# 1. Property based testing (QuickTest)

- The inner workings of QuickTest do not really matter. The important thing is coming up and writing properties to verify correctness.
- Remember to explicitly write the types of the predicates so that QuickCheck can provide appropriate examples.

# 2. Lazy evaluation

# 2.1. WHNF (= Weak Head Normal Form)

An expression is in WHNF if any of the following are true:

- 1. The expression is a **constructor**;
  - If the expression is a constructor that is being pattern matched on (e.g. a constructor inside a case ... of, then it will be reduced depending on the value of the constructor.
- 2. The expression is an **anonymous function**, i.e. a lambda expression;
  - If lambda is being applied to all of its arguments, then it is reduced depending on their value.
- 3. The expression is a function applied to too few arguments.

# 3. Higher-kinded abstractions

#### 3.1. Functor

```
class Functor m where
  fmap :: (a -> b) -> f a -> f b
```

- Generalisation of map for arbitrary containers
- Examples:
  - [] (List) where fmap = map,
  - Maybe where fmap mapMaybe, but rather fmap f Nothing = Nothing, fmap f (Just x) = f x, and
  - $\bullet~$  IO where fmap comes from the IO Monad.
- There is an alias for fmap namely <\$>, e.g. map (+1) [1..3] == (+1) <\$> [1..3].

## 3.1.1. Functor laws

```
fmap id = id`
fmap (f . g) = fmap f . fmap g`
```

## 3.2. Aplicative

```
class Functor f => Applicative f where
  pure :: a -> f a
  <*> :: f (a -> b) -> f a -> f b
```

- Sometimes called *Applicative functor*
- Found in the Control. Applicative module.
- Examples:
  - Maybe
  - List
  - IO
- liftA2 :: (Applicative f)  $\Rightarrow$  (a  $\Rightarrow$  b  $\Rightarrow$  c)  $\Rightarrow$  f a  $\Rightarrow$  f b  $\Rightarrow$  f c takes a binary function an *lifts* it to a function that operates on two functors.

#### 3.3. Monoid

```
class Monoid a where
  mempty :: a
  mappend :: a -> a -> a
  mconcat :: [a] -> a
  mconcat = foldr mconcat mempty
```

• We often write mappend using infix notation with <>.

### 3.3.1. Monoid laws

```
-- mempty is the identity element for <>
mempty <> x = x
x <> mempty = x

-- The <> operator is left and right associative
(x <> y) <> z = x <> (y <> z)
```

### 3.4. Monad

```
class Functor m => Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
  return :: a -> m a
```

### 3.4.1. Do notation

• Remember that each do block maps to a monad, and some monads don't do what we intuitively think they do. Consider

where the list monad comes into place (think of list comprehensions in this case).

## 3.4.2. Aplicative

#### 3.4.3. Traversable