Transmit Antenna Selection for Multi-User Underlay Cognitive Transmission With Zero-Forcing Beamforming

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***Abstract*—We present a transmit antenna subset selection scheme for an underlay cognitive system serving multiple secondary receivers. The secondary system employs zero-forcing beamforming to nullify the interference to multiple primary users and eliminate inter-user interference to the secondary users simultaneously. Simulation results show that the proposed scheme achieves near-optimal performance with low computational complexity. Lastly, an optimal power allocation strategy is also introduced to improve the secondary network throughput.**

***Index Terms*—Antenna subset selection, Gram-Schmidt orthogonalization, underlay cognitive radio, water-filling, zero-forcing beamforming.**

I. Introduction

Cognitive radio (CR) technology addresses the problem of licensed spectrum under-utilization either by opportunistically accessing or by sharing the spectrum with licensed users [1]–[3]. In the spectrum sharing mode, also referred to as underlay mode of operation, the cognitive (or secondary) users ensure that the interferences caused by their transmission to the licensed users do not exceed some fixed level determined from, for example, the quality of service (QoS) requirement of the primary network [2], [3]. As such, different methodologies for limiting the interference caused by the secondary transmitter (ST) to the primary receivers (PR) have been proposed in the literature for an underlay CR. For example, [4] proposed adapting ST's transmit power to maximize the secondary network throughput and limit the interference to the PR simultaneously. Interference to the PR can either be eliminated or limited by using multiple antennas at the secondary transmitter and proper beamforming [5], [6]. Although employing multiple antennas at the ST can effectively control or nullify the interference, it entails high hardware cost. This hardware complexity, however, can significantly be reduced by judicious transmit antenna subset selection [7]–[11].

Various transmit antenna selection strategies have been proposed for an underlay secondary transmission. The authors in [7] presented a *maximum signal-to-leak interference ratio (MSLIR)* based single transmit antenna selection scheme. The MSLIR scheme outperforms the *unconstrained selection* and *minimum interference selection* by considering both the ST to SR and ST to PR channel gains for improving the overall system capacity [7], [8]. The *difference antenna selection* scheme introduced in [9] outperforms the MSLIR scheme in many scenarios by adapting ST power along with the antenna selection. Another scheme targeting the minimization of SR's symbol error probability (SEP) was proposed in [10]. Recently, [12] proposed a transmit antenna selection scheme that leads to the maximum signal-to-interference ratio at the SR. These schemes, however, select only single ST antenna to serve a single SR and rely on power adaptation to meet the interference constraint at the PR.

The secondary network throughput can be improved by selecting more than one antenna at the ST. The authors in [7] presented three antenna selection strategies based on secondary channel's norm for different linear precoding schemes. Two antenna subset selection schemes for an underlay cognitive radio employing MRT were also proposed in [13]. However, these schemes are applicable for a single SR case and cannot be generalized to a multiple SRs and PRs case. To the best of the authors' knowledge, no antenna subset selection scheme has been proposed for a case where ST serves multiple SRs while either limiting or eliminating the interference to the primary receivers.

In this paper, we propose a novel antenna subset selection scheme for an underlay cognitive radio serving multiple secondary receivers. The ST employs zero-forcing (ZF) beamforming to eliminate the inter-user interference amongst the secondary receivers and nullify the interference to the primary receivers. Simulation results show that the proposed scheme achieves near-optimal performance while reducing the selection process computational complexity significantly. Optimal power allocation at the ST in a water-filling fashion is also introduced to improve the throughput of the secondary network.

The rest of the paper is organized as follows. Section II describes the system model. The proposed scheme is introduced in Section III, which is followed by Section IV on optimal power allocation. Lastly, concluding remarks are given

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in Section V.

II. System Model

Consider an underlay CR setup where the ST, equipped with N antennas, transmits data to K secondary users and nullifies interference to the R primary receivers as shown in Fig. 1. Here, for the sake of simplicity, all the primary and secondary receivers are equipped with single antenna only. The matrix $\widetilde{\mathbf{H}}$ denotes the multi-user MIMO (MU-MIMO) channel between the secondary receivers and the ST, while $\widetilde{\mathbf{G}}$ denotes the MU-MIMO channel between the primary receivers and the ST. We use $\mathbf{h}_i$ and $\mathbf{g}_i$ to denote the ith column of the matrices $\widetilde{\mathbf{H}}$ and $\widetilde{\mathbf{G}}$, respectively, where $i = 1, 2, \cdots, N$. Perfect channel knowledge of the matrices $\widetilde{\mathbf{H}}$ and $\widetilde{\mathbf{G}}$ at the ST is assumed. The ST may obtain such channel state information, for example, by uplink transmission of the primary and secondary receivers in TDD mode of communication.

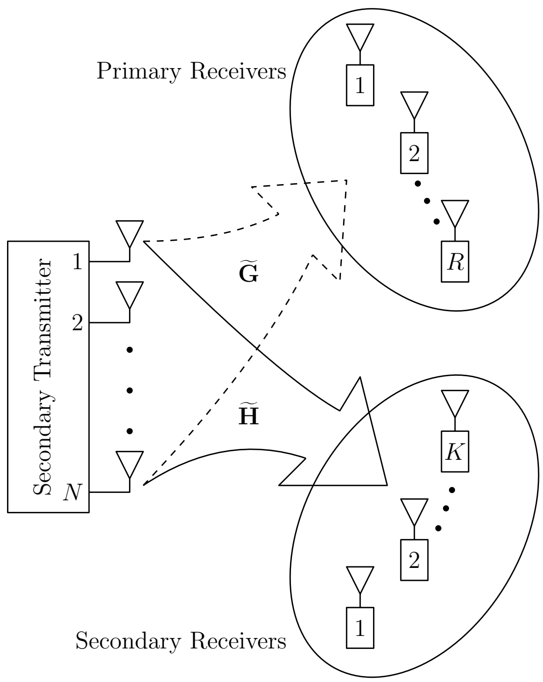

Fig. 1. An underlay cognitive radio with R primary receivers, K secondary receivers, and a single ST equipped with N antennas.

The ST selects $R + K$ antennas and performs ZF beamforming to transmit data to the SRs and eliminate interference to the PRs. Let $\mathbf{H}$ and $\mathbf{G}$ be respectively the sub-matrices of $\widetilde{\mathbf{H}}$ and $\widetilde{\mathbf{G}}$ comprising the channel gains corresponding to the selected antennas only. Denoting the first K columns of the matrix $\left(\left[\mathbf{H}^T \ \mathbf{G}^T\right]^T\right)^{-1}$ by $\mathbf{B}$, the ST precodes the unity power data symbols intended for K SRs, $\mathbf{s}$, as

$$\mathbf{x} = \sqrt{\frac{P_t}{\mathrm{Tr}\left(\mathbf{B}\mathbf{B}^H\right)}}\mathbf{B}\mathbf{s}, \tag{1}$$

where P_t denotes the transmit power of the ST, and $\mathbf{x}$ is the precoded vector of length $K + R$. Here the scaling factor is determined from the transmit power constraint at the ST, i.e. $\mathrm{Tr}(\mathbf{x}\mathbf{x}^H) = P_t$, and the independence of the data symbols, i.e. $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}$. As $\mathbf{H}\mathbf{B} = \mathbf{I}$ and $\mathbf{G}\mathbf{B} = \mathbf{O}$, where $\mathbf{O}$ denotes a null matrix, the resulting transmission eliminates both the interferences to the primary receivers and the inter-user interference amongst the secondary receivers. The sum-rate, R_{sum}, of the secondary network can be written as

$$R_{\mathrm{sum}} = K \log_2\left(1 + \frac{P_t}{\sigma^2 \,\mathrm{Tr}\left(\mathbf{B}\mathbf{B}^H\right)}\right), \tag{2}$$

where σ^2 is the common noise variance at each SR. According to (2), R_{sum} can be maximized by selecting an antenna subset that results in the minimum value of $\mathrm{Tr}\left(\mathbf{B}\mathbf{B}^H\right)$. This can be achieved by selecting the subset through an exhaustive search, but it entails a high computational cost. As such, we propose a low-complexity sub-optimal antenna subset selection strategy that results in near-optimal performance.

III. Proposed Scheme

We will first consider a case when the ST is serving only a single ST and nullifies interference to only one PR by selecting two antennas. After developing a sub-optimal antenna subset selection scheme for that case, we will extend the scheme to a case where ST serves multiple SRs and eliminates interference to multiple PRs.

A. Single-SR/ Single-PR Case

In this case, the ST selects two antennas to serve the SR and eliminate the interference to the PR. By denoting the indices of the chosen antennas by i and j, the beamformed vector, $\mathbf{x}$, can be simplified to

$$\mathbf{x} = \sqrt{\frac{P_t}{|g_i|^2 + |g_j|^2}}\begin{bmatrix} g_j \\ -g_i \end{bmatrix}\mathbf{s}. \tag{3}$$

The resultant R_{sum} is given as

$$R_{\mathrm{sum}} = \log_2\left(1 + \frac{P_t}{\sigma^2}\frac{|h_i g_j - h_j g_i|^2}{|g_i|^2 + |g_j|^2}\right). \tag{4}$$

The optimal antenna pair that maximizes the sum rate can be found by the exhaustive search over all the $\binom{N}{2}$ antenna pairs. The chosen pair (i^*, j^*) can be mathematically described as

$$(i^*, j^*) = \underset{(i,j)}{\mathrm{argmax}} \frac{|h_i g_j - h_j g_i|^2}{|g_i|^2 + |g_j|^2}. \tag{5}$$

However, the exhaustive search based antenna subset selection incurs a high computational burden. Since the search is performed over $\binom{N}{2}$ pairs, the computational complexity of the optimal scheme is $\mathcal{O}(N^2)$. This computational complexity can be significantly reduced by selecting only one antenna at a time. In the proposed scheme, we first of all select an antenna based on the secondary channel gains only. After selecting the first antenna, the antenna that maximizes R_{sum} amongst the unselected antennas is selected as the second antenna. Mathematically, we select antennas i^* and j^* as

$$i^* = \underset{i}{\mathrm{argmax}}\, |h_i|^2, \tag{6}$$

and

$$j^* = \underset{j}{\operatorname{argmax}} \frac{|h_{i^*} g_j - h_j g_{i^*}|^2}{|g_{i^*}|^2 + |g_j|^2}. \quad (7)$$

The computational complexity of the proposed scheme is $\mathcal{O}(N)$ implying that the proposed scheme results in a complexity reduction of an order of N, which is significant especially in a massive MIMO setting [14].

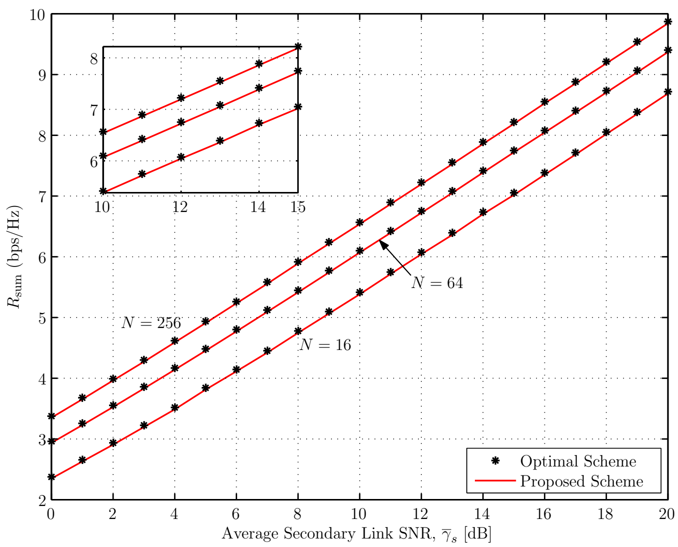

Fig. 2. R_{sum} for the optimal and the proposed antenna subset selection schemes against secondary link average SNR, $\overline{\gamma}_s$, in an i.i.d Rayleigh fading environment.

Fig. 2 shows the trend of R_{sum} against average secondary link SNR, $\overline{\gamma}_s = E[P_t|h_i|^2/\sigma^2]$, in an independent and identically distributed (i.i.d) Rayleigh fading environment. The secondary to primary link average SNR, $\overline{\gamma}_p = E[P_t|g_i|^2/\sigma^2]$, is 5 dB. A total of 10^4 Monte-Carlo trials were used to generate the plot. Fig. 2 shows that the proposed sub-optimal scheme, while reducing the computational complexity significantly, achieves near optimal performance for the complete average SNR range and different values of N.

B. Multiple-SR/ Multiple-PR Case

The selection procedure described above can be extended to the case where the ST serves multiple SRs and eliminates interference to multiple PRs by minimizing an approximation of $\operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right)$. To this end, we define $\mathbf{W}$ as the composite matrix of channel gains, i.e. $\mathbf{W} = \left[\mathbf{H}^T\ \mathbf{G}^T\right]^T$. Since $\mathbf{B} = \mathbf{W}^{-1}\left[\mathbf{I}\ \mathbf{O}\right]^T$, $\operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right)$ is equivalent to the sum of the first K diagonal entries of the matrix $\left(\mathbf{W}\mathbf{W}^H\right)^{-1}$. This implies that $\operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right)$ can be minimized by reducing the value of $\operatorname{Tr}\left(\mathbf{W}\mathbf{W}^H\right)^{-1}$. Furthermore, Gram-Schmidt orthogonalization can be performed on the columns of $\mathbf{W}$ as

$$\mathbf{w}_1 = a_{11}\mathbf{u}_1; \quad (8)$$

$$\mathbf{w}_2 = a_{12}\mathbf{u}_1 + a_{22}\mathbf{u}_2; \quad (9)$$

$$\vdots$$

$$\mathbf{w}_{K+R} = \sum_{j=1}^{K+R} a_{K+Rj}\mathbf{u}_j, \quad (10)$$

where $\mathbf{w}_k = [\mathbf{h}_k^T\ \mathbf{g}_k^T]^T$ is the composite channel vector corresponding to the kth selected antenna, and $\mathbf{u}_k$'s are orthonormal basis vectors. As shown in the Appendix, the trace of $\mathbf{B}\mathbf{B}^H$ can be approximated as

$$\operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right) \approx \sum_{k=1}^{K+R} \frac{1}{|a_{kk}|^2}\left(1 + \sum_{r=1}^{k-1}\frac{|a_{rk}|^2}{|a_{rr}|^2}\right). \quad (11)$$

Similarly, by performing Gram-Schmidt orthogonalization on $\mathbf{H}$,

$$\operatorname{Tr}\left(\mathbf{H}\mathbf{H}^H\right)^{-1} \approx \sum_{i=1}^{K} \frac{1}{|\beta_{ii}|^2}\left(1 + \sum_{j=1}^{i-1}\frac{|\beta_{ji}|^2}{|\beta_{jj}|^2}\right), \quad (12)$$

where β_{ij} is the component of $\mathbf{h}_j$ along the direction of the ith orthonormal basis vector computed by Gram-Schmidt orthogonalization of $\mathbf{H}$.

In the proposed scheme, like for the single-SR and single-PR case, we select the first K antennas based on the secondary channel gains, whereas the remaining R antennas are selected based on both the primary and secondary channel gains. Let i^* and j^* denote the indices of the antennas selected in the ith and jth steps respectively, where $i = 1, 2, \cdots, K$ and $j = K+1, K+2, \cdots, K+R$. We propose to select those antennas as the first K antennas that result in small values of $\operatorname{Tr}\left(\mathbf{H}\mathbf{H}^H\right)^{-1}$. Specifically, in the ith step, the index of the chosen antenna is computed as

$$i^* = \underset{i \in \mathcal{I}_i'}{\operatorname{argmax}} \frac{1}{|\beta_{ii}|^2}\left(1 + \sum_{r=1}^{i-1}\frac{|\beta_{ri}|^2}{|\beta_{rr}|^2}\right), \quad (13)$$

where $\mathcal{I}_i'$ is the set of antenna indices that have not been selected till the ith step.

For the remaining R antennas, the selection is carried out such that the approximate value of $\operatorname{Tr}\left(\mathbf{B}\mathbf{B}^H\right)$ given in (11) is minimized. Mathematically, the index of the selected antenna in the jth step is given as

$$j^* = \underset{j \in \mathcal{I}_j'}{\operatorname{argmax}} \frac{1}{|a_{jj}|^2}\left(1 + \sum_{r=1}^{j-1}\frac{|a_{rj}|^2}{|a_{rr}|^2}\right). \quad (14)$$

In the proposed scheme, Gram-Schmidt orthogonalization is performed on a sub-matrix of $\mathbf{W}$ for $K+R$ times. Since orthogonalization has the computational complexity of $\mathcal{O}\big(N(K+R)\big)$, the computational complexity of the proposed scheme is $\mathcal{O}\big(N(K+R)^2\big)$. The optimal scheme, on the other hand, finds the best subset by performing an exhaustive search over $\binom{N}{K+R}$ antenna subsets and computes the inverse of the matrix $\mathbf{W}$ for each subset. Since the matrix inversion requires $\mathcal{O}\big((K+R)^3\big)$ computations, the computational complexity of the optimal scheme is $\mathcal{O}\big(N^{K+R}(K+R)^3\big)$. As a result, the proposed scheme brings significant computational complexity savings of an order of $N^{K+R-1}(K+R)$ over the optimal scheme.

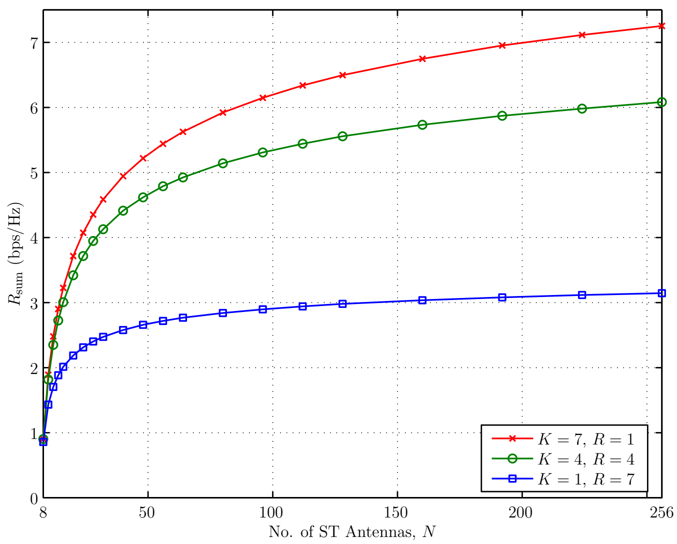

Fig. 3. R_{sum} for the proposed antenna subset selection scheme against the number of ST antennas, N, in an i.i.d Rayleigh fading environment with $\overline{\gamma}_s = 15$ dB and $\overline{\gamma}_p = 10$ dB.

Fig. 3 shows the behaviour of R_{sum} against the number of ST antennas, N, in an i.i.d Rayleigh fading environment with $\overline{\gamma}_s = 15$ dB and $\overline{\gamma}_p = 10$ dB. The plots were generated by averaging the sum rate over 10^5 different channel realizations. A total of 8 ST antennas are selected for serving K SRs and eliminating the interference to R PRs. The sum rate increases by increasing the number of ST antennas as equipping the ST with more antennas increases the chances of selecting a good subset. For large values of N, increasing N brings only diminishing results as is the case with conventional single-user transmit antenna selection scheme [11], [15]. Lastly, increasing K and reducing R simultaneously bring an overall performance gain as evident from the figure.

Fig. 4 shows the trend of the sum rate against the number of secondary receivers, K. Here the ST nullifies the interference to the same number of primary users as the secondary receivers, i.e. $R = K$, by selecting $2K$ antennas. As evident from the figure, R_{sum} increases by increasing the number of ST antennas, N. Furthermore, the sum rate generally increases by increasing the number of secondary receivers, especially for large N and small K. For small values of N and large K, increasing the number of SRs may result in a decrease in the sum rate. This is because adding an extra secondary receiver and a primary receiver results in the distribution of the total power in more users and a large-norm beamforming matrix. This in turn results in a loss in the individual SNR which may outweigh the performance gain of adding an extra user.

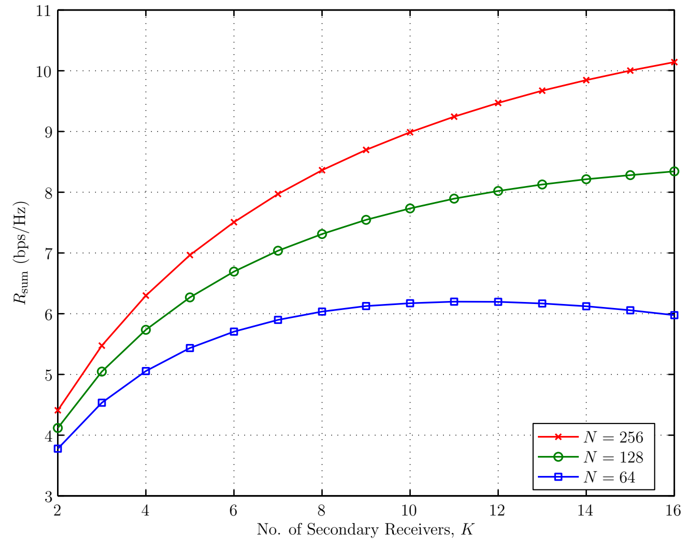

Fig. 4. R_{sum} for the proposed antenna subset selection scheme against the number of secondary receivers, K, in an i.i.d Rayleigh fading environment with $\overline{\gamma}_s = \overline{\gamma}_p = 0$ dB.

IV. Optimal Power Allocation

The sum rate of the secondary network can further be improved by optimally distributing the total transmit power amongst the secondary receivers. Specifically, let $\mathbf{b}_i$ denote the ith column of the beamforming matrix $\mathbf{B}$, where $i = 1, 2, \cdots, K$. If P_i is the power allocated to the ith SR, then the beamformed vector $\mathbf{x}$ can be written as

$$\mathbf{x} = \sum_{i=1}^{K} \sqrt{P_i}\mathbf{b}_i s_i, \tag{15}$$

where s_i is the unity power transmit data symbol intended for the ith SR. Consequently, the secondary system's sum rate is

$$R_{\text{sum}} = \sum_{i=1}^{K} \log_2\left(1 + \frac{P_i}{\sigma^2}\right). \tag{16}$$

The optimal power allocation can be found by maximizing (16) with the constraints $\sum_{i=1}^{K} P_i \|\mathbf{b}_i\|^2 \leq P_t$ and $P_i \geq 0$ for $i = 1, 2, \cdots, K$. Using Lagrange multipliers, the optimal solution turns out to be

$$\frac{P_i}{\sigma^2} = \left(\frac{\mu}{\|\mathbf{b}_i\|^2} - 1\right)^+, \tag{17}$$

where $(x)^+ = \max(x, 0)$, and μ can be found by

$$\sum_{i=1}^{K} (\mu - \|\mathbf{b}_i\|^2)^+ = \frac{P_t}{\sigma^2}. \tag{18}$$

By distributing the total power in the water-filling fashion as in (17), the achievable sum rate can be shown to be

$$R_{\text{sum}} = \sum_{\{i:\|\mathbf{b}_i\|^2<\mu\}} \log_2\left(\frac{\mu}{\|\mathbf{b}_i\|^2}\right). \tag{19}$$

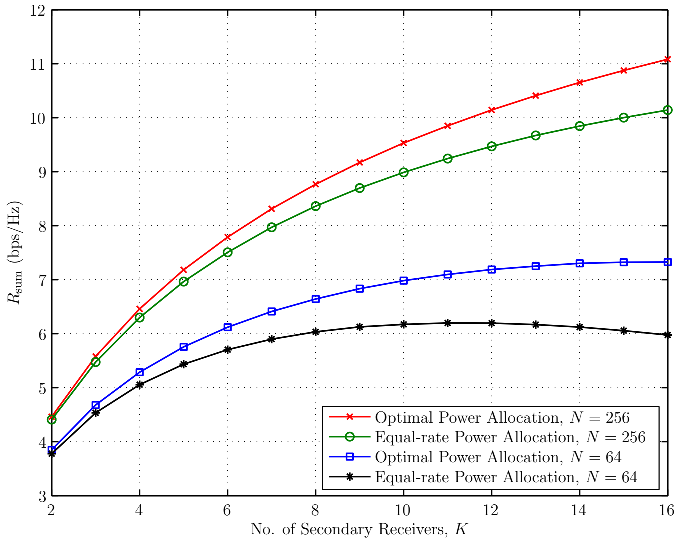

Fig. 5. R_{sum} of the proposed antenna subset selection scheme against the number of secondary receivers, K, for both equal-rate and optimal power allocations in an i.i.d Rayleigh fading environment with $\overline{\gamma}_s = \overline{\gamma}_p = 0$ dB.

Remark: Allocating equal power to each user, i.e. $P_i = P_t/\operatorname{Tr}(\mathbf{B}\mathbf{B}^H)$, achieves the same data rate for each secondary receiver, and the sum rate in that scenario is given by (2).

Fig. 5 shows the behaviour of the sum rate of the proposed antenna subset selection scheme for both the optimal and equal-rate power allocation strategies. Same parameters are used for the simulation as for Fig. 4. The optimal power allocation results in the higher value of the sum rate for all values of K and N. Futhermore, the gap between the sum rates for optimal and equal-rate power allocation strategies becomes wider for higher values of K.

V. Conclusion

Zero-forcing beamforming at the secondary transmitter of a cognitive radio can nullify the interference to the primary receivers and eliminate inter-user interference in the secondary network. A novel sub-optimal antenna subset selection is introduced in this paper for both the single-SR/single-PR and multiple-SR/multiple-PR scenarios. The proposed scheme, while achieving near-optimal performance, brings huge hardware cost savings. Furthermore, an optimal power allocation strategy based on the water-filling principle is introduced to further improve the secondary system throughput.

Appendix

Consider Gram-Schmidt orthogonalization of a 2×2 matrix $\mathbf{A}$ as

$$\mathbf{A} = \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix} [\mathbf{v}_1 \ \mathbf{v}_2]. \tag{20}$$

The $\operatorname{Tr}(\mathbf{A}\mathbf{A}^H)^{-1}$ can be shown to be

$$\operatorname{Tr}\left(\mathbf{A}\mathbf{A}^H\right)^{-1} = \frac{1}{|c_{11}|^2} + \frac{1}{|c_{22}|^2}\left(1 + \frac{|c_{12}|^2}{|c_{11}|^2}\right). \tag{21}$$

Similarly, for a 3×3 matrix $\mathbf{A}$, we have

$$\operatorname{Tr}\left(\mathbf{A}\mathbf{A}^H\right)^{-1} = \frac{1}{|c_{11}|^2} + \frac{1}{|c_{22}|^2}\left(1 + \frac{|c_{12}|^2}{|c_{11}|^2}\right) + \frac{1}{|c_{33}|^2}\left(1 + \frac{\left|c_{13} - \frac{c_{12}c_{23}}{c_{22}}\right|^2}{|c_{11}|^2} + \frac{|c_{23}|^2}{|c_{22}|^2}\right). \tag{22}$$

We approximate $\operatorname{Tr}(\mathbf{A}\mathbf{A}^H)^{-1}$ by ignoring the fraction in the numerator of the last term and arrive at

$$\operatorname{Tr}\left(\mathbf{A}\mathbf{A}^H\right)^{-1} \approx \sum_{m=1}^{3} \frac{1}{|c_{mm}|^2}\left(1 + \sum_{r=1}^{m-1} \frac{|c_{rm}|^2}{|c_{rr}|^2}\right). \tag{23}$$

Following a similar reasoning, we can approximate $\operatorname{Tr}(\mathbf{A}\mathbf{A}^H)^{-1}$ for an $L \times L$ matrix $\mathbf{A}$ as

$$\operatorname{Tr}\left(\mathbf{A}\mathbf{A}^H\right)^{-1} \approx \sum_{m=1}^{L} \frac{1}{|c_{mm}|^2}\left(1 + \sum_{r=1}^{m-1} \frac{|c_{rm}|^2}{|c_{rr}|^2}\right). \tag{24}$$

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