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Numerical Methods

Project Report

Numerical Analysis of Purchase Intention in Marketing Science

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1 Problem Statement

In today's interconnected global landscape, the swift spread of information, products, and concepts has emerged as a defining feature of contemporary society. The sudden and widespread adoption of a particular element within a population, whether it's a viral social media post, a popular topic, or the swift adoption of a new technology, is of paramount importance in various domains, including marketing, social sciences, and technological advancement.

The objective of this undertaking is to formulate an extensive mathematical model that replicates and examines the interactions among individuals in the context of this phenomenon. Specifically, we aim to apply numerical techniques like 7-point Runge-Kutta, to analyse the mathematical model. We then aim to understand popularity of movie Pirates of Caribbean at the time of release.

2 Key Objectives

- Develop a mathematical framework that represents human interactions as a stochastic process, considering factors like advertisement budget time as an input, impact of direct communication(word of mouth) and indirect communication(including the effects of social media posts), and individual behavior.
- Implement numerical methods, like 7-point Runge-Kutta, to simulate the proposed mathematical model and analyze its predictive capabilities.
- Investigate the impact of various parameters on the initiation, propagation, and saturation of trends within a given population.
- Validate the model's accuracy and reliability through comparisons with real-world data and existing empirical studies.
- Explore the potential applications of the model in predicting trends, optimizing marketing strategies, and understanding the societal implications of trending phenomena.

3 Significance of Problem

This project addresses a critical knowledge gap in our understanding of human dynamics and the rapid diffusion of trends, ideas, and innovations. The development of a robust mathematical model, combined with numerical methods, will not only contribute to the academic understanding of the "hit" phenomenon but also offer practical insights for businesses, organizations, and policymakers seeking to harness the power of trends and viral phenomena.

4 Mathematical Model

The dynamic human interactions can be analyzed by modeling the willingness of the individuals to anticipate the popularizing element (For example : buying the product after its release). So, we build a mathematical model to understand and measure this “willingness” of individuals.

4.1 Purchase Intention

We define Purchase Intention as the intent of individuals to adopt the popularizing element. Let's denote Purchase intention of individual i at time t after the release of product, as $I_i(t)$.

4.2 Factors Affecting Purchase Intent

We are interested in modelling purchase intention ($I_i(t)$) of individuals. So, to model, the following factors have been taken into account that critically affect Purchase Intention.

- Advertisements
- Direct Communication
- Indirect Communication
- Intent Decline after buying

4.2.1 Advertisements

It refers to promotion of the product through mass media such as TV, newspapers, magazines, the Web, Facebook and Twitter.

$$\frac{dI_i(t)}{dt} = c_i \cdot A(t) \tag{1}$$

where $A(t)$ is the measure of the advertisement effect at time t, and c_i represents how effective the advertisement was on the person i.

4.2.2 Direct Communication

Direct communication refers to one-on-one interactions between individuals. These personal interactions can significantly influence a person's purchase intention. The impact of direct communication on an individual i is determined by the impressions (D_{ij}) left by other individual j.

$$\frac{dI_i(t)}{dt} = \sum_{j=1}^{N} d_{ij} \cdot I_j(t) \tag{2}$$

where, D_{ij} represents the impression of communication on person i.

The equation models how direct communication affects an individual's purchase intention.

4.2.3 Indirect communication

Indirect communication involves the spread of information through rumors, conversations overheard in public places, or online sources such as blogs and social media. These indirect sources can influence an individual's purchase intention through complex interactions involving multiple individuals.

$$\frac{dI_i(t)}{dt} = \sum_{j=1}^{N} \sum_{k=1}^{N} h_{ijk} \cdot I_j(t) \cdot I_k(t) \tag{3}$$

The Equation models the effect of conversation between individual k and j on individual i.

4.2.4 Intent Decline after anticipation

An individual after adopting the product is no more interested in anticipating it again. For Example, A buyer doesn't buy same TV again.

$$\frac{dI_i(t)}{dt} = -a \cdot I_i(t) \tag{4}$$

The above equation models this effect.

4.3 Combining the Equations

On combining Equations we get the following model that relates the discussed factors and purchase Intention.

$$\frac{dI_i(t)}{dt} = c_i \cdot A(t) + \sum_{j=0}^{n} d_{ij} \cdot I_j(t) + \sum_{j=0}^{n} \sum_{k=0}^{n} h_{ijk} \cdot I_j(t) \cdot I_k(t) - a \cdot I_i(t) \tag{5}$$

5 Final Model

In the previous section, we modeled the relation between Purchase Intention and various factors affecting it. With this we can model the dynamic interactions among individuals and understand popularisation of the products.

$$\frac{dI_i(t)}{dt} = c_i \cdot A(t) + \sum_{j=0}^{n} d_{ij} \cdot I_j(t) + \sum_{j=0}^{n}\sum_{k=0}^{n} h_{ijk} \cdot I_j(t) \cdot I_k(t) - a \cdot I_i(t) \tag{6}$$

Now in the following sections, we will use this model to analyze the popularisation of the movie Pirates of the Caribbean.

6 Assumptions

The following assumptions were taken to simplify the problem.

- The coefficient of Direct Communication between individuals, D_{ij}, is same for any two individuals and is D.
- The coefficient of Indirect Communication, h_{ijk}, is same for any three individuals.

7 Governing Equations

Now, we further develop model in Equation (6) for understanding popularity of movie Pirates of Caribbean at its releasing time.

7.1 Modelling Advertisement Effect

First, we model the advertisement impressions with random effect for i th person as $f_i(t)$ which represents the distribution and effect of advertisements on the individual. So, the Equation (6) becomes :

$$\frac{dI_i(t)}{dt} = f_i(t) + \sum_{j=0}^{n} d_{ij} \cdot I_j(t) + \sum_{j=0}^{n}\sum_{k=0}^{n} h_{ijk} \cdot I_j(t) \cdot I_k(t) - a \cdot I_i(t) \tag{6}$$

7.2 Taking Ensemble Average

Now, we will take the ensemble average of the purchase intention.

$$I(t) = \frac{1}{N}\sum_{i} I_i(t) \tag{7}$$

Now, taking the average of equation (6), the left side is:

$$\frac{dI_i(t)}{dt} \equiv \frac{1}{N}\sum_i \frac{dI_i(t)}{dt} \ = \frac{d}{dt}\left(\frac{1}{N}\sum_i I_i(t)\right) \ = \frac{dI}{dt} \tag{8}$$

For the right-hand side, we will separately see the average of the first, second and third term

$$-a \cdot dI_i = -a\frac{1}{N}\sum_i dI_i(t) = -a \cdot dI_i$$

$$\sum_j \ \mathrm{d}_{ij} I_j(t) = \sum_j \ \mathrm{d}I_j(t) = \frac{1}{N}\sum_i\sum_j \ \mathrm{d}I_j(t) = \sum_i \ \mathrm{d}\frac{1}{N}\sum_j I_j(t) = N \ \mathrm{d}I(t) \tag{9}$$

$$\begin{aligned}
\sum_j\sum_k p_{ijk} I_j(t) I_k(t) &= p\sum_j\sum_k I_j(t)I_k(t) \\
&= \frac{1}{N}\sum_i p\sum_j\sum_k I_j(t)I_k(t) \\
&= \sum_i p\frac{1}{N}\sum_j\sum_k I_j(t)I_k(t) \\
&= Np\sum_i \frac{1}{N}\sum_j I_j(t)\frac{1}{N}\sum_k I_k(t) \\
&= N^2 pI(t)^2,
\end{aligned}$$

For the random effect term, we can divide it into two parts, the collective random effect and the individual effect

$$\begin{aligned}
f_i(t) &= f(t) + \Delta f_i(t), \\
f_i(t) &= \frac{1}{N}\sum_i f_i(t) = f(t),
\end{aligned}$$

Here, δ the deviation of the individual effects from the collective effect.As f(t) corresponds to the publicity garnered by advertisement to public in society.We can assume these deviations to average out to nil.

$$\Delta f_i(t) = \frac{1}{N}\sum_i \Delta f_i(t) = 0 \tag{11}$$

The equation below is the final equation we get from plugging all the terms obtained.

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -aI(t) + DI(t) + PI(t)^2 + f(t) \tag{12}$$

8 Boundary Conditions

In order to initiate the model, we consider the start of the advertisement campaign for the films or products. This can vary from 20 to 60 days before the film release or product launch. At this initial time, either people are excited for the launch or have no prior knowledge or intention to purchase it. Therefore, we analysed the purchase intention for a variety of Initial Purchase Intention,

$$I(0) = 0, 20, 40, 60, 80, 100 \tag{13}$$

These boundary conditions signify the absence of purchase intention at the beginning of the advertisement campaign.

9 Parameters

Now, the derived Equation (12) involves the following coefficients :

- a
- D
- P

These parameters were derived from the Box Office data of movie Pirates of Caribbean. The Table shows its values :

Table 1: Parameters Used

Parameter Name	Parameter Value
a	0.3
D	0.0179
P	9.99E - 07

10 Analytical Solution

For the solution of the equation 5, we cannot proceed analytically if f(t) is not a constant function. this is because otherwise we would have got the below term of which right hand side cannot be solved further:

$$\int I(t)\left(-a + D + \frac{2PI(t)}{2}\right) dI(t) = \int dt + \int I(t)\left(-a + D + \frac{2PI(t)}{2}\right) f(t)dt$$

General Solution:
If we assume $f(t) = 0$, the differential equation becomes:

$$\frac{dI(t)}{dt} = -aI(t) + DI(t) + PI(t)^2.$$

This is a first-order nonlinear homogeneous ordinary differential equation. We can attempt to solve it analytically. To do this, we'll separate the variables and integrate:

$$\frac{dI(t)}{-aI(t)+DI(t)+PI(t)^2}=dt.$$

Now, let's use partial fraction decomposition to simplify the left-hand side:

$$\frac{dI(t)}{I(t)(D-a+PI(t))}=dt.$$

Now, we'll integrate both sides:

$$\int \frac{1}{I(t)(D-a+PI(t))}dI(t)=\int dt.$$

To solve the left-hand side integral, we can use a technique called partial fraction decomposition. We'll assume that $I(t)$ does not have any roots that make the denominator zero. The decomposition might look like this:

$$\frac{A}{I(t)}+\frac{B}{D-a+PI(t)}=\frac{1}{I(t)(D-a+PI(t))}.$$

Where A and B are constants to be determined. Now, we can find A and B by equating coefficients:

$$1=A\left(D-a+PI(t)\right)+BI(t).$$

Now, we'll solve for A and B:

$$A(D-a)=1 \Rightarrow A=\frac{1}{D-a},$$

$$B=\frac{P}{a-D}.$$

Now, we can rewrite the integral as:

$$\int \left(\frac{1}{D-a}\frac{1}{I(t)}+\frac{P}{a-D}\frac{1}{D-a+PI(t)}\right)dI(t)=\int dt.$$

Now, we can integrate both terms separately:

$$\frac{1}{D-a}\int \frac{1}{I(t)}dI(t)+\frac{P}{a-D}\int \frac{1}{D-a+PI(t)}dI(t)=\int dt.$$

Integrate the first term with respect to $I(t)$:

$$\frac{1}{D-a}\ln|I(t)|,$$

Now, for the second term, let $u=D-a+PI(t)$, then $du=PdI(t)$. So, the integral becomes:

$$\frac{1}{P}\int \frac{1}{u}du = \frac{1}{P}\ln|u|.$$

Now, substitute back for u:

$$\frac{1}{P}\ln|D - a + PI(t)|.$$

So, the general solution to the differential equation is:

$$\frac{1}{D-a}\ln|I(t)|+\frac{1}{a-D}\ln|D-a+PI(t)|-1D - a\ln|I(0)|+\frac{1}{a-D}\ln|D-a+PI(0)| = t,$$

Which boils down to below final equation:

$$\frac{1}{D-a}\ln|I(t)/|D - a + PI(t)||- = t,$$

This is the analytical solution for the differential equation when $f(t) = 0$.

11 Non-Dimensionalisation of the Model

In order to analyze the behavior of the model and gain insights into its underlying dynamics, we employ a non-dimensionalisation technique. This process involves introducing dimensionless variables and scales to simplify the differential equation.

Given the original differential equation representing the "hit" phenomenon:

$$\frac{dI(t)}{dt} = -aI(t) + DI(t) + PI(t)^2 + f(t)$$

where: $I(t)$ represents the ensemble-averaged purchase intention,
a is a rate constant,
D and P are coefficients of direct and indirect communication respectively.

We calculate the daily purchase-intention using equation (21) with the daily advertisement cost as input data for $\langle f(t)\rangle$.

We introduce dimensionless variables for time and purchase intention:

$$t' = \frac{t}{T}, \quad I' = \frac{I(t)}{I_c}$$

And their derivatives become:

$$\frac{dI}{dt} = \frac{1}{T}\frac{dI'}{dt'}$$

This is derived from the chain rule, considering $t' = \frac{t}{T}$:

$$\frac{dI'}{dt'} = \frac{dI'}{dt}\cdot\frac{dt}{dt'} = T\frac{dI'}{dt}$$

Substituting these into the original equation, we obtain the non-dimensionalised form:

$$\frac{1}{T}\frac{dI'}{dt'} = -aI' + DI' + P(I')^2 + \frac{f(t)}{f_c}$$

Here, T, I_c, and f_c are characteristic scales for time, purchase intention, and the advertisement cost, respectively.

This non-dimensionalised form allows us to analyze the system in terms of dimensionless parameters, providing valuable insights into the relative importance of different terms in the equation.

12 Algorithm Used

For the numerical analysis of the problem, we've picked up Runge-Kutta 7th Order 9 Stages Method.

A typical updation through the R-K Method looks like this:

$$y_i{+}_1 = y_i + Q \cdot step \qquad ()$$

where one Q is chosen as per the accuracy required while solving for the ODE.

For R-K 7th Order 9 Stages Method, the value of Q corresponds to,

$$Q = \frac{41K_1 + 216K_4 + 27K_5 + 272K_6 + 27K_7 + 216K_8 + 41K_9}{840} \qquad (1)$$

where,

$$K_1 = f(x_n, y_n) \qquad (2)$$

$$K_2 = f(x_n + \frac{h}{12}, y_n + \frac{K_1}{12}) \qquad (3)$$

$$K_3 = f(x_n + \frac{h}{12}, y_n + \frac{-10K_1 + 11K_2}{12}) \qquad (4)$$

$$K_4 = f(x_n + \frac{2h}{12}+, y_n + \frac{2K_3}{12}) \qquad (5)$$

$$K_5 = f(x_n + \frac{4h}{12}, y_n + \frac{157K_1 - 318K_2 + 4K_3 + 160K_4}{9}) \qquad (6)$$

$$K_6 = f(x_n + \frac{6h}{12}, y_n + \frac{-322K_1 + 199K_2 + 108K_3 - 131K_5}{30}) \qquad (7)$$

$$K_7 = f(x_n + \frac{8h}{12}, y_n + \frac{3158K_1}{45} - \frac{638K_2}{6} - \frac{23K_3}{2} + \frac{157K_4}{3} + \frac{157K_6}{45}) \qquad (8)$$

$$K_8 = f(x_n + \frac{10h}{12}, y_n - \frac{53K_1}{14} + \frac{38K_2}{7} - \frac{3K_3}{14} - \frac{65K_5}{72} + \frac{29K_7}{90}) \qquad (9)$$

$$K_9 = f(x_n+h, y_n+\frac{56K_1}{25}-\frac{283K_2}{14}-\frac{119K_3}{6}-\frac{26K_4}{7}+\frac{13K_5}{15}+\frac{149K_6}{32}-\frac{25K_7}{9}+\frac{27K_8}{25}) \tag{10}$$

Here h is the step value for the iteration.

Therefore, the Final Equation for the Numerical Analysis becomes:

$$y_i+_1 = y_i + \frac{41K_1 + 216K_4 + 27K_5 + 272K_6 + 27K_7 + 216K_8 + 41K_9}{840} \cdot h \tag{11}$$

13 Numerical Solution and Results

The plot of the variation of Purchase Intention with time for different initial purchase intentions comes as follows:

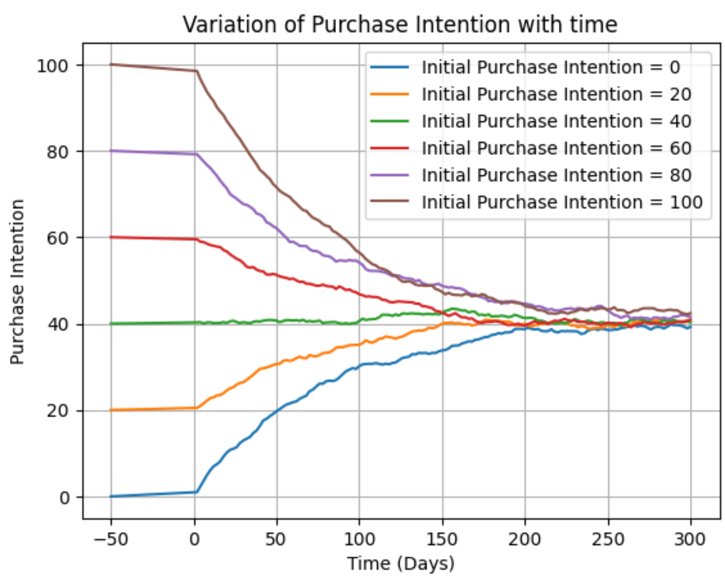

Figure 1: Numerical Analysis of the problem

1. It can be seen that even for different initial purchase intentions, the final P.I. is converging.

2. For negative time, the drop in purchase intention is not much significant because there's no word-of-mouth and random effect at play.

14 References

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