See discussions, stats, and author profiles for this publication at: https://www.researchgate.net/publication/301854201

## Online Ski Rental for ON/OFF Scheduling of Energy Harvesting, Millimeter Wave Base Stations

	7 IEEE Transactions on Wireless Communications •	Februar	y 2016	
CITATION		READS		
1		18		
5 author	s, including:			
	Walid Saad	SIGN AND OFPLOYMEN OF SMALL CELL	Mehdi Bennis	
	Virginia Polytechnic Institute and State University	NETWORKS  ED THE BEN AND AND AND AND AND AND AND AND AND AN	University of Oul	u
	291 PUBLICATIONS 4,082 CITATIONS		<b>199</b> PUBLICATIONS	<b>2,212</b> CITATIONS
	SEE PROFILE		SEE PROFILE	
	Fumiyuki Adachi			
	Tohoku University			
	789 PUBLICATIONS 8,858 CITATIONS			
	SEE PROFILE			

Some of the authors of this publication are also working on these related projects:



All content following this page was uploaded by Walid Saad on 23 June 2016.

The user has requested enhancement of the downloaded file. All in-text references underlined in blue are added to the original document and are linked to publications on ResearchGate, letting you access and read them immediately.

# Online Ski Rental for ON/OFF Scheduling of Energy Harvesting, Millimeter Wave Base Stations

Gilsoo Lee<sup>†</sup>, Walid Saad<sup>†</sup>, Mehdi Bennis<sup>‡</sup>, Abolfazl Mehbodniya<sup>§</sup>, and Fumiyuki Adachi<sup>§</sup>

† Wireless@VT, Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA, USA, Emails: {gilsoolee, walids}@vt.edu.

<sup>‡</sup> Centre for Wireless Communications, University of Oulu, Finland, Email: bennis@ee.oulu.fi.

§ Dept. of Communication Engineering, Graduate School of Engineering, Tohoku University, Sendai, Japan, Emails: mehbod@mobile.ecei.tohoku.ac.jp,adachi@ecei.tohoku.ac.jp.

Abstract—The co-existence of millimeter wave (mmW) small cell base stations (SBSs) with conventional microwave macrocell base stations is a promising approach to boost the capacity and coverage of cellular networks. However, densifying the network with a viral deployment of mmW SBSs can significantly increase energy consumption. To reduce the reliance on unsustainable energy sources, one can adopt self-powered, mmW SBSs that rely solely on energy harvesting. Due to the uncertainty of energy arrival and the finite capacity of energy storage systems, self-powered SBSs must smartly optimize their ON and OFF schedule. In this paper, the problem of ON/OFF scheduling of self-powered mmW SBSs is studied, in the presence of energy harvesting uncertainty with the goal of minimizing the power consumption and transmission delay of a network. To solve this problem, a novel approach based on the ski rental framework, a powerful online optimization tool, is proposed. Using this approach, each SBS can effectively decide on its ON/OFF schedule autonomously, without any prior information on future energy arrivals. By using competitive analysis, a deterministic algorithm (DOA) and a randomized online algorithm (ROA) are developed. The proposed ROA is then shown to achieve the optimal competitive ratio. Simulation results show that, compared to the DOA and a baseline approach, the ROA can yield significant performance gains reaching, respectively, up to 31.8% and 25.3% in terms of reduced power consumption and up to 41.5% and 36.8%, in terms of delay reduction. The results also shed light on the factors that affect the ON time of SBSs while demonstrating how the proposed ROA can eliminate up to 96.2% of the ON/OFF switching overhead compared to a baseline approach.

#### I. Introduction

To support 24.3 Exabytes of mobile traffic in 2019, new cellular networking architectures are needed. One promising solution is by densely deploying low-power, low-cost, small cell base stations (SBSs), which significantly boost the coverage and performance of cellular systems [1]. Coupled with this network densification, to overcome the spectrum scarcity in wireless bandwidth, one can exploit millimeter wave (mmW) frequency bands which promise high bandwidth, high capacity wireless communications [2]. In particular, the use of mmW spectrum promises a 20 fold increase of the capacity due to the large available bandwidth [3]. Due to the possibility of using fully directional links in mmW communications, the interference between cellular transmissions can be significantly reduced, further increasing capacity [4].

Due to these advantages, mmW communication is seen as a promising way to improve the performance of both radio access and backhaul in small cell networks (SCNs) [5]. In particular, mmW communications are well-suited for

SCNs in which the average distance between users and their serving SBSs is relatively closer than in traditional cellular networks, thus making it possible to overcome the propagation challenges of mmW communications [3]. To further reap the benefits of mmW for cellular communications, one can simultaneously deploy mmW SBSs along with conventional microwave ( $\mu$ W) macrocell base stations, thus overcoming the significant propagation loss and blockage effects of mmW [4]. Consequently, it is envisioned that heterogeneous SCNs in which both mmW and  $\mu$ W spectrum frequencies co-exist will be a key feature of next-generation cellular networks.

However, deploying the density of such SCNs can also increase the overall power consumption of a cellular system. The power consumption from the access network and edge facilities account for up to 83% of mobiles' operator power consumption. Additionally, the processing power consumption in mmW cellular systems can be significant due to the high computational complexity of multiantenna systems [4]. To this end, enhancing the energy efficiency of dense SCNs has emerged as a major research challenge [6]. In particular, there has been a recent significant interest, not only in minimizing energy consumption, but also in maximizing the use of green energy [7]. In this regard, one can realize the vision of truly green cellular networks by deploying self-powered, energy harvesting SBSs that rely solely on renewable energy for their operation [8]. However, operating a system that consists of self-powered, energy harvesting SBSs mandates effective and self-organizing ways to optimize the ON and OFF schedules of such SBSs, depending on uncertain and intermittent future energy arrivals.

Recently, numerous works have focused on the use of energy harvesting techniques in cellular networks [9]–[14]. For instance, the authors in [9] provide a model to measure the performance of heterogeneous networks with self-powered SBSs. Also, the work in [10] overviews key design issues for adopting energy harvesting into cellular networks. Along with energy harvesting, base station (BS) ON/OFF scheduling has been actively studied to enhance energy efficiency. In [11], the authors propose algorithms to minimize grid power consumption when considering hybrid-powered BSs. For solving a capital expenditure minimization problem, the authors in [12] propose an ON/OFF scheduling method for self-powered BSs. The work in [13] investigates the problem of minimizing grid power consumption and blocking probability by using statistical information for traffic and renewable energy. The

authors in [14] study the optimal BS sleep policy based on dynamic programming with the statistical energy arrival information.

In addition, several recent works were developed to enhance energy efficiency considering mmW communications [15]–[18]. The authors in [15] consider heterogeneous SCNs in which SBSs are connected to the MBS by using mmW backhaul networks. The energy efficiency of wireless backhauls is evaluated for distributed and centralized architectures of SBSs. In [16], the authors study energy efficient relaying scheme when relaying stations use mmW communications. Also, when considering coexistence of both  $\mu$ W MBS and mmW SBS, the author in [17] shows that energy efficiency can be enhanced by adjusting communication range of mmW SBSs. In [18], the authors propose a traffic-aware BS ON/OFF scheduling algorithm to reduce power consumption of dense SCNs using mmW communications.

Most of the existing works that are focused on the energy efficiency of mmW SCNs [15]-[18] have not taken into account the use of energy harvesting in such mmW cellular networks. On the other hand, in the existing body of literature that addresses ON/OFF scheduling in energy harvesting networks [11]–[14], it is generally assumed that statistical or complete information about the amount and arrival time of energy is perfectly known. However, in practice, energy arrivals are largely intermittent and uncertain since they can stem from multiple sources. Moreover, turning SBSs ON and OFF based on every single energy arrival instance can lead to significant handovers and network stoppage times. Further, the existing works [10], [11], and [13] on energy harvesting networks often assume the presence of both smart grid and energy harvesting sources at every SBS. In contrast, here, we focus on cellular networks in which mmW SBSs are completely self-powered and reliant on energy harvesting. Unlike [9] which focuses on the global performance analysis of self-powered  $\mu W$  SBSs, our goal is to develop self-organizing and online algorithms for optimizing the ON/OFF schedule of self-powered, mmW SBSs.

The main contributions of this paper is to develop a novel framework for optimizing the ON and OFF schedule of selfpowered SBSs in a cellular network in which multiple mmW SBSs coexist with a  $\mu W$  MBS. In particular, we formulate a global optimization problem that seeks to minimize the power and delay of the system by appropriately turning SBSs ON and OFF, in the presence of energy harvesting uncertainty. We show that the problem can be decomposed into a set of distributed online optimization problems that are run at each SBS. To solve the per-SBS online optimization problem, a novel approach based on the ski rental problem, a powerful online optimization tool [19], is proposed. In particular, we propose two schemes to solve the ski rental problem: a deterministic algorithm (DOA) and a randomized online algorithm (ROA). On the one hand, the DOA is a benchmark scheme designed to turn each SBS OFF at a predetermined time. On the other hand, the ROA enables the SBSs to make a decision according to a probability distribution that is derived by using competitive analysis. Then, we show that the proposed ROA can achieve the optimal competitive ratio e/(e-1) which pro-

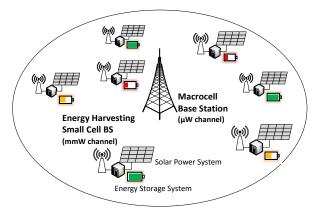


Fig. 1: System model of a heterogeneous deployment with self-powered SBSs.

vides an upper bound of the optimization problem. Moreover, by using ROA, each SBS can effectively decide on its ON/OFF schedule, without knowing any prior information on future energy arrivals. To the best of our knowledge, this is the first work that exploits the online ski rental problem for managing energy uncertainty in cellular systems with self-powered, mmW SBSs. Simulation results show that the proposed ROA can reduce power consumption and delay compared to DOA or a baseline that turns SBSs ON by using an energy threshold. This performance advantage is shown to reach up to 31.8%and 25.3% in reducing the power consumption relative to DOA and the baseline, respectively. The ROA also decreases the delay up to 41.5% and 36.8% relative to DOA or the baseline, respectively. Finally, our results show that ROA can decrease the ON/OFF switching overhead. In particular, we observe that the ON time of each SBS is affected by various factors including the number of SBSs or UEs, the harvested energy, and the power consumption of BSs.

The rest of this paper is organized as follows. In Section II, the system model is presented. In Section III, we present the problem formulation. In Section IV, we propose online algorithms based on the ski rental framework. In Section V, the performance of the proposed algorithm is demonstrated with using extensive simulations. Finally, conclusions are drawn in Section VI.

### II. SYSTEM MODEL

Consider the downlink of a two-tier heterogeneous small cell network in which a macrocell base station (MBS) is located at the center of a service area. In this network, a set  $\mathcal J$  of J self-powered SBSs are deployed. The SBSs use mmW frequency bands whereas the MBS uses  $\mu$ W frequency bands. Consequently, we define the set of all base stations (BSs) as  $\mathcal B=\{0,1,2,\cdots,J\}$  where the MBS is indexed by 0 and SBSs are indexed by  $j\in\{1,\cdots,J\}$ . In this system, mmW SBSs can offload traffic from the  $\mu$ W MBS thus reducing the overall network congestion. Also, a set  $\mathcal I$  of I UEs is randomly distributed in the coverage of the MBS. We assume that each UE can access mmW or  $\mu$ W frequency bands. Moreover, any given UE is connected with only one of the BSs (SBS or MBS) at a given time.

An illustration of our system model is shown in Fig. 1. In our considered system, while the MBS is connected to

the conventional power grid, SBSs are self-powered and rely exclusively on energy harvesting sources. For example, SBSs can be equipped with solar panels to procure energy for their operation, or alternatively, they can use wireless power transfer from MBS transmissions. To enhance the overall energy efficiency of the system, we assume that the SBSs can dynamically turn ON or OFF, depending on the network state, energy harvesting state, and other related parameters. To manage the intermittent and uncertain nature of energy harvesting, energy storage systems (ESSs) can be used. Energy harvesting is assumed to be done irrespective on whether an SBS is turned ON or OFF. Thus, an SBS will store energy in its ESS when it is turned OFF, and this stored energy can be used when it is turned ON to service users. Also, when it is turned ON, an SBS can store the excess of harvested energy if instantaneous harvested energy is enough to operate an SBS.

### A. Network Performance Model

Next, we model the performance of the mmW connection between SBS and UE. By using mmW links, SBSs can provide high data rate. Due to the small wave length, the number of antennas can be increased, which make it possible to have high directivity gain and fully directional communications [20]. However, a mmW link can be significantly impaired due to blockage. To model the blockage effect, we define a Bernoulli random variable  $L_{ij}$  that indicates whether a link between BS  $k \in \mathcal{B}$  and UE i is line-of-sight (LOS) or non-line-of-sight (NLOS), and its realization  $l_{ij}$  is given by:

$$l_{ij} = \begin{cases} 1, & \text{if a link is LOS,} \\ 0, & \text{if a link is NLOS.} \end{cases}$$
 (1)

Recent studies [21]–[23] proposed different LOS probability functions. For our model, we use the stochastic blockage model derived in [23] based on random shape theory. In such a model, the LOS probability is given by:

$$\Pr[L_{ij} = 1] = p_{ij} \triangleq e^{-(\rho_1 x_{ij} + \rho_2)},$$
 (2)

where  $x_{ij}$  is Euclidean distance between SBS j and UE i,  $\rho_1$  is the parameter related to the average perimeter of a building blockage, and  $\rho_2$  is a parameter related to the area of a blockage. The LOS probability function is non-increasing with respect to the distance  $x_{ij}$ . Then, the NLOS probability will be

$$\Pr[L_{ij} = 0] = 1 - p_{ij}. (3)$$

When a link becomes NLOS, the corresponding UE experiences a significant degradation in the QoS.

In our system, without loss of generality, we consider a fixed user association rule, that is given by:

$$k^* = \operatorname{argmin}_{k \in \mathcal{B}} x_{ik}. \tag{4}$$

Thus, a given UE i is associated with the BS having the smallest distance  $x_{ik}$ . Here, we note that, under this user association rule, one can maximize the LOS probability. The set of UEs associated with the same BS k will be denoted by

$$\mathcal{I}_k = \{i | k = \operatorname{argmin}_{k \in \mathcal{B}} x_{ik}, \forall i \in \mathcal{I}\}. \tag{5}$$

For example, if  $k \neq 0$  and  $k \in \mathcal{B}$ , then  $\mathcal{I}_k$  indicates the set of UEs associated with an SBS. Similarly, when k=0, then  $\mathcal{I}_0$  is the set of UEs connected to the MBS. Subsequently,  $\mathcal{I}$  can be divided into J+1 subsets, each of which is denoted by  $\mathcal{I}_k$ ,  $k \in \mathcal{B}$ . By doing so, once all UEs are associated with a BS, we have:  $\mathcal{I} = \cup_{j=1}^J \mathcal{I}_j \cup \mathcal{I}_0$ .

Then, we define the channel model for mmW as

$$h_{ij}(l_{ij}) = l_{ij} \left( 10^{-\frac{\beta_L + 10\alpha_L \log_{10} x_{ij}}{10}} \right) + (1 - l_{ij}) \left( 10^{-\frac{\beta_N + 10\alpha_N \log_{10} x_{ij}}{10}} \right), (6)$$

where  $\alpha_L$  and  $\alpha_N$  are, respectively, the path loss exponent for LOS and NLOS, and  $\beta_L$  and  $\beta_N$  are parameters used to fit intercepts of path loss curves of LOS and NLOS, respectively. The parameters of (6) are derived based on the propagation measurements done in [22].

Due to the narrow beam width of mmW, the interference between neighboring SBSs can be significantly reduced. For instance, in the downlink of mmW networks, it can be observed that thermal noise is similar to or greater to interference [4]. Thus, aligned with this observation, we assume a noise-limited environment, and the signal to noise ratio (SNR) of UE i that is associated to BS j is given by

$$\gamma_{ij}(l_{ij}) = \frac{P_j^{tx} h_{ij}(l_{ij}) G_{ij}}{N_0 B_m},\tag{7}$$

where  $P_j^{\rm tx}$  is the transmit power of the connected SBS j,  $G_{ij}$  is the overall antenna gain of antenna arrays of mmW transceivers,  $B_m$  is the bandwidth of mmW channel, and  $N_0$  is the noise power spectral density. Then, the achievable data rate of UE i is

$$c_{ij}(l_{ij}) = B_m \log_2(1 + \gamma_{ij}(l_{ij})).$$
 (8)

Since the quality of a mmW link depends on whether a link is LOS or NLOS, the expected data rate is derived by

$$\mathbb{E}[c_{ij}(l_{ij})] = \sum_{l_{ij} = \{0,1\}} c_{ij}(l_{ij}) \Pr[L_{ij} = l_{ij}]. \tag{9}$$

Next, the performance of the  $\mu W$  connection between MBS and UE is modeled. When UE i is associated with the MBS, the SNR of UE i becomes

$$\gamma_{i0} = \frac{P_0^{\text{tx}} h_{i0}}{N_0 B_{\mu}},\tag{10}$$

where  $P_0^{\rm tx}$  is the transmit power of the MBS,  $B_\mu$  is the bandwidth of  $\mu{\rm W}$  channel,  $h_{i0}=10^{-\frac{\beta+10\alpha\log_{10}x_{i0}}{10}}$  is the channel gain from the MBS to UE i where  $\alpha$  and  $\beta$  are the parameters related to the path loss attenuation. Then, the achievable data rate of UE i is

$$c_{i0} = B_{\mu} \log_2(1 + \gamma_{i0}). \tag{11}$$

We consider a time slotted system in which each operation slot period is T. Within period T, time is further divided into N slots indexed by n which is an integer between 0 and T. We define  $\delta = T/N$ , so the slotted time n indicates the time  $\delta n$ . At the beginning of each operation slot, we assume that user association is done by (4). Therefore, we define the user association at time n by using the notations for initial user association (4). At time n, the UEs associated with BS j are

given by the set  $\mathcal{I}_{j,n}$ . In particular, we let  $\mathcal{I}_{0,n}$  be the set of UEs served by the MBS at time n, and  $\mathcal{I}_{j,n}$ ,  $j \neq 0, j \in \mathcal{B}$ , be the set of UEs served by SBS j at time n. Then, all UEs should be associated with one of BSs at any time  $n \in \{0, T\}$  from (4), so it can be shown as  $\mathcal{I} = \bigcup_{j=1}^J \mathcal{I}_{j,n} \ \cup \ \mathcal{I}_{0,n}$ .

We provide an example of user association at time n with considering a two-cell network consisted of an SBS and a MBS. Note that the initial user association (5) will need to be dynamically changed when an SBS is turned OFF. Suppose that SBS j is turned OFF at time  $t_j$ . Therefore,  $\mathcal{I}_{0,n}$  and  $\mathcal{I}_{j,n}$  are changed back and forth when SBS j is turned OFF at time  $t_j$ . During which SBS j is ON  $(n < t_j)$ ,  $\mathcal{I}_{j,n} = \mathcal{I}_j$  and  $\mathcal{I}_{0,n} = \mathcal{I}_0$ . However, if SBS j is turned OFF  $(n \ge t_j)$ ,  $\mathcal{I}_{j,n} = \emptyset$ , and  $\mathcal{I}_{0,n} = \mathcal{I}_j \cup \mathcal{I}_0$ , which means that the UEs served by SBS j are handed over to the MBS.

In the considered model, whenever a file of K bits needs to be transmitted to each UE, we can define the total transmission delay between SBS j and all UE in  $\mathcal{I}_{j,n}$  at time n as

$$\phi_j^{\mathcal{I}_{j,n}} = \sum_{i \in \mathcal{I}_{j,n}} \frac{K}{\mathbb{E}[c_{ij}(l_{ij})]}.$$
(12)

Similarly, the total transmission delay between the MBS and UEs in  $\mathcal{I}_{0,n}$  is given by

$$\phi_0^{\mathcal{I}_{0,n}} = \sum_{i \in \mathcal{I}_{0,n}} \frac{K}{c_{i0}}.$$
 (13)

Thus, the total delay needed to serve the all UEs at time n is captured via the following cost function:

$$\Phi_n = \sum_{j=1}^{J} \phi_j^{\mathcal{I}_{j,n}} + \phi_0^{\mathcal{I}_{0,n}}$$
 (14)

$$= \sum_{j=1}^{J} \phi_{j}^{\mathcal{I}_{j}} \sigma_{j}^{n} + \sum_{j=1}^{J} \phi_{m}^{\mathcal{I}_{j}} (1 - \sigma_{j}^{n}) + \phi_{0}^{\mathcal{I}_{0}}, \quad (15)$$

where the ON or OFF state of SBS j is denoted by  $\sigma_j^n$  which is defined as follows:

$$\sigma_j^n = \begin{cases} 1, & \text{if SBS } j \text{ is turned ON at time } n, \\ 0, & \text{otherwise.} \end{cases}$$
 (16)

### B. Power Consumption Model

Next, we define the power consumption models for the MBS and SBSs. When modeling the power consumption of the MBS, one must account for the hardware system of the MBS which includes complex components such as RF transmission, signal processing, battery backup, power supply, and cooling. We assume that the resource utilization of the MBS is proportionally increased as the number of UE connections is increased. Thus, the power consumption model for the MBS includes two components: the utilization-proportional power consumption and the fixed power consumption. The power consumption of the MBS at time n is therefore given by:

$$\psi_0^{\mathcal{I}_{0,n}} = \frac{|\mathcal{I}_{0,n}|}{M} (1-q) P_0^{\text{op}} + q P_0^{\text{op}}, \tag{17}$$

where  $|\mathcal{I}_{0,n}|$  is the number of UEs in  $\mathcal{I}_{0,n}$ , M is the maximum number of UE connections, q is a weighting parameter

that captures the tradeoff between the utilization-proportional power consumption and the fixed power, and  $P_0^{\rm op}$  is the maximum power consumption when the MBS is fully utilized. Using (16), the power consumption (17) can be rewritten as

$$\psi_0^{\mathcal{I}_{0,n}} = \frac{\sum_{j=1}^{J} |\mathcal{I}_j| (1 - \sigma_j^n) + |\mathcal{I}_0|}{M} (1 - q) P_0^{\text{op}} + q P_0^{\text{op}}.$$
 (18)

Also,  $P_0^{\rm tx} = a P_0^{\rm op}$  where the constant a denotes the fraction of the transmit power  $P_0^{\rm tx}$  out of the total the maximum operational power  $P_0^{\rm op}$ . For example, if q=1, the MBS consumes constant power regardless of the utilization level of the MBS. On the other hands, if q=0, the power consumption of the MBS is proportional to the utilization, which is a more realistic BS power consumption model.

For the SBSs, a constant power consumption model is used since SBS hardware has lower complexity than the MBS. If SBS j is turned ON, it consumes the operational power  $P_j^{\text{op}}$  that includes the transmit power  $P_j^{\text{tx}}$ . An SBS consumes power when it is ON, so the power consumption of SBS j at time n is

$$\psi_j^n = P_j^{\text{op}} \sigma_j^n. \tag{19}$$

Since SBSs use energy harvesting as a primary energy source, an ESS can be used to store the excess energy for future use. Thus, the available amount of energy at time n is given by

$$E_i^n = \min(E_i^{n-1} - \psi_i^{n-1}\delta + \Omega_i^n, E_{\text{max}}), \ \forall j \in \mathcal{J}, \quad (20)$$

where  $E_j^n \geq 0$  is the stored energy of SBS j at n,  $\psi_j^{n-1}\delta$  is the consumed energy of SBS j during a small fractional time  $\delta$  that is the time duration between  $n^{\text{th}}$  and  $n-1^{\text{th}}$  time slots,  $\Omega_j^n$  is the amount of energy arrival of SBS j at n, and  $E_{\text{max}}$  is the maximum capacity of ESS.

Then, the total power consumption of the networks at time n is:

$$\Psi_n = \sum_{j \in \mathcal{J}} \psi_j^n + \psi_0^{\mathcal{I}_{0,n}}.$$
 (21)

### III. PROBLEM FORMULATION

Given the defined delay cost, power cost, and energy state, our goal is to analyze the optimal ON and OFF scheduling problem for the SBSs. In cellular networks consisting of self-powered SBSs, the amount of available energy is fluctuating and very limited. To be able to operate using energy harvesting as a primary energy source of SBSs, self-powered SBSs should intelligently manage their ON and OFF states considering delay, power, and energy state. Moreover, since future energy arrivals can be highly unpredictable, optimizing the ON and OFF schedule of SBSs is a very challenging problem. By properly scheduling its OFF duration, an SBS can reduce its the power consumption while also storing more energy for future use. However, at the same time, the SBS must

 $^1$  While it is out of scope of this work, if an SBS has high stored energy, then it could be possible to increase its transmit and operational powers. For example, it can be readily applied by increasing power  $E_j^0 P_j^{\rm ix}/E_{\rm max}$  and  $E_j^0 P_j^{\rm op}/E_{\rm max}$  where  $E_j^0$  is initially stored energy of SBS j, and  $E_{\rm max}$  is the maximum capacity of ESS.

turn ON for a sufficient period of time to service users and offload MBS traffic. To cope with the inherent uncertainty of energy harvesting while balancing the tradeoff between power consumption and cost, we introduce a novel, self-organizing online optimization framework for optimizing the ON and OFF schedule of self-powered SBSs.

As a first step, we formulate the global ON and OFF scheduling problem whose goal is to minimize the sum of the delay cost function and the power cost function, as follows:

$$\min_{\boldsymbol{\sigma}^n} \qquad \sum_{n=1}^N (\Phi_n + \eta \Psi_n), \tag{22}$$

s.t. 
$$\mathcal{I}_{j,n} = \mathcal{I}_j$$
, if  $\forall j \in \mathcal{B}_n^{\text{on}}$ , (23)

$$\mathcal{I}_{0,n} = (\cup_{j \in \mathcal{B} \setminus \mathcal{B}_n^{\text{on}}} \mathcal{I}_j) \cup \mathcal{I}_0, \tag{24}$$

$$\mathcal{I} = \cup_{k \in \mathcal{B}_{on}} \mathcal{I}_{k,n}, b \in \mathcal{B} \tag{25}$$

$$\mathcal{I}_{k,n} \cap \mathcal{I}_{k',n} = \emptyset, k \neq k', \ \forall k, k' \in \mathcal{B}_n^{\text{on}},$$
 (26)

$$E_j^n = \min(E_j^{n-1} - \psi_j^{n-1} \delta + \Omega_j^n, E_{\text{max}}), \forall j \in \mathcal{J}, (27)$$

where  $\eta$  is a weighting parameter that captures the powerdelay tradeoff between the delay cost function and the power cost function, and  $\mathcal{B}_n^{\text{on}}$  is the set of ON BSs at time n. The optimization variables are shown as a vector  $\boldsymbol{\sigma}^n = \{\sigma_j^n | \forall j \in \mathcal{J}\}$  that indicates the ON and OFF states of SBSs at time n. In (22), we want to minimize the total cost during a given time period T, however, the network may not know the uncertain future energy arrival information. If information about energy arrival is given, an offline algorithm can be used to find a solution. However, to solve this problem in a dynamically changing environment with uncertain energy arrivals, an online algorithm is more appropriate.

Therefore, to overcome these challenges, we need to develop a self-organizing approach in which the solution to (22) is done locally at each SBS. By decomposing a problem into smaller, per SBS subproblems, we have an advantage in reducing complexity of the problem. As shown next, each SBS can solve an individual optimization subproblem, so the global problem in (22) can be solved in a distributed way.

**Proposition 1.** The main optimization problem in (22) can be decomposed into  $|\mathcal{J}|$  subproblems.

*Proof.* The objective function of the problem (22) can be shown to be a sum of functions of  $\sigma_j^n$  as shown as (28). In (28),  $\sum_{n=1}^N (\phi_0^{\mathcal{I}_0} + \eta(\frac{|\mathcal{I}_0|}{M}(1-q)P_0^{\mathrm{op}} + qP_0^{\mathrm{op}})) \text{ is the sum of costs of the MBS, and it becomes a constant value. Thus, subtracting this part from (22) does not affect to the solution of the original problem. Therefore, the original objective function (22) can be separated into <math>|\mathcal{J}|$  functions. Also, each constraint in the main problem contains the variables that only depends on each SBS j. Hence, the original problem (22) can be decomposed into  $|\mathcal{J}|$  subproblems.

Now, we have  $|\mathcal{J}|$  subproblems derived from the main problem. The solution of the main problem is shown as the form of a vector  $\sigma^n$ , and the elements of the vector are  $\sigma^n_j$ . Since  $\sigma^n_j$  is the solution of subproblems,  $\sigma^n$  can be found by solving the subproblems. Each subproblem can be solved independently. In other words, the ON or OFF decision of an

SBS does not affect the decision of another SBS, so we can solve  $|\mathcal{J}|$  subproblems in a parallel way. Consequently, each SBS will solve its local version of (22) that seeks to minimize its individual cost function given by

$$\begin{split} \min_{\sigma_{j}^{n}} & & \sum_{n=1}^{N} \phi_{j}^{\mathcal{I}_{j}} \sigma_{j}^{n} + \phi_{0}^{\mathcal{I}_{j}} (1 - \sigma_{j}^{n}) \\ & & + \eta (P_{j}^{\text{op}} \sigma_{j}^{n} + \frac{|\mathcal{I}_{j}| (1 - \sigma_{j}^{n})}{M} (1 - q) P_{0}^{\text{op}}), \\ \text{s.t.} & & E_{j}^{n} = \min(E_{j}^{n-1} - \psi_{j}^{n-1} \delta + \Omega_{j}^{n}, E_{\text{max}}). \end{split}$$

In the local optimization problem, we can observe that the sets  $\mathcal{I}_j$  and  $\mathcal{I}_0$  are used instead of the notation  $\mathcal{I}_{j,n}$  or  $\mathcal{I}_{0,n}$ , satisfying the constraints in (23)-(26). Thus, these constraints can be relaxed in the local optimization problem. The objective function of (29) is changed according to the state of SBS j,  $\sigma_j^n = \{0,1\}$ . If  $\sigma_j^n = 1$  indicating ON state of SBS j at time n, the objective function shows the cost of SBS j in ON state. Similarly, if  $\sigma_j^n = 0$  that means OFF state of SBS j at time n, the output of objective function is the cost of SBS j in OFF state. Thus, it can be found that the value of individual cost function is changed only when an ON/OFF state transition occurs.

Now, we show that the ON/OFF scheduling problem (29) can be redefined as the problem of finding the optimal OFF time. To derive the power and delay costs of ON or OFF state, we let  $t_j$  be the OFF time for each SBS j. When  $n < t_j$ , SBS j is ON, and the cost becomes  $\Phi_n + \Psi_n = \phi_j^{\mathcal{I}_j} + \phi_0^{\mathcal{I}_0} + P_j^{\mathrm{op}} + \psi_0^{\mathcal{I}_0}$ . Similarly when  $n \geq t_j$ , SBS j is in an OFF state, and the cost is  $\Phi_n + \Psi_n = \phi_0^{\mathcal{I}_0 \cup \mathcal{I}_j} + \psi_0^{\mathcal{I}_0 \cup \mathcal{I}_j}$ . By using this fact, we can map the ON/OFF scheduling optimization problem in (29) into the problem of finding the optimal, per SBS OFF time  $t_j$ . In essence, in the proposed solution, each SBS j will individually solve (29), given its own local information. The per SBS problem of finding the optimal OFF time  $t_j$  is redefined as follows:

$$\min_{t_j} \qquad F_j(t_j) = r_j t_j + b_j, \tag{30}$$

s.t. 
$$E_{j}^{n} = \min(E_{j}^{n-1} - \psi_{j}^{n-1} \delta + \Omega_{j}^{n}, E_{\text{max}}),$$

$$r_{j} = \phi_{j}^{\mathcal{I}_{j}} - \phi_{0}^{\mathcal{I}_{j}} + \eta(P_{j}^{\text{op}} - P_{0}^{\text{op}, \mathcal{I}_{j}}),$$

$$b_{i} = (\phi_{0}^{\mathcal{I}_{j}} + \eta P_{0}^{\text{op}, \mathcal{I}_{j}})T,$$
(31)

where  $P_0^{\text{op},\mathcal{I}_j}=\frac{|\mathcal{I}_j|}{M}(1-q)P_0^{\text{op}}$ .  $r_j$  is the difference between the costs yielded by ON or OFF state of SBS j. Also,  $b_j$  is the delay and power cost when the MBS services the UEs that were associated with SBS j. Due to the uncertainty of energy harvesting, SBS j does not have any knowledge about future energy arrivals, however, it has to make a decision for how long to turn ON. Since SBS j does not know the whole input sequence (e.g., uncertain energy arrivals), the SBS cannot know the optimal schedule of ON and OFF before time elapses. Thus, (30) can be formulated as an online optimization problem, for which an online algorithm is needed to make a decision in real time under an uncertain future. Remarkably, the problem in (30) is analogous to the so-called *ski rental problem* [19], an online optimization framework that enables such decision making in face of uncertainty, as discussed next.

$$\sum_{n=1}^{N} \sum_{j=1}^{J} (\phi_{j}^{\mathcal{I}_{j}} \sigma_{j}^{n} + \phi_{0}^{\mathcal{I}_{j}} (1 - \sigma_{j}^{n}) + \eta (P_{j}^{\mathsf{op}} \sigma_{j}^{n} + \frac{|\mathcal{I}_{j}|(1 - \sigma_{j}^{n})}{M} (1 - q) P_{0}^{\mathsf{op}}) + \sum_{n=1}^{N} (\phi_{0}^{\mathcal{I}_{0}} + \eta (\frac{|\mathcal{I}_{0}|}{M} (1 - q) P_{0}^{\mathsf{op}} + q P_{0}^{\mathsf{op}})). \tag{28}$$

### IV. ON/OFF SCHEDULING AS AN ONLINE SKI RENTAL PROBLEM

First, we will explicitly define the analogy between ski rental and self-powered BS scheduling. In the classical online ski rental problem, an individual is going skiing for an unknown number of days [24]. The uncertainty on the skiing period is due to factors such as nature or whether this individual will enjoy skiing or not. Here, the individual must decide on whether to rent skis over a short period of time or, alternatively, buy them for a long period of time, depending on the costs of renting and buying, the number of days that he/she will end up skiing, and on whether the skiing activity will be enjoyable. The online ski rental framework provides online optimization techniques that allows one to understand how an individual will make a "rent" or "buy" decision in such a scenario while facing uncertainty due to nature and while accounting for the tradeoff between the costs of rental and purchase and the benefits of skiing.

In this regard, our problem in (30) is similar to the ski rental decision making process. In our model, each SBS is an individual that must *rent* its resources (turn ON) to the network under the uncertainty of energy harvesting or alternatively *buy* more reliable MBS resources (and turn OFF). From (31) and (32), we can see that  $r_j$  and  $b_j$  will represent the prices for rent and buy, respectively. Thus, the decision of an SBS on how long to turn ON is essentially a decision on how long to rent its resources which require paying  $r_j$  per unit time. Once the SBS turns OFF, the network must buy the more expensive but more reliable MBS resources at a price  $b_j$ . Given this analogy, we can develop efficient online algorithms to solve (30) [25]. An *online algorithm* can solve the problem at each present time without having whole information about future energy harvesting results.

### A. Competitive Analysis and Optimal Offline Strategy

We use online algorithms to solve the optimization problem, and competitive analysis is used to study the performance of the online algorithms. Competitive analysis [24] is a method used to compare between the performance of online algorithms and an performance of the optimal offline algorithm.

**Definition 1.** The *competitive ratio* of an online algorithm is defined as

min 
$$\kappa$$
, s.t.  $\kappa \ge \frac{\beta_{\text{ALR}}(u_j)}{\beta_{\text{OPT}}(u_j)}$ , (33)

where  $u_j$  is a random time instant when harvested energy is depleted,  $\beta_{\rm ALR}(u_j)$  is the cost of an online algorithm that corresponds to the total cost of the problem (30), and  $\beta_{\rm OPT}(u_j)$  is the optimal cost achieved by using an offline algorithm that knows all input information.

The performance of an online algorithm can be measured by comparing it with that of the optimal offline algorithm.

### Algorithm 1 Deterministic Online Algorithm (DOA)

- 1: Initialization: SBS  $j \in \mathcal{J}$  has a predetermined value  $t_j$ .
- 2: while  $n \leq T$
- 3: Update  $n \leftarrow n + 1$ .
- 4: If ((31) is unsatisfied) or  $(n = t_i)$ ,
- 5: **then** SBS j is turned OFF.
- 6: **else** SBS j maintains its ON state.
- 7: end while
- 8: At n=T, update  $P_j^{\text{op}}, P_j^{\text{tx}}, \forall j \in \mathcal{J}$ , and user association.

Therefore, in competitive analysis, the competitive ratio is meaningful since it shows the performance of an online algorithm [26]. For this analysis, we assume that an arbitrary input sequence, which corresponds to uncertain energy arrivals, is used to evaluate the competitive ratio of online algorithms. An arbitrary input sequence is characterized by  $u_j$  that is the moment of energy depletion. SBS j should be turned OFF at time  $u_j$  even if the intended OFF time  $t_j$  is later than  $u_j$ . This is suitable for our ON/OFF scheduling problem since the input sequence is related to the energy arrivals at a given SBS, which are often *unknown and uncertain*. Even though an SBS does not know the input sequence, the use of online algorithms will give a solution that can at least achieve the cost of  $\kappa\beta_{\rm OPT}(u_j)$ .

The optimal cost  $\beta_{\mathrm{OPT}}(u_j)$  is calculated by assuming an offline scenario where energy arrival information over the entire period is given. Thus, the amount of stored energy at each moment becomes known information. The offline optimal cost can be shown as

$$\beta_{\text{OPT}}(u_j) = \begin{cases} r_j u_j, & 0 \le u_j \le \frac{b_j}{r_j}, \\ b_j, & \frac{b_j}{r_j} \le u_j \le T. \end{cases}$$
(34)

The optimal solution is using the rent option until  $b_j/r_j$  if energy is depleted earlier than  $b_j/r_j$ . Otherwise, the buy option should be chosen with one time payment  $b_j$  at time 0.

### B. Deterministic Online Algorithm

To design an online algorithm that can achieve a close performance to optimal, we first investigate how close performance a deterministic online algorithm can yield. A deterministic approach is mainly operated by a predetermined parameter when making decision of ON/OFF scheduling. In a deterministic online algorithm (DOA), SBS j is turned OFF at a predetermined time  $t_j$ ,  $0 \le t_j \le T$ . From time 0 to  $t_j$ , the rent option is used, and the cost is increased along with the rental cost  $r_j$  per time. Then, at time  $t_j$ , the buy option is purchased for the one time cost  $b_j$ . DOA can be shown as Algorithm 1. The competitive ratio  $\kappa$  of DOA is given by

$$\frac{\beta_{\text{DOA}}(u_j)}{\beta_{\text{OPT}}(u_j)} = \begin{cases} \frac{r_j u_j}{\min\{r_j u_j, b_j\}}, & 0 \le u_j \le t_j, \\ \frac{r_j t_j + b_j}{\min\{r_j u_j, b_j\}}, & t_j \le u_j \le T, \end{cases}$$
(35)

where  $\beta_{DOA}$  is the cost of DOA. We want to minimize  $\kappa$ subject to  $\beta_{DOA}(u_j) \leq \kappa \beta_{OPT}(u_j)$  for every  $u_j$  from 0 to T. Therefore, when  $u_j = t_j = b_j/r_j$ , the competitive ratio becomes 2 known as the best possible competitive ratio of a deterministic, online algorithm [27].

### C. Randomized Online Algorithm

To handle uncertainty, a rent or buy decision will be made by using a randomized online algorithm (ROA) by means of a probability distribution for ON/OFF scheduling designed to solve our cost-minimization problem. For instance, it is known that, when a randomized approach is used to address a ski rental problem, it is possible to achieve a lower competitive ratio of  $\frac{e}{e-1}$  [19], [27], while DOA achieves the competitive ratio of 2.

To develop an ROA for our problem, a competitive analysis should be used. For an arbitrary input, ROA computes an output (i.e., the turn OFF time,  $t_i$ ) based on a probability distribution. We want to design an ROA that satisfies  $\mathbb{E}[F_i(t_i)] < \kappa \beta_{OPT}(u_i)$  where  $\mathbb{E}[F_i(t_i)]$  is the expected cost of the problem (30) provided that unknown time of energy depletion is given by  $u_i$ . This will be adequate for our problem in that the input sequence is the unknown and uncertain energy arrivals at a given SBS. Even though an SBS does not know the input sequence, the use of an ROA will give a solution that can at least achieve the expected cost of  $\kappa \beta_{OPT}$ .

In this section, the cost of ROA is the sum of delay cost and power consumption to serve UEs in the coverage of an SBS during a given period T. We emphasize that the rental price  $r_j$  and the buying price  $b_j$  are values related to the cost of using an SBS and the MBS, respectively. First, we will compute the expected cost of ROA. Suppose that the desired ON time of SBS j is  $t_i$  where  $t_i$  is determined by SBS j. Also,  $u_i$  is the possible ON time of SBS j since SBS j can be turned ON up to the moment when energy is depleted at time  $u_j$ . It implies that the possible ON time is limited by the uncertainty of energy arrivals. At time  $t_i$ , the state of the SBS can be either ON or OFF with probability distribution  $p_i^{on}(t_i)$ or  $p_j^{\text{off}}(t_j) = 1 - p_j^{\text{on}}(t_j)$ . When an SBS decides to turn OFF at  $t_i$ , we have

$$\mathbb{E}[F_{j}(t_{j})] = \int_{0}^{u_{j}} (r_{j}t_{j} + b_{j})p_{j}^{\prime \text{off}}(t_{j})dt_{j} + \int_{u_{j}}^{T} r_{j}u_{j}p_{j}^{\prime \text{off}}(t_{j})dt_{j}, (36)$$

where  $p_j^{\text{off}}(t_j)$  is the first-order derivative of  $p_j^{\text{off}}(t_j)$ . Then, from  $\frac{d}{du_j}\mathbb{E}[F_j(t_j)]=R_j(u_j)$ , the rate of increase of the cost will be expressed by

$$R_j(u_j) = r_j p_j^{\text{on}}(u_j) + r_j u_j p_j^{\text{on}}(u_j) + (r_j u_j + b_j) p_j^{\text{off}}(u_j),$$

where  $p_i^{\text{on}} = -p_i^{\text{off}}$ . To find an upper bound on  $F_j(t_j)$ , we focus on the case in which the expected cost is at its largest value. Naturally, this is the same as finding the worst case in the online ski rental problem which corresponds to the case in which the individual buys the skis on one day, but is unable to use them in the next day. In our model, this corresponds to the case in which the SBS pays for the MBS resources at a price  $b_i$  at  $u_i$  due to the uncertainty of energy. However, at Algorithm 2 Proposed Randomized Online Algorithm (ROA)

- 1: Initialization: SBS  $j \in \mathcal{J}$  determines  $r_j$  and  $b_j$ .
- 2: Find  $t_j$  s.t.  $p_j^{\text{off}}(t_j) = \mu_j, \ \mu_j \sim U(0,1), \forall j \in \mathcal{J}$ . 3: **while**  $n \leq T$
- 4: Update  $n \leftarrow n + 1$ .
- 5: If ((31) is unsatisfied) or  $(n = t_i)$ ,
- then SBS j is turned OFF.
- **else** SBS *j* maintains its ON state. 7:
- 8: end while
- 9: At n = T, update  $P_i^{op}, P_j^{tx}, \forall j \in \mathcal{J}$ , and user association.

 $u_j = t_j$ , the SBS does not need to turn OFF if new energy arrives suddenly at that moment. In this worst case, the costincreasing rate  $R_i(u_i)$  becomes

$$R_{j}(t_{j}) = r_{j}p_{j}^{\text{on}}(t_{j}) + r_{j}t_{j}p_{j}^{\prime\text{on}}(t_{j}) + (r_{j}t_{j} + b_{j})p_{j}^{\prime\text{off}}(t_{j})$$
$$= r_{j}p_{j}^{\text{on}}(t_{j}) - b_{j}p_{j}^{\prime\text{on}}(t_{j}).$$

By using the relationship  $\mathbb{E}[F_i(t_i)] < \kappa \beta_{OPT}$ , the costincreasing rate of  $\mathbb{E}[F_j(t_j)]$  cannot be larger than the costincreasing rate of  $\kappa \beta_{OPT}$ . The cost-increasing rate of  $\beta_{OPT}$  with respect to  $u_i$  can be readily derived by choosing the rent or buy option that yields smaller cost. Now, we divide the range of  $u_i, t_i$  into two cases.

First, if  $0 < u_j < b_j/r_j$  and  $0 < t_j < b_j/r_j$ , then the optimal cost-increasing rate is  $r_j$  which means that an SBS should be turned ON during  $t_i$ . Thus, the cost-increasing rate of ROA cannot be lower than  $\kappa$  times the optimal costincreasing rate, we have

$$r_j \kappa = r_j p_j^{\text{on}}(t_j) - b_j p_j'^{\text{on}}(t_j).$$

Since this is a first-order linear ordinary differential equation, the solution  $p_i^{on}(t_i)$  is given by:

$$p_j^{\text{on}}(t_j) = ce^{\frac{r_j}{b_j}t_j} + \kappa, \tag{37}$$

where c is a constant that can be found by using two boundary conditions. If an SBS starts with the ON state, then  $p_i^{on}(0) =$  $\kappa + c = 1$ , and then  $c = 1 - \kappa$ .

Second, if  $b_j/r_j < u_j$  and  $b_j/r_j < t_j$ , then using the MBS is the optimal choice. In this case, an SBS should buy the MBS resource before  $b_i/r_i$ . Thus, the SBS should remain in the OFF state at  $b_j/r_j$ . This fact leads us to find  $p_j^{\rm on}(b_j/r_j)=(1-\kappa)e+\kappa=0$ , and we find  $\kappa=\frac{e}{e-1}$ . Therefore, we have the ON probability  $p_j^{\rm on}(t_j)=\frac{e-e^{\frac{r_j}{b_j}t_j}}{e-1}$ .

**Remark 1.** At  $t_j$ , SBS j will turn OFF according to the following probability distribution,

$$p_j^{\text{off}}(t_j) = \begin{cases} \frac{e^{\frac{r_j}{b_j}t_j} - 1}{e - 1}, & 0 \le t_j \le \frac{b_j}{r_j}, \\ 1, & \frac{b_j}{r_j} \le t_j \le T. \end{cases}$$
(38)

The proposed online ski rental algorithm is summarized in Algorithm 2. From (38), we observe the tradeoff between rent and buy. As mentioned, the rental price is a cost related to using an SBS while the buying price reflects the cost of using the MBS. For example, the rental price is reduced if using an SBS yields lower delay cost, or the power consumption of an SBS is reduced. Also, the buying price is increased if the delay from using the MBS is increased, or the power consumption of the MBS is increased. Therefore, if  $r_j$  is low and  $b_j$  is high, then it implies that using SBS will reap benefits in terms of delay cost or power consumption, so the rent time becomes longer. In contrast, the rent time becomes shorter if  $r_j$  is high and  $b_j$  is low. The short rent time means an SBS turns OFF early because buying the MBS resource would be more beneficial than using the SBS resource with the rent price. Each SBS will now run Algorithm 2 and decide at time t=0 when to turn OFF, without knowing any information on energy arrivals, by using the distribution in (38). Thus, any knowledge of energy arrivals is not needed in the following theorem that will show the bound of the cost ratio. Now, we verify the performance of the proposed ROA.

**Theorem 1.** The expected competitive ratio of ROA is  $\frac{e}{e-1}$  if  $(\phi_j^{\mathcal{I}_j} - 2\phi_0^{\mathcal{I}_j}) + \eta(P_j^{op} - 2P_0^{op,\mathcal{I}_j}) \geq 0$  is satisfied where  $\eta$  is a positive value.

*Proof.* When the rental option is chosen during the whole period T, the total cost is  $r_jT$ . If the total cost is smaller than selecting the buy option such that  $r_jT < b_j$ , then this leads to a special case. For such a case, since the optimal solution is always choosing the rental option, the SBS is not turned OFF until the energy is exhausted. Therefore, to find a solution of our interest, we should consider the case in which  $r_jT \geq b_j$ .

From (31) and (32), we have

$$T \geq \frac{b_j}{r_j} = \frac{(\phi_0^{\mathcal{I}_j} + \eta P_0^{\text{op},\mathcal{I}_j})T}{\phi_i^{\mathcal{I}_j} - \phi_0^{\mathcal{I}_j} + \eta (P_i^{\text{op}} - P_0^{\text{op},\mathcal{I}_j})}.$$

By dividing both sides by T that is positive, we can find the following condition:

$$(\phi_i^{\mathcal{I}_j} - 2\phi_0^{\mathcal{I}_j}) + \eta(P_i^{\text{op}} - 2P_0^{\text{op},\mathcal{I}_j}) \ge 0, \tag{39}$$

where  $\eta$  is a positive weighting parameter defined in (22).

Then, to show the expected competitive ratio, we calculate the expected cost of ROA. First, let us consider when  $0 \le u_j < b_j/r_j$  and  $b_j/r_j < T$ . By using (36), the expected cost is

$$\mathbb{E}[F_{j}(t_{j})] = \int_{0}^{u_{j}} (r_{j}t_{j} + b_{j})p_{j}^{\prime \text{off}}(t_{j})dt_{j}$$

$$+ \int_{u_{j}}^{\frac{b_{j}}{r_{j}}} r_{j}u_{j}p_{j}^{\prime \text{off}}(t_{j})dt_{j}$$

$$+ \int_{\frac{b_{j}}{r_{j}}}^{T} r_{j}u_{j}p_{j}^{\prime \text{off}}(t_{j})dt_{j} \qquad (40)$$

$$= \frac{r_{j}u_{j}e}{e-1},$$

where

$$p_j^{\text{off}}(t_j) = \begin{cases} \frac{r_j}{b_j} \frac{e^{\frac{i_j}{b_j} t_j}}{e-1}, & 0 \le t_j \le \frac{b_j}{r_j}, \\ 0, & \frac{b_j}{r_j} \le t_j \le T. \end{cases}$$

The third integration in (40) becomes zero since  $p_j'^{\text{off}}(t_j) = 0$  in  $b_j/r_j \le t_j \le T$ . Second, by letting  $b_j/r_j \le u_j < T$ , we have the expected cost shown as

$$\mathbb{E}[F_{j}(t_{j})] = \int_{0}^{\frac{b_{j}}{r_{j}}} (r_{j}t_{j} + b_{j})p_{j}^{'\text{off}}(t_{j})dt_{j}$$

$$+ \int_{\frac{b_{j}}{r_{j}}}^{u_{j}} (r_{j}t_{j} + b_{j})p_{j}^{'\text{off}}(t_{j})dt_{j}$$

$$+ \int_{u_{j}}^{T} r_{j}u_{j}p_{j}^{'\text{off}}(t_{j})dt_{j}$$

$$= \frac{b_{j}e}{e-1}.$$
(41)

The second and third terms in (41) become zero since  $p_j^{\text{off}}(t_j) = 0$  in  $b_j/r_j \le t_j \le T$ . By using Definition 1 and the optimal cost given by (34), the expected competitive ratio of ROA  $\kappa = \frac{e}{e-1}$ .

**Corollary 1.** The power costs of an SBS and the MBS satisfy  $P_j^{op} \geq P_0^{op,\mathcal{I}_j}$  provided that the delay cost is given by  $\phi_j^{\mathcal{I}_j} < \phi_0^{\mathcal{I}_j}$ .

*Proof.* Since  $\phi_j^{\mathcal{I}_j}$  and  $\phi_0^{\mathcal{I}_j}$  have non-negative values, we have  $\phi_j^{\mathcal{I}_j} < \phi_0^{\mathcal{I}_j} < 2\phi_0^{\mathcal{I}_j}$ . Thus, we have  $\phi_j^{\mathcal{I}_j} - 2\phi_0^{\mathcal{I}_j} < 0$ . According to Theorem 1,  $\eta(P_j^{\text{op}} - 2P_0^{\text{op},\mathcal{I}_j}) > (2\phi_0^{\mathcal{I}_j} - \phi_j^{\mathcal{I}_j})$ . To prove the statement, we use contrapositive argument. Suppose that  $P_j^{\text{op}} - 2P_0^{\text{op},\mathcal{I}_j} < 0$ . Then, we have a boundary of  $\eta$  given by  $0 < \eta < (2\phi_0^{\mathcal{I}_j} - \phi_j^{\mathcal{I}_j})/(P_j^{\text{op}} - 2P_0^{\text{op},\mathcal{I}_j})$ . Here, we use the fact that  $\eta$  is positive. We can find that if  $P_j^{\text{op}}/P_0^{\text{op},\mathcal{I}_j} < 2$ , then  $\phi_j^{\mathcal{I}_j}/\phi_0^{\mathcal{I}_j} > 2$ . Now, the equivalent contrapositive argument is that if  $\phi_j^{\mathcal{I}_j}/\phi_0^{\mathcal{I}_j} \leq 2$ , then  $P_j^{\text{op}}/P_0^{\text{op},\mathcal{I}_j} \geq 2$ . Hence, we have  $P_j^{\text{op}} \geq 2P_0^{\text{op},\mathcal{I}_j} \geq P_0^{\text{op},\mathcal{I}_j}$  where  $P_0^{\text{op},\mathcal{I}_j}$  has a non-negative value.

In heterogeneous SCNs overlaying mmW SBSs and the μW MBS, due to high average data rate of mmW links, the transmission delay cost of an SBS is smaller than that of the MBS,  $\phi_j^{\mathcal{I}_j} < \phi_0^{\mathcal{I}_j}$ . Meanwhile, to achieve the optimality condition from Corollary 1, the power consumption of an SBS should be larger than the utilization-proportional power consumption of the MBS (that depends on the load of the MBS),  $P_i^{\text{op}} \geq P_0^{\text{op},\mathcal{I}_j}$ . If the portion of fixed power consumption of the MBS is high, i.e., q is close to 1, from (17), we can observe that the utilization-proportional power consumption of the MBS can be smaller than the power consumption of an SBS. Therefore, given that a conventional  $\mu$ W MBS can have a high portion of fixed power consumption, the optimality condition of Corollary 1 can be satisfied if self-powered mmW SBSs are deployed along with conventional  $\mu W$  cellular networks. In such a case, the proposed ROA can be readily implemented and will yield significant benefits in terms of energy efficiency.

**Corollary 2.** If  $\phi_j^{\mathcal{I}_j}/\phi_0^{\mathcal{I}_j}=1$  and  $P_j^{op}/P_0^{op,\mathcal{I}_j}=1$ , the buying option is not chosen during the slot period T.

*Proof.* Since we have  $r_j = 0$  under the given conditions, the result directly becomes a special case of Theorem 1.

If using an SBS or the MBS yields the same costs, the objective function of the global ON/OFF scheduling problem (22) becomes a constant regardless of choosing any ON/OFF

TABLE I: Simulation parameters

Notation	Value
$\alpha_L, \beta_L, \alpha_N, \beta_N$	2,61.4,2.92,72
$\alpha$ , $\beta$	3.5, 35.75
$\rho_1,  \rho_2$	$5.6 \times 10^{-3}, 4.4 \times 10^{-2}$
$P_0^{\text{op}}, P_j^{\text{op}}$	100 W, 13 W
$P_0^{\text{tx}}, P_j^{\text{tx}}$	30 dBm, 13 dBm
$B_{\mu}, B_{m}$	10 MHz, 1 GHz
T, N	10 sec, 100 samples
$G_{ij}$	15 dB
$N_0$	-174 dBm/Hz
$f_{c,\mu}, f_{c,m}$	2.1 GHz, 28 GHz

scheduling strategy. In this case, our solution effectively suggests that the SBS will be turned ON until the moment of total energy depletion.

### V. SIMULATION RESULTS AND ANALYSIS

For our simulations, we assume that the SBSs and UEs are randomly distributed in a 1 km  $\times$  1 km area with one MBS located at the center of the area as shown in Fig 2. We use typical parameters from [6] and [22] as shown in Table I. Without loss of generality, we assume that energy arrivals follow a Poisson process in which energy arrival rate is 20, and each arrived energy is 0.2 J during T = 10 s. Also it is assumed that initially stored energy of SBS j is set to  $E_i^0 = 20 J$ . We use  $\eta = 0.5$  if  $K = 10^4$  bits, or  $\eta = 10$  if  $K \ge 10^5$  bits. Also, the maximum number of UE connections of the MBS is M=50, and we use q=0.9. Statistical results are averaged over a large number of independent simulation runs during two time periods 2T. We compare our online ski rental approach ROA to the DOA approach as well as to a baseline approach that turns an SBS ON if and only if the percentage of charged energy in storage is greater than a threshold K. We set K=45or 50 such that an SBS maintains its ESS half-charged given that the maximum capacity of ESS is  $E_{\rm max} = 100~J$ .

Fig. 2 shows a snapshot example for 25 SBSs, 50 UEs,  $\eta=0.5$ , and T=0.4. In Fig. 2, 16 SBSs are turned ON while 9 SBSs are turned off. Here, user association is shown as dotted lines between ON SBSs and UEs. Five SBSs out of the 9 SBSs that are OFF, stay in the OFF state since they do not have any associated UE as shown in Fig. 2. The other 4 SBSs are turned OFF due to the OFF scheduling of ROA. We observe that the 4 OFF SBSs following ROA are located near the MBS. In contrast, most of the ON SBSs are located far from the MBS. In Fig. 2, as UEs in  $\mathcal{I}_j$  are located closer to the MBS, the delay cost of using the MBS,  $\phi_0^{\mathcal{I}_j}$ , is decreased. Therefore, the rent price in (31) becomes higher. Thus, as the use of an SBS becomes more expensive, the SBS tends to buy the MBS resource earlier.

Fig. 3 shows, jointly, the power consumption and the total sum of network delay, for various numbers of SBSs with 50 UEs and  $\eta=0.5$ . From Fig. 3, we can see that, for all algorithms, as the network size increases, the overall delay will decrease, but the energy consumption will increase. This is due to the fact that having more SBSs turned ON will enable the network to service users more efficiently, however, this comes with an increase in power consumption. From Fig. 3, we can clearly see that the proposed algorithm significantly reduces both the delay and the energy consumption as compared to the

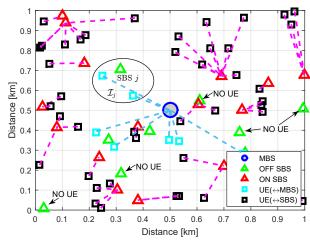


Fig. 2: Snapshot example of network resulting from the proposed ROA approach.

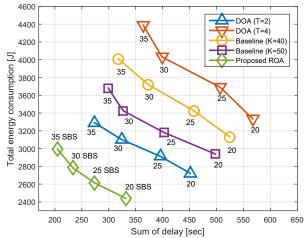


Fig. 3: Total power consumption and delay cost for the proposed algorithm, DOA, and a baseline.

baselines. This performance advantage, reaches up to 41.5% reduction in the delay relative to DOA T=4 at 20 SBSs and 31.8% reduction in energy consumption relative to DOA T=4 at 35 SBSs. Also, when we compare ROA to the baseline, the delay is reduced by 36.8% relative to the baseline K=40 at 20 SBSs, and the energy consumption is reduced by 25.3% relative to the baseline K=40 at 35 SBSs.

In Fig. 4, we show the total number of ON/OFF operations within time period T for 50 UEs and  $\eta = 0.5$ . The proposed ROA and DOA exhibit lower number of SBS ON/OFF switchings whereas the baseline turns SBSs ON and OFF more frequently. This is mainly due to the fact that the baseline (K = 40) will turn ON all SBSs that have more than 40% of energy. Thus, in the baseline approach, the ON/OFF operation depends on energy arrivals which can be intermittent. However, the proposed approach and DOA turns an SBS ON and OFF only once in period T. Furthermore, it is interesting to compare the results of the proposed ROA and DOA. In DOA, the number of ON/OFF switching per SBS is exactly one, so the number of switching in networks equals to the number of SBSs. However, in ROA, some SBSs can remain in OFF state during all period. If the rental and buying prices are similar for an SBS, it motivates the system to choose

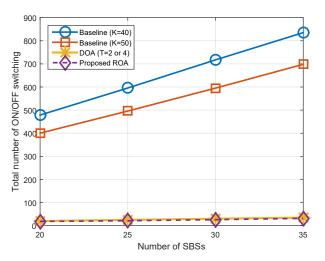


Fig. 4: The number of ON/OFF switchings of the network during one period T.

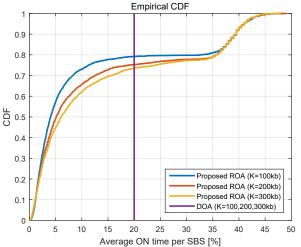


Fig. 5: The distribution of ON time per SBS for different file

the buy option earlier rather than choosing the rent option; thus, the SBS stays in OFF state from the beginning. Thus, the number of switchings of ROA is less than the number of SBSs. Hence, Fig. 4 shows that the performance advantage of ROA reaches up to 96.2% of reduction in the number of ON/OFF switchings when compared to the baseline K=40 in the network consisted of 35 SBSs and 50 UEs.

Fig. 5 shows the CDF of ON time per SBS when we change the file size K from 100 kbits to 300 kbits for 20 SBSs, 50 UEs, and  $\eta = 10$ . First, we can see that, for the DOA, the ON time is fixed as 20% of period T since the DOA scheme has a predetermined OFF time regardless of other parameters including the file size K. In the proposed ROA, each SBS, however, has different ON/OFF switching time. When the file size becomes larger, Fig. 5 shows that the ON time per SBS is increased. For example, when K = 100 kbits, 70% of SBSs have an ON time that is less than 8.22% of period T. However, if K is increased to 300 kbits, then the ON time increases to 15.49% of T. Thus, the overall ON time per SBS increases with the file size K. This result have also shown that, when K is large, SBSs should be used more often. However, if K is small, using the MBS is enough to serve all UEs without

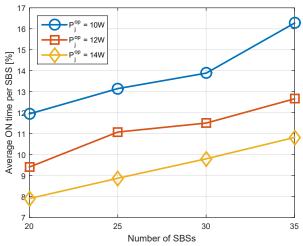


Fig. 6: The average ON time per SBS for the various SBS operational powers.

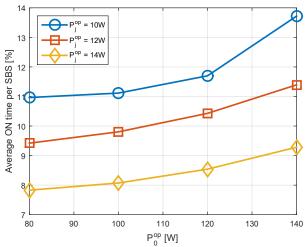


Fig. 7: The average ON time per SBS with respect to the operational power of an SBS and the MBS during period T.

resulting in a high delay cost. Hence, the ON time is effectively controlled by the proposed ROA as shown in the simulation results, so it would be possible to adopt the proposed ROA in various environments.

In Fig. 6 and Fig. 7, the average ON time per SBS within time period T is shown for different operational powers of an SBS with 50 UEs and  $\eta=0.5$ . We compare three different values for the operation power of an SBS,  $P_j^{\text{op}}$ : 10, 12, and 14 W. If an SBS uses a high  $P_j^{\text{op}}$ , then the rent price becomes higher. As the use of an SBS becomes more expensive, the SBS tends to buy the MBS resource. This, in turn, results in a shorter ON time as shown in Fig. 6. For example, the average ON time is reduced by 33.6% if  $P_j^{\text{op}}$  is increased from 10 W to 14 W when 35 SBSs are in a network. Also, the same effect can be shown in Fig. 7; for instance, the average ON time is reduced by 25.2% if  $P_j^{\text{op}}$  is increased from 10 W to 14 W when the MBS consumes 140 W.

Furthermore, Fig. 6 shows the relationship between the average ON time per SBS and the number of SBSs in the network. We observe that the ON time per SBS becomes longer if the number of SBSs is increased. This is because the average number of UEs per SBS is decreased as the number

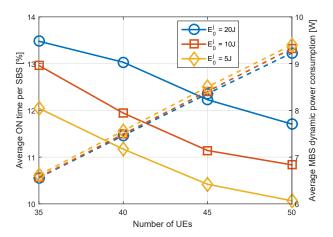


Fig. 8: Average ON time per SBS (solid line) and average MBS utilization-proportional power consumption (dashed line) for different initial energy levels in ESS.

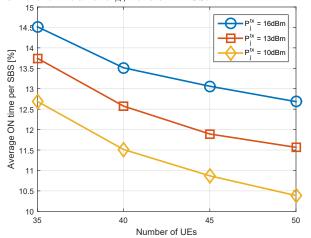


Fig. 9: Average ON time per SBS for different transmission power of an SBS.

of SBSs is increased. Thus, due to the small number of the assocated UE, the sum of delay per SBS decreases. As a result, the rent price become lower. This means that choosing the rent option becomes more affordable, thus resulting in a longer average ON time per SBS. From Fig. 6, the average ON time per SBS is increased by 36.2% when the number of SBSs is increased from 20 to 35 in the case in which the operational power is 14 W.

In Fig. 7, we observe that the ON time per SBS can be prolonged if the MBS consumes a high  $P_0^{\rm op}$ . This can be explained as follows: if  $P_0^{\rm op}$  is high, then the rent price (31) becomes higher, so the ON time per SBS becomes longer. The result coincides with the intuition that, if the MBS consumes more power, then it would be beneficial to use SBS rather than using the MBS. The simulation result shows that the average ON time is increased by 36.3% if  $P_0^{\rm op}$  is increased from 80 W to 140 W when an SBS consumes 10 W.

In Fig. 8, we show the effect of the initial energy levels on the average ON time for 50 UEs and  $\eta=0.5$ . We compare three different values for  $E_0^{\rm j}$ : 5, 10, and 20 J. As an SBS has high  $E_0^{\rm j}$ , an increase in the average ON time is observed. The result is due to the fact that a high  $E_0^{\rm j}$  can help an SBS maintain in ON state for a longer period. For instance, the

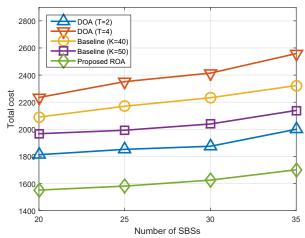


Fig. 10: Comparison of total network cost when using the proposed online ski rental algorithm, DOA, and a baseline.

average ON time per SBS is increased by 16.3% if  $E_0^{\rm J}$  is increased from  $5\,J$  to  $20\,J$  for a network with 50 UEs. Also, when the ON time per SBS becomes longer by using high  $E_0^{\rm J}$ , the utilization-proportional power consumption of the MBS is reduced since SBSs offloads UEs from the MBS.

Fig. 9 shows the average ON time per SBS within time period T for different transmission powers of an SBS. We compare three different values for  $P_j^{\rm tx}$ : 10, 13, and 16 dBm for 20 SBSs and  $\eta=0.5$  while the transmission power of the MBS is fixed. From Fig. 9, we can see that higher  $P_j^{\rm tx}$  results in longer average ON time per SBS. This is due to the fact that higher  $P_j^{\rm tx}$  yields higher data rate for UEs and lower delay cost for an SBS. Therefore, the rental cost of the SBS becomes lower, so it provides incentives to use the SBS resources for a longer time. For instance, the average ON time can be increased by 22% if the transmission power of an SBS is increased from 10 dBm to 16 dBm in the case of 50 UEs.

In Fig. 8 and Fig. 9, the average ON time per SBS within time period T is decreased as the number of UEs is increased in the network. The large number of UEs in the networks induces a higher number of connected UEs per SBS. Therefore, the sum of delay per SBS becomes larger, and consequently the rental cost is increased, thus bringing the results of the shorter ON time. For example, in Fig. 8, the average ON time is reduced by 16.4% if the number of UEs is increased from 35 to 50 when  $E_0^{\rm i}=10~J$ . Similarly, Fig. 9 shows that the average ON time is also reduced by 15.8% if the number of UEs is increased from 35 to 50 when  $P_i^{\rm i}=13~{\rm dBm}$ .

In Fig. 10, we show the total cost of the network as the network size varies for  $50 \text{ UEs}^2$  and  $\eta = 0.5$ . From Fig. 10, we can first see that the overall cost of the network will increase as the number of SBSs increases. This is mainly due to the fact that increasing the number of SBSs will increase the overall power consumption of the network. Fig. 10 shows that the cost increase of the proposed ROA is much slower than the increase

<sup>&</sup>lt;sup>2</sup>We consider dense SCNs, such that the average number of associated UEs per SBSs is typically very small. Thus, in such a dense scenario, the resource utilization level of a given BS is not high, and, thus, a BS can adopt any well-known resource scheduling scheme.

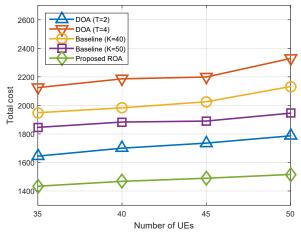


Fig. 11: Comparison of total network cost with respect to the number of UEs.

of the DOA and the baseline approach. This demonstrates the effectiveness of the proposed approach in maintaining a low network cost. In particular, Fig. 10 shows that, at all network sizes, the proposed online ski rental approach yields reduction in the overall cost of the network. This performance advantage reaches up to 33.5% reduction of the average cost for 35 SBSs compared to DOA T=4.

In Fig. 11, the total cost of the network is shown when the number of UEs varies for a network with 20 SBSs and  $\eta=0.5$ . Fig. 11 shows that the total cost of the network increases along with the number of UEs. This is because of the fact that increasing the number of UEs will naturally lead to a higher network delay. Nonetheless, we can clearly see that the cost increase of the proposed ROA is slower than that of the DOA and the baseline approach. This shows that the increase of the overall cost is limited by using the proposed ROA. Fig. 11 shows that the performance advantage of ROA can yield a reduction of up to 34.9% of the average cost for 35 UEs compared to DOA T=4.

### VI. CONCLUSION

In this paper, we have proposed a novel approach to optimize the ON/OFF schedule of self-powered, mmW SBSs. We have formulated the problem as an online ski rental problem which enables the network to operate effectively in the presence of energy harvesting uncertainty. To solve this online problem, we have proposed a randomized online algorithm that is shown to achieve the optimal competitive ratio. Indeed, we have shown that by using the proposed ROA, each SBS can autonomously decide on its ON time without knowing any prior information on future energy arrivals. Simulation results have shown that the proposed approach can reduce the power consumption up to 31.8% and 25.3% relative to a DOA and a conventional baseline, respectively. The results have also shown that both delay and the ON/OFF switching overhead are significantly reduced when one adopts the online ski rental approach. REFERENCES

- I. Hwang, B. Song, and S. S. Soliman, "A holistic view on hyper-dense heterogeneous and small cell networks," *IEEE Commun. Mag.*, vol. 51, no. 6, June 2013.
- [2] Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broad-band systems," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 101–107, June 2011.

- [3] A. Ghosh, T. Thomas, M. C. Cudak, R. Ratasuk, P. Moorut, F. W. Vook, T. S. Rappaport, G. R. MacCartney, S. Sun, and S. Nie, "Millimeterwave enhanced local area systems: A high-data-rate approach for future wireless networks," *IEEE J. Sel. Areas in Commun.*, vol. 32, no. 6, pp. 1152–1163, June 2014.
- [4] S. Rangan, T. S. Rappaport, and E. Erkip, "Millimeter-wave cellular wireless networks: Potentials and challenges," *Proc. of IEEE*, vol. 102, no. 3, pp. 366–385, Mar. 2014.
- [5] S. Hur, T. Kim, D. J. Love, J. V. Krogmeier, T. Thomas, and A. Ghosh, "Millimeter wave beamforming for wireless backhaul and access in small cell networks," *IEEE Trans. on Commun.*, vol. 61, no. 10, pp. 4391–4403, Oct. 2013.
- [6] G. Lee and H. Kim, "Green small cell operation using belief propagation in wireless networks," in *Proc. of IEEE Global Commun. Conference* (GLOBECOM) Wkshp., Austin, TX, USA, Dec. 2014, pp. 1266–1271.
- [7] N. Ansari, T. Han, and M. Taheri, "GATE: Greening at the edge," arXiv preprint arXiv:1508.06218, Sep 2015.
- [8] Y. Mao, Y. Luo, J. Zhang, and K. B. Letaief, "Energy harvesting small cell networks: Feasibility, deployment and operation," CoRR, vol. abs/1501.02620, 2015. [Online]. Available: http://arxiv.org/abs/1501. 02620
- [9] H. S. Dhillon, Y. Li, P. Nuggehalli, Z. Pi, and J. G. Andrews, "Fundamentals of heterogeneous cellular networks with energy harvesting," *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2782–2797, May 2014.
- [10] T. Han and N. Ansari, "Powering mobile networks with green energy," IEEE Wireless Commun., vol. 21, no. 1, pp. 90–96, Feb. 2014.
- [11] D. Liu, Y. Chen, K. Chai, T. Zhang, and M. Elkashlan, "Two dimensional optimization on user association and green energy allocation for hetnets with hybrid energy sources," *IEEE Trans on Commun.*, vol. PP, no. 99, pp. 1–1, Dec. 2015.
- [12] T. Han and N. Ansari, "Provisioning green energy for small cell BSs," in *Proc. of IEEE Global Commun. Conference (GLOBECOM)*, Austin, TX, USA, Dec. 2014, pp. 4935–4940.
- [13] J. Gong, J. Thompson, S. Zhou, and Z. Niu, "Base station sleeping and resource allocation in renewable energy powered cellular networks," *IEEE Trans. on Commun.*, vol. 62, no. 11, pp. 3801–3813, Nov. 2014.
- [14] S. Zhou, J. Gong, and Z. Niu, "Sleep control for base stations powered by heterogeneous energy sources," in *Proc. of International Conference* on *ICT Convergence (ICTC) 2013*, Jeju, South Korea, Oct. 2013, pp. 666–670.
- [15] X. Ge, H. Cheng, M. Guizani, and T. Han, "5G wireless backhaul networks: challenges and research advances," *IEEE Network*, vol. 28, no. 6, pp. 6–11, Nov. 2014.
- [16] S. Yun, S. H. Jeon, J. K. Choi, and A.-S. Park, "Energy efficiency of relay operation in millimeter-wave mobile broadband systems," in *Proc.* of IEEE Vehicular Technology Conference (VTC) Spring, Seoul, Korea, May 2014, pp. 1–5.
- [17] J. Choi, "Energy efficiency of a heterogeneous network using millimeterwave small-cell base stations," in *Proc. of PIMRC 2016*, Hong Kong, China, Aug. 2015, pp. 293–297.
- [18] C. Araujo, I. Lian, and A. Klautau, "Traffic-aware sleep mode algorithm for 5G networks," in *Proc. in IEEE Int. Wksp on Telecommun. (IWT)*, Santa Rita, Brazil, June 2015, pp. 1–5.
- [19] Z. Lotker, B. Patt-Shamir, and D. Rawitz, "Ski rental with two general options," *Inf. Process. Lett.*, vol. 108, no. 6, pp. 365–368, Nov. 2008.
- [20] H. Shokri-Ghadikolaei, C. Fischione, G. Fodor, P. Popovski, and M. Zorzi, "Millimeter wave cellular networks: A MAC layer perspective," arXiv preprint arXiv:1503.00697, Jul. 2015.
- [21] 3GPP. TR 36.814. [Online]. Available: http://www.3gpp.org/dynareport/ 36814.htm
- [22] M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Sun, S. Rangan, T. S. Rappaport, and E. Erkip, "Millimeter wave channel modeling and cellular capacity evaluation," *IEEE J. Sel. Areas in Commun.*, vol. 32, no. 6, pp. 1164–1179, June 2014.
- [23] T. Bai, R. Vaze, and R. W. Heath, "Analysis of blockage effects on urban cellular networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 9, pp. 5070–5083, Sept. 2014.
- [24] M. Grötschel, S. O. Krumke, and J. Rambau, Online optimization of large scale systems. Springer Science & Business Media, 2013.
- [25] S. Albers, "Energy-efficient algorithms," Commun. of the ACM, vol. 53, no. 5, pp. 86–96, May 2010.
- [26] A. Borodin and R. El-Yaniv, Online computation and competitive analysis. Cambridge University Press, 2005.
- [27] A. R. Karlin, M. S. Manasse, L. A. McGeoch, and S. Owicki, "Competitive randomized algorithms for nonuniform problems," *Algorithmica*, vol. 11, no. 6, pp. 542–571, June 1994.