61. ~~3~~ 305 ~~41a,б~~
~~4~~ 32 ~~57~~
21 ~~25~~ 59
~~24~~ ~~30~~
~~27a~~ ~~40~~

No: 61.3

а) $Ax = \sum_{i=1}^{n} (x, e_i) f_i \quad \forall x \in V$

$Ax = A(x_1 e_1 + \ldots + x_n e_n) = A((x, e_1)e_1 + \ldots + (x, e_n)e_n) = (x, e_1)Ae_1 + \ldots + (x, e_n)Ae_n = \sum (x, e_i) f_i$

б) $A^* y = \sum (y, f_i) e_i = A^*(y_1 f_1 + \ldots + y_n f_n)$

$A^* y = v \in V$

$v = v_1 e_1 + \ldots + v_n e_n = (v, e_1)e_1 + \ldots + (v_n, e_n)e$

$v_i = (v, e_i) = (A^* y, e_i) = (y, Ae_i) = (y, f_i)$

No: 61.4

$\operatorname{tr} A = \sum_{i=1}^{n} (Ae_i, e_i)$

$\operatorname{tr} A = \sum_{i=1}^{n} a_{ii}$

$(Ae_i, e_j) = (Ae_i)_j = a_{ji}$

No: 61.21

$(A_e)^* = (A_e)^H \quad \forall e$

$Ae_i = \sum_{k=1}^{n} a_{ki} e_k \qquad A^* e_i = \sum_{k=1}^{n} \overline{a_{ik}} e_k$

$e_1, \ldots, e_n$ - ОНБ

$f_1, \ldots, f_n : \; f_i = e_i \quad i \neq k$

$(Ae_i, e_j) = \left(\sum_{k=1}^{n} a_{ki} e_k, e_j\right) = \sum_{k}^{i=1} a_{ki} (e_k, e_j)$

(

No: 61.24

$A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$(A^*)_e = (A_e)^H = A^T \Rightarrow A^*_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

No: 61.27

а) $Ax = Bx$

n-мерное арифм. пр-во => B матрица оператора в естественном базисе => Значит, что

$A^* = A^H \Rightarrow A^* x = B^T x$

No: 61.30

б)

No: 61.35

$S: e \to f \qquad S = (f_1, f_2, f_3)$

$A_f = S^{-1} A_e S$

$A_e = S A_f S^{-1} \qquad A_e^T = S^{-1^T} A_f^T S^T$

$$A_f^* = S^{-1} S^{-1^T} A_f^T S^T S = \begin{pmatrix} -36 & -37 & -15 \\ 30 & 30 & 19 \\ 26 & 27 & 9 \end{pmatrix}$$

No: 61.39

а) $A(\overset{B}{a_1, a_2}) = (\overset{C}{b_1, b_2})$

$A = CB^{-1} = \begin{pmatrix} -7 & 3 \\ 0 & 1 \end{pmatrix}$

$A^* = \begin{pmatrix} -7 & 0 \\ 3 & 1 \end{pmatrix}$

б) $A = \begin{pmatrix} 1 & -1 \\ -5 & 3 \end{pmatrix}$

$A^* = \begin{pmatrix} 1 & -5 \\ -1 & 3 \end{pmatrix}$

No: 61.40

а) $\frac{1}{2}\begin{bmatrix} 0 & -5 & 0 \\ 6 & 0 & 2 \\ 0 & 15 & 0 \end{bmatrix}$ см. пример 61.4

б) $S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad S^{-1}AS = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 6 & 0 & 0 \\ 0 & 5 & 0 \end{bmatrix}$

No: 61.41

$(f,g) = f(-1)g(-1) + f(0)g(0) + f(1)g(1)$

а) $(e_1, e_2) = -1+0+1 = 0$

$(e_1, e_3) = 2$ $\Rightarrow$ не ортонормированный

$(e_1, e_1) = 3$

$(e_2, e_2) = 2$

$(e_2, e_3) = 0$

$(e_3, e_3) = 2$

$(De_1, e_1) = 0 \quad (De_2, e_1) = 3 \quad (De_3, e_1) = 2$

$(De_1, e_2) = 0 \quad (De_2, e_2) = 0 \quad (De_3, e_2) = 4$

$(De_1, e_3) = 0 \quad (De_2, e_3) = 2 \quad (De_3, e_3) = 0$

$$\left(\begin{array}{ccc|ccc} 3 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 & 0 & 2 \\ 2 & 0 & 2 & 0 & 4 & 0 \end{array}\right)$$

$\alpha_1\ (0, 3/2, 0)$

$\alpha_2\ (-4, 0, 6)$

$\alpha_3\ (0, 1, 0)$

$$D_e^* = \begin{bmatrix} 0 & -4 & 0 \\ 3/2 & 0 & 1 \\ 0 & 6 & 0 \end{bmatrix} = A$$

б) $S = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \qquad S^{-1}AS = \begin{bmatrix} 0 & -2 & 0 \\ 3/2 & 0 & 3/2 \\ 0 & 2 & 0 \end{bmatrix}$

No: 61.51

а) $\subset$] $x \in \ker(A^*A) \Rightarrow A^*Ax = 0 \Rightarrow (Ax, Ax) = (x, A^*Ax) = 0 \Rightarrow Ax = 0 \Rightarrow x \in \ker A$

$\supset$ $x \in \ker A \Rightarrow Ax = 0 \Rightarrow A^*(Ax) = (A^*A)x = 0$

б) $\subset$] $x \in \operatorname{Im}(A^*A) \Rightarrow x = (A^*A)y = A^*(Ay) \Rightarrow x \in \operatorname{Im} A^*$

$\supset$ $\operatorname{Im} A = \ker(A^*)^\perp = \ker(AA^*)^\perp = \operatorname{Im} AA^*$