# Trapping and Beamloading in hybrid Plasma Wakefield Accelerator schemes

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$$
\begin{aligned}
f_{\mathrm{f}}(\xi) &= f(\xi_{\mathrm{f}}(\xi_{\mathrm{i}}))\frac{\partial \xi_{\mathrm{i}}}{\partial \xi_{\mathrm{f}}} && (0.1)\\
&= \frac{1}{\sigma_\xi\sqrt{2\pi}}e^{-\frac{(\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}})^2}{2\sigma_\xi^2}}\frac{\xi_{\mathrm{i}}}{\sqrt{\xi_{\mathrm{i}}^2+\alpha_{\mathrm{t}}}} && (0.2)\\
&= \frac{1}{\sigma_\xi\sqrt{2\pi}}e^{-\frac{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}{2\sigma_\xi^2}}\frac{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}}{\sqrt{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}^2+\alpha_{\mathrm{t}}}} && (0.3)\\
&= \frac{1}{\sigma_\xi\sqrt{2\pi}}e^{-\frac{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}{2\sigma_\xi^2}}\frac{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}}{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}+\alpha_{\mathrm{t}}}} && (0.4)\\
&= \frac{1}{\sigma_\xi\sqrt{2\pi}}e^{-\frac{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}{2\sigma_\xi^2}}\frac{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}}{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}+\alpha_{\mathrm{t}}}} && (0.5)\\
&= \frac{1}{\sigma_\xi\sqrt{2\pi}}e^{-\frac{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}{2\sigma_\xi^2}}\frac{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}}{\sqrt{\xi_{\mathrm{f}}^2}} && (0.6)\\
&= \frac{1}{\sigma_\xi\sqrt{2\pi}}e^{-\frac{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}{2\sigma_\xi^2}}\frac{\sqrt{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}}{\xi_{\mathrm{f}}} && (0.7)\\
&= \frac{1}{\sigma_\xi\sqrt{2\pi}}e^{-\frac{\xi_{\mathrm{f}}^2-\alpha_{\mathrm{t}}}{2\sigma_\xi^2}}\sqrt{1-\frac{\alpha_{\mathrm{t}}}{\xi_{\mathrm{f}}^2}} && (0.8)\\
& && (0.9)
\end{aligned}
$$

asdasd

$$\sqrt{\gamma\epsilon}$$

$$-\alpha\sqrt{\epsilon/\beta}$$

$$\sqrt{\beta\epsilon}$$

$$I = \hat{I}_{x,y}W_{\mathrm{Laser}}/(2\sqrt{\pi}\sigma_t(\Delta x_{\mathrm{res}}\times 10^-6)^2)$$

In order to derive an expression for the trapping condition of a single electron in PWFA, one has to start with the equation of motion for such a single electron.

$$F = \frac{d\vec{p}}{dt} = q(\vec{E}\times\vec{B}) \tag{0.10}$$

with the electron charge $q$ electric field $\vec{E}$ and magnetic field $\vec{B}$

This leads to the single particle electron hamiltonian.

$$\frac{dH}{dt} = \frac{d}{dt}(\gamma m_e c^2) + \frac{d}{dt}(q\Phi) \tag{0.11}$$
$$= \vec{v}\frac{d\vec{p}}{dt} + \frac{d}{dt}(q\Phi) \tag{0.12}$$
$$= q\vec{v}(-\nabla\Phi - \frac{\partial\vec{A}}{\partial t}) + \frac{\vec{v}\times\vec{B}}{c} + \frac{d}{dt}(q\Phi) \tag{0.13}$$
$$= q(\frac{d}{dt}\Phi - \vec{v}\vec{\nabla}\Phi - \vec{v}\frac{\partial\vec{A}}{\partial t}) \tag{0.14}$$
$$= q(\frac{\partial\Phi}{\partial t} - \vec{v}\frac{\partial\vec{A}}{\partial t}) \tag{0.15}$$

If one assumes now, that the wake fields are constant during the trapping process, then

$$H - v_\phi P_z = \text{const.} \tag{0.16}$$
$$\gamma mc^2 + \Psi - v_\phi p_z - v_\phi q A_z = \text{const.} \tag{0.17}$$
$$\gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} = \text{const.} \tag{0.18}$$
$$\gamma - v_\phi \frac{p_z}{mc^2} \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\hat{\Psi}} = \text{const.} \tag{0.19}$$

which is especially true for the hamiltonian.

$$\frac{d}{dt}H = q(\frac{\partial\Phi}{\partial t} - \vec{v}\frac{\partial\vec{A}}{\partial t})$$
$$= -qv_\phi(\frac{\partial\Phi}{\partial z} - \vec{v}\frac{\partial\vec{A}}{\partial z})$$

The trapping of a single electron in PWFA happens in a short time (i.e. a short propagation distance) compared to the timescales on which the wakefield changes its shape. An example for a distance over which the wakefield is modified is the betatron length FORUMLA !!!. This gives the convenient possibility to treat the problem in a frame moving along with the phase velocity of the wake $v_\phi$. Mathematically this can be done by finding a constant $C_\mathrm{H}$ with $\frac{dC_\mathrm{H}}{dt} = 0$ , so that $\frac{d}{dt}(H - C_\mathrm{H}) = 0$. W. Lu suggested in his thesis [citation needed !!!]

$$\frac{d}{dt}(H - v_\phi P_z) = -qv_\phi(\frac{\partial\Phi}{\partial z} - \vec{v}\frac{\partial\vec{A}}{\partial z}) - qv_\phi(v_z\frac{\partial A_z}{\partial z} - \frac{\partial\Phi}{\partial z})$$
$$\approx qv_\phi(v_z\frac{\partial A_z}{\partial z} - v_z\frac{\partial A_z}{\partial z}) = 0$$

$$
\begin{aligned}
H - v_\phi P_z &= const. \\
\gamma m c^2 + q\Phi - v_\phi p_z - v_\phi q A_z &= const. \\
\gamma + \frac{q\Phi}{mc^2} - v_\phi \frac{p_z}{mc^2} - v_\phi q \frac{A_z}{mc^2} &= const \\
\gamma - v_\phi \frac{p_z}{mc^2} - \underbrace{\frac{q}{mc^2}(\Phi - v_\phi A_z)}_{\Psi} &= const. \\
-const. + \gamma + v_\phi \frac{p_z}{mc^2} &= \Psi
\end{aligned}
$$

$$
\Psi_f - \Psi_i = \gamma_f - \gamma_i - v_\phi \frac{\gamma_f m v_f}{mc^2}
$$

$$
Q'(z) = \int_{-\infty}^{\infty} 1 - exp(W_{\mathrm{ADK}}(z,t))\ dt
$$

# LITERATURVERZEICHNIS