Trapping and Beamloading in hybrid Plasma Wakefield Accelerator schemes

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[?] It is worth noting that the Debye Shielding assumes a thermalized plasma and is not the right idea of a plasma reaction to a rapid change in charge on a femtosecond timescale. The more appropriate figure of merit in that case is the depth an electromagnetic wave

with frequency ω can permeate the plasma fluid.

The transparency of a plasma to a electromagnetic wave can be calculated by applying the Maxwell equations **??** so that one gets the differenctial equation.

$$\frac{E_z}{E_0} = \frac{k_p}{2}\xi \tag{0.1}$$

$$\frac{E_r - B_\theta}{E_0} = \frac{k_p}{2}r \tag{0.2}$$

$$\frac{\partial E_z}{\partial r} = \frac{\partial E_r}{\partial z} \approx 0 \tag{0.3}$$

If one assumes now, that the wake fields are constant during the trapping process, then

$$\begin{aligned} H - v_\phi P_z &= \text{const.} && (0.4)\\ \gamma mc^2 + \Psi - v_\phi p_z - v_\phi q A_z &= \text{const.} && (0.5)\\ \gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} &= \text{const.} && (0.6)\\ \gamma - v_\phi \frac{p_z}{mc^2} \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\hat{\Psi}} &= \text{const.} && (0.7) \end{aligned}$$

which is especially true for the hamiltonian.

$$\begin{aligned} \frac{d}{dt}H &= q(\frac{\partial \Phi}{\partial t} - \vec{v}\frac{\partial \vec{A}}{\partial t}) \\ &= -qv_\phi(\frac{\partial \Phi}{\partial z} - \vec{v}\frac{\partial \vec{A}}{\partial z}) \end{aligned}$$

The trapping of a single electron in PWFA happens in a short time (i.e. a short propagation distance) compared to the timescales on which the wakefield changes its shape. An example for a distance over which the wakefield is modified is the betatron length FORUMLA !!!. This gives the convenient possibility to treat the problem in a frame moving along with the phase velocity of the wake v_ϕ. Mathematically this can be done by finding a constant C_H with $\frac{dC_\mathrm{H}}{dt} = 0$, so that $\frac{d}{dt}(H - C_\mathrm{H}) = 0$. W. Lu suggested in his thesis [citation needed !!!]

$$\begin{aligned}\frac{d}{dt}(H - v_\phi P_z) &= -qv_\phi(\frac{\partial \Phi}{\partial z} - \vec{v}\frac{\partial \vec{A}}{\partial z}) - qv_\phi(v_z\frac{\partial A_z}{\partial z} - \frac{\partial \Phi}{\partial z}) \\ &\approx qv_\phi(v_z\frac{\partial A_z}{\partial z} - v_z\frac{\partial A_z}{\partial z}) = 0\end{aligned}$$

$$\begin{aligned}H - v_\phi P_z &= const. \\ \gamma mc^2 + q\Phi - v_\phi p_z - v_\phi q A_z &= const. \\ \gamma + \frac{q\Phi}{mc^2} - v_\phi\frac{p_z}{mc^2} - v_\phi q\frac{A_z}{mc^2} &= const \\ \gamma - v_\phi\frac{p_z}{mc^2} - \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\Psi} &= const. \\ -const. + \gamma + v_\phi\frac{p_z}{mc^2} &= \Psi\end{aligned}$$

$$\Psi_f - \Psi_i = \gamma_f - \gamma_i - v_\phi\frac{\gamma_f m v_f}{mc^2}$$

$$Q'(z) = \int_{-\infty}^{\infty} 1 - exp(W_\mathrm{ADK}(z,t))\ dt$$

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