

Trapping and Beamloading in hybrid Plasma Wakefield Accelerator schemes

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17. Oktober 2016

INHALTSVERZEICHNIS

$$I = \hat{I}_{x,y} W_{\text{Laser}} / (2\sqrt{\pi} \sigma_t (\Delta x_{\text{res}} \times 10^{-6})^2)$$

In order to derive an expression for the trapping condition of a single electron in PWFA, one has to start with the equation of motion for such a single electron.

$$F = \frac{d\vec{p}}{dt} = q(\vec{E} \times \vec{B}) \quad (0.1)$$

with the electron charge q electric field \vec{E} and magnetic field \vec{B}
This leads to the single particle electron hamiltonian.

$$\frac{dH}{dt} = \frac{d}{dt}(\gamma m_e c^2) + \frac{d}{dt}(q\Phi) \quad (0.2)$$

$$= \vec{v} \frac{d\vec{p}}{dt} + \frac{d}{dt}(q\Phi) \quad (0.3)$$

$$= q\vec{v}(-\nabla\Phi - \frac{\partial \vec{A}}{\partial t}) + \frac{\vec{v} \times \vec{B}}{c} + \frac{d}{dt}(q\Phi) \quad (0.4)$$

$$= q(\frac{d}{dt}\Phi - \vec{v}\nabla\Phi - \vec{v}\frac{\partial \vec{A}}{\partial t}) \quad (0.5)$$

$$= q(\frac{\partial \Phi}{\partial t} - \vec{v}\frac{\partial \vec{A}}{\partial t}) \quad (0.6)$$

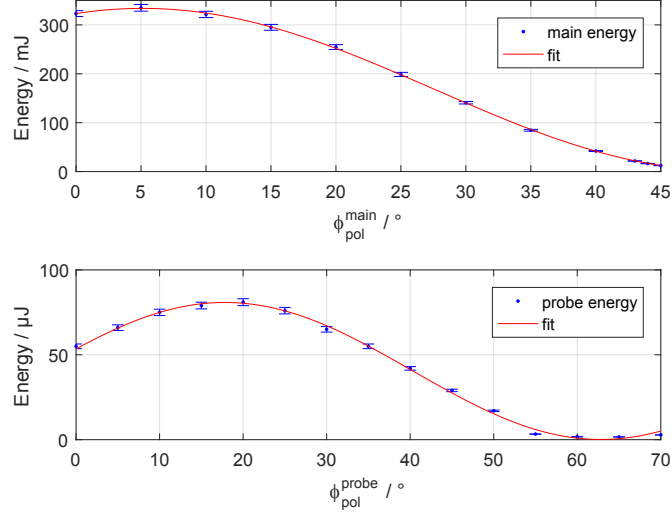
If one assumes now, that the wake fields are constant during the trapping process, then

$$H - v_\phi P_z = \text{const.} \quad (0.7)$$

$$\gamma mc^2 + \Psi - v_\phi p_z - v_\phi q A_z = \text{const.} \quad (0.8)$$

$$\gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} = \text{const.} \quad (0.9)$$

$$\gamma - v_\phi \frac{p_z}{mc^2} \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\hat{\Psi}} = \text{const.} \quad (0.10)$$



which is especially true for the hamiltonian.

$$\begin{aligned} \frac{d}{dt}H &= q\left(\frac{\partial\Phi}{\partial t} - \vec{v}\frac{\partial\vec{A}}{\partial t}\right) \\ &= -qv_\phi\left(\frac{\partial\Phi}{\partial z} - \vec{v}\frac{\partial\vec{A}}{\partial z}\right) \end{aligned}$$

The trapping of a single electron in PWFA happens in a short time (i.e. a short propagation distance) compared to the timescales on which the wakefield changes its shape. An example for a distance over which the wakefield is modified is the betatron length $L_{\beta} = \frac{2\pi}{k_{\beta}}$. This gives the convenient possibility to treat the problem in a frame moving along with the phase velocity of the wake v_ϕ . Mathematically this can be done by finding a constant C_H with $\frac{dC_H}{dt} = 0$, so that $\frac{d}{dt}(H - C_H) = 0$. W. Lu suggested in his thesis [citation needed !!!]

$$\begin{aligned} \frac{d}{dt}(H - v_\phi P_z) &= -qv_\phi\left(\frac{\partial\Phi}{\partial z} - \vec{v}\frac{\partial\vec{A}}{\partial z}\right) - qv_\phi\left(v_z\frac{\partial A_z}{\partial z} - \frac{\partial\Phi}{\partial z}\right) \\ &\approx qv_\phi\left(v_z\frac{\partial A_z}{\partial z} - v_z\frac{\partial A_z}{\partial z}\right) = 0 \end{aligned}$$

$$\begin{aligned}
H - v_\phi P_z &= \text{const.} \\
\gamma mc^2 + q\Phi - v_\phi p_z - v_\phi q A_z &= \text{const.} \\
\gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} - v_\phi q \frac{A_z}{mc^2} &= \text{const} \\
\gamma - v_\phi \frac{p_z}{mc^2} - \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\Psi} &= \text{const.} \\
-\text{const.} + \gamma + v_\phi \frac{p_z}{mc^2} &= \Psi
\end{aligned}$$

$$\Psi_f - \Psi_i = \gamma_f - \gamma_i - v_\phi \frac{\gamma_f m v_f}{mc^2}$$

$$Q'(z) = \int_{-\infty}^{\infty} 1 - \exp(W_{\text{ADK}}(z, t)) \, dt$$

LITERATURVERZEICHNIS