Trapping and Beamloading in hybrid Plasma Wakefield Accelerator schemes

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$$\Phi_\mathrm{f} = R_\mathrm{i,j}\Phi_\mathrm{f}$$

$$\begin{pmatrix} x_\mathrm{f} \\ x'_\mathrm{f} \\ y_\mathrm{f} \\ y'_\mathrm{f} \\ z_\mathrm{f} \\ \delta_\mathrm{f} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix} \begin{pmatrix} x_\mathrm{i} \\ x'_\mathrm{i} \\ y_\mathrm{i} \\ y'_\mathrm{i} \\ z_\mathrm{i} \\ \delta_\mathrm{i} \end{pmatrix} \tag{0.1}$$

$$\sqrt{\gamma\epsilon}$$

$$-\alpha\sqrt{\epsilon/\beta}$$

$$\sqrt{\beta\epsilon}$$

$$I = \hat{I}_{x,y} W_\mathrm{Laser}/(2\sqrt{\pi}\sigma_t(\Delta x_\mathrm{res} \times 10^-6)^2)$$

In order to derive an expression for the trapping condition of a single electron in PWFA, one has to start with the equation of motion for such a single electron.

$$F = \frac{d\vec{p}}{dt} = q(\vec{E} \times \vec{B}) \tag{0.2}$$

with the electron charge q electric field $\vec{E}$ and magnetic field $\vec{B}$

This leads to the single particle electron hamiltonian.

$$\begin{aligned} \frac{dH}{dt} &= \frac{d}{dt}(\gamma m_e c^2) + \frac{d}{dt}(q\Phi) && (0.3) \\ &= \vec{v}\frac{d\vec{p}}{dt} + \frac{d}{dt}(q\Phi) && (0.4) \\ &= q\vec{v}(-\nabla\Phi - \frac{\partial\vec{A}}{\partial t}) + \frac{\vec{v} \times \vec{B}}{c} + \frac{d}{dt}(q\Phi) && (0.5) \\ &= q(\frac{d}{dt}\Phi - \vec{v}\vec{\nabla}\Phi - \vec{v}\frac{\partial\vec{A}}{\partial t}) && (0.6) \\ &= q(\frac{\partial\Phi}{\partial t} - \vec{v}\frac{\partial\vec{A}}{\partial t}) && (0.7) \end{aligned}$$

If one assumes now, that the wake fields are constant during the trapping process, then

$$H - v_\phi P_z = \text{const.} \tag{0.8}$$
$$\gamma mc^2 + \Psi - v_\phi p_z - v_\phi q A_z = \text{const.} \tag{0.9}$$
$$\gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} = \text{const.} \tag{0.10}$$
$$\gamma - v_\phi \frac{p_z}{mc^2} \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\hat{\Psi}} = \text{const.} \tag{0.11}$$

which is especially true for the hamiltonian.

$$\begin{aligned} \frac{d}{dt} H &= q(\frac{\partial \Phi}{\partial t} - \vec{v}\frac{\partial \vec{A}}{\partial t}) \\ &= -q v_\phi (\frac{\partial \Phi}{\partial z} - \vec{v}\frac{\partial \vec{A}}{\partial z}) \end{aligned}$$

The trapping of a single electron in PWFA happens in a short time (i.e. a short propagation distance) compared to the timescales on which the wakefield changes its shape. An example for a distance over which the wakefield is modified is the betatron length FORUMLA !!!. This gives the convenient possibility to treat the problem in a frame moving along with the phase velocity of the wake v_ϕ. Mathematically this can be done by finding a constant C_H with $\frac{dC_\mathrm{H}}{dt} = 0$, so that $\frac{d}{dt}(H - C_\mathrm{H}) = 0$. W. Lu suggested in his thesis [citation needed !!!]

$$\begin{aligned} \frac{d}{dt}(H - v_\phi P_z) &= -q v_\phi (\frac{\partial \Phi}{\partial z} - \vec{v}\frac{\partial \vec{A}}{\partial z}) - q v_\phi (v_z \frac{\partial A_z}{\partial z} - \frac{\partial \Phi}{\partial z}) \\ &\approx q v_\phi (v_z \frac{\partial A_z}{\partial z} - v_z \frac{\partial A_z}{\partial z}) = 0 \end{aligned}$$

$$\begin{aligned} H - v_\phi P_z &= const. \\ \gamma mc^2 + q\Phi - v_\phi p_z - v_\phi q A_z &= const. \\ \gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} - v_\phi q \frac{A_z}{mc^2} &= const \\ \gamma - v_\phi \frac{p_z}{mc^2} - \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\Psi} &= const. \\ -const. + \gamma + v_\phi \frac{p_z}{mc^2} &= \Psi \end{aligned}$$

$$\Psi_f - \Psi_i = \gamma_f - \gamma_i - v_\phi \frac{\gamma_f m v_f}{mc^2}$$

$$Q'(z) = \int_{-\infty}^{\infty} 1 - exp(W_\mathrm{ADK}(z,t))\ dt$$

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