# Trapping and Beamloading in hybrid Plasma Wakefield Accelerator schemes

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$$I = \hat{I}_{x,y} W_{\mathrm{Laser}} / (2\sqrt{\pi}\sigma_t (\Delta x_{\mathrm{res}} \times 10^-6)^2)$$

In order to derive an expression for the trapping condition of a single electron in PWFA, one has to start with the equation of motion for such a single electron.

$$F = \frac{d\vec{p}}{dt} = q(\vec{E} \times \vec{B}) \tag{0.1}$$

with the electron charge $q$ electric field $\vec{E}$ and magnetic field $\vec{B}$

This leads to the single particle electron hamiltonian.

$$\begin{aligned}
\frac{dH}{dt} &= \frac{d}{dt}(\gamma m_e c^2) + \frac{d}{dt}(q\Phi) && (0.2)\\
&= \vec{v}\frac{d\vec{p}}{dt} + \frac{d}{dt}(q\Phi) && (0.3)\\
&= q\vec{v}(-\nabla\Phi - \frac{\partial\vec{A}}{\partial t}) + \frac{\vec{v} \times \vec{B}}{c} + \frac{d}{dt}(q\Phi) && (0.4)\\
&= q(\frac{d}{dt}\Phi - \vec{v}\vec{\nabla}\Phi - \vec{v}\frac{\partial\vec{A}}{\partial t}) && (0.5)\\
&= q(\frac{\partial\Phi}{\partial t} - \vec{v}\frac{\partial\vec{A}}{\partial t}) && (0.6)
\end{aligned}$$

If one assumes now, that the wake fields are constant during the trapping process, then

$$\begin{aligned}
H - v_\phi P_z &= \mathrm{const.} && (0.7)\\
\gamma mc^2 + \Psi - v_\phi p_z - v_\phi q A_z &= \mathrm{const.} && (0.8)\\
\gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} &= \mathrm{const.} && (0.9)\\
\gamma - v_\phi \frac{p_z}{mc^2} \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\hat{\Psi}} &= \mathrm{const.} && (0.10)
\end{aligned}$$

which is especially true for the hamiltonian.

$$\begin{aligned}\frac{d}{dt}H &= q(\frac{\partial\Phi}{\partial t} - \vec{v}\frac{\partial\vec{A}}{\partial t}) \\ &= -qv_\phi(\frac{\partial\Phi}{\partial z} - \vec{v}\frac{\partial\vec{A}}{\partial z})\end{aligned}$$

The trapping of a single electron in PWFA happens in a short time (i.e. a short propagation distance) compared to the timescales on which the wakefield changes its shape. An example for a distance over which the wakefield is modified is the betatron length FORUMLA !!!. This gives the convenient possibility to treat the problem in a frame moving along with the phase velocity of the wake $v_\phi$. Mathematically this can be done by finding a constant $C_\mathrm{H}$ with $\frac{dC_\mathrm{H}}{dt} = 0$ , so that $\frac{d}{dt}(H - C_\mathrm{H}) = 0$. W. Lu suggested in his thesis [citation needed !!!]

$$\begin{aligned}\frac{d}{dt}(H - v_\phi P_z) &= -qv_\phi(\frac{\partial\Phi}{\partial z} - \vec{v}\frac{\partial\vec{A}}{\partial z}) - qv_\phi(v_z\frac{\partial A_z}{\partial z} - \frac{\partial\Phi}{\partial z}) \\ &\approx qv_\phi(v_z\frac{\partial A_z}{\partial z} - v_z\frac{\partial A_z}{\partial z}) = 0\end{aligned}$$

$$\begin{aligned}H - v_\phi P_z &= const. \\ \gamma mc^2 + q\Phi - v_\phi p_z - v_\phi qA_z &= const. \\ \gamma + \frac{q\Phi}{mc^2} - v_\phi\frac{p_z}{mc^2} - v_\phi q\frac{A_z}{mc^2} &= const \\ \gamma - v_\phi\frac{p_z}{mc^2} - \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\Psi} &= const. \\ -const. + \gamma + v_\phi\frac{p_z}{mc^2} &= \Psi\end{aligned}$$

$$\Psi_f - Psi_i = \gamma_f - \gamma_i - v_\phi\frac{\gamma_f m v_f}{mc^2}$$