Wideband transmit beamforming using integer-time-delayed and phase-shifted waveforms

Yang Gao✉, Defu Jiang and Ming Liu

A method for wideband transmit beamforming using integer-time-delayed and phase-shifted waveforms is proposed. Unlike traditional true-time-delayed digital methods, which employ variable fractional delay filters or discrete Fourier transforms to create fractional time delays, the proposed method uses only integer-time-delayed waveforms with phase shifts corresponding to fractional time delays. Furthermore, the shifted phase can be directly generated; this generation can be simply achieved by controlling the parameters of a direct digital synthesiser. A beam pattern simulation confirms that the integer-time-delayed and phase-shifted waveforms are capable of synthesising wideband transmitting beam patterns. Finally, a power synthesis efficiency analysis demonstrates that the synthesis loss caused by phase shifting is negligible.

Introduction: In antenna array applications, the generation of true-time-delayed (TTD) waveforms to feed the antenna elements is an essential requirement for successful transmit beamforming and power synthesis, when using wideband signals [1, 2]. The required delay can in general be divided into integer and fractional parts relative to the processing clock. The integer delay can be easily achieved through digital processing. The fractional delay is, therefore, the critical element in wideband transmit beamforming. Two digital methods can be used to obtain fractional delays in wideband waveforms. One is a time-domain method using variable fractional delay (VFD) filters to calculate arbitrary inter-sample values [3–5]. The other is a frequency-domain method using discrete Fourier transforms (DFTs), and is based on the time-shifting property of the Fourier transform [6]. Although both methods are effective in producing arbitrary delays, they have very high computational complexity [7, 8]. Consequently, implementing fractional delays using these methods requires large amounts of resources, especially hardware multipliers. In this Letter, we propose a new method to generate waveforms for wideband transmit beamforming using both phase shifting and integer delays. The main feature of the proposed method is the direct generation of the shifted phase corresponding to the fractional delay, which can be further simplified by controlling the parameters of a programmable digital waveform generator or direct digital synthesiser (DDS). Therefore, no hardware requirements for VFD filters or DFT modules exist in this method.

In the following sections, the proposed global approach to delayed waveform generation is first presented. The proposed method is described in detail and illustrated using linear frequency modulated (LFM) waveforms, to show how the parameters of a DDS can be controlled to meet fractional delay requirements. For validation, a wideband transmit beam pattern is simulated using a set of delayed LFM waveforms generated with the proposed method. Finally, the power synthesis efficiency is discussed, to evaluate the synthesis loss caused by phase shifting.

Waveform generation using the proposed method: Assume that D_m is the mth transmitted waveform ($s_m(t)$) time delay required to steer the maximum transmitting beam power in a desired direction θ_t [9]. Here, $m = 0, 1, \ldots, M-1$, and M is the number of antennas. This delay can be divided into integer and fractional delay terms, using the processing interval T_s:

$$D_m = T_m + \tau_m \tag{1}$$

where the integer delay T_m and fractional delay τ_m are given by:

$$T_m = \mathrm{Round}\left(\frac{D_m}{T_s}\right) \cdot T_s = I_m \cdot T_s \tag{2}$$

$$\tau_m = D_m - T_m \tag{3}$$

Here, I_m is an integer and Round(·) denotes rounding to the nearest integer. A diagram of delayed waveforms generated with the proposed method is shown in Fig. 1. In this figure, $s_m(t)$ has a τ_m with the same sign of D_m, while $s_k(t)$ has a τ_k with a sign opposite to that of D_k. Compared with the reference waveform $s_0(t)$, the starting times of $s_m(t)$ and $s_k(t)$ are delayed only by integer multiples of T_s, and not by the nominal delay times (D_m and D_k) of TTD waveforms. As shown in Fig. 1, these start time differences (corresponding to the residual fractional delays) imply extensions or truncations at the beginning of the generated waveforms, and are compensated by another set of truncations or extensions at the end of the generated waveforms, to maintain a constant pulse duration. The effective intervals between the generated waveforms and the reference used to synthesise the signal power are also shown in Fig. 1.

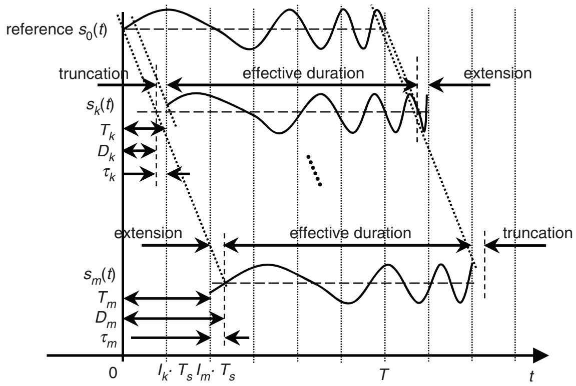

Fig. 1 *Waveforms generated using the proposed method, with both positive and negative fractional delays*

The transmitting waveform $s_0(t)$ of the reference element 0 can be expressed as:

$$s_0(t) = a_0(t)\exp\{\mathrm{j}\varphi_0(t)\} \cdot \mathrm{Rect}\left\{\frac{t - T/2}{T}\right\} \tag{4}$$

Here $\mathrm{Rect}\{t\} = 1$ when $-1/2 \le t \le 1/2$, or 0 when others; T is the duration of the transmitted waveforms, and $a_0(t)$ and $\varphi_0(t)$ are the envelope and phase of $s_0(t)$, respectively. Waveform $s_m(t)$, which has both an integer delay and a phase shift, can be written as:

$$\begin{aligned} s_m(t) &= \left[a_0(t - D_m)\exp\{\mathrm{j}\varphi_0(t - D_m)\}\right] \cdot \mathrm{Rect}\left\{\frac{t - T_m - T/2}{T}\right\} \\ &= \left[a_0(t - \tau_m)\exp\{\mathrm{j}\varphi_0(t - \tau_m)\} \cdot \mathrm{Rect}\left\{\frac{t - T/2}{T}\right\}\right] \otimes \delta(t - T_m) \end{aligned} \tag{5}$$

where ⊗ denotes convolution. The generation of $s_m(t)$ can be divided into two steps. The first step is the generation of the integer delay, which can be easily implemented by simply waiting an integer number of T_s periods during processing. The second step generates the delayed envelope $a_0(t-\tau_m)$ and phase $\varphi_0(t-\tau_m)$ directly, instead of using the traditional digital VFD filters or DFT modules to delay $s_0(t)$. In the particular case of LFM waveform transmission, the envelope and phase of $s_0(t)$ are:

$$\begin{cases} a_0(t) = A \\ \varphi_0(t) = 2\pi f_0 t + \pi\mu_0 t^2 + \theta_0 \end{cases} \tag{6}$$

where A is the LFM waveform constant amplitude, μ_0 is the frequency modulation slope, f_0 the starting frequency, and θ_0 the initial phase. Therefore, the delayed envelope and phase of $s_m(t)$ to be generated in the proposed method's second step are:

$$\begin{cases} a_0(t - \tau_m) = A \\ \varphi_0(t - \tau_m) = 2\pi f_0(t - \tau_m) + \pi\mu_0(t - \tau_m)^2 + \theta_0 \\ \qquad = 2\pi\left(f_0 - \mu_0\tau_m\right)t + \pi\mu_0 t^2 + \pi\mu_0\tau_m^2 - 2\pi f_0\tau_m + \theta_0 \end{cases} \tag{7}$$

and the $s_m(t)$ parameters μ_m, f_m, and θ_m should be chosen as:

$$\begin{cases} \mu_m = \mu_0 \\ f_m = f_0 - \mu_0\tau_m \\ \theta_m = \pi\mu_0\tau_m^2 - 2\pi f_0\tau_m + \theta_0 \end{cases} \tag{8}$$

In summary, using the true delay D_m required for beam steering and the pre-selected T_s, we can derive the required number of waiting cycles I_m and the fractional delay τ_m shown in (2) and (3); then, using τ_m and the reference parameters μ_0, f_0, and θ_0, we can calculate parameters μ_m, f_m, and θ_m using (8), and thus generate the phase-shifted waveform $s_m(t)$ using a DDS [10, 11].

For other waveforms, parameters μ_m, f_m, and θ_m must be obtained differently. For example, for phase-coded waveforms with carrier frequency f_c and phase set P, we can obtain the parameters for all code elements using $\mu_0 = 0$, $f_0 = f_c$, and $\theta_0 = P\{C_k\}$ for the kth code element. Here, C_k is the phase code of the kth code element. For a more complicated example using non-LFM (NLFM) waveforms with frequency modulation function $f(t) = f_0 + k(t)t$, where $k(t)$ is the modulated frequency slope, we can obtain parameters μ_m, f_m, and θ_m using $\mu_0 = k(t - \tau_m)$, $f_0 = f_c - B/2$, and $\theta_0 = 0$. Here, B is the waveform bandwidth. The implementation cost required to obtain $k(t - \tau_m)$ using the same method used for $\varphi_0(t - \tau_m)$ in (7) depends on the computational complexity of $k(t)$.

Wideband beam pattern validation: According to (5), $s_m(t)$ can be further expressed as:

$$s_m(t) = [s_0(t)\exp\{\mathrm{j}(\varphi_0(t - \tau_m) - \varphi_0(t))\}] \otimes \delta(t - T_m) \tag{9}$$

where $\varphi_0(t - \tau_m) - \varphi_0(t) = [-2\pi\cdot\mu_0\cdot\tau_m\cdot t + \varphi_0(-\tau_m) - \varphi_0(0)]$ when using the phase expression of LFM. Assume that $S_0(f)$ is the Fourier domain representation of $s_0(t)$. Applying basic Fourier transform properties, the corresponding frequency domain expression of $s_m(t)$ can be expressed as:

$$S_m(f) = S_0\left(f + \Delta f_m\right)\exp\left\{-\mathrm{j}2\pi f\cdot T_m\right\}\exp\left\{\varphi_0(-\tau_m) - \varphi_0(0)\right\} \tag{10}$$

where $\Delta f_m = \mu_0\cdot\tau_m$. The beam pattern formed by the LFM waveform set $\{s_m(t),\ m = 0,\ 1,\ \ldots,\ M - 1\}$ is given by:

$$G(\theta, f) = \left|\sum_{m=0}^{M-1} S_m(f) d_m(f)\right|^2 \tag{11}$$

where $d_m(f)$ is the mth factor in the array steering vector, and $d_m(f) = \exp\{-\mathrm{j}2\pi f\cdot\Delta\Gamma_m\}$. Here, $\Delta\Gamma_m = m\cdot d\cdot\sin(\theta)/c$ for an arbitrary direction θ, c is the speed of light, and d is the inter-element spacing in a uniform linear array.

MATLAB R2013b was used to generate delayed LFM waveforms using the proposed integer time delay and phase shifting method and simulate the transmitting beam pattern. Fig. 2 shows the normalised wideband beam pattern synthesised with the beamforming parameters listed in Table 1. As can be observed, the transmitting beam is well synthesised in the steering direction over a wide working frequency range, with no obvious distortions.

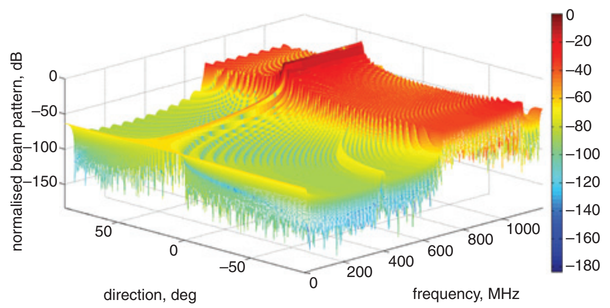

Fig. 2 *Wideband beam steering pattern for $\theta_t = 30°$*

Table 1: Beamforming parameters

Steering direction	$\theta_t = 30°$
Number of antennas	$M = 64$
Centre frequency	$f_c = 900$ MHz
Inter-element spacing	$d = \lambda_c/2$ ($\lambda_c = c/f_c$)
Bandwidth	$B = 400$ MHz
Pulse duration	$T = 10$ µs
Sampling frequency	$f_s = 1200$ MHz ($T_s = 1/f_s$)

Spatial power synthesis efficiency: Considering two arbitrary waveforms in the generated waveform set $\{s_m(t),\ m = 0,\ 1,\ \ldots,\ M - 1\}$, $s_m(t)$ and $s_k(t)$, $m \neq k$ and $\tau_m \geq \tau_k$; the effective time interval between them is:

$$T_e(m, k) = T - (\tau_m - \tau_k) \tag{12}$$

The common effective interval in the entire waveform set is the minimum value of T_e, and the worst case is when τ_m approaches $T_s/2$ and τ_k approaches $-T_s/2$. That is,

$$T_{e\_\min} \geq T - T_s \tag{13}$$

Assuming that all transmitters have equal, constant power during the transmission, the power synthesis efficiency of the wideband waveform beamforming satisfies:

$$\eta > \frac{T_{e\_\min}}{T} \geq \frac{T - T_s}{T} \tag{14}$$

Generally, T_s is <1 ns, while T is in the order of several microseconds. For instance, the efficiency η calculated with the parameters shown in Table 1 is 0.999917. Therefore, the synthesis loss caused by phase shifting is negligible.

Conclusion: The method for wideband transmit beamforming using integer-time-delayed and phase-shifted waveforms proposed in this Letter was shown to be very effective. The phase-shifted waveforms could be conveniently generated using a programmable DDS. Even though the LFM waveform was used as an example, other waveforms, such as NLFM and phase-coded waveforms whose instantaneous phases can be explicitly obtained, can also be generated using the proposed method for transmit beamforming.

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One or more of the Figures in this Letter are available in colour online.

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