Optimizing Random Unitary Beamforming for Energy Efficiency in MIMO Broadcast Channels

(Invited Paper)

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***Abstract*—Random unitary beamforming (RUB) achieves multiuser diversity gain over multiple-input multiple-output (MIMO) broadcast channels with channel state information (CSI). In this paper, we optimize RUB schemes in terms of energy efficiency (EE) which is defined as "capacity/power consumption". The key idea of our approach is to derive EE-optimal number of antennas and transmit power based on mathematical analysis. Using simple approximation expression, we derive efficient approach to find global optimum. Simulation results demonstrate that analytical results are accurate.**

***Index Terms*—energy efficiency, random unitary beamforming, power allocation, multiplexing gain, circuit power consumption**

I. Introduction

In the past years, multiple-input multiple-output (MIMO) technology has been used to improve the performance of wireless communications systems. By deploying many antennas at both base stations (BSs) and users, MIMO takes the advantage of spatial multiplexing gain. In the downlink cellular network scenario, unfortunately, there exists a performance limitation since mobile users generally have limited number of antennas due to the size/battery constraints. To exploit the gain of additional antennas at the BS, MIMO broadcast channel techniques, which allow multiple users to communicate simultaneously at the same frequency and time resource have been studied [1].

Random unitary beamforming (RUB) is one of the transmission schemes for MIMO broadcast channels with partial channel state information (CSI) [2]. Compared to beamforming strategies using full CSI, such as zeroforcing beamforming (ZFBF) [3], RUB is a low-complexity and low-delay scheme, where the BS schedules users to explore a multiuser diversity gain based on channel quality and beam index feed back from users. According to [2], RUB can asymptotically achieve the same growth rate $M \log\log K$ as the optimal dirty paper coding (DPC) schemes, when $K \to \infty$.

In this paper, we focus on optimizing RUB in terms of energy efficiency (EE). Recently, EE defined as "data rate/power consumption" [4] is considered as an important performance metrics on next-generation green cellular networks. Many studies on the EE of MIMO broadcast channels based on full CSI have been performed [5]–[7]. However, the research on the RUB has been limited.

To optimize EE for RUB, we need to obtain the optimal system configuration parameters, such as the number of antennas, M, and the total transmit power, P, based on sum rate analysis of RUB. For a given number of users, K, the closed-form expression of ergodic sum rate for RUB was derived [8]. However, this result is hard to analyze due to the complex integration involved. Hence we derive an approximated result of sum rate for the analysis here. Applying the realistic power consumption model which was introduced in [9] and our approximation of sum rate, we determine optimal M and P in terms of maximizing EE. Based on our results, BS is able to transmit signals to selected users by activating optimal M antennas and allocating optimal P.

The rest of this paper is organized as follows. In section II, we present system and power consumption model. In section III, we derive optimal number of antennas and transmit power based on sum rate approximation. In section IV, simulation results are given. Finally, section V summarizes the paper.

II. System and Power Consumption Model

A. System Model

We consider MIMO broadcast channels in a single cell senario where BS has M transmit antennas and K ($K \geq M$) users have single antenna. BS serves M selected users simultaneously using M beams. The transmitted signal vector from BS with precoding can be written as

$$\mathbf{x} = \sum_{j=1}^{M} \mathbf{f}_j s_j \qquad \text{for } tr\{\mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{H}}]\} = P, \qquad (1)$$

where s_j and $\mathbf{f}_j$ are data symbol and precoding vector for the jth selected user, respectively, and P is the total transmit power. The received signal at the ith user is written as

$$y_i = \mathbf{h}_i{}^{\mathrm{T}}\mathbf{x} + n_i = \mathbf{h}_i{}^{\mathrm{T}}\mathbf{f}_i s_i + \sum_{j=1, j\neq i}^{M} \mathbf{h}_i{}^{\mathrm{T}}\mathbf{f}_j s_j + n_i, \qquad (2)$$

where $\mathbf{h}_i = [h_{i1}, h_{i2}, \cdots, h_{iM}]^{\mathrm{T}}$ is $M \times 1$ Rayleigh fading channel vector between the serving BS and the ith user. Note that h_{ik} is modeled as independent identically distributed (i.i.d) complex Gaussian random variable with zero mean and unit variance and n_i is an additive white gaussian noise with spectral density, N_o.

In RUB systems, BS uses M random orthonormal beams, which are column vectors of the unitary matrix $\mathbf{U} = \{\mathbf{u}_1\ \mathbf{u}_2\ \cdot$

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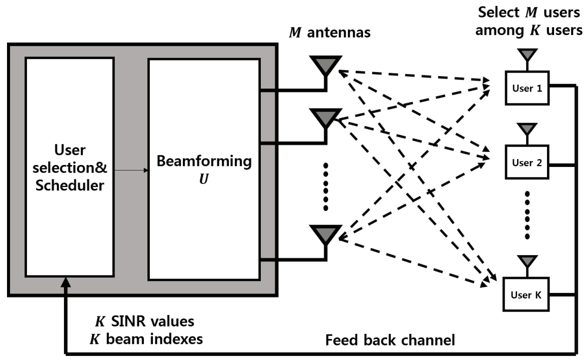

Fig. 1. System model of random unitary beamforming

$\cdots \mathbf{u}_M\}$, generated from an isotropic distribution. The received signal to interference plus noise ratio (SINR) of the selected ith user served by the mth beam is calculated as

$$\gamma_{i,m} = \frac{\frac{\rho}{M}\left|\mathbf{h}_i{}^{\mathrm{T}}\mathbf{u}_m\right|^2}{\frac{\rho}{M}\sum_{j=1,j\neq m}^{M}\left|\mathbf{h}_i{}^{\mathrm{T}}\mathbf{u}_j\right|^2+1}. \quad (3)$$

Note that $\rho = \frac{P}{WN_o}$ is a signal-to-noise ratio (SNR) where W is system bandwidth. The precoding vector of the jth user, $\mathbf{f}_j$, is selected as

$$\mathbf{f}_i = \arg\max_{\mathbf{u}_m} \gamma_{i,m}, \quad (4)$$

which means that each M user will be served by the best beam among M beams in terms of SINR. For the purpose of multi-user diversity gain, BS selects and schedules the best M users among K users who feed back their best SINR values and corresponding beam indexes and assigns M beams to each of M users. With $\gamma_i^* = \max_m \gamma_{i,m}$, the ergodic sum rate with user selection is given by

$$\begin{aligned}\mathrm{E}[R] &= \mathrm{E}\left[\sum_{i=1}^{M}\log_2(1+\gamma_i^*)\right] = M\mathrm{E}\left[\log_2\left(1+\gamma^*\right)\right] \quad (5)\\ &= M\int_0^\infty \log_2(1+x)f_{\gamma^*}(x)dx\\ &\le M\int_0^\infty \log_2(1+x)d\left\{F_{\mathrm{x}}(x)\right\}^K. \quad (6)\end{aligned}$$

where $F_{\mathrm{x}}(x) = 1-\frac{e^{-Mx/\rho}}{(1+x)^{M-1}}$ [2]. There is a small probability that some users among selected M users feed back the same best beam index. If it happens, BS selects another user with second best quality on the beam. Therefore, (6) is upper bound of the exact sum rate. Note that without loss of generality, upper bound is very tight when K is moderately large. Although the exact PDF was introduced in [10], we will use (6) for mathematical tractability.

B. Power Consumption Model

The power consumption of the wireless communication systems consists of transmit power, P, circuit power, $P_{circuit}$, and static power (system-cooling, control signaling, etc.), P_{static}, as [9]

$$P_{tot} = \frac{1}{\eta}P + P_{circuit} + P_{static} \qquad \text{(watt)}, \quad (7)$$

where η is a power amplifier (PA) efficiency of the transmitter. Even though the circuit power consumption is often ignored in [11] or simplified in [12], we follow the circuit power consumption model which is introduced in [9], where $P_{circuit}$ is given as

$$\begin{aligned}P_{circuit} = &\underbrace{M(P_{DAC}+P_{mix}+P_{filt})+P_{syn}}_{\text{consumed by the BS}}\\ &+\underbrace{M(P_{LNA}+P_{mix}+P_{IFA}+P_{filr}+P_{ADC})}_{\text{consumed by } M \text{ users}}. \quad (8)\end{aligned}$$

In (8), P_{mix}, P_{filt}, P_{filr}, P_{LNA}, P_{IFA}, P_{sync}, P_{DAC} and P_{ADC} are the coefficients of the power consumption in the mixer, filters of transmitter, filters of receiver, low noise amplifier, intermediate frequency amplifier, frequency synthesizer, DAC, and ADC, respectively. Thus, the total power consumption model in (7) can be simply expressed as

$$P_{tot} = \mathcal{A}P + \mathcal{B}M + \mathcal{C}, \quad (9)$$

where $\mathcal{A} = 1/\eta$, $\mathcal{B} = P_{DAC}+P_{mix}+P_{filt}+P_{LNA}+P_{mix}+P_{IFA}+P_{filr}+P_{ADC}$ and $\mathcal{C} = P_{sync}+P_{static}$. The total power consumption is a function of P and M.

III. Analysis of Energy Efficiency

The EE (bits/Joule), η_{EE}, is generally defined as data rate divided by the total power consumption. The EE-optimization problem can be formulated as

$$\max \eta_{EE} = \max\left(\frac{W\mathrm{E}[R]}{P_{tot}}\right) = W\times\max\left(\frac{\mathrm{E}[R]}{P_{tot}}\right). \quad (10)$$

For fixed K, η_{EE} is a function of M and P. To maximize η_{EE}, we need to determine the optimal values of the number of antennas, M^*, and the optimal power, P^*, by solving the optimization problem for two variables,

$$\{M^*, P^*\} = \arg\max_{\{M,P\}} \frac{\mathrm{E}[R]}{\mathcal{A}P+\mathcal{B}M+\mathcal{C}}. \quad (11)$$

Lemma 1: There exists the unique solutions for M^* and P^* in (11) and can be derived by solving $\frac{\partial\eta_{EE}}{\partial M} = 0$ and $\frac{\partial\eta_{EE}}{\partial P} = 0$.

Proof: see Appendix.

With above lemma, we can simply obtain M^* and P^* respectively by computing partial derivatives. Note that if M^* derived by $\frac{\partial\eta_{EE}}{\partial M} = 0$ is smaller than 1, we set $M^* = 1$ due to the $M \ge 1$ constraint, which means that activating only one antenna and serving single user can be the EE-optimum in this case. To solve (11), we first analyze the ergodic sum rate.

A. Asymptotic Analysis

The integration of (6) is still difficult due to the mathematically untractable form of PDF. To investigate the effect of M and P on the EE, we first analyze the ergodic sum rate and EE asymptotically in the low SNR and high SNR regions.

1) Low SNR region: Over the low SNR region, a RUB system becomes interference-limited. The CDF of received SINR, $F_{\mathrm{x}}(x)$, can be approximated as

$$F_{\mathrm{x}}(x) \approx 1 - e^{-\frac{M}{\rho}x}. \tag{12}$$

Using Jensen's inequality and substituting $u = \{F_x(x)\}^K$, the upper bound of sum rate in (5) can be computed as

$$\begin{aligned}\mathrm{E}[R] &= M\mathrm{E}\left[\log_2\left(1+\gamma^*\right)\right] \le M\log_2\left(1+\mathrm{E}[\gamma^*]\right)\\ &= M\log_2\left(1+\frac{\rho}{M}\int_0^1 \ln\left(\frac{1}{1-u^{\frac{1}{K}}}\right)du\right)\\ &= M\log_2\left(1+\frac{\rho}{M}\Omega_K\right),\end{aligned} \tag{13}$$

where $\Omega_K = \int_0^1 -\ln\left(1-u^{\frac{1}{K}}\right)du$ is the function of K. It is observed that the multiplexing gain is M and the sum rate is proportional to $\log P$ in the low SNR region. Therefore, increasing M or P can improve the sumrate performance and EE-optimization problem can be simplified as

$$\max_{M,P}\eta_{EE} \approx \max_{M,P}\frac{M\log_2\left(1+\frac{P\Omega_K}{MWN_o}\right)}{\mathcal{A}P+\mathcal{B}M+\mathcal{C}}. \tag{14}$$

For fixed parameters in the optimization problem of (14), each M^* and P^* can be obtained as

$$M^* \approx \max\left\{\left\lceil\frac{\frac{\Omega_K P}{WN_o}}{e^{\mathcal{W}\left(\frac{\Omega_K\mathcal{A}P}{WN_o(\mathcal{A}P+\mathcal{C})e}-\frac{1}{e}\right)+1}-1}\right\rfloor,\ 1\right\}, \tag{15}$$

$$P^* \approx \frac{WN_oM\left(e^{\mathcal{W}\left(\frac{\Omega_K(\mathcal{B}M+\mathcal{C})}{WN_oM\mathcal{A}e}\right)+1}-1\right)}{\Omega_K}. \tag{16}$$

Note that $\mathcal{W}(x)$ in (15) and (16) is *Lambert W function* and the notation $\lceil z\rfloor$ means the nearest integer of z. If RUB is operated over the low SNR to minimize inter user interference (IUI), these results are asymptotically valid.

2) High SNR region: In the high SNR region, noise is negligible and IUI limits the sum rate. Contrary to the case of low SNR, we can approximate $F_{\mathrm{x}}(x)$ as

$$F_{\mathrm{x}}(x) \approx 1 - \frac{1}{(1+x)^{M-1}}. \tag{17}$$

Then the sum rate can be expressed as

$$\begin{aligned}\mathrm{E}[R] &= M\mathrm{E}\left[\log_2\left(1+\gamma^*\right)\right]\\ &= \frac{M}{M-1}\int_0^1 \log_2\left(\frac{1}{1-u^{\frac{1}{K}}}\right)du \qquad (18)\\ &= \frac{M\log_2 e}{M-1}\Omega_K. \qquad (19)\end{aligned}$$

As above, there is no gain in high SNR region since the IUI becomes dominant. Furthermore, the sum rate decreases and converges to $\Omega_K/\ln 2$ as M increases. Therefore, we can conclude that EE-optimal solution is activating only one antenna.

B. Analysis Based on Approximation

Asymptotic results of M^* and P^* derived in (15) and (16) are not accurate as system design parameters. To obtain general M^* and P^*, it is necessary to simplify the sum rate expression which is described by integration from 0 to ∞. (6) can be calculated as follows.

$$\begin{aligned}\mathrm{E}[R] &= M\int_0^\infty \log_2(1+x)d\{F_{\mathrm{x}}(x)\}^K\\ &\overset{\text{(a)}}{=} M\log_2 e\int_0^1 \ln(1+x)du\\ &\overset{\text{(b)}}{=} \frac{M}{\ln 2}\int_0^1 -\mathcal{W}\left(\lambda\left(1-u^{\frac{1}{K}}\right)^{\frac{1}{1-M}}\right)du \qquad (20)\\ &\quad + \frac{M^2}{\ln 2(M-1)\rho} + \frac{M\Omega_K}{\ln 2(M-1)}. \qquad (21)\end{aligned}$$

Note that we substitute $u = \{F(x)\}^K$ in (a) and $\lambda = \frac{M}{\rho(M-1)}e^{\frac{M}{\rho(M-1)}}$ in (b). Unfortunately, to the best of our knowledge, since the integration of $\mathcal{W}(f(x))$ for an arbitrary function $f(x)$ is not known, we use the following simple approximation of $\mathcal{W}(x)$.

$$\mathcal{W}(x) \approx a\ln(x+b)+c, \tag{22}$$

where a,b and c are constants which are given by Levenberg-Marquardt curve-fitting method.[1] Then (21) can be rewritten as

$$\mathrm{E}[R] \approx \frac{1}{\ln 2}\left[-a\int_0^1 G(u,M,P)du + H(M,P)\right], \tag{23}$$

where

$$G(u,M,P) \triangleq M\ln\left(1+\frac{b}{\lambda}(1-u^{\frac{1}{K}})^{\frac{1}{M-1}}\right), \tag{24}$$

$$H(P,M) \triangleq M\left[-a\ln\lambda - c + \frac{M+\rho\Omega_K(1+a)}{(M-1)\rho}\right]. \tag{25}$$

We now focus on $\int_0^1 G(u,M,P)du$. If K is moderately large, $G(u,M,P)$ is very sharply decreased w.r.t u in $[0,\epsilon]$ and $[1-\epsilon,1]$, where ϵ is very small number. Then we can apply that

$$\int_0^1 G(u,M,P)du \approx \int_\epsilon^{1-\epsilon} G(u,M,P)du. \tag{26}$$

In $[\epsilon,1-\epsilon]$, we consider *trapezoidal's rule* which is known as one of numerical approximation of definite integration. This approach approximates the region under the graph of a function to the sum of trapezoids. For the area discretized into L equally spaced panels, (26) can be approximated as

$$\begin{aligned}&\int_\epsilon^{1-\epsilon} G(u,M,P)du\\ &\quad\approx \frac{1-2\epsilon}{2L}\sum_{k=1}^{L}\left\{G\left(u_{k-1},M,P\right)+G\left(u_k,M,P\right)\right\}.\end{aligned} \tag{27}$$

[1]In this work, we simply compute constants a,b,c by using Matlab curve fitting tools with 95% confidence interval.

Let us denote that $u_k = \epsilon + k(1-2\epsilon)/L$. Since $G(u,M,P)$ is smoothly decreased, it is feasible to use $L=1$. The sum rate is expressed as

$$E[R] \approx \left[\frac{H(P,M)}{\ln 2} - \frac{aG(1-\epsilon,M,P)}{\ln 4} - \frac{aG(\epsilon,M,P)}{\ln 4}\right], \quad (28)$$

We validate our results by simulation in section IV. Through $\frac{\partial \eta_{EE}}{\partial M}=0$ and $\frac{\partial \eta_{EE}}{\partial P}=0$, P^* and $M^{(o)}$ can be computed in (29) and (30). We can obtain an integer M^* from a real number $M^{(o)}$ as $M^* = \max\left[1, \lceil M^{(o)} \rfloor\right]$.

C. Joint Optimization

Utilizing the results of (29) and (30) enables maximization of EE by separately optimizing M and P when other parameters are fixed. However, our ultimate objective is to obtain the joint optimum values of M and P. We can find the joint global optimum by an exhaustive search over all candidates of M and P. However, it requires high computational complexity to find a real number P while searching an integer M does not. Therefore, we take into account a practical method to use the convergence property of this problem.

Algorithm 1 Joint optimization algorithm

Initialize $M=1$, $P=0$ and $\widetilde{M}=0$.
while $M \neq \widetilde{M}$ **do**
  $\widetilde{M} \leftarrow M$.
  Compute P from (30).
  Compute $M^{(o)}$ from (29) and $M = \max\left[1, \lceil M^{(o)} \rfloor\right]$.
end while
return $M^* \leftarrow M$ and $P^* \leftarrow P$.

Convergence is declared when an integer M is unchanged in an iteration. This approach can lead to a global optimum.

IV. Numerical Results

This section validates our results by simulations. Circuit power consumption parameters are listed in [13]. We consider $W=20$MHz and $N_o=-110$dBm/Hz. In Fig. 2, we show the sum rate performance of RUB as a function of SNR with respect to different K and different M and demonstrate that approximation of (28) is near to exact sum rate. It is observed that the sum rate increases over the low SNR region but saturates in the high SNR region.

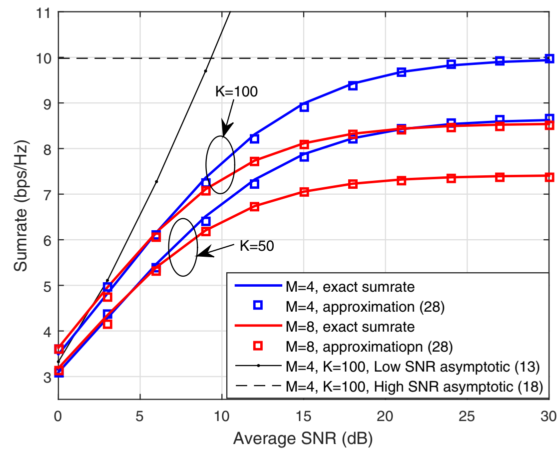

Fig. 2. Sum rate performance for different number of antennas and/or number of users

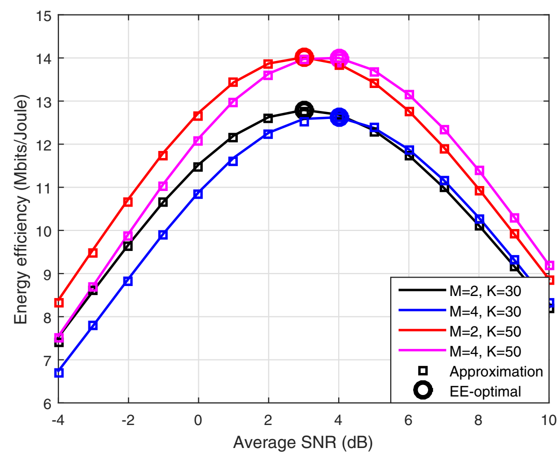

Fig. 3. Energy efficiency performance for different number of antennas with $K=30$ and $K=50$.

Fig. 3 illustrates that there exists optimal SNR. This simulation is performed at every 1dB. According to simulations, maximum energy efficiency is obtained when we apply $\rho=3$dB in $M=2$ case and $\rho=4$dB in $M=4$ case. In the comparison between analytical results and simulation results which are listed in table. I, we can observe that there is little error between them. In addition, low SNR asymptotic analysis in (16) can be valid when EE-optimal solution is near to low SNR region.

$$M^{(o)} = \arg\min_M \left| \frac{-a\ln\left(\lambda + b\left(1-(1-\epsilon)^{\frac{1}{K}}\right)^{\frac{1}{M-1}}\right) - a\ln\left(\lambda + b\left(1-\epsilon^{\frac{1}{K}}\right)^{\frac{1}{M-1}}\right) + 2\left(\frac{M+\rho\Omega_K(1+a)}{(M-1)\rho} - c\right)}{2\left(-\frac{a\ln\lambda + a(M-1) - c}{M} + \frac{M}{\rho(1-M)} + \frac{\Omega_K(1+a)}{(M-1)M} + \frac{a - WN_o + P\Omega_K(1+a)}{P(M-1)^2}\right) - \frac{a}{M}(g_{1-\epsilon}+g_\epsilon)} - M - \frac{\mathcal{A}P+\mathcal{C}}{\mathcal{B}} \right|. \quad (29)$$

$$P^* = \arg\min_P \left| \frac{-a\ln\left(\lambda + b\left(1-(1-\epsilon)^{\frac{1}{K}}\right)^{\frac{1}{M-1}}\right) - a\ln\left(\lambda + b\left(1-\epsilon^{\frac{1}{K}}\right)^{\frac{1}{M-1}}\right) + 2\left(\frac{M+\rho\Omega_K(1+a)}{(M-1)\rho} - c\right)}{\left(\frac{1}{\rho} + \frac{M}{\rho^2(M-1)}\right)\left(2 - \left(\lambda + b(1-\epsilon^{\frac{1}{K}})^{\frac{1}{M-1}}\right)^{-1} - \left(\lambda + b(1-\epsilon^{\frac{1}{K}})^{\frac{1}{M-1}}\right)^{-1}\right)} - P - \frac{\mathcal{B}M+\mathcal{C}}{\mathcal{A}} \right|. \quad (30)$$

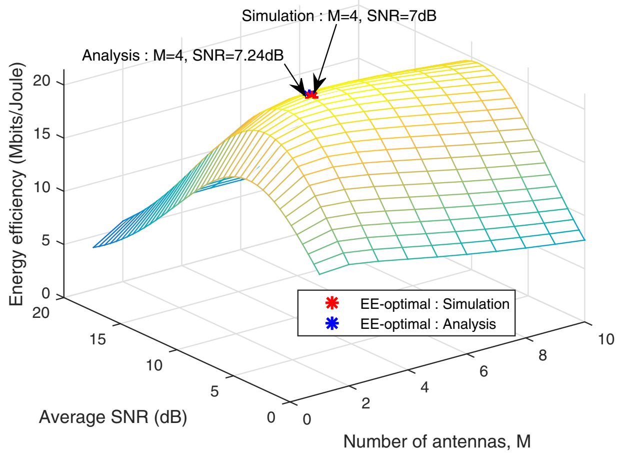

Fig. 4. Energy efficiency performance with joint global optimum in $K = 1000$.

TABLE I
OPTIMAL SNR IN FIG. 3

	Simulation	Analytic (30)	Low SNR (16)
$M = 2, K = 30$	3dB	2.9dB	3.7dB
$M = 4, K = 30$	4dB	3.7dB	4.9dB
$M = 2, K = 50$	3dB	3.3dB	3.6dB
$M = 4, K = 50$	4dB	4.1dB	4.5dB

In Fig. 4, EE performance for different M and SNR is described. The global EE-optimum is achieved at $M = 4$ and $\rho = 7$dB from simulation. Also, we can obtain analytical optimal of $M = 4$ and $\rho = 7.24$dB.

V. CONCLUSION

In this paper, we analyzed energy efficiency of RUB schemes. According to statistical characteristics of RUB, EE-optimization problem was transformed to decide the optimal number of antennas and optimal transmit power. First of all, energy efficiency was asymptotically investigated over the low SNR and high SNR region. For general analysis, we approximated an ergodic sum rate of RUB. The key technique was to apply an approximation of *Lambert W* function and the numerical integral method. Based on this result, we derive optimal M and P analytically when the other parameter is fixed and achieve joint optimum using a property of convergence. These results which depend on the parameter setting may not be general, but for a given values of parameters we can optimize RUB in terms of the energy efficiency.

APPENDIX

Note that $\mathrm{E}[R] \geq 0$, $P_{tot} > 0$ and $\frac{\mathrm{E}[R]}{P_{tot}} \geq 0$. We can simply show that there exists EE-optimal number of antennas by following results of two limitations.

$$\lim_{M \to 0} \frac{\mathrm{E}[R]}{P_{tot}} = 0 \quad \text{and} \quad \lim_{M \to \infty} \frac{\mathrm{E}[R]}{P_{tot}} = 0.$$

The EE with regard to M is not convex function. However, we can verify that there is only one M which satisfies $\frac{\partial \eta_{EE}}{\partial M} = 0$. $\frac{\partial \eta_{EE}}{\partial M}$ is calculated as

$$\frac{\partial \eta_{EE}}{\partial M} = M \frac{\partial T_M}{\partial M} \left(\frac{T_M}{M P_{tot} \frac{\partial T_M}{\partial M}} + \frac{1}{P_{tot}} - \frac{B T_M}{(P_{tot})^2 \frac{\partial T_M}{\partial M}} \right).$$

We denote that $T_M = \int_0^\infty \log_2(1+x) f_{\gamma^*}(x) dx$ is monotonically decreasing function and $\frac{\partial T_M}{\partial M}$ is a monotonically increasing function w.r.t M. Hence, first term and second term are monotonically decreasing functions and third term is monotonically increasing function. Therefore, equation $\frac{\partial \eta_{EE}}{\partial M} = 0$ has only one solution in $M > 0$ and $\frac{\partial T_M}{\partial M} < 0$.

Similarly, following two limitations about transmit power can be computed as

$$\lim_{P \to 0} \frac{\mathrm{E}[R]}{P_{tot}} = 0 \quad \text{and} \quad \lim_{P \to \infty} \frac{\mathrm{E}[R]}{P_{tot}} = 0.$$

EE-optimal transmit power P^* also exists in $[0, \infty]$ and we can verify that the solution of $\frac{\partial \eta_{EE}}{\partial P} = 0$ is unique through similar procedure.

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