

DZ 1

③ a)

SJEČIŠTE ZRAKE I TROKUTA

Dabc, zrakom pogleda e+td

baricentrične koordinate $\Rightarrow \alpha + \beta + \gamma = 1$:

$$e+td = \alpha a + \beta b + \gamma c = \alpha + \beta(b-a) + \gamma(c-a)$$

Razpišemo po koordinatama u sustav

$$\begin{bmatrix} x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix}$$

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ x_a - x_b & x_a - x_c & x_d \\ y_a - y_b & y_a - y_c & y_d \\ z_a - z_b & z_a - z_c & z_d \end{bmatrix} = \begin{bmatrix} A_1 & A_2 & A_3 \\ a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} = A \quad \begin{bmatrix} \beta \\ \gamma \\ t \end{bmatrix} = X \quad \begin{bmatrix} x_a - x_e \\ y_a - y_e \\ z_a - z_e \end{bmatrix} = \begin{bmatrix} i \\ j \\ k \end{bmatrix} = b$$

Cramerovo pravilo:

$$\beta = \frac{\det[b A_2 A_3]}{\det A}$$

$$\gamma = \frac{\det[A_1 b A_3]}{\det A}$$

$$t = \frac{\det[A_1 A_2 b]}{\det A}$$

$$\beta = \frac{j(e-hf) + k(gf-di) + l(dh-eg)}{a(ei-hf) + b(gf-di) + c(dh-eg)}$$

$$\gamma = \frac{i(bk-jb) + h(jc-al) + g(bl-kc)}{a(ei-hf) + b(gf-di) + c(dh-eg)}$$

$$t = -\frac{f(bk-jb) + e(jc-al) + d(bl-kc)}{a(ei-hf) + b(gf-di) + c(dh-eg)}$$

, ako $\alpha, \beta, \gamma \in (0, 1)$: $t > 0$

\Rightarrow ZRAKA SJEČE Δ u točki

$$\beta = e+td$$

SJECIŠTE ZRAKE I SFERE

$S(c, R)$, centar sfere $c = (x_c, y_c, z_c)$, R radijus

$$(p-c)(p-c) - R^2 = 0$$

$$\Rightarrow (e+td-c)(e+td-c) - R^2 = 0$$

$$(d \cdot d)t^2 + 2d(e-c)t + (e-c)^2 - R^2 = 0$$

kv. jedn.

$$t_{1,2} = \frac{-d(e-c) \pm \sqrt{d \cdot (e-c)^2 - (d \cdot d) \cdot D}}{d \cdot d}$$

ako $D \geq 0$ zraka siječe sferu.

b) zraka $e + \lambda d$

pozitivan cilindar

$$0 \leq z \leq h$$

$$e = (e_1, e_2, e_3)$$

$$d = (d_1, d_2, d_3)$$

$$\frac{x^2}{r^2} + \frac{y^2}{s^2} = 1$$

projiciramo zraku na ravninu

$$e_{xy} = (e_1, e_2) \quad , \quad d_{xy} = (d_1, d_2)$$

odredimo tačku sjecišta projekcije zrake s eliptičnom površinom pomoću parametarskih jednačina:

$$x = e_1 + \lambda d_1$$

$$y = e_2 + \lambda d_2$$

$$\frac{(e_1 + \lambda d_1)^2}{r^2} + \frac{(e_2 + \lambda d_2)^2}{s^2} = 1 \quad (\text{riješimo za } \lambda) \quad (\lambda \in [0, \infty) \text{ jer } 0 \leq z \leq h)$$

\Rightarrow Ako postoji realno pozitivno rješenje za λ , zraka siječe cilindar, u suprotnom ne siječe