Transmit Beamforming Optimization for Wireless Information and Power Transfer in MISO Interference Channels with Signal Cooperation

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***Abstract*—In simultaneous wireless information and power transfer (SWIPT) systems, dedicated energy signals only convey wireless energy, but not information. For this reason, the energy-carring signals in the SWIPT can be pre-determined in advance and is shared among communication nodes. By exploiting this nature, this paper designs the optimal transmit beamforming vectors for the multiple-input single-output SWIPT interference channel with *signal cooperation* (IFC-SC), where the energy-carrying signal waveforms are known to transmitters and receivers. Specifically, we aim to identify the optimal tradeoff between the information rate and the harvested energy. To this end, an information rate maximization problem is formulated under minimum required harvested energy constraint, which is non-convex in general. To solve the problem, a new parameterization technique is introduced, and we can decouple the original problem into two subproblems, which yields closed-form beamforming solutions by addressing the line search method for the parameter. Simulation results confirms that the proposed optimal IFC-SC beamforming vectors outperform conventional SWIPT IFC systems.**

I. INTRODUCTION

There have been intensive researches on simultaneous wireless information and power transfer (SWIPT) which exploits the dual usage of radio frequency (RF) signal, namely wireless information transmission and wireless energy transfer [1]–[4]. Unlike traditional communication systems where the role of the RF signals has been confined to conveying information, in the SWIPT systems, the RF signal can carry both information and energy at the same time. The SWIPT was first introduced in [1] for single-input single-output (SISO) point-to-point communication systems. In [2], the non-trivial rate-energy (R-E) tradeoff of the SWIPT systems in the frequency-selective channels was studied via power allocation in frequency domain. In two-user multiple-input multiple-output (MIMO) broadcast channels (BC), the optimal R-E tradeoff curve was identified in [3]. Also, the authors in [4] proposed the optimal beamforming vectors which maximize the weighted sum harvested energy for multi-user multiple-input single-output (MISO) BC systems under signal-to-noise-plus-interference ratio constraint.

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According to the cooperation level among transmitters and the receivers in interference channels (IFC), the SWIPT systems can be classified into the following two categories: IFC with *partial cooperation* (IFC-PC) [5] [6] and with *signal cooperation* (IFC-SC) [7]. In the IFC-PC, it is assumed that only channel state information (CSI) is available at transmitters and receivers, but not the transmitted signal. On the other hand, in the IFC-SC systems, both the CSI and the signal waveforms are known at the transmitters and the receivers since the energy-carrying RF signal can be determined in advance [4], [7]. Then, we can improve the achievable R-E performance by constructing the transmitted signals as a superposition of the information and the energy signals as in [4] and [7]. Also, with the pre-determined energy signal, the receivers can eliminate the interference caused by the energy signal. Therefore, the capability of SWIPT is significantly improved in the IFC-SC. In MIMO IFC-PC, suboptimal SWIPT transmission schemes and their achievable R-E regions were investigated in [5]. In our previous work [6], the optimal SWIPT beamforming vector was proposed for a two-user MISO IFC assuming the PC setup. In the SC scenario, the authors in [7] introduced the collaborative signal transmission and interference cancelling protocols for the general K-user SISO IFC.

In this paper, we propose the optimal linear beamforming vectors for the SWIPT systems in two-user MISO IFC-SC, where each transmitter sends message or transfers energy to its associated receiver. In this configuration, we completely characterize the optimal tradeoff between the information rate and the harvested energy by investigating the Pareto boundary of the achievable R-E region. To this end, the information rate maximization problem is constructed under minimum required harvested energy constraint, which is non-convex in general. To solve the problem efficiently, we introduce a *power splitting* parameter which splits the transmit power constraint at the transmitter, and enables the signaling cooperative transmission. Through this new parameterization technique, we can decouple the original non-convex problem into two sequential subproblems. Then, the first subproblem identifies the optimal parameter by a simple line search method, whereas the second one designs beamforming vectors with a closed-form. Simulation results confirm that the proposed IFC-SC beamforming vectors outperforms the MISO IFC-PC systems.

Throughout this paper, we employ uppercase boldface letters, lowercase boldface letters, and normal letters for matri-

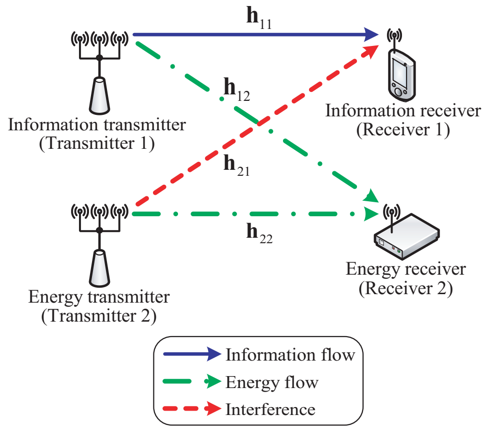

Fig. 1. Schematic diagram for the SWIPT system in two-user MISO IFC

ces, vectors, and scalar quantities, respectively. A set of all complex matrices of size m-by-n is represented by $\mathbb{C}^{m\times n}$, and conjugate transpose of a matrix or a vector is denoted by $(\cdot)^H$. $\mathbf{I}_m$ stands for an identity matrix of size m-by-m. The absolute value of a scalar is given by $|\cdot|$, and $\|\cdot\|$ accounts for the 2-norm operation of a vector. Also, the matrix $\mathbf{\Pi}_{\mathbf{x}}^{\perp} = \mathbf{I}_m - \mathbf{x}\mathbf{x}^H/\|\mathbf{x}\|^2$ reflects the projection matrix onto the nullspace of a vector $\mathbf{x} \in \mathbb{C}^{m\times 1}$.

II. System Model

In this section, we describe a system model for two-user MISO SWIPT IFC-SC where two transmitters each equipped with M antennas radiate the RF signals to single antenna receivers as illustrated in Figure 1. In this scenario, the information decoding (ID) receiver decodes message transmitted from the information transmitter, while the energy harvesting (EH) receiver collects energy of the RF signals radiated from both the energy and information transmitters.[1]

For notational conveniences, we denote the information and the energy transmitter as transmitter 1 and 2, respectively, and the ID and the EH receiver as receiver 1 and 2, respectively. Assuming frequency-flat fading, the channel vector from transmitter i to receiver j is denoted by $\mathbf{h}_{ij} \in \mathbb{C}^{M\times 1}$ for $i, j = 1, 2$. Also, we define $s_1 \sim \mathcal{CN}(0, 1)$ as the scalar information symbol for the ID receiver, and s_2 with $\mathbb{E}[|s_2|^2] = 1$ as the energy symbol for the EH receiver, which is assumed to be independent with s_1. It is worth noting that since the energy symbol s_2 does not carry any information, we can pre-determine s_2 as an arbitrary random variable or a fixed symbol. Therefore, this pre-determined energy symbol s_2 can be known at all the transmitters and the ID receiver in advance.

At the information transmitter, we apply the signal splitting approach which constructs the signal vector sent from the information transmitter $\mathbf{x}_1 \in \mathbb{C}^{M\times 1}$ as a superposition of the information symbol s_1 and the energy symbol s_2. Denoting $\mathbf{g}_{1,\mathrm{I}} \in \mathbb{C}^{M\times 1}$ and $\mathbf{g}_{1,\mathrm{E}} \in \mathbb{C}^{M\times 1}$ as the beamforming vectors at the information transmitter for the information and the energy symbol, respectively, $\mathbf{x}_1$ is expressed as

$$\mathbf{x}_1 = \mathbf{g}_{1,\mathrm{I}} s_1 + \mathbf{g}_{1,\mathrm{E}} s_2.$$

[1]For conveniences, we normalize the time slot duration to unity, and thus the power and energy terms are interchangeably used in this paper.

To satisfy the power constraint at the information transmitter $\mathbb{E}[\|\mathbf{x}_1\|^2] \leq P_1$, we have $\|\mathbf{g}_{1,\mathrm{I}}\|^2 + \|\mathbf{g}_{1,\mathrm{E}}\|^2 \leq P_1$.

Then, the received signals r_1 and r_2 are written by

$$r_1 = \mathbf{h}_{11}^H \mathbf{g}_{1,\mathrm{I}} s_1 + (\mathbf{h}_{11}^H \mathbf{g}_{1,\mathrm{E}} + \mathbf{h}_{21}^H \mathbf{g}_2) s_2 + n_1, \tag{1}$$

$$r_2 = \mathbf{h}_{12}^H \mathbf{g}_{1,\mathrm{I}} s_1 + (\mathbf{h}_{22}^H \mathbf{g}_2 + \mathbf{h}_{12}^H \mathbf{g}_{1,\mathrm{E}}) s_2 + n_2, \tag{2}$$

where $\mathbf{g}_2 \in \mathbb{C}^{M\times 1}$ is the beamforming vector at the energy transmitter with the transmit power constraint $\|\mathbf{g}_2\|^2 \leq P_2$ and $n_i \sim \mathcal{CN}(0, 1)$ represents the complex Gaussian noise at receiver i.

With the energy symbol s_2 at hand, in order to improve the information rate, the ID receiver can perfectly cancel the interference caused by the energy symbol s_2, i.e., $(\mathbf{h}_{11}^H \mathbf{g}_{1,\mathrm{E}} + \mathbf{h}_{21}^H \mathbf{g}_2) s_2$ in (1), as in [4] and [7]. Therefore, in the IFC-SC scenario, the achievable information rate at the ID receiver is obtained as

$$R_{\text{IFC-SC}} = \log_2(1 + |\mathbf{h}_{11}^H \mathbf{g}_{1,\mathrm{I}}|^2).$$

Meanwhile, the EH receiver harvests the energy of the received RF signal (2), and the harvested energy in the IFC-SC can be given by [3]

$$E_{\text{IFC-SC}} = \mathbb{E}[|r_2|^2] = |\mathbf{h}_{22}^H \mathbf{g}_2 + \mathbf{h}_{12}^H \mathbf{g}_{1,\mathrm{E}}|^2 + |\mathbf{h}_{12}^H \mathbf{g}_{1,\mathrm{I}}|^2.$$

The optimal tradeoff between two quantities $R_{\text{IFC-SC}}$ and $E_{\text{IFC-SC}}$ is characterized by the Pareto boundary of the achievable R-E region $\mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$ given by (3), shown at the top of the next page. The Pareto boundary is defined as a R-E point $(R, E) \in \mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$ which does not have another R-E point with $(R', E') \geq (R, E)$ and $(R', E') \neq (R, E)$, where the inequality is componentwise [8].

The Pareto boundary point of $\mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$ can be identified by finding the maximum information rate under minimum required harvested energy constraint [3]. To this end, we construct the information rate maximization problem for given EH constraint U.

$$R^{\star}_{\text{IFC-SC}}(U) = \max_{\mathbf{g}_{1,\mathrm{I}}, \mathbf{g}_{1,\mathrm{E}}, \mathbf{g}_2} \log_2(1 + |\mathbf{h}_{11}^H \mathbf{g}_{1,\mathrm{I}}|^2) \tag{4}$$

$$s.t. \ \|\mathbf{g}_{1,\mathrm{I}}\|^2 + \|\mathbf{g}_{1,\mathrm{E}}\|^2 \leq P_1, \ \|\mathbf{g}_2\|^2 \leq P_2, \tag{5}$$

$$|\mathbf{h}_{22}^H \mathbf{g}_2 + \mathbf{h}_{12}^H \mathbf{g}_{1,\mathrm{E}}|^2 + |\mathbf{h}_{12}^H \mathbf{g}_{1,\mathrm{I}}|^2 \geq U, \tag{6}$$

where (5) stands for the transmit power constraints at the transmitters, and (6) indicates the minimum required harvested energy constraint at the EH receiver. Due to the EH constraint (6), the existing methods in conventional MISO IFC [9]–[12] cannot be applied to problem (4). Also note that, unlike the SISO IFC-SC [7], we should optimize the beamforming vectors for both the information and the energy signals. Thus, a solution for (4) can be viewed as a generalization of the two-user SISO IFC-SC systems with one information transmitter-receiver pair and one energy transmitter-receiver pair.

By solving problem (4) for all valid EH constraint U, all the Pareto boundary points $(R^{\star}_{\text{IFC-SC}}(U), U) \in \mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$ can be identified. However, problem (4) is non-convex due to the EH constraint (6), and thus it is difficult to find the globally optimal solution.

$$\mathcal{C}_{\text{IFC-SC}}(P_1, P_2) = \Big\{ (R_{\text{IFC-SC}}, E_{\text{IFC-SC}}) : R_{\text{IFC-SC}} = \log_2(1 + |\mathbf{h}_{11}^H \mathbf{g}_{1,\text{I}}|^2),$$

$$E_{\text{IFC-SC}} = |\mathbf{h}_{22}^H \mathbf{g}_2 + \mathbf{h}_{12}^H \mathbf{g}_{1,\text{E}}|^2 + |\mathbf{h}_{12}^H \mathbf{g}_{1,\text{I}}|^2, \|\mathbf{g}_{1,\text{I}}\|^2 + \|\mathbf{g}_{1,\text{E}}\|^2 \le P_1, \|\mathbf{g}_2\|^2 \le P_2 \Big\}. \tag{3}$$

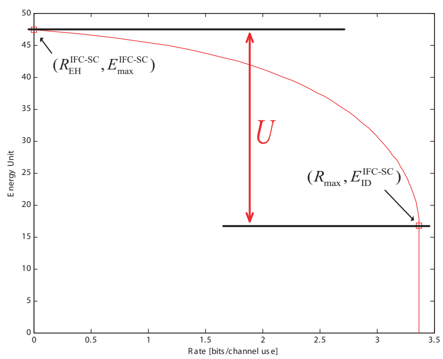

Fig. 2. Pareto boundary and valid EH constraint

A. Valid EH constraint U

Before solving the problem in (4), in this subsection, we address the valid region for the EH constraint U. To this end, we first compute two special R-E boundary points, namely the maximum harvested energy and the maximum information rate points for the IFC-SC systems. Then, the valid EH constraint U will be addressed.

As illustrated in Figure 2, we have two special Pareto points $(R_{\text{EH}}^{\text{IFC-SC}}, E_{\text{max}}^{\text{IFC-SC}})$ and $(R_{\text{max}}, E_{\text{ID}}^{\text{IFC-SC}})$ in the IFC-SC. First, the maximum harvested energy point can be computed as

$$(R_{\text{EH}}^{\text{IFC-SC}}, E_{\text{max}}^{\text{IFC-SC}}) = \left(0, \left(\sqrt{P_1}\|\mathbf{h}_{12}\| + \sqrt{P_2}\|\mathbf{h}_{22}\|\right)^2\right),$$

where this point is achieved with $\mathbf{g}_{1,\text{I}} = \mathbf{0}$, $\mathbf{g}_{1,\text{E}} = \sqrt{P_1}\frac{\mathbf{h}_{12}}{\|\mathbf{h}_{12}\|}$, and $\mathbf{g}_2 = \sqrt{P_2}\frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}$, i.e., all the beamforming vectors are aligned to the EH receiver. Second, the the maximum information rate point is given by

$$(R_{\text{max}}, E_{\text{ID}}^{\text{IFC-SC}}) = \left(\log_2(1 + P_1\|\mathbf{h}_{11}\|^2), P_1\frac{|\mathbf{h}_{12}^H\mathbf{h}_{11}|^2}{\|\mathbf{h}_{11}\|^2} + P_2\|\mathbf{h}_{22}\|^2\right).$$

To attain this point, we apply $\mathbf{g}_{1,\text{I}} = \sqrt{P_1}\frac{\mathbf{h}_{11}}{\|\mathbf{h}_{11}\|}$, $\mathbf{g}_{1,\text{E}} = \mathbf{0}$, and $\mathbf{g}_2 = \sqrt{P_2}\frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}$, i.e., the information beamforming vector $\mathbf{g}_{1,\text{I}}$ is aligned to the ID receiver, while the energy transmitter employs the maximum energy beamforming vector $\mathbf{g}_2 = \sqrt{P_2}\frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}$ since it does not interfere with the ID receiver due to the interference cancellation. Thus, as illustrated in Figure 2, the Pareto boundary points of the R-E region $\mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$ are corresponding to the boundary points (R, E) for $R_{\text{EH}}^{\text{IFC-SC}} \le R \le R_{\text{max}}$ and $E_{\text{ID}}^{\text{IFC-SC}} \le E \le E_{\text{max}}^{\text{IFC-SC}}$. As a result, if we solve problem (4) for $E_{\text{ID}}^{\text{IFC-SC}} \le U \le E_{\text{max}}^{\text{IFC-SC}}$, all Pareto boundary points for the IFC-SC can be identified.

III. Parameterization of the Pareto Boundary

In this section, we provide the parameterization technique for the Pareto boundary of the R-E region $\mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$ by investigating problem (4). To this end, we first introduce a power splitting parameter θ, which represents the transmit power of the energy signal at the information transmitter as illustrated in Figure 3. Then, the information transmitter allocates the power θ for the energy signal, while the remaining power $P_1 - \theta$ is utilized for the information signal.

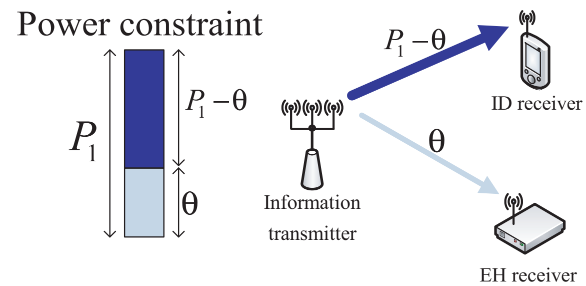

Fig. 3. Graphical interpretation of the power splitting parameter θ

Defining the power splitting parameter $\theta = \|\mathbf{g}_{1,\text{E}}\|^2$ and a unit norm vector $\mathbf{f} \in \mathbb{C}^{M\times 1}$ as $\mathbf{f} = \frac{\mathbf{g}_{1,\text{E}}}{\sqrt{\theta}}$, the original problem (4) is reformulated as

$$R_{\text{IFC-SC}}^\star(U) = \max_{\mathbf{g}_{1,\text{I}},\theta,\mathbf{f},\mathbf{g}_2} \log_2(1 + |\mathbf{h}_{11}^H\mathbf{g}_{1,\text{I}}|^2) \tag{7}$$

$$s.t.\ \|\mathbf{g}_{1,\text{I}}\|^2 \le P_1 - \theta,\ \|\mathbf{g}_2\|^2 \le P_2,\ \|\mathbf{f}\|^2 = 1,$$
$$|\mathbf{h}_{12}^H\mathbf{g}_{1,\text{I}}|^2 \ge U - |\mathbf{h}_{22}^H\mathbf{g}_2 + \sqrt{\theta}\mathbf{h}_{12}^H\mathbf{f}|^2. \tag{8}$$

For a given θ, we first present the optimal solutions $\mathbf{f}^\star$ and $\mathbf{g}_2^\star$ of problem (7). Note that $\mathbf{f}$ and $\mathbf{g}_2$ do not affect the objective function of (7), but the feasible set of $\mathbf{g}_{1,\text{I}}$. The feasible set of $\mathbf{g}_{1,\text{I}}$ is defined as

$$\mathcal{Q}(\theta, \mathbf{f}, \mathbf{g}_2) = \{\mathbf{g}_{1,\text{I}} : \|\mathbf{g}_{1,\text{I}}\|^2 \le P_1 - \theta,$$
$$|\mathbf{h}_{12}^H\mathbf{g}_{1,\text{I}}|^2 \ge U - |\mathbf{h}_{22}^H\mathbf{g}_2 + \sqrt{\theta}\mathbf{h}_{12}^H\mathbf{f}|^2\}.$$

Therefore, the optimal $\mathbf{f}^\star$ and $\mathbf{g}_2^\star$ for the problem (7) should enlarge the set $\mathcal{Q}(\theta, \mathbf{f}, \mathbf{g}_2)$. This leads to the following maximization problem:

$$\varepsilon(\theta) = \max_{\mathbf{f},\mathbf{g}_2} |\mathbf{h}_{22}^H\mathbf{g}_2 + \sqrt{\theta}\mathbf{h}_{12}^H\mathbf{f}|^2 \tag{9}$$
$$s.t.\ \|\mathbf{f}\|^2 = 1,\ \|\mathbf{g}_2\|^2 \le P_2,$$

where $\varepsilon(\theta)$ represents the optimal value of problem (9).

One can show that the optimal solution to problem (9) can be obtained as

$$\mathbf{g}_2^\star = \sqrt{P_2}\frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|} \text{ and } \mathbf{f}^\star = \frac{\mathbf{h}_{12}}{\|\mathbf{h}_{12}\|}, \tag{10}$$

i.e., the both beamforming vectors are aligned to the EH receiver, and the corresponding optimal value $\varepsilon(\theta)$ is given by $\varepsilon(\theta) = (\sqrt{P_2}\|\mathbf{h}_{22}\| + \sqrt{\theta}\|\mathbf{h}_{12}\|)^2$

Thus, the original problem (4) can be recast to

$$R_{\text{IFC-SC}}^\star(U) = \max_{\mathbf{g}_{1,\text{I}},\theta} \log_2(1 + |\mathbf{h}_{11}^H\mathbf{g}_{1,\text{I}}|^2)$$
$$s.t.\ \|\mathbf{g}_{1,\text{I}}\|^2 \le P_1 - \theta,\ |\mathbf{h}_{12}^H\mathbf{g}_{1,\text{I}}|^2 \ge U - \varepsilon(\theta).$$

Now, we present the following theorem which enables the parameterization technique for the IFC-SC systems.

Theorem 1: The maximum information rate $R^\star_{\text{IFC-SC}}(U)$ in problem (7) is identical to the optimal value of the following maximization problem:

$$\Psi(U) = \max_{\theta} \tilde{R}_{\text{IFC-SC}}(\theta, U) \quad (11)$$
$$s.t.\ \theta_{\min} \le \theta \le P_1, \quad (12)$$

where

$$\theta_{\min} = \left(\frac{(U - P_1\|\mathbf{h}_{12}\|^2 - P_2\|\mathbf{h}_{22}\|^2)^+}{2\sqrt{P_2}\|\mathbf{h}_{12}\|\|\mathbf{h}_{22}\|} \right)^2.$$

Also, $\tilde{R}_{\text{IFC-SC}}(\theta, U)$ is defined as

$$\tilde{R}_{\text{IFC-SC}}(\theta, U) = \max_{\mathbf{g}_{1,\text{I}}} \log_2(1 + |\mathbf{h}_{11}^H \mathbf{g}_{1,\text{I}}|^2) \quad (13)$$
$$s.t.\ \|\mathbf{g}_{1,\text{I}}\|^2 \le P_1 - \theta,\ |\mathbf{h}_{12}^H \mathbf{g}_{1,\text{I}}|^2 \ge U - \varepsilon(\theta). \quad (14)$$

Proof: See Appendix A. ■

Theorem 1 implies that the optimal information rate $R^\star_{\text{IFC-SC}}(U)$ for a given EH constraint U, which is originally found by solving (4), can be equivalently attained from (11) and (13). In other words, the Pareto boundary point $(R^\star_{\text{IFC-SC}}(U), U)$ of $\mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$ can be computed by performing the following two step approach: First, we identify $\tilde{R}_{\text{IFC-SC}}(\theta, U)$ for all feasible θ, and then $R^\star_{\text{IFC-SC}}(U)$ is determined via one-dimensional line search over $\theta_{\min} \le \theta \le P_1$.

Therefore, the Pareto boundary of the R-E region in the IFC-SC systems is characterized by the parameter θ, which represents the transmit power of the energy signal $\mathbf{g}_{1,\text{E}} s_2$ at the information transmitter. In Section IV, we will show that the optimal power splitting parameter $\theta^\star$ can be computed by the bisection method without loss of optimality.

IV. Optimal Power Splitting Parameter $\theta^\star$

Now, the remaining part for finding the Pareto boundary point $(R^\star_{\text{IFC-SC}}(U), U)$ is solving problems (11) and (13). To this end, we first derive the optimal solution $\tilde{\mathbf{g}}_{1,\text{I}}(\theta)$ for problem (13), and then prove that the optimal $\theta^\star$ can be obtained via the bisection method. Note that problem (13) is equivalent to the rate maximization problem for the BC and a closed-form solution is given by [3]

$$\tilde{\mathbf{g}}_{1,\text{I}}(\theta) = \begin{cases} \sqrt{\frac{U-\varepsilon(\theta)}{\|\mathbf{h}_{12}\|^2}} \frac{\mathbf{h}_{12}^H \mathbf{h}_{11}}{|\mathbf{h}_{12}^H \mathbf{h}_{11}|} \frac{\mathbf{h}_{12}}{\|\mathbf{h}_{12}\|} & \\ + \sqrt{P_1 - \theta - \frac{U-\varepsilon(\theta)}{\|\mathbf{h}_{12}\|^2}} \frac{\mathbf{\Pi}^\perp_{\mathbf{h}_{12}} \mathbf{h}_{11}}{\|\mathbf{\Pi}^\perp_{\mathbf{h}_{12}} \mathbf{h}_{11}\|}, & \text{for } \theta_{\min} \le \theta < \theta_1, \\ \sqrt{P_1 - \theta} \frac{\mathbf{h}_{11}}{\|\mathbf{h}_{11}\|}, & \text{for } \theta_1 \le \theta \le P_1. \end{cases}$$

Here, the threshold θ_1 which determines the formula for $\tilde{\mathbf{g}}_{1,\text{I}}(\theta)$ is different from the BC case [3] due to the parameter θ, and it is expressed as

$$\theta_1 = \min\left\{ \left(\frac{\sqrt{B^2 + AC} - B}{A} \right)^2, P_1 \right\},$$

where

$$A = \|\mathbf{\Pi}^\perp_{\mathbf{h}_{11}} \mathbf{h}_{12}\|^2, B = \sqrt{P_2}\|\mathbf{h}_{12}\|\|\mathbf{h}_{22}\|, C = U - E^{\text{IFC-SC}}_{\text{ID}}.$$

Next, we present the method for identifying $\theta^\star$. By using the proposed beamforming solution $\tilde{\mathbf{g}}_{1,\text{I}}(\theta)$, $|\mathbf{h}_{11}^H \tilde{\mathbf{g}}_{1,\text{I}}(\theta)|^2$ can be written as (15) at the top of the next page. One can prove

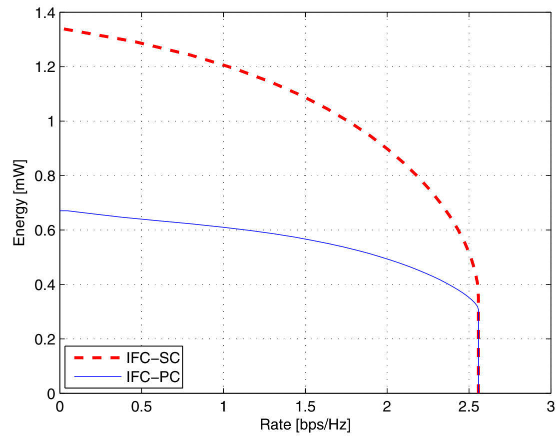

Fig. 4. Achievable R-E regions of the IFC-SC and the IFC-PC systems

that (15) is concave by checking the second derivative, i.e., $\frac{\partial |\mathbf{h}_{11}^H \tilde{\mathbf{g}}_{1,\text{I}}(\theta)|^2}{\partial \theta} \le 0$, and thus $\tilde{R}_{\text{IFC-SC}}(\theta, U)$ is also concave because the logarithm is non-decreasing concave. In addition, since $\tilde{R}_{\text{IFC-SC}}(\theta, U)$ is decreasing for $\theta_1 \le \theta \le P_1$, the optimal $\theta^\star$ must lie in $[\theta_{\min}, \theta_1]$. As a result, we can search the optimal $\theta^\star$ by applying the bisection method.

We summarize the method which identifies the Pareto boundary of the IFC-SC as Algorithm 1 below.

Algorithm 1: Optimal algorithm for the IFC-SC

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1. Set the EH constraint $E^{\text{IFC-SC}}_{\text{ID}} \le U \le E^{\text{IFC-SC}}_{\max}$.
2. Find the optimal $\theta^\star \in [\theta_{\min}, \theta_1]$ by the bisection method.
3. Compute the optimal beamforming vectors as $\mathbf{g}^\star_{1,\text{I}} = \tilde{\mathbf{g}}_{1,\text{I}}(\theta^\star)$, $\mathbf{g}^\star_{1,\text{E}} = \sqrt{\theta^\star} \frac{\mathbf{h}_{12}}{\|\mathbf{h}_{12}\|}$, and $\mathbf{g}^\star_2 = \sqrt{P_2} \frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}$.
4. Obtain the Pareto boundary point $(R^\star_{\text{IFC-SC}}(U), U)$.

In this algorithm, we first find the optimal power splitting parameter $\theta^\star$ based on the closed-form expression $\tilde{R}_{\text{IFC-SC}} = \log_2(1 + |\mathbf{h}_{11}^H \tilde{\mathbf{g}}_{1,\text{I}}(\theta)|^2)$ with the given EH constraint $E^{\text{IFC-SC}}_{\text{ID}} \le U \le E^{\text{IFC-SC}}_{\max}$. Then, from (15), the corresponding optimal information beamforming vector can be obtained as $\mathbf{g}^\star_{1,\text{I}} = \tilde{\mathbf{g}}_{1,\text{I}}(\theta^\star)$, and from (10), the optimal energy beamforming vectors are given by $\mathbf{g}^\star_{1,\text{E}} = \sqrt{\theta^\star}\mathbf{f}^\star$ and $\mathbf{g}^\star_2 = \sqrt{P_2} \frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}$. Finally, we can attain the Pareto boundary point $(R^\star_{\text{IFC-SC}}(U), U)$ of the achievable R-E region $\mathcal{C}_{\text{IFC-SC}}(P_1, P_2)$.

V. Numerical Examples

In this section, we present numerical examples for the achievable R-E regions of the IFC-SC and compare with the IFC-PC systems [6]. In the simulations, we assume 10 m distance between transmitter i and receiver j $(i, j = 1, 2)$ with 30 dB signal attenuation at 1 m, and the path loss exponent is set to 3. Also, we set the transmit power constraint and the noise variance as $P_1 = P_2 = 20$ dBm and -50 dBm, respectively.

In Figure 4, we provide the Pareto boundary of the IFC-SC and the IFC-PC systems for $M = 3$ antennas. Here, the

$$|\mathbf{h}_{11}^H\tilde{\mathbf{g}}_{1,\mathrm{I}}(\theta)|^2 = \begin{cases} \left(\sqrt{\frac{U-\varepsilon(\theta)}{\|\mathbf{h}_{12}\|^2}}\frac{|\mathbf{h}_{12}^H\mathbf{h}_{11}|}{\|\mathbf{h}_{12}\|} + \sqrt{P_1-\theta-\frac{U-\varepsilon(\theta)}{\|\mathbf{h}_{12}\|^2}}\|\mathbf{\Pi}_{\mathbf{h}_{12}}^{\perp}\mathbf{h}_{11}\|\right)^2, & \text{for } \theta_{\min}\le\theta<\theta_1, \\ (P_1-\theta)\,\|\mathbf{h}_{11}\|^2, & \text{for } \theta_1\le\theta\le P_1. \end{cases} \tag{15}$$

channel vectors are given by $\mathbf{h}_{ij} = \sqrt{10^{-6}}\tilde{\mathbf{h}}_{ij}$, where

$$\tilde{\mathbf{h}}_{11} = [0.693-0.068j, -0.188-0.976j, -0.388-0.515j]^T,$$
$$\tilde{\mathbf{h}}_{12} = [1.334-0.843j, -2.078-0.310j, 0.693+0.967j]^T,$$
$$\tilde{\mathbf{h}}_{21} = [-0.617+0.288j, 0.063+0.458j, 0.175-0.116j]^T,$$
$$\tilde{\mathbf{h}}_{22} = [0.574+0.681j, 0.289+0.461j, -0.912-0.291j]^T.$$

We can see that the achievable R-E region in the IFC-SC becomes larger than the IFC-PC in [6].

VI. Conclusion

In this paper, we have completely characterized the Pareto boundary of the achievable R-E regions for the SWIPT IFC-SC systems. To this end, an information rate maximization problem was constructed under the harvested energy constraint. By using the power splitting parameter θ, we can decouple the original problem into two sequential problems. Then, the Pareto boundary is identified with closed-form beamforming vectors and the line search for θ. Simulation results have confirmed that the proposed optimal beamforming vectors in the IFC-SC provides a lager R-E region than the IFC-PC.

Appendix A
Proof of Theorem 1

First, we verify that the feasible region of the power splitting parameter θ is written by (12). From the constraint in (14), θ is bounded as

$$\frac{(\sqrt{U-|\mathbf{h}_{12}^H\mathbf{g}_{1,\mathrm{I}}|^2}-\sqrt{P_2}\|\mathbf{h}_{22}\|)^2}{\|\mathbf{h}_{12}\|^2} \le \theta \le P_1 - \|\mathbf{g}_{1,\mathrm{I}}\|^2. \tag{16}$$

Let us denote the feasible set of the problem (13) as

$$\tilde{\mathcal{G}}(\theta,U,P_1) = \left\{\mathbf{g}_{1,\mathrm{I}} : \|\mathbf{g}_{1,\mathrm{I}}\|^2 \le P_1-\theta, |\mathbf{h}_{12}^H\mathbf{g}_{1,\mathrm{I}}|^2 \ge U-\varepsilon(\theta)\right\}.$$

Then, the condition (16) should be satisfied for any $\mathbf{g}_{1,\mathrm{I}} \in \tilde{\mathcal{G}}(\theta,U,P_1)$.

Thus, it follows

$$\begin{aligned} \theta &\ge \min_{\mathbf{g}_{1,\mathrm{I}}} \frac{(\sqrt{U-|\mathbf{h}_{12}^H\mathbf{g}_{1,\mathrm{I}}|^2}-\sqrt{P_2}\|\mathbf{h}_{22}\|)^2}{\|\mathbf{h}_{12}\|^2} \\ &= \frac{(\sqrt{U-(P_1-\theta)\|\mathbf{h}_{12}\|^2}-\sqrt{P_2}\|\mathbf{h}_{22}\|)^2}{\|\mathbf{h}_{12}\|^2}, \end{aligned} \tag{17}$$

$$\theta \le P_1 - \min_{\mathbf{g}_{1,\mathrm{I}}} \|\mathbf{g}_{1,\mathrm{I}}\|^2 = P_1, \tag{18}$$

where the equality in (17) is achieved with $\mathbf{g}_{1,\mathrm{I}} = \sqrt{P_1-\theta}\frac{\mathbf{h}_{12}}{\|\mathbf{h}_{12}\|}$ since the valid U is upper-bounded by $U \le E_{\max}^{\text{IFC-SC}}$. From (17), we have

$$\sqrt{\theta} \ge \frac{U - P_1\|\mathbf{h}_{12}\|^2 - P_2\|\mathbf{h}_{22}\|^2}{2\sqrt{P_2}\|\mathbf{h}_{12}\|\|\mathbf{h}_{22}\|}. \tag{19}$$

Since θ represents the transmit power of the energy signal at the information transmitter, θ is basically lower-bounded by $\theta \ge 0$. Combining this with (19) and (18), the feasible range of θ is given by (12).

Next, we show that the optimal rate $R_{\text{IFC-SC}}^{\star}(U)$ in problem (7) is the same as $\Psi(U)$ in (11). The feasible set of the problem in (7) can be expressed as

$$\mathcal{G}(U,P_1) = \left\{(\mathbf{g}_{1,\mathrm{I}},\theta) : \|\mathbf{g}_{1,\mathrm{I}}\|^2 \le P_1-\theta, |\mathbf{h}_{12}^H\mathbf{g}_{1,\mathrm{I}}|^2 \ge U-\varepsilon(\theta)\right\}.$$

It is easy to show that $\tilde{\mathcal{G}}(\theta,U,P_1) \subseteq \mathcal{G}(U,P_1)$ must be true for any feasible θ. Based on this fact, we will first prove that $R_{\text{IFC-SC}}^{\star}(U) \le \Psi(U)$. Let us define $\mathbf{g}_{1,\mathrm{I}}^{\star}$ as the optimal solution for the problem (7), then there exists at least one $\hat{\theta}$ such that $\mathbf{g}_{1,\mathrm{I}}^{\star} \in \tilde{\mathcal{G}}(\hat{\theta},U,P_1)$, for instance, $\hat{\theta} = P_1 - \|\mathbf{g}_{1,\mathrm{I}}^{\star}\|^2$. Thus, the optimal solution of problem (13) with $\theta = \hat{\theta}$ must be $\mathbf{g}_{1,\mathrm{I}}^{\star}$ since $\tilde{\mathcal{G}}(\hat{\theta},U,P_1) \subseteq \mathcal{G}(U,P_1)$. Therefore, it follows

$$R_{\text{IFC-SC}}^{\star}(U) = \tilde{R}_{\text{IFC-SC}}(\hat{\theta},U) \le \max_{\theta} \tilde{R}_{\text{IFC-SC}}(\theta,U) = \Psi(U). \tag{20}$$

Now, in order to prove the fact $R_{\text{IFC-SC}}^{\star}(U) \ge \Psi(U)$, we express $\tilde{\mathbf{g}}_{1,\mathrm{I}}(\theta)$ as the optimal beamforming vector for problem (13) with a given θ. Denoting $\theta^{\star}$ as the optimal solution for (11), it follows $\tilde{\mathcal{G}}(\theta^{\star},U,P_1) \subseteq \mathcal{G}(U,P_1)$, which implies that the feasible region of problem (7) always contains $\tilde{\mathbf{g}}_{1,\mathrm{I}}(\theta)$. Then $\Psi(U)$ is bounded by $R_{\text{IFC-SC}}^{\star}(U) \ge \tilde{R}_{\text{IFC-SC}}(\theta^{\star},U) = \Psi(U)$. By combining this and (20), we attain $R_{\text{IFC-SC}}^{\star}(U) = \Psi(U)$. This completes the proof.

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