fig. 5(a).

V (mm/min)	τ (s)	k_1 (N/m)	k_2 (N/m)	η (Pa-s)
200	0.10	8,000	380	8.9×10^4
500	0.10	6,700	602	7.5×10^4
1000	0.08	10,000	548	8.9×10^4

Ideally, the data obtained at the three compression speeds could all be fit by the same three parameters but this was not possible. The most likely explanation is that the cylinder was reconstructed by hand after each compression and it distorted slightly with time while mounted in the testing machine. Consequently, the dimensions of the cylinder were controlled to an accuracy of only about ± 1 mm. Nevertheless, the model provides a very good qualitative description of the data and indicates that the relaxation time during the compression phase was 0.09 ± 0.01 s and that the viscosity was $(8.2 \pm 0.7) \times 10^4$ Pa-s, in good agreement with previous results for silly putty.[10] A slightly longer relaxation time was found during the 1.2 s interval while the compression remained constant. The initial stiffness of the cylinder, given by $k_1 + k_2$, was up to a factor of two smaller than that found for a short impact, indicating that the stiffness of silly putty increases when the compression rate is increased by a large factor.

The results shown in Fig. 5 are commonly plotted for other materials as a graph of σ vs $d\epsilon/dt$, the slope being equal to the "extensional viscosity" of the material (as opposed to the shear viscosity). If the graph is not linear then the ratio $\sigma/(d\epsilon/dt)$ at any point represents the apparent viscosity.[5] In the present case it is clear that the viscosity or the apparent viscosity depends on both the compression and the compression rate so that neither definition is especially useful. For example if we take a typical point in Fig. 5(a) on the 500 mm/min curve, then the apparent viscosity at $F = 5$ N is 6.7×10^4 Pa-s and the apparent viscosity at $F = 10$ N is twice that value, or 1.34×10^5 Pa-s. The model shown in Fig. 1 appears to offer a better operational definition of the viscosity of silly putty since the result does not depend strongly on the compression or the compression rate.

VI. RAPID COMPRESSION RESULTS

A typical result from the rapid compression experiment is shown in Fig. 6. The wood block did not bounce when it was pushed onto the stops since it was pushed and held down firmly by hand. In Fig. 6, the block approached at a speed of 0.59 m/s and compressed the putty by 10 mm over a time interval of 17 ms. The impact was therefore similar to that shown in Fig. 4(c) for the 2 kg ball, apart from the fact that the compression was maintained at its maximum value of 10 mm for several seconds. During that time, the force decreased with time due to stress relaxation, with a time constant of about 0.05 s.

During the rapid compression, F increased linearly with time to 168 N, so the stiffness $k_1 + k_2 = 16.8$ N/mm, slightly higher than the 15 N/mm result found from the data in Fig. 4. After 0.35 s, F approached a constant value 7.0 N, giving $k_2 = 700$ N from Eq. (5), also slightly higher than the values for k_2 shown in Table 1. The result in Fig. 6 is therefore in reasonable agreement with the previous results and is also in very good agreement with results obtained by others[10], but it was not possible to obtain an accurate fit to the relaxation data in Fig. 6 using Eq. (4). A logarithmic plot of the relaxation data indicated that τ varied from 25 ms at the start of the relaxation phase to about 60 ms when $t > 0.05$ s. The latter effect is commonly observed when studying viscoelastic materials. That is, the relaxation time tends to increase with time, a result that is presumably associated with the fact the stress also decreases with time. For that reason, the model shown in Fig. 1 is commonly extended to include several parallel Maxwell elements, each with a different time constant.[3,10]

VII. CONCLUSION

Several different techniques were used to measure the elastic and viscous properties of a cylindrical sample of silly putty. Impact tests revealed that a steel ball can bounce off silly putty even if the impact duration is as long as about 40 ms. The coefficient of restitution was found to decrease for longer impacts with heavier balls, but insufficient data were obtained to determine the exact cause. Further experiments would be needed to determine whether the reduction in the COR and associated additional loss of elastic energy was due primarily to the longer impact duration or to the larger deformation of the cylinder when using heavier balls. Both factors could be equally important. Young's modulus for a rapid deformation was found to about $1.7 \times 10^6\,\mathrm{N/m^2}$. Silly putty is sufficiently elastic during a short impact that it vibrates as a result of the impact.

Relatively slow compression tests in a materials testing machine indicated that the force required to compress a cylinder of silly putty by a given amount decreases when the compression rate is reduced. For that reason, it is easier to squash or stretch silly putty if a small force is applied for a long time rather than applying a large force for a small time. The results were compared with a standard viscoelastic model consisting of two springs and a dashpot. The model provided a good qualitative description of the behavior of silly putty, and indicated that the room temperature viscosity of silly putty is about 8×10^4 Pa-s and the stress relaxation time is about 0.1 s during a slow compression (or at low stress levels) but can be as short as 0.025 s during a rapid compression (or at high stress levels).

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Figure Captions

Fig. 1 Standard model used to describe a viscoelastic material, consisting of two springs and a dashpot. The compression, x, is a function of time in this model, even if the applied force, F, remains constant.

Fig. 2 Method used to measure the impact duration and coefficient of restitution for a mass M impacting on a 35 mm long, 20 mm diameter cylinder of silly putty.

Fig. 3 Method used to measure the force, F, applied to a 35 mm long cylinder of silly putty, and its compression, x.

Fig. 4 Results obtained by dropping a 28 g and a 2.0 kg ball on a cylinder of silly putty. Parts (a) and (c) show the respective forces on the ball vs time, while (b) and (d) show the corresponding force vs compression (x) curves.

Fig. 5 Results obtained by compressing a cylinder of silly putty by about 10 mm at three different speeds in a materials testing machine, showing (a) F vs x and (b) F vs t. The vertical section of the $F(x)$ plot indicates that F decreased to zero when x was maintained at a constant value for 1.2 s. Dot points in part (a) are best fit solutions given by Eq. (4).

Fig. 6 Result obtained by compressing a 35 mm tall cylinder of silly putty by 10 mm at a compression speed of 35,400 mm/min, and then holding the compression constant for a few seconds.