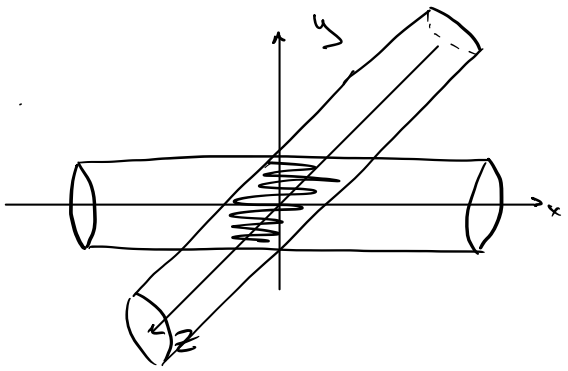
38.1 б
2 б
4 б
5 а,б
6 а,б



№ 38.1

б) $x^2+z^2=a^2$

$y^2+z^2=a^2$

Тело вокруг Oz $\quad z\in[0;a]$

$x^2=a^2-z^2=y^2 \Rightarrow x=y=\sqrt{a^2-z^2}$

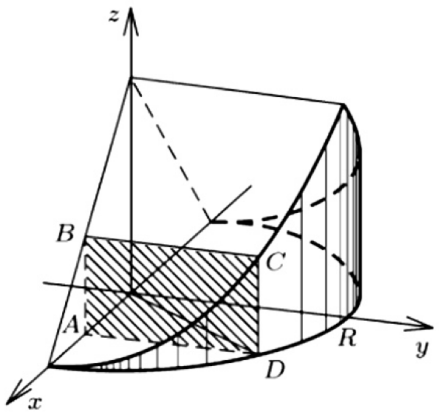
$S(z)=xy=a^2-z^2$ $\qquad V=8\int_0^a (a^2-z^2)\,dz=8\left(a^2z-\frac{z^3}{3}\right)\Big|_0^a=\frac{16}{3}a^3$

№ 38.2

б) $x^2+y^2=R^2$, $\frac{x}{R}+\frac{z}{H}=1$, $\frac{x}{R}-\frac{z}{H}=-1$, $y=0$, $z=0$

$y=\sqrt{R^2-x^2}$ $\qquad z=H\left(1-\frac{x}{R}\right)$

$S=yz=H\left(1-\frac{x}{R}\right)\sqrt{R^2-x^2}$



$$\frac{H}{R}\int_0^R (R-x)\sqrt{R^2-x^2}\,dx = H\int_0^R \sqrt{R^2-x^2}\,dx-\frac{H}{2R}\int_0^R \sqrt{R^2-x^2}\,dx^2 =$$

$$= H\left(\frac{x}{R}\sqrt{R^2-x^2}+\frac{R^2}{2}\arcsin\frac{x}{R}\right)\Big|_0^R+\frac{H}{2R}\cdot\frac{2}{3}\left(R^2-x^2\right)^{3/2}\Big|_0^R=\frac{HR^2}{2}\cdot\frac{\pi}{2}-\frac{H}{3}R^2=$$

$$=HR^2\left(\frac{\pi}{4}-\frac{1}{3}\right)$$

№ 38.4

б) $y=e^{-x}\sqrt{\sin x}$ $\qquad 0\le x<+\infty$ $\qquad V-?$

$$\pi\int_0^{+\infty} y^2(x)\,dx=\pi\int_0^{+\infty} e^{-2x}\sin x\,dx=\frac{\pi}{5}$$

$$I=-e^{-2x}\cos x-2\int_0^{+\infty} e^{-2x}\cos x\,dx=-e^{-2x}\cos x-2\left(e^{-2x}\sin x+2\int_0^{+\infty} e^{-2x}\sin x\,dx\right)=-e^{-2x}(\cos x-2\sin x)-4I$$

$$I=-\frac{e^{-2x}}{5}(\cos x+2\sin x)\Big|_0^{+\infty}=\frac{1}{5}$$

$$2\pi\int_0^{+\infty} xe^{-x}\sqrt{\sin x}\,dx$$??

№ 38.5

б) $x=a\cos^3 t$, $y=a\sin^3 t$ $\qquad 0\le t\le 2\pi$

$x'=-3a\cos^2 t\sin t$

$y'=3a\sin^2 t\cos t$

$$-\pi\int_{T_0}^{T} y^2(t)x'(t)\,dt=-\pi\int_0^{\pi/2} a^2\sin^6 t(-3a\cos^2 t\sin t)=3\pi a^3\int_0^{\pi/2}(1-\cos^2 t)^3\cos^2 t\,d(\cos t)=-3\pi\left(\frac{\cos^3 t}{3}-\frac{3\cos^5 t}{5}+\right.$$

$$\left.+\frac{3\cos^7 t}{7}-\frac{\cos^9 t}{9}\right)\Big|_0^{\pi/2}=\frac{16\pi a^3}{105} \qquad V=\frac{32\pi a^3}{105}$$

№ 38.6

а) $S = 2\pi a^2 \int_{-b}^{b} \cos\frac{\pi x}{2b} \sqrt{\frac{1}{a^2} + \left(\frac{\pi}{2b}\sin\frac{\pi x}{2b}\right)^2}\, dx = 2\pi a^2 \cdot \frac{4b^2}{\pi^2} \int_{-b}^{b} \sqrt{\frac{1}{a^2} + \left(\frac{\pi}{2b}\sin\frac{\pi x}{2b}\right)^2}\, d\left(\frac{\pi}{2b}\sin\frac{\pi x}{2b}\right) =$

$$2a\sqrt{4b^2 + a^2\pi^2} + \frac{8b^2}{\pi} \ln\left(\sqrt{1 + \frac{a^2\pi^2}{4b^2}} + \frac{a\pi}{2b}\right)$$

б) $x^{2/3} + y^{2/3} = a^{2/3}$

$x = a\cos^3 t \qquad x' = -3a\cos^2 t \sin t$

$y = a\sin^3 t \qquad y' = 3a\sin^2 t \cos t$

$$S = 2\pi \int_0^{\pi/2} a\sin^3 t \sqrt{9a^2\cos^4 t \sin^2 t + 9a^2 \sin^4 t \cos^2 t}\, dt = 6a^2\pi \int_0^{\pi/2} \sin^3 t \sin t \cdot \cos t\, dt = 6a^2\pi \int_0^{\frac{\pi}{2}} \sin^4 t\, d(\sin t) = \frac{6a^2\pi}{5} \sin^5 t \Big|_0^{\pi/2} =$$

$$= \frac{6a^2\pi}{5} \qquad S = \frac{12a^2\pi}{5}$$