EE6150: Stochastic modeling and the theory of queues (Sep-Dec 2020)
Assignment 6 (Simulation Exercise)

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Due: Jan 10, 2020 Total Marks: 38

Problem 1. In Mumbai city, consider two adjacent shops selling pav-bhajis. The first shop has an 'automated' pav-bhaji dispenser, while the second one has two human pav-bhaji makers. In each shop, customers arrive according to a Poisson process of rate 20 customers per hour. Further, in each shop, if there are too many customers queued up, then the customers leave. For both shops, the probability that a customer moves away is as follows:

$$\begin{cases} \frac{k}{16} \text{ if } k \leq 16 \text{ customers are already in the system} \\ 1 \text{ when } k > 16 \text{ customers are there in the system} \end{cases}$$

The service time in either shop is exponentially distributed with the following rates: 10 customers per hour for automated shop, and 6 customers per hour for each server in the other shop. Model the above system with a queuing model, simulate using your preferred coding language/software, and report the following from the simulation results:

(a) In both shops,

  (i) Find the rate at which customers move away. (2 marks)

  (ii) Find the average number of customers waiting to be served. (2 marks)

  (iii) Assume each customer orders a single pav-bhaji in either shop, with each pav-bhaji priced 5 INR. Further, the charge of each maker in a system per hour is 40 INR. Find the expected profit per hour in both the systems. (2 marks)

(b) Suppose you plan to start a pav-bhaji business in Mumbai city. From the results obtained, suggest which system would you choose, i.e., an automated one or the one with two human servers. (2 marks)

Problem 2. A TA group named *EE6150 TA helpline* offers a student helpline for the immensely popular course *Stochastic Modelling and Theory of Queues* at IIT Madras. Several hundred students, who are crediting the course can call and talk to a TA who answers the questions. Currently, there are only two TAs to handle the calls individually, and each call takes 3 minutes on the average. When both the TAs are busy, students who called are placed in a 'hold queue'. The TAs have observed that no student has abandoned the call so far, but the TAs are worried that releasing 'Assignment 6' may significantly increase the number of calls to the helpline, leading to significant increase in the size of the hold queue. Suppose the students' calls arrive according to a Poisson process with rate 20 per hour. Develop a queuing model, simulate using your preferred coding language/software, and answer the following questions from the simulation results obtained:

(a) How large can the arrival rate become before the TAs are forced to add another TA to the helpline. (2 marks)

(b) With only two TAs,

  (i) How large can the arrival rate be so that the expected time on hold of a student is at most 4 minutes? (2 marks)

  (ii) How large can the arrival rate be so that the percentage of the time that there are more than five people in the hold queue is at most 20 percent of the time. (2 marks)

(c) Suppose the students on hold lose their patience after 5 minutes, and abandon the call. What is the expected number in the hold queue for this 'impatient' model? (2 marks)

Problem 3. 'Pay Less - Eat More' is a new restaurant outside at IIT Madras. At this restaurant, there are two waiters. The first waiter is on duty at all times, while the second waiter goes on duty if there are 3 or more tables occupied. Assume that the tables get occupied according to a Poisson process with rate 11 per hour. Both waiters have a service at the rate of 7 tables per hour. Service times are exponentially distributed. Develop a queuing model, simulate using your preferred coding language/software, and answer the following questions:

(a) Find the fraction of time that the second waiter is on duty. (2 marks)

(b) Find the average number of tables occupied. (2 marks)

(c) When should the second agent go on duty to keep the expected number of tables occupied under 5 ? (2 marks)

Problem 4. An idli shop on the Chennai-Bengaluru highway has two service options. Customers could get served at a window outside, or enter the shop and avail the services of two waiters inside the shop. The idli shop is open from 7 a.m. to 10 p.m. The inter-arrival times of the customers using the outside window is exponentially distributed with mean 15 minutes. But the outside window can have only 3 customers waiting, while one gets served. The customers who find the outside queue full have to go inside the shop, and this takes 2 minutes. In addition, there are customers who prefer having food inside the shop and not at the outside window. The inter-arrival time of such customers is exponentially distributed with mean as 10 minutes. All the customers who enter the shop either get served by one of the two waiters, or else wait in a queue. The service time follows a Gaussian distribution with the following parameters: Both the waiters inside have mean 5 minutes and standard deviation 1 minute, while the server outside has mean 3 minute and standard deviation 1 minute.

Simulate this idli shop using your preferred coding language/software, and answer the following questions:

(a) The average service of a customer outside/inside the shop. (2 marks)

(b) Is it optimal to add a waiter inside, or another window outside? Answer this question using simulation results. (2 marks)

Problem 5. Consider the DTMCs, whose transition matrices given in the two files MC1 and MC2. For each DTMC, do the following:

(a) Simulate the DTMC for N steps, where N takes values in the set $\{100, 1000, 100000\}$. (3 marks) For each value of N, repeat the simulation 100 times.

(b) Estimate the steady-state distribution, say π, using empirical averages from the parts above. Let $\hat{\pi}_N$ denote the empirical estimate of π, obtained after simulating the DTMC for N steps. For MC1, Tabulate/plot $\hat{\pi}_N(10)$ with N taking values as in part (a), and compare these estimates to π_{10} be a random variable with distribution π. Repeat the exercise for MC2. (2 marks)

(c) Letting X_n denote the state at time instant n, plot $\mathbb{P}\left(X_n = 10 | X_0 = 1\right), \mathbb{E}\left[X_n | X_0 = 1\right]$, $\mathrm{Var}\{X_n | X_0 = 1\}$ as functions of n in the case of MC1. (2 marks)

(d) Plot $\mathbb{P}\left(X_n = 50 | X_0 = 1\right), E\{X_n | X_0 = 1\}$, $\mathrm{Var}\{X_n | X_0 = 1\}$ as functions of n in the case of MC2. (2 marks)

Interpret the numerical results, and answer the following:

1. How many time steps are required to be within 1% relative error of the limiting values for each quantity in parts (d) and (e) above? (1 marks)

2. Do the probability, expected value and variance estimates in parts (d) and (e) converge to within 1% relative error of the limiting values at the same time step? (2 marks)