Domácí úkol č. 4

B0B01MA1

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1. Spočtěte

$$\int_0^{\ln\sqrt{2}} \frac{6e^{6x}-2e^{4x}}{e^{4x}-3e^{2x}+4}\,\mathrm{d}x = \left|\begin{matrix} u = e^{2x} \\ \mathrm{d}u = 2e^{2x}\,\mathrm{d}x \end{matrix}\right| = \int_1^2 \frac{\cancel{2e^{2x}}(3u^2-u)}{u^2-3u+4}\cdot\frac{\mathrm{d}u}{\cancel{2e^{2x}}} = \int_1^2 \frac{3u^2-u}{u^2-3u+4}\,\mathrm{d}u =$$

$$= \left|\begin{matrix} \text{dělení} \\ \text{polynomů} \end{matrix}\right| = \int_1^2 \left(3+\frac{8u-12}{u^2-3u+4}\right)\mathrm{d}u = [3u]_1^2 + \int_1^2 \frac{8u-12}{u^2-3u+4}\,\mathrm{d}u = \left|\begin{matrix} t = u^2-3u+4 \\ \mathrm{d}t = (2u-3)\,\mathrm{d}u \end{matrix}\right| =$$

$$= [3u]_1^2 + \int_2^2 \frac{4\cdot\cancel{(2u-3)}}{t}\cdot\frac{\mathrm{d}t}{\cancel{2u-3}} = [3u]_1^2 + \underbrace{4\int_2^2 \frac{\mathrm{d}t}{t}}_{=0} = [3u]_1^2 = 6-3 = \underline{\underline{3}}$$

2. Spočtěte

$$\int_{\frac{e}{2}}^{\infty} \frac{3}{x(\ln^2 2x - 6\ln 2x + 13)}\,\mathrm{d}x = \left|\begin{matrix} u = \ln 2x \\ \mathrm{d}u = \frac{1}{x}\,\mathrm{d}x \end{matrix}\right| = 3\int_1^{\infty} \frac{1}{\cancel{x}(u^2-6u+13)}\cdot\cancel{x}\,\mathrm{d}u = 3\int_1^{\infty} \frac{1}{(u^2-6u+9)+4}\,\mathrm{d}u =$$

$$= 3\int_1^{\infty} \frac{1}{(u-3)^2+4}\,\mathrm{d}u = \left|\begin{matrix} t = u-3 \\ \mathrm{d}t = 1\,\mathrm{d}u \end{matrix}\right| = 3\int_{-2}^{\infty} \frac{1}{t^2+4}\,\mathrm{d}t = \frac{3}{4}\int_{-2}^{\infty} \frac{1}{\frac{t^2}{4}+1}\,\mathrm{d}t = \left|\begin{matrix} v = \frac{t}{2} \\ \mathrm{d}v = \frac{1}{2}\,\mathrm{d}t \end{matrix}\right| = \frac{3}{2}\int_{-1}^{\infty} \frac{1}{v^2+1}\,\mathrm{d}v =$$

$$= \frac{3}{2}[\arctan v]_{-1}^{\infty} = \frac{3}{2}\left(\lim_{v\to\infty}(\arctan v) - \arctan(-1)\right) = \frac{3}{2}\cdot\frac{\pi}{2} - \frac{3}{2}\left(-\frac{\pi}{4}\right) = \underline{\underline{\frac{9\pi}{8}}}$$

3. Spočtěte

$$\int_0^{\frac{\pi}{2}} \frac{\sin x\cdot\cos x - 4\cos x}{1-2\sin x-\cos^2 x}\,\mathrm{d}x = \left|\begin{matrix} u = \sin x \\ \mathrm{d}u = \cos x\,\mathrm{d}x \end{matrix}\right| = \int_0^1 \frac{\cancel{\cos x}(u-4)}{u^2-2u}\cdot\frac{\mathrm{d}u}{\cancel{\cos x}} = \int_0^1 \frac{\cancel{u-2}}{u(\cancel{u-2})}\,\mathrm{d}u - 2\int_0^1 \underbrace{\frac{1}{u(u-2)}}_{g(u)}\,\mathrm{d}u =$$

$$= \underbrace{[\ln|u|]_0^1}_{\boldsymbol{S_f}} + \underbrace{\int_0^1 -2g(u)\,\mathrm{d}u}_{\boldsymbol{S_g}} = \boldsymbol{S_f} + \boldsymbol{S_g} = \boldsymbol{S}$$

$$\underline{g(u)} = \frac{1}{u(u-2)} = \frac{A}{u} + \frac{B}{u-2}$$

$$A + B = 0 \Rightarrow A = -B$$

$$-2A = 1 \Rightarrow A = -\frac{1}{2} \Rightarrow B = \frac{1}{2}$$

$$g(u) = -\frac{1}{2}\cdot\frac{1}{u} + \frac{1}{2}\cdot\frac{1}{u-2}$$

$$\boldsymbol{S_g} = -2\left(\int_0^1 -\frac{1}{2}\cdot\frac{1}{u}\,\mathrm{d}u + \int_0^1 \frac{1}{2}\cdot\frac{1}{u-2}\,\mathrm{d}u\right) = [\ln|u|]_0^1 - [\ln|u-2|]_0^1$$

$$\boldsymbol{S} = \boldsymbol{S_f} + \boldsymbol{S_g} = [\ln|u|]_0^1 + [\ln|u|]_0^1 - [\ln|u-2|]_0^1 = 0 - \lim_{u\to 0}\ln|u| + 0 - \lim_{u\to 0}\ln|u| - 0 + \ln 2 = +\infty + \infty + \ln 2 = \underline{\underline{+\infty}}$$

Diverguje $\Rightarrow$ existuje.

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4. Spočtěte

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{5\sin x - \sin x \cdot \cos x}{\underbrace{\sin^2 x}_{1-\cos^2 x} + \cos x + 1}\,\mathrm{d}x = \begin{vmatrix} u = \cos x \\ \mathrm{d}u = -\sin x\,\mathrm{d}x \end{vmatrix} = \int_{\frac{1}{2}}^{0} = \frac{\cancel{\sin x}(5-u)}{1-u^2+u+1} \cdot \frac{\mathrm{d}u}{\cancel{-\sin x}} = \int_{\frac{1}{2}}^{0} \frac{5-u}{(u-2)(u+1)}\,\mathrm{d}u =$$

$$= 5\int_{\frac{1}{2}}^{0} \underbrace{\frac{1}{(u-2)(u+1)}}_{f(u)}\,\mathrm{d}u + \int_{\frac{1}{2}}^{0} \underbrace{\frac{-u}{(u-2)(u+1)}}_{g(u)}\,\mathrm{d}u = \underbrace{\int_{\frac{1}{2}}^{0} f(u)\,\mathrm{d}u}_{\boldsymbol{S_f}} + \underbrace{\int_{\frac{1}{2}}^{0} -g(u)\,\mathrm{d}u}_{\boldsymbol{S_g}} = \boldsymbol{S_f} + \boldsymbol{S_g} = \boldsymbol{S}$$

$$\underline{f(u)} = \frac{1}{(u-2)(u+1)} = \frac{A}{u-2} + \frac{B}{u+1}$$

$$A + B = 0 \Rightarrow A = -B$$

$$A = -B \wedge A - 2B = 1 \Rightarrow -3B = 1 \Rightarrow B = -\frac{1}{3} \wedge A = \frac{1}{3}$$

$$f(u) = \frac{1}{3} \cdot \frac{1}{u-2} - \frac{1}{3} \cdot \frac{1}{u+1}$$

$$\boldsymbol{S_f} = 5\int_{\frac{1}{2}}^{0} f(u) = \frac{5}{3}\int_{\frac{1}{2}}^{0} \frac{1}{u-2}\,\mathrm{d}u - \frac{5}{3}\int_{\frac{1}{2}}^{0} \frac{1}{u+1}\,\mathrm{d}u = \frac{5}{3}\Big([\ln|u-2|]_{\frac{1}{2}}^{0}\Big) - \frac{5}{3}\Big([\ln|u+1|]_{\frac{1}{2}}^{0}\Big) =$$

$$= \frac{5}{3}(\ln 2 - \cancel{(\ln 3 - \ln 2)}) - \frac{5}{3}(\ln 1 - \cancel{(\ln 3 - \ln 2)}) = \frac{5}{3}(\ln 2 - 0) = \boldsymbol{S_f}$$

$$\underline{g(u)} = \frac{u}{(u-2)(u+1)} = \frac{\cancel{(u-2)}}{\cancel{(u-2)}(u+1)} + \frac{2}{(u-2)(u+1)} = \frac{1}{u+1} + \frac{2}{3}\left(\frac{1}{u-2} - \frac{1}{u+1}\right)$$

$$\boldsymbol{S_g} = -\int_{\frac{1}{2}}^{0} g(u) = -\int_{\frac{1}{2}}^{0} \frac{1}{u+1}\,\mathrm{d}u - \frac{2}{3}\int_{\frac{1}{2}}^{0} \frac{1}{u-2}\,\mathrm{d}u + \frac{2}{3}\int_{\frac{1}{2}}^{0} \frac{1}{u+1}\,\mathrm{d}u =$$

$$= -[\ln|u+1|]_{\frac{1}{2}}^{0} - \frac{2}{3}[\ln|u-2|]_{\frac{1}{2}}^{0} + \frac{2}{3}[\ln|u+1|]_{\frac{1}{2}}^{0} = -(\ln 1 - (\ln 3 - \ln 2)) - \frac{2}{3}(\ln 2 - \cancel{(\ln 3 - \ln 2)}) + \frac{2}{3}(\ln 1 - \cancel{(\ln 3 - \ln 2)}) =$$

$$= \ln 3 - 0 - \ln 2 - \frac{2}{3}(\ln 2 - 0) = \boldsymbol{S_g}$$

$$\boldsymbol{S} = \boldsymbol{S_f} + \boldsymbol{S_g} = \frac{5}{3}\ln 2 + \ln 3 - \ln 2 - \frac{2}{3}\ln 2 = \frac{3}{3}\ln 2 - \ln 2 + \ln 3 = \underline{\underline{\ln 3}}$$

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