No: 37.1

б) $y = a\ln\frac{a^2}{a^2-x}$ $\quad 0\le x\le b<a$

$y' = \frac{2ax}{a^2-x^2}$ $\quad \int_0^b \sqrt{\frac{a^4 - \cancel{2a^2x^2} + x^4 + \cancel{4}a^2x^2}{(a^2-x^2)^2}}\,dx = \int_0^b \frac{a^2+x^2}{a^2-x^2}dx = \int_0^b \frac{2a^2}{a^2-x^2}dx - \int_0^b \frac{\cancel{a^2-x^2}}{\cancel{a^2-x^2}}dx =$

$= \cancel{2}a^{\cancel{2}}\cdot\frac{1}{\cancel{2a}}\ln\left|\frac{x+a}{x-a}\right|\Big|_0^b - x\Big|_0^b = a\ln\left|\frac{b+a}{b-a}\right| - b$

г) $x = a\ln\frac{a+\sqrt{a^2-y^2}}{y} - \sqrt{a^2-y^2}$

$x' = a\frac{\cancel{y}}{\cancel{a+\sqrt{a^2-y^2}}}\cdot \frac{-\frac{1}{\cancel{2}\sqrt{a^2-y^2}}\cancel{2}y^2 - (a+\sqrt{a^2-y^2})}{y^2} = -\frac{a^2}{\sqrt{a^2-y^2}\,y} + \frac{y}{\sqrt{a^2-y^2}} = \frac{-a^2+y^2}{y\sqrt{a^2-y^2}} = -\frac{\sqrt{a^2-y^2}}{y}$

$\frac{\frac{\cancel{y^2} + \sqrt{a^2-y^2}\,a + a^2 - \cancel{y^2}}{\sqrt{a^2-y^2}}}{y} = \frac{a\left(\sqrt{a^2-y^2}\cancel{+a}\right)}{\sqrt{a^2-y^2}\;y}$

$\int_b^a \sqrt{\frac{\cancel{y^2} + a^2 - \cancel{y^2}}{y^2}}\,dy = \int_b^a \frac{a}{y}dy = a\ln|y|\Big|_b^a = a\ln\left(\frac{a}{b}\right)$

е) $y = \frac{1}{2}\left(\ln(\cos x) + \ln(\sin x)\right)$

$y' = \frac{1}{2}\left(-\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) = \frac{-\sin^2 x + \cos^2 x}{2\cos x\sin x} = \frac{\cos 2x}{\sin 2x} = \operatorname{ctg} 2x$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1+\operatorname{ctg}^2 2x}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin 2x}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{1-\cos^2 2x}dx = -\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1-\cos^2 2x}d\cos 2x = -\frac{1}{2}\left(-\frac{1}{2}\ln\left|\frac{\cos 2x-1}{\cos 2x+1}\right|\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} =$

$= -\frac{1}{2}\left(-\frac{1}{2}\ln\left|\frac{-\frac{1}{2}-1}{-\frac{1}{2}+1}\right| + \frac{1}{2}\ln\left|\frac{\frac{1}{2}-1}{\frac{1}{2}+1}\right|\right) = \frac{1}{4}\ln 3 + \frac{1}{4}\ln 3 = \frac{1}{2}\ln 3$

No: 37.2

б) $x = \cos^4 t \qquad y = \sin^4 t \qquad t\in\left[0;\frac{\pi}{2}\right]$

$x' = -4\cos^3 t\sin t$ $\qquad (x')^2 + (y')^2 = 16\sin^2 t\cos^2 t(\sin^4 t + \cos^4 t) = \cancel{4}^{2}\sin^2 2t\left(\frac{2-\sin^2 2t}{\cancel{2}}\right) =$

$y' = 4\sin^3 t\cos t$

$= 2\sin^2 2t(1+\cos^2 2t)$

$\int_0^{\frac{\pi}{2}} \sqrt{2\sin^2 2t(1+\cos^2 2t)}\,dt = \sqrt{2}\int_0^{\frac{\pi}{2}} \sin 2t\sqrt{1+\cos^2 2t}\,dt = -\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \sqrt{1+\cos^2 2t}\,d(\cos 2t) =$

$= \cancel{-}\frac{1}{2\sqrt{2}}\left(\cos 2t\sqrt{1+\cos^2 2t} + \ln|\cos 2t + \sqrt{1+\cos^2 2t}|\right)\Big|_0^{\frac{\pi}{2}} = \frac{1}{2\sqrt{2}}\left(2\sqrt{2} + \ln\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = 1 + \frac{1}{\sqrt{2}}\ln(\sqrt{2}+1)$

$-\sqrt{2} + \ln\sqrt{2}-1 - (\sqrt{2} + \ln\sqrt{2}+1)) =$

$= \cancel{-}\left(2\sqrt{2} + \ln\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

e) $x = e^t\cos t$ $\quad x' = e^t\cos t - e^t\sin t$ $\quad (x')^2 + (y')^2 = e^{2t}\cos^2 t - \cancel{2e^{t}\cos t\sin t} + e^{2t}\sin^2 t + e^{2t}\sin^2 t + \cancel{2e^{t}\cos t\sin t} +$

$y = e^t\sin t$ $\quad y' = e^t\sin t + e^t\cos t$ $\quad + e^{2t}\cos^2 t = e^{2t}(\cos^2 t + \sin^2 t + \sin^2 t + \cos^2 t) = 2e^{2t}$

$$\int_0^{\ln\pi} \sqrt{2e^{2x}}\,dx = \sqrt{2}\,e^x\Big|_0^{\ln\pi} = \sqrt{2}\pi - \sqrt{2}$$

No: 37.3

b) $r = a\sin^3\frac{\varphi}{3}$

$$\int_0^{3\pi} \sqrt{a^2\sin^6\frac{\varphi}{3} + a^2\sin^4\frac{\varphi}{3}\cos^2\frac{\varphi}{3}}\,d\varphi = a\int_0^{3\pi} \sin^2\frac{\varphi}{3}\sqrt{\sin^2\frac{\varphi}{3} + \cos^2\frac{2\varphi}{3}}\,d\varphi = \frac{a}{2}\int_0^{3\pi}\left(1 - \cos\frac{2\varphi}{3}\right)d\varphi = \frac{a}{2}\varphi\Big|_0^{3\pi} - 0 = \frac{3\pi a}{2}$$

e) $r = \frac{1}{1+\cos\varphi}$ $\quad -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$ $\quad r' = \frac{\sin\varphi}{(1+\cos\varphi)^2}$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\frac{1}{(1+\cos\varphi)^2} + \frac{\sin^2\varphi}{(1+\cos\varphi)^4}}\,d\varphi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+2\cos\varphi+\cos^2\varphi+\sin^2\varphi}{(1+\cos\varphi)^4}\right)^{1/2}d\varphi = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\frac{\varphi}{2}}{(1+\cos\varphi)^2}\,d\varphi = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cancel{\cos\frac{\varphi}{2}}}{\cos^4\frac{\varphi}{2}}\,d\varphi =$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\cos^3\frac{\varphi}{2}}\,d\varphi = \left\{\begin{array}{l} t = \frac{\varphi}{2} \\ d\varphi = 2dt \end{array}\right\} = \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 t}{\cos^3 t}\,dt}_{I_1} + \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\cos t}\,dt}_{I_2} = \left(\frac{\sin t}{2\cos^2 t} + \frac{1}{4}\ln\left|\frac{\sin t - 1}{\sin t + 1}\right| - \frac{1}{2}\ln\left|\frac{\sin t - 1}{\sin t + 1}\right|\right) \textcircled{=}$$

$$I_1 = \sin t\frac{1}{2\cos^2 t} - \int \frac{1}{2\cos^2 t}\cos t\,dt = \sin t\frac{1}{2\cos^2 t} - \frac{1}{2}\int\frac{\cos t}{1-\sin^2 t}\,dt = \sin t\frac{1}{2\cos^2 t} + \frac{1}{4}\ln\frac{\sin t - 1}{\sin t + 1}$$

$$I_2 = \int\frac{1}{1-u^2}\,du$$

$$\frac{1}{2}\ln\left|\frac{u-1}{u+1}\right|$$

$$\textcircled{=}\ 4\frac{\sin\frac{\varphi}{2}}{4\cos^2\frac{\varphi}{2}} - \frac{1}{4}\ln\left|\frac{\sin\frac{\varphi}{2} - 1}{\sin\frac{\varphi}{2} + 1}\right|\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{4}\left(2\sqrt{2} + \ln(3+2\sqrt{2}) + 2\sqrt{2} + \ln(3+2\sqrt{2})\right) = 4\sqrt{2} + 2\ln(3+2\sqrt{2}) =$$

$$= \sqrt{2} + \ln(\sqrt{2}+1)$$

No: 37.4

a) $y = \sqrt{x}$ $\quad y' = \frac{1}{2\sqrt{x}}$

$y = \sqrt{2-x^2}$ $\quad y' = \frac{-x}{\sqrt{2-x^2}}$

$$\int_0^1 \sqrt{1+\frac{1}{4x}}\,dx = \left\{\begin{array}{l} t = \sqrt{x} \\ x = t^2 \\ dx = 2t\,dt \end{array}\right\} = 2\int_0^1 t\sqrt{1+\frac{1}{4t^2}}\,dt = 2\int_0^1 \sqrt{t^2+\frac{1}{4}}\,dt = 2\left(\frac{t}{2}\sqrt{t^2+\frac{1}{4}} + \frac{1}{8}\ln\left|t+\sqrt{t^2+\frac{1}{4}}\right|\right)\Big|_0^1 =$$

$$= \left(t\sqrt{t^2+\frac{1}{4}} + \frac{1}{4}\ln\left|t+\sqrt{t^2+\frac{1}{4}}\right|\right)\Big|_0^1 = \frac{\sqrt{5}}{2} + \frac{1}{4}\ln\left|1+\frac{\sqrt{5}}{2}\right| - \frac{1}{4}\ln\left|\frac{1}{2}\right| = \frac{\sqrt{5}}{2} + \frac{1}{4}\ln\left|2+\sqrt{5}\right|$$

$$\int_0^1 \sqrt{1+\frac{x^2}{2-x^2}}\,dx = \sqrt{2}\int_0^1\frac{dx}{\sqrt{2-x^2}} = \sqrt{2}\arcsin\frac{x}{\sqrt{2}}\Big|_0^1 = \frac{\sqrt{2}\pi}{4}$$

$$P = \sqrt{5} + \frac{\sqrt{2}\pi}{2} + \frac{1}{2}\ln|2+\sqrt{5}|$$

No: 37.8

a) $(y - \arcsin x)^2 = 1 - x^2$

$y^2 - 2y\arcsin x + \arcsin^2 x = 1 - x^2$

$D = \cancel{4\arcsin^2 x} - \cancel{4\arcsin^2 x} + 4 - 4x^2 \Rightarrow$ $\quad y = \arcsin x \pm \sqrt{1-x^2}$

$$y_1' = \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} = \frac{1+x}{\sqrt{1-x^2}} = \frac{(1+x)\sqrt{1-x^2}}{1-x^2} =$$

$$y_2' = \frac{\sqrt{1-x^2}}{1+x} = \frac{(1+x)\sqrt{1-x^2}}{(1+x)(1-x)}$$

$$\int_0^1 \sqrt{1+\left(\frac{\sqrt{1-x^2}}{1-x}\right)^2}\,dx = \int_0^1 \sqrt{\frac{2}{1-x}}\,dx = \left\{\begin{matrix} t=1-x \\ dx=-dt \end{matrix}\right\} = \int_1^0 -\frac{\sqrt{2}}{\sqrt{t}}\,dt = \sqrt{2}\int_0^1 \frac{1}{\sqrt{t}}\,dt = \sqrt{2}\; 2\sqrt{t}\Big|_0^1 = 2\sqrt{2}$$

$$\int_0^1 \sqrt{\frac{2}{1+x}}\,dx = 2\sqrt{2+2x}\Big|_0^1 = 4-2\sqrt{2}$$

$$\ell = 2(4-2\sqrt{2}+2\sqrt{2}) = 8$$