University of Colorado **Boulder**

Department of Computer Science

CSCI 5622: Machine Learning

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Lecture 5:
SGD Tips and Tricks and Multiclass Classification

Tips and Tricks for SGD Roadmap

- Feature Scaling
- Learning Rate Schedules
- Momentum

Feature Scaling with Regularization

$$NLL(\mathbf{w}) = -\sum_{i=1}^{m} \left[y_i \log \text{sigm}(\mathbf{w}^T \mathbf{x}_i) + (1 - y_i) \log(1 - \text{sigm}(\mathbf{w}^T \mathbf{x}_i)) \right] + \lambda \underbrace{\sum_{k=1}^{D} w_k^2}_{\text{REGULARIZATION}}$$

x_2 = # floors

x_5 = Age

$w_2 x_2$ vs $w_5 x_5$

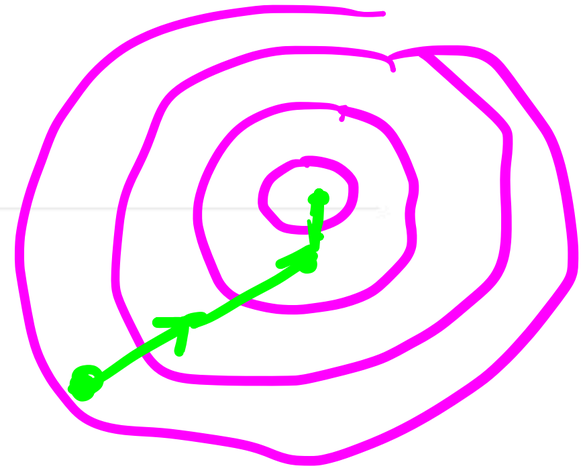
w_2 ↗ Big, x_2 ↑ small

w_5 ↑ small, x_5 ↑ BIG

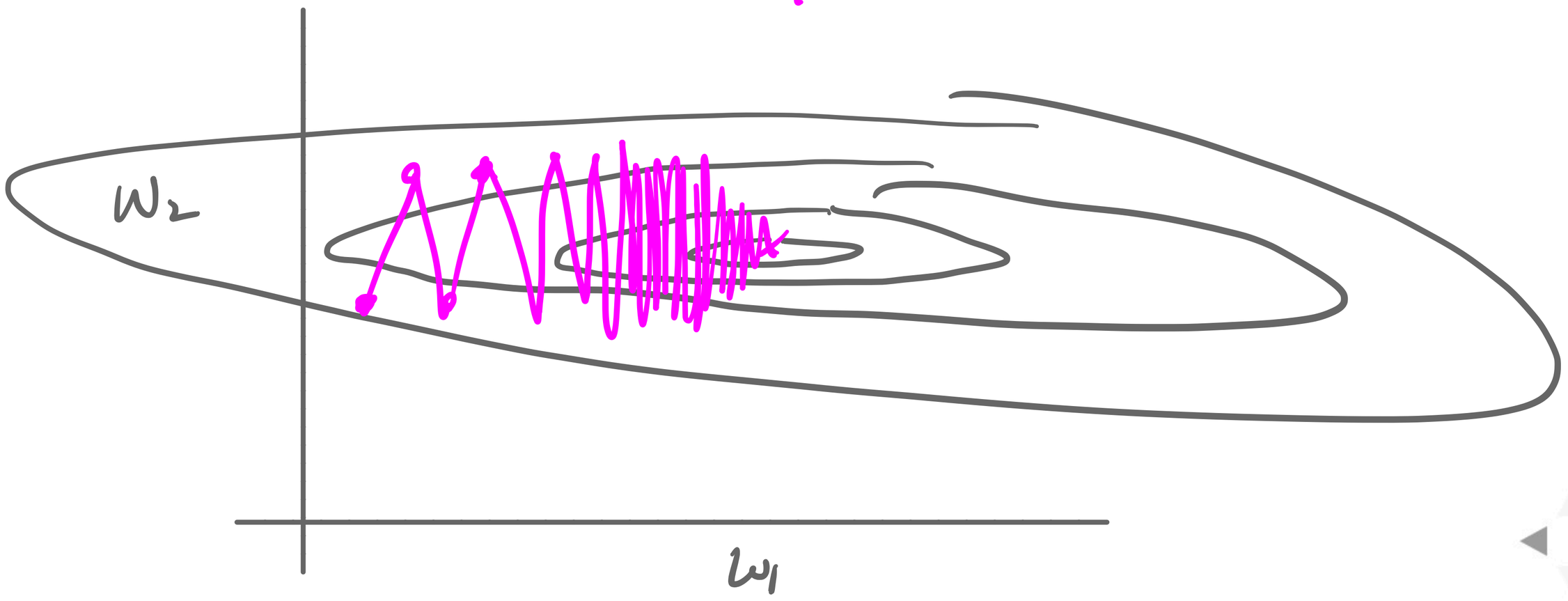
REG SEES w_2 AS ENEMY BEFORE w_5

Feature Scaling for SGD Convergence

$$RSS = \sum_{i=1}^{m} (w_1 x_{i1} + w_2 x_{i2} - y_i)^2$$

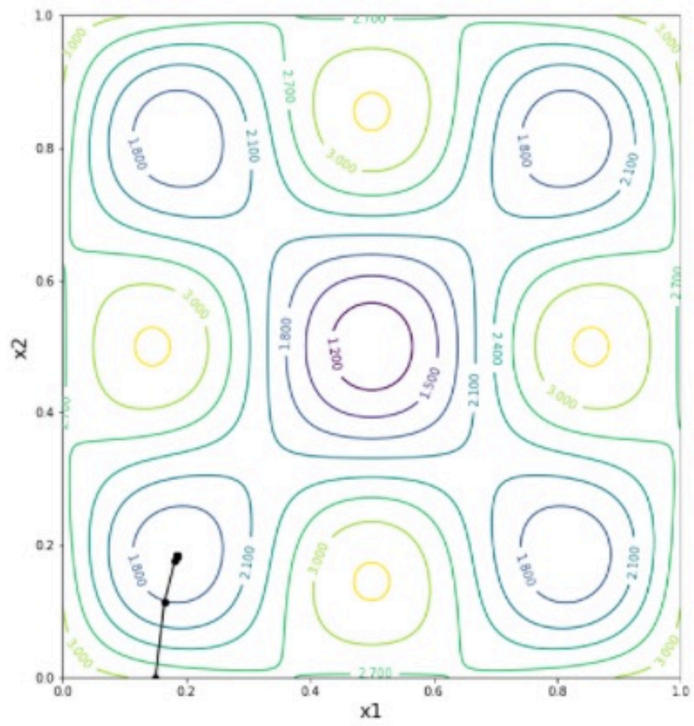


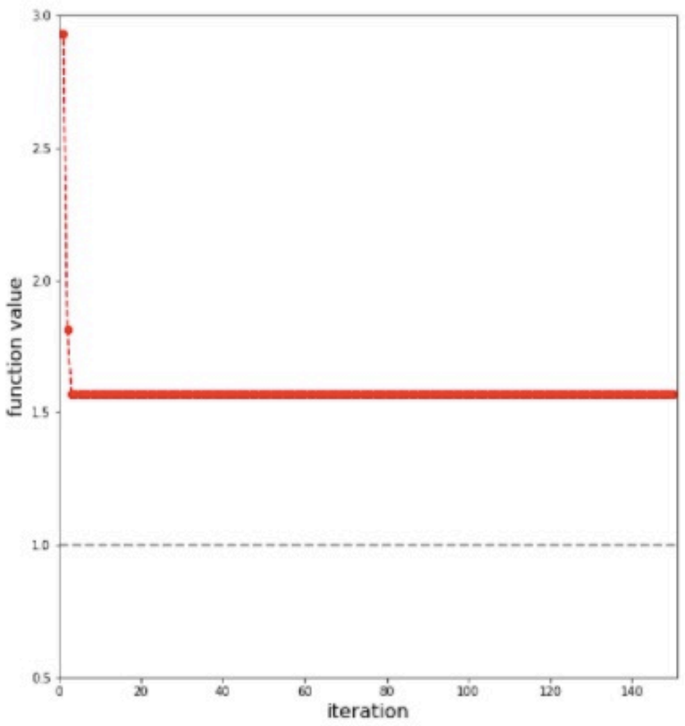
$$\overset{Ex}{=} (w_1 \cdot 1 + w_2 \cdot 100 - 1)^2 + (w_1 \cdot 1 + w_2 \cdot (-100) - 0)^2$$



Learning Rate Schedules

Allowing the learning rate to evolve (usually shrink) over time can aid convergence in both the convex and nonconvex cases





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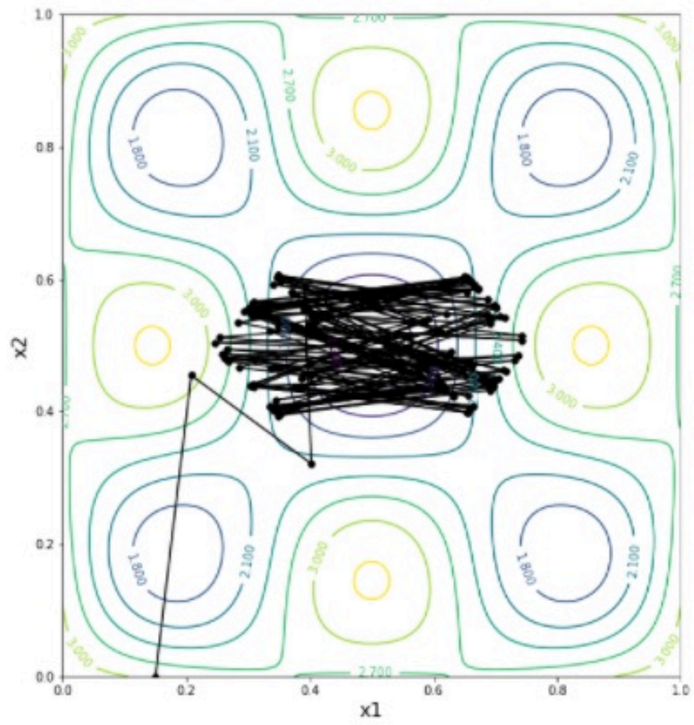

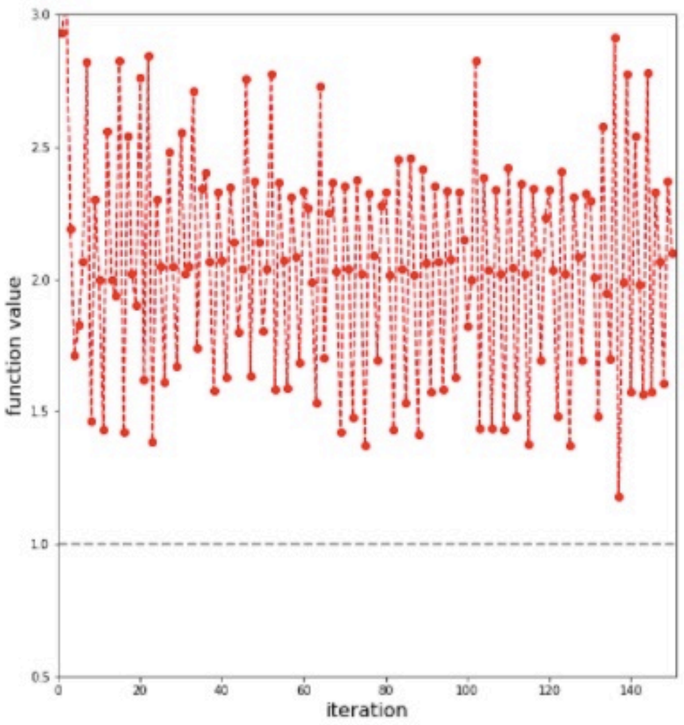

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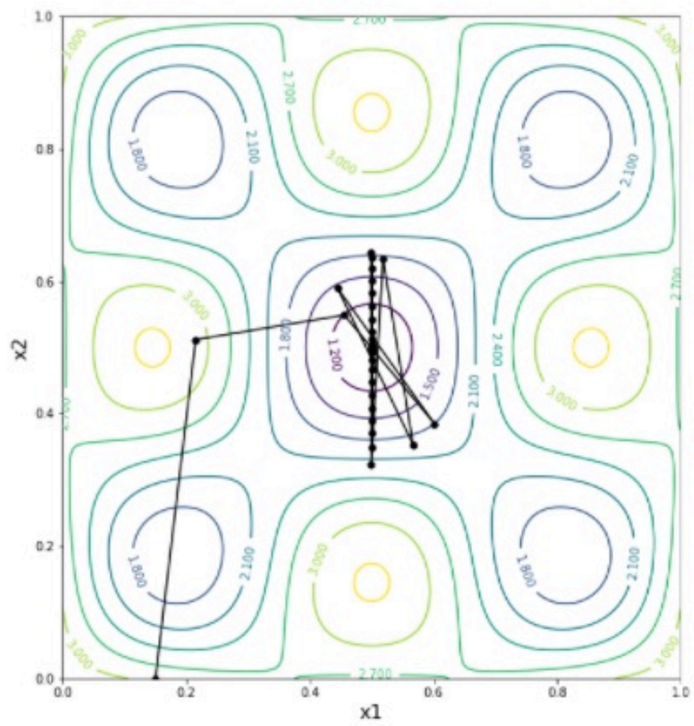
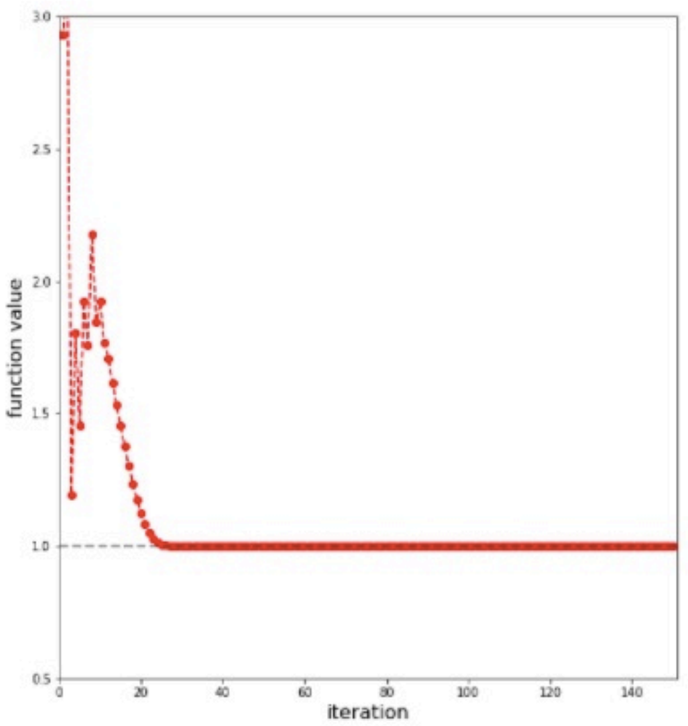
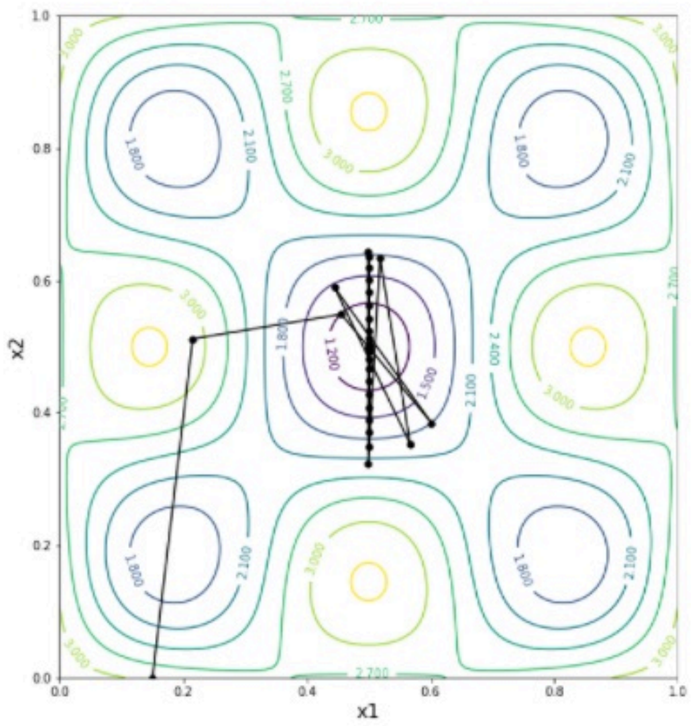
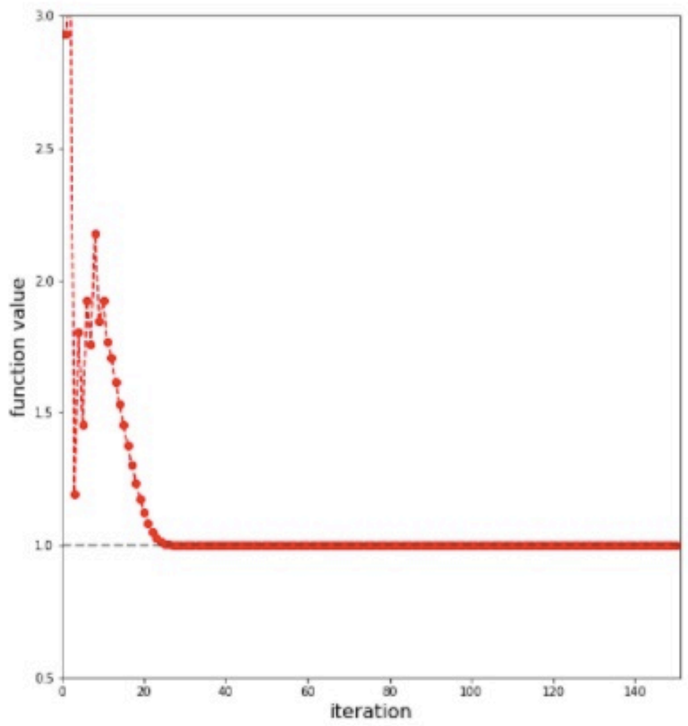




Learning Rate Schedules

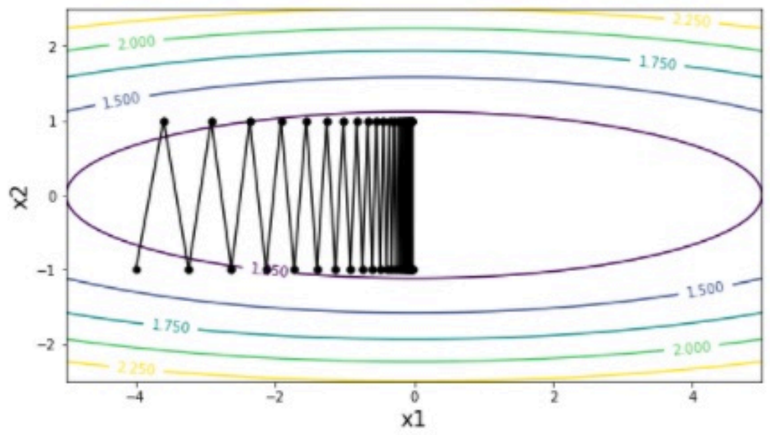
A common, simple learning rate schedule: $\eta_k = \dfrac{\eta_0}{1 + \alpha\, k\,/\,n}$

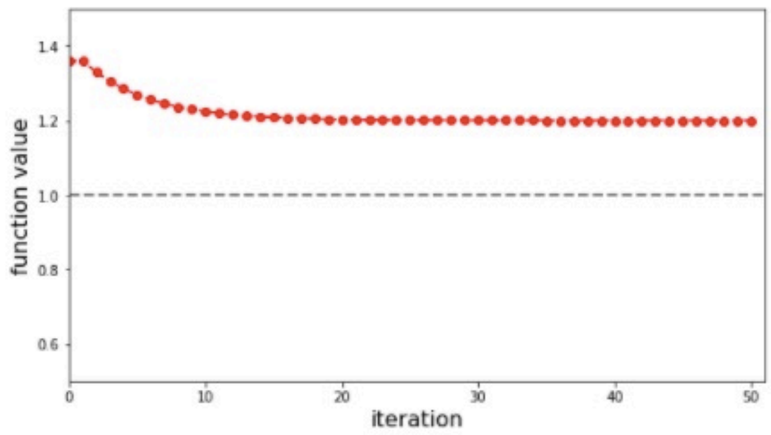




Momentum

SGA tends to hop around quite a bit when determining step direction based on one or a small sample of training examples



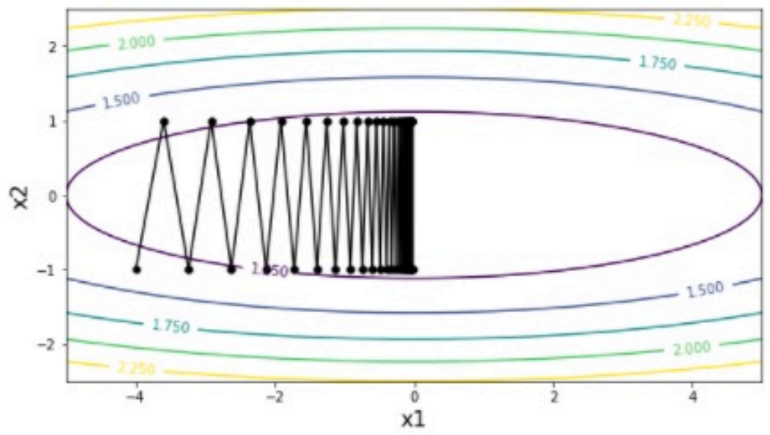


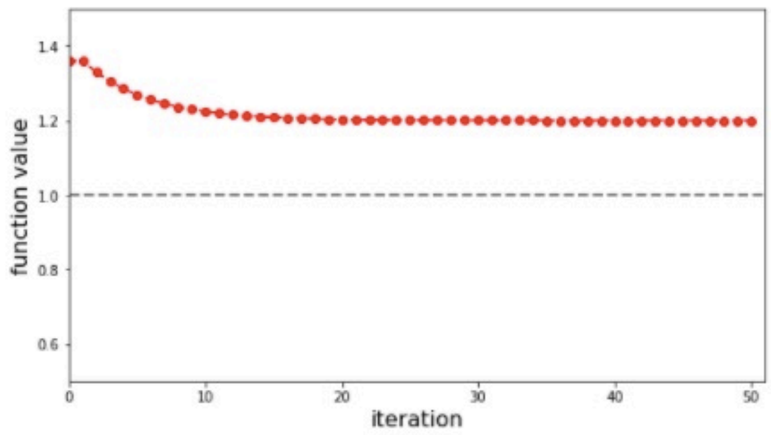
SGA is a *memoryless* algorithm

Keeping and using information about the past is a common acceleration technique in optimization

Momentum

SGA tends to hop around quite a bit when determining step direction based on one or a small sample of training examples





Adding a **momentum** term is popular:

-NLL

$$\mathbf{v}^{(k)} = \eta \, \nabla_{\mathbf{w}} J(\mathbf{w}^{(k-1)}, \mathbf{x}_i)$$

$$\mathbf{w}^{(k)} = \mathbf{w}^{(k-1)} - \mathbf{v}^{(k)}$$

Momentum

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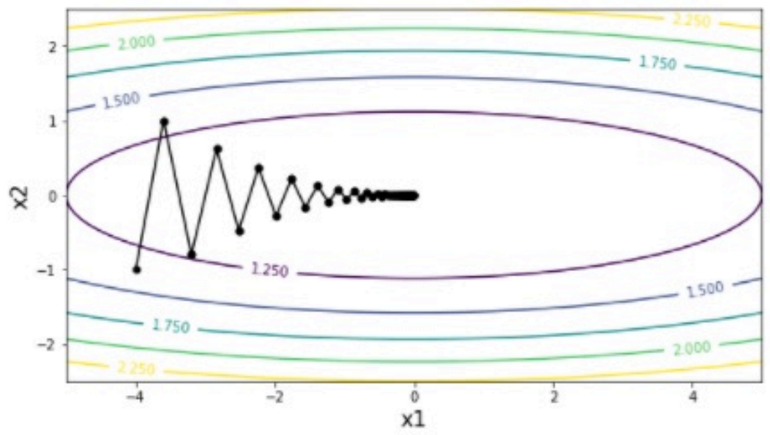

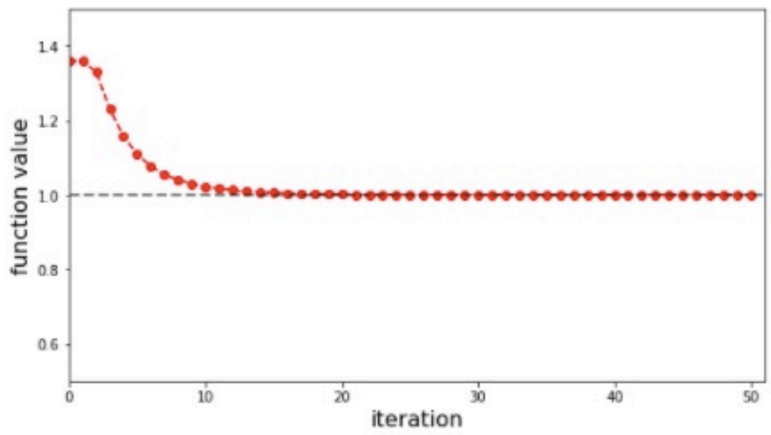

Adding a **momentum** term is popular:

$$\mathbf{v}^{(k)} = \eta \nabla_{\mathbf{w}} J(\mathbf{w}^{(k-1)}, \mathbf{x}_i) + \mu \mathbf{v}^{(k-1)}$$

$$\mathbf{w}^{(k)} = \mathbf{w}^{(k-1)} - \mathbf{v}^{(k)}$$

GRADIENT

PREVIOUS STEP

μ: tuning PARAM

Classification Roadmap

- Inherently multiclass classifiers
- Multiclass classifiers from binary classifiers
  - One-vs-All
  - All Pairs
  - Error Correcting Codes (in-class exercises)

General Classification

Given: $\mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{m}$ training examples

$$\mathbf{x}_i \in \mathbb{R}^D \qquad y_i \in \{1, 2, \ldots, K\}$$

Goal: Given new data $\mathbf{x}$, predict its label y

Some problems are inherently binary:

- Spam vs Ham

Some problems are inherently multiclass:

- Is there a car/truck/bike/person in this image
- Should self-driving car speed up / slow down / hold speed

Inherently Multiclass Classification

Some methods we've seen are inherently multiclass

Naive Bayes

For each class c_k, estimate

$$p(y = c_k \mid \mathbf{x}, \mathcal{D})$$

Assign to $\mathbf{x}$ the class with highest probability

$$\hat{y} = \arg\max_c p(y = c \mid \mathbf{x}, \mathcal{D})$$

Inherently Multiclass Classification

Some methods we've seen are inherently multiclass

K-Nearest Neighbors

1. Find $N_K(\mathbf{x}, \mathcal{D})$: the K training examples in $\mathcal{D}$ "nearest" to $\mathbf{x}$
2. Assign $\hat{y}$ the majority label of N_K

Inherently Multiclass Classification

Some methods we've seen are inherently multiclass

Logistic Regression

ODDS

For binary classification we modeled

$$\log \frac{p(y=1 \mid \mathbf{x}, D)}{p(y=0 \mid \mathbf{x}, D)} = w_0 + w_1 x_1 + \cdots + w_D x_D = \mathbf{w}^T \mathbf{x}$$

$$\log \frac{p(y=1 \mid \mathbf{x}, D)}{1 - p(y=1 \mid \mathbf{x}, D)} = w_0 + w_1 x_1 + \cdots + w_D x_D = \mathbf{w}^T \mathbf{x}$$

with

$$p(y = 1 \mid \mathbf{x}) = \text{sigm}(\mathbf{w}^T \mathbf{x}), \quad p(y = 0 \mid \mathbf{x}) = 1 - \text{sigm}(\mathbf{w}^T \mathbf{x})$$

Inherently Multiclass Classification

RATIO: $y=1$ to $y=K$

RATIO: $y=2$ to $y=K$

For multiclass classification with K classes we can do

$$\log \frac{p(y=1 \mid \mathbf{x},D)}{p(y=K \mid \mathbf{x},D)} = w_{10} + w_{11}x_1 + \cdots + w_{1D}x_D = \mathbf{w}_1^T\mathbf{x}$$

$$\log \frac{p(y=2 \mid \mathbf{x},D)}{p(y=K \mid \mathbf{x},D)} = w_{20} + w_{21}x_1 + \cdots + w_{2D}x_D = \mathbf{w}_2^T\mathbf{x}$$

$$\vdots$$

$$\log \frac{p(y=K-1 \mid \mathbf{x},D)}{p(y=K \mid \mathbf{x},D)} = w_{(K-1)0} + w_{(K-1)1}x_1 + \cdots + w_{(K-1)D}x_D = \mathbf{w}_{K-1}^T\mathbf{x}$$

with

$$p(y=k \mid \mathbf{x}) = \frac{\exp(\mathbf{w}_k^T\mathbf{x})}{1+\sum_{\ell=1}^{K-1}\exp(\mathbf{w}_\ell^T\mathbf{x})} \quad k=1,\ldots,K-1, \qquad p(y=K \mid \mathbf{x}) = \frac{1}{1+\sum_{\ell=1}^{K-1}\exp(\mathbf{w}_\ell^T\mathbf{x})}$$

Inherently Binary Classification

Some methods can be implemented as binary classification

- Logistic Regression

Some common methods we'll see later are inherently binary

- Perceptron
- Support Vector Machines
- Boosted Decision Stumps

Want to leverage these to do multiclass labeling

Inherently Binary Classification

Example Data:

$$\langle \texttt{name=Cindy, age=5, sex=F} \rangle \quad ,$$
$$\langle \texttt{name=Marcia, age=15, sex=F} \rangle \quad ,$$
$$\langle \texttt{name=Bobby, age=6, sex=M} \rangle \quad ,$$
$$\langle \texttt{name=Jan, age=12, sex=F} \rangle \quad ,$$
$$\langle \texttt{name=Peter, age=13, sex=M} \rangle \quad ,$$

We'll look at two strategies

- **One-vs-All**
- **All-Pairs**

One-vs-All

BLACK VS NOT BLACK

	■	■	■	■
$\mathbf{x}_1$ ■	−	+	−	−
$\mathbf{x}_2$ ■	−	−	+	−
$\mathbf{x}_3$ ■	−	−	−	+
$\mathbf{x}_4$ ■	−	+	−	−
$\mathbf{x}_5$ ■	+	−	−	−
	h_1	h_2	h_3	h_4

- Build K binary problems of form *Class k vs Not Class k*
- Evaluate all h's. Choose w/ highest probability / confidence

All-Pairs

	■ v ■	■ v ■	■ v ■	■ v ■	■ v ■	■ v ■
$\mathbf{x}_1$ ■	−			−		−
$\mathbf{x}_2$ ■		−	+			+
$\mathbf{x}_3$ ■			−	+	−	
$\mathbf{x}_4$ ■	−			−		−
$\mathbf{x}_5$ ■	+	+			+	
	h_{12}	h_{13}	h_{34}	h_{42}	h_{14}	h_{32}

- Build one binary problem for each pair of classes
- Define h_{ij} case when $i \rightarrow +$ and $j \rightarrow -$
- **Note**: $h_{ij} = -h_{ji}$
- Classify with $H(\mathbf{x}) = \arg\max_i(\sum_j h_{ij}(\mathbf{x}))$

One-vs-All vs All-Pairs

m = # TRAINING EXAMPLES

Time Complexity:

Suppose training time is $\mathcal{O}(m^\alpha)$ and testing time is $\mathcal{O}(T)$

	Train	Test
OVA	$\mathcal{O}(km^\alpha)$	$\mathcal{O}(kT)$
APs	$\mathcal{O}(k^2(m/k)^\alpha)$	$\mathcal{O}(k^2T)$

- One-vs-All wins at Test time
- All-Pairs wins at Training time
- All-Pairs generally faster and more accurate

Multiclass Wrap-Up

- Lots of choices for multiclass classification
- Pretty much plug-and-play with binary classifiers
- Can come up with multiclass theory based on binary theory

In Class

- **Get out laptops and get in groups!**

In Class

In Class

In Class

In Class

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