Beamforming in Coexisting Wireless Systems with Uncertain Channel State Information

(Invited paper)

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***Abstract*—This paper considers an underlay access strategy for coexisting wireless networks where the secondary system utilizes the primary spectrum to serve its users. We focus on the practical cases where there is uncertainty in the estimation of channel state information (CSI). Here the throughput performance of each system is limited by the interference imposed by the other, resulting in conflicting objectives. We first analyze the fundamental trade-off between the tolerance interference level at the primary system and the total achievable throughput of the secondary users. We then introduce a beamforming design problem as a multi-objective optimization to minimize the interference imposed on each of the primary users while maximizing the intended signal received at every secondary user, taking into account the CSI uncertainty. We then map the proposed optimization problem to a robust counterpart under the maximum CSI estimation error. The robust counterpart is then transformed into a standard convex semi-definite programming. Simulation results confirm the effectiveness of the proposed scheme against various levels of CSI estimation error. We further show that in the proposed approach, the trade-off in the two systems modelled by Pareto frontier can be engineered by adjusting system parameters. For instance, the simulations show that at the primary system interference thresholds of -10 dBm (-5 dBm) by increasing number of antennas from 4 to 12, the secondary system throughput is increased by 3.3 bits/s/channel-use (5.3 bits/s/channel-use).**

I. Introduction

In an underlay spectrum access cognitive radio network, where the primary and secondary networks share the same radio resources [1], there exist fundamentally contradictory interests between the licensed, i.e., primary, and unlicensed, i.e., secondary, networks. The performance of each network, e.g., system throughput, is limited by the interference imposed by the other. Since the radio resource is owned by the primary network, the secondary system has to operate in a way that its interference inflicted on the primary users is less than the interference threshold/tollerance level defined by the primary system. The interference threshold is usually a fixed maximum tolerable level, see, e.g., [1] and references therein. It has been shown that beamforming is an efficient method for the secondary system to manage its interference [2]–[6].

In many cases, the primary system can tolerate a higher interference level. For instance in cases where the primary system experiences a lower traffic load or the primary receiver has a sophisticated coding technique [7]. Motivated by primary systems' higher tolerance we recently introduced in [8] a multi-objective optimization problem (MOP) [9], [10] that simultaneously optimizes two contradicting objectives. One objective is to minimize the interference due to the transmission of a cognitive base station (BS) on each of the primary users (PUs). The second set of objectives is to maximize the intended signal received at each of the secondary users (SUs). Since the SUs share the same resource, maximizing received signal at any SU conflicts with that of the others. Therefore, we incorporated a set of signal-to-interference-plus-noise-ratio (SINR) constraints to ensure that each SU is provided with its required level at least. To protect the primary system against the secondary system communication activity, interference levels at the PUs are kept below their thresholds.

Our preliminary work in [8] was based on the perfect channel state information (CSI). Considering practical scenarios, the available CSI at the BSs is imperfect due to several reasons including estimation error, delay, and the quantization error. Uncertain CSI may further arise as a result of limited feedback from a user terminal to a BS [11]. Uncertain CSI significantly affect the performance of the beamforming schemes as the constraints in the corresponding optimization problem might be violated under CSI estimation error, see, e.g., [11] and [12]. Hence it is of utmost importance to develop beamforming schemes robust against imperfect CSI.

This paper takes a further step by introducing a robust beamforming design to the original MOP proposed in [8]. We model the uncertainty of CSI obtained by the transmitter confined in hyper-spherical sets. We derive a robust counterpart for the proposed optimization problem for the worst case of CSI estimation error, and then transform the robust counterpart into a standard semidefinite programming (SDP) form which is convex and can be solved using standard optimization packages. Simulation results indicate the trade-off between the interference tolerance at PUs and the total achievable throughput at SUs. The proposed approach provides robustness against uncertainty in CSI estimation at the cost of a decrease in the total attainable SUs' throughput. In return the proposed scheme guarantees all the SUs' SINR, and PUs' interference constraints.

Notations: $\mathbf{x}$: column vector $\mathbf{x}$; $\mathbf{X}$: matrix $\mathbf{X}$; $\mathrm{Tr}(\cdot)$: trace operator; $\mathbf{Y} \succeq 0$: matrix $\mathbf{Y}$ is positive semi definite; $\preccurlyeq$: element-wise inequality; $(y_i)_{i=1}^{U}$: $\begin{bmatrix} y_1 & y_2 & \cdots & y_U \end{bmatrix}^T$;

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$\mathbb{E}(x)$: expected value of x; $\|\cdot\|$: Frobenius norm operator; $(\cdot)^T$: transpose operator; $(\cdot)^H$: complex conjugate transpose operator; $\widetilde{\mathbf{x}}$ or $\widetilde{\mathbf{X}}$: estimated value of $\mathbf{x}$ or $\mathbf{X}$.

II. System Model

We consider the primary system as a cellular communication network with N PUs. Utilizing underlay spectrum access [1], a cognitive BS serves U SUs by sharing the spectrum of the primary network subject to interference constraints at the PUs. The beamforming technique is adopted at the cognitive BS which is equipped with M antennas, i.e., $M \geq U$. We assume single-antenna setting at the SUs and PUs. The received signal at the ith SU is

$$y_i = \mathbf{h}_{s,i}^H \mathbf{w}_i s_i + \sum_{j=1, j\neq i}^{U} \mathbf{h}_{s,i}^H \mathbf{w}_j s_j + n_i, \tag{1}$$

where $\mathbf{h}_{s,i}^H = \widetilde{\mathbf{h}}_{s,i}^H + \mathbf{e}_{s,i}^H$ is the actual channel between the cognitive BS and the ith SU; $\widetilde{\mathbf{h}}_{s,i}^H \in \mathbb{C}^{1\times M}$ and $\mathbf{e}_{s,i}^H \in \mathbb{C}^{1\times M}$ are, respectively, the estimated channel and its corresponding estimation error; $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ is the beamforming vector for the ith SU; s_i is the data symbol to be sent to the ith SU; and n_i is a zero mean circularly symmetric complex Gaussian noise with variance σ_i^2, i.e., $n_i \sim \mathcal{CN}(0, \sigma_i^2)$. The primary system imposed interference at the SUs is considered as an additive background noise [13]. For brevity, the average transmitted symbol energy to the ith SU at the cognitive BS is assumed to be unity. Let $\mathbf{R}_{s,i} = \mathbb{E}\left(\mathbf{h}_{s,i}\mathbf{h}_{s,i}^H\right)$, then $\mathbf{R}_{s,i} = \widetilde{\mathbf{R}}_{s,i} + \mathbf{\Delta}_{s,i}$ where $\widetilde{\mathbf{R}}_{s,i} = \mathbb{E}_{\widetilde{\mathbf{h}}_{s,i}}\left(\widetilde{\mathbf{h}}_{s,i}\widetilde{\mathbf{h}}_{s,i}^H\right)$, $\mathbf{\Delta}_{s,i} = \mathbb{E}_{\mathbf{e}_{s,i}}\left(\mathbf{e}_{s,i}\mathbf{e}_{s,i}^H\right)$. Furthermore, let $\mathcal{W} = \{\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_U\}$ be the set of candidate beamforming vectors in the cognitive BS for all SUs. The SINR at the ith SU is

$$g_i(\mathcal{W}) = \frac{\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i} + \mathbf{\Delta}_{s,i}\right)\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\left(\widetilde{\mathbf{R}}_{s,i} + \mathbf{\Delta}_{s,i}\right)\mathbf{w}_j + \sigma_i^2}. \tag{2}$$

Let $\mathbf{R}_{p,t} = \mathbb{E}\left(\mathbf{h}_{p,t}\mathbf{h}_{p,t}^H\right)$ where $\mathbf{h}_{p,t} = \widetilde{\mathbf{h}}_{p,t}^H + \mathbf{e}_{p,t}^H$ is the correct channel between the cognitive BS and the tth PU, $\widetilde{\mathbf{h}}_{p,t}^H \in \mathbb{C}^{1\times M}$ and $\mathbf{e}_{p,t}^H \in \mathbb{C}^{1\times M}$ are, respectively, the estimated channel and its corresponding error. We can write $\mathbf{R}_{p,t} = \widetilde{\mathbf{R}}_{p,t} + \mathbf{\Delta}_{p,t}$ where $\widetilde{\mathbf{R}}_{p,t} = \mathbb{E}_{\widetilde{\mathbf{h}}_{p,t}}\left(\widetilde{\mathbf{h}}_{p,t}\widetilde{\mathbf{h}}_{p,t}^H\right)$, $\mathbf{\Delta}_{p,t} = \mathbb{E}_{\mathbf{e}_{p,t}}\left(\mathbf{e}_{p,t}\mathbf{e}_{p,t}^H\right)$. We aim to design beamforming vectors for the cognitive BS such that the total interference imposed on every t PU, i.e., $\sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{R}_{p,t}\mathbf{w}_i$, is kept below its threshold I_t. We assume that the primary system is able to update its interference thresholds, i.e., I_t, and provide that information to the secondary system. Hereafter, if otherwise stated, $i \in \{1, \cdots, U\}$ and $t \in \{1, \cdots, N\}$.

III. Robust Beamforming

The aim is to design the beamforming vector $\mathbf{w}_i$ for each SU in the secondary BS considering their required SINR. In this paper, our objective is to maximize the intended signal power received at each SU i, i.e., $\mathbf{w}_i^H\mathbf{R}_{s,i}\mathbf{w}_i$, while minimizing the corresponding interference inflicted at each PU t, i.e., $\sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{R}_{p,t}\mathbf{w}_i$. We set

$$f_{s,i}(\mathcal{W}) = -\mathbf{w}_i^H\mathbf{R}_{s,i}\mathbf{w}_i = -\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i} + \mathbf{\Delta}_{s,i}\right)\mathbf{w}_i, \tag{3}$$

$$f_{p,t}(\mathcal{W}) = \sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{R}_{p,t}\mathbf{w}_i = \sum_{i=1}^{U}\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{p,t} + \mathbf{\Delta}_{p,t}\right)\mathbf{w}_i. \tag{4}$$

The objective vector is then defined as

$$\begin{aligned}\mathbf{f}(\mathcal{W}) \quad = \quad & [f_{p,1}(\mathcal{W}), \cdots, f_{p,N}(\mathcal{W}), \\ & f_{s,1}(\mathcal{W}), \cdots, f_{s,U}(\mathcal{W})].\end{aligned} \tag{5}$$

We now define the decision space

$$\begin{aligned}\mathcal{D} \triangleq \Big\{ & \mathcal{W} \mid (\gamma_i)_{i=1}^{U} \preccurlyeq (g_i(\mathcal{W}))_{i=1}^{U}, \\ & (f_{p,t}(\mathcal{W}))_{t=1}^{N} \preccurlyeq (I_t)_{t=1}^{N}, \sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{w}_i \leq P_{\mathrm{m}}\Big\}\end{aligned} \tag{6}$$

where γ_i is the required SINR level at the ith SU and P_{m} is the maximum transmit power of the secondary BS . We propose the following MOP:

$$\min_{\mathcal{W}\in\mathcal{D}} \quad \mathbf{f}(\mathcal{W}). \tag{7}$$

In (7), the set of SINR constraints guarantees each SU being served with its required level at least. The optimization problem then tries to tune the beam to further improve each SU's received signal strength and thus to raise the achievable throughput above the required level as far as possible.

Let $\lambda_{p,t} > 0\ \forall t$, $\lambda_{s,i} > 0\ \forall i$ and $\sum_{t=1}^{N}\lambda_{p,t} + \sum_{i=1}^{U}\lambda_{s,i} = 1$. According to [10], the Properly Pareto optimal solution[1], i.e., $\widehat{\mathcal{W}}$, to the MOP defined in (7) can be obtained as the optimal solution to the following problem:

$$\begin{aligned}\min \quad & \sum_{t=1}^{N}\lambda_{p,t}f_{p,t}(\mathcal{W}) + \sum_{i=1}^{U}\lambda_{s,i}f_{s,i}(\mathcal{W}), \\ \text{s. t.} \quad & g_i(\mathcal{W}) \geq \gamma_i, \ \forall i, \\ & f_{p,t}(\mathcal{W}) \leq I_t, \forall t, \\ & \sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{w}_i \leq P_{\mathrm{m}}.\end{aligned} \tag{8}$$

To account for the imperfection of channel estimation, here we assume that the uncertainty in the estimation of channel covariance matrices $\delta_{s,i}$ and $\delta_{p,t}$ are confined within hyper-spherical sets $\mathcal{E}_{s,i}$ and $\mathcal{E}_{p,t}$, i.e.,

$$\mathcal{E}_{s,i} = \left\{\mathbf{\Delta}_{s,i} \in \mathbb{C}^{M\times M} : \|\mathbf{\Delta}_{s,i}\| \leq \delta_{s,i}\right\}, \forall i \tag{9}$$

$$\mathcal{E}_{p,t} = \left\{\mathbf{\Delta}_{p,t} \in \mathbb{C}^{M\times M} : \|\mathbf{\Delta}_{p,t}\| \leq \delta_{p,t}\right\}, \forall t, \tag{10}$$

where $\delta_{s,i}$ and $\delta_{p,t}$ are the radius of $\mathcal{E}_{s,i}$ and $\mathcal{E}_{p,t}$, respectively. Furthermore, for any $M \times M$ Hermitian positive semidefinite

[1] *Properly Pareto optimal* solutions are defined as Pareto optimal solutions with bounded trade-offs amongst the objectives [10].

matrix, $\mathbf{Y}$, given $\|\mathbf{Y}\| \leq \delta$, and an $M \times 1$ arbitrary vector $\mathbf{x}$, we have

$$\mathbf{x}^H \mathbf{Y} \mathbf{x} \leq \mathbf{x}^H \delta \mathbf{I} \mathbf{x}. \tag{11}$$

Utilizing (11), we then evaluate the worst case effect of the channel estimation error on $f_{s,i}(\mathcal{W})$ and $f_{p,t}(\mathcal{W})$ as follows:

$$\max_{\|\mathbf{\Delta}_{s,i}\| \leq \delta_{s,i}} f_{s,i}(\mathcal{W}) = -\mathbf{w}_i^H \left(\widetilde{\mathbf{R}}_{s,i} - \delta_{s,i} \mathbf{I} \right) \mathbf{w}_i, \tag{12}$$

and

$$\max_{\|\mathbf{\Delta}_{p,t}\| \leq \delta_{p,t}} f_{p,t}(\mathcal{W}) = \sum_{i=1}^{U} \mathbf{w}_i^H \left(\widetilde{\mathbf{R}}_{p,t} + \delta_{p,t} \mathbf{I} \right) \mathbf{w}_i. \tag{13}$$

Similarly, utilizing (11) we then write the worst case of error on $g_i(\mathcal{W})$ as (14)[2] given at the top of next page. Hence, in the worst case (8) can be cast as (15) shown at the top of next page.

We then proceed by defining beamforming matrix $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H$, where $\mathbf{W}_i \succeq 0$ and $\mathbf{W}_i$ is a rank-one matrix. By rearranging the constraints along with $\mathbf{x}^H \mathbf{Y} \mathbf{x} = \mathrm{Tr}\left(\mathbf{Y} \mathbf{x} \mathbf{x}^H\right)$, problem (15) is then converted to the SDP form in (16) shown in the next page, where $\{\mathbf{W}_i\} = \{\mathbf{W}_1, \cdots, \mathbf{W}_U\}$ is the set of beamforming matrices. When transforming (15) into (16), we have dropped the rank-one condition on $\mathbf{W}_i$. Following the same approach in the proof of Theorem 1 in [8], one can prove that the optimal solutions to problem (16) are rank one. Therefore, the transformed problem (16) maintains the optimality of the original problem (15). The optimization problem in (16) can be solved by the SeDuMi solver, provided by CVX optimization package [15], to obtain the set of optimal beamforming matrices $\mathbf{W}_i^\star$.

IV. Simulation Results

In this section, the performance of the proposed scheme is investigated and compared against a baseline introduced in [8]. It is worth mentioning that when the error levels are set to zero, i.e., $\delta_{s,i} = \delta_{p,t} = 0, \ \forall i, t$, the proposed robust scheme reduces to the baseline approach.

Here we consider a cognitive cellular network with 2 PUs and 2 SUs. The PUs are located at -50° and 50° while the SUs are located at -10° and 10° relative to the array broadside. The distances from the SUs and PUs to the cognitive BS are 0.5 km and 1 km, respectively. The (p, q)th entry of the $M \times M$ channel matrice $\mathbf{R}_{s,i}$ or $\mathbf{R}_{p,t}$ is obtained using [16]:

$$\xi e^{\frac{j2\pi\Delta}{\ell}[(q-p)\sin\phi]} e^{-2\left[\frac{\pi\Delta\sigma_a}{\ell}\{(q-p)\cos\phi\}\right]^2}, \tag{17}$$

where ξ represents the channel gain coefficient, ϕ is the angle of departure, Δ is the antenna spacing at the BS, σ_a is the angular spread and ℓ is the carrier wavelength. In (17), we set $\Delta = \ell/2$, $\sigma_a = 2^\circ$, $\xi = 34.5 + 35\log_{10}(d)$ captures the distance-dependent path-loss, where d is the distance in meters with $d \geq 35$ m, a log-normal shadow fading with 8 dB standard deviation, and a Rayleigh component for the multi-path fading channel. The noise power spectral density, the noise figure at each SU, and antenna gain are assumed to be -174 dBm/Hz, 5 dB, and 15 dBi, respectively.

[2]The worst-case-evaluation approach for SINR was first introduced in [14].

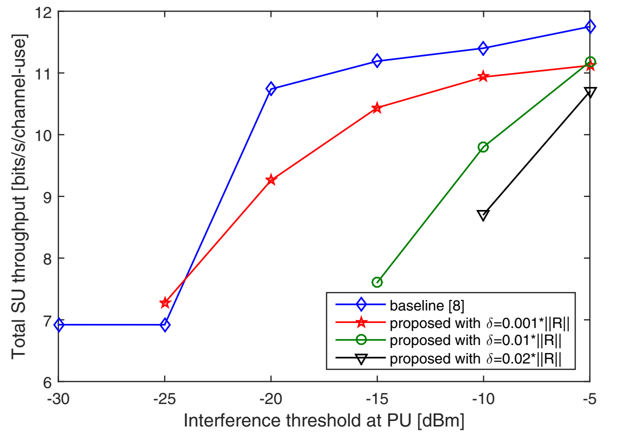

Fig. 1. Total throughput of the SUs *vs.* the interference threshold at the PUs for the proposed and baseline schemes. SUs' SINR requirements $\gamma_i = 10$ dB $\forall i$. The proposed scheme is shown with different error levels. The number of antenna $M = 6$.

Fig. 1 illustrates the approximation of Pareto frontiers of the proposed and the baseline scheme. For a given I_m, Fig. 1 depicts the maximum achievable SU throughput. The proposed scheme is shown with different error levels δ. Without loss of generality we have set the same error level for all users, i.e., $\delta = \delta_{s,i} = \delta_{p,t}, \ \forall i, t$. Fig. 1 also indicates the fact that there is a trade-off between the interference tolerance level at PUs and the total achievable throughput of the SUs. The higher the interference tolerance level at the PU is, the higher total throughput would be attained by the SUs.

Fig. 1 also shows that the performance of the proposed robust scheme decreases as the error level in the estimation of CSI increases. This can be explained as follows. By increasing the uncertainty of users' CSI, the proposed approach has to reduce its transmit power to protect the PUs, consequently, the total throughput at SUs is reduced. The proposed scheme also maintains the SINR constraint of 10 dB at each SU as it can be seen from the figure that the total throughput at the SUs is always greater than $2\log_2(1 + 10) = 6.92$ bits/s/channel-use. Furthermore, Fig. 1 reveals that the baseline provides higher total SU throughput than the proposed approach.

The achieved higher performance however, comes at a cost of harming the PUs as shown latter in Figs. 2 and 3. Fig. 1 indicates that the lower the error level is, the stricter interference threshold can be guaranteed by the proposed approach. In the interference threshold range from -30 dBm to just under -25 dBm, due to strict interference constraints, the baseline can only support the SUs with their minimum throughput requirement, i.e., indicated by the constant throughput, while the proposed approach fails to operate.

In order to investigate the effect of the cognitive BS's

$$\min_{\|\mathbf{\Delta}_{s,i}\|\leq\delta_{s,i}} g_i\left(\mathcal{W}\right)=\frac{\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i}-\delta_{s,i}\mathbf{I}\right)\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\left(\widetilde{\mathbf{R}}_{s,i}+\delta_{s,i}\mathbf{I}\right)\mathbf{w}_j+\sigma_i^2} \tag{14}$$

$$\begin{aligned}
\min_{\{\mathbf{w}_i\}}\quad & \sum_{t=1}^{N}\lambda_{p,t}\sum_{i=1}^{U}\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{p,t}+\delta_{p,t}\mathbf{I}\right)\mathbf{w}_i-\sum_{i=1}^{U}\lambda_{s,i}\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i}-\delta_{s,i}\mathbf{I}\right)\mathbf{w}_i\\
\text{s. t.}\quad & \frac{\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{s,i}-\delta_{s,i}\mathbf{I}\right)\mathbf{w}_i}{\sum_{j=1,j\neq i}^{U}\mathbf{w}_j^H\left(\widetilde{\mathbf{R}}_{s,i}+\delta_{s,i}\mathbf{I}\right)\mathbf{w}_j+\sigma_i^2}\geq\gamma_i,\ \forall i\\
& \sum_{i=1}^{U}\mathbf{w}_i^H\left(\widetilde{\mathbf{R}}_{p,t}+\delta_{p,t}\mathbf{I}\right)\mathbf{w}_i\leq I_t,\forall t,\\
& \sum_{i=1}^{U}\mathbf{w}_i^H\mathbf{w}_i\leq P_{\mathrm{m}}
\end{aligned} \tag{15}$$

$$\begin{aligned}
\min_{\{\mathbf{W}_i\}}\quad & \mathrm{Tr}\left(\sum_{i=1}^{U}\left\{\left[\sum_{t=1}^{N}\lambda_{p,t}\widetilde{\mathbf{R}}_{p,t}+\sum_{t=1}^{N}\lambda_{p,t}\delta_{p,t}-\lambda_{s,i}\widetilde{\mathbf{R}}_{s,i}+\lambda_{s,i}\delta_{s,i}\right]\mathbf{W}_i\right\}\right),\\
\text{s. t.}\quad & \left(1+\frac{1}{\gamma_i}\right)\mathrm{Tr}\left(\widetilde{\mathbf{R}}_{s,i}\mathbf{W}_i\right)-\sum_{j=1}^{U}\mathrm{Tr}\left(\widetilde{\mathbf{R}}_{s,i}\mathbf{W}_j\right)-\sum_{j=1,j\neq i}^{U}\delta_{s,i}\mathrm{Tr}\left(\mathbf{W}_j\right)-\frac{\delta_{s,i}}{\gamma_i}\mathrm{Tr}\left(\mathbf{W}_i\right)-\sigma_i^2\geq0,\ \forall i,\\
& I_t-\mathrm{Tr}\left(\widetilde{\mathbf{R}}_{p,t}\sum_{i=1}^{U}\mathbf{W}_i\right)-\mathrm{Tr}\left(\delta_{p,t}\sum_{i=1}^{U}\mathbf{W}_i\right)\geq0,\ \forall t,\\
& P_{\mathrm{m}}-\sum_{i=1}^{U}\mathrm{Tr}\left(\mathbf{W}_i\right)\geq0,\\
& \mathbf{W}_i\succeq0,\ \forall i,
\end{aligned} \tag{16}$$

transmission on PUs, we define a normalized interference constraint value, $f_{p,t}\left(\mathcal{W}\right)/I_t$, where $f_{p,t}\left(\mathcal{W}\right)$ is given in (4) and I_t is the interference threshold/tollerance at tth PU. We randomly generate 1000 error matrices $\mathbf{\Delta}_{p,t}$ for each value of t to test the performance of the proposed and baseline approach. If the normalized interference constraint value is less than 1, then the interference constraint at each PU is maintained. Otherwise, interference constraint at each PU is violated.

Figs. 2 and 3 depict the histograms of the normalized interference constraint values, respectively, for PU 1 and PU 2 at $I_t=-15$ dBm $\forall t$, SINR level at SU $\gamma_{s,i}=10$ dB $\forall i$, and $\delta_{p,t}=\delta_{s,i}=\delta=0.001\times||\mathbf{R}||$ where $\mathbf{R}\in\{\mathbf{R}_{s,i},\mathbf{R}_{p,t}\}$, $\forall i,t$. It is observed in Figs. 2 and 3 that the proposed scheme effectively keeps the imposed interference on each PU less than the required level while the baseline approach fails to protect the interference constraints for around half of the cases. This confirms the effectiveness of the proposed scheme against the error in the channel estimation.

As shown in Fig. 4, the Pareto frontier obtained by the proposed approach can be designed by varying the number of antennas. With 4 antennas, the proposed approach can maintain the required throughput for 2 SUs while taking care the interference threshold for 2 PUs in the range from -10 dBm to -5 dBm. By increasing the number of antennas the proposed approach can provide much higher achievable throughput than the original SU requirement. For example, with 12 antennas, it offers 1 and 5.6 bits/s/channel-use higher than the original SU requirement at the PU interference thresholds of -15 dBm and -5 dBm, respectively.

V. Conclusion

We considered the performance of two coexisting wireless networks adopting an underlay access strategy. We then formulated the beamforming design problem as a linear combination of two contradictory objectives, i.e., minimizing the interference imposed on each primary user while maximizing the intended signal received at every secondary user. The proposed beamforming approach takes into account the uncertainty in the estimation of CSI. The proposed optimization problem was then reformulated to a robust optimization problem under the worst case of CSI estimation error. We then transformed

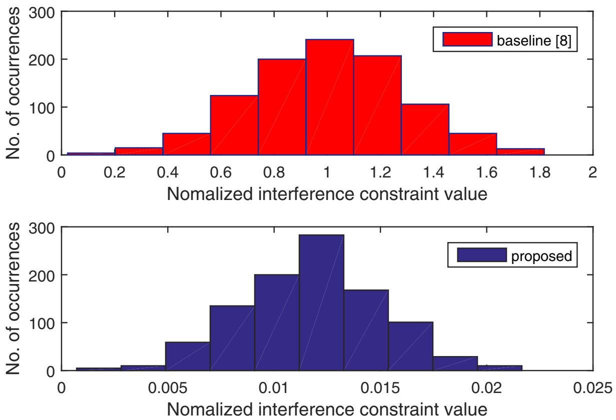

Fig. 2. Histograms of the normalized interference constraint value for PU 1 at $I_t = -15$ dBm $\forall t$, SINR level at SU $\gamma_{s,i} = 10$ dB $\forall i$ and $\delta_{p,t} = \delta_{s,i} = \delta = 0.001 \times ||\mathbf{R}||$, $\forall i, t$. The number of antenna $M = 6$. $P_{\text{m}} = 40$ dBm.

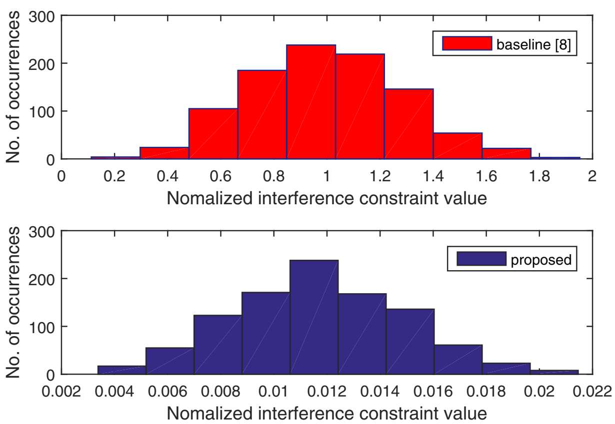

Fig. 3. Histograms of the normalized interference constraint value for PU 2 at $I_t = -15$ dBm $\forall t$, SINR level at SU $\gamma_{s,i} = 10$ dB $\forall i$ and $\delta_{p,t} = \delta_{s,i} = \delta = 0.001 \times ||\mathbf{R}||$, $\forall i, t$. The number of antenna $M = 6$. $P_{\text{m}} = 40$ dBm.

the robust optimization problem into a standard SDP form which is convex and can be solved by standard optimization packages. Simulation results confirmed the effectiveness of the proposed scheme against various levels of CSI estimation error. Although all the constraints imposed on the secondary system, i.e., SUs' SINR levels and PUs' interference thresholds, are maintained, the robustness comes at the cost of lower total SUs' throughput. Simulation results also indicated that the baseline scheme can provide higher total SUs' throughput than the proposed approach however, the former fails to maintain the interference tolerance level of the PUs for around half of the occurrences.

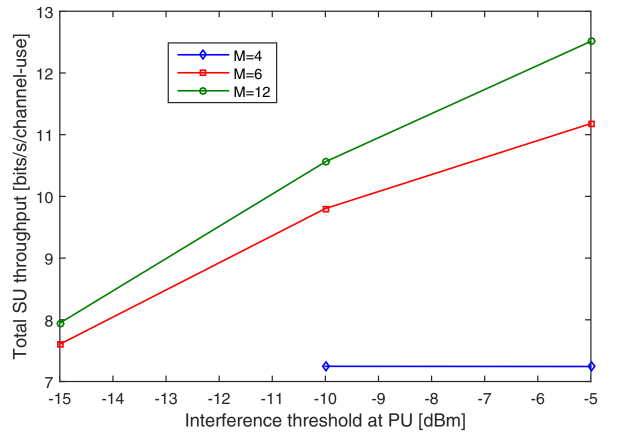

Fig. 4. Total throughput of the SUs *vs.* the interference threshold at the PUs for the proposed scheme with different number of antennas. SUs' SINR requirements $\gamma_i = 10$ dB $\forall i$. $\delta_{p,t} = \delta_{s,i} = \delta = 0.01 \times ||\mathbf{R}||$, $\forall i, t$. $P_{\text{m}} = 40$ dBm.

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