The UMAP Journal

Vol. 27, No. 3

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The UMAP Journal is published quarterly by the Consortium for Mathematics and Its Applications (COMAP), Inc., Suite 3B, 175 Middlesex Tpke., Bedford, MA, 01730, in cooperation with the American Mathematical Association of Two-Year Colleges (AMATYC), the Mathematical Association of America (MAA), the National Council of Teachers of Mathematics (NCTM), the American Statistical Association (ASA), the Society for Industrial and Applied Mathematics (SIAM), and The Institute for Operations Research and the Management Sciences (INFORMS). The Journal acquaints readers with a wide variety of professional applications of the mathematical sciences and provides a forum for the discussion of new directions in mathematical education (ISSN 0197-3622).

Periodical rate postage paid at Boston, MA and at additional mailing offices.

Send address changes to: info@comap.com

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Publisher's Editorial

Because Math Matters

Solomon A. Garfunkel
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The President has recently appointed a National Mathematics Advisory Panel. National newspapers carry lead editorials on math education. Why and why now? For many years, there has been a debate on how best to teach mathematics in our nation's schools. There are a number of reasons why this discussion has gone on so long and become so heated.

The first is that there is a great deal at stake. From Sputnik on, we have worried about our ability to compete in science and industry—first with Russia, then with Japan, and now with India and China (not to mention Western Europe). And math matters. Mathematics is at the heart of technological innovation, advances in engineering, physics, medicine, biology, and on and on. Mathematical models can forecast environmental change and monitor energy supply and demand. Without mathematics we wouldn't have MRI's or maps of the human genome.

Second, we are not doing a very good job. U.S. students are falling behind students in most industrial countries as measured on any number of international tests. And again math matters. We know that the careers of the 21st century will require more and more quantitative reasoning. We know that in this global economy, companies can and will outsource jobs to countries with more mathematically skilled work forces. To quote CBS news great Fred Friendly, we don't want to become a country "in which we take in each other's laundry."

The third reason the debate is so heated is that it has become very political. We hear terms like "back to basics" and "fuzzy math." But what's lost in all of this is the kids. Education debates need at their heart to be about education. We want our children to learn, to understand and be able to use mathematics as they go through school and work. Not all students will go on to be mathematicians, but they will all be called upon to use the mathematics they know.

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I can't emphasize this point strongly enough: The half-life of students in mathematics courses remains one year from 10th grade on. In other words, the number of students taking math in 11th grade is half those taking math in 10th, and so on for every year right up until the Ph.D. What happens to the other half? We simply cannot afford to throw away half of our students each year because they don't have serious prospects of becoming research mathematicians.

We can continue to ask students problems of the form: Solve for x in the equation $x^2 - 3x + 1 = 0$. Or we can ask at what proportion of performance enhancing drug use in the population is it cheaper to test two athletes by pooling their samples—a real-life question that leads to the same equation. We can teach mathematics through engaging contexts that students will see as real and important, or we can continue to insist on honing skills. Learning to hammer a nail before trying to build a house sounds right. But hammering nails for six years before even knowing that there's such a thing as a house just doesn't make sense. If mathematics is a life skill, then students need to see mathematical skills at work in their lives.

Math matters. We cannot afford partisan politics. The National Science Foundation, staffed by independent scientists and mathematicians, has led the effort for innovation in mathematics education since the 1950s. Innovation is desperately needed. We must not go back to methods that have consistently failed us. After all, the reason that the current reform movement began in the first place was that we were unhappy with student performance. What we need are serious people who recognize the importance and difficulties in getting a quantitatively literate citizenry and who are willing to put aside any specific political agenda.

Articles in the *Wall Street Journal* and the *New York Times* have declared that the Math Wars are over and that the Back to Basics movement has won. Well, I have news. The Math Wars are not over. It doesn't matter that these newspapers declare, "Mission Accomplished." The mission is about helping children learn, not about winning a political battle or finding common ground. We will continue to fight for

- the introduction of new and relevant content,
- the appropriate use of new technologies,
- showing students important contemporary applications, and
- using innovative pedagogical approaches.

And we will do so because math matters.

About the Author

Solomon Garfunkel, previously of Cornell University and the University of Connecticut at Storrs, has dedicated the last 25 years to research and development efforts in mathematics education. He has served as project director for the Undergraduate Mathematics and Its Applications (UMAP) and the High School Mathematics and Its Applications (HiMAP) Projects funded by NSF, and directed three telecourse projects including *Against All Odds: Inside Statistics*, and *In Simplest Terms: College Algebra*, for the Annenberg/CPB Project. He has been the Executive Director of COMAP, Inc. since its inception in 1980. Dr. Garfunkel was the project director and host for the series, *For All Practical Purposes: Introduction to Contemporary Mathematics*. He was the Co-Principal Investigator on the ARISE Project, and is currently the Co-Principal Investigator of the CourseMap, ResourceMap, and WorkMap projects. In 2003, Dr. Garfunkel was Chair of the National Academy of Sciences and Mathematical Sciences Education Board Committee on the Preparation of High School Teachers.

About This Issue

Paul J. Campbell
Editor

This issue of *The UMAP Journal* continues the practice inaugurated in Vol. 26.

This issue runs longer than 92 pp—in fact, it runs over 200 pp. Not all of the articles in this issue are printed in the paper copy. Some articles appear only in the *Tools for Teaching* 2006 CD-ROM (and at `http://www.comap.com` for COMAP members), which will reach members and subscribers at a later time and will also contain the entire 2006 year of *Journal* issues.

However, all articles of this issue on the CD-ROM appear in the printed table of contents and are regarded as published in the *Journal*. In addition, the abstract of each Outstanding paper appears in the printed version. Pagination of the issue runs continuously, including in sequence articles that do not appear in printed form. *So, if you notice that, say, page 350 in the printed copy is followed by page 403, your copy is not necessarily defective!* The articles corresponding to the intervening pages will be on the CD-ROM.

We hope that you find this arrangement, if not entirely satisfying, at least satisfactory. It means that we do not have to procrusteanize the content of the *Journal* to fit a fixed number of allocated pages. For example, we might otherwise need to select only two or three Outstanding MCM papers to publish (a hard task indeed!). Instead, we continue to bring you the full content as in the past.

Modeling Forum

Results of the 2006 Mathematical Contest in Modeling

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Introduction

A total of 747 teams of undergraduates, from 270 institutions and 403 departments in 12 countries, spent the first weekend in February working on applied mathematics problems in the 22nd Mathematical Contest in Modeling (MCM).

The 2006 MCM began at 8:00 P.M. EST on Thursday, February 2 and ended at 8:00 P.M. EST on Monday, February 6. During that time, teams of up to three undergraduates were to research and submit an optimal solution for one of two open-ended modeling problems. Students registered, obtained contest materials, downloaded the problems at the appropriate time, and entered completion data through COMAP's MCM Website. After a weekend of hard work, solution papers were sent to COMAP on Monday. The top papers appear in this issue of *The UMAP Journal*.

Results and winning papers from the first 21 contests were published in special issues of *Mathematical Modeling* (1985–1987) and *The UMAP Journal* (1985–2005). The 1994 volume of *Tools for Teaching*, commemorating the tenth anniversary of the contest, contains all of the 20 problems used in the first 10 years of the contest and a winning paper for each year. Limited quantities of that volume and of the special MCM issues of the *Journal* for the last few years are available from COMAP. That volume is available on COMAP's special Modeling Resource CD-ROM (`http://www.comap.com/product/?idx=613`). In addition, available from COMAP is a new volume, *The MCM at 21*, which contains all of

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the 20 problems from the second 10 years of the contest and a winning paper from each year.

This year's Problem A asked teams to develop a strategy for irrigating a field. Problem B asked teams to prepare a bid and a strategy for managing the provision of wheelchairs and escorts at airports. The 11 Outstanding solution papers are published in this issue of *The UMAP Journal*, along with commentary from problem authors, contest judges, and outside experts.

In addition to the MCM, COMAP also sponsors the Interdisciplinary Contest in Modeling (ICM) and the High School Mathematical Contest in Modeling (HiMCM). The ICM, which runs concurrently with MCM, offers a modeling problem involving concepts in operations research, information science, and interdisciplinary issues in security and safety. Results of this year's ICM are on the COMAP Website at `http://www.comap.com/undergraduate/contests`; results and Outstanding papers appeared in Vol. 26 (2005), No. 2. The HiMCM offers high school students a modeling opportunity similar to the MCM. Further details about the HiMCM are at `http://www.comap.com/highschool/contests`.

Problem A: Positioning and Moving Sprinkler Systems for Irrigation

There are a wide variety of techniques available for irrigating a field. The technologies range from advanced drip systems to periodic flooding. One of the systems that is used on smaller ranches is the use of "hand move" irrigation systems. Lightweight aluminum pipes with sprinkler heads are put in place across fields, and they are moved by hand at periodic intervals to insure that the whole field receives an adequate amount of water. This type of irrigation system is cheaper and easier to maintain than other systems. It is also flexible, allowing for use on a wide variety of fields and crops. The disadvantage is that it requires a great deal of time and effort to move and set up the equipment at regular intervals.

Given that this type of irrigation system is to be used, how can it be configured to minimize the amount of time required to irrigate a field that is 80 meters by 30 meters? For this task you are asked to find an algorithm to determine how to irrigate the rectangular field that minimizes the amount of time required by a rancher to maintain the irrigation system. One pipe set is used in the field. You should determine the number of sprinklers and the spacing between sprinklers, and you should find a schedule to move the pipes, including where to move them.

A pipe set consists of a number of pipes that can be connected together in a straight line. Each pipe has a 10-cm inner diameter with rotating spray nozzles that have a 0.6-cm inner diameter. When put together the resulting pipe is 20 meters long. At the water source, the pressure is 420 kilo-Pascals and has a flow rate of 150 liters per minute. No part of the field should receive more than 0.75 cm per hour of water, and each part of the field should receive at least 2 centimeters of water every 4 days. The total amount of water should be applied as uniformly as possible.

Problem B: Wheelchair Access at Airports

One of the frustrations with air travel is the need to fly through multiple airports, and each stop generally requires each traveler to change to a different airplane. This can be especially difficult for people who are not able to easily walk to a different flight's waiting area. One of the ways that an airline can make the transition easier is to provide a wheelchair and an escort to those people who ask for help. It is generally known well in advance which passengers require help, but it is not uncommon to receive notice when a passenger first registers at the airport. In rare instances an airline may not receive notice from a passenger until just prior to landing.

Airlines are under constant pressure to keep their costs down. Wheelchairs wear out and are expensive and require maintenance. There is also a cost for making the escorts available. Moreover, wheelchairs and their escorts must be constantly moved around the airport so that they are available to people when their flight lands. In some large airports, the time required to move across the airport is nontrivial. The wheelchairs must be stored somewhere, but space is expensive and severely limited in an airport terminal. Also, wheelchairs left in high traffic areas represent a liability risk as people try to move around them. Finally, one of the biggest costs is the cost of holding a plane if someone must wait for an escort and becomes late for their flight. The latter cost is especially troubling because it can affect the airline's average flight delay which can lead to fewer ticket sales as potential customers may choose to avoid an airline.

Epsilon Airlines has decided to ask a third party to help them obtain a detailed analysis of the issues and costs of keeping and maintaining wheelchairs and escorts available for passengers. The airline needs to find a way to schedule the movement of wheelchairs throughout each day in a cost effective way. They also need to find and define the costs for budget planning in both the short term and in the long term.

Epsilon Airlines has asked your consultant group to put together a bid to help them solve their problem. Your bid should include an overview and analysis of the situation to help them decide if you fully understand their problem. They require a detailed description of an algorithm that you would like to implement which can determine where the escorts and wheelchairs should be and how they should move throughout each day. The goal is to keep the total costs as low as possible. Your bid is one of many that the airline will consider. You must make a strong case as to why your solution is the best and show that it will be able to handle a wide range of airports under a variety of circumstances.

Your bid should also include examples of how the algorithm would work for a large (at least four concourses), a medium (at least two concourses), and a small airport (one concourse) under high and low traffic loads. You should determine all potential costs and balance their respective weights. Finally, as populations begin to include a higher percentage of older people who have more time to travel but may require more aid, your report should include projections of potential costs and needs in the future with recommendations to meet future needs.

The Results

The solution papers were coded at COMAP headquarters so that names and affiliations of the authors would be unknown to the judges. Each paper was then read preliminarily by two "triage" judges at either Appalachian State University (Irrigation Problem) or at the National Security Agency (Wheelchair Problem). At the triage stage, the summary and overall organization are the basis for judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree on a score, a third judge evaluated the paper.

This year, again an additional Regional Judging site was created at the U.S. Military Academy to support the growing number of contest submissions.

Final judging took place at Harvey Mudd College, Claremont, California. The judges classified the papers as follows:

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Irrigation Problem	6	85	132	293	516
Wheelchair Problem	5	37	56	133	231
	11	122	188	426	747

The 11 papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with commentaries. We list those teams and the Meritorious teams (and advisors) below; the list of all participating schools, advisors, and results is in the **Appendix**.

Outstanding Teams

Irrigation Papers

Institution and Advisor	Team Members
"Sprinkler Systems for Dummies: Optimizing a Hand-Moved Sprinkler System"	
Carroll College Helena, MT Mark Parker	Ben Dunham Steffan Francischetti Kyle Nixon
"Fastidious Farmer Algorithms (FFA)"	
Duke University Durham, NC William G. Mitchener	Matthew A. Fischer Brandon W. Levin Nikifor C. Bliznashki
"A Schedule for Lazy but Smart Ranchers"	
Shanghai Jiaotong University Shanghai, China Song Baorui	Wang Cheng Wen Ye Yu Yintao
"Optimization of Irrigation"	
University of California Davis, CA Sarah A. Williams	Bryan J.W. Bell Yaroslav Gelfand Simpson H. Wong
"Sprinkle, Sprinkle, Little Yard"	
University of Colorado Boulder, CO Bengt Fornberg	Brian Camley Bradley Klingenberg Pascal Getreuer
"Developing Improved Algorithms for Irrigation Systems"	
Zhejiang University of Technology Hangzhou, China Wang Shiming	Ying Yujie Jin Qiwei Zhou Kai

Wheelchair Papers

"Profit Maximizing Allocation of Wheelchairs in a Multi-Concourse Airport"
Harvard University
Cambridge, MA
Clifford H. Taubes

Christopher Yetter
Neal Gupta
Benjamin Conlee

"Minimization of Cost for Transfer" Escorts in an Airline Terminal"
Harvard University
Cambridge, MA
Michael Brenner

Elaine Angelino
Shaun Fitzgibbons
Alexander Glasser

"Application of Min-Cost Flow to Airline Accessibility Services"
Massachusetts Institute of Technology
Cambridge, MA
Martin Z. Bazant

Dan Gulotta
Daniel Kane
Andrew Spann

"Cost Minimization of Providing a Wheelchair Escort Service"
Rensselaer Polytechnic Institute
Troy, NY
Peter Roland Kramer

Matthew J. Pellicione
Michael R. Sasseville
Igor Zhitnitsky

"A Simulation-Driven Approach for a Cost-Efficient Airport Wheelchair Assistance Service"
Rice University
Houston, TX
Mark Embree

Samuel F. Feng
Tobin G. Isaac
Nan Xiao

Meritorious Teams

Irrigation Problem (85 teams)
Asbury College, Wilmore, KY (David Coulliette)
Austin Peay State University, Clarksville, TN, (Nell Rayburn)
Beijing Jiaotong University, Beijing, China (three teams) (Wang Xiaoxia) (Wang Zhouhong) (Zhang Shangli)
Beijing University of Posts and Telecommunications, Beijing, China (He Zuguo)

Bethel University, St. Paul, MN (William Kinney)

Cal-Poly Pomona University, Pomona, CA (three teams) (Ioana Mihaila) (Hubertus von Bremen) (Kurt Vandervoort, Physics)

California Polytechnic State University, San Luis Obispo, San Luis Obispo, CA (Lawrence Sze)

California State University at Monterey Bay, Seaside, CA (Hongde Hu)

Carroll College, Helena, MT (Holly Zullo)

Central South University, Changsha, Hunan, China (Qin Xuanyun)

China University of Mining and Technology, School of Computer Science and Technology, Xuzhou, Jiangsu, China (Jiang Shujuan)

Chongqing University, Dept. of Statistics and Actuarial Science, Chongqing, China Zhengmin Duan)

College of Mount St. Joseph, Cincinnati, OH (Scott Sportsman)

Columbia University, New York, NY (David Keyes)

Cornell University, Ithaca, NY (two teams) (Alexander Vladimirsky) (Shane Henderson, Operations Research and Industrial Engineering)

Dalian Nationalities Innovation College, Dalian, Liaoning, China (Rixia Bai)

Drury University, Springfield, MO (Keith Coates)

Duke University, Dept. of Computer Science, Durham, NC (Owen Astrachan)

Harvey Mudd College, Claremont, CA (two teams) (Jon Jacobsen) (Ran Libeskind-Hadas, Computer Science)

Hefei University of Technology, Hefei, Anhui, China (Xueqiao Du)

Helsingin Matematiikkalukio, Helsinki, Finland (Esa Lappi)

Humboldt State University, Dept. of Environmental Resources Engineering, Arcata, CA (Brad Finney)

Jilin University, Institute of Mechanical Sciece and Engineering, Changchun, Jilin, China (Fang Peichen)

Johns Hopkins University, Baltimore, MD (Fred Torcaso)

Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea (two teams) (Dongsup Kim, Biosystems) (Ki Hyoung Ko)

Lawrence Technological University, Southfield, MI (Ruth Favro)

Maggie Walker Governor's School, Richmond, VA (two teams) (Harold Houghton, Science) (John Barnes)

Nanchang University, Nanchang, Jiangxi, China (Chen Tao)

National University of Defense Technology, School of Humanity and Management, Changsha, Hunan, China (Wang Dan)

National University of Ireland, Galway, Ireland (Niall Madden)

Northeastern University, Dept. of Computer Science, Shenyang, Liaoning, China (Zhao Shuying Zhao)

Northwestern Polytechenical University, Dept. of Applied Physics, Xi'an, Shaanxi, China (Zhe Liu)

PLA University of Science and Technology, Institute of Command Automation, Nanjing, Jiangsu, China (Liu Shousheng)

Rensselaer Polytechnic Institute, Troy, NY (Peter Kramer)

Rowan University, Glassboro, NJ (Hieu Nguyen) (two teams)

Shanghai Foreign Language School, Shanghai, China (Pan Liqun)

Shanghai Jiaotong University, Shanghai, China (Song Baorui)

Shanghai Jiaotong University, Minhang Branch, Shanghai, China (Huang Jianguo)

Slippery Rock University, Slippery Rock, PA (Richard Marchand)

South-China Normal University, Dept. of Probability and Statistics, Guangzhou, Guangdong, China (Zhang Shaohui)
Southern Connecticut State University, New Haven, CT (Therese Bennett)
Sun Yat-sen University, Dept. of Computer Science, Guangzhou, Guangdong, China (Liu Xiaoming)
Truman State University, Kirksville, MO (Steve Smith)
Tsinghua University, Beijing, China (Zhiming Hu)
University of Alaska Fairbanks, Dept. of Computer Science, Fairbanks, AK (Orion Lawlor)
University College Cork, Cork, Ireland (three teams) (Dmitrii Rachinskii) (Andrew Usher) (James Grannell, Applied Mathematics)
University of California, Berkeley, CA (Nicolai Reshetikhin)
University of Colorado, Boulder, CO (Anne Dougherty)
University of Colorado at Denver, Denver, CO (Gary Olson)
University of Delaware, Newark, DE (Louis Rossi)
University of Helsinki, Helsinki, Finland (Petri Ola)
University of Massachusetts Lowell, Lowell, MA (James Graham-Eagle)
University of Oxford, Oxford, United Kingdom, (Jeffrey Giansiracusa)
University of San Diego, San Diego, CA (Diane Hoffoss) (two teams)
University of Saskatchewan, Saskatoon, SK, Canada (James Brooke)
University of Science and Technology of China, Hefei, Anhui, China (two teams) (Meng Qiang) (Yang Zhouwang)
University of Stellenbosch, Stellenbosch, Western Cape, South Africa (Jan van Vuuren)
University of West Georgia, Carrollton, GA (Scott Gordon)
Victoria University of Wellington, Wellington, New Zealand (Mark McGuinness)
Wake Forest University, Winston Salem, NC (two teams) (Edward Allen)
Western Washington University, Bellingham, WA (Tjalling Ypma)
Xi'an Communication Institute, School of Information, Xi'an, Shaanxi, China (two teams) (Zhang Jianhang Zhang) (Kang Jinlong)
Xi'an Communication Institute School of Science, Xi'an, Shaanxi, China (two teams) (Yang Dongsheng) (Li Guo)
Xi'an Jiaotong University, Xi'an, Shaanxi, China (Dai Yonghong)
Xidian University, Xi'an, Shaanxi, China (two teams) (Feng Hailin) (Li Wei)
Youngstown State University, Youngstown, OH (Angela Spalsbury)
Zhejiang University, Hangzhou, Zhejiang, China (Yang Qifan)
Zhejiang University City College, Hangzhou, Zhejiang, China (Zhang Huizeng)

Wheelchair Problem (37 teams)
Beijing University of Posts and Telecommunications, Dept. of Applied Physics, Beijing, China (Ding Jinkou)
Central University of Finance and Economics, Beijing, China (Fan Xiaoming)
Central Washington University, Ellensburg, WA (Stuart, Boersma)
Davidson College, Davidson, NC (three teams) (Timothy Chartier, two teams) (Mark Foley, Economics)
Duke University, Durham, NC (two teams) (William Mitchener) (Owen Astrachan, Computer Science)
Eastern Oregon University, La Grande, OR (David Allen)
Harbin Institute of Technology, Harbin, Heilongjiang, China (Jiao Guanghong)
Harvey Mudd College, Dept. of Computer Science, Claremont, CA (Ran Libeskind-Hadas)

Humboldt State University Dept. of Environmental Resources Engineering, Arcata, CA (Brad Finney)
Lewis and Clark College, Portland, OR (Liz Stanhope)
Maggie Walker Governor's School, Richmond, VA (John Barnes)
Minhang Branch of Shanghai Jiaotong University, Shanghai, China (Huang Jianguo)
Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu, China (Kong Gaohua)
Nanjing University of Science and Technology, Dept. of Statistics, Nanjing, Jiangsu, China (Liu Liwei)
Northern Kentucky University, Highland Heights, KY (Gail Mackin)
Päivölä College, Tarttila, Finland (Janne Puustelli)
Regis University, Denver, CO (Jim Seibert)
Shanghai Jiaotong University, Shanghai, China (Zhou Gang)
Sichuan University, Dept. of Statistics, Chengdu, Sichuan, China (Zhou Jie)
Simpson College, Indianola, IA (Murphy Waggoner)
South China University of Technology, Guangzhou, Guangdong, China (Quan Liu)
Southwest University of Finance and Economics, Dept. of Economic Mathematics, Chengdu, Sichuan, China (Sun Jiangming)
Tsinghua University, Beijing, China (Chi Chi Hung)
University of Alaska Fairbanks, Dept. of Computer Science, AK (Orion Lawlor)
University of California, Davis, CA (Sarah, Williams)
University of Colorado at Boulder, Dept. of Physics, Boulder, CO (Michael Ritzwoller)
University of Electronic Science and Technology of China, Dept. of Information and Computation Science, Chengdu, Sichuan, China (Xu Quanzi)
University of Guangxi, Dept. of Information Science, Nanking, Guangxi, China (Wang Xing)
University of South Florida Dept. of Industrial and Management Systems Engineering, Tampa, FL (Nan Kong)
University of Washington, Seattle, WA (two teams) (Anne Greenbaum) (James Morrow)
Wake Forest University Dept. of Economics, Winston Salem, NC (Claire Hammond)
Xi'an Jiaotong University, Xi'an, Shaanxi, China (He Xiaoliang)
Xidian University, Xi'an, Shaanxi, China (Zhu Qiang)

Awards and Contributions

Each participating MCM advisor and team member received a certificate signed by the Contest Director and the appropriate Head Judge.

INFORMS, the Institute for Operations Research and the Management Sciences, recognized the teams from Duke University (Irrigation Problem) and Massachusetts Institute of Technology (Wheelchair Problem) as INFORMS Outstanding teams and provided the following recognition:

- a letter of congratulations from the current president of INFORMS to each team member and to the faculty advisor;
- a check in the amount of $300 to each team member;

- a bronze plaque for display at the team's institution, commemorating their achievement;
- individual certificates for team members and faculty advisor as a personal commemoration of this achievement;
- a one-year student membership in INFORMS for each team member, which includes their choice of a professional journal plus the *OR/MS Today* periodical and the INFORMS society newsletter.

The Society for Industrial and Applied Mathematics (SIAM) designated one Outstanding team from each problem as a SIAM Winner. The teams were from University of Colorado (Irrigation Problem) and Harvard University (team of Christopher Yetter, Neal Gupta and Benjamin Conlee; advisor Clifford H. Taubes) (Wheelchair Problem). Each of the team members was awarded a $300 cash prize and the teams received partial expenses to present their results in a special Minisymposium at the SIAM Annual Meeting in Boston, MA in July. Their schools were given a framed hand-lettered certificate in gold leaf.

The Mathematical Association of America (MAA) designated one Outstanding team from each problem as an MAA Winner. The teams were from University of Colorado (Irrigation Problem) and Rice University (Wheelchair Problem). With partial travel support from the MAA, the Rice University team presented their solution at a special session of the MAA Mathfest in Knoxville, TN in August. Each team member was presented a certificate by Richard S. Neal of the MAA Committee on Undergraduate Student Activities and Chapters.

Ben Fusaro Award

One Meritorious paper was selected for each problem for the Ben Fusaro Award, named for the Founding Director of the MCM and awarded for the second time this year. It recognizes an especially creative approach; details concerning the award, its judging, and Ben Fusaro are in *The UMAP Journal* 25 (3) (2004): 195–196. The Ben Fusaro Award winners were from Shangahai Jiaotong University (Shanghai, China) (Irrigation Problem) and the Maggie L. Walker Governor's School (Richmond, VA) (Wheelchair Problem).

Judging

Director
Frank R. Giordano, Naval Postgraduate School, Monterey, CA

Associate Directors
Robert L. Borrelli, Mathematics Dept., Harvey Mudd College, Claremont, CA
Patrick J. Driscoll, Dept. of Systems Engineering, U.S. Military Academy, West Point, NY
William P. Fox, Mathematics Dept., Francis Marion University, Florence, SC

Irrigation Problem

Head Judge
Marvin S. Keener, Executive Vice-President, Oklahoma State University, Stillwater, OK (MAA)

Associate Judges
William C. Bauldry, Chair, Dept. of Mathematical Sciences, Appalachian State University, Boone, NC (Triage)
Kelly Black, Mathematics Dept., Union College, Schenectady, NY
Steve Horton (MAA), Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY
Mario Juncosa, RAND Corporation, Santa Monica, CA (retired)
Michael Moody, Olin College of Engineering, Needham, MA
David H. Olwell (INFORMS), Naval Postgraduate School, Monterey, CA
John L. Scharf, Mathematics Dept., Carroll College, Helena, MT
Richard Douglas West, Francis Marion University, Florence, SC (Ben Fusaro Award)
Daniel Zwillinger, Newton, MA (SIAM)

Wheelchair Problem

Head Judge
Maynard Thompson, Mathematics Dept., University of Indiana, Bloomington, IN

Associate Judges
Peter Anspach, National Security Agency, Ft. Meade, MD (Triage)
Karen D. Bolinger, Mathematics Dept., Clarion University of Pennsylvania, Clarion, PA
Jim Case (SIAM)
Lisette de Pillis, Mathematics Dept., Harvey Mudd College, Claremont, CA
J. Douglas Faires, Youngstown State University, Youngstown, OH
Jerry Griggs, Mathematics Dept., University of South Carolina, Columbia, SC
Veena Mendiratta, Lucent Technologies, Naperville, IL

Kathleen M. Shannon, Dept. of Mathematics and Computer Science,
Salisbury University, Salisbury, MD (MAA)
Dan Solow, Mathematics Dept., Case Western Reserve University,
Cleveland, OH (INFORMS)
Michael Tortorella, Dept. of Industrial and Systems Engineering,
Rutgers University, Piscataway, NJ
Marie Vanisko, Dept. of Mathematics, California State University—Stanislaus,
Turlock, CA (Ben Fusaro Award)

Regional Judging Session

Head Judge
Patrick J. Driscoll, Dept. of Systems Engineering, United States Military Academy
(USMA), West Point, NY

Associate Judges
Merrill Blackman, Dept. of Systems Engineering, USMA
Darrall Henderson, Dept. of Mathematical Sciences, USMA
Michael Jaye, Dept. of Mathematical Sciences, USMA
John Kobza, Dept. of Industrial and Systems Engineering,
Texas Tech University, Lubbock, TX
Ed Pohl, Dept. of Industrial and Systems Engineering, University of Arkansas,
Fayetteville, AR

Triage Sessions:

Irrigation Problem

Head Triage Judge
William C. Bauldry, Chair, Dept. of Mathematical Sciences,
Appalachian State University, Boone, NC

Associate Judges
Mark Ginn,
Jeff Hirst,
Rick Klima,
and
Vicky Klima
—all from Dept. of Math'l Sciences, Appalachian State University, Boone, NC

Wheelchair Problem

Head Triage Judge
Peter Anspach, National Security Agency (NSA), Ft. Meade, MD

Associate Judges
Stewart Saphier, Dept. of Defense, Washington, DC
Dean McCullough, High Performance Technologies, Inc.
Craig Orr, NSA
and other members of NSA.

Sources of the Problems

Both problems were contributed by Kelly Black (Mathematics Dept., University of New Hampshire, Durham, NH).

Acknowledgments

Major funding for the MCM is provided by the National Security Agency and by COMAP. We thank Dr. Gene Berg of NSA for his coordinating efforts. Additional support is provided by the Institute for Operations Research and the Management Sciences (INFORMS), the Society for Industrial and Applied Mathematics (SIAM), and the Mathematical Association of America (MAA). We are indebted to these organizations for providing judges and prizes.

We also thank for their involvement and support:

- IBM Business Consulting Services, Center for Business Optimization; and
- Two Sigma Investments. (This group of experienced, analytical, and technical financial professionals based in New York builds and operates sophisticated quantitative trading strategies for domestic and international markets. The firm is successfully managing several billion dollars using highly automated trading technologies. For more information about Two Sigma, please visit `http://www.twosigma.com`.)

We thank the MCM judges and MCM Board members for their valuable and unflagging efforts. Harvey Mudd College, its Mathematics Dept. staff, and Prof. Borrelli were gracious hosts to the judges.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the student papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Editing (and sometimes substantial cutting) has taken place: Minor errors have been corrected, wording has been altered for clarity or economy, and style has been adjusted to that of *The UMAP Journal*. Please peruse these student efforts in that context.

To the potential MCM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

COMAP's Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling are the only international modeling contests in which students work in teams. Centering its educational philosophy on mathematical modeling, COMAP uses mathematical tools to explore real-world problems. It serves the educational community as well as the world of work by preparing students to become better-informed and better-prepared citizens.

Appendix: Successful Participants

KEY:
P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

INSTITUTION	CITY	ADVISOR	A	B
ALASKA				
U. Alaska Fairbanks (CS)	Fairbanks	Orion Lawlor	M	M
ARIZONA				
University of Arizona	Tucson	Bruce Bayly		H
CALIFORNIA				
Cal Poly Pomona	Pomona	Ioana Mihaila	M	
		Hubertus von Bremen	M	
Cal-Poly Pomona U. (Phys)	Pomona	Kurt Vandervoort	M	
Calif. Polytechnic State Univ.	San Luis Obispo	Lawrence Sze	M	
Cal. State U. at Monterey Bay	Seaside	Hongde Hu	M	
Cal. State Univ., Bakersfield	Bakersfield	Maureen Rush		P
Cal. State Univ., Stanislaus	Turlock	Brian Jue	P	
Harvey Mudd College	Claremont	Jon Jacobsen	M,H	
(CS)		Ran Libeskind-Hadas	M	M
Humboldt State U. (Env Eng)	Arcata	Brad Finney	M	M
University of California	Berkeley	Nicolai Reshetikhin	M	
	Davis	Sarah Williams	O	M
University of San Diego	San Diego	Diane Hoffoss	M,M	
COLORADO				
Regis University	Denver	Jim Seibert		M
University of Colorado	Boulder	Anne Dougherty	M	
		Bengt Fornberg	O,O	
(Phys)		Michael Ritzwoller	H	M
	Colorado Springs	Radu Cascaval		P
	Denver	Gary Olson	M	
CONNECTICUT				
Sacred Heart University	Fairfield	Peter Loth	H	
Southern Conn. State U.	New Haven	Therese Bennett	M	
DELAWARE				
University of Delaware	Newark	Louis Rossi	M	
DISTRICT OF COLUMBIA				
George Washington Univ.	Washington	Daniel Ullman		H

INSTITUTION	CITY	ADVISOR	A	B
FLORIDA				
Embry-Riddle University	Daytona Beach	Greg Spradlin	H	
Jacksonville University	Jacksonville	Robert Hollister	H P	
University of South Florida	Tampa	Brian Curtin	P	
(Ind'l & Mgmnt Sys Eng)		Nan Kong		M
GEORGIA				
University of West Georgia	Carrollton	Scott Gordon	M,H	
Wesleyan College	Macon	Joseph Iskra		H,P
(Chem & Phys)		Charles Benesh		P
ILLINOIS				
Greenville College	Greenville	George Peters	H,H	
Wheaton College	Wheaton	Paul Isihara	H	H
INDIANA				
Franklin College	Franklin	John Boardman	P	
Goshen College	Goshen	Charles Crane	P	
Indiana Univ. South Bend	South Bend	Morteza Shafii-Mousavi	P	
Rose-Hulman Inst. of Tech.	Terre Haute	David Rader	H	P
Saint Mary's College	Notre Dame	Joanne Snow	H,P	
IOWA				
Grand View College	Des Moines	Sergio Loch	P	
Grinnell College	Grinnell	Karen Shuman	H,H	
Iowa State University	Ames	Stephen Willson	P	
Luther College	Decorah	Reginald Laursen	H,H	
		Steve Hubbard		P
Mt. Mercy College	Cedar Rapids	K. Knopp		H
Simpson College	Indianola	Murphy Waggoner	P	M
(Bio)		Jeff Parmelee	H,P	
KANSAS				
Benedictine College	Atchison	Linda Herndon	H	
Kansas State University	Manhattan	Dave Auckly		P
KENTUCKY				
Asbury College	Wilmore	David Coulliette	M,H	
Northern Kentucky Univ.	Highland Heights	Gail Mackin	H	M
(Phys & Geology)		Sharmanthie Fernando	P	
MAINE				
Colby College	Waterville	Paul Cohen	H	P

INSTITUTION	CITY	ADVISOR	A	B
MARYLAND				
Johns Hopkins University	Baltimore	Fred Torcaso	M,H	
Loyola College	Baltimore	Christos Xenophontos	H	P
Mount St. Mary's University	Emmitsburg	Fred Portier	P	
Salisbury University	Salisbury	Divya Devadoss		P
Towson University	Towson	Michael O'Leary	P	
MASSACHUSETTS				
Emmanuel College	Boston	Matthew Tom		P
Harvard University	Cambridge	Clifford Taubes		O
(Eng)		Michael Brenner		O
Masschusetts Institute of Tech.	Cambridge	Martin Bazant	P	O
(Phys)		Leonid Levitov		H
Simon's Rock College	Great Barrington	Allen Altman	P	P
(Phys)		Michael Bergman	H,P	
Smith College	Northampton	Ruth Haas		P
Univ. of Massachusetts Lowell	Lowell	James Graham-Eagle	M	
Western New England College	Springfield	Lorna Hanes	P	
Worcester Polytechnic Institute	Worcester	Suzanne Weekes	H	P
(CS)		Stanley Selkow	H	P
MICHIGAN				
Albion College	Albion	Darren Mason	H	
Lawrence Technological Univ.	Southfield	Ruth Favro	M,P	
MISSOURI				
Drury University	Springfield	Keith Coates	M	P
(Phys)		Bruce Callen	H,P	
Saint Louis University	St. Louis	David Jackson	P	
(Aero & Mech Eng)		Sanjay Jayaram	H	
Southeast Missouri State Univ.	Cape Girardeau	Robert Sheets	H	
Truman State University	Kirksville	Steve Smith	M	P
MINNESOTA				
Bethel University	St. Paul	William Kinney	M	
Coll. of St. Benedict / St. John's U.	Collegeville	Robert Hesse	P	P
Macalester College	St. Paul	Daniel Kaplan		P
Minnesota State University	Moorhead	Ellen Hill	P,P	
MONTANA				
Carroll College	Helena	Holly Zullo	M	
		Mark Parker	O	
(Chem)		Dawn Bregel	H	
University of Montana	Missoula	George McRae		H

INSTITUTION	CITY	ADVISOR	A	B
NEBRASKA				
Nebraska Wesleyan University	Lincoln	Kristin Pfabe	P	
NEW JERSEY				
Rowan University	Glassboro	Hieu Nguyen	M,M	
NEW MEXICO				
NM Inst. of Mining and Tech.	Socorro	John Starrett	H	
New Mexico State University	Las Cruces	Mary Ballyk	P	
		Tiziana Giorgi	P	
NEW YORK				
Clarkson University	Potsdam	Kathleen Fowler		P,P
Columbia University	New York	Peter Bank	H	
(Appl Phys & Appl Math)		David Keyes	M,P	
Concordia College	Bronxville	John Loase		P
Cornell University	Ithaca	Alexander Vladimirsky	M	P
(Operations Res & Ind'l Eng)		Shane Henderson	M,P	
Hobart and William Smith Colls.	Geneva	Scotty Orr	H,P	
(Geoscience)		Tara Curtin	H	
Iona College	New Rochelle	Srilal Krishnan	H	
Ithaca College (CS)	Ithaca	Ali Erkan		P
(Phys)		Bruce Thompson	H	
Nazareth College	Rochester	Daniel Birmajer		P
Rensselaer Polytechnic Institute	Troy	Peter Kramer	M	O
(Chem & Bio Eng)		Shekhar Garde	H	
Roberts Wesleyan College	Rochester	Gary Raduns	P,P	
Union College	Schenectady	Peter Otto	H,H	
United States Military Acad.	West Point	Joseph Lindquist		P
		Kerry Moores	H	
Westchester Community Coll.	Valhalla	Marvin Littman	P,P	
NORTH CAROLINA				
Appalachian State University	Boone	Eric Marland	H,P	
Brevard College	Brevard	Clarke Wellborn		P
Davidson College	Davidson	Timothy Chartier		M,M
(Econ)		Mark Foley		M,P
Duke University	Durham	William Mitchener	O	M
(CS)		Owen Astrachan	M	M
Meredith College	Raleigh	Cammey Cole	P,P	
NC School of Sci. & Math	Durham	Daniel Teague	H	P
Wake Forest University	Winston Salem	Edward Allen	M,M	
(CS)		David John	P	
(Econ)		Claire Hammond		M
Western Carolina University	Cullowhee	Erin McNelis	H	

INSTITUTION	CITY	ADVISOR	A	B
OHIO				
Bowling Green State University	Bowling Green	Juan Bes	H,P	
College of Mount St. Joseph	Cincinnati	Scott Sportsman	M	
Malone College	Canton	David Hahn	H	
Xavier University	Cincinnati	Michael Goldweber		P,P
Youngstown State University	Youngstown	Angela Spalsbury	M,H	
(Civil Eng)		Scott Martin		H
OKLAHOMA				
Oklahoma State University	Stillwater	Lisa Mantini	H	P
OREGON				
Eastern Oregon University	La Grande	David Allen		M
Lewis & Clark College	Portland	Liz Stanhope		M
Linfield College	McMinnville	Jennifer Nordstrom	H	P
(CS)		Daniel Ford	H	H
Pacific University	Forest Grove	John August	H	
		Nancy Neudauer		H
(Phys)		James Butler	H	
Southern Oregon University	Ashland	Kemble Yates	H	
Willamette University	Salem	Inga Johnson	P	
PENNSYLVANIA				
Bloomsburg University	Bloomsburg	Kevin Ferland	H,P	
Chatham College	Pittsburgh	Japheth Wood	P,P	
Gannon University	Erie	Michael Caulfield	P,P	
Gettysburg College (Phys)	Gettysburg	Sharon Stephenson	H	
Juniata College	Huntingdon	John Bukowski		P
Lafayette College	Easton	Ethan Berkove		P
Slippery Rock University	Slippery Rock	Richard Marchand	M	
University of Pittsburgh	Pittsburgh	Jonathan Rubin	H	P
Westminster College (CS)	New Wilmington	Barbara Faires		P
SOUTH CAROLINA				
College of Charleston	Charleston	Amy Langville	P	P
Midlands Technical College	Columbia	John Long	H,P	
University of South Carolina	Columbia	Lili Ju		H
SOUTH DAKOTA				
South Dakota School of Mines & Tech.	Rapid City	Kyle Riley	H	
TENNESSEE				
Austin Peay State University	Clarksville	Nell Rayburn	M	
Tennessee Technological University	Cookeville	Andrew Hetzel		P
(CS)	Cookeville	Martha Kosa	H	

INSTITUTION	CITY	ADVISOR	A	B
TEXAS				
Angelo State University	San Angelo	Karl Havlak	P	P
Rice University	Houston	Mark Embree		O
UTAH				
University of Utah	Salt Lake City	Don Tucker	P	
VIRGINIA				
James Madison University	Harrisonburg	Caroline Smith	P	
Longwood University	Farmville	M. Leigh Lunsford	H	
Maggie Walker Governor's School	Richmond	John Barnes	M	M
(Sci)		Harold Houghton	M	
Radford University	Radford	Laura Spielman	P	
Randolph-Macon College	Ashland	Bruce Torrence	P	
University of Richmond	Richmond	Kathy Hoke	P	
Virginia Western Community Coll.	Roanoke	Steve Hammer	H	
		Ruth Sherman	P	
WASHINGTON				
Central Washington University	Ellensburg	Stuart Boersma		M
Heritage University	Toppenish	Richard Swearingen	H	
Pacific Lutheran University	Tacoma	Jeffrey Stuart	H	P
University of Puget Sound	Tacoma	DeWayne Derryberry		P
University of Washington	Seattle	James Morrow	H	M
(Appl Comp'l Math'l Sci)		Anne Greenbaum	H	M
Western Washington University	Bellingham	Tjalling Ypma	M	
WISCONSIN				
Beloit College	Beloit	Paul J. Campbell	H	
St. Norbert College	De Pere	John Frohliger	P	
University of Wisconsin	River Falls	Kathy Tomlinson	H	
Edgewood College	Madison	Steven Post		P
AUSTRALIA				
University of New South Wales	Sydney	James Franklin		P
Univ. of Southern Queensland	Toowoomba	Dmitry Strunin	A	H
CANADA				
Dalhousie University	Halifax	Dorothea Pronk	P	
University of Western Ontario	London	Allan MacIsaac	H	
York University	Toronto	Hongmei Zhu		H
		Huaiping Zhu	P	
Queen's University	Kingston	David Steinsaltz		P
McGill University	Montreal	Nilima Nigam	H,P	
		Nilima	Nigam	P
University of Saskatchewan	Saskatoon	James Brooke	M	P

INSTITUTION	CITY	ADVISOR	A	B
CHINA				
Anhui				
Anhui University	Hefei	Wang Xuejun	P	
(Info)		Wang Jian	P	
		Chen Mingsheng	P	
(Stat)		Xiang Junping	P	
Hefei University of Technology	Hefei	Su Huaming	P	
		Du Xueqiao	M	
(Comp'l Math)		Zhou Yongwu	P	
		Huang Youdu		P
University of Science and Technology of China	Hefei	Huang Chuan		P
		Yang Zhouwang	M	
(Automation)		Meng Qiang	M	
Beijing				
BeiHang University (BHU) (Eng)	Beijing	Sun HaiYan	P	
(Sci)		Liu Hongying	P	
(CS)		Wu Sanxing	P	
		Li ShangZhi	P,P	
Beijing Forestry University	Beijing	Gao Mengning	P	
		Li Hongjun	P,P	
(Bio)		Gao Ning	P	
Beijing Institute of Technology	Beijing	Wang Hongzhou		P
		Ren Qun	P	
		Yan Xiaoxia	H	
		Li Xuewen	P	
		Yan Guifeng	P	
Beijing Jiaotong University	Beijing	Wang Xiaoxia	M	
		Wang Zhouhong	M,P	
		Feng Guochen	P	
		Zhang Shangli	M	
		Yu Yongguang	P	P
(Comm. Eng)		Liu Xiao	P	
(Info)		Fan Bingli	P	
		Yu Jiaxin		P
Beijing Language and Culture Univ. (Accounting)	Beijing	Xun Endong	H	
(Finance)		Song Rou		H
(Info)		Zhao Xiaoxia	H,H	
Beijing Normal University	Beijing	Cui Hengjian		H
		Huang Haiyang	P	
		Qing He		H,P
		Liu Laifu		H,P
(Phys)		Peng Fanglin)		H
(Psych)		Lin Danhua	H	

INSTITUTION	CITY	ADVISOR	A	B
Beijing University of Chemical Tech. (Sci)	Beijing	Liu Damin	P	
		Liu Hui	P	
(Chem. Eng.)		Jiang Guangfeng		H
		Huang Jinyang	P	
Beijing University of Posts and Telecomm.	Beijing	Yuan Jianhua		P
(Info)		Zhang Wenbo	P	
		He Zuguo	M,H	
		Hongxiang	Sun	H,H
(CS)		Wang Xiaoxia		H
(Phys)		Ding Jinkou		M
Beijing University of Technology	Beijing	Xue Yi	H	
Beijing Wuzi University	Beijing	Li Zhenping		P,P
Beijing Wuzi Xueyuan (Jichubu)	Beijing	Cheng Xiaohong	P	P
Central Univ. of Finance and Economics	Beijing	Li Donghong		P
		Yin Xianjun	H	
		Fan Xiaoming		M
		Yin Zhao	Yin	P
China Agricultural University	Beijing	Liu Junfeng		P,P
China University of Geosciences (Info)	Beijing	Yan Deng	H,P	
(Eng)		Huang Guangdong	P,P	
College of Info. Sci. and Tech.	Beijing	Shen Fuxing		P
North China Electr. Power U. (Automation)	Beijing	Wen Tan	P	
Peking University	Beijing	Deng Minghua	P	P
		Liu Xufeng	P	P
		Liu Yulong	H	
(CS)	Beijing	Tang Huazhong	P	
(Financial Math)		Wu Lan	P	
(Sci Comp)		Wang Ming	H,P	
Renmin University of China	Beijing	Han Litao	P,P	
(Info)		Yang Yunyan	P	
School of Computer and Information Tech.	Beijing	Ren Liwei		P
Tsinghua University	Beijing	Ye Jun	P,P	
		Hu Zhiming	M,P	
(Sftwr)		Chi Chi Hung		M
Chongqin				
Chongqin University	Chongqing	He Renbin	P	
(Info)		Yang Dadi	P	
(Appl. Sftwr)		Li Fu		H
(Stat)		Rong Tengzhong	P	
		Duan Zhengmin	M	
Fujian				
Fujian Normal Univ	Fuzhou	Zhang Shenggui	P	

INSTITUTION	CITY	ADVISOR	A	B
Xiamen University	Xiamen	Zhong Tan	P,P	
(Life Sci)		Long Minnan		P
Gansu				
Lanzhou Commercial College (Info)	Lanzhou	Li Bode	P	
Guangdong				
Guangzhou University	Guangzhou	Fu Lin	P	
		Shang Dong	P	
		Xiong Jian		P
		Zhong Bin	P	
Jinan University	Guangzhou	Hu Daiqiang		H
(CS)		Zhang Chuanlin	P	
		Luo Shizhuang	P	
(Electronics)		Ye Shiqi	P	
South-China Normal Univ. (Comp Apps)	Guangzhou	Li Hunan	H	
(CS)		Wang Henggeng	H,H	
(Stat)		Zhang Shaohui	M	H
South China University of Technology	Guangzhou	Liu Quan	P	M
(Electr. Power)	Guangzhou	Qin Yong An	P	
(CS)		Tao Zhi Sui	P	
Sun Yat-sen University	Guangzhou	Feng Guocan	P	
(CS)		Liu Xiaoming	M	
(Phys)		Bao Yun		H
(Geography)		Yuan ZuoJian		P
Guangxi				
University of Guangxi (Op'ns Research)	Nanning	Wu Ru	P,P	
(Info)		Wang Xing		M,P
Guizhou				
Guizhou University for Nationalities	Guiyang	Suo Hongmin		P
		Hong Zhensheng	P	
Hebei				
Hebei Polytechnic University	Tangshan	Liu Baoxiang	P	
		Jin Dianchuan		P
		Meng Junbo	P	
North China Electric Power University	Baoding	Gu Gendai	H	
		Shi Huifeng		P
		Zhang Po		P
		Zhang Yagang	P	
(Electr. Eng.)		Wang Shenghua	H	
Heilongjiang				
Biomedical Research Institute	Harbin	Wang Qi		P

INSTITUTION	CITY	ADVISOR	A	B
Daqing Petroleum Institute	Daqing	Zhang Chang	P	
		Kong Ling	P,P	
		Yang Yun	P	
Harbin Engineering University	Harbin	Zhu Lei	P	
		Yu Tao	P	
(Comp'l Math)	Harbin	Yu Fei	P,P	
(Phys)		Shen Jihong	P,P	
		Zhang Xiaowei	P,P	
(Sci)	Harbin	Luo Yuesheng	P	
		Gao Zhenbin	P,P	
		Hong Ge	P	
Harbin Institute of Technology	Harbin	Liu Kean	P	
		Shang Shouting	H,P	
		Wang Xilian	P,P	
		Jiao Guanghong		M
		Zhang Chiping		H
Harbin Medical University (Bioinformatics)	Harbin	Liu SaLi	P	P
		Wang QiangHu	P	H
Harbin Normal University (Info)	Harbin	Yao Huanmin	P	
Harbin University of Science and Technology	Harbin	Chen Dongyan	P	
		Tian Guangyue	P,P	
Jia-Musi University	Jia-Musi	Wen Bin	P	
		Zhang Hong	P,P	
(Dean's Ofc)	Jia-Musi	Fan Wei	P	P
Northeast Agri. U. of China, Chengdong Inst.	Harbin	Zhang YaZhuo	P	
(Accounting)		Zhang YaZhuo	P	
Hubei				
Huazhong Univ. of Sci. and Tech. (Ind'l Eng.)	Wuhan	Gao Liang	P	
		Dong Yan		P
Wuhan University	Wuhan	Zhong Liuyi	H	
		Ming Hu	P	
		Hu Mingyuan	P	
		Chu Luo	H	
		Hu Xingqi		H
(Remote Sensing)	Wuhan	Hu Xinqi		P
		Luo Chu		P
(Electr. Eng.)	Wuhan	Hu Xinqi	P	
		Hu Yuan		P
(Info)	Zhong Liuyi	P		
(River Eng.)		Hu Yuanming	P,P	
(Banking & Financial Math)		Luo Zhuangchu	P	P
(Dynamics and Machinery)		Zhong Liu		P
(Mech. Eng.)		Qi Huxin	P	
(Stat)		Zhong Liuyi		P

INSTITUTION	CITY	ADVISOR	A	B
Wuhan University of Technology	Wuhan	Chen Jianye	P	
		Chu Yangjie		P
		He Lang	P	
		Huang Xiaowei	P	
		Huang Zhangcan	P	
		Liu Yang	P	
		Zhu Huaping		P,P
(Stat)		Li Yuguang	P	
		Mao Shuhua		P
Hunan				
Central South University	Changsha	Qin Xuanyun	M	H
(Traffic & Transp Eng)		Yi Laoshi	P	
(Civil Eng & Arch)		Yi Kunnan	P	
Railway Campus (Eng)	Changsha	Yi Kunnan	P	
Changsha University of Sci. and Technology	Changsha	Tong QingShan	P	
Hunan University (Info)	Changsha	Ma Chuanxiu	P	
(Stat)		Wang Liping	P,P	
National University of Defense Technology	Changsha	M.D. Wu	P,P	
(CS)		Xiong Yueshan		P
(Mgmnt Sci & Eng)		Wang Dan	M,P	
(Sys Sci)		Mao Ziyang	H	
Inner Mongolia				
Inner Mongolia University	Hohhot	Han Hai	P	
		Wang Mei	P	
		Ma Zhuang	P	
Jiangsu				
China University of Ming and Technology	Xuzhou	Zhang Xingyong	P	
(Ed Admin)		Wu Zongxiang	P	
Jiang Su University	Zhenjiang	Yang Honglin	P	P
		Gui Guilong	P,P	
		Xu Gang	P	
(Sys Eng)		Li Yimin	P,P	
Nanjing University	Nanjing	Huang Weihua	H	P
		Yao Tianxing	P	P
(Chem)		Duan Chunying	P	
(Phys)		Gao Jian	P	
(CS)		Li Ning	P	
(Finance)		Lin Hui		H
Nanjing Univ. of Sci. & Tech.	Nanjing	Xu Chungen		P
		Wang Pinling	P	
(Stat)		Liu Liwei	P	M

INSTITUTION	CITY	ADVISOR	A	B
Nanjing University of Posts and Telecomm.	Nanjing	Kong Gaohua		M
		Qiu Zhonghua	P	
		Xu LiWei	P	
PLA U. of Sci. and Tech. (Command Auto.)	Nanjing	Liu Shousheng	M	
(Eng)		Shen Jinren	P	
		Teng Jiajun		P
(Meteorology)		Qin Zheng		P
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		Dong Xiangyu	P	
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		Zhao Lizhong	H	P
		He Mingfeng	H,P	
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Software College (Sftwr Eng)		Xu Jianzhong	H	
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University of Stellenbosch	Stellenbosch	Jan van Vuuren	M,H	
UNITED KINGDOM				
University of Oxford	Oxford	Jeffrey Giansiracusa	M ,P	

Abbreviations for Organizational Unit Types (in parentheses in the listings)

(none)	Mathematics	M; Pure M; Applied M; Computing M; M and Computer Science; M and Computational Science; M and Information Science; M and Statistics; M, Computer Science, and Statistics; M, Computer Science, and Physics; Mathematical Sciences; Applied Mathematical and Computational Sciences; Natural Science and M; M and Systems Science; Applied M and Physics
Bio	Biology	B; B Science and Biotechnology; Biomathematics; Life Sciences
Chm	Chemistry	C; Applied C; C and Physics; C, Chemical Engineering, and Applied C
CS	Computer	C Science; C and Computing Science; C Science and Technology; C Science and (Software) Engineering; Software; Software Engineering; Artificial Intelligence; Automation; Computing Machinery; Science and Technology of Computers
Econ	Economics	E; E Mathematics; Financial Mathematics; Financial Mathematics and Statistics; Management; Business Management; Management Science and Engineering
Eng	Engineering	Civil E; Electrical Eng; Electronic E; Electrical and Computer E; Electrical E and Information Science; Electrical E and Systems E; Communications E; Civil, Environmental, and Chemical E; Propulsion E; Machinery and E; Control Science and E; Mechanisms; Operations Research and Industrial E; Automatic Control
Info	Information	I Science; I and Computation(al) Science; I and Calculation Science; I Science and Computation; I and Computer Science; I and Probability; I and Computing Science; I Engineering; Computer and I Technology; Computer and I Engineering; I and Optoelectronic Science and Engineering
Phys	Physics	P; Applied P; Mathematical P; Modern P; P and Engineering P; P and Geology; Mechanics; Electronics
Sci	Science	S; Natural S; Applied S; Integrated S
Stat	Statistics	S; S and Finance; Mathematical S; Probablity and S

EDITOR'S NOTE: For team advisors from China, I have endeavored to list family name first, and I thank Jie Fu (Beloit College '10) for her help in this regard.

Sprinkler Systems for Dummies: Optimizing a Hand-Moved Sprinkler System

Ben Dunham
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Kyle Nixon
Carroll College
Helena, MT

Advisor: Mark Parker

Summary

"Hand move" irrigation, a cheap but labor-intensive system used on small farms, consists of a movable pipe with sprinklers on top that can be attached to a stationary main. Our goal is a schedule that meets specific watering requirements and minimizes labor, given flow parameters and pipe specifications.

We apply Bernoulli's energy-conservation equation to the flow characteristics to determine sprinkler discharge speeds, ranges, and flow rates. Using symmetry and a model of sprinkler coverage, we find that three sprinklers, operating 57 min at 9 consecutive cycling stations during four 11-hour workdays, with the sprinklers 9 m apart on the 20 m mobile pipe and six mainline stations spaced 15 m apart, will water more than 99% of the field. Our computer model uses a genetic algorithm to improve the efficacy to 100% by changing sprinkler spacing to 10 m and adjusting the mainline station spacing accordingly.

The text of this paper appears on pp. 237–254.

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Fastidious Farmer Algorithms (FFA)

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Brandon W. Levin
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Duke University
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Advisor: William Mitchener

Summary

Summary

An effective irrigation plan is crucial to "hand move" irrigation systems. "Hand move" systems consist of easily movable aluminum pipes and sprinklers that are typically used as a low-cost, low-scale watering system. Without an effective irrigation plan, the crops will either be watered improperly, resulting in a damaged harvest, or watered inefficiently, using too much water.

We determine an algorithm for "hand move" irrigation systems that irrigates as uniformly as possible in the least amount of time. We physically characterize the system, determine a method of evaluating various irrigation algorithms, and test these algorithms to determine the most effective strategy.

Using fluid mechanics, we find that we can have at most three nozzles on the 20-m pipe while maintaining appropriate water pressure. We model our sprinkler system after the Rain Bird 70H 1" impact sprinkler, which works at the desired pressure and has approximately a 0.6 cm diameter. Combining data and analysis, we confirm that the radius of the sprinkler will be 19.5 m. Researchers have proposed several models for the water distribution pattern about a sprinkler; we consider a triangular distribution and an exponential distribution.

We do not consider schemes that do not water all areas of the field at least 2 cm every 4 days or water areas more than 0.75 cm/h. The largest cost in time and labor is in moving the pipe. Thus, we look for a small number of moves that still gives the desired time and stability. From these configurations, computer analysis determines which is most uniform.

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For various situations, we propose an optimal solution. The bases of the sprinkler placement patterns are triangular and rectangular lattices. We craft three patterns to maximize application to the difficult edges and corners.

- For calm conditions and a level field, the field can be watered with just two moves (the "Lazy Farmer" configuration). However, this approach is unstable, and even weak wind would leave parts of the field dry. With three moves, little stability is gained; so four positions is best.

- The "Creative Farmer" triangular lattice gives both stability and uniformity. The extra time is warranted because of its ability to adapt.

- We obtain even more stability using the "Conservative Farmer" model but at the price of a decrease in uniformity from the "Creative Farmer" approach.

The text of this paper appears on pp. 255–268.

A Schedule for Lazy but Smart Ranchers

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Wen Ye
Yu Yintao
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Shanghai, China

Advisor: Song Baorui

Summary

We determine the number of sprinklers to use by analyzing the energy and motion of water in the pipe and examining the engineering parameters of sprinklers available in the market.

We build a model to determine how to lay out the pipe each time the equipment is moved. This model leads to a computer simulation of catch-can tests of the irrigation system and an estimation of both distribution uniformity (DU) and application efficiency of different schemes of where to move the pipe. In this stage, DU is the most important factor. We find a schedule in which one sprinkler is positioned outside of the field in some moves but higher resulting DU (92%) and saving of water.

We determine two schedules to irrigate the field. In one schedule, the field receives water evenly during a cycle of irrigation (in our schedule, 4 days), while the other schedule costs less labor and time. Our suggested solution, which is easy to implement, includes a detailed timetable and the arrangement of the pipes. It costs 12.5 irrigation hours and 6 equipment resets in every cycle of 4 days to irrigate the field with DU as high as 92%.

The text of this paper appears on pp. 269–283.

Optimization of Irrigation

Bryan J.W. Bell
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Simpson H. Wong
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Davis, CA

Advisor: Sarah A. Williams

Summary

We determine a schedule for a hand-move irrigation system to minimize the time to irrigate a 30 m $\times$ 80 m field, using a single 20 m pipeset with 10 cm-diameter tube and 0.6 cm-diameter rotating spray nozzles. The schedule should involve a minimal number of moves and the resulting application of water should be as uniform as possible. No part of the field should receive water at a rate exceeding 0.75 cm per hour, nor receive less than 2 cm in a four-day irrigation circle. The pump has a pressure of 420 KPa and a flow-rate of 150 L/min.

The sprinklers have a throw radius of 14.3 m. With a riser height of 30 in, the field can be irrigated in 48 h over four days. Moreover, a single sprinkler is optimal. The pipes should be moved every 5 h and be at least 21 m apart. The resulting irrigation has precipitation uniformity coefficient of .89 (where 1 would be maximum uniformity).

We deal with each constraint in turn. Using geometrical analysis, we convert the coverage problem to determining the least number of equal-sized circles that could cover the field. We perturb the solution to optimize uniformity, by applying a Simultaneous Perturbation Stochastic Approximation (SPSA) optimization algorithm. We perturb this solution further to find the minimal number of pipe setups, by experimentally "fitting" the pipesets through the sprinklers. The rationale for perturbation is that some drop in uniformity can be tolerated in favor of minimizing the number of setups while still ensuring that we irrigate the entire field. We feed the optimal layout of pipe setups to another algorithm that generates an irrigation schedule for moving the pipes.

The text of this paper appears on pp. 285–294.

The UMAP Journal 27 (3) (2006) 225.

Sprinkle, Sprinkle, Little Yard

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Bradley Klingenberg
Pascal Getreuer
University of Colorado
Boulder, CO

Advisor: Bengt Forberg

Summary

We determine an optimal algorithm for irrigating an 80 m $\times$ 30 m field using a hand-move 20-m pipe set, using a combination of analytical arguments and simulated annealing. We minimize the number of times that the pipe is moved and maximize the Christiansen uniformity coefficient of the watering.

We model flow from a sprinkler as flow from a pipe combined with projectile motion with air resistance; doing so predicts a range and distribution consistent with data from the literature. We determine the position of sprinkler heads on a pipe to optimize uniformity of watering; our results are consistent with predictions from both simulated annealing and Nelder-Mead optimization.

Using an averaging technique inspired by radial basis functions, we prove that periodic spacing of pipes maximizes uniformity. Numerical simulation supports this result; we construct a sequence of irrigation steps and show that both the uniformity and number of steps required are locally optimal.

To prevent overwatering, we cannot leave the pipe in a single location until the minimum watering requirement for that region is met; to water sufficiently, we must water in several passes. The number of passes is minimized as uniformity is maximized.

We propose watering the field with four repetitions of five steps, each step lasting roughly 30 min. We place two sprinkler heads on the pipe, one at each end. The five steps are uniformly spaced along the long direction of the field, with the first step at the field boundary. The pipe locations are centered in the short direction. This strategy requires only 20 steps and has a Christiansen uniformity coefficient of 94, well above the commercial irrigation minimum of 80. Simulated annealing to maximize uniformity of watering re-creates our solution from a random initialization.

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The consistency between solutions from numerical optimization and from analytical techniques suggests that our result is at least a local optimum. Moreover, the solution remains optimal upon varying the sprinkler profile, indicating that the results are not overly sensitive to our initial assumptions.

The text of this paper appears on pp. 295–314.

Developing Improved Algorithms for Irrigation Systems

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Jin Qiwei
Zhou Kai
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Hangzhou, China

Advisor: Wang Shiming

Summary

Our goal is an algorithm that minimizes the time to irrigate a relatively small field under given conditions.

We focus on minimization of time, uniformity of irrigation, and feasibility. Our effort is divided into five basic parts:

- **We assess the wetted radius** based on experimental results for several typical rotating spray sprinklers.
- **We determine the number of sprinklers** from an empirical formula for sprinkler flow.
- **We simulate the water distribution pattern,** using a 0.25 m $\times$ 0.25 m grid.
- **We evaluate the uniformity of water distribution** by Christiansen's uniformity coefficient.
- **We find an optimal irrigation schedule including when and where to move the pipes:** We devise a single-lateral-pipe scheme and a multiple-lateral-pipes scheme; the latter gives better results. To irrigate more uniformly, we adjust the spacing between sprinklers and the spacing from the edge. Using our grid, we move the sprinklers symmetrically on both sides, node by node, to find the optimal positions for an improved multiple-lateral-pipes scheme.

Simulations show that all three schemes perform acceptably in realistic conditions. The improved multiple-lateral-pipes scheme is superior, with minimum time and the highest Christiansen's uniformity coefficient (CU). We conclude that four sprinklers are required, the minimal amount of time is 732 min, and the CU is 90%.

We do a sensitivity analysis of the variation of CU and of minimum time with wetted radius, which shows that our model is robust.

The text of this paper appears on pp. 315–328.

Profit Maximizing Allocation of Wheelchairs in a Multi-Concourse Airport

Christopher Yetter
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Cambridge, MA

Advisor: Clifford H. Taubes

Summary

To minimize Epsilon Airlines' cost of providing wheelchair assistance to its passengers, we examine the trade-off between explicit costs (chairs and personnel) and implicit costs (losses in market share). Our *Multi-Concourse Airport Model* simulates the interactions between escorts, wheelchairs, and passengers. Our *Airline Competition Model* takes a game-theoretic perspective in representing the profit-seeking behavior of airline companies. To ground these models in reality, we incorporate extensive demographic data and run a case study on 2005 Southwest Airlines flight data from Midland TX, Columbus OH, and St. Louis MO. We conclude that Epsilon Airlines should employ a "hub and spokes" strategy that uses "wheelchair depots" in each concourse to consolidate the movement of chairs. Across different airport sizes and strategies, we find that two escorts per concourse and two wheelchairs per escort are optimal.

The text of this paper appears on pp. 329–344.

Minimization of Cost for Transfer Escorts in an Airport Terminal

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Cambridge, MA

Advisor: Michael Brenner

Summary

We minimize the cost for Epsilon Airlines to provide a wheelchair escort service for transfers in an airport terminal. We develop probabilistic models for flow of flight traffic in and out of terminal gates, for the number of passengers on flight who require service, and for transfer destinations within the terminal.

We develop an economic model to quantify both the short- and long-term costs of operating such a service, including the salaries of escorts, the maintenance and storage of wheelchairs, and the costs incurred when late escorted transfers delay a departing flight.

We develop a simulated annealing (SA) algorithm that uses our economic models to minimize cost by optimizing the number and allocation of escorts to passengers. Having indexed the space of all possible escort allocations to be accessible to our SA, we selectively search the space of allocations for a global optimum. Although the space is too large to find a global optimum, our simulations suggest that the SA is effective at approximating this optimum.

Using current airport and airline data, we break our analysis down into short- and long-term costs, simulating escort service operation under dynamic airport conditions, varying air traffic, airport size, and the fraction of traveling population that requests wheelchair-aided transfer (simulating a greater future abundance of elderly travelers).

The text of this paper appears on pp. 345–361.

The UMAP Journal 27 (3) (2006) 231.

Application of Min-Cost Flow to Airline Accessibility Services

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Advisor: Martin Z. Bazant

Summary

We formulate the problem as a network flow in which vertices are the locations of escorts and wheelchair passengers. Edges have costs that are functions of time and related to delays in servicing passengers. Escorts flow along the edges as they proceed through the day. The network is dynamically updated as arrivals of wheelchair passengers are announced.

We solve this min-cost flow problem using network flow techniques such as price functions and the repeated use of Dijkstra's algorithm. Our algorithm runs in an efficient polynomial time. We prove a theorem stating that to find a no-delay solution (if one exists), we require advance notice of passenger arrivals only equal to the time to traverse the farthest two points in the airport.

We run our algorithm on three simulated airport terminals of different sizes: linear (small), Logan A (medium), and O'Hare 3 (large). In each, our algorithm performs much better than the greedy "send-closest-escort" algorithm and requires fewer escorts to ensure that all passengers are served.

The average customer wait time under our algorithm with a 1-hour advance notice is virtually the same as in the full-knowledge optimal solution. Passengers giving only 5-min notice can be served with only minimal delays.

We define two levels of service, Adequate and Good. The number of escorts for each level scales linearly with the number of passengers.

One hour of advance notice is more than enough. Epsilon Airlines can make major improvements by using our algorithm instead of "send-closest-escort" ; it should hire a number of escorts somewhere between the numbers for Adequate and Good service.

The text of this paper appears on pp. 363–381.

Cost Minimization of Providing a Wheelchair Escort Service

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Advisor: Peter Roland Kramer

Summary

Epsilon Airlines provides a wheelchair escort service to passengers to who require aid. We use an optimized earliest-due-date-first (EDD) algorithm to minimize the overall cost. Our algorithm is broad enough to accommodate various airport concourses, flight schedules, and flight delays. In addition, it allows for wheelchair escorts to perform other tasks beneficial to the airline, such as provide information at a kiosk, to help reduce the overall cost. Moreover, it creates schedules for each employee.

A naive strategy would be to employ the minimum number of escorts to guarantee that all passengers reach their gates on time. We show that this strategy is not optimal but can be improved by assigning different numbers of escorts to shifts based on expected traffic. For example, if Delta Airlines were to utilize the naive strategy at Atlanta International Airport, the cost would be over \$5 million/yr, whereas our strategy reduces this cost to under \$4 million/yr. A similar reduction in cost could be expected for Epsilon Airlines.

The text of this paper appears on pp. 383–394.

The UMAP Journal 27 (3) (2006) 233.

A Simulation-Driven Approach for a Cost-Efficient Airport Wheelchair Assistance Service

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Advisor: Mark P. Embree

Summary

Although roughly 0.6% of the U.S. population is wheelchair-bound, the strain of travel is such that more than twice that amount relies on wheelchairs in airports [Haseltine Systems Corp. 2006].

Two issues have the greatest impact on the cost and effectiveness of this service: the number of wheelchairs and how they should be deployed. The proper number of escorts and wheelchairs is not only a question of the airport but of the volume of passengers, which can vary greatly. If escorts detemine their own movements within the airport, lack of coordination could result in areas being unattended; however, fluctuation in requests could be so great that a territory-based plan could overwork some escorts and underwork others.

We present an algorithm for scheduling of the movement of escorts that is both simple in implementation and effective in maximizing the use of available time in each escort's schedule. Then, given the implementation of this algorithm, we simulate the scheduling of requests in a given airport to find the number of wheelchair/escort pairs that minimizes cost.

The text of this paper appears on pp. 395–407.

The Median is the Message for Efficient Wheelchair Service

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Advisor: John Barnes

Summary

Every year, more than 17 million disabled passengers travel on commercial airlines, some with special needs that the airlines must meet. Meeting these needs can be challenging, because such requests are relatively rare and occur unexpectedly. However, if an airline does not set aside adequate provisions to help the disabled, flights may become delayed as handicapped passengers struggle to reach their destination.

We model the situation and devise a protocol for an airline to use minimal resources to respond efficiently to such requests, balancing the costs of additional personnel against delayed flights.

The model consists of three parts:

- an algorithm for finding the number of escorts that an airport should hire (based on balancing costs);
- establishing that the best ratio of wheelchairs to escorts is one-to-one (even if a wheelchair-bound individual does not request an escort, someone is needed to transport the chair);
- showing that wheelchair service is most efficient when escorts work out of a central "hub" (a hub shortens the time for an escort to travel to the disabled passenger's gate). The ideal location for a hub is generally the median gate, the gate with an equal number of gates on either side; if there is no gate there, then a nearby lounge would suffice.

To test the efficiency of our model, we simulated wheelchair service in large, medium, and small airports. Our model was successful in reducing

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delay times. However, the model is not perfect. It assumes that all escorts are perfectly efficient in their occupation and that all passengers are completely cooperative; the human element is a significant complication that our model does not address. A strength of our model is its flexibility: The algorithm for the number of escorts can adjust to changes in the population and in the airline industry. Thus, as the nation ages and the airline industry grows, our model will still be applicable.

[EDITOR'S NOTE: This Meritorious paper won the Ben Fusaro Award for the Wheelchair Problem. The full text of the paper does not appear in this issue of the *Journal*, but a Judges' Commentary on the paper is on pp. 413–414.]

Pp. 237–254 can be found on the *Tools for Teaching* 2006 CD-ROM.

Sprinkler Systems for Dummies: Optimizing a Hand-Moved Sprinkler System

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Summary

"Hand moved" irrigation, cheap but labor-intensive systems used on small farms, consists of a movable pipe with sprinklers on top which can be attached to a stationary main. Our goal is a schedule that meets specific watering requirements and minimizes labor, given flow parameters and pipe specifications.

We apply Bernoulli's energy-conservation equation to the flow characteristics to determine sprinkler discharge speeds, ranges, and flow rates. Using symmetry and a model of sprinkler coverage, we find that three sprinklers, operating 57 min at 9 consecutive cycling stations during four 11-hour workdays, with the sprinklers 9 m apart on the 20 m mobile pipe and six mainline stations spaced 15 m apart, will water more than 99% of the field. Our computer model uses a genetic algorithm to improve the efficacy to 100% by changing sprinkler spacing to 10 m and adjusting the mainline station spacing accordingly.

Introduction

Our challenge is to design a movable sprinkler system to meet a set of watering criteria on an 80 m $\times$ 30 m field with specified flow characteristics of the main pipe. We must

- decide how a stationary main water pipe should be positioned on the field;

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- determine the number of sprinklers, the sprinkler spacing on the movable pipe, and the spacing of the attachments for the movable pipe; and
- schedule moving the pipe.

We make simplifications and formulate models. We determine the stationary pipe position and, ignoring friction, formulate first a simple one-sprinkler model, using the principles of conservation of flow and the Bernoulli energy equation. We use the results in the jet equation to find trajectory and range of a single sprinkler. We apply the same principles to multisprinkler systems.

The area watered can be modeled as either a uniform disk or a ring. We minimize the overlapping areas in the disk model to eliminate underwatered areas and maximize uniform water distribution, for different numbers of sprinklers. This optimization determines sprinkler spacing, movable pipe spacing, and the number of moves to water the entire field.

Using calculated values for flow rates and depth accumulation over time, we determine a schedule that minimizes time and effort.

Assumptions

- The field is flat, so there is no change in energy (or head) of the system due to changes in elevation.
- One 80-m fixed pipe runs the length of the field, to which a lateral arm can be attached perpendicularly.
- There is only one lateral pipe 20 m long.
- The angle at which the water leaves the sprinkler is 30°.
- Sprinklers can be modeled as 360° rotary jets.
- The entire flow does not need to leave the sprinklers; a return system is implied.
- There is no wind; the sprinklers always water evenly over a circular area.
- Rain is not modeled; the system is turned off during rain.
- The sprinkler system is inactive for at least 8 h every night.

Models

System Requirements and Statistics

The aluminum pipe system consists of a stationary main pipe with a diameter of 10 cm. Perpendicular to this pipe, a single 20-m-long (10-cm-diameter) lateral or movable pipe can be attached. On this pipe are a number of sprinklers (0.6 cm diameter) that can either sit directly on the movable pipe or connect to the pipe with 0.6 cm diameter vertical pipes. Water flow at the source is 150 L/min at a pressure of 420 kPa. Every area should receive no more than 0.75 cm depth of water in an hour and no less than 2 cm over four days.

Field Layout

The field is 80 m by 30 m. The lateral or movable pipe is 20 m long when assembled with a number of rotating sprinklers attached to it. We interpret the sprinkler set to mean we can use only one 20-m-long pipe as the lateral movable pipe (multiple 20-m sections cannot be attached to one another). The lateral pipe can be connected perpendicularly to a fixed main pipe with the same diameter (10 cm). We find it efficient and symmetric when the main pipe runs the length of the 80 m field and is inset 5 m onto the field. This ensures that if sprinklers are set on both ends of the lateral pipe, the field can still be watered symmetrically. We can optimize the number of connection points and spacing on the main pipe based on the number of sprinklers on the lateral pipe. We assume that valves can shut off flow to excess length of the main pipe in order to direct the entire flow into the lateral pipe. Our model also assumes that the entire flow does not exit the sprinklers, but instead only the pressure of the system forces the water from the sprinklers; the rest of the water is returned to the reservoir from either a flexible return pipe or irrigation ditch.

One-Sprinkler System

To understand the outflow of the sprinklers, we simplify the model to include only one sprinkler and study the exit speed of the water leaving the rotating sprinkler head. This analysis can be done in terms of conservation of flow, or in terms of conservation of energy.

Conservation of Flow

We assume that there is no speed loss due to friction, the bend in the pipe, or the condition and angle of the sprinkler head.

Flow ($Q = vA = 150\ \text{L/min} = 0.0025\ \text{m}^3/\text{s}$) through the pipe must be conserved, so the flow through the pipes satisfies

$$v_2 A_2 = v_1 A_1,$$

where A_i is the area of pipe i and v_i is the speed in it [Walski et al. 2004, 3]. Taking the main pipe as pipe 1 and measuring in meters, we have

$$v_2 = \frac{0.318\ \text{m/s} \times \pi(0.05)^2}{\pi(0.003)^2} = 88.3\ \text{m/s},$$

which—at almost 200 mph—is faster than a sprinkler could probably handle.

Conservation of Energy

To take energy into consideration, we must make a bold assumption: Not all of the water running through the main pipe ends up on the field. To conserve energy, we must use the Bernoulli equation and neglect friction head loss at both positions 1 (the main pipe) and 2 (the sprinkler head) [Walski et al. 2004, 6]:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g},$$

where

P = pressure (N/m^2),

γ = specific weight of fluid (N/m^3),

z = elevation above a reference point (m),

v = fluid speed (m/s), and

g = gravitational acceleration (m/s^2).

For our parameter values, we have

$$\frac{420 \text{ kN/m}^2}{9.81 \text{ kN/m}^3} + 0 + \frac{(0.318 \text{ m/s}^2)}{2(9.81 \text{ m/s}^2)} = 0 + z_2 \text{ m} + \frac{v_2^2 \text{ (m/s)}^2}{2(9.81) \text{ m/s}^2}.$$

The pressure at point 2 (the sprinkler) is zero because at this point the water is being expelled through the nozzle and is under no pressure from the pipes [Finnemore 2002, 511]. So the exit speed of water out of the sprinkler, as a function of the height z_2 of the sprinkler off the ground is, after some algebra:

$$v_2(z_2) \approx 4.43\sqrt{42.83 - z_2}.$$

The height of sprinklers is usually between 6 in and 4 ft depending on the crop (assuming no braces to support the sprinklers) [National Resources Conservation Service 1997]; therefore, we test the sensitivity of the speed based on height (for our purposes using a range of 0 m to 1 m):

$$v_2(0) = 29.0 \text{ m/s}, \qquad v_2(1) = 28.7 \text{ m/s}.$$

So the speed out of the sprinkler is not sensitive to height. For the remainder of the study, we use $z_2 = 0.5$ m.

Trajectory

First, we make some assumptions:

- The speed found using energy conservation is the same initial speed that would occur through the nozzle at any discharge angle.

- The maximum speed is not affected by the means of dispersing the water, even if the spray is more similar to a fan than a jet.
- The nozzle is frictionless.

We use the single-sprinkler speed to find the range of the water discharged through the nozzle, via the jet equation [Finnemore 2002, 169]:

$$z = \frac{v_{z0}}{v_{x0}}\,x - \frac{g}{2v_{x0}}\,x^2.$$

For each additional sprinkler, the speed is cut down proportionally, based on the conservation of flow equation $v_2 = v_1 A_1 / A_2$; the effective outflow area (A_2) is increased proportionally, so the speed decreases to $v = v_0/n = 28.8/n$ m/s. When rearranged, the jet equation gives the range for the number of sprinklers:

$$x = \frac{v\cos\theta\sqrt{v^2\sin^2\theta - 2gn^2 z} + v\sin\theta}{gn^2}, \tag{1}$$

where

x is the outer radius of the sprinkler coverage (range);

θ is the angle to the horizontal at which the sprinkler discharges;

v is the speed of the water at the sprinkler head, found using the conservation of energy equation;

z is the change in height from the jet to the ground;

g is the acceleration due to gravity; and

n is the number of sprinklers on the lateral pipe.

Since most rotary crop sprinklers discharge between 18° to 28° above the horizontal axis, but up to 35°, we assume that $\theta = 30°$ [Fipps and Dainello 2001].

Multiple-Sprinkler System

Given the design, a minimum 5 m radius is needed to reach the edges of the field. Based on our calculations from **(1)**, we find with four sprinkler heads that the radius is 5.32 m. Therefore, no more than four sprinklers can be put on the lateral pipe and still have a sufficient range to reach the edge of the field.

We need to know the rate at which the water flows out of the sprinklers to determine the rate of watering. Since $Q = vA$, the flow Q_n from each sprinkler when there are n sprinklers is

$$Q_n = v_n \pi (0.003)^2.$$

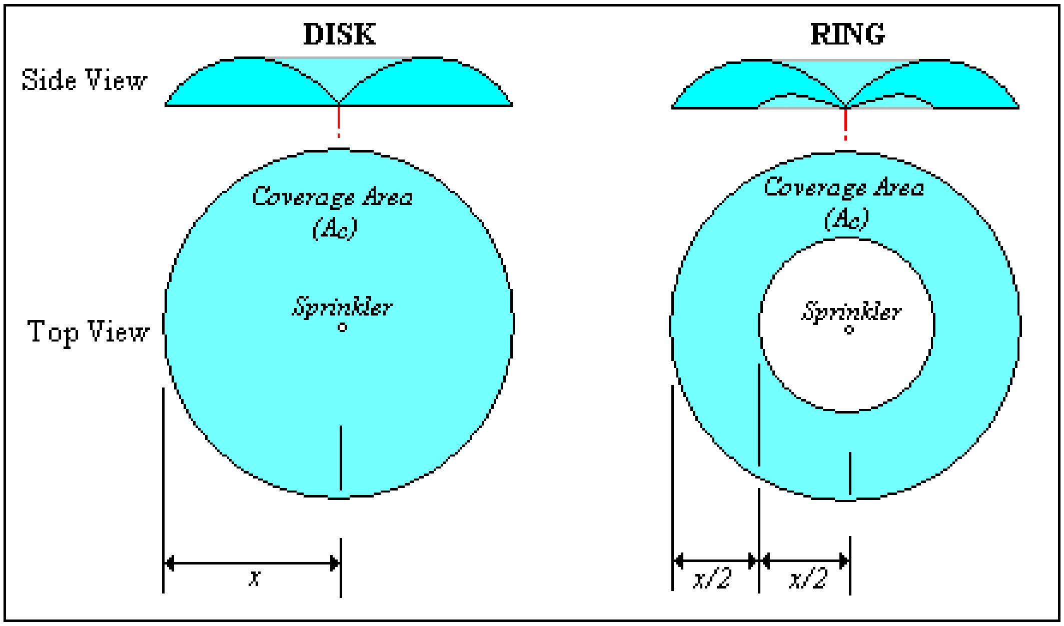

Figure 1. The two sprinkler coverage model extremes.

Two options represent the extremes of sprinkler coverage (**Figure 1**); we assume for both cases that the sprinkler is a well-distributed fan of water, meaning that the entire designated area for each sprinkler is watered uniformly.

- The sprinkler head discharges uniformly over a disk, with outer radius x m and inner radius 0 m.
- The sprinkler discharges uniformly over a ring, with outer radius x m and inner radius $x/2$ m. We can justify this ratio because, realistically, a sprinkler discharging onto an area narrower than this would require too many additional sprinklers to hydrate the unwatered area around the center.

We then find the rate (depth over time) D (cm/h) at which the area covered by the sprinkler receives water:

$$D = \frac{Q_n}{A_c},$$

where A_c is the area covered by the sprinkler, with

$$A_c = \pi(x_o^2 - x_i^2),$$

where $x_o,\ x_i$ are the outer and inner radii of the ring ($x_i = 0$ m for disk).

We calculate the depth over time D distributed over the discharge area A_c (**Table 1**) for both disk and ring models.

As sprinklers are added to the lateral arm, the area covered by each sprinkler decreases; consequently, D increases greatly. Recall our constraints:

- No part of the field should receive more than 0.75 cm/h.

Table 1.
Disk Model: Effective radius, flow/sprinkler, and depth/time for n sprinklers.

Sprinklers (on lateral pipe)	Effective radius (m)	Q_n (m^3/h/sprinkler)	Depth over area (cm/h)	
			Disk	Ring
1	74	2.9	0.02	0.02
2	19	1.5	0.13	0.17
3	9	1.0	0.39	0.52
4	5	0.7	0.82	1.10

- Each part of the field should receive at least 2 cm every 4 days.

One sprinkler is not time-efficient, since the lateral pipe would need to sit about four days to fulfill the minimum. Two sprinklers is also not efficient, since they cover an area far exceeding the field boundaries. Either one or two sprinklers would likely result in far too much pressure for a sprinkler to handle, for both the disk model and the ring model. We study only three- and four-sprinkler systems beyond this point.

Four sprinklers causes a depth rate in excess of 0.75 cm/h. However, we can interpret this to mean the rate can exceed 0.75 cm/h as long as the pipe does not sit long enough for the cumulative depth to exceed 0.75 cm in one hour. In other words, the water can be shut off before the full hour is up.

Optimal Overlap Model (for Disks)

We define a *station* to be one of the lateral positions, a *cycle* to be each station being watered once, and a *watering* to be the process of watering a single station.

We try to optimize the system by arranging overlap of sprinklers so that no area gets hit more than twice in any cycle; doing this maximizes the time that the system can stay in one spot.

We arrange the sprinklers to cover the edges of the field as nearly as possible—though possibly missing some small triangles along the edges of the field as we try to optimize the speed of watering. We assume enough soil permeability so that water seeps from surrounding watered areas.

With four sprinklers, the outer two need to be at the ends of the pipe so that they can cover the edge as much as possible and unwatered area is minimized. With three sprinklers, the two outside sprinklers are 1.1 m from the end point of the lateral line and the third is in the center. It is ideal if the radius is large enough to cover the edge of the field and also hit the next sprinkler over on the mobile line; then the sprinklers can sprinkle a little less water over the edge, minimizing waste.

To determine the move distance from station to station, we use the Pythagorean theorem:

$$R^2 = \left(\tfrac{1}{2}L_s\right)^2 + \left(\tfrac{1}{2}S_s\right)^2, \qquad L_s = 2\sqrt{R^2 - \left(\tfrac{1}{2}S_s\right)^2},$$

where L_s is the lateral spacing of the movable pipe, S_s is the spacing between the sprinklers, and R is the radius of the spray for each sprinkler (**Figure 2**).

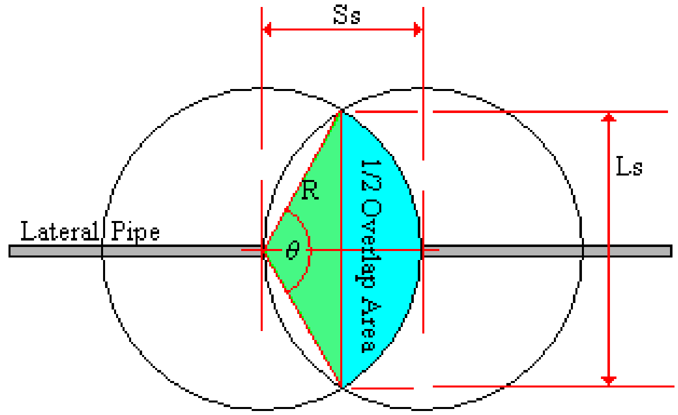

Figure 2. Sprinkler overlap.

Lateral spacing is kept constant for as long as possible through the middle of the field. At the edge of the field, the two connections to the stationary line should be equidistant from the edge while still maintaining enough coverage. The lateral pipe spacing, sprinkler spacing, and number of moves required to cover the field can be seen in **Table 2**.

Table 2.
Pipe spacing, sprinkler spacing, and number of moves for 3 and 4 sprinklers.

Sprinklers	Spray radius (m)	Sprinkler spacing (m)	Pipe spacing (m)	Moves to cover field
3	8.9	8.9	15.4	6
4	5.3	6.7	8.3	10

To calculate the area inside the field that is double-watered, we use L_s to find the length of the opposite edge of the isosceles triangle. The two equal sides are the radius of the disk. From the law of cosines $L_s^2 = 2R^2 - 2R^2 \cos\theta$, we can easily find θ; for laterally overlapping wedges, we substitute S_s for L_s. The area of the wedge is found using $A = \theta\pi R^2/360°$ and double the area of the triangle is subtracted from it.

The resulting areas of double, single, missed, and outside property watered are in **Table 3**. The area double-watered by three sprinklers during a cycle is greater than that for four sprinklers, but the area missed is much smaller.

A strength of these models is that there is no specific order in which the field must be watered, since no area is hit more than twice in a cycle. Moving the sprinkler progressively from station to station across the field both minimizes move distance and reduces move time.

The time in each position is calculated such that the lateral pipe is relocated before the overlapping sections receive more than 0.75 cm in an hour. The combined flow rates of the sprinklers for both the three- and the four-sprinkler

Table 3.
Sprinkling imperfections (areas in m^2).

Sprinklers	Double-watered	Single-watered	Missed	Watered outside field
3	1131	1250	19	717
4	1057	1281	59	140

arrangements is faster than the allowed rate, especially in overlapping areas. The water must be shut off in time and left to seep in for the rest of the hour.

Sprinkling Time and Schedule

We calculate the time to accumulate 0.75 cm in the areas hit by two sprinklers (the overlapping sprinkler areas). The D-values for the overlap areas are twice those in **Table 1**. Thus, the time t_n that the lateral pipe with n sprinklers must water an area is

$$t_3 = \frac{0.75}{2D_3} = 0.960 \text{ h} = 58 \text{ min}, \qquad t_4 = \frac{0.75}{2D_4} = 0.455 \text{ h} = 27 \text{ min}.$$

As seen in the **Appendix**, the number of lateral pipe stations to cover the entire field is 6 for three sprinklers and 10 for four sprinklers. [EDITOR'S NOTE: We omit the **Appendix**.] This result is important in realizing how many total moves are required to meet the requirement of the entire area getting at least 2.0 cm in four days.

To find the number of cycles needed to be repeated over four days, we consider the areas that are not overlapped. These areas receive (0.75 cm / cycle) / 2 = 0.375 cm / cycle; so to receive 2 cm over four days requires 2 cm / (0.375 cm / cycle) = 5.3 cycles. Therefore, six complete cycles must occur over four days to meet the minimum water requirement, for either three or four sprinklers.

For simplicity, we assume that the time to move the lateral pipe from one station to the next is uniformly 15 min. We display several statistics about our models in **Table 4**.

Table 4.
Sprinkler statistics for scheduling.

Sprinklers	Watering time (min)	Stations	Cycles in 4 d	Total waterings per 4 d	Waterings per d
3	58	6	6	36	9
4	27	10	6	60	15

Finally, we can make watering schedules for three and for four sprinklers. [EDITOR'S NOTE: We omit the schedules.]

Each model requires six cycles over four days. Therefore, a cycle and a half should be completed each day to keep the workload balanced. For either number of sprinklers, the daily watering time is roughly the same (11 h), as is the total work time over four days (43 h). However, a considerably larger amount of time is spent moving the pipes using four sprinklers (15 stations/d vs. 9). Since one goal is to minimize time moving equipment, we recommend the three-sprinkler system, since watering takes roughly the same amount of time with much less effort and also leaves only 19 m^2 unwatered instead of 59 m^2.

Ring Method of Area Estimation

Some sprinklers water a ring-like area; we assume uniform water distribution over the area and that the outer spray radius is twice the inner radius. Considering the ring areas complicates the model significantly:

- Preventing the same area being hit by the sprinklers more than twice in a cycle is impossible.
- To cover the entire area requires a significant amount of area to be watered three or four times as often as the areas watered once in a cycle (**Figure 3**).

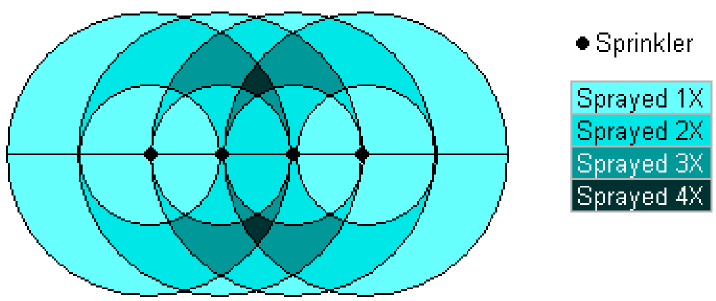

Figure 3. The ring model shows a great imbalance in the area watered.

- The lateral pipe must be moved more frequently and must water for a much smaller length of time to prevent the accumulation in heavily watered areas from exceeding 0.75 cm/h.
- Many more cycles must occur to ensure that lightly watered areas receive a cumulative depth of at least 2.0 cm every four days.
- To prevent areas from getting overwatered while minimizing the underwatered area requires a staggered station progression, thereby increasing the distance that the lateral line must be moved.
- The watering of the field can no longer be considered uniform.

The ring model assumes equally spaced sprinklers arranged such that no area in the center of any radius is left unwatered. We do not explore sprinkler spacing but merely note that the spacing depends on the outer spray radius.

Once the sprinkler spacing is determined, the lateral pipe spacing can be determined as in the disk model (optimal overlap model), as well as the number of stations required for a given number of sprinklers.

For an algorithm on station progression for a cycle, there are several options:

- Minimize the total distance that the lateral pipe must be moved in one cycle. This option
  - progresses similarly to the disk model (simply progress to the next station, with down-time at the endpoints of the main watering line);
  - likely requires some wait time between waterings to meet the maximum watering requirement (no more than 0.75 cm/h on area), since there is significant overlap of watered areas;
  - is not time-efficient.
- Minimize wait time between waterings. This option
  - requires the pipe to be moved to another station immediately after a watering is complete;
  - requires, to avoid overwatering certain areas, that the lateral pipe must be moved beyond an adjacent station upon completing another watering;
  - requires a complex station-progression algorithm to ensure that the field is watered quickly and efficiently.
  - has the weakness that the total distance that the lateral pipe is moved between waterings (as well as total distance moved during a complete cycle) is much greater than in the first option. In theory, the time required to move this extra distance could exceed the time that the farmer would need to wait using the first option.
- Compromise by allowing some wait time between stations and allowing the lateral pipe to be moved beyond the adjacent station during the progression.

Since one objective is to minimize the time and effort moving the pipes, we suggest the first option.

Head Loss

A major weakness of our models is that they do not account for energy losses due to friction between the pipe and the water moving through it, also known as *head loss*. We apply the Hazen-Williams equation solved for meters of head loss per meter of pipe or friction slope [Walski et al. 2004, 17]:

$$S_f = \frac{10.7}{D^{4.87}} \left(\frac{Q}{C} \right)^{1.852}, \tag{2}$$

where

S_f is friction slope (m/m),

D is the diameter of the pipe (m),

Q is the flow rate (m^3/s), and

C is the Hazen-Williams friction coefficient (for aluminum, $C = 130$).

With our given values of $D = 0.1$ m and $Q = 0.0025$ m^3/s, we have $S_f = 0.00146$ m/m, or a total head loss over the 100 m of pipe of $100S_f = 0.15$ m, which is insignificant.

For the farthest sprinkler position, there will be multiple tees with valves where the lateral line ties into the main line, and each has an associated loss of 0.3–0.4 meters of head [Walski et al. 2004]. These losses can add up and reduce the speed of the water exiting the sprinkler when the lateral arm is far from the water source. The problem is remedied by decreasing the distance between stations as the attachment points progress farther down the mainline.

When we use **(2)** to check the head loss on the small sprinkler pipes, assuming that there is only one outlet for all of the flow, we obtain an astronomical friction slope, $S_f = 1305$ m/m. This calculation is another justification for our assumption that not all of the flow exits the sprinklers and a return line of some sort is necessary. When adjusted for the proper flow based on the number of sprinklers, we obtain a friction slope of 12.5 m/m for four sprinklers and 21.4 m/m for three sprinklers. When we use the Bernoulli equation with losses due to friction accounted for, the appropriate losses are added to each side of the equation and once again solved for v_2. The new speeds are:

Three sprinklers: v = 8.9 m/s instead of 9.6 m/s (for no friction)

Four sprinklers: v = 6.7 m/s instead of 7.2 m/s (for no friction)

Three sprinklers can cover the field, but with four sprinklers the speed is too low to cover the edges reliably.

Computer Modeling and Solution Approach

We created a computer program to model the effectiveness of solutions and to find its own near-ideal solution via a genetic algorithm.

The Computer Model

We divide the field into a 200×75 cell grid of cells 0.4 m on a side and time into 5-min discrete intervals; each cell records how much water it receives during each time interval over the course of four days. Inputs are:

- The direction that the mainline runs (cross-wise or length-wise). (We later found that length-wise leads to insurmountable pressure-loss.)
- The inset of the mainline from the edge of the field.

- The inset of the first sprinkler from the beginning of the lateral line.
- The inset of the last sprinkler from the end of the lateral line.
- The total number of sprinklers on the lateral line (spaced evenly between first and last).
- A list of steps in the watering schedule. Each step consists of:
  - The distance in cells down the mainline to where the lateral is attached.
  - The number of time intervals that the lateral operates at that location.

In addition, the model accounts for several other variables that are hard-coded into the program but could be changed:

- The time to move the lateral line (15 min).
- The "up-time" per day (16 h), that is, ensuring that the system stops at night.

The program assigns a radius range to the sprinklers and a rate of water accumulation for every cell in that range, based on earlier calculations on sprinkler pressure and speed. After the entire schedule has been simulated, each cell is queried and assigned one of three conditions based on water accumulation:

- Overwatered: If during any one-hour period the cell received more than 0.75 cm of water, it is considered overwatered, even if the total water over four days was less than 2 cm.
- Underwatered: If the cell is not overwatered, and it received less than 2 cm over the course of four days, it is considered underwatered.
- Ideally-watered: If the cell is neither overwatered nor underwatered, it is considered within the ideal watering range.

The Genetic Algorithm

The genetic algorithm attempts to find optimal solutions through evolution. It creates 100 random sets of input for the testing model and tests each set, or "genome"; the 10 best-ranked genomes (see the **Appendix** for the ranking system) are selected as "parents" for another collection of 100 input sets (another "generation") [Editor's Note: We omit the **Appendix**.]. Ninety of the new genomes are created from two parents (10 parents allow for 45 unique combinations—two of each are used); the input values are the averages of their parents' values, with a small percentage of random variation added in. The remaining 10 genomes are created directly from one parent, with a slight amount of variation added. The computer proceeds through many generations, constantly improving the solution sets—in theory. In practice, the large number of input variables and the relatively small number of effective solutions means

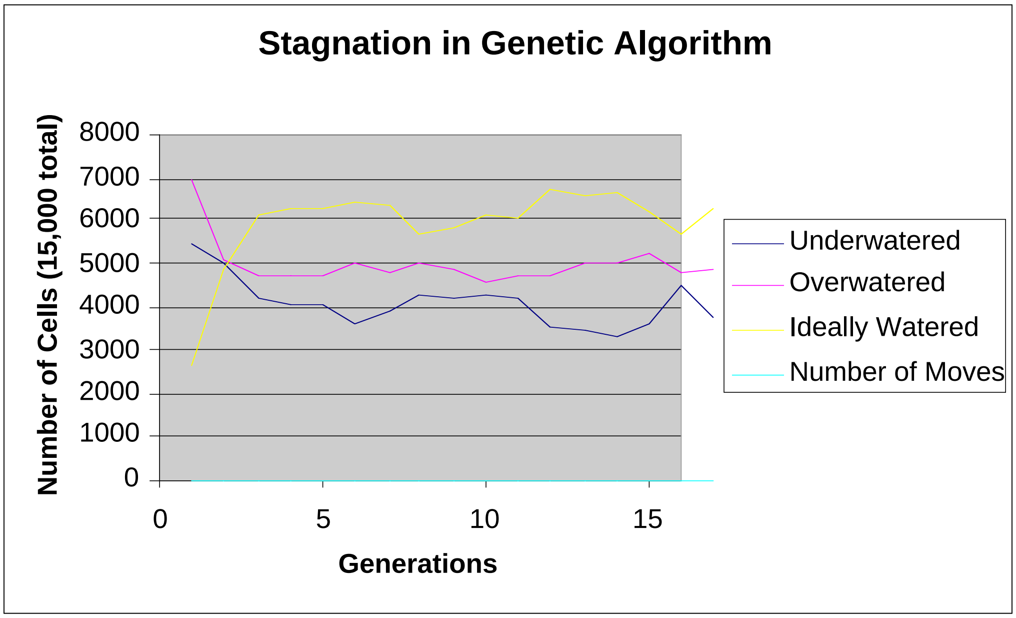

Figure 4. After three to four generations, progress ceases. (Number of Moves hugs the horizontal axis.)

that it is extremely easy to worsen a solution but very difficult to improve it. After a few generations of progress, the model stagnates (**Figure 4**).

With a much larger population size (tens of thousands), the chances increase of finding one or two better solutions in every generation; with our computing power, we are limited to a smaller population and thus cannot use the full range of input variables.

The evolution simulation always favors certain values for particular variables: For three or four sprinkler heads, the simulation always picks three as optimal in the very first generation. In addition, the inset value of the mainline always tends towards centering the length of the lateral in the field, and the inset value of the first and last sprinklers always approaches zero within five to ten generations, suggesting that these sprinklers are best placed at the very beginning and end of the lateral line so as to leave less area unwatered. The simulation does not keep track of water that lands off of the field. By decreasing the areas that are watered twice and dumping a little extra water over the edge, unwatered area decreases to zero if lateral spacing is optimized.

The Station-Based Model

Using noncomputer methods, we had determined that to water the entire field adequately with three sprinklers, six stations had to be visited six times each, with a watering time of less than 57 min for each visit (see **Table 5**). We tested the boundaries of this model by running it with a 60-min and with a 55-min operating time at each schedule step, a situation that should cause overwatering where sprinkler coverage overlaps. **Figure 5** shows the results.

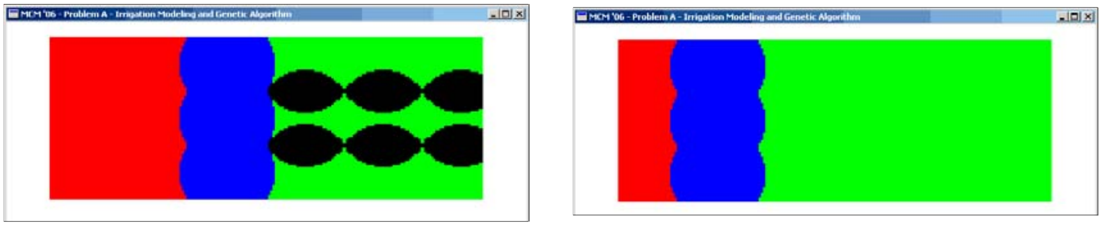

Figure 5. Station-based schedule with 60 min per station (left) and with 55 min per station (right). KEY: Red (leftmost region): cells underwatered or not yet watered. Blue (middle): cells being watered. Black (ovoids; none for 55 min): cells overwatered. Green (right): cells in the ideal range.

For 60 min, our prediction of overwatering in overlaps is proven true; for 55 min, the lack of overwatering shows that our model is accurate.

The next question is, Is our sequence of stations the most optimal? We start at one end of the field and move down by one station every 55 min until we reach the other end, where we stop for one cycle of down-time and then move back up the field. This cycle continues until we hit every station six times, requiring moving the line a total of 432 m. Cycling back and forth seems an obvious and efficient schedule, but is it possible to find something faster?

The Station-Based Genetic Algorithm

We restructured the genetic algorithm to find the optimal station schedule.

The simulation is set up to allow genetic change only in the station number to be visited at each schedule step; each visit must be 55 min long, and each station has to be visited 6 times, but the stations can be visited in any order. All other variables are fixed at the values that we had determined to be the most efficient. The scoring system is adjusted so that the only criterion for selection is moving the lateral line a lesser distance than the other genomes.

The genetic algorithm stabilizes at a single solution within five elapsed generations, as shown in **Figure 6**. The mean distance required to move the lateral line over a four-day cycle drops from more than 900 m to a surprising 72 m, a major improvement over our "optimal" 432 m.

The computer took an approach so simple as to be overlooked: Rather than moving on after turning off the water, the irrigation system is simply turned off for 15 min before resuming at the same location. The lateral line starts at one end of the field and operate for six on-off cycles before moving to the next station.

Of course, this may not be an option for fields that cannot quickly absorb water, but it satisfies the conditions of the problem statement. Over any given hour, the total depth of water applied within the overlapping areas is less than 0.75 cm because of watering for only 55 min plus the 15-min move time.

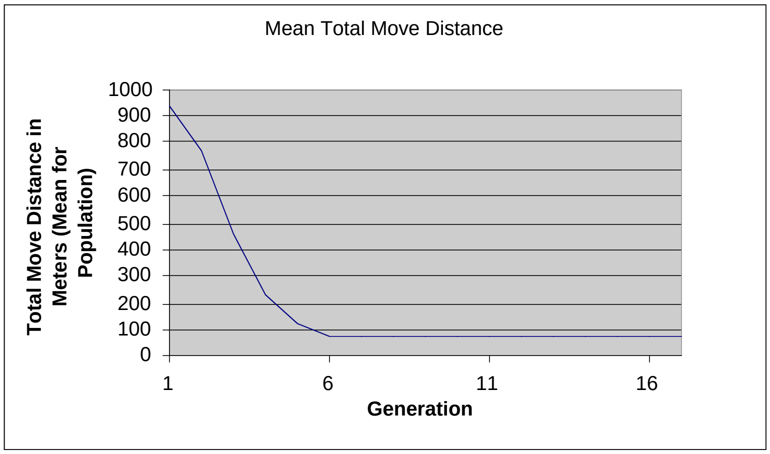

Figure 6. Mean total move distance.

Results and Conclusions

We found spray ranges based on the number of sprinklers in the system. This model assumes no friction loss, and we found speed not to be sensitive to height. We investigated two models of coverage: disk and ring areas. Using the areas and flow rates from sprinklers to approximate depth accumulated in the areas, we eliminated one and two sprinklers based on the overshooting range, poor time efficiency, excessive waste, and large speeds. Calculating the range for five sprinklers showed that the spray would not reach the edge of the field. Therefore, we limited our models to three or four sprinklers on the lateral pipe.

Using geometric methods, we found an optimized spacing of three and four sprinklers along the lateral pipe (using disks to model areas):

- Three sprinklers: placed 1.1 m inset from the end of the 20-m pipe, spaced evenly at 8.9 m between sprinklers.

- Four sprinklers: a sprinkler at each end point of the 20-m pipe, spaced evenly at 6.7 m between sprinklers.

Similarly, the lateral pipe spacing and number of stations/moves required to cover the entire field was calculated for the two cases:

- Three sprinklers: 15.4 m between stations; six stations.

- Four sprinklers: 8.3 m between stations; ten stations.

Advantages to using three sprinklers are fewer moves per cycle and minimization of the area underwatered.

We found that to meet the minimum water requirement of at least 2 cm depth over four days, the areas watered once per cycle are the driving factor, resulting in six cycles required every four days for both three and four sprinklers. Therefore, to balance the workload evenly over four days, one-and-a-half cycles must be completed each day. In accordance with the maximum rate that water can accumulate (0.75 cm/h), the time that a sprinkler could water an overlapping area was calculated to be:

- Three sprinklers: 58 min per station; 9 stations per day.
- Four sprinklers: 27 min per station; 15 stations per day.

We also found lateral pipe can be progressively moved from station to station during a cycle. However, at end stations the best option (while maintaining uniform coverage) involves leaving the pipe at a station for two consecutive turns while scheduling a wait time between the two waterings.

The total work day for these models is approximately the same. Using three sprinklers requires less move time, as well as fewer moves each day.

The ring model requires more frequent relocation of the lateral pipe, many more cycles to ensure the proper amount of moisture in all areas, and a staggered progression of the lateral pipe.

The biggest weakness of our initial models is assuming no friction head loss in the system. After some calculation of head loss, and reapplication of the Bernoulli energy equation, we found the new sprinkler discharge speeds and compared them to speeds calculated with no head loss:

- Three sprinklers: $v = 8.9$ m/s instead of 9.6 m/s (no friction)
- Four sprinklers: $v = 6.7$ m/s instead of 7.2 m/s (no friction)

Though the discharge speeds decrease, the speed of the optimized three-sprinkler system is still large enough to reach the edge of the field when the sprinklers are positioned on the end of the movable pipe.

Our computer simulation found an ideal watering over a four-day period using the parameters of our previous three-sprinkler model, eliminating unwatered area by placing sprinklers on the ends of the lateral pipe and changing the spacing of the lateral attachments accordingly.

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Summary

An effective irrigation plan is crucial to "hand move" irrigation systems. "Hand move" systems consist of easily movable aluminum pipes and sprinklers that are typically used as a low-cost, low-scale watering system. Without an effective irrigation plan, the crops will either be watered improperly, resulting in a damaged harvest, or watered inefficiently, using too much water.

We determine an algorithm for "hand move" irrigation systems that irrigates as uniformly as possible in the least amount of time. We physically characterize the system, determine a method of evaluating various irrigation algorithms, and test these algorithms to determine the most effective strategy.

Using fluid mechanics, we find that we can have at most three nozzles on the 20-m pipe while maintaining appropriate water pressure. We model our sprinkler system after the Rain Bird 70H 1" impact sprinkler, which works at the desired pressure and has approximately a 0.6 cm diameter. Combining data and analysis, we confirm that the radius of the sprinkler will be 19.5 m. Researchers have proposed several models for the water distribution pattern about a sprinkler; we consider a triangular distribution and an exponential distribution.

We do not consider schemes that do not water all areas of the field at least 2 cm every 4 days or water areas more than 0.75 cm/h. The largest cost in time and labor is in moving the pipe. Thus, we look for a small number of moves that still gives the desired time and stability. From these configurations, computer analysis determines which is most uniform.

For various situations, we propose an optimal solution. The bases of the sprinkler placement patterns are triangular and rectangular lattices. We craft three patterns to maximize application to the difficult edges and corners.

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- For calm conditions and a level field, the field can be watered with just two moves (the "Lazy Farmer" configuration). However, this approach is unstable, and even weak wind would leave parts of the field dry. With three moves, little stability is gained; so four positions is best.
- The "Creative Farmer" triangular lattice gives both stability and uniformity. The extra time is warranted because of its ability to adapt.
- We obtain even more stability using the "Conservative Farmer" model but at the price of a decrease in uniformity from the "Creative Farmer" approach.

Description of Problem

The goal is to irrigate a 30 m $\times$ 80 m field as uniformly as possible while minimizing the labor/time required. We assume the following equipment and specifications:

- Pipes of 10-cm diameter with rotating spray nozzles of 0.6 cm diameter
- Nozzles are raised about 1 m from the pipe and can spray at angles ranging from 20° to 30°.
- Total length of the pipe is 20 m.
- A water source with a pressure of 420 kPa and a flow rate of 150 L/min.

We consider the following guidelines and assumptions:

- No part of the field should receive more than 0.75 cm/h.
- Every part of the field should receive at least 2 cm every four days.
- Overwatering should be avoided.
- Sprinklers are in working order and rotate 360°, spraying uniformly with respect to rotational symmetry.
- The soil is approximately uniform and the terrain is flat.
- Wind is considered only in terms of stability.
- We can place a water supply pipe through the field along either its width or its length, which has multiple connection spots for the movable pipes.
- For such a small field, any move requires approximately equal time, so we need only minimize the total number of moves M_T.
- In particularly arid areas, evaporation reduces the total water application but with no more than 5 % loss.
- We ignore rainfall, assuming that it is accounted for by delaying waterings.

Definitions and Notation

Let D be a distribution of sprinklers, including placement on the pipe and locations in the field. We consider accumulation over a region R. Let

$M_T(D)$ = total number of moves by the farmer required for a distribution,

$Aver(D, R)$ = average application rate over region R,

$\text{Var}(D, R)$ = variance of the rate of application over region R,

$\max(D, R)$ = maximum rate of application over the region R, and

$\min(D, R)$ = minimum rate of application over the region R.

Pipe Capacity and Resulting Pressure/Radii

Watch out for the Rain Bird

We derive the exit speed, flow, and water-drop drag coefficient for a sprinkler with our conditions and show that it agrees with the Rain Bird 70H 1″ Brass Impact Sprinkler [Rain Bird Agricultural Products n.d.]. We assume laminar flow and use Bernoulli's equation:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2,$$

where

P_i is absolute pressure,

ρ is the density of water,

v_i is speed,

g is the gravitational constant, and

y_i is height.

Because our field is flat, we have $y_1 = y_2$, so the height of our source relative to our sprinklers does not affect the exit speed v_2:

$$v_2 = \sqrt{\frac{2}{\rho}P + v_1^2},$$

where P is the relative pressure. We must first find the speed v_1 of water at our source:

$$v_1 = \frac{150\ \text{L}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} \times \frac{1\ \text{m}^3}{1000\ \text{L}} \times \frac{1}{\pi(0.05)^2\ \text{m}^2} = \frac{1}{\pi}\ \text{m/s}.$$

Plugging v_1 into the equation for v_2, we obtain

$$v_2 = \sqrt{\frac{2}{1000} \times 420 \times 1000 + \frac{1}{\pi^2}} \approx \sqrt{840} \approx 28.9 \text{ m/s}.$$

That's fast (about 60 mph)! It may be too fast. This exit speed does not take into account friction in the pipes, for which we propose an attenuation factor. The volume out of the sprinkler is the speed times the cross-sectional area of the sprinkler times the attenuation factor:

$$Q = C_s A_c \sqrt{\frac{2}{\rho} P},$$

where

Q is the discharge in (m^3/s),

C_s is the attenuation factor, and

A_c is the cross-sectional area (m^2).

Using pressure and discharge data from Rain Bird Agricultural Products [n.d.], we find the attenuation factor to be

$$C_s = \frac{Q}{A_c \sqrt{\frac{2}{\rho P}}} = \frac{3.17 \times \frac{1}{3600}}{\pi (0.003175)^2 \sqrt{800}} \approx 0.983.$$

This value shows very little loss due to friction. The escape speed with friction is

$$v = 0.983 \times 28.9 \approx 28.5 \text{ m/s}.$$

How many liters flow out of each sprinkler per minute is simply the speed multiplied by the area, converted to liters per minute:

$$\frac{\text{Volume}}{\text{unit time}} = 28.5 \times \pi (0.003)^2 \times \frac{1000 \text{ L}}{\text{m}^3} \times \frac{60\text{s}}{\text{min}} = 48.35 \text{ L/min}.$$

We can therefore use up to three sprinklers without using more than 150 L/min; with more than 3 sprinklers, there will be a pressure drop. To find the new pressure, we use the continuity principle, which states that the volume of water flowing in equals the volume of water flowing out:

$$A_s v_s = n A_N v_N,$$

where

A_s is cross-sectional area of the source,

v_s is speed of water at our source,

n is number of sprinklers,

A_N is cross-sectional area of the sprinkler nozzle, and

v_N is speed out of the sprinkler nozzle.

Solving for v_N, we obtain

$$v_N = \frac{r_s^2}{n\pi r_N^2} = \frac{(5 \times 10^{-2})^2}{n\pi (3 \times 10^{-3})^2} \approx \frac{88}{n} \text{ m/s},$$

where $n > 3$, r_s is the radius of the pipe at the source, and r_N is the radius of the sprinkler nozzle.

For four sprinklers, the exit speed would be 22 m/s and the pressure would be 252 kPa. The pressure needs to be above 280 kPa [Rain Bird Agricultural Products n.d.]. Since too low a pressure would result in a low degree of uniformity, we limit ourselves to at most three sprinklers.

Kinematics Equations

Because water droplets are small and the escape speed is above the terminal speed, drag must be taken into account. We have the following differential equations for speeds in the x- and y-directions:

$$\frac{dv_x}{dt} = -kv_x, \qquad \frac{dv_y}{dt} = -g - kv_y,$$

whose solutions are

$$y(t) = \frac{-g}{k}t + \left(\frac{v_0 k \sin\theta + g}{k^2}\right)\left(1 - e^{-kt}\right) + y_0,$$
$$x(t) = \frac{v_0 \cos\theta}{k}\left(1 - e^{-kt}\right) + x_0.$$

We use the following initial conditions (with some from Rain Bird Agricultural Products [n.d.]) to determine the drag constant:

$$y_0 = 1 \text{ m}, \qquad x_0 = 0, \qquad v_0 = 1 \text{ m}, \qquad \theta = 21^\circ.$$

The published value for the radius for our system is approximately 19.5 m [Rain Bird Agricultural Products n.d.]. Using this distance and the above initial conditions, we determine the drag constant to be $k = 1.203$.

We thus have an equation for how the radius of the water emitted by the sprinkler is determined by the height and angle of the sprinkler. Although we keep our sprinkler at factory settings, the farmer could modify the sprinkler to adjust the radius if needed.

Distribution from Standard Sprinkler

While the sprinklers under consideration cover a disk of radius 19.5 m, the distribution need not be uniform over that area. Large droplets tend to travel farther, but the area near the perimeter is much larger than near the sprinkler head. We discuss various models for this behavior based on empirical data.

Triangular Model

Smajstrla et al. [1997] propose that the water distribution can be modeled as a triangle. That is, the application rate falls linearly as a function of distance from the sprinkler head, disappearing outside the radius. **Figure 1** shows an example with radius 25 ft.

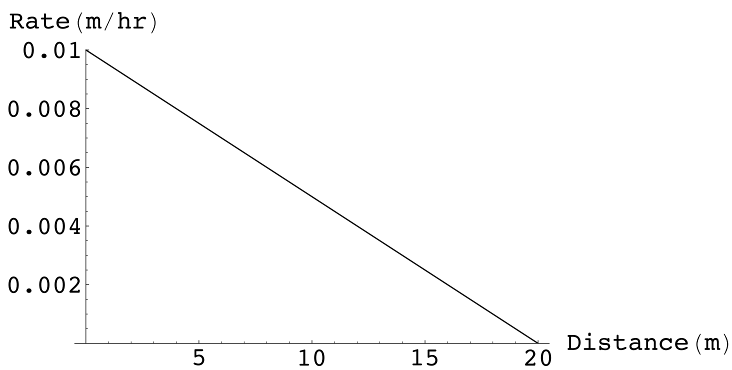

Figure 1. Triangular water distribution (redrawn from Smajstrla et al. [1997]).

In three dimensions, the distribution is cone-shaped, centered about the sprinkler nozzle. To analyze a grid pattern, we sum the water distribution over a cone and analyze the resulting surface.

Experimental/Exponential Decay Model

Louie and Selker [2000] experimentally tested the performance of a Rain Bird 4.37-mm nozzle. The distribution spikes within 2 m of the sprinkler head, then maintains an approximately uniform rate before decaying near the edge of the radius (**Figure 2**). We use exponentials to fit a curve to this graph. We then scale the width and the height of the function to correspond to the radius and water flow of our larger sprinkler:

$$f(r) = (3 \times 0.00267 \times e^{-0.7r} + 0.00267) \times e^{-(r/19.5)^{20}}.$$

To get the three-dimensional distribution, we rotate the function about the z-axis by replacing r with $\sqrt{(x-a)^2 + (y-b)^2}$ for a sprinkler centered at (a, b). In some ways, this distribution is a worst-case scenario because of the large peak about the sprinkler. The curve is based experimentally on where drops landed but does not take into account possible spread on landing.

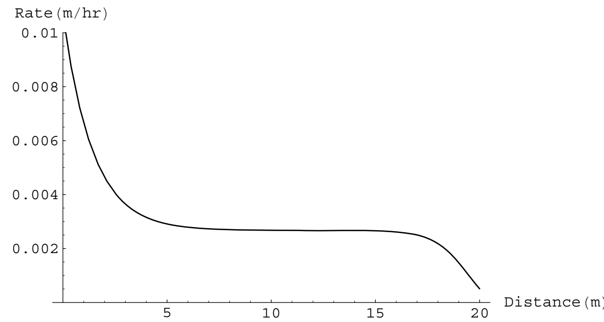

Figure 2. Exponential water distribution (redrawn from Louie and Selker [2000].

Comparison of Models

The exponential decay model is the more realistic of the two models. It forces careful consideration of how long a sprinkler can be left. Any configuration acceptable for this model will most likely work under the triangular model too.

Conclusions

With the exponential model, the application rate near the sprinkler head increases to 0.01 m/h = 10 cm/h. We are constrained to a maximum rate of 7.5 cm/h to avoid damage to the soil and crops. Thus, using the exponential model results in configurations where sprinklers run for less than the full 60 min each hour. We later discuss several methods to minimize the inconvenience that this constraint causes the farmer.

A similar difficulty arises in the triangular model with three sprinklers. The best that we can do for the sprinkler in the middle is to space the sprinkler heads evenly, with one at each end. The distance of separation is then 10 m. Scaling the triangle for the values of the Rain Bird (3.2 m^3/h, 19.5-m radius), we get a peak height of about 8 cm/h; so at 10 m, we get 4 cm/h. The middle sprinkler head would be receiving 16 cm/h, which is over twice the acceptable amount.

For either model, three sprinklers can only be run for a limited time every hour. Thus, our proposed solutions have exactly two sprinklers on the pipe.

Analysis of Standard Grid Patterns

We analyze standard grid patterns. Symmetric designs that cover a rectangular field include squares, rectangles, and triangles. To counteract the effects of varying models of distribution (triangular or exponential), all patterns employ overlapping sprinkler patterns. In most cases, researchers recommend 40–60% overlap of radii to obtain the most uniform distribution, which also tends to be the most stable under windy conditions [Eisenhauer et al. n.d.].

We use the triangular distribution to evaluate grid patterns with the goal of finding the ideal side lengths as a ratio of the radius. Mathematical analysis shows that in terms of uniformity, the ideal rectangle is a square with side $1.1\times$(radius); however, a triangle grid pattern obtains better uniformity, though with smaller spacing, $0.85 \times$ (radius).

Evaluation Methods

We have two primary concerns in evaluating a grid pattern. The minimum value on the surface determines the time required to water the field, so we must watch for too low a minimum value. We measure uniformity by calculating the variance of the distribution. In each case, we consider a unit of the grid, that is, one square or one triangle, and plot the distributions of all sprinklers that water that square. The average rate and variance are

$$\begin{aligned} Aver(D,R) &= \frac{1}{\text{Area}(R)} \int_R D(x,y), \\ \text{Var}(D,R) &= \frac{1}{\text{Area}(R)} \int_R (D(x,y) - Aver(D,R))^2, \end{aligned}$$

where $D(x,y)$ is the distribution and R is the unit region. A large $\text{Var}(D,R)$ means water will be applied nonuniformly and could result in poor growth. To aid in assessing the extent of variation, we also calculate $\max(D,R)$. The difference between $\max(D,R)$ and $\min(D,R)$ gives a measure of how large the variation is.

Rectangular Grids

For each of several vertical separations between sprinklers, from 0.8 to 1.2 times the radius, we consider a range of possible horizontal separations. Generally, the variance decreases and then increases as the horizontal separation increases, defining a clear minimum value, which for all configurations is approximately a horizontal separation of 1.1 times the radius. The best rectangular configurations turns out to be a square of side length $1.1\times$(radius). In the tests, the difference in maximum and minimum correlates closely with the variance, so we use that as the basis for comparison.

Triangular Grids

For the triangular lattice, we must also model surrounding triangles because nearby sprinklers have a significant effect.

We do not consider distances less than $0.8 \times$ (radius) because significant overwatering would take place. Thus, the most uniform configuration is for $0.85 \times$ (radius). This separation distance results in higher uniformity than the rectangular configuration.

The exponential distribution gave similar results on the tests, indicating that triangular set-ups are a good enough approximation for comparing uniformity.

Proposed Irrigation Methods

We proceed to design the pipe network and watering schemes. We make the following assumptions:

- Water is distributed according to the exponential water distribution.
- The sprinklers are unmodified, and we place two of them at the ends of the pipe, separated by 20 m.
- The efficiency of the sprinkler irrigation system is at least 95%, no more than 5% is lost to evaporation and other factors.
- There exists infrastructure that can supply water along the center of the field.

Our goal is a system that provides at least 2 cm of water at every point of the field every 4 days, and no more than 0.75 cm during any single hour. In addition, we would like the pattern of watering to be periodic with period of four days. We compare the different systems using the following criteria:

- required number of moves M_T of the pipes;
- hours of operation of the system;
- stability with respect to factors like wind and equipment malfunctions; and
- uniformity of irrigation.

The water falling right next to the sprinkler is 1 cm/h, which means we cannot have a sprinkler operational for more than 45 min in an hour; so the farmer must come and stop the sprinklers 45 min after they were turned on and to turn them on again (or move them) 15 min later. Since the pipes have valves that can easily be closed and opened, even under pressure, turning them on or off doing consumes only a few minutes. In addition, if one sprinkler is within the radius of another, the time that they can be operational will be severely reduced; we want to avoid such a situation by positioning the sprinklers at the ends on the 20 m pipe.

We divided the field into four 20 m $\times$ 30 m rectangular pieces, each of which is further subdivided into triangles by the two diagonals (**Figure 3**).

It is impossible to water the whole field using our pipe of length 20 m and our sprinklers with their radius of irrigation of 19.5 m, since we cannot water two points separated by more than 19.5 m + 20 m + 19.5 m = 59 m. Therefore, we must move the pipe at least once; and since after 4 days the pipe should be in its initial position, we must move it twice per period. Therefore, our lower

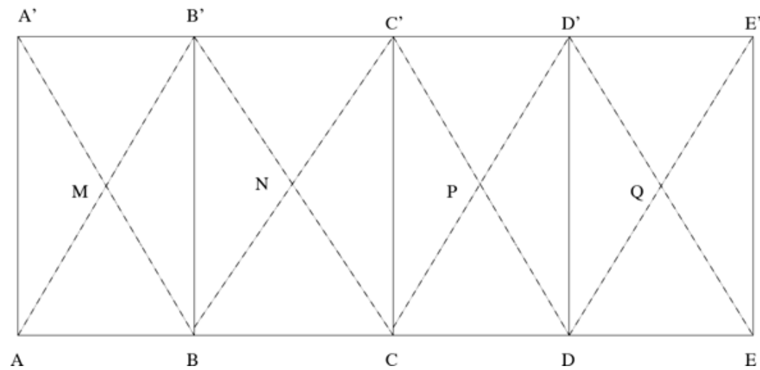

Figure 3. The field, subdivided.

bound on M_T is 2. It would be nice to achieve this minimum. Insight into how to do this can be obtained by drawing circles with radius 19.5 at the points A, C, E, A', C', and E' in **Figure 4**. We must position the sprinklers so that in each circle there is at least sprinkler. This leads to a scheme with two moves.

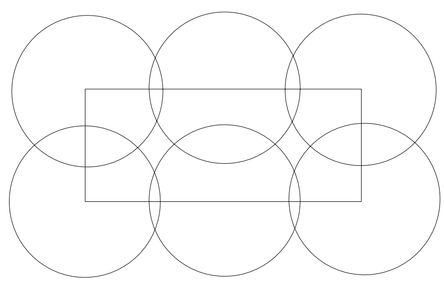

Figure 4. Covering the edges.

The Case $M_T = 2$

Suppose that we position the pipe as shown in **Figure 5**.

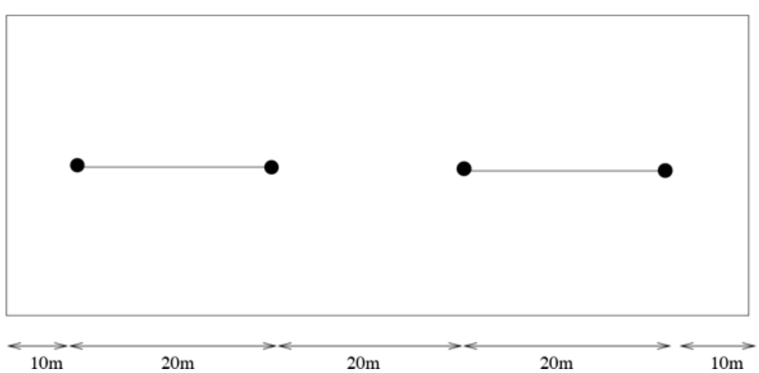

Figure 5. The Lazy Farmer configuration.

The greatest distance from a sprinkler (denoted by a dot) is $\sqrt{10^2 + 15^2} = 18.02$ m, and these points are precisely A, C, E, A', C', and E' in **Figure 5**.

From our exponential water distribution graph (**Figure 2**), the amount of water falling at those points is 2.25 mm/h; but since we operate a sprinkler for at most 45 min, the actual value is 1.68 mm/h. Thus, if we operate the system for 13 hours at each location, we get a minimum of 2.18 cm of water at every point, for more than 2 cm everywhere when we subtract loss due to evaporation. Therefore, the total time the system would be operational is 26 hours, the pipes would have to be moved twice, and the amount of water used would be $(2)(26\text{ h})(45\text{ min/h})(48.35\text{ L/min}) = 113 \times 10^3$ L per period. If the watering were optimal, the required amount of water would be $(30\text{ m})(80\text{ m})(2\text{ cm}) = 48 \times 10^3$ L of water. Therefore, the water efficiency is 42%. As for uniformity, we calculate that the variance is 1.7×10^{-6}, which corresponds to a high degree of uniformity. However, this configuration has a major disadvantage: Even a small change of 2 m in the sprinkler radius (for instance, due to wind or to decrease in pressure in the pipes) can result in distant points such as A receiving no water. Therefore, this configuration, although very uniform and with minimal M_T, is not very stable.

The case $M_T = 3$

We want to have a smaller maximum distance $d_{\max}$ between a point in the field and the nearest sprinkler. Using a similar argument to the the previous case, the resulting configuration should look something like **Figure 6**.

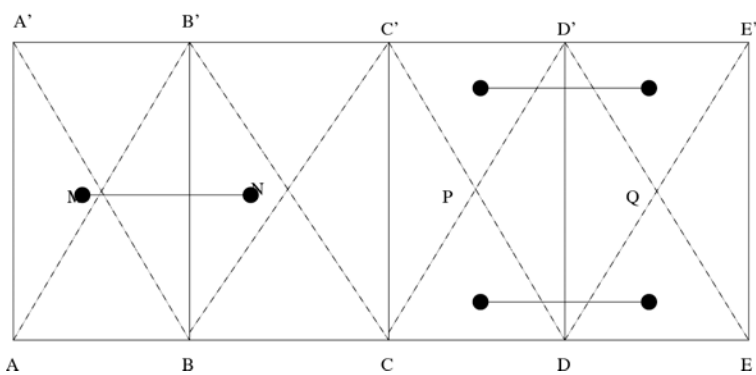

Figure 6. Configuration for $M_T = 3$.

In such a configuration, $d_{\max} \geq 16.5$, so the gain in stability is slight. In addition, there is a huge increase in operational time and water required, to 39 h and 169×10^3 L. Therefore, the case $M_T = 3$ results in bad configurations.

The case $M_T = 4$

With the increase in the number of times that we can move the pipes, the complexity of positioning them increases dramatically, making it nearly impossible to consider all configurations. However, since we want a stable configuration, we should have sprinklers close to the points A, A', E, and E'. In addition, for uniformity, we should preserve some symmetry. The earlier triangular and rectangular patterns can be successfully applied in this case. The best way to reduce peaks in watering is to use a triangular pattern, like the one in **Figure 7**.

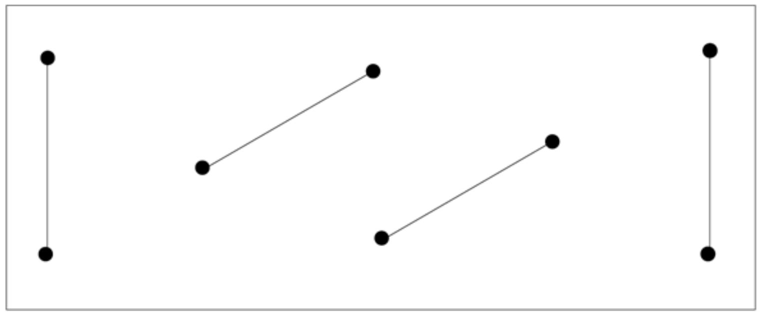

Figure 7. Creative Farmer layout.

The sprinklers are first set at the vertices of equilateral triangles of side 20 m. After that, to minimize instability, the leftmost pipe is translated 5 m to the right and the rightmost 5 m to the left. Then $d_{\max} \leq 14$, which implies that this scheme would work well provided that the wind does not result in more than 25% deviation. This layout has a variance of 3.35×10^{-6}, period of operation of 52 h, and water consumption of 226×10^3 L per period.

Another possibility is sprinklers in a rectangular pattern (**Figure 8**).

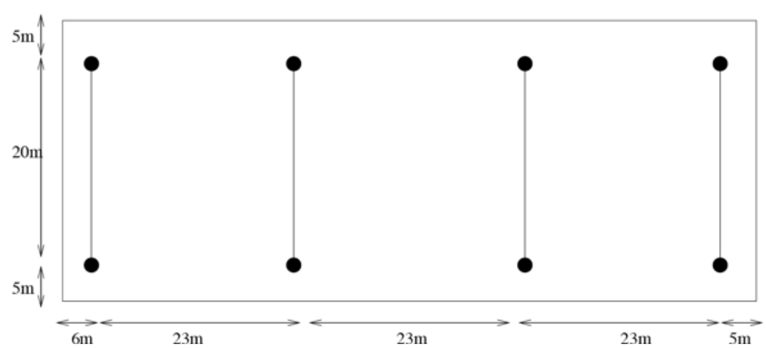

Figure 8. Conservative Farmer layout.

The distance from the sprinklers to the points on the sides is less than 12 m, and the area between two pipe positions is within the radius of four sprinklers and thus would be watered no matter what the direction of the wind is. This irrigation would be good provided that the wind doesn't alter the area covered by more than 7 m. The layout's variance is 4.17×10^{-6}, and the hours of operation and water consumption are the same as in the previous case.

The case $M_T > 4$

Since moving the pipes takes a lot of time, and in addition we have observed how the variance increases, even for the triangular configuration, we can conclude that the case $M_T > 4$ would not lead to a good layout.

Numerical Analysis of Proposed Strategies

With the same criteria used to evaluate the standard grid patterns, we diagnose the algorithms on our 30 m $\times$ 80 m test field. We set up the field on a grid with endpoints (0,0), (80,0), (0,30) and (80,30). This setup allows us to evaluate the entire field and take into account edge effects. The following four strategies are the best performers of several that we analyzed. The corresponding sprinkler placements are shown in **Figure 9**.

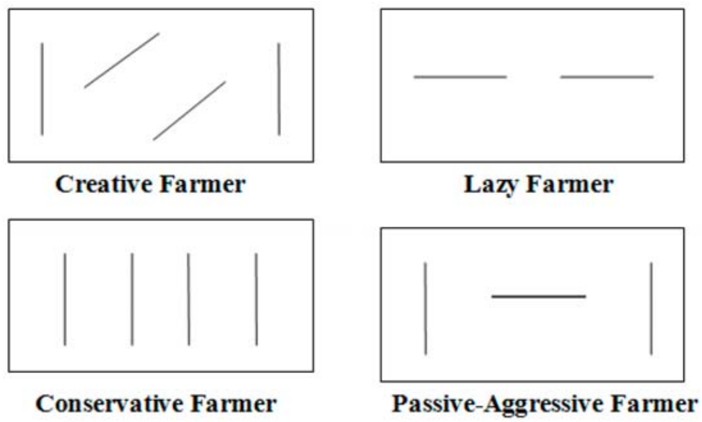

Figure 9. Sprinkle placements for several strategies.

Lazy Farmer System

This strategy uses few moves, has a high degree of uniformity, and can irrigate the entire field—perfect for farmers who would rather be shooting soda cans off of a fence post than lugging around a heavy aluminum tube. It takes 26 h and uses the least amount of water.

The Passive-Aggressive Farmer System

This approach neither improves much upon the stability of the Lazy Farmer system nor saves time by using few pipe moves. Therefore, it would be perfect for an indecisive or passive-aggressive farmer.

The Conservative Farmer System

This strategy is very stable, perfect for a farmer who is very careful and untrusting of the wind. It takes twice as much time as the Lazy Farmer approach and uses twice as much water.

The Creative Farmer System

This is the second most uniform system. The setup is somewhat complicated, but some farmers may be up to the task. It is perfect for a farmer who regularly plays Sudoku and stopped watching the TV show *MacGyver* (1985–1992) because the farmer felt MacGyver lacked ingenuity. It takes just as long as the Conservative Farmer system and uses just as much water.

Conclusion

There are only two worthwhile strategies. The Conservative Farmer system should be used in windy conditions or if the level of the field is somewhat nonuniform. The Lazy Farmer system should be used otherwise, because it is the fastest, easiest, and most uniform.

We base our strategies and conclusions on data from a sprinkler manufacturer We also examined specifications of sprinklers from other manufacturers and found little change in our results.

The methods that we used to evaluate proposed strategies are general. Our method of analysis could be repeated to obtain optimal strategies in other cases with different parameter values (a different pipe, field, or water pressure).

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Team members Matthew Fischer, Brandon Levin, and Nikifor Bliznashki, in Duke apparel.

Team advisor William Mitchener (on right), with friend.

Pp. 269–328 can be found on the *Tools for Teaching* 2006 CD-ROM.

A Schedule for Lazy but Smart Ranchers

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Summary

We determine the number of sprinklers to use by analyzing the energy and motion of water in the pipe and examining the engineering parameters of sprinklers available in the market.

We build a model to determine how to lay out the pipe each time the equipment is moved. This model leads to a computer simulation of catch-can tests of the irrigation system and an estimation of both distribution uniformity (DU) and application efficiency of different schemes of where to move the pipe. In this stage, DU is the most important factor. We find a schedule in which one sprinkler is positioned outside of the field in some moves but higher resulting DU (92%) and saving of water.

We determine two schedules to irrigate the field. In one schedule, the field receives water evenly during a cycle of irrigation (in our schedule, 4 days), while the other schedule costs less labor and time. Our suggested solution, which is easy to implement, includes a detailed timetable and the arrangement of the pipes. It costs 12.5 irrigation hours and 6 equipment resets in every cycle of 4 days to irrigate the field with DU as high as 92%.

Assumptions and Definitions

- The weather is "fine" and the influence of wind can be neglected.
- The whole system is "ideal" in that evaporation, leaking, and other water loss can be neglected.

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- The water source can be put at any position of the field. In practice, a tube can be used to transport water from the pump to the pipe set.
- No mainline exists, so that all pipes join together and can be put at any position of the field.
- The time for a rancher to uncouple, move, and reinstall the pipe set is half an hour.
- The discharge of any sprinkler is the same.
- The design pressure of sprinklers is about 400 kPa and the sprinkler is an impact-driven rotating sprinkler.
- The diameter of the riser is the same as that of the pipe.
- The water pressures in pipes are assumed to be the same. In practice, there is a slight difference.

Table 1.
Variables and constants.

Variable	Definition	Units
p_{in}	Water pressure in the pipe before a junction	kPa
p_{out}	Water pressure in the pipe after a junction	kPa
p_{up}	pup Water pressure at the sprinkler at a junction	kPa
v_{in}	vin Water speed in the pipe before a junction	m/s
v_{out}	Water speed in the pipe after a junction	m/s
v_{up}	Water speed at the sprinkler at a junction	m/s
A_{in}	Section area in the pipe before a junction	cm^2
A_{out}	Section area in the pipe after a junction	cm^2
A_{up}	Section area of the sprinkler at a junction	cm^2
h	Height of the sprinkler above the pipe	m
Δt	Change in time	s
v_{source}	Speed of the water source	m/s
n	Number of sprinklers	–
$\text{distr}(r)$	Distribution function of precipitation profile	–
p	Precipitation rate	–
R	Sprinkling range	m
r	Distance from a sprinkler	m
r_i	Distance from the ith sprinkler	m
α	Obliquity of the precipitation profile	rad
$\text{pr}(r)$	Precipitation function of one sprinkler	cm/min
DU	Distribution uniformity of an irrigation system	–
Constant	**Definition**	**Value**
ρ	Density of water	1.0 kg/L
g	Acceleration of gravity	9.8 m/s^2

Problem Analysis

Our goal is to determine the number of sprinklers and the spacing between them and find a schedule to move the pipes, including where to move them. Our approach can be divided into three stages:

- Determine the number of sprinklers. We figure out the pressure and speed of water from each sprinkler and then determine possible sprinkler numbers from engineering data.
- Determine where to put the pipes. We consider major factors, such as sprinkling time, moving time and distribution uniformity (DU). Since the pipe positions depend on the number of sprinklers and the precipitation profile, we just work out some problem-specific cases. However, our method can be used to solve any practical case.
- Determine the schedule to move the pipes. Referring to the water need of the field, we make a schedule that minimizes the time cost, which, obviously, is closely related to the number of moves of the pipes.

Model Development

Stage 1: Water Pressure and Speed

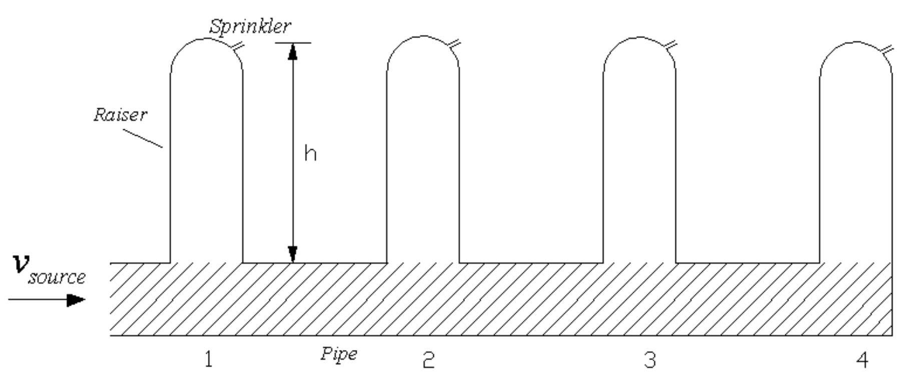

Figure 1. Overall sketch for four sprinklers and four junctions. The pressure throughout the shaded area is the same, due to our assumption.

We apply the law of conservation of energy. The work done by the forces is

$$F_{\text{in}}s_{\text{in}} - F_{\text{up}}s_{\text{up}} - F_{\text{out}}s_{\text{out}} = p_{\text{in}}A_{\text{in}}v_{\text{in}}\Delta t - p_{\text{up}}A_{\text{up}}v_{\text{up}}\Delta t - p_{\text{out}}A_{\text{out}}v_{\text{out}}\Delta t.$$

The decrease in potential energy is

$$-mgh = -\rho g A_{\text{up}} v_{\text{up}} \Delta t h.$$

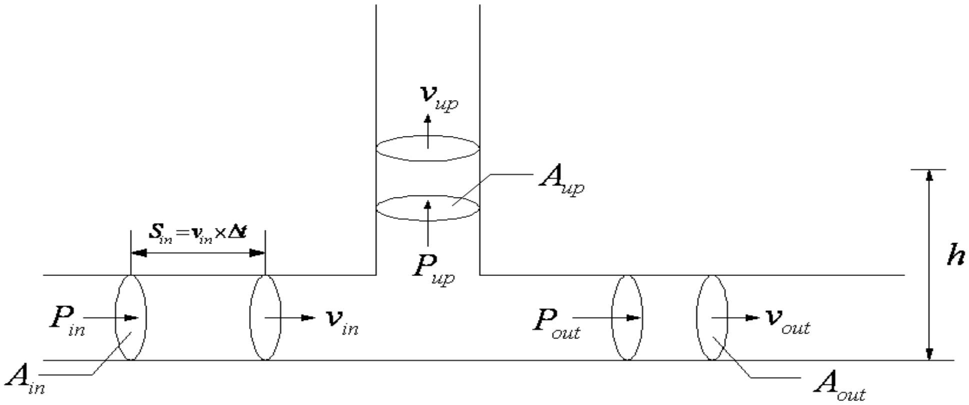

Figure 2. Sketch of one junction.

The increase in kinetic energy is

$$\tfrac{1}{2}mv_{\text{up}}^2+\tfrac{1}{2}mv_{\text{out}}^2-\tfrac{1}{2}mv_{\text{in}}^2=\tfrac{1}{2}\rho A_{\text{up}}v_{\text{up}}\Delta t v_{\text{up}}^2+\tfrac{1}{2}\rho A_{\text{out}}v_{\text{out}}\Delta t v_{\text{out}}^2-\tfrac{1}{2}\rho A_{\text{in}}v_{\text{in}}\Delta t v_{\text{in}}^2.$$

Putting these together, because of the law of conservation of energy, yields

$$\begin{aligned} p_{\text{in}}A_{\text{in}}v_{\text{in}}\Delta t - p_{\text{up}}A_{\text{up}}v_{\text{up}}\Delta t - p_{\text{out}}A_{\text{out}}v_{\text{out}}\Delta t - \rho g A_{\text{up}}v_{\text{up}}\Delta t h = \\ \tfrac{1}{2}\rho A_{\text{up}}v_{\text{up}}\Delta t v_{\text{up}}^2+\tfrac{1}{2}\rho A_{\text{out}}v_{\text{out}}\Delta t v_{\text{out}}^2-\tfrac{1}{2}\rho A_{\text{in}}v_{\text{in}}\Delta t v_{\text{in}}^2. \end{aligned} \tag{1}$$

Since the fluid is incompressible, we have

$$A_{\text{in}}v_{\text{in}} = A_{\text{up}}v_{\text{up}} + A_{\text{out}}v_{\text{out}}. \tag{2}$$

The diameters are all the same:

$$A_{\text{in}} = A_{\text{up}} = A_{\text{out}} = \pi\left(\frac{10\text{ cm}}{2}\right)^2. \tag{3}$$

According to the assumptions, at every junction we have

$$p_{\text{in}} = p_{\text{out}} = 420\text{ kPa}, \tag{4}$$

$$v_{\text{up}} = \frac{v_{\text{source}}}{n}, \tag{5}$$

where

$$v_{\text{source}} = \frac{150\text{ L/min}}{\pi\left(\frac{10\text{ cm}}{2}\right)^2} = 0.318\text{ m/s}.$$

Therefore, from **(2)**, **(3)**, and **(5)**, we have for the ith junction

$$v_{\text{in}} = v_{\text{source}}\left(1-\frac{i-1}{n}\right), \qquad v_{\text{out}} = v_{\text{source}}\left(1-\frac{i}{n}\right).$$

Putting **(1)–(5)** together, we can obtain p_{up} at every junction. In fact, at the last (i.e., the nth) junction, we have

$$v_{\text{in}} = v_{\text{up}} = \frac{v_{\text{source}}}{n}, \qquad v_{\text{out}} = 0.$$

Putting these into **(1)**, we get

$$p_{\text{up}} = p_{\text{in}} - \rho g h,$$

which means that the pressure at the last sprinkler is independent of n.

Commonly, h is about 0.5 m to 1.5 m, and even if we assume that $h = 1.5$ m, the v_{up} at the last junction will be 405 kPa, not far from 420 kPa. (If $h = 0.5$ m, the last v_{up} will be 415 kPa.)

From these equations, we know that v_{up} at the last junction differs the most from 420 kPa, that at the first junction is the closest to 420 kPa (and below 420 kPa), and the values are decreasing slowly from junction 1 to junction n. We conclude that the values of v_{up} at every junction are all below 420 kPa but very close to 420 kPa, no matter how many sprinklers. This fact explains our assumption that the design pressure of sprinklers is about 400 kPa.

Information and Analysis of Sprinklers

The impact-driven sprinkler is the most widely used rotating sprinkler and the one that we assume is used. Some rotating sprinklers have a sector mechanism that can wet either a full circle or a circular sector. There are three main structure parameters of sprinklers: intake line diameter, nozzle diameter, and nozzle elevation angle. An empirical formula gives the spraying range of an impact-driven sprinkler:

$$R = 1.70 d^{\,0.487} h_p^{0.45},$$

where d is the nozzle diameter and h_p is the operational pressure head.

Table 2 shows data on impact sprinklers. Since for our problem the design pressure of the sprinklers is 400 kPa, we have medium-pressure sprinklers; in fact, they have the best application uniformity.

Table 2.

Data on sprinklers [Zhu et al. 1989].

Type	Design pressure (kPa)	Range (m)	Discharge (m^3/h)
Low pressure	<200	<15.5	<2.5
Medium pressure	200–500	15.5–42	2.5–32
High pressure	>500	>42	>32

Table 3 shows data on medium-pressure impact-driven sprinklers with 6-mm nozzle diameter. For sprinklers working at 400 kPa (as assumed), the discharge is 2.5–3.5 m^3/h and spraying range is 18.5, 19, or 19.5 m; we use 19 m as the range. The discharge of the source is 150 L/min = 9 m^3/h; thus, to fit every sprinkler's actual discharge to the design discharge, the number of sprinklers should be 3 or 4, because 9/3=3 or 9/4=2.25, which are within the range 2.5–3.5 m^3/h (or close to it).

Table 3.
Data on nozzles with diameter 6 mm [Zhu et al. 1989].

Model	Nozzle diameter (mm)	Design pressure (kPa)	Discharge (m^3/h)	Range (m)
PY-$_1$15	6	200	1.23	15.0
		300	1.51	16.5
PY-$_1$20	6	300	2.17	18.0
		400	2.50	19.5
PY-$_1$S20A (four nozzles)	6 ($\times$ 4)	300	2.99	17.5
		400	3.41	19.0
PY$_1$S20	6	300	2.22	18.0
		400	2.53	19.5
15PY$_2$22.5	6	350	2.40	17.0
		400	2.56	17.5
15PY$_2$30	6	350	2.40	18.0
		400	2.56	18.5

Sprinklers with higher design pressure tend to have larger wetted diameters. However, deviations from the manufacturer's recommended pressure may have the opposite effect (increase in pressure, decrease in diameter), and uniformity will probably be compromised. **Figure 3** shows typical precipitation distribution of one sprinkler with low, correct, and high sprinkler pressure.

In practice, people use "catch-can" data to generate a precipitation profile of a "hand-move" irrigation system. That is, they put cans evenly in the field to catch the water; the precipitation profile of the irrigation is given by the amounts of water in the catch-cans.

One measure of how uniformly the water is applied to the field is Distribution Uniformity (DU) [Merkley and Allen 2004]:

$$\text{DU} = \frac{\text{average precipitation of low quarter}}{\text{average precipitation rate}} \times 100\%. \qquad \textbf{(6)}$$

Usually, DUs of less than 70% are considered poor, DUs of 70–90% are good, and DUs greater than 90% are excellent. A bad DU means that either too much water is applied, costing unnecessary expense, or too little water is applied, causing stress to crops. There must be good DU before there can be good irrigation efficiency [Rain Bird Agricultural Products n.d.]. To simplify our calculation, we approximate the precipitation profile of a single sprinkler (in **Figure 3b**) to a function $\text{distr}(r)$, which means that the relative precipitation rate in the position with a distance r from the sprinkler (**Figure 5**).

Stage 2: Scheduling the Irrigation

A schedule to move the pipes includes both where to move them and how long to leave them. We imagine a fixed irrigation system consisting of several 20 m pipes. If the system can meet the needs of the crops nicely—that is,

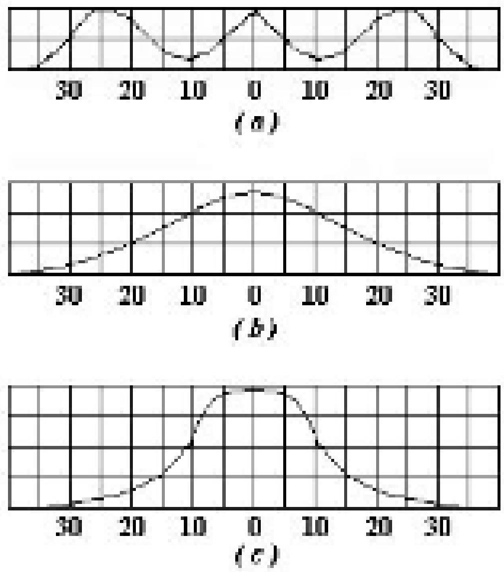

Figure 3. Relation between pressure and precipitation distribution (redrawn from Zhu et al. [1989]). **a)** Pressure is too low. **b)** Pressure is OK. **c)** Pressure is too high.

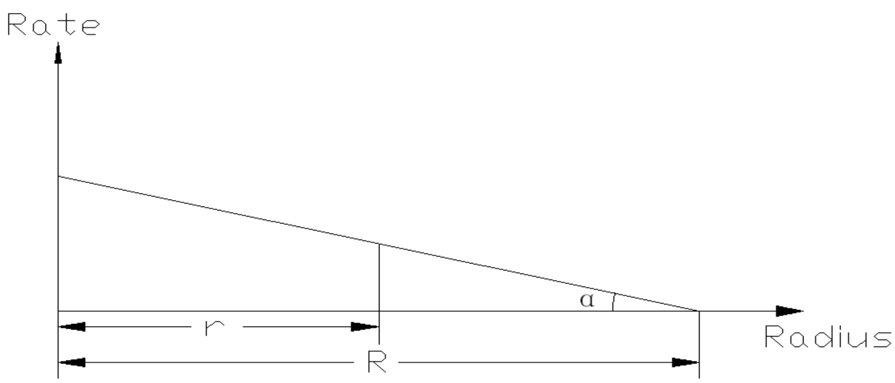

Figure 5. Precipitation rate vs. distance to the sprinkler.

with high Distribution Uniformity (DU)—then we just move the pipe from one position to another. So, we determine where to move the pipe by laying out a system of several 20 m pipes, and then decide for how long we should water the field before making the next move. First, we use a simulation of catch-can analysis to choose a layout with a high DU.

Catch-can Analysis

Since the water sprayed by a sprinkler has a determined distribution distr(r), we use the following method to simulate the catch-can test.

For rectangular spacing (**Figure 6a**), we consider the rectangular region between four adjacent sprinklers. We pick 900 positions evenly distributed in the region. For each position, we calculate its relative precipitation rate p:

$$p = \sum_i \mathrm{distr}(r_i),$$

where $r_i$is the distance from the ith position to the sprinkler. Using **(6)**, we calculate the DU of this irrigation system.

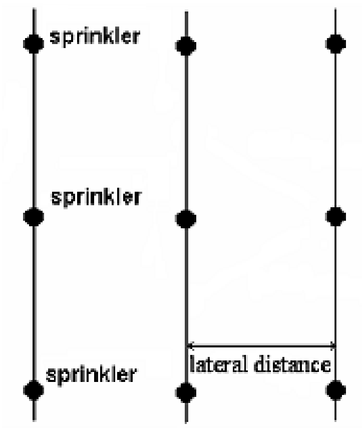

Figure 6a. Rectangular spacing.

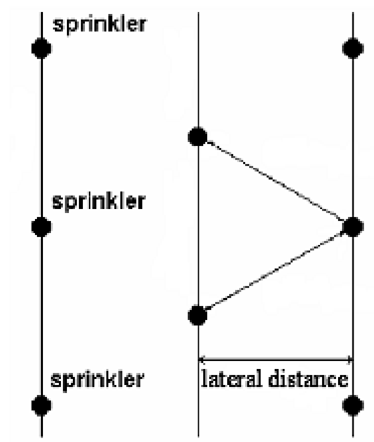

Figure 6b. Triangular spacing.

As we've already deduced, the number of the sprinklers should be 3 or 4, thus the sprinkler distance will be either 10 m ($= 20$ m$/(3-1)$) or 6.67 m ($= 20$ m$/(4-1)$). So the DU is a function of the lateral distance. And this can also be applied to triangular spacing (**Figure 6b**). From the simulation, we get the results in **Figure 7**.

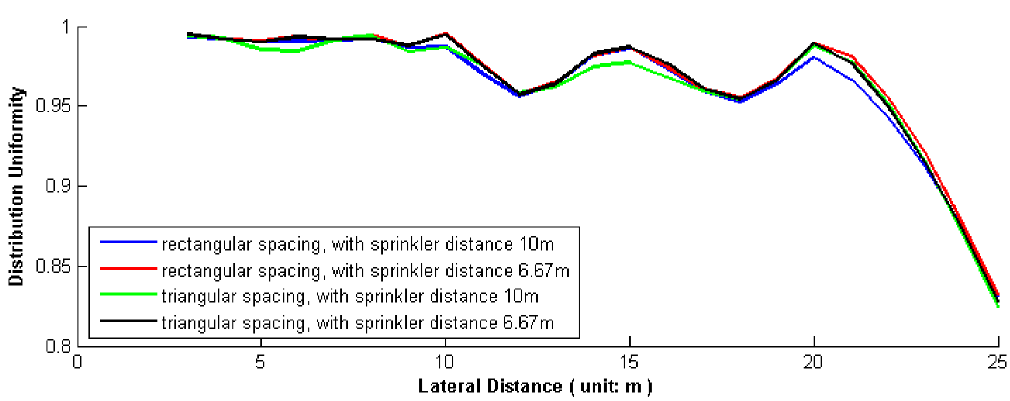

Figure 7. DU vs. lateral distance, in 4 different situations.

The simulation shows that when lateral distance is ≤ 20, DU is acceptable ($\geq 90\%$), regardless of the spacing and whether the sprinkler distance is 6.67 m or 10 m. But since larger lateral distance results in smaller amount of time required to irrigate the field (the number of moves to make will be fewer, we pick 20 m as the lateral distance. **Figure 8** and **9** show the precipitation profile for the irrigation systems with sprinkler distance 10 m and lateral distance 20 m.

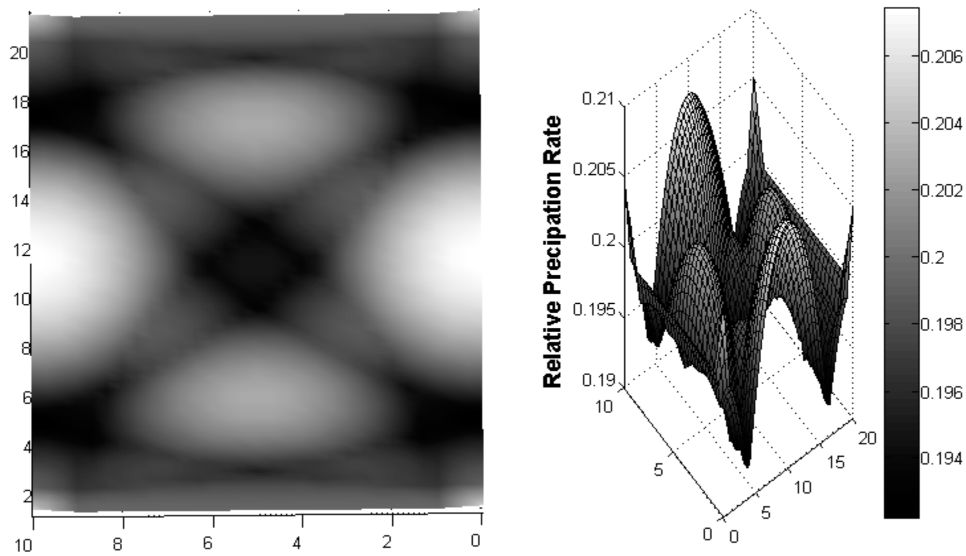

Figure 8. DU=98.1%. Left: Precipitation profile for rectangular spacing with sprinkler distance 10 m and lateral distance 20 m. Right: The 3D form of the precipitation profile.

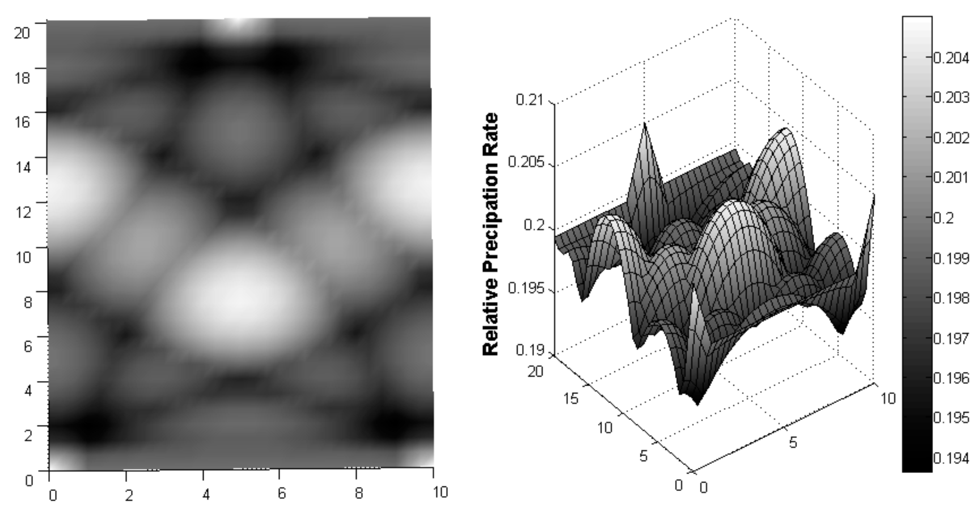

Figure 9. DU=98.7%. Left: Precipitation profile for triangular spacing with sprinkler distance 10 m and lateral distance 20 m. Right: The 3D form of the precipitation profile.

Considering that the field (30 m $\times$ 80 m) is not large enough to implement triangular spacing when the pipe is 20 m long, we use rectangular spacing with only 0.7% negligibly weaker DU. Before we layout the pipe set, we should first determine the maximum distance from the edge of the field to the sprinklers so that the DU is acceptable. We simulate a catch-can test on the rectangular region on the edge of the field (**Figure 10**).

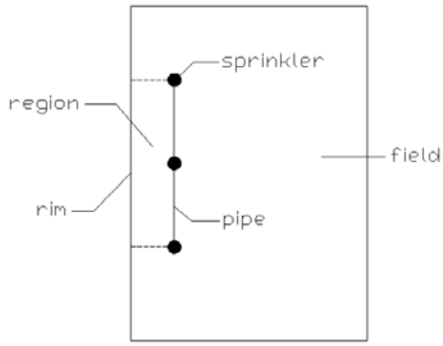

Figure 10. Region to simulate a catch-can test.

The result is in **Figure 11**. The maximum length between the edge of the field and the sprinklers is 5 m, with an acceptable DU of 83%.

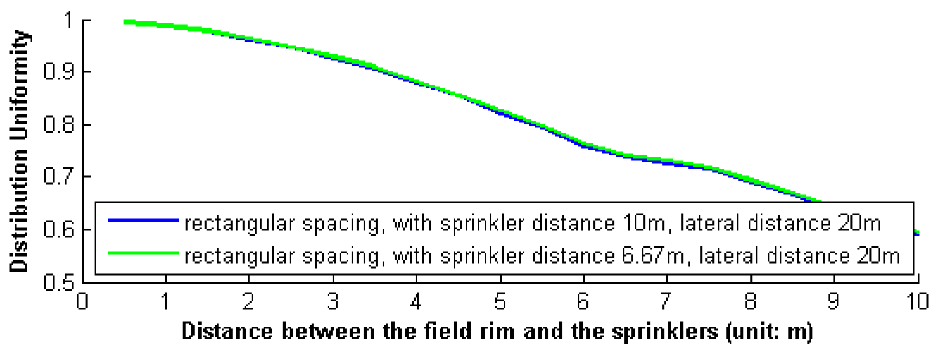

Figure 11. DU vs. distance from the edge to the sprinkler in two different situations.

Layout of the Pipe Set

Having three or four sprinklers along the pipe makes only a slight difference. Considering that the spraying radius (19 m) is too large compared with the sprinkler distance 6.67 m for four sprinklers, we choose to have three. Thus, there are only two feasible layouts (**Figures 12** and **13**). Layout 1 requires five moves and setups of the pipes, while Layout 2 requires six.

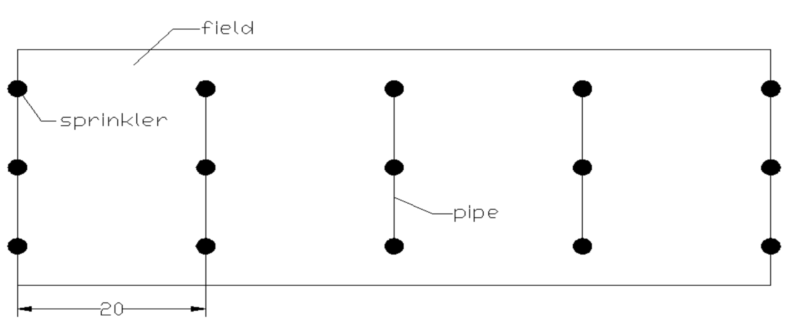

Figure 12. Layout 1.

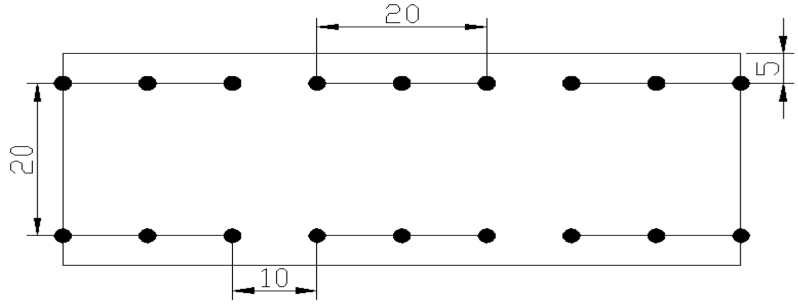

Figure 13. Layout 2.

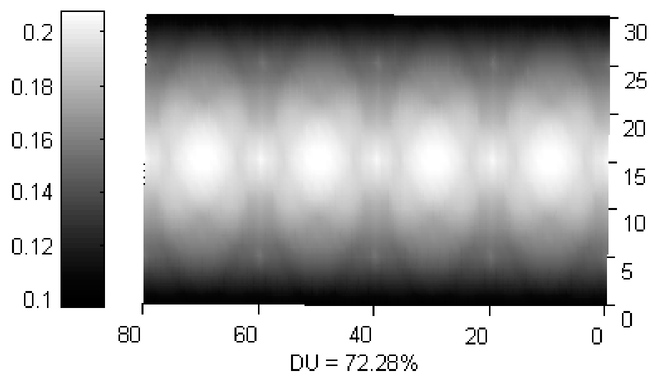

Figure 14. Catch-can test simulation result of Layout 1.

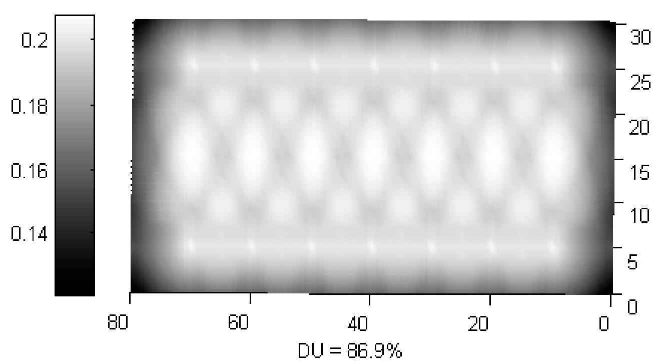

Figure 15. Catch-can test simulation result of Layout 2.

We abandon Layout 1 because it has a very poor DU (**Figure 14**). After slightly changing the lateral distance in Layout 2 (**Figure 15**), we finally achieve a best DU of 89.5% in Layout 3 (**Figure 16**).

Then, if we are brave enough to move some sprinklers outside of of the field, we achieve an even higher DU with Layout 4 (**Figure 17**).

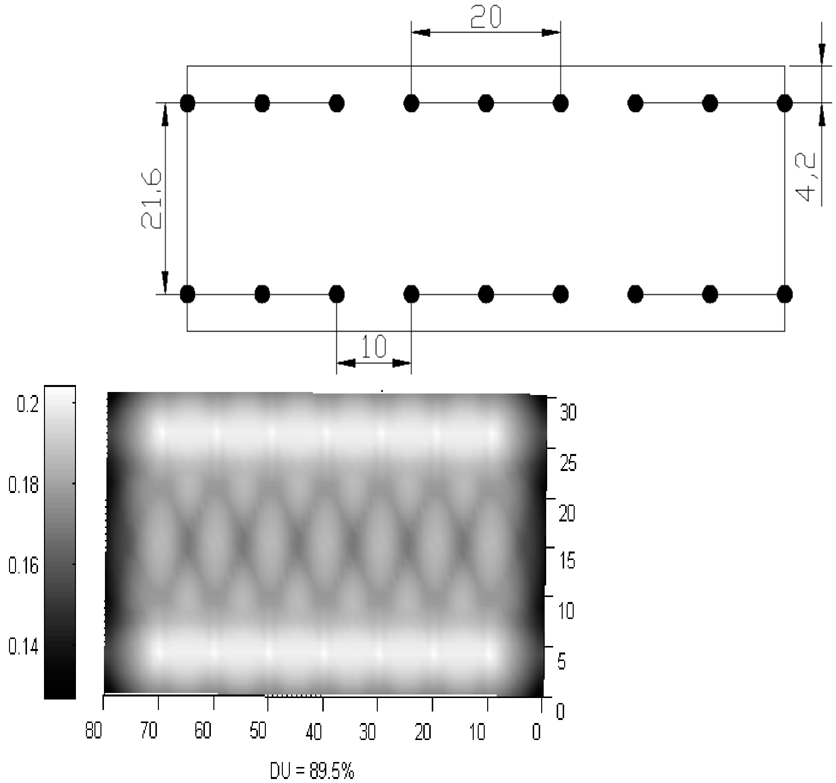

Figure 16. Upper: Layout 3. Lower: Catch-can test simulation result of Layout 3.

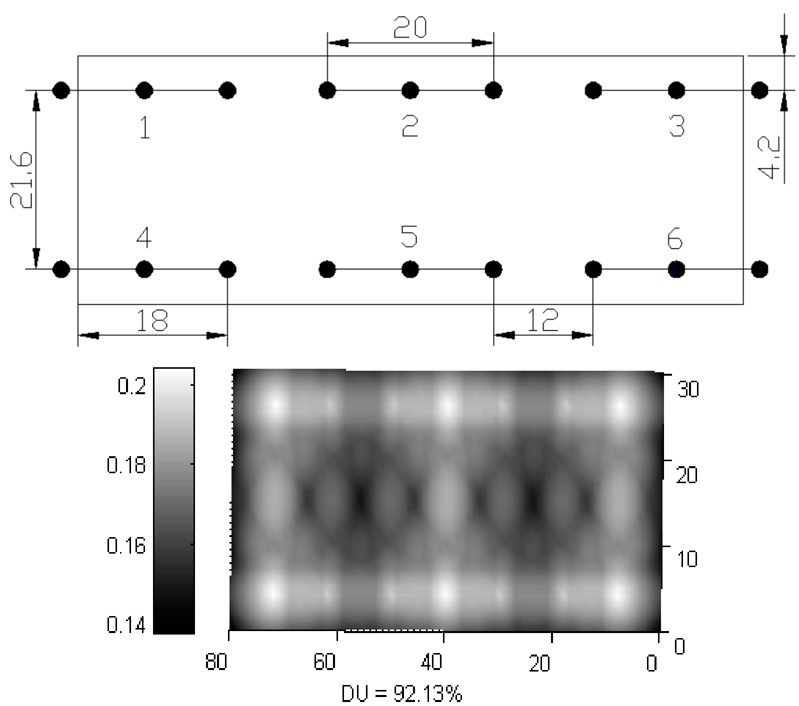

Figure 17. Upper: Layout 4. Lower: Catch-can test simulation result of Layout 4.

Calculation of the Precipitation

To meet the problem's constraints that in any part of the field,

Constraint A: The precipitation rate is ≥ 2 cm/4 d;

Constraint B: The precipitation rate is ≤ 0.75 cm/h,

we should calculate the precipitation rate of the system in Layout 4 before scheduling the interval to irrigating the field and to move the pipe. The precipitation area of a sprinkler should be a circle with a radius R, as **Figure 18** shows.

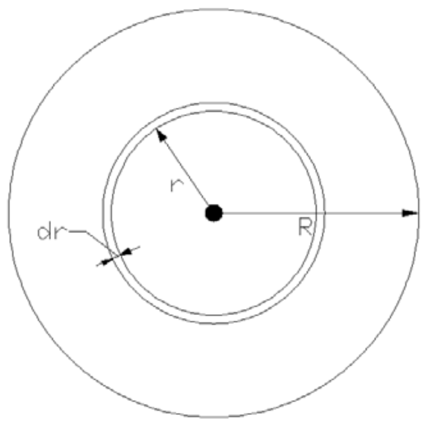

Figure 18. Precipitation area of a single sprinkler.

The profile of the precipitation rate distribution is in **Figure 5**. To figure out the precipitation rate at a point a distance r from the sprinkler, we first calculate the angle α in **Figure 5**. As we normalize the distribution, we get

$$\int_0^R [(R-r)\tan\alpha] 2\pi r \, dr = 1,$$

so

$$\tan\alpha = \frac{3}{\pi R^3}.$$

Then the precipitation rate is

$$\mathrm{pr}(r) = v(R-r)\tan\alpha = \frac{3(R-r)}{\pi R^3} v,$$

where $R = 19$ m, $v = 50$ L/min. With Matlab, we calculated the precipitation rate at each point in the 80 m $\times$ 30 m field, with the irrigation system working only once (**Figure 19 Right**) and after a complete cycle of moving the equipment and irrigating (**Figure 19 Left**).

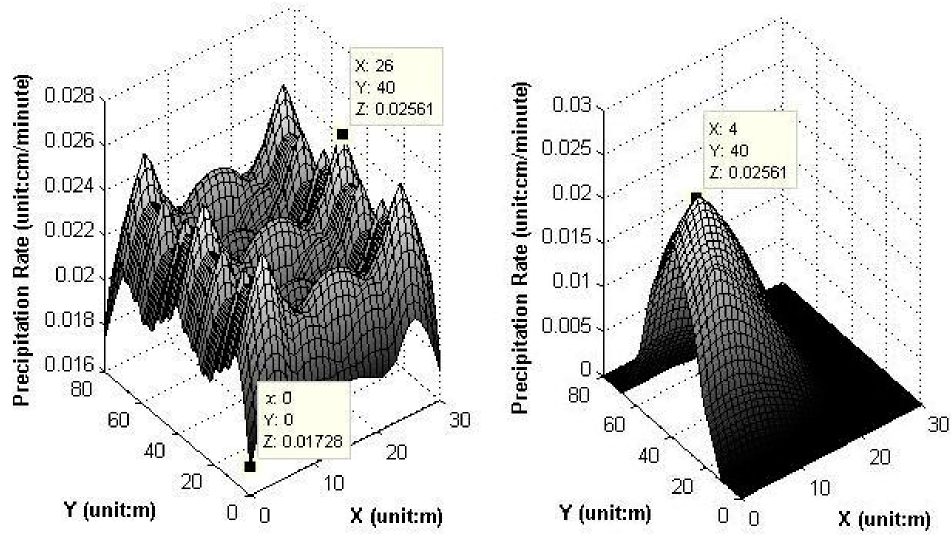

Figure 19. Precipitation rate of water in the field with Layout 4.
Left: The effect of six pipes together. Right: The effect of a 20 m pipe working alone.

Scheduling the Irrigation Time

Figure 19 Right shows that when the sprinklers (only one 20-m pipe at a time) are working, the maximum precipitation rate is 0.02561 cm/min. To satisfy Constraint B, the period of irrigation should be less than

$$\frac{0.75 \text{ cm/h}}{0.02561 \text{ cm/min}} \approx 29 \text{ min/h}.$$

Because the shorter the interval of irrigation, the more frequently the farmer must move the pipe, we choose a large but easy to implement interval: 25 min/h. **Figure 19 Left** shows that after a complete cycle of irrigation of the whole field the minimum precipitation rate is 0.0173 cm/min. To satisfy Constraint A, the period of irrigation should be longer than

$$\frac{2 \text{ cm/4d}}{0.0173 \text{ cm/min}} \approx 116 \text{ min/4d}.$$

To meet this requirement, we irrigate the same place five times, each time for 25 min, or 125 min in all. Using the same method, we calculate the same parameters for Layout 3.

Layout 4 not only has higher DU than Layout 3 but also saves 17% of the water, because Layout 3 has a smaller minimum precipitation rate (0.0160 vs. 0.0173), which leads to more irrigation time (150 min vs. 125 min) in order to satisfy Constraint A. So we choose Layout 4 as our solution.

Stage 3: Schedule Design

Our schedules can achieve a DU as high as 92.1%. We give two concrete timetables; one requires considerably less moving time (3 h vs. 15 h, per 4-days) but waters less evenly on average. Both schedules have a 12.5-h irrigation time in one cycle with a DU of 92%. Using a sprinkler with a sector mechanism, we can control the range of the sprinkler at the edges of the field, to reduce water waste. [EDITOR'S NOTE: We omit the tables.]

Strengths and Weaknesses

Strengths

- We use real data on sprinklers to determine the number of sprinklers.
- We establish a model based on the engineering knowledge of sprinklers, find out the overall precipitation distribution, and then find an optimal schedule.
- Our model for the layout of the irrigation system is sprinkler-independent. If a sprinkler's precipitation profile is known, we can determine the precipitation profile across the whole field.
- The placement and schedule is very clear and easy to implement.

Weaknesses

- Water pressure in the pipe may vary, and so the discharge of the sprinklers may not be exactly the same.

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Optimization of Irrigation

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Summary

We determine a schedule for a hand-move irrigation system to minimize the time to irrigate a 30 m $\times$ 80 m field, using a single 20-m pipeset with 10-cm-diameter tube and 0.6-cm-diameter rotating spray nozzles. The schedule should involve a minimal number of moves and the resulting application of water should be as uniform as possible. No part of the field should receive water at a rate exceeding 0.75 cm per hour, nor receive less than 2 cm in a four-day irrigation circle. The pump has a pressure of 420 KPa and a flow-rate of 150 L/min.

The sprinklers have a throw radius of 13.4 m. With a riser height of 30 in, the field can be irrigated in 48 h over four days. Moreover, a single sprinkler is optimal. The pipes should be moved every 5 h and be at least 21 m apart. The resulting irrigation has precipitation uniformity coefficient of .89 (where 1 would be maximum uniformity).

We deal with each constraint in turn. Using geometrical analysis, we convert the coverage problem to determining the least number of equal-sized circles that could cover the field. We perturb the solution to optimize uniformity, by applying a Simultaneous Perturbation Stochastic Approximation (SPSA) optimization algorithm. We perturb this solution further to find the minimal number of pipe setups, by experimentally "fitting" the pipesets through the sprinklers. The rationale for perturbation is that some drop in uniformity can be tolerated in favor of minimizing the number of setups while still ensuring that we irrigate the entire field. We feed the optimal layout of pipe setups to another algorithm that generates an irrigation schedule for moving the pipes.

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Assumptions

Main Assumptions

- The sprinkler has a throw radius of 13.4 m.
- Zero wind conditions. While wind affects precipitation uniformity, we do not explore this option.
- The field is reasonably flat, which allows us to assume equal water pressure at sprinkler nozzles.
- The rancher operates in 12-h workdays.
- All sprinklers operate on a 30-in riser. This is the most common riser configuration that we found.
- The rancher does not use the sprinklers when it rains.
- The sprinkler application rate profile is semi-uniform for the rotating spray nozzle sprinkler.

Other Assumptions

- There is only one accessible water source.
- The boundary of the field is lined with pipes that connect to the water source.
- Each pipe placement must be perpendicular to and touching one of the field boundaries.
- Set up time for the pipeset does not take more than an hour.
- The flow rate and water pressure from the source remains constant.
- As nozzle size increases, so does the flow rate loss.
- All sprinklers operate identically and do not malfunction.
- The throw radius of the sprinklers may exceed the bounds of the field.
- No sprinkler is placed outside the boundaries of the field.
- At the end of the workday, the sprinklers are shut off; the pipeset need not be disassembled.

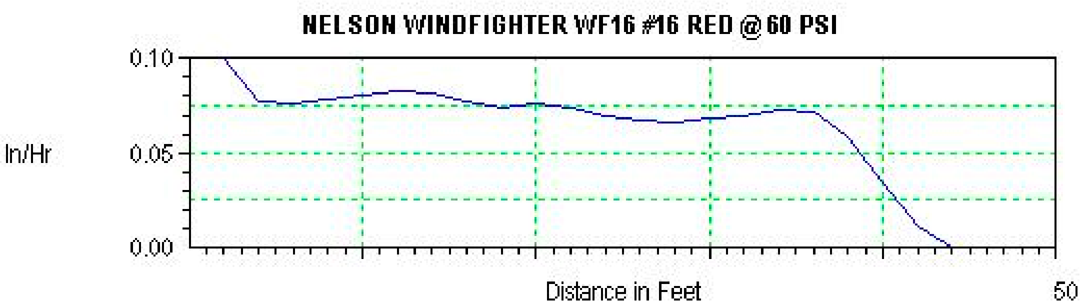


Figure 1. Sprinkler profile [Zoldoske and Solomon 1988].

Possible Sprinkler Profiles

We did not find any sprinkler application rate profiles for a 0.6 cm nozzle but we did find a profile for the Nelson Wind Fighter WF16 with a #16 Red nozzle (1/8″ ≈ 0.3 cm); **Figure 1** shows its application rate profile at 60 psi.

We assume that the profile for a 1/4″ (≈ 0.6 cm) nozzle would be similar but with a higher application rate. The flow rate for the WF16 with a 1/8″ nozzle is 3.42 gal/min; for our nozzle diameter of 0.6 cm = 0.236″, we have a flow rate of 12.57 gal/min, so we estimate that the application rate is 3.3 times as great, taking into account an increased loss due to the sprinkler (see later for the formulas used).

Model

Overall Approach

First, we tackle the requirement of sprinkling the entire field. Using geometrical analysis, we reduced this problem to a covering problem, which translates to finding the least number of equally-sized circles that can cover any given area.

However, this solution results in placing the sprinklers outside the field boundaries, so we perturb the solution to readjust the placement while maintaining complete coverage of the field.

We then use this new solution as a blueprint for finding the minimal number of pipe setups by experimentally "fitting" the pipesets through the sprinklers (if possible). We use an algorithm that iteratively perturbs the sprinkler layout and finds the minimum number of pipe setups. After a specified number of iterations, the algorithm outputs the minimum found. The rationale for perturbation is that we are willing to sacrifice some uniformity in order to find the least number of setups, while simultaneously ensuring that we still irrigate the entire field.

We feed the layout of pipe setups to another algorithm to generate an irrigation schedule.

Simulating Sprinkler Irrigation

Given the sprinkler positions, the sprinkler precipitation profile, and the length of time that they are on, we simulate in Matlab the sprinkler irrigation of the field. **Figure 2** shows the the output for the sprinkler profile of **Figure 1** with the sprinkler running for 1 h.

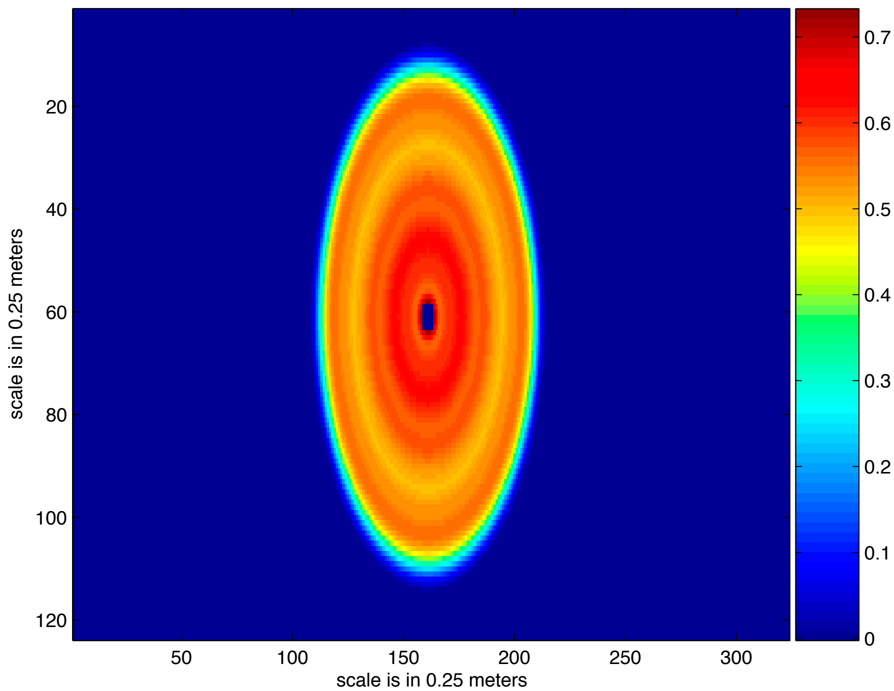


Figure 2. Matlab simulation of sprinkler irrigation over 1 h.

To represent the field, we use a matrix of cells. For the simulation, we have a list of sprinkler positions, and for each sprinkler specify how long it runs.

We iterate through the list of sprinkler positions; for each, we simulate the precipitation due to the sprinkler.

To simulate precipitation from a sprinkler, we use a simple nested `for` loop to iterate through the cells within the wetted radius of the sprinkler. For each cell, we compute the distance to the sprinkler and then use the given sprinkler precipitation rate profile and the length of time the sprinkler runs to calculate the additional precipitation received by that cell.

Complete Sprinkler Coverage of Field

No area of the field may receive less than 2 cm of water every four days. To accomplish this, we think of a sprinkler's wetted area as a circular disk located in a rectangle representing the field. The problem then reduces to covering this rectangle with disks. However, because the distribution profile for the sprinkler is nearly uniform (see **Figure 2**), allowing for radial overlaps will disturb overall uniformity and increase the number of sprinklers needed to cover the field. Hence, it is best to minimize overlap by minimizing the number of sprinklers while ensuring that every part of the field is completely covered. This can, however, be restated as a covering problem, in which we find the smallest number of equally-sized circular disks that can cover a given rectangle. **Figure 3** displays this solution; no other configuration of circular disks can cover this rectangle more efficiently [Kershner 1939].

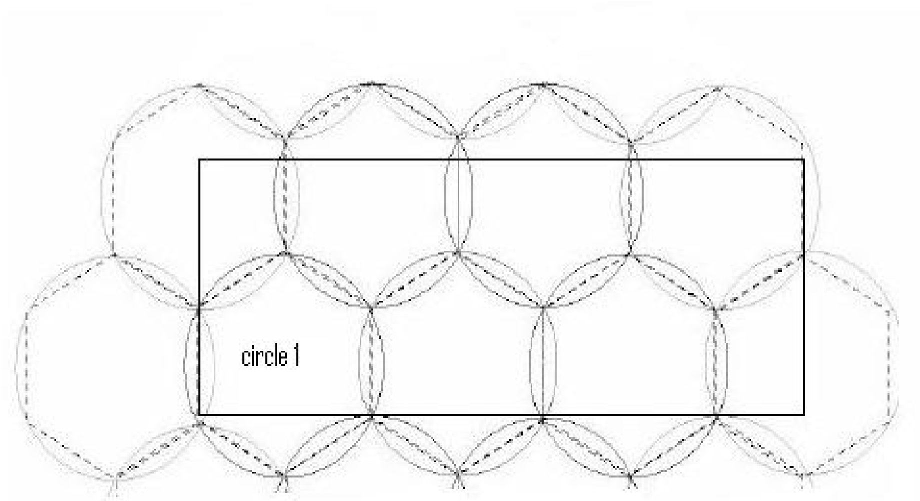


Figure 3. Hexagonal covering.

Adjustment of the Covering Problem Solution

This solution, however, is not completely useful, since it would result in some sprinklers being positioned outside the field. Moreover, adjusting sprinkler positions can also increase uniformity. In **Figure 4**, we show a possible placement of the field with the solution of the covering problem.

We need to adjust this solution, to ensure that the sprinklers are inside the field while covering it as uniformly as possible. From the hexagonal covering pattern (**Figure 3**), we know that the center of circle 1 is supposed to be located at a distance of $X = R\sqrt{3}/2$ to the right of field's left boundary. From spline interpolation of the data from the sprinkler coverage profile, we estimate $R \approx 13.4$ m. But at 13.4 m, the precipitation rate is zero. We not only want the rate greater than zero but also for the total application to reach 2 cm in 5 h. Placing this constraint on the precipitation rate, we find that it has a radius of only 11.5 m; and anything beyond this will not provide the required coverage. Thus, to achieve the most complete coverage, we need to calculate the x-coordinate of circle 1, using $R = 11.5$ m, and then place all the other circles on the left row with the regular spacing of 13.4 m. This will result in the shift of the left

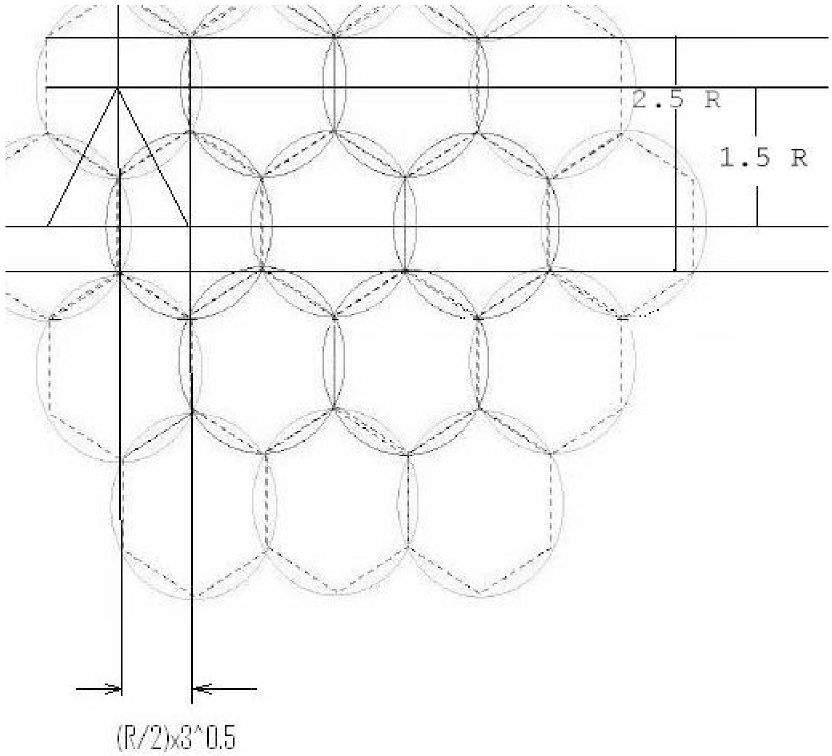


Figure 4. Hexagonal covering of the field.

row $13.4 \text{ m} - 11.5 \text{ m} = 1.93 \text{ m}$ to the left, thus not only keeping the sprinklers within bounds but also increasing precipitation uniformity.

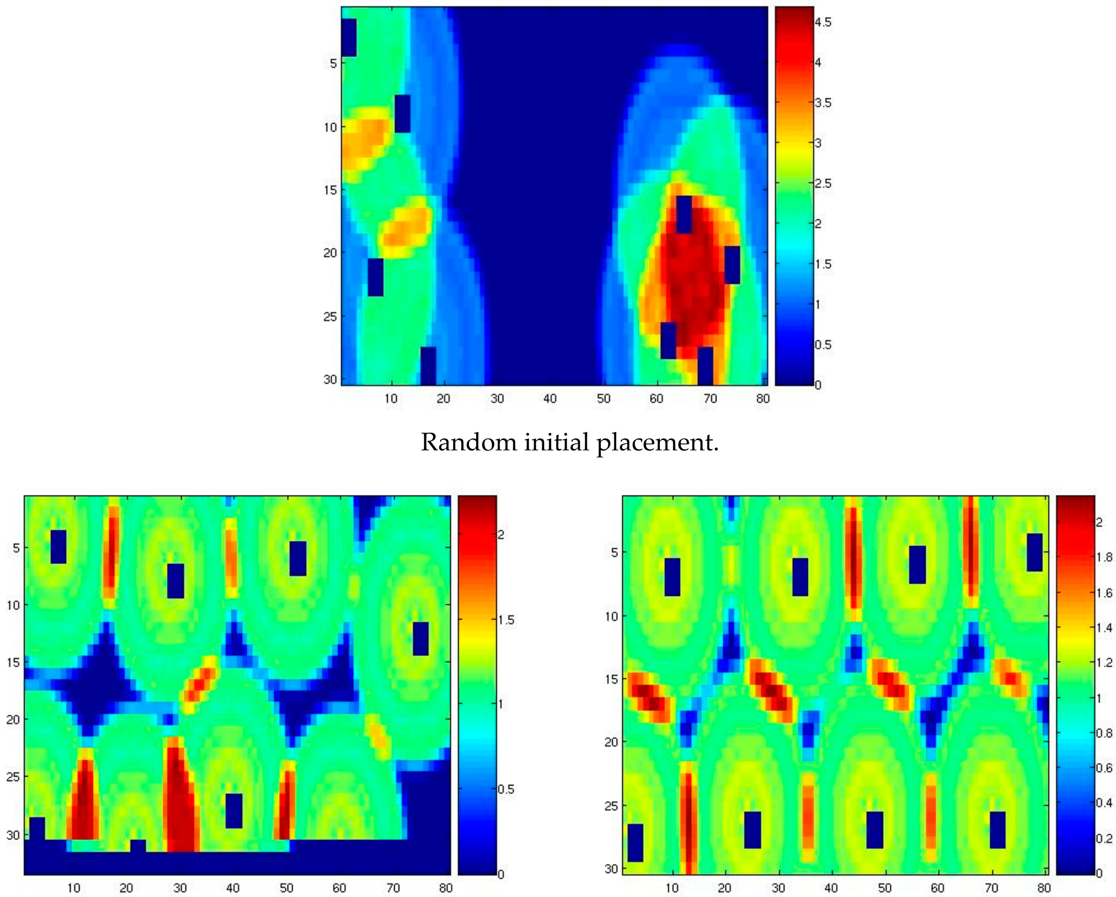
Maximizing Precipitation Uniformity

Applying water to the field as uniformly as possible is a main concern, since doing so leads to efficient crop yields [Lamm 1998]. We tackle this optimization problem using a Simultaneous Perturbation Stochastic Approximation (SPSA) optimization algorithm [Spall 2996], with which we minimize the standard deviation from the average precipitation in the field. **Figure 5** shows the result.

After 5,000 iterations, the solution seems to mimic the shifted covering problem solution—that is, our shifting method achieves a uniformity coefficient that adequately approximates the one from the SPSA, hence yields a sprinkler output that approximately maximizes uniformity.

Algorithm: Minimization of Pipe Setups

This algorithm takes as input the coordinates of sprinklers positions as determined by our approximation to the SPSA solution. From this layout, it minimizes the number of pipe setups by first selecting a sprinkler closest to the upper corner of the field, calculating the distances of all other sprinklers from it, and selecting the sprinkler with the shortest *lateral* distance (we cannot place a pipe diagonally). If this distance is less than or equal to the pipe length



Random initial placement.

After 500 iterations.

After 5,000 iterations.

Figure 5. Sprinkler placement from the SPSA algorithm after a specified number of iterations.

(20 m), the algorithm calculates the precipitation rates at points located within the overlapping radii; if a rate exceeds 0.75 cm/h, the algorithm goes onto the next closest sprinkler.

Irrigation System Calculations

The problem statement specifies that the mainline pipe used in the hand-move system is aluminum with diameter 10 cm, the sprinkler nozzle size is 0.6 cm, and the water source has pressure of 420 kPa and possible flow rate 150 L/min. For our calculations, we use the following formulas from Rain Bird Agri-Products Co. [2001].

Hazen-Williams

$$\text{Pressure Loss (psi)} = 4.55\,\frac{\left(\frac{Q}{C}\right)^{1.852}}{(ID)^{4.87}}\,L,$$

where

Q = pipe flow (gal/min),
C = roughness coefficient (aluminum w/ couplers = 120),
ID = pipe inside diameter (in), and
L = pipe length (ft).

Nozzle Discharge

$$\text{Discharge (gpm)} = 29.82\sqrt{P}D^2C_d,$$

where

P = nozzle pressure (psi),
D = nozzle orifice diameter (in), and
C_d = nozzle discharge coefficient (tapered ≈ 0.96 or 0.98).

Since 0.145 kPa = 1 psi, the system pressure is at most 60.9 psi. The nozzle size is $(0.6 \text{ cm})/(2.54 \text{ cm/in}) = 0.236$ in; and assuming a nozzle discharge coefficient of 0.97, we obtain a flow rate per sprinkler of 12.6 gal/min= 47.58 L/min. The pressure loss due to the mainline pipe assuming four sprinklers is only 0.012 psi, which can be neglected. We assume that each sprinkler is on a 30 in riser and that the riser is a 1 in-diameter steel pipe. The pressure loss assuming a flow of 12.6 gal/min is 0.058 psi; thus, we also can neglect pressure loss due to the riser.

Results

Our model generates the optimal pipeset configuration shown in **Figure 6**. This configuration consists of 8 pipe movements each in intervals of 5 h (with an assumed 1 h time for moving and set up of equipment) and results in a total irrigation time of 48 h every 4 days, or approximately 12 h/d. Each pipeset contains only one sprinkler; if the sprinklers were closer than 21 m apart, the overlap of their wetted areas would yield precipitation greater than 75 cm/h. Less than 1% of the field receive an insufficient amount (2 cm) of water, in areas at the edges, where the crop could easily be damaged by other factors.

Table 1 shows the generated irrigation schedule for the repositioning of the sprinklers, given a 12-h work day for a rancher. Each pipe is set in place for 5 h. [EDITOR'S NOTE: We omit the table giving the irrigation schedule and coordinates for the sprinklers.]

Based on our assumptions and the design of our algorithm, there is no faster way to irrigate this field while maintaining such a high measure of uniformity.

As expected, this result is consistent with the earlier analysis, given our assumed sprinkler distribution profile. Our solution yields a uniformity coefficient of 0.89, unsurprisingly close to the optimal value of .90 generated by the SPSA after 5,000 iterations.

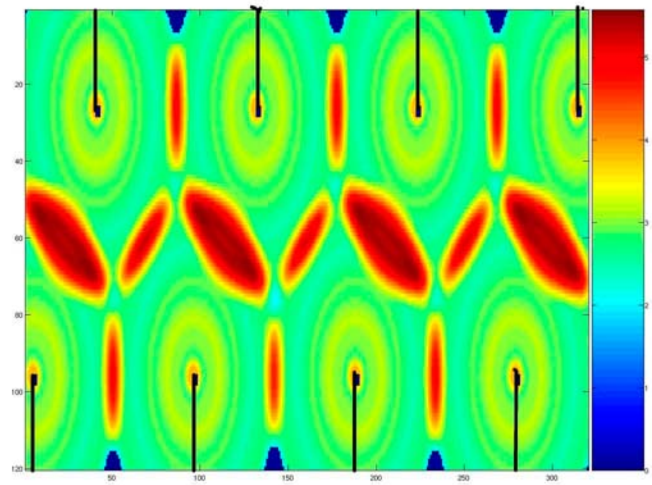


Figure 6. Best sprinkler placement, with pipe sets shown as black lines.

Weaknesses

- The model does not account for change in the sprinklers' profile due to wind, which could lead to a completely different model for sprinkler placement.
- SPSA ideally could have produced the best possible solution, but we did not have time to run it for enough iterations. Using FORTRAN would increase the speed of calculations by a factor of about 10,000.
- The rancher has to work 12 h/d, not 8 h/d.
- The rotating spray nozzle profile in our model is half the size of the one prescribed in the problem statement. Even though we scale the precipitation rate, there is no guarantee that the sprinkler profile would not change with flow rate.
- Coefficient of Uniformity (CU) provides an average deviation from mean coverage. Iff the area is overwatered due to the overlap of different sprinklers placed at different times, or overwatered in one spot and underwatered at another, the CU may come out the same [Zoldoske and Solomon 1988]. Thus, CU is not the most accurate way to measure the uniformity and water application, especially for our purposes, since we care less about minor overwatering than about underwatering.
- We could not validate our model in real-life conditions.

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Summary

We determine an optimal algorithm for irrigating an 80 m $\times$ 30 m field using a hand-move 20-m pipe set, using a combination of analytical arguments and simulated annealing. We minimize the number of times that the pipe is moved and maximize the Christiansen uniformity coefficient of the watering.

We model flow from a sprinkler as flow from a pipe combined with projectile motion with air resistance; doing so predicts a range and distribution consistent with data from the literature. We determine the position of sprinkler heads on a pipe to optimize uniformity of watering; our results are consistent with predictions from both simulated annealing and Nelder-Mead optimization.

Using an averaging technique inspired by radial basis functions, we prove that periodic spacing of pipe locations maximizes uniformity. Numerical simulation supports this result; we construct a sequence of irrigation steps and show that both the uniformity and number of steps required are locally optimal.

To prevent overwatering, we cannot leave the pipe in a single location until the minimum watering requirement for that region is met; to water sufficiently, we must water in several passes. The number of passes is minimized as uniformity is maximized.

We propose watering the field with four repetitions of five steps, each step lasting roughly 30 min. We place two sprinkler heads on the pipe, one at each end. The five steps are uniformly spaced along the long direction of the field, with the first step at the field boundary. The pipe locations are centered in the short direction. This strategy requires only 20 steps and has a Christiansen uniformity coefficient of 94, well above the commercial irrigation minimum of 80. Simulated annealing to maximize uniformity of watering re-creates our solution from a random initialization.

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The consistency between solutions from numerical optimization and from analytical techniques suggests that our result is at least a local optimum. Moreover, the solution remains optimal upon varying the sprinkler profile, indicating that the results are not overly sensitive to our initial assumptions.

Introduction

Maximizing the uniformity of irrigation reduces the amount of water needed [Ascough and Kiker 2002]. We use a 420-kPa, 150-L/min water source and a 20-m hand-move pipe set for a 80 m $\times$ 30 m field. We determine the number and placement of sprinkler heads on the pipe, together with a schedule of watering locations and times that maximizes uniformity and minimizes time required.

Initial Assumptions

- **We live in a boring place.** We have a flat, windless, weatherless field. Though wind is often an influential factor, uniformity can be corrected to compensate for wind [Tarjuelo Martín-Benito et al. 1992].
- **Time required = number of moves.** We attempt to minimize the number of moves; we do not consider any other kind of effort, such as minimizing the total distance that the pipe must be moved.
- **Our sprinkler heads are ideal.** The distribution of water from the sprinkler heads is radially symmetric and the same for every head.
- **Average, not instantaneous overwatering.** We can water an area for half an hour at a rate of 1.5 cm/h, then leave it for half an hour, and this would not constitute overwatering. Without this assumption, it is impossible to meet the constraints on watering.
- **The pipe must stay within the field.** Our pipe locations remain completely on the field, though we allow water to fall off the field.

Slow-Watering and Fast-Watering

We break watering techniques into two categories:

- **Slow-watering.** Keep the pipe in one section of the field until it has been watered sufficiently, that is, we water the field in one pass.
- **Fast-watering.** Make multiple short passes, waiting for the field to absorb the water between runs.

Slow-watering minimizes effort (the number of times we move the pipe) while fast-watering requires extra moves. Fast-watering also carries the physical risk of washing away the topsoil, but we ignore this.

With the given constraints, we cannot create a slow-watering solution. To irrigate the field in one pass, we must keep the pipe in one position until the minimum is met. We should water at a rate no greater than 0.75 cm/h. But the rate of water flow, 150 L/min = 9×10^6 cm^3/h, amounts to 0.375 cm/h over the entire field, or half the field at the maximum rate. However, our sprinkler cannot cover so great an area, hence cannot help overwatering the area that it reaches if we water for an hour or more.

We are forced to choose a fast-watering technique, which involves several passes over the field.

Judging the Quality of Solutions for Fast Watering

What solution is best? We want to minimize the number of times that we move the pipe, which is number of passes required times the number of pipe locations in each pass.

How many passes? The number of passes is determined by the minimum watering criterion. If the minimum application rate is S_{min}, then to make sure that every location receives the minimum 2 cm of water, we need

$$S_{\text{min}} t \times (\text{number of passes}) = 2 \text{ cm},$$

where t is the watering time.

How long to water? We choose t so that we don't overwater. With, in one pass, a maximum application rate of S_{max} cm/h, we can water only long enough to apply 0.75 cm, the maximum possible in an hour:

$$S_{\text{max}} t = 0.75 \text{ cm}.$$

Combining the two equations, we find

$$\text{number of passes} = \left\lceil \frac{8}{3} \frac{S_{\text{max}}}{S_{\text{min}}} \right\rceil . \qquad \textbf{(1)}$$

The ratio $S_{\text{max}}/S_{\text{min}}$ decreases with increasing uniformity. In other words, *increase in uniformity decreases the number of moves required.*

Christiansen Coefficient of Uniformity

$S_{\text{min}}/S_{\text{max}}$ is not a typical measurement of uniformity. We also use the Christiansen coefficient of uniformity, the most broadly used and well-recognized criterion for uniformity of watering [Ascough and Kiker 2002; Tarjuelo Martín-Benito et al. 1992]:

$$CU = 100 \left(1 - \frac{\sigma_S}{\langle S \rangle} \right),$$

where σ_S is the standard deviation of the application rate during a pass and $\langle S \rangle$ is the mean.

Summing over Sprinklers

We determine $S(\vec{x})$ by superimposing the water flows from the sprinkler heads, using the expression $\varphi(|\vec{x} - \vec{x_1}|)$ to denote application of 1 cm/h at position $\vec{x}$ from the sprinkler head at $\vec{x_1}$:

$$S(\vec{x}) = \sum_k t_k \varphi\left(|\vec{x} - \vec{x_k}|\right),$$

where t_k is the time spent at sprinkler head k.

Determining the Sprinkler Profile $\varphi(r)$

To optimize the layout, we must know how the sprinkler applies water as a function of distance, $\varphi(r)$. This is a complicated function that depends on the sprinkler type and pressure in the line; its form is not well-known and it is often is simulated numerically [Carrión et al. 2001].

The Linear Model

A first guess at the sprinkler function would be a simple decreasing linear function. In fact, this is reasonably consistent with measured data (**Figure 1**).

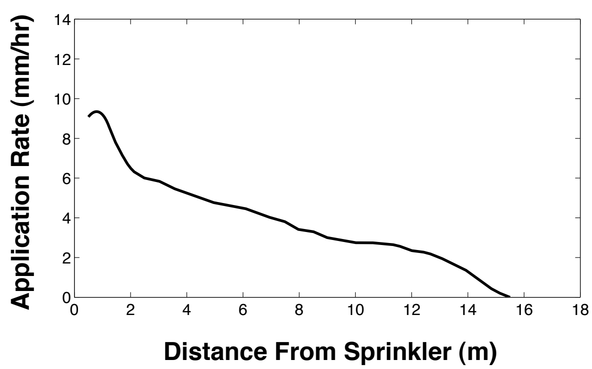


Figure 1. The sprinkler function is approximately linear (redrawn from Carrión et al. [2001]).

The linear approximation allows for simple solutions; in one dimension, it is possible to combine linear functions to lead to a uniform water distribution [Smajstrla et al. 1997]. Several other empirical models have been used for $\varphi(r)$; Mateos uses, among others, $\varphi(r) \sim \left(1 - r^2/R^2\right)$ [Mateos 1998].

Model of Water Distribution from a Sprinkler

Output Speed of Sprinklers

We model a sprinkler head, ignoring rotational effects and angle, as a hole in the pipe. Bernoulli's equation states that along a streamline,

$$P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant},$$

where

P is the pressure,

ρ is the density of water,

v is the fluid velocity,

g is the acceleration of gravity at the Earth's surface, and

h is the height of the fluid.

We assume that the variation in height is negligible and consider a point within the pipe and a point at the hole. Then, with P_w the water pressure and P_a the atmospheric pressure, $P_w + \frac{1}{2}\rho v_w^2 = P_a + \frac{1}{2}\rho v_o^2$ implies that

$$v_o^2 = v_w^2 + \frac{2(P_w - P_a)}{\rho}$$

is the speed of outgoing water for one sprinkler. Typically, this result is stated as $v_o = \sqrt{2gH}$, where $H = 2(P_w - P_a)/\rho$ is the pressure head and the speed of the water is considered negligible. In our case, however, v_w is significant.

With n sprinklers, the continuity property requires that the flux in any one section of the tube is $J/A_i n$, where J is the total flux (150 L/min) and A_i is the cross-sectional pipe area. Therefore, the output speed is

$$v_n = \sqrt{\left(\frac{J}{A_i n}\right)^2 + \frac{2(P_w - P_a)}{\rho}},$$

which ranges from 25 to 40 m/s, depending on the number of sprinkler heads.

Sprinkler Range

The spray remains coherent for a while before breaking up into particles [Carrión et al. 2001; Kranz et al. 2005]. We treat the motion of the outgoing water drop as a projectile problem, first without air resistance, then with air resistance proportional to the square of the speed.

Without air resistance, direct integration of the equations of motion gives the range and flight time for initial speed v_o at angle θ to the horizontal:

$$\text{range} = \frac{v_o^2}{g}\sin 2\theta, \qquad \text{time of flight} = \frac{2 v_o \sin\theta}{g}. \qquad \textbf{(2)}$$

Air resistance is often represented using a damping force quadratic in speed; the resulting equations cannot be solved analytically in general [Marion and Thornton 1988; Tan and Wu 1981]. However, in our system, the droplets have very large horizontal speeds and only a small vertical distance to fall. In the limit, we can ignore the vertical drag force, writing

$$\frac{dv_x}{dt} = -kv_x^2, \qquad v_x(0) = v_o; \qquad \frac{dv_y}{dt} = -g, \qquad v_y(0) = 0.$$

The equation in x has solution $v_x(t) = 1/(kt + v_o^{-1})$, which yields

$$x(t) = \tfrac{1}{k} \ln(ktv_o + 1).$$

This gives us one solution, but we need to consider variations. Different drop sizes have different drag forces, according to the Prandtl expression [Carrión et al. 2001; Marion and Thornton 1988]:

$$k = \frac{C_d \rho_a A}{2m},$$

where

C_d is the dimensionless drag coefficient (on the order of 1),

ρ_a is the density of air,

A is the cross-sectional area of the drop, and

m is the mass of the drop.

We model the drop as a sphere of water, so $m = \rho_w (4/3)\pi R^3$, where R is the radius of the drop. Using $A = \pi R^2$, we get

$$k = \tfrac{3}{8}\, C_d \, \frac{\rho_{\text{air}}}{\rho_{\text{water}}} \, \frac{1}{R}. \tag{3}$$

This means that the distribution of $1/k$ is *exactly* the distribution of R, the droplet size distribution. The distance $x(t)$ is, to first order, proportional to $1/k$, so *the size distribution of the drops directly controls the distribution of their distance!*

The probability that a drop flies a distance x, in terms of the radius, is approximately

$$P(X = x) \approx \frac{8}{3}\left(\frac{\rho_{\text{water}}}{\rho_{\text{air}}}\right)\frac{R}{C_d}\ \ln(ktV_o + 1)\, P(\mathcal{R} = R).$$

Unfortunately, the distribution $P(\mathcal{R} = R)$ is not known. Raindrops follow the empirical distribution $\lambda \exp(-\lambda R)$ [Marshall and Palmer 1948], but there is no a priori reason to assume that sprinkler droplets do. The droplets are roughly spherical because of their surface tension, so we could also assume a Maxwell-Boltzmann distribution based on surface tension energy, $P(\mathcal{R} = R) = 1/Z \exp\left(-J\pi r^2/kT\right)$, yielding a normal distribution. The drop-size distribution from fire sprinklers is described as log-normal [Sheppard 2002].

Since we are not certain about the exact distribution, we combine the physical intuition gained from this model with the simplicity of the linear model.

Making the Linear Model Physically Consistent

We use the simple linear distribution but choose its properties to conserve water volume. We choose a linear shape such that:

- the width of the shape depends on the speed, and
- the total leaving the sprinklers is the total water supplied.

These two conditions determine the slope and intercept of the linear approximation. To do this in a realistic way, we must estimate the drag force using **(3)**. We guess that C_d for a water drop in air is 0.2 and assume that the largest radius of a water droplet is 0.05 cm; these are reasonable values [Tan and Wu 1981] that lead to $k = 0.15\text{ m}^{-1}$.

We now develop the x-intercept and y-intercept of the linear system in terms of the physics. A water drop that falls for t seconds travels

$$x_n(t) = \tfrac{1}{k}\ln\left(ktv_n + 1\right)$$

meters horizontally. This fixes the width of our linear distribution. Now, we ask: What is t? Normally, we would just calculate the amount of time for a drop to fall. For the 10 cm from the top of the pipe to the bottom of the pipe, this would be 0.14 s; however, no sprinkler throws out drops horizontally. When we calculated the distance traveled, we assumed horizontal initial speed; we now correct for this by using the no-air-resistance theory. We choose t as the no-resistance flight time from **(2)** with the sprinkler at a 45° angle. This is physically reasonable because air resistance makes only a small correction to flight time [Marion and Thornton 1988].

Using this approximation, we find 18–21 m as typical values for x_n, the sprinkler "throw," depending on the number of heads on the pipe. These results are consistent with typical sprinklers at pressures around 400 kPa [Carrión et al. 2001; Tarjuelo Martín-Benito et al. 1992].

Conserving Volume

Now that we know x_n, we can determine the y-intercept for the linear function. We know that in any unit of time, the amount of water coming into the pipe (J) is the amount of water sprayed by the sprinklers. For a sprinkler profile $\varphi(r)$, we can write this assumption as:

$$n\int_{r=0}^{\infty}\int_{\theta=0}^{2\pi}\varphi(r)r\,dr\,d\theta = 2\pi n\int_{r=0}^{\infty}\varphi(r)r\,dr = J,$$

where J is the incoming flux of water, 150 L/min. For our linear $\varphi(r)$, with $\varphi(0) = h_n$ and $\varphi(x_n) = 0$, this is equivalent to

$$n\left(\text{volume of cone with height } h_n \text{ and radius } x_n\right) = n\left(\frac{\pi}{3}h_n x_n^2\right) = J.$$

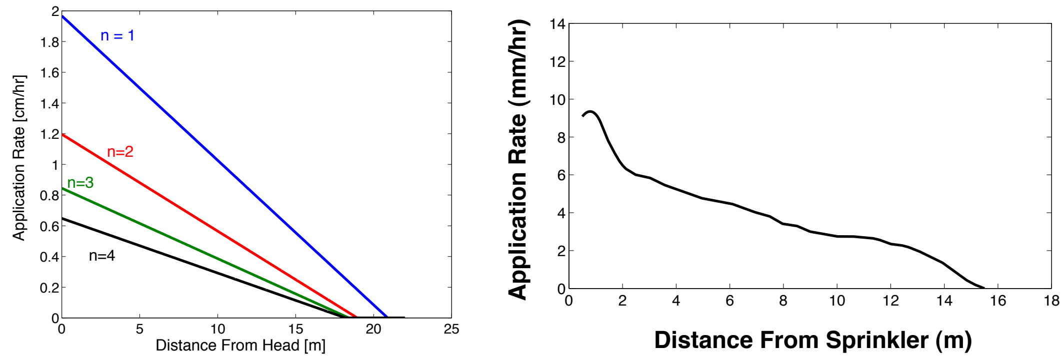


Figure 2. The modeled water distribution $\varphi(r)$ changes with the number of heads on the pipe (left) and is consistent with the experimental results of **Figure 1** (right).

This equation lets us fix the height, and completely determine the water distribution from a sprinkler,

$$h_n = \frac{3J}{\pi n x_n^2}.$$

The sprinkler profile $\varphi(r)$ is

$$\varphi(r) = \max\left(\frac{1}{k}\ln\left(ktv_n+1\right) - r\left\{\frac{3Jk^2}{\pi n\left[\ln\left(ktv_n+1\right)\right]^2}\right\}, 0\right). \qquad \textbf{(4)}$$

We illustrate it for a few values of n in **Figure 2**.

Radial Approximation

For sprinkler heads along a pipe, we get a roughly elliptical distribution of water application rates. If we approximate this as a radial distribution, we get a one-dimensional profile function. We can then find a set of pipe locations by superimposing these functions and maximizing uniformity.

Determining the Radial Profile

Let L be the length of the pipe. We require $\varphi(r)$ to be monotonically decreasing, as is the case for our approximate linear sprinkler profile and for some other distributions, such as the exponential and the normal centered at zero (**Figure 3**).

Let P be the number of times that we move the pipe (which has n sprinkler heads on it). Let also

$\vec{p}_i$ and θ_i be the position and angle of the pipe,

t_i be the length of time that the pipe remains at the ith position, and

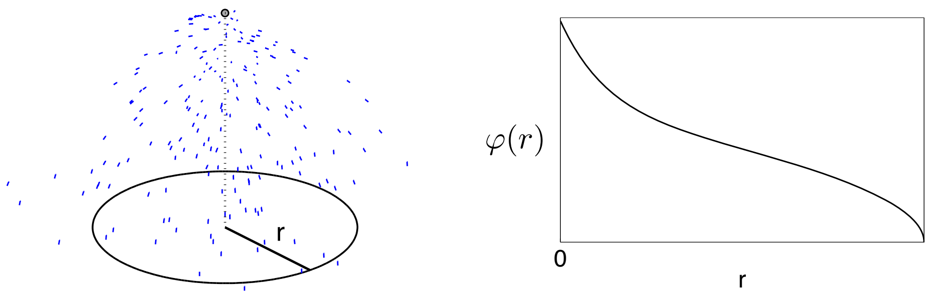


Figure 3. Sprinkler heads deliver water with a radially-symmetric distribution $\varphi(r)$.

s_j be the position of the jth head along the pipe, where $-\frac{L}{2} \leq s_j \leq \frac{L}{2}$.

We write a position on a pipe as the position vector of the pipe's center plus a part along the pipe's axis:

$$\vec{l}(\tau) = \vec{p} + \tau \hat{u}_\theta,$$

where $\vec{u}_\theta$ is an unit vector with angle θ from the positive x-axis,

$$\hat{u}_\theta = \hat{e}_x \cos\theta + \hat{e}_y \sin\theta.$$

In this notation, the position of the jth sprinkler on the ith pipe location is $\vec{l}_i(s_j)$, and the sum of water over the field is

$$S(\vec{x}) = \sum_{i=1}^{P} t_i \sum_{j=1}^{n} \varphi\left(|\vec{x} - \vec{l}_i(s_j)|\right).$$

This $S(\vec{x})$ can be interpreted as a radial basis function interpolant [Powell 1987]. We write it as a sum over pipe locations rather than over sprinkler heads:

$$S(\vec{x}) = \sum_{i=1}^{P} t_i G_i(\vec{x}),$$

where $G_i(\vec{x})$ is the water distribution from the ith pipe position,

$$G_i(\vec{x}) = \sum_{j=1}^{n} \varphi\left(|\vec{x} - \vec{l}_i(s_j)|\right).$$

We define an approximation to $G_i(\vec{x})$ by breaking the sprinkler heads into infinitesimal pieces:

$$\begin{aligned} \widetilde{G}_i(\vec{x}) &= n \lim_{k\to\infty} \sum_{j=0}^{k-1} \tfrac{1}{k}\, \varphi\left(|\vec{x} - \vec{l}_i(s_j)|\right) \\ &= n \int_{-L/2}^{L/2} \varphi\left(|\vec{x} - \vec{l}_i(\tau)|\right) d\tau. \end{aligned} \quad \textbf{(5)}$$

The second equality follows from assuming that the heads are sufficiently uniform to approximate an integral. The quantity $\widetilde{G}_i$ is approximately G_i; it is the limit of the water distribution as the number of heads becomes infinite while keeping constant the total volume of water that the pipe delivers.

If $\vec{x}$ is a point along the pipe, $\vec{x} = \vec{p} + t\vec{u}_\theta$, then **(5)** reduces to

$$\widetilde{G}_i(\vec{x}) = n \int_{-L/2}^{L/2} \varphi(|t-\tau|)\, \mathrm{d}\tau.$$

If φ is zero (or approximately zero) outside a radius r, and if $|t| < L - r$, then the integral is constant (or approximately constant) with respect to t. Hence the water distribution is dominantly characterized by the orthogonal distance from the pipe. Define $\mu(x)$, the pipe radial water distribution function, as

$$\mu(x) = \widetilde{G}(\hat{e}_x x) = n \int_{-L/2}^{L/2} \varphi(\sqrt{x^2 + \tau^2})\, d\tau.$$

The function $\mu(r)$ is the water distribution for the pipe, in analogy with the distribution $\varphi(r)$ for a sprinkler head. We use this function to approximate G_i, again using the analogy with radial basis functions:

$$G_i(\vec{x}) \approx \mu(|\vec{x} - \vec{z}|), \qquad \text{where } \vec{z} \text{ is the closest point to } \vec{x} \text{ on the pipe.}$$

We then use μ to approximate the total water sum,

$$\widetilde{S}(\vec{x}) = t \sum_{i=1}^{P} \mu(|\vec{x} - \vec{z}_i|), \qquad \text{where } \vec{z}_i \text{ is the closest point to } \vec{x} \text{ on the } i\text{th pipe.}$$

The function μ is proportional to a smoothed version of φ. With $\varphi(x) = \max(\lambda - |x|, 0)$, we obtain the profile

$$\begin{aligned} \mu(x) &= n \int_{-L/2}^{L/2} (\lambda - \sqrt{x^2 + \tau^2})_+ \, \mathrm{d}\tau \\ &= \begin{cases} n\left[\lambda\sqrt{\lambda^2 - x^2} + \frac{x^2}{2} \ln \frac{\lambda - \sqrt{\lambda^2 - x^2}}{\lambda + \sqrt{\lambda^2 - x^2}}\right], & |x| \le \lambda; \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

The result is a unimodal function symmetric over the domain $[-\lambda, \lambda]$.

To indicate the flexibility of the method, we note that if φ follows a normal distribution, so does μ; or if φ is monotonically decreasing, then so is μ.

Optimality of Periodic Solutions

Consider an irrigation sequence where the pipe is oriented at the same angle θ and for the same duration t at each step for P steps. At each step, the pipe is

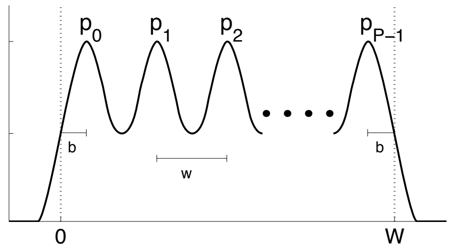


Figure 4. $S(\hat{e}_x x)$.

moved laterally w meters, so $\vec{p}_i = \vec{p}_0 + iw\vec{u}_{\theta+\pi/2}$. If, in the direction parallel to the pipe, $\vec{x}$ is not beyond the endpoints, then

$$S(\vec{x}) = t\sum_{i=1}^{P} \mu(|\vec{u}_{\theta+\pi/2} \cdot (\vec{p}_0 - \vec{x}) + iw|). \tag{6}$$

Consider watering a $W \times L$ rectangular area. Define the boundary margin b as the distance between the boundary and the first pipe and the step widths $w = (W - 2b)/(P-1)$ (**Figure 4**). Let $\vec{p}_0 = b\hat{e}_x$, $\theta = \frac{\pi}{2}$, and $\vec{p}_1 = p_0 + iwu_0$, such that the sequence irrigates the region $R = [0, W] \times [-\frac{L}{2}, \frac{L}{2}]$ (**Figure 5**).

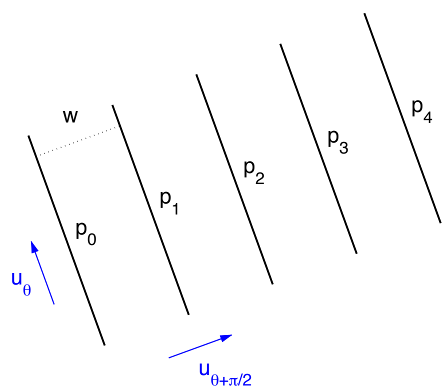


Figure 5. A five-step irrigation sequence with parallel pipe orientations.

We apply the "number of passes" uniformity criterion to $S(\vec{x})$. We maximize this criterion by maximizing its approximation in terms of $\mu(r)$:

$$\mathcal{C}_u = \frac{\min_{\vec{x}\in R} S(\vec{x})}{\max_{\vec{x}\in R} S(\vec{x})}.$$

A higher criterion value indicates more uniform irrigation of R.

Since $S(\vec{x})$ is nearly constant in the direction along the pipe, $\mathcal{C}$ is closely approximated by

$$\widetilde{\mathcal{C}}_u = \frac{\min_x S(\hat{e}_x x)}{\max_x S(\hat{e}_x x)}, \tag{7}$$

that is, $\mathcal{C}_u$ restricted to the east-west line through the middle of R. Suppose that w is wide enough relative to the decay of μ such that the overlap between non-adjacent pipe terms in **(6)** is negligible. Then for $\lfloor \frac{y-b}{w} \rfloor = k$, $k = 1, 2, \ldots, (P-1)$, we have

$$\widetilde{S}(\hat{e}_x x) = \mu\big(x - b - (k-1)w\big) + \mu(b + kw - x). \qquad \textbf{(8)}$$

Since

$$\frac{\partial}{\partial x}\widetilde{S}\left(\hat{e}_x\left[b + \left(k + \tfrac{1}{2}\right)w\right]\right) = \mu'\left(\tfrac{w}{2}\right) - \mu'\left(\tfrac{w}{2}\right) = 0,$$

the midpoints $x = b + \left(k + \frac{1}{2}\right)w$ between pipe positions are local extrema of the water sum $\widetilde{S}$. Furthermore, by **(8)**, each extremum attains the same value

$$\widetilde{S}\left(\hat{e}_x\left[b + \left(k + \tfrac{1}{2}\right)w\right]\right) = 2\mu\left(\tfrac{w}{2}\right).$$

Since μ is monotonically decreasing, another set of extrema are the pipe positions $x = b + kw$, each attaining the value $\mu(0)$. At the ends of the field,

$$\widetilde{S}(\hat{e}_x x) = \begin{cases} \mu(b - x), & 0 \le x \le b; \\ \mu\big(x - b - (P-1)w\big), & b + (P-1)w \le x \le W. \end{cases}$$

For the physically-derived sprinkler distribution **(4)**, μ is simple enough that overlaps do not produce any other extrema. Therefore, $\widetilde{\mathcal{C}}_u$ is maximized by the choice of b and w such that all minima are equal and all maxima are equal.

For wider w, the pipe positions are maxima, the midpoints are minima, and the boundary margin b is selected such that $\widetilde{S}(0\hat{e}_x) = \widetilde{S}(W\hat{e}_x) = 2\mu(\frac{w}{2})$ (as in **Figure 4**). For narrower w, there is more overlap and the midpoints become maxima and b is set to zero.

Thus, the periodic solution maximizes the approximate criterion $\widetilde{C}_u$: *the periodic watering is locally optimal in uniformity of water delivery.*

Complete Solution

We restrict ourselves to periodic solutions; by symmetry, we place the pipe locations in the center of the field. We must now optimize over the number of pipe locations in one sweep of the field, the number of sprinkler heads, and the distribution of sprinkler heads along the pipe.

Analytical Prediction of Sprinkler Head Distribution

We analytically determine the location of sprinklers on a pipe to maximize uniformity. Let $\varphi(r)$ be the radial water distribution

$$\varphi(r) = h_n\left(1 - \tfrac{r}{x_n}\right)_+ = \max\big(h_n(1 - \tfrac{r}{x_n}), 0\big).$$

The sprinkler delivers a maximum of h_n cm/h at its center and delivery decays linearly to zero at radius x_n m. For $n = 2$, we have $h_n = 1.20$ cm/h and $x_n = 19.0$ m. The variables h_n and x_n decrease as the number of sprinkler heads n increases. Asymptotically, $x_n \to 17.9$ m (we need this lower bound later). The water distribution of the pipe is, as before,

$$G_i(\vec{x}) = \sum_{j=1}^{n} \varphi\left(|\vec{x} - \vec{l}_i(s_j)|\right).$$

We select the sprinkler head locations s_j that minimize our uniformity criterion $S_{\max}/S_{\min}$ of **(1)**:

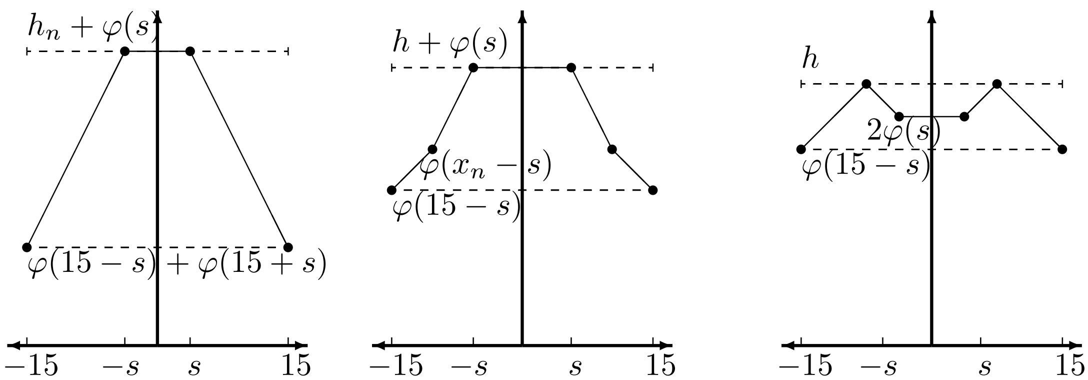
$$\mathcal{C}_u = \frac{\min_{|x|\le 15} G(x\hat{e}_x)}{\max_{|x|\le 15} G(x\hat{e}_x)}.$$

For $n = 2$, the pipe water distribution is

$$G(x\hat{e}_x) = \varphi(|x - s_1|) + \varphi(|x - s_2|).$$

The symmetry of the optimization problem implies that the heads are best placed symmetrically on the pipe. Let $s = s_1 = -s_2$, $s \ge 0$. Since the heads must be on the pipe, $s \le 10\,\text{m} < x_n$. Evaluating $G(x\hat{e}_x)$ reduces to three cases:

Case 1: $s \le x_n - 15$ Case 2: $x_n - 15 < s \le \frac{x_n}{2}$ Case 3: $\frac{x_n}{2} \le s \le 10$



In the first two cases, $\mathcal{C}_u$ improves as s increases. In the third case, G increases at the endpoints and the value in the middle decreases as s increases. Therefore, the uniformity criterion is optimized when

$$\begin{aligned} \varphi(15 - s) &= 2\varphi(s), \\ h_n\left(1 - \tfrac{1}{x_n}(15 - s)\right) &= 2h\left(1 - \tfrac{1}{x_n}s\right), \\ s &= 5 + \tfrac{1}{3}x_n; \end{aligned}$$

but this s places the heads beyond the endpoints of the pipe. Since $x_n > 17.9$ m, s is greater than 10.95 m.

The optimal choice is $s = 10$ m, placing the heads at the ends of the pipe.

For n even, $n > 2$, the solution is the same. Since $17.9\,\text{m} < x_n < 21.0\,\text{m}$ for all n, the same restrictions apply and the optimal choice is $s_j = (-1)^j 10$ m. For odd n, the symmetry requirement places the last sprinkler head in the center.

Optimization of Sprinkler Head Distribution

Sprinkler heads should be positioned as close as possible to the end of the pipe. For a fixed number of heads, we determine the distribution of the sprinkler heads that minimizes the number of passes required, that is, minimizes $S_{\text{max}}/S_{\text{min}}$ and thus maximizes uniformity. Simulated annealing and the downhill simplex (Nelder-Mead) method [Hiller and Lieberman 2005; Press et al. 1992] determine essentially the same results as we predicted above from analytical considerations!

Pipe Number, Initial Position, Number of Heads

We vary three parameters:

- **Pipe number** P controls the density of pipe positions in the field;
- **Initial position** b is the offset, how the solution interacts with the boundaries; and
- **Number of sprinkler heads** n.

The allowable ranges for these parameters are narrowly restricted. For instance, with fewer than three pipe locations per pass, we always have a region that never gets watered. Also, solutions requiring more than ten pipe locations per pass are suboptimal by the limitations on fast-watering.

The small range of parameters allows us to brute-force the optimization, calculating all possible cases (quantizing the variable b, which controls the distance of the first pipe from the boundary).

Results of Brute-Force Variation

We propose using five steps in a pass, with two sprinkler heads per pipe and periodic spacing of pipe locations, with the first pipe location on the boundary ($b = 0$). This requires four passes, or 20 moves, and has a uniformity coefficient of 94 (**Figures 6–8**); this solution is at least locally optimal.

We determine the watering time from the constraints. We make four passes, at each pass staying 32 min at each of the locations in **Figure 7**; the total amount of water applied in 96 h is given in **Figure 8**. The total watering time required is around 11 h, though the farmer does not need to be present for all of this time. The steps could also easily be split up over a four-day period.

Simulated annealing methods reproduce these values quickly, indicating that the solution space is reasonable.

We also calculated the Christiansen uniformity coefficient for these states (**Figure 9**), which shows that our best solution maximizes uniformity as well as minimizing the number of moves.

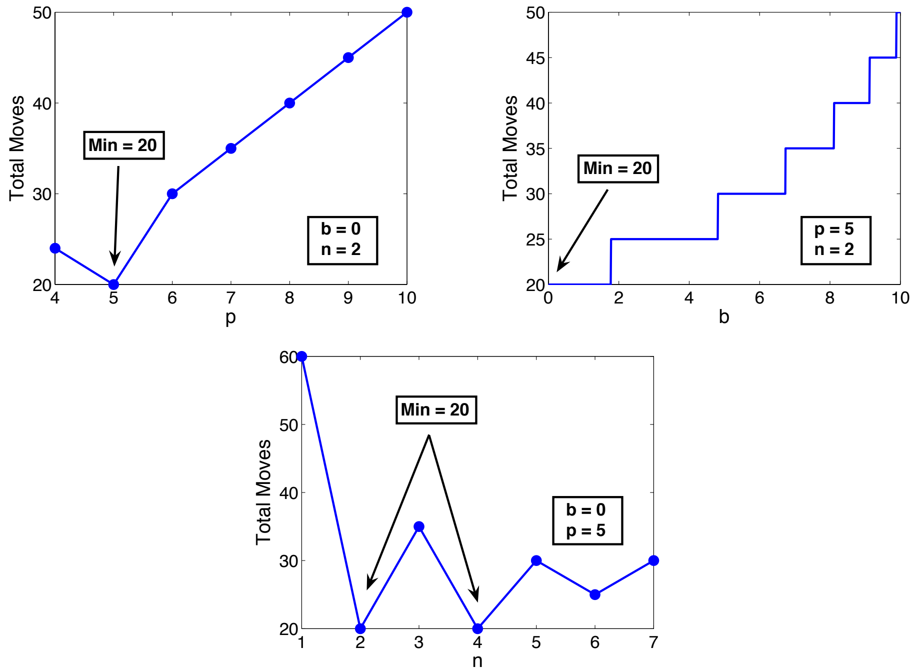


Figure 6. The number of moves required is minimal for $P = 5$, $b = 0$, and $n = 2$.

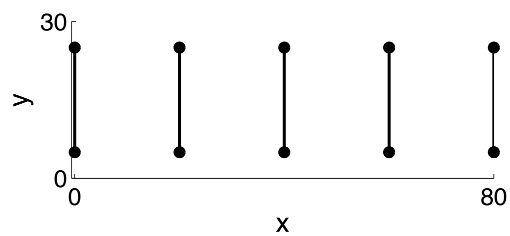


Figure 7. The layout of moves for our solution $P = 5$, $b = 0$, and $n = 2$.

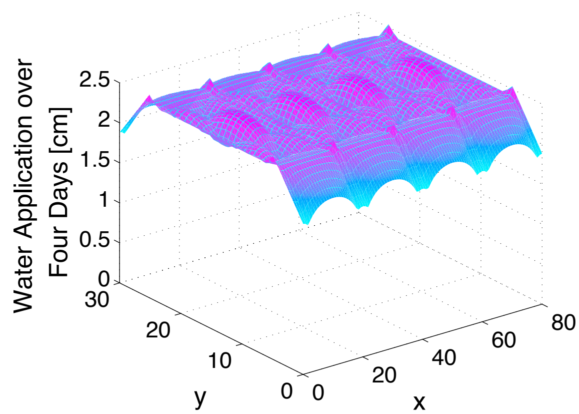


Figure 8. Our best solution has high uniformity (CU = 94), and meets the minimum watering criterion.

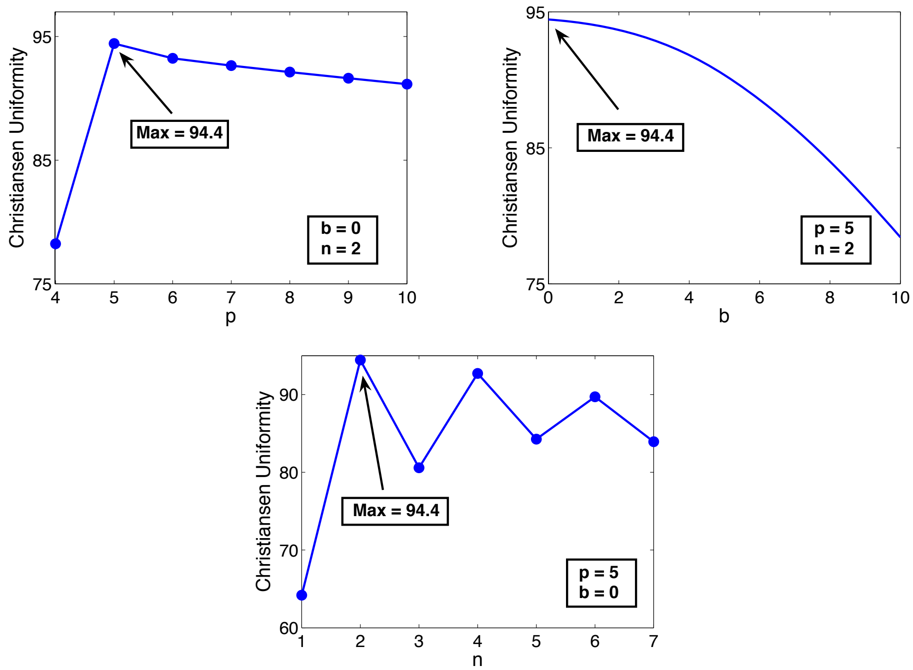


Figure 9. The uniformity of watering is at a maximum when the number of steps is at a minimum.

We are at a minimum in all three orthogonal directions in parameter space, since $b > 0$ by our constraint that all pipe locations must be within the field. This is the halting condition for Powell's optimization method, so we have, once again, a local optimal point [Press et al. 1992].

General Simulated Annealing Results

We start with a completely random distribution of pipe locations and use simulated annealing to optimize for maximum uniformity. This, in many cases, reproduces our geometrically determined distribution (**Figure 10**).

Stability of Model under Alterations

What if we change $\varphi(r)$? For $\varphi(r) = A_n \exp(r/x_n)$, where x_n is the "throw" of the sprinkler and A_n is determined so as to conserve volume, simulated annealing to maximize uniformity gives results very similar to that of our linear φ (**Figure 11**). The same occurs with φ a normal distribution.

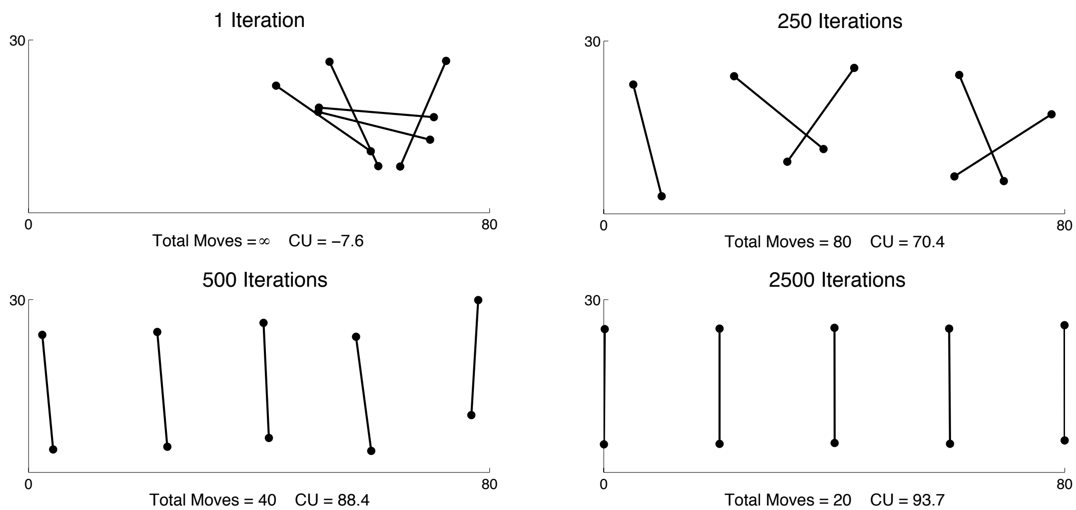


Figure 10. Simulated annealing converging to our geometrically-proposed solution.

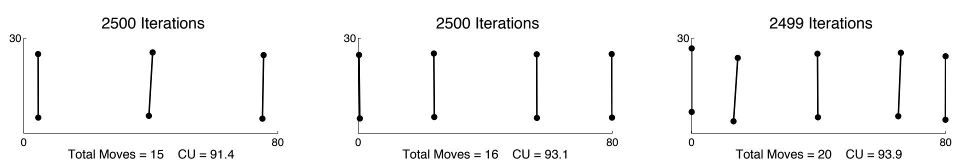


Figure 11. $\varphi(r) = A_n \exp(-r/x_n)$, for $P = 3, 4, 5$. Even when $\varphi(r)$ is changed, the general form of the optimum solution remains.

A Slow-Watering Solution

If we relax the restrictions, we can create a slow-watering solution. We take our optimum solution and treat it in the case where the flow rate is cut in half, $J = 75$ L/min. This leads to a watering solution with CU of 95 that requires moving the pipe only five times, with watering shifts of around 2 h instead of four shifts of 30 min each.

Conclusion

- We develop a physical model of sprinklers on a "hand move" pipe.
- Our model of the sprinklers predicts values for the range and distribution that are consistent with experiment.
- Using an averaging argument reduces the problem to one dimension and predicts a periodic solution.

- Through a combination of optimization and geometric arguments, we develop a fast-watering solution that requires only 20 moves with uniformity coefficient of 94, far better than the "market-worthy threshold" of 80.
- We have shown that a periodic solution is a local maximum in uniformity of watering.
- The optimum distribution of sprinkler heads has the heads near the end of the pipe.
- Simulated annealing recreates our best solution.
- Reducing the water flow would allow a slow-watering solution that required only five total moves, with CU = 95.

Strengths and Weaknesses

Strengths

- **Solution quality.** We have created a solution that works, with a relatively small number of moves and with a high uniformity.
- **Consistency.** Our analytical predictions are consistent with results from numerical optimization. Both approaches show that our solution is at least locally optimal.
- **Stability.** Searching the parameter space with simulated annealing reproduces our solution.
- **Feasibility and simplicity.** Our solution can be easily implemented by a rancher.
- **Physical consistency.** Our physical arguments produce sprinkler profiles very close to those measured experimentally.
- **Flexibility.** Our optimization techniques do not depend strongly on the sprinkler profile $\varphi(r)$.

Weaknesses

- **Constant attention.** Our system cannot be left for long periods of time—the water must be shut off, or the pipe moved, every 30 min.
- **Lack of geometric flexibility.** Though our general simulated annealing approach can adapt to different boundary conditions, our best solution depends strongly on the symmetry of the problem.
- **No global optimum.** Despite our extensive simulations, we cannot guarantee a global optimum.

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Developing Improved Algorithms for Irrigation Systems

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Summary

Our goal is an algorithm that minimizes the time to irrigate a relatively small field under given conditions.

We focus on minimization of time, uniformity of irrigation, and feasibility. Our effort is divided into five basic parts:

- **We assess the wetted radius** based on experimental results for several typical rotating spray sprinklers.
- **We determine the number of sprinklers** from an empirical formula for sprinkler flow.
- **We simulate the water distribution pattern,** using a 0.25 m $\times$ 0.25 m grid.
- **We evaluate the uniformity of water distribution** by Christiansen's uniformity coefficient.
- **We find an optimal irrigation schedule including when and where to move the pipes:** We devise a single-lateral-pipe scheme and a multiple-lateral-pipes scheme; the latter gives better results. To irrigate more uniformly, we adjust the spacing between sprinklers and the spacing from the edge. Using our grid, we move the sprinklers symmetrically on both sides, node by node, to find the optimal positions for an improved multiple-lateral-pipes scheme.

Simulations show that all three schemes perform acceptably in realistic conditions. The multiple-lateral-pipes scheme is superior, with minimum time and

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the highest Christiansen's uniformity coefficient (CU). We conclude that four sprinklers are required, the minimal amount of time is 732 min, and the CU is 90%.

We do a sensitivity analysis of the variation of CU and of minimum time with wetted radius, which shows that our model is robust.

Introduction

Structure of a Hand-Move Irrigation System

A hand-move irrigation system has two kinds of pipes: a portable or buried mainline pipe, and a portable aluminum (sometimes plastic) lateral pipe with quick couplers and spray nozzles.

Definitions and Key terms

Pipeset: Pipes that can be connected together in a straight line.

Working pressure: Pressure at the water source (kPa).

Hydraulic pressure: Equivalent to working pressure but with measurement in meters (m).

Flow rate: Volume of water discharged per unit of time at the water source (m^3/h).

Sprinkler flow: Volume of water discharged per unit of time by a sprinkler (m^3/h).

Rotating spray nozzle: Water distribution device equipped with a rotating deflection pad to distribute water.

Wetted radius: Farthest distance measured while the spray nozzle is rotating normally, from the spray nozzle centerline to the point where water is deposited (m).

Precipitation: How much water reaches the ground, equivalent to natural rainfall (mm/h).

Distribution pattern: Pattern showing precipitation by location in the field.

Uniformity of distribution: Evenness of water throughout a field.

Symbols are listed in **Table 1**.

Table 1.
Symbols.

Symbol	Description	Units
Q	Flow rate	m^3/h
Q'	Sprinkler flow	m^3/h
A	Cross-sectional area of nozzle	m^2
n	The number of sprinklers	
μ	Discharge coefficient	
H_p	Hydraulic pressure	m
d	Diameter of nozzle	mm
α	Trajectory angle of nozzle	$^\circ$
g	Acceleration due to gravity	m/s^2
P	Precipitation	mm/h
R	Wetted radius	m
r	Distance from the sprinkler	m
ρ	Average precipitation over the area covered by one sprinkler	mm/h
CU	Christiansen's uniformity coefficient	
(x_i, y_j)	Coordinate of grid node in a network	
N	Total number of grid nodes	
T	Irrigation time	h
$h_{i,j}$	Precipitation at (x_i, y_j)	mm

General Assumptions

- There is no infiltration, evaporation, or wind.
- Time spent on moving the pipes is negligible.
- Sprinklers used in a pipe set are of the same type.
- The diameter of the sprinkler is small compared to the dimensions of the watered area.
- We ignore the height of sprinklers [Carrión et al. 2001].
- Pressure at each sprinkler equals working pressure.
- Sprinkler flow rate remains stable.

Model Design

Wetted Radius

The wetted radius mainly factors in working pressure and nozzle characteristics such as size, trajectory angle, and rotating velocity. Accelerating the rotating speed reduces wetted radius. Wetted radius depends on hydraulic pressure, diameter, and (not significantly) on trajectory angle, of nozzles via the relationship

$$R = f(\alpha, h_p, d).$$

When the trajectory angle α is stationary, an empirical formula commonly used by manufacturers is

$$R = \xi h_p^m d^n, \tag{1}$$

where ξ, m, and n are parameters evaluated by the manufacturer's testing at various water pressures.

Applying least-squares to the experimental wetted radius of three typical rotation spray sprinklers, we get parameter values for four different trajectory angles (**Table 2**). We can substitute parameter values and easily obtain wetted radius.

Table 2.

Values of parameters.

Trajectory angle $(^\circ)$	ξ	m	n
7	11.46	0.369	0.319
15	5.61	0.225	0.734
22.5	8.63	0.140	0.476
30	4.52	0.128	0.844

Number of Sprinklers

Sprinkler flow depends mainly on two factors: working pressure and diameter of the nozzle. Flow rate from a nozzle increases with working pressure and can normally be fitted to the equation [Zhao 1999]:

$$Q' = 3600\,\mu A \sqrt{2gH_p}, \tag{2}$$

where

Q' is sprinkler flow (m^3/h);

μ is the discharge coefficient, usually between 0.75 and 0.98; and

A is the cross-sectional area of nozzle (m^2).

The number of sprinklers required is $n \approx Q/Q'$.

Water Distribution

Distribution Pattern of a Single Sprinkler

Water distribution patterns are usually obtained under controlled no-wind conditions. **Figure 1** is a common pattern for simulating precipitation from a single sprinkler [Mateos 1998].

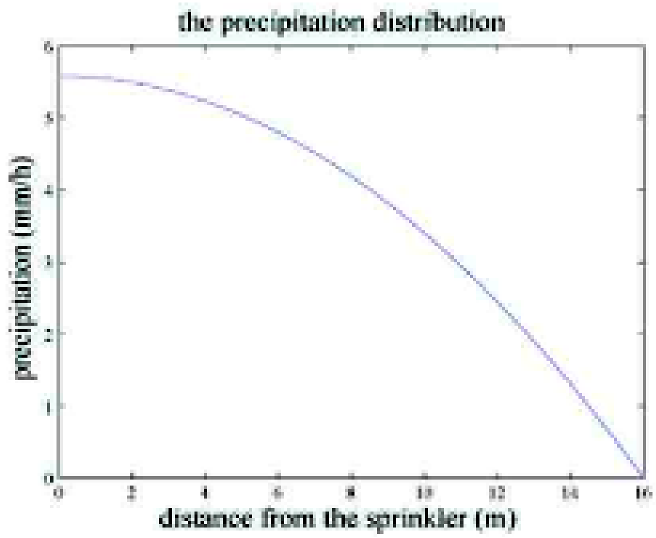

Figure 1. Distribution pattern of an individual sprinkler.

The mathematical function is

$$P = \frac{2Q'T}{R^2\pi}\left(1 - \frac{r^2}{R^2}\right),$$

where

P is the precipitation (mm/h),

r is distance from the sprinkler (m),

$Q^{'}$ is sprinkler flow (m^3/h),

R is wetted radius (m), and

T is irrigation time (h).

Water Distribution over the Whole Field

We divide the field uniformly into sufficiently small grid squares (0.25 m $\times$ 0.25m) and overlap the precipitation from each sprinkler.

Average Precipitation

To adjust the amount of water received on each part of the field per hour or per day, we introduce the concept of average precipitation (mm/h):

$$\rho = \frac{Q^{'}}{R^2 \pi}, \tag{3}$$

where $Q^{'}$ is sprinkler flow (m^3/h) and R is wetted radius (m).

Uniformity of Water Distribution

Irrigation uniformity is a major factor in maintaining proper crop growth. We calculate a uniformity coefficient for the field [Wilcox and Swailes 1947], using that of Christiansen [1941]:

$$\text{CU} = \left(1 - \frac{\sum_i \sum_j (h_{i,j} - \bar{h})}{N \times \bar{h}}\right) \times 100\%, \tag{4}$$

where

$h_{i,j}$ is precipitation at (x_i, y_i) (mm/h),

$\bar{h}$ is the average value of all $h_{i,j}$, and

N is the total number of grid nodes.

Model Validation on a Small Ranch

The Specifications

The field is 80 m $\times$ 30 m. Each pipe has a 10-cm inner diameter with rotating spray nozzles with 6 mm inner diameter, and the pipes connected together are 20 m long. The pressure of the water source is 420 kPa, with a flow rate of 150 L/min. No part of the field should receive more than 0.75 cm/h of water, and each part should receive at least 2 cm of water every 4 days.

Number of Sprinklers

Given the flow rate and diameter of the nozzle, we calculate the sprinkler flow using **(2)**, then assess that the number of sprinklers should be four.

The Conditions on the Ranch

Our simulation has 121×321 total grid nodes, with grid size 0.25 m $\times\, 0.25$ m. Equation **(4)** becomes

$$\text{CU} = \left(1 - \frac{\sum_{i=0}^{120} \sum_{j=0}^{320} (h_{i,j} - \bar{h})}{121 \times 321 \bar{h}}\right) \times 100\%.$$

Schemes of Positioning and Moving

We examine several typical workable schemes and compare them to find the optimal configuration.

Single Lateral Pipe

If all pipes are connected into one lateral pipe, then we have the approximate average precipitation rate by **(3)** that should be satisfied:

$$\frac{4 \times \frac{1}{4} Q}{\pi R^2} < 7.5 \text{ mm}. \tag{5}$$

If we use a rotating spray sprinkler with a trajectory angle of $30°$, then by **(1)** and **Table 2**, the wetted radius is $R \approx 20$ m.

Description

The mainline pipe is located across the field as shown in **Figure 2**. The lateral pipe is moved across through the field at right angles to the row direction. This lateral pipe has four sprinklers 6.67 m apart.

Results

We design a schedule for four days. The minimum time is 1228 min, with four cycles, and CU $= 78\%$. The distribution pattern is shown in **Figure 3**.

Multiple Lateral Pipes

We try to improve the uniformity of the precipitation by changing the position of the lateral pipe and the spacing between sprinklers, but we can't get a satisfactory result, since CU cannot be improved. In addition, wind normally has a significant impact on sprinklers with a higher trajectory angle. We conclude that more than one lateral pipe should be used.

We conclude that two lateral pipes with two sprinklers on each are appropriate. In light of **(3)**, the approximate average precipitation rate should be satisfied:

$$\frac{2 \times \frac{1}{4} Q}{\pi R^2} < 7.5 \text{ mm}. \tag{6}$$

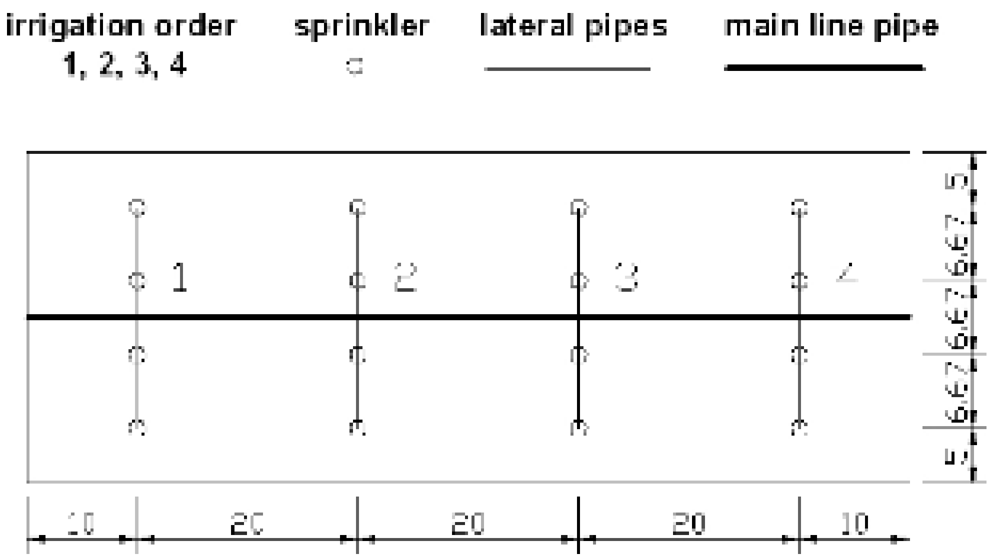

Figure 2. Single-lateral-pipe scheme, with measurements in meters.

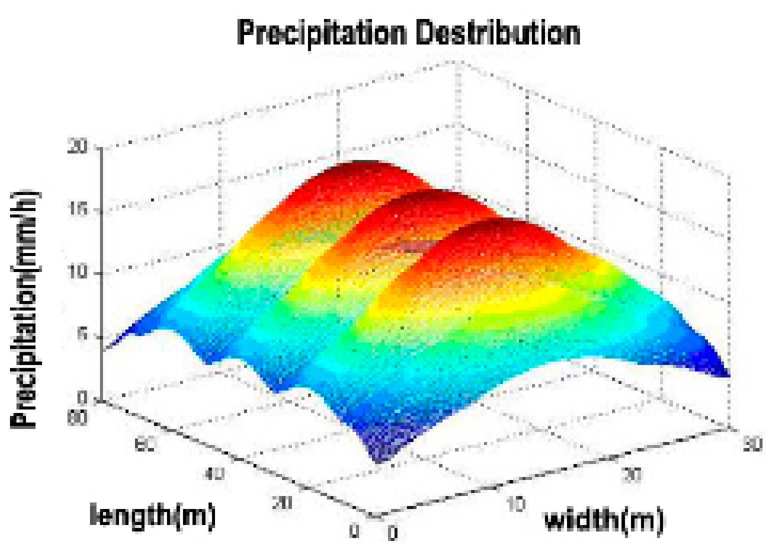

Figure 3. Distribution pattern for the single-lateral-pipe scheme.

If we use a rotating spray sprinkler with a trajectory angle of $15°$, then by **(1)** and **Table 2**, the wetted radius is $R \approx 16$ m.

Description

The mainline pipe goes along the edge of field, connected to two lateral pipes. Each lateral has two sprinklers 5 m apart. The two lateral pipes are moved crossways. The irrigation order and positions of sprinklers are presented in **Figure 4**.

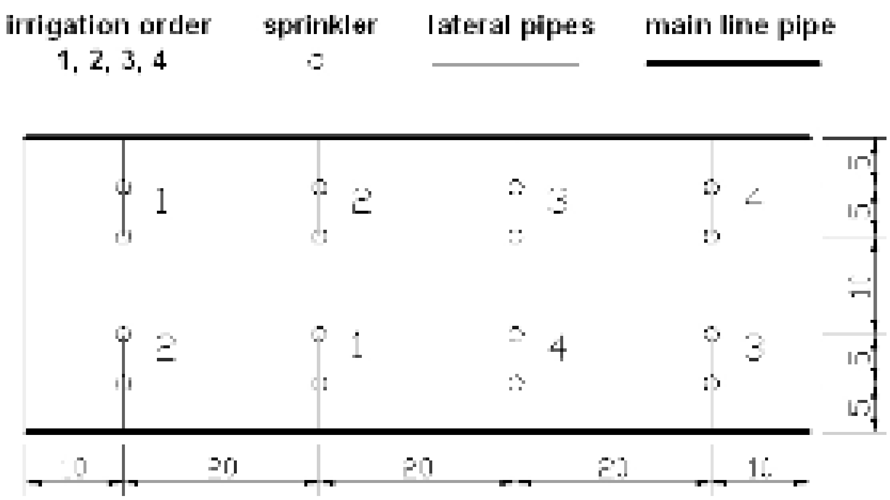

Figure 4. Multiple-lateral-pipes scheme, with measurements in meters.

Results

We design a schedule for four days. The minimum time is 920 min, with four cycles, with CU $= 83\%$. The distribution pattern is shown in **Figure 5**.

Improved Multiple-Lateral-Pipes Scheme

Description

With multiple lateral pipes, precipitation is relatively excessive in the middle of the field, due to overlap. We move the sprinkler closer to the edge to uniformize the precipitation. Using a 0.25 m $\times$ 0.25 m network, we move sprinklers on both sides, node by node symmetrically, to determine the optimal position. Two sprinklers 5 m apart on the same lateral, at 3 m or 8 m from the edge of the field, are optimal (**Figure 6**).

Results

We design a schedule for four days. The minimum time is 732 min, with four cycles, and CU $= 90\%$. The distribution pattern is shown in **Figure 7**.

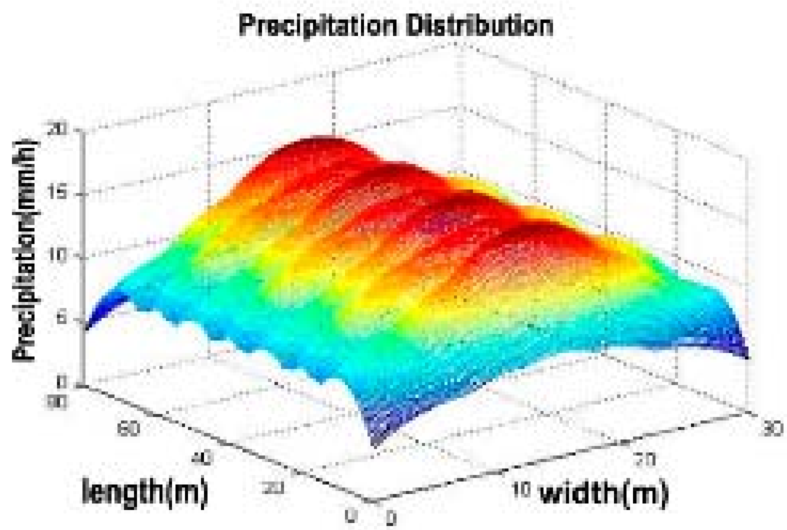

Figure 5. Distribution pattern for the multiple-lateral-pipes scheme.

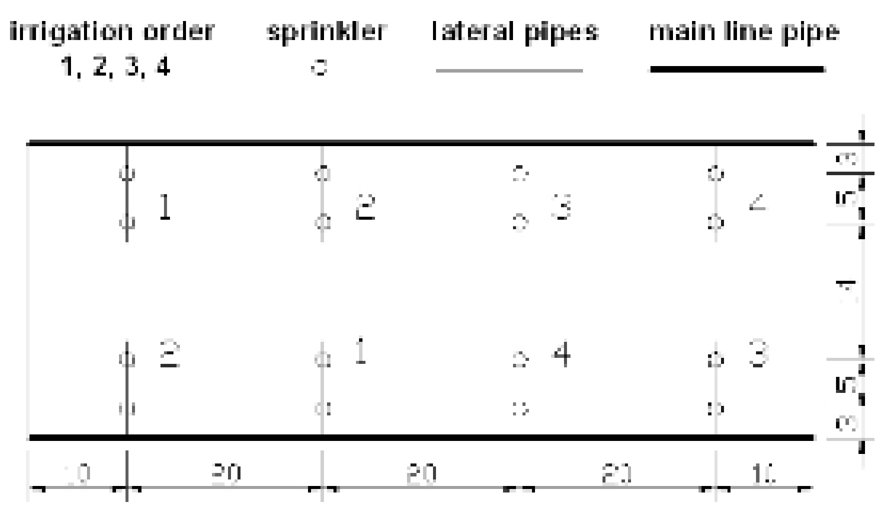

Figure 6. Improved-multiple-lateral pipes scheme, with measurements in meters.

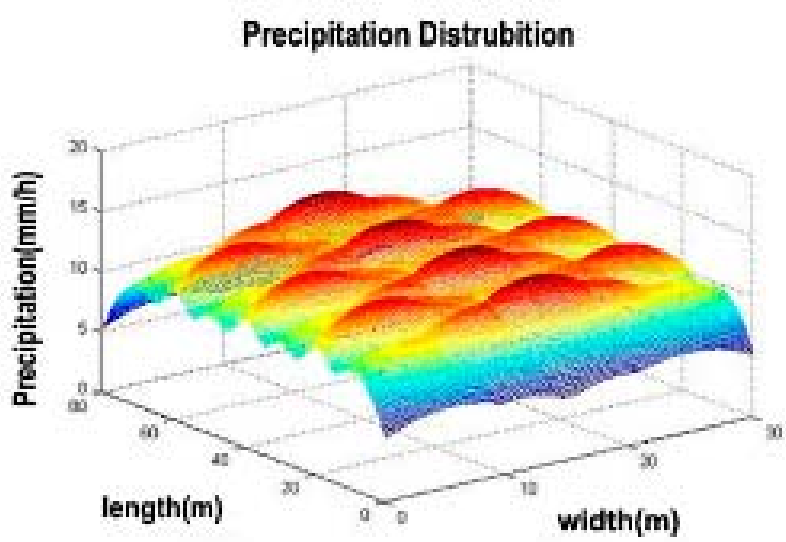

Figure 7. Distribution pattern for the improved multiple-lateral-pipes scheme.

Conclusions

Irrigation Schedule

[EDITOR'S NOTE: We omit the schedule.]

Comparison of Schemes

The multiple-lateral-pipes scheme and the improved multiple-lateral-pipes scheme take less time (920 min and 732 min) than the single-lateral-pipe scheme (1228 min).

The improved multiple-lateral-pipes scheme is superior, with both minimum time and the highest Christiansen's uniformity coefficient.

Sensitivity Analysis

We do a sensitivity analysis of the variation in CU and in minimum time with value of wetted radius. (**Figures 8–9**). Both figures show that our model is robust.

To obtain the optimal scheme, we use an algorithm to move sprinklers on both sides node by node symmetrically. **Figure 10** shows the sensitivity of CU to distance of the sprinkler from the main line. In our model, the difference of CU between two grid nodes is no more than 1.3%.

The sprinklers in the middle of the field can be shut off selectively and don't need to work continuously during one cycle; the minimum time is determined

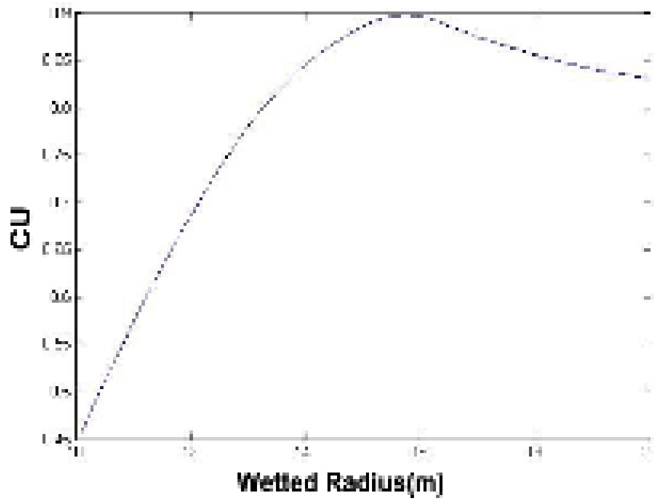

Figure 8. The variation in CU caused by wetted radius (for minimum time). Our improved multiple-lateral-pipes scheme uses a wetted radius of 16 m.

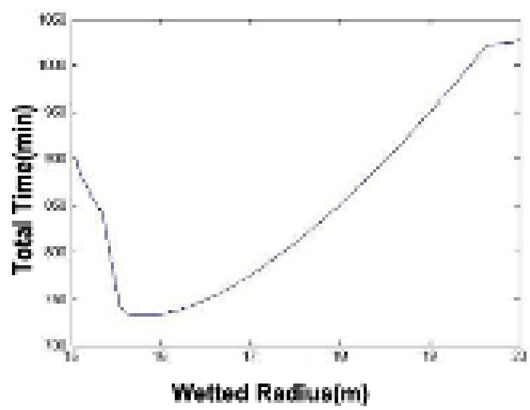

Figure 9. The variation of the minimum time caused by wetted radius. Our improved multiple-lateral-pipes scheme uses a wetted radius of 16 m.

by the sprinklers near the edge. That way, we can not only carry out the irrigation more uniformly but also save much water.

Strengths and Weaknesses

Strengths

- We find the water distribution pattern from the sprinklers, using simulation.
- We investigate different numbers of lateral pipes and different values of the wetted radius.
- We provide a good result and find an optimal scheme.

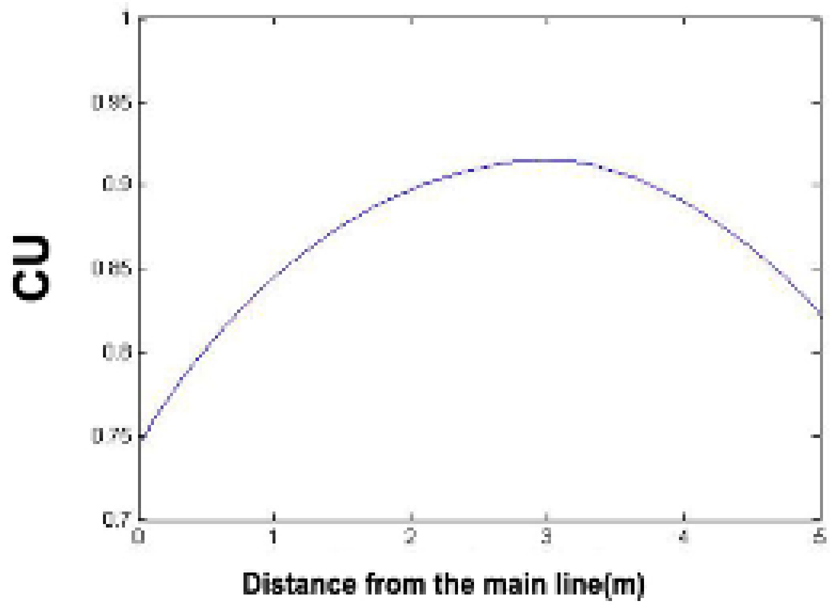

Figure 10. Sensitivity of uniformity to distance of sprinkler from the main line. In our improved multiple-lateral-pipes scheme, the distance is 3 m.

- We examine various approaches and modifications to find the best design for the irrigation system.

Weaknesses

- We did not incorporate into our model some factors that might have effect in real life, such as infiltration, evaporation, and wind.

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Judge's Commentary: The Outstanding Irrigation Problem Papers

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Introduction

Irrigation planning is a real-life activity with many complexities; good system design can demonstrate profound water savings. For the contest problem, an entire region must be minimally watered but not overwatered, and trade-offs between fixed and periodically moved equipment must be made.

As in any real-life modeling activity, the approaches, metrics, and results of others can be obtained with little effort—when applicable, this earlier work should be used, or improved upon. For example, the most widely used measure of irrigation uniformity in the turf industry is Christiansen's uniformity coefficient. Also, manufacturers' specifications of sprinkler characteristics are easily obtained.

The components appearing in a solution must be identified. For this problem, the judges were looking for the following components:

- Defined constraints on the problem, such as the needed water flow rate.
- Subjective constraints on the problem, such as what "optimal" is.
- Created constraints on the problem, such as the water distribution pattern from a single sprinkler.
- One or more metrics by which a solution can be evaluated.
- A procedure for obtaining an optimal solution.
- A description of the optimal solution.

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Problem Specifics

One of the first considerations is the meaning of "optimal" for this problem. It could be the number of times that the pipes must be moved, or it could be related to the distance that the pipes must be moved. Either of these, or other similar metrics, are reasonable. Minimizing the actual time of watering—selected as a metric by some teams—does not seem to be as useful; it does not obviously correlate to cost.

The contest problem was stated without much detail. In fact, the problem didn't state exactly where the water outlet was in the field. For this reason, a high-level model, appropriate for simple problem descriptions, is warranted. The judging focused on resolving the difficulties—for this problem, the difficulty was in determining the pipe layout. Excessive detail in, say, the distribution of water from a sprinkler head, is not warranted. Use of either a realistic model (easily available from manufacturers) or a simple model is appropriate.

In a real situation, the actual sprinkler head water distribution, wind, and other secondary considerations could be important. A description of how they affect the high-level model, and its solution, is warranted—even for a high-level model. However, only if the high-level model solution is complete is it appropriate to incorporate their effects.

Problem Areas

Most of the teams approached the problem well and identified most of the components noted in the **Introduction**. There were two areas, however, that confused several teams.

- The maximal soaking rate of 0.75 cm/h was intended to be (in the words of the Colorado team) an "average, not instantaneous overwatering" constraint. While mathematically equivalent to 0.0125 cm/min, it was not the problem's intent to prohibit a solution that watered at the rate of 0.025 cm/min, if this watering occurred for less than 30 min in an hour.

- Care is needed to determine the flow rate and pressure from the sprinkler heads when there is more than one. The Duke team had a very clean derivation of this result (although atmospheric pressure of approximately 100 kPa is missing in their computations). In summary: When the flow is *pressure-limited* (i.e., few sprinkler heads), then energy balance (Bernoulli's equation) can be used to determine the output speed. When the flow is *volume-limited* (i.e., several sprinkler heads), then mass conservation can be used to determine the output speed.

What Made Them Outstanding

The Outstanding papers obtained solutions that could be shown to a customer. These papers obtained schedules by using analytical thinking and by numerical optimizations (using, for example, simulated annealing, genetic algorithms, or methodical searches); some did both. The length of the Outstanding papers, as submitted, varied from 17 to 60 pages, with an average of 29 pages.

Further Comments and Advice

Some overall comments on the submissions and the judging process:

- The summary should include:
  - problem synopsis,
  - description of analysis, and
  - results.

  This section is worth writing, and then rewriting; it gets much attention.
- An ideal paper concluded with an explicit watering/movement schedule and a statement about the effectiveness of the schedule. While a result may be exact for a given model, the model is only an approximation to reality. As such, it is unrealistic to report many decimal places in the results. In practice, pipe placement may be accurate to a foot or so; a placement schedule should not require centimeter accuracy.
- Many submissions created very detailed models. It is always an advantage to start with a simple, perhaps idealized, version of the problem. Even an approximate solution to this idealized problem, perhaps obtained by hand, can be used as a bound when comparing the results from more detailed models. Such back-of-the-envelope checks can be vital in checking the reasonableness of a solution.
- When there are different ways to attack a problem, try using several techniques. If they lead to the same answer, then the answer is probably close to correct. And when computer models are used, sensitivity analysis is especially important (and it should be relatively easy to carry out). For example, what happens to the field watering if the pipes are not placed in the exact right positions?
- "Dead ends" are typically useful only if they lead to an insight or constrain a model in some way. Details on such "dead ends" rarely contribute to a paper's overall ranking; at some point, more is less.

- Graphics indicating the pipe layout and the resulting water distribution were created by most teams. These graphics reveal much information about a pipe layout and its solution.
- As is usually the case, the judges wanted to see justifications for the assumptions made. Reusing standard results (say, those obtained from a book or from the Web) is appropriate; but justifying their applicability is an important aspect of reuse. Note that re-deriving standard results adds little value. Also, the only assumptions that should appear are those used in the problem analysis.
- Finally, details must appear somewhere; if they appear in two places, then error-checking can occur. For example, when a mathematical statement appears suspect, the judges will often locate the computation in the code to see exactly what was implemented.

About the Author

Daniel Zwillinger attended MIT and Caltech, where he obtained a Ph.D. in applied mathematics. He taught at Rensselaer Polytechnic Institute, worked in industry (Sandia Labs, Jet Propulsion Lab, Exxon, IDA, Mitre, BBN, Ratheon), and has been managing a consulting group for the last dozen years. He has worked in many areas of applied mathematics (signal processing, image processing, communications, and statistics) and is the author of several reference books.

Pp. 333–366 can be found on the *Tools for Teaching* 2006 CD-ROM.

Profit Maximizing Allocation of Wheelchairs in a Multi-Concourse Airport

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Summary

To minimize Epsilon Airlines' cost of providing wheelchair assistance to its passengers, we examine the trade-off between explicit costs (chairs and personnel) and implicit costs (losses in market share). Our *Multi-Concourse Airport Model* simulates the interactions between escorts, wheelchairs, and passengers. Our *Airline Competition Model* takes a game-theoretic perspective in representing the profit-seeking behavior of airline companies. To ground these models in reality, we incorporate extensive demographic data and run a case study on 2005 Southwest Airlines flight data from Midland TX, Columbus OH, and St. Louis MO. We conclude that Epsilon Airlines should employ a "hub and spokes" strategy that uses "wheelchair depots" in each concourse to consolidate the movement of chairs. Across different airport sizes and strategies, we find that two escorts per concourse and two wheelchairs per escort are optimal.

Introduction

We study the procedures used by airlines to shuttle passengers from arriving flight to connecting flight. According to the U.S. Department of Transportation, "The delivering carrier shall be responsible for assistance in making flight connections and transportation between gates [for passengers needing assistance]" [2003]. With an aging population, more passengers need help.

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We develop two models. The *Multi-Concourse Airport Model* simulates the interactions of passengers, wheelchairs, and airport staff within an airport, following each passenger and tracking delays. Passengers needing wheelchair assistance are shuttled through the airport using one of three algorithms.

The *Airline Competition Model* simulates a competitive marketplace over a 40-year period. Profit depends on both costs and market share; the latter fluctuates based on customer satisfaction. The *Airline Competition Model* provides a way to judge accurately and objectively the merit of wheelchair allocation strategies.

Key Terminology

- **Gate:** A location where air passengers board flights. A given gate can be represented as an ordered pair (i, j), where i indexes the concourse and j indexes the gates in a concourse.
- **Concourse:** A collection of gates. Concourse i contains k_i gates and is represented as a vector $c_i = \langle (i,1), (i,2), ..., (i,k_i) \rangle$.
- **Airport:** A collection of concourses, which we consider as a graph.
- **Passenger:** A traveler in an airport, associated with an arriving flight and with a connecting flight. We distinguish WPs (wheelchair passengers) from non-wheelchair passengers.
- **Traffic:** The mass of passengers in an airport. The level of traffic affects the number of WPs needing transport between gates.
- **Wheelchair depot:** A location where wheelchairs are stored while not in use. In the *hub and spokes* strategy, there is a depot in each concourse.
- **Escort:** An airline employee responsible for picking up WPs from arrival gates and transporting them to connecting gates.
- **Missed flight:** When a passenger arrives more than 15 min after the connecting flight's departure time, the flight leaves without them.
- **Strategy:** An algorithm for the flow of escorts and wheelchairs throughout the airport.

Basic Assumptions

Airport Layouts

An airport consists of 1 to 10 concourses, each of with consists of 2 to 50 gates. Gates in the same concourse are generally located close to one another, while the travel time between concourses can be quite lengthy. Hence, we assume that inter-concourse travel is much lengthier than intra-concourse travel.

Our model represents concourses and gates as nodes in a graph.

Table 1.
Variables and their meanings.

Variable	Definition
A	Airport, graph of concourses
c_i	Concourse i, a node of A
C	The number of concourses in A
(i, j)	Gate j within concourse i
k_i	Number of gates in c_i
Day	Type of day modeled by the simulation
Year	The year modeled by the simulation
$N_t^{(i)}$	Market share of airline i at time t
$f_t^{(i)}$	Fraction of customers defecting for airline i at time t
P_t	Year t population of the airport market
χ	A strategy
W^*	Ratio of wheelchairs to wheelchair needing customers
E^*	Ratio of escorts to wheelchair needing customers
W_t	Number of wheelchairs in year t
E_t	Number of escorts in year t
U_M	Utility loss of missing a flight (> 0)

Table 2.
Constants and their values.

Constant	Definition	Value
p_E	Escort annual salary (including benefits)	\$40,000 /yr
p_W	Purchase price of a wheelchair	\$ 135
x^*	Maximum time that a plane will wait for a passenger	15 minutes
ω	Proportion of passengers who are WPs	1.6%
p_{inf}	Proportion of WPs informing of arrival	95%
v_F	Velocity of an escort walking alone	250 ft/min
v_S	Velocity of an escort pushing a wheelchair	180 ft/min
U_m	Utility loss for being delayed one minute on the runway	$0.0005U_m$
U_G	Utility loss for a chair idling one minute by a gate	$0.0001U_m$
p_S	Storage cost of a wheelchair	\$50/year
π	Airline profit per customer	\$4.51

Wheelchair Passenger Needs

WPs comprise a proportion ω of the total passenger pool (1.5% in 1996 [Conway 2001] and 1.6% in 2006 (the starting year of our model).

A proportion p_{inf} of WPs inform our airline of their need before arrival; we assume $p_{\text{inf}} = 95\%$. Knowing p_{inf} and $\omega = .015$ and $p_{\text{inf}} = .95$, we use a binomial distribution to find a probability mass function for the arrival of WPs on a flight (**Table 3**).

There are two ways that a WP can miss a connecting flight:

- Late incoming flight: Roughly one-third of flights arrive late and about 5%

Table 3.
Unexpected wheelchair passengers in a flight of 120 people.

Unexpected Passengers	Probability (%)
0	91.39
1	8.23
2	0.37
3	0.01

are at least an hour late [U.S. Department of Transportation n.d.].

- Slow arrival of escorts: We try to minimize this risk.

Intergate Transportation

- Average fast walking speed is 250 ft/min (3 mph), but average speed when arms are immobilized (as when pushing a wheelchair) is only 180 ft/min (2 mph) [Gross and Shi 2001]. We assume that an escort walks at these speeds.
- An escort can operate only one wheelchair at a time. U.S. Dept. of Transportation guidelines discourage leaving WPs unattended. Hence, the escort takes a WP to the connecting flight and remains until the flight leaves.
- Airport customer service employees (escorts) earn on average \$11.80/h; the annualized cost with benefits per escort is $p_E = \$40{,}000$ [Bureau of Labor Statistics 2004]. Transport wheelchairs bought in large batches cost \$135/chair [Transport Wheelchairs 2005].
- Passengers who arrive more than 15 min late to their connecting flight are left behind; airlines wait just this long for delayed arriving flights.
- Escorts are in contact with one other via radio.

Market Share and Delays

Wheelchair service is not just a legal responsibility but a good idea from a customer-relations standpoint and enhances competition for market share. A passenger who misses their connecting flight (due to poor wheelchair allocation or a delayed arriving flight) must wait (perhaps several hours or overnight) for the next outbound flight. Additionally, that wait could make that passenger and others miss a subsequent connecting flight.

Multi-Concourse Airport Model

Formal Definition

Let A be an airport with C concourses $c_1, ..., c_C$ and gates $\langle(1,1), ..., (C, k_C)\rangle$, where k_i is the number of gates in concourse c_i. Further, let E^* be the ratio of escorts to total WPs and W^* be the ratio of wheelchairs to total WPs. Escorts and chairs are assigned by some strategy χ.

Several factors in the airport system are beyond our control:

- ω, the proportion of passengers who are WPs;
- Day, i.e., high- or low-traffic days;
- Year (passengers in years past 2006 have different demographics); and
- costs of wheelchairs (p_W) and of wages (p_E) for escorts,

We take these factors as exogenous to our model, so the only control variables are χ, W^*,and E^*.

We seek to minimize cost by reducing both explicit costs (escort wages, wheelchair purchases) and implicit costs (lost business due to frequent delays).

The *Multi-Concourse Airport Model (MCAM)* relates the three control variables to explicit costs and total delays. Delays include both missed flights and late departures as a result of missing passengers.

Let C be daily cost and D be disutility of a delay. We want a function f such that

$$f(\chi, W^*, E^*) \longmapsto (C, D).$$

The MCAM model runs a Monte Carlo simulation for several different days. This process gives the expected daily delays and costs, so the results of MCAM serve as a suitable proxy for f.

Aggregating Delays and Disutility

The airline has a set policy that a plane will wait up to x^* min for a passenger heading toward the gate. Increasing x^* decreases delay due to missing flights but increases delay for passengers waiting aboard planes; similarly, lowering x^* favors boarded passengers at the expense of late passengers.

The airline seeks an optimal value for x^* to balance the discomfort of waiting passengers against the probability that the late passenger will arrive in time.

Let disutility for small unexpected delays be linear in time, so if 120 passengers on a plane wait 15 min, the total utility loss is proportional to the $120 \times 15 = 1{,}800$ min of delay. Also, let the utility loss from missing a flight be $-U_m < 0$ and set time $t = 0$ to be the flight's planned departure time. If a passenger is not at the gate at $t = 0$ but we know that they are on their way, then we wait up to x^* min for them.

Let L be a random variable for the lateness of our passenger; the probability of arrival after $t = 0$ is $P(L \leq x^*|L \geq 0)$. This means the late passenger benefits by $U_m P(L \leq x^*|L \geq 0)$, while the others expect to wait $E[T|0 \leq T \leq x^*]$, since they will leave in at most 15 min.

With x^* chosen optimally, lost utility from waiting equals benefit to the late passenger. So, when N passengers are waiting, optimality is achieved when

$$NE[T|0 \leq T \leq x^*] = U_m P(L \leq x^*|L \geq 0).$$

Given our past experience, we assume that $x^* = 15$, so that

$$U_m = \frac{N \times E[T|0 \leq T \leq 15]}{P(L \leq 15|L \geq 0)}.$$

We determine average $P(L \leq x^*|L \geq 0)$ and $E[T|0 \leq T \leq x^*]$ from our simulation results of an airport with a large enough supply of escorts and wheelchairs that every WP is immediately taken to their connecting flight (**Table 4**).

Table 4.

Benefit of waiting 15 min for late passengers.

Airport	*Day*	$P(L \leq 15\|L \geq 0)$ (%)	$E[T\|0 \leq T \leq 15]$ (min)
Midland	Low-Delay	31.5	5.0
Columbus	Low-Delay	23.2	6.9
St. Louis	Low-Delay	19.8	8.5
Midland	High-Delay	25.9	2.5
Columbus	High-Delay	26.9	6.6
St. Louis	High-Delay	26.3	7.7
Average		**25.6**	**6.2**

An average plane has capacity 120 and load factor .695 (69.5% of seats occupied), so the effective N is $120 \times .695$, we have $U_m = (.695)(120)(6.2)/(.256) \approx 2000$, which means that missing a flight is 2,000 times as bad as one person waiting 1 min—a reasonable result.

A Third Source of Disutility

Idle wheelchairs near gates are an inconvenience to passengers and a liability risk to airlines. Every minute that a wheelchair sits at a gate, it contributes disutility equal to 20% of the disutility of a single individual being delayed 1 min.

Aggregate Disutility

To combine the three disutilities, note that 1 min of flight delay affects the N people waiting at the equivalent of $\frac{N}{2000} U_m = 0.0005 N U_m$, where U_m is the disutility of missing a flight. Also, 1 min of a wheelchair idling by a gate provides disutility 20% as large, or $0.0001 U_m$.

The Strategy

The strategy set χ governs the rules that escorts follow in making their decisions. These include:

- How do escorts choose which WP to pick up?
- How do escorts find a chair to use?
- What do escorts do after they've dropped off their WP?
- Where do escorts leave a wheelchair when they are done using it?

The MCAM tests three strategies, one random and two "intelligent."

Random Strategy

When a WP arrives at the airport, a free escort is randomly chosen to shuttle them to the connecting gate. The escort stays with the wheelchair at the gate until the next assigned WP.

Intelligent Strategies

The two intelligent strategies borrow their names from airline industry terminology: *direct transfer* and *hub and spokes*.

The airport knows in advance about most WPs. Each escort and each gate agent (the airline representative at a gates) has an ordered list of expected WPs. The heuristic for ordering WPs waiting to be taken to connecting gates is:

$$H = \text{time until flight leaves} - \text{time to reach gate (via wheelchair)}.$$

If a WP is expected, an escort anticipates their arrival by waiting at the gate. (In our implementation, expected WPs are inserted into the waiting queue 20 min before their flight lands.) When unexpected WPs arrive, the gate agent there radios over an open channel so that everyone can update their lists.

An escort who becomes free reports to the group. The WP at the top of the list is assigned to the closest free escort, who radios to find available wheelchairs, preferring one close to the WP or on the way to the WP.

Our intelligent strategies differ about what escorts do after shuttling a WP.

- **Direct Transfer:** The gate-based strategy runs all operations out of the gates in an airport. After an escort drops a WP off at gate (i, j), the escort and chair remain at that gate until assigned another WP. For the next assignment, say at gate (i', j'), the escort radios to find a chair closer to (i', j'). The strategy spreads the wheelchairs out among the gates so that any gate likely has a wheelchair nearby. A disadvantage is unattended chairs near gates.

- **Hub and Spokes**: Each concourse has a wheelchair depot, where wheelchairs are stored. After an escort drops off a WP at gate (i, j), the escort returns the wheelchair to the depot in concourse i. When escorts are idle and there are no WPs to be shuttled, the chairs wait in the depots (instead of at the gates). Wheelchair depots eliminate leaving chairs in high-traffic areas near the gates themselves, and escorts know that all available chairs are at depots.

Long-Term Concerns

The MCAM simulates a single day of airport activities, but to choose a wheelchair/escort strategy we should consider long-term factors. Ideally, our model will take the data from the MCAM and use it to simulate several years of airline operation. The problem is that several factors that are constant in MCAM could change over 10 or 40 years.

Aging Population

In 2000, one-sixth of the US population was over 60 years old [U.S. Census Bureau 2005] and 72% of wheelchair users are in this age group [Conway 2001]; a person over 60 is 13 times as likely to need a wheelchair as a person under 60.

The over-65 age group will grow by 40% in the next 20 years [U.S. Census Bureau 2005]. We assume that in 2006 1.6% of passengers are WPs; using the Census Bureau's demographic data, we calculate the fraction of future air travelers who will be WPs.

It is not clear whether this growing older group will take proportionately more or fewer plane flights in the future (a question of saved income and free time vs. health). We assume a middle ground: The proportion of wheelchair users in the country and on flights are directly proportional. In our models, we use the appropriate ω for each year.

Lower Profit Margins

In 2004 and 2005, airline earnings were reduced by high jet-fuel prices.If high prices persist, airlines will be forced to raise ticket prices (sales will fall) or cut profit margins. We simulate this effect by using a smaller marginal profit per passenger in the long-run model than in the short-run model.

Airline Competition Model

The MCAM model simulates the flows of escorts, chairs, and WPs. But we are searching for a cost-minimizing strategy. To do this, we need to model the changing market share of our airline (based on customer satisfaction) and derive a profit function for its operations. Maximizing profit is equivalent to minimizing costs if we view lost future business as a cost.

The *Airline Competition Model (ACM)* uses the output of the MCAM to determine market share and profitability for a group of competing airlines.

Market Share

A principal factor in an airline's long-run profitability is market share, the proportion of the market the airline holds at an airport. After experiencing flight delays or missed flights due to limited availability of wheelchairs, WPs and non-WPs may defect to other airlines. We model defection as a function of total disutility to an airline's passenger base.

Let there be M airlines and let $N_t^{(i)}$ be airline i's market share at time t, with $f_t^{(i)}$ the fraction of its customers who defect at the end of period t. Assuming that defecting passengers choose one of the $M-1$ other airlines with equal frequency, the stochastic process for market share is

$$\Delta N_t^{(i)} = N_{t+1}^{(i)} - N_t^{(i)} = \left(\sum_{j \neq i} \frac{1}{M-1} N_t^{(j)} f_t^{(j)} \right) - N_t^{(i)} f_t^{(i)}. \qquad \textbf{(1)}$$

Alternatively, we could suppose that defecting passengers distribute themselves among the $M-1$ other airlines in proportion to airline market share:

$$\Delta N_t^{(i)} = \left(\sum_{j \neq i} \frac{N_t^{(i)}}{1 - N_t^{(j)}} N_t^{(j)} f_t^{(j)} \right) - N_t^{(i)} f_t^{(i)}$$

Without data on how consumers choose airlines, we assume that ticket price is the overwhelming factor. Ticket pricing is complex but does not vary with market share—the giant Southwest and tiny Vanguard Airlines offer comparable rates [Airline pricing data 2005]. Because of the importance of prices, we believe that **(1)** is more accurate.

If the total market grows by a proportion r, new passengers choose carriers so that market shares remain unchanged. We can express this assumption nicely using matrix notation. Letting $\mathbf{N_t}$ denote the vector of market shares and define the elements of $M \times M$ matrix $\mathbf{A}$:

$$a_{ij} = \begin{cases} \dfrac{1}{M-1}, & \text{if } i \neq j; \\ -1, & \text{if } i = j. \end{cases}$$

This simplifies **(1)** to:

$$\mathbf{N_{t+1}} = (\mathbf{I_M} + \mathbf{AF_t})\mathbf{N_t},$$

where $\mathbf{F^{(t)}}$ is the matrix whose diagonal entries are $f_t^{(j)}$, and whose off-diagonal entries are zero and $\mathbf{I_M}$ is the $M \times M$ identity matrix. This formula iterates nicely to give the closed form

$$\mathbf{N_T} = \left[\prod_1^T (\mathbf{I_M} + \mathbf{AF_t}) \right] \mathbf{N_0}.$$

The distribution of the multivariate random variable $\mathbf{F_t}$ is fixed in the short run while the proportion of handicapped individuals remains constant. In the short run, therefore, $\mathbf{N_T}$ is (up to a constant) the product of T uniformly distributed random variables, all distributed as $(\mathbf{I_M} + \mathbf{AF})$ (we drop the subscript on $\mathbf{F}$ because its distribution is independent of time in the short run).

Passenger Defection

The rationale behind a profit-based model is to quantify the trade-off between the cost of accommodating WPs and the loss in market share associated with customer dissatisfaction. In the short term, airlines would be more prone to provide less accommodations because the resultant effect on market share would not be seen until the next period. However, in the long term, the market share for airlines providing poor service will suffer and profit will fall. Moreover, short-run costs include the fixed cost of purchasing wheelchairs, while long-run costs are the smaller costs of chair maintenance and replacement. Our profit function measures only the profit gained (or lost) from wheelchair allocation strategy.

Let there be J different types of days, each of with its distribution of delays. For example, days before holidays and weekends probably have longer delays. A day of type j experiences a defection of $f_t^{(i,j)}$ from total market share, based solely on total disutility of passengers of airline i traveling on that day. We also assume that traffic is constant over days of each type but varies across types; these traffic differences affect the underlying value for $f_t^{(i,j)}$. Let there be V_j days of type j per year.

The MCAM simulates total disutility of passengers given the parameters (χ, W, E), and we obtain $f_t^{(i,j)}$, the daily defection rate. This gives a distribution for the random variable $g_t^{(i,j)} = \log\left(1 - f_t^{(i,j)}\right)$, for various days t, implying a mean and variance for such a distribution, which we denote by the ordered pair $\left(\mu(i,j), \sigma(i,j)^2\right)$:

$$g_t^{(i,j)} \sim \left(\mu^{(i,j)}, (\sigma^{(i,j)})^2\right).$$

The retention rate for a day is given by $1 - f_t^{(i,j)}$, so total retention for a year is given by

$$1 - F_t^{(i)} = \prod_{j=1}^{J} \prod_{v=1}^{V_j} \left(1 - f_v^{(i,j)}\right).$$

Taking logarithms yields

$$\log\left(1 - F_t^{(i,j)}\right) = \sum_{j=1}^{J} \sum_{v=1}^{V_j} g_v^{(i,j)}.$$

By the Central Limit Theorem, if the values for V_j are sufficiently large (they are 35 and 330 for our program), we have the approximate distribution

$$\sum_{v=1}^{V_j} g_v^{(i,j)} \sim \mathrm{N}\left[V_j \mu^{(i,j)}, V_j \left(\sigma^{(i,j)}\right)^2\right],$$

where $N[\mu, \sigma^2]$ is the normal distribution. In our implementation, we use random draws for the realizations of $g_v^{(i,j)}$.This implies that $F_t^{(i)}$ is approximately distributed as

$$F_t^{(i)} \sim 1 - \exp\left(\sum_{j=1}^{J} N\left[V_j \mu^{(i,j)}, V_j \left(\sigma^{(i,j)}\right)^2\right]\right).$$

Our profit model is constructed to hold all factors constant except wheelchair strategy. Because of this feature, when a WP misses a flight, it is always the result of poor wheelchair allocation and not of another factor. We assume that missing a flight causes a WP to defect from the airline with probability $p_d = 1/4$. This high probability is reasonable, since to the WP it appears as if the airline has neglected them by not shuttling them to their connecting gate.

On day t, airline i has $P_t N_t^{(i)}$ passengers in its market share, of whom $n_t^{(i)}$ total are traveling on day t with airline i.

The probability of not defecting after missing a flight is $1 - p_d$. We assume that the probability of not defecting is multiplicative in the number of missed flights, that is, after missing m flights, there is a $1 - (1 - p_d)^m$ probability of defection.

We also assume that the disutility D_t is uniformly distributed across all passengers, so each passenger has disutility $u_t = D_t / n_t^{(i)}$, measured as a multiple of U_m, the disutility of missing a flight. An individual defects with probability $1 - (1 - p_d)^{u_t}$.

By the Law of Large Numbers, the total number of individuals defecting on a particular day approximately equals the expected value of this random variable. We therefore have:

$$f_t^{(i,j)} = n_t^{(i)}[1 - (1 - p_d)^{u_t}].$$

Implementation of Delay Distributions

In our implementation, we use actual 2005 daily data on average delays for Southwest Airlines. At each of three airports, about 10% of days have particularly high delays [Southwest Airlines 2006]. So we categorize days as *high-delay* (the top 10%, 35 days) or *low-delay* (the bottom 90%, 330 days).

Southwest Airlines reports an average load factor of 69.5% (the percentage of occupied seats on a flight). Since high-delay days are often during peak travel times, we assume that flights operate at full capacity on such days; a weighted average calculation gives 100% and 66% for the load factors on high- and low-delay days, respectively.

Present Value of Profits

We assume that costs and airline profits per passenger grow at a constant annual rate r_C, and we let r_D be the nominal interest rate.

Let $\Pi_t^{(i)}$ denote the real (adjusted for inflation) profit in year t for airline i. If costs and profits per unit of good grow at a constant inflation rate, then $\Pi_t^{(i)}$ can be determined assuming zero inflation. Inflation will be included in the discount factor for the long-term calculation of firm profit.

So, airline i maximizes the expected present value of profits given the discount rate δ:

$$\Pi^{(i)} = \sum_{t=0}^{T} \delta^t \Pi_t^{(i)}.$$

We let $T = 40$ and make projections out to 2046. The current inflation rate is approximately 2%, so we estimate r_C as 2%, and r_D is the forward risk-free rate in the future, which we assume to be constant and equal to 4.5%.

$$\delta = \frac{1}{1 - r_C + r_D} = \frac{1}{1.025} = \approx 0.975.$$

The Profit Function

Profit related to wheelchair policy can be split up into the following contributive factors:

- total profits from passengers, which is proportional to market share;
- wheelchair purchase and replacement costs;
- wheelchair storage costs; and
- escort salary and benefits payments.

We derive from these contributive factors a formula for profit, given by:

$$\Pi_t^{(i)} = \begin{cases} P_t N_t^{(i)} \pi - p_W R_t - p_S W_t - p_E E_t, & \text{for } t > 0; \\ P_t N_t^{(i)} \pi - (p_W + p_S) W_t - p_E E_t, & \text{for } t = 0, \end{cases}$$

where:

P_t = total number of airline passengers in the market,

$N_t^{(i)}$ = market share of airline i,

π = profit per passenger,

R_t = number of wheelchairs replaced in year t,

p_W = price of a wheelchair,

p_S = annual storage cost of a wheelchair (\$50), and

p_E = annual cost of an escort.

Earlier, W^* and E^* were the proportions of chairs and escorts to the population, but here we need W_t and E_t, the actual number of chairs and escorts used.

The cost in the first period differs because there is an initial purchase cost of wheelchairs. In later periods, wheelchairs need replacement at 20% per year, so at the end of year t we have $0.8W_t$ usable wheelchairs. Due to changes in market share, we may desire more or fewer wheelchairs in year $t+1$; the target number of chairs is

$$W_{\text{target},t+1} = W^* P_t N_t^{(i)},$$

and similarly, $E_{\text{target},t+1} = E^* P_t N_t^{(i)}$. We won't throw away good chairs, so the number of chairs we use in year $t+1$ is:

$$W_{t+1} = \max\{0.8W_t, W_{\text{target},t+1}\}.$$

This gives the necessary number of chair replacements:

$$R_{t+1} = \max\{0, W_{\text{target},t+1} - 0.8W_t\}.$$

Implementation

The computer simulation of ACM relies heavily on the results produced by MCAM. We take M airlines, each with a different (χ, W^*, E^*), and for a given year we estimate their total costs and total disutilities.

Starting with the first year, ACM simulates the operation of airline i by using its strategy (which remains fixed through the end year, T) as the input for $MCAM$. Realizing that high-delay and low-delay days affect airport operations differently, we simulate 35 high days and 330 low days in each year.

This gives and output of total disutility which determines gain or loss in market share by **(1)**. (In the first year, we start every airline with an equal market share.) We can calculate each company's profit for year t and use the updated market shares in the calculation for year t.

This simulation runs for 40 years and result is a profit vector (across time) for each airline. We discount future periods at the rate δ and compare the present values that the various strategies (χ, W^*, E^*) produce. Recall that our profit function does not determine the airline's actual profit (which involves buying planes, pilots, flight attendants, etc.) but only the profit related to wheelchair use in airports. The relative value of discounted profit is how we gauge which strategy is most attractive.

Case Study

Southwest Airlines reported a 2005 profit of \$313 million from 70.9 million passengers, or \$4.41 / passenger. However, this number is already reduced by costs included in our profit function, namely wheelchair and escort costs, which we estimate at \$0.10 / passenger.

The data for our case study are quite extensive, including flight times, airport layouts, load factors, and average delays per airport, for airports in Midland TX, Columbus OH, and St. Louis MO.

Results and Observations

Multi-Concourse Airport Model

We applied the random, direct-transfer, and hub-and-spokes strategies to the case-study data, using 5 escorts with 5, 10, or 20 wheelchairs, and 10 escorts with 20 chairs. There are several notable results:

- Low-delay days and high-delay days give about the same disutility across all strategies when only 5 escorts are used. When 10 escorts are used, high-delay days give an average of 40 more disutility equivalents than low-delay days. A possible explanation is that 5 escorts are kept busy on both low- and high-delay days, but on a low-delay day 10 escorts is sufficient to handle all of the wheelchair traffic. On a high-delay day, some every late passengers will miss their flights even with 10 escorts available.

- The random strategy performs nearly identically to direct transfer under all chair / escort configuration combinations and both delay types. This could be due to the fact that the direct transfer strategy distributes wheelchairs so sparsely (at all the gates) that the assignment is essentially random.

- The hub-and-spokes strategy is less effective than the other two strategies when 5 escorts are used but more effective with 10 escorts. The hub-and-spokes algorithm involves streamlining wheelchair movement, so without adequate personnel it may not be efficient. In essence, the hub-and-spokes strategy requires a base number of escorts to be effective.

Airline Competition Model

For each of the three strategies and four combinations of chairs and escorts, we calculate the market share (reported as N_{year}) as well as the final profit.

The *Hub and Spokes* strategy with 10 escorts and 20 chairs earns the top market share in 2046 (11.1%), while the same strategy with 5 escorts and 20 chairs does the worst (7.2%). We believe that this is because the hub-and-spokes system is labor-intensive, since escorts must walk longer in taking chairs back to their depots.

In terms of profit, the strategy (Hubs, 20, 10) wins again—by a lot! (106 vs. 101.9 for second-best). Interestingly the strategy (Random, 20, 10), which had the 3rd-best market share, also had the 2nd-worst profit.

The winner is the hub-and-spokes configuration with 20 chairs and 10 escorts.

Analysis of the Models

Strengths of Model

The model uses actual data, including flight delay distributions for all 365 days in 2005, to rank wheelchair policies in terms of market share and long-term profits. Over the long-term, it also accommodates projections of changes in the proportion of WPs in the airline passenger market.

The model realistically captures uncertainty about prior information about need for wheelchairs.

The model converts total passenger disutility into a defection rate, which captures the effects of quality of service in a competitive market. The dynamics show both the long-term and short-term effects of the trade-offs between budgeting and loss/gain in market share.

Weaknesses of Model

Growth in passengers depends on many factors, including changing demographics, which may change from predicted values.

Conclusion

We recommend the hub-and-spokes configuration with two escorts per concourse and two wheelchairs per escort, to maximize both gain in market share and long-term profit. These suggestions are a fast and effective formula for higher profits. After all, at the end of the day, all we want is to make Epsilon greater than Delta.

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Minimization of Cost for Transfer Escorts in an Airport Terminal

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Summary

We minimize the cost for Epsilon Airlines to provide a wheelchair escort service for transfers in an airport terminal. We develop probabilistic models for flow of flight traffic in and out of terminal gates, for the number of passengers on flight who require service, and for transfer destinations within the terminal.

We develop an economic model to quantify both the short- and long-term costs of operating such a service, including the salaries of escorts, the maintenance and storage of wheelchairs, and the costs incurred when late escorted transfers delay a departing flight.

We develop a simulated annealing (SA) algorithm that uses our economic models to minimize cost by optimizing the number and allocation of escorts to passengers. Having indexed the space of all possible escort allocations to be accessible to our SA, we selectively search the space of allocations for a global optimum. Although the space is too large to find a global optimum, our simulations suggest that the SA is effective at approximating this optimum.

Using current airport and airline data, we break our analysis down into short- and long-term costs, simulating escort service operation under dynamic airport conditions, varying air traffic, airport size, and the fraction of traveling population that requests wheelchair-aided transfer (simulating a greater future abundance of elderly travelers).

The UMAP Journal 27 (3) (2006) 349–365.

Introduction

We describe, quantify, and optimize a cost-effective approach to wheelchair-aided escorted transfers at Epsilon Airlines.

We investigate the airport layout, particulars of flight travel and passengers, and the economics of the escort service. We use a simulated annealing (SA) algorithm to find a cost-optimized solution, so that Epsilon's many satisfied customers will continue to say: "Epsilon will always be in my neighborhood!"

Definitions and Key Terms

- An **airport terminal** is an ordered pair $(\tau, d_{\text{terminal}})$, where τ is an ordered collection of concourses each of which is an ordered collection of gates, and d_{terminal} is a metric defining the distance between any two gates of the terminal (**Figure 1**).
- A **concourse** is an ordered collection of gates.
- A **gate** is an element of a concourse at which *arrivals* and *departures* take place. A gate's position in a terminal is determined by an ordered pair (concourse#, gate#)
- A **wheelchair passenger (WP)** is a passenger who arrives at a gate, has a connecting flight at another gate, and who can move between gates only by means of a wheelchair pushed by an airline escort. A WP normally notifies the airline in advance of the need for an escort.
- An **escort** is an airline employee who wheels WPs to connecting flights, with salary per shift of P_E.
- A **shift** is an 8-hour period during peak operating hours of the terminal.
- A **transfer job** is an ordered 4-tuple

$$(\text{concourse} \times \text{gate}_{\text{arr}}, \text{time}_{\text{arr}}, \text{concourse} \times \text{gate}_{\text{dep}}, \text{time}_{\text{dep}})$$

  describing the time and place of arrival and of departure of a WP.
- The **delay time** of a departure is the time difference between the actual departure time and the scheduled departure time of a given flight, given that the actual departure time is later. The actual departure time is when the last WP booked on the flight arrives at the departure gate.
- A shift's **transfer schedule** is the set of escort-required transfers that are scheduled throughout the shift. New WP announcements and unexpected non-escort-related delays cause the transfer schedule to be updated throughout the day.

- An **escort job list** is the ordered set of transfer jobs performed by a particular escort over the length of the shift.
- An **escort allocation algorithm** is a general method of determining the number of escorts and specifying the escorts' behavior throughout a shift.

Model Assumptions and Key Concepts

Geometry of an Airport Terminal

We demonstrate our optimization over one airport terminal geometry (with a varying number of concourses.)

We represent a terminal by the union of a central hub with the cartesian product of n concourses and m gates per concourse: $\{0\} \cup \{1, ..., n\} \times \{1, ..., m\}$. (**Figure 1**). We denote the distance between the start of the concourse to the concourse's first gate, as well as the distance between any two adjacent gates of a concourse, by r_g, and the distance from the central hub to the start of each concourse by r_h. The distance between any two *non-hub* points (n_1, m_1) and (n_2, m_2) of a terminal is given by the metric

$$d_{\text{terminal}} = \delta(n_1, n_2)|r_g(m_1 - m_2)| + (1 - \delta(n_1, n_2))|r_g(m_1 + m_2) + 2r_h|.$$

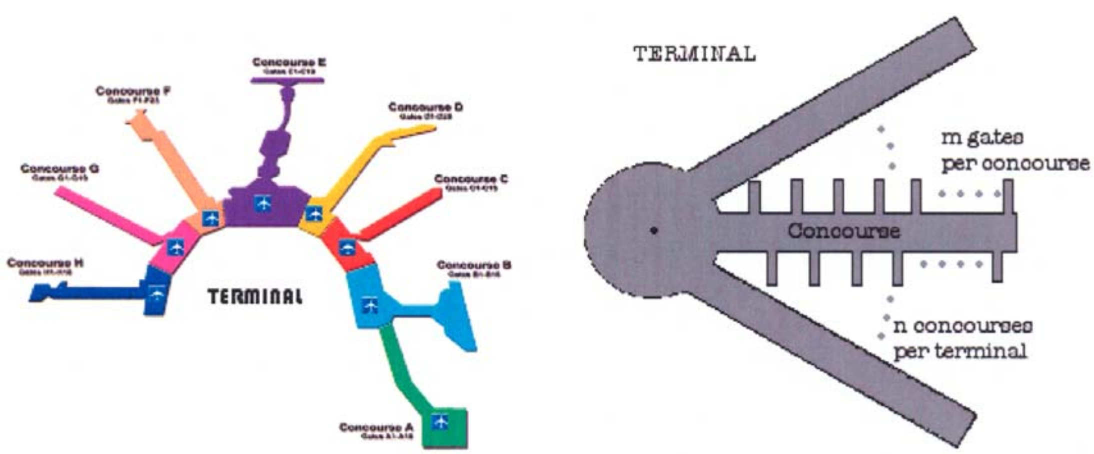

Figure 1. Terminal map of Miami International Airport [n.d.] and a schematic 2-D diagram of our essentially 1-D representation of an airport terminal.

Flight Specifications

Flow of Air Traffic at Terminal

- We take as *departure time* the latest time that a passenger can board a flight. We define a parameter t_{sit} to be the time that a plane sits at a gate after boarding and before takeoff.

- Every arrival flight becomes a departure flight at the same gate some fixed time later, and we call this fixed interval of time the *turnover time* t_{to}. We also call the departure flight *incident to* the arrival flight.

- We define a parameter O_G, a terminal's *gate occupancy*, as the fraction of terminal gates at which planes are docked at any given time.

- We describe the flow of flight traffic through a terminal via five parameters:

  - the number n of concourses per terminal,
  - the number m of gates per concourse,
  - the sit time t_{sit},
  - the turnover time t_{to}, and
  - the terminal gate occupancy O_G.

  The time between terminal arrivals is

$$t_{\text{gap}} = \left\lceil \frac{t_{\text{to}} + t_{\text{sit}}}{mnO_G} \right\rceil .$$

- We generate a terminal's daily flight schedule as follows: We begin with no airplanes docked at the terminal, and let them arrive one at a time, each at a time t_{gap} apart, randomly assigning them to free gates as they come in. We generate a schedule of departures by having each arrival flight depart a time t_{to} after arrival. We run this simulation for 12 simulated hours and select the middle 8 to form our terminal shift schedule.

Delays

- Arrival delays occur according to a normal distribution with mean and standard deviation $(\bar{x}_{\text{delay}}, \sigma_{\text{delay}})$, though we consider all delays under 15 min to be negligible, following the practice of most available flight delay statistics [U.S. Department of Transportation n.d.].

- A departure flight incident to a delayed arrival flight is delayed by the same amount. However, we there are no other causes of departure delay (such other delays are tangential to the problem of transfer escorts).

Transfer Specifications

WPs per Flight

- We fix a parameter P_F as the number of passengers per flight.

- We let a parameter S_{transfer} be the fraction of passengers who transfer from a flight upon arrival, and a parameter $S_{\text{wheelchair}}$ be the fraction who request wheelchair assistance. Therefore, the probability that a given passenger requires wheelchair service (the probability of a WP) is $P(\text{WP}) = S_{\text{transfer}} \times S_{\text{wheelchair}}$. (We neglect passengers who require wheelchair service but do not transfer.) Given this probability, we determine the probability of r WPs on a flight, assuming a binomial distribution:

$$P(r) = \frac{P_F!}{r!(P_F - r)!} P(\text{WP})^r [1 - P(\text{WP})]^{P_F - r}. \qquad \textbf{(1)}$$

We cap r at 3, since the probability of more is negligible.

Allowable Transfers

- Given a schedule of n transfers, and a general escort allocation algorithm, we define the ith component of a delay time n-vector $\vec{dt}$ to be the delay time of the ith transfer.
- We fix a buffer time t_{buffer} such that no airline will book a transfer time less than t_{buffer}.
- We fix a greatest allowable transfer time $t_{\max}$, to capture the fact that the transfer time between flights is not usually very long.
- The transfer time is uniformly distributed between t_{buffer} and $t_{\max}$.

Method for WP Assignment

For every flight, we choose the number of WPs according to **(1)**. To each WP, we assign a randomly chosen occupied terminal gate as the destination for the transfer. If the gate's flight's departure time falls within the allowable transfer interval, we schedule the transfer; otherwise, we search at random for a new terminal gate.

At the Gate

- For simplicity, we assume immediate deplaning of all passengers upon arrival.
- An escort may drop off a WP at a gate any time before departure. That is, WPs can board without the escort and escort's wheelchair.
- The time that it takes a WP to load in and out of a wheelchair is negligible.

Traversing the Terminal

- An escort who completes a transfer job moves directly toward the arrival gate of the next job, even if required to wait there for the flight's arrival. An escort always takes the shortest route and has complete informational contact with the manager assigning jobs.

- Escorts have two walking speeds: v_{walk} (with an empty wheelchair) and $v_{\text{wheel}} = \frac{3}{4}\, v_{\text{walk}}$ (with an occupied wheelchair).

Wheelchairs and Wheelchair Storage

A wheelchair in the hands of an escort presents no liability risk to other terminal traffic. Therefore, we stipulate that an escort walks around with a wheelchair at all times.

Economic Model of an Escort Service

Associated Costs

- Flight delays: If there are not enough escorts, our service will cause flight delays.
- Salary of escorts.
- Maintenance of wheelchairs.
- Storage of wheelchairs: Wheelchairs cannot be left lying haphazard about a terminal before or after a shift—they pose a liability risk. So we stipulate that outside of the escort shift, wheelchairs are stored in a storage facility.

Quantification of Escort and Wheelchair Costs

- Let K be the number of escorts and P_E be an escort's pay per shift.
- Let $W = K$ be the number of wheelchairs and let P_W be their maintenance cost per shift.
- Let A_W be the area that a wheelchair takes up in storage, and let R_A be the daily rent of airport storage space per square foot.

The Short Term Dollar Cost of Delay Time

- There is no effect of delay time on ticket sales.
- Escorts are paid on an annual basis, independent of extra hours logged due to flight delay. Therefore, delay times incur no overtime costs.
- The only short-term cost of flight delay, therefore, is the high cost of operating the plane during the delay.
- Therefore, given operating cost C_{pl} of a plane (\$/min) and the delay time dt of a flight (min), the short-term cost of a flight delay for duration dt is $C_{\text{pl}}\, dt$.

The Long Term Dollar Cost of Delay Time

- We attribute the long-term cost of delay time to two factors: the added operating cost of planes, and a long-term reduction in ticket sales.

- We quantify the reduction in ticket sales. Consider the standard Microeconomics 101 two-good problem: A consumer is faced with purchasing t_ϵ flight tickets on Epsilon Airlines or t_A flight tickets on Airline A. Assume that Epsilon Airlines and Airline A are indistinguishable except for ticket prices (P_ϵ, P_A) and total average delay times per shift (D_ϵ, D_A). The dollar-utility function of a consumer is

$$U(t_\epsilon, t_A) = E_{\text{pa}}(t_\epsilon + t_A) - P_\epsilon t_\epsilon - P_A t_A - C_{\text{pa}}(D_\epsilon t_\epsilon + D_A t_A),$$

  where

  - t_i is the number of airline$_i$ tickets purchased per shift;
  - P_i is the price of an airline$_i$ ticket;
  - E_{pa} is the dollar utility that a consumer receives from a flight (assumed to be the same for all flights, for simplicity);
  - C_{pa} is the dollar utility of passenger time in dollars/time; and
  - D_i is the average WP-related delay time per shift of airline$_i$.

  Optimizing this dollar utility with respect to the t_is, we find:

$$\frac{\partial U}{\partial t_\epsilon^*} = 0 \Longrightarrow P_\epsilon = E_{\text{pa}} - C_{\text{pa}} D_\epsilon, \qquad \frac{\partial U}{\partial t_A^*} = 0 \Longrightarrow P_A = E_{\text{pa}} - C_{\text{pa}} D_A.$$

  Note that

$$D_\epsilon = \frac{\sum_i dt_i}{F_{\text{shift}}},$$

  where the index i runs over all the delay times of all transfers in a shift, and F_{shift} is the number of flights per shift. Now, recalling that P_F is the passengers per flight, we find that

$$P_F F_{\text{shift}} P_\epsilon = E_{P\text{pa}} P_F F_{\text{shift}} - C_{\text{pa}} P_F \sum_i dt_i.$$

  Since the product on the left side of the equation is total revenue per shift, we can associate the summands on the right with all contributing subrevenues and subcosts. (These numbers denote the contribution of single shifts to long-term revenue, after ticket prices have responded to changing market demand and market demand has responded to airline conditions—delay times, etc.). The effect of transfer delay times $\{dt_i\}_i$ per shift on long term revenue is given by $-C_{\text{pa}} P_F \sum_i dt_i$.

Summary: The Economic Model

By the above arguments, we may finally write the short- and long-term expense functions of our escort service per shift:

$$E_{\text{st}} = KP_E + KP_W + KA_W R_A + C_{Pl} \sum_i dt_i,$$

$$E_{\text{lt}} = KP_E + KP_W + KA_W R_A + C_{Pl} \sum_i dt_i + C_{Pa} P_F \sum_i dt_i.$$

Expectations of Our Model

We expect our model to exhibit the following behaviors:

- It should optimize allocation of escorts by minimizing costs.
- As passenger traffic increases, the number of escorts should increase.
- The number of escorts should increase with the number of concourses.
- Since long-term costs affect ticket costs but short-term costs do not, the number of escorts that optimizes long-term budget concerns should exceed the number that optimizes short-term budget concerns.
- The number of escorts should increase with life expectancy.

Annealing-Based Optimization

We consider the complete-information case where the transfer schedule is fully specified at the beginning of a shift.

Reduction to an Ordering Problem

We use the assumptions about escorts to reduce the space of escort deployments to a simple permutation group. In this formulation, with just a single escort, the problem of finding an optimal allocation is equivalent to finding an optimal hamiltonian path (a path in an undirected graph which visits each node exactly once) through a fully connected graph (one in which each pair of nodes is connected by an edge).

We define a *job node* as a transfer job indexed in a way convenient for our algorithm. It is characterized by arrival time (t_1), arrival gate (x_1), departure time (t_2), and departure gate (x_2). We index all this information by two numbers: node type (1 for a *job* node—we will see later that other node types are required to solve the multi-escort problem) and its position in the transfer schedule. We

Table 1.

Variables and parameters.

Variable	Meaning	Units	Estimate
n	Number of concourses per terminal		2
m	Number of gates per concourse	s	20
O_G	Average terminal gate occupancy		50%
K	Number of escorts hired		2
$W = K$	Number of wheelchairs		2
$S_{\text{wheelchair}}$	Fraction of passengers who require a wheelchair		0.4%

Parameter	Definition	Value	Units	(source)
P_E	Escort salary per shift	80	\$/shift	(1)
r_g	Distance between adjacent gates	30	m	(2)
r_h	Distance between hub and concourse	30	m	(3)
t_{sit}	Time a plane remains at gate after boarding	10	min	(4)
t_{to}	Time between arrival and incident departure	60	min	(5)
$\bar{x}_{\text{delay}}$	Mean flight delay time	35.6	min	(6)
σ_{delay}	Standard deviation of flight delay time	24.4	min	(7)
P_F	Passengers per flight	90		(8)
S_{transfer}	Fraction of passengers who transfer	0.5		(9)
t_{buffer}	Minimum allowable transfer	30	min	(10)
$t_{\max}$	Maximum allowable transfer	120	min	(11)
v_{walk}	Average human walking speed	1.3	m/s	(12)
v_{wheel}	Speed at which escort wheels full wheelchair	1	m/s	(13)
P_W	Maintenance cost of wheelchair per shift	1	\$	(14)
A_W	Floor area that a wheelchair takes up	9	sq. ft	(15)
R_A	Daily rent of airport commercial real estate	2	\$/sq. ft	(16)
C_{pl}	Operating cost of a plane	1495	\$/h	(17)
C_{pa}	Value of passenger time	44	\$/h	(18)

Sources for Parameter Values

(1) `http://www.avjobs.com/table/airsalry.asp`
(2) Half the Boeing–747 wingspan (since gates alternate sides of concourse),
`http://airportbusiness.cygnus.proteus.com/article/article.jsp?id=1291\&siteSection=1`
(3) Estimated to be same as (2)
(4) Approximate time for flight-attendant preparation, from personal experience
(5) Approximate time to fuel a Boeing-747,
`http://www.uk-trucking.net/?page=news\&mode=view\&id=113`
(6) `http://news.bbc.co.uk/1/hi/business/1833213.stm`
(7) `http://news.bbc.co.uk/1/hi/business/1833213.stm`
(8) `www.faa.gov/library/reports/delay_analysis/media/DCOS1995.doc`
(9) Gatersleben, Michel R., and Simon W. van der Weij, Analysis and simulation of passenger flows in an airport terminal, in *Simulation—A Bridge to the Future, Proceedings of the 31st Conference on Winter Simulation*, vol. 2, 1226–1231; 1999
(10) Common airport practice
(11) Common airport practice
(12) `http://phyun5.ucr.edu/~wudka/Physics7/Notes_www/node18.html`
(13) 3/4 walking speed of (12)
(14) `http://www.1800wheelchair.com/asp/view-category-products.asp?category_id=498`,
roughly annually replaced
(15) Measured at home
(16) `http://rebuildnewyork.nreimag.com/ar/real_estate_cw_report_new/`,
assuming that concourse rent is comparable to highest NYC commercial rents
(17) `www.faa.gov/library/reports/delay_analysis/media/DCOS1995.doc`
(18) `www.faa.gov/library/reports/delay_analysis/media/DCOS1995.doc`

refer to such a node as job node i, and refer to the information that it contains with double indices (i.e., x_{i1}).

In general, we have multiple escorts. For k escorts, we try to find an optimal set of $\leq k$ distinct paths through the graph defined by the set of job nodes such that every job node is included in exactly one path. In this formulation, the problem is formidable. However, a reformulation transforms the problem back into a straightforward hamiltonian-path optimizing problem [Bellmore and Hong 1974]. An *escort node* is characterized (and indexed) by two numbers: its node type (0 for an escort node) and its placement within the ordered list of escorts. Adding $k-1$ escort nodes to our graph of job nodes partitions the transfer schedule into individual escort job lists for every hamiltonian path.

We now define a tool for labelling hamiltonian paths through this extended graph. A *configuration* is an ordered sequence containing exactly one each of the job and escort nodes of the extended graph.

Traveling from a job node to an escort node ends the escort's job list and opens up the job list for the escort indexed by the escort node that is landed on. The first escort in the list is always assumed to be selected at the beginning of the node sequence. Traveling to a job node from any node type, by contrast, adds that job to the escort job list of the current escort. This implies that any time that an escort node occurs at the beginning/end of a configuration, or two escort nodes occur consecutively within a configuration, an escort's job list is terminated with no jobs—equivalent to that escort never being hired for the shift. This formulation allows us to optimize over the number of escorts hired (up to a certain maximum number of escorts) simultaneously with optimizing over escort job assignments.

Having thus formulated our problem as a choice of an optimal hamiltonian path through a graph of $n+k-1$ nodes, we move on to solution via simulated annealing.

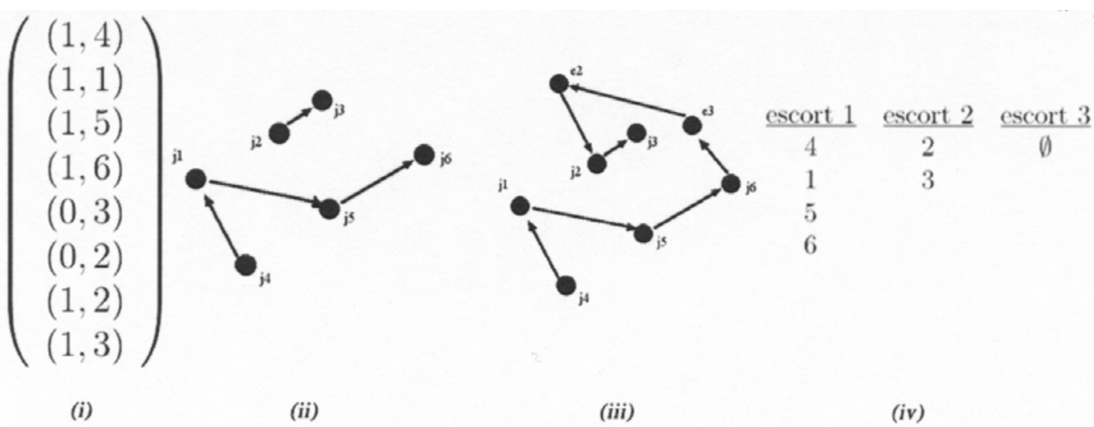

Figure 2. Various ways to represent an escort deployment configuration: (i) configuration vector, (ii) $k \leq 3$ paths through the graph of job nodes such that every job node is contained in exactly one path, (iii) hamiltonian path through the graph of job and escort nodes, and (iv) specification of each of the $(k=3)$ individual escort job lists.

Simulated Annealing Preliminaries

The inspiration for the simulated annealing (SA) approach is found in nature, in particular in the phenomenon of observable crystallization that occurs in certain systems in solid state thermodynamics when such a system is slowly cooled. The underlying idea is that if a system is cooled slowly enough from a disordered configuration at high temperature, it will tend strongly toward organizing itself into a highly optimized low-energy state. The subsequent discussion of annealing methods is adapted from Bentner et al. [2001].

One computes the number of microstates available to the total system. The fundamental assumption is that all available microstates occur with equal probability, given by the Boltzmann distribution. Normalizing this weighting distribution to obtain a probability distribution gives for microstate σ

$$\pi(\sigma) = \frac{1}{Z} \exp\left(-\frac{H(\sigma)}{k_B T}\right),$$

where

$$Z = \sum_{\tau} \exp\left(-\frac{H(\tau)}{k_B T}\right)$$

and $H(\sigma)$ is the energy of the system when in microstate σ. The function H is the *hamiltonian function* for the system. If we cool the system slowly enough to $T = 0$ so that equilibrium holds at all times (adiabatic cooling), then at $T = 0$ the system will be "frozen" into a lowest energy microstate.

For our problem, the microstates are configuration vectors, and our hamiltonian is the expense function (E_{st} (short-term) or E_{lt} (long-term). If we can simulate an adiabatic cooling, starting from an initial random microstate and moving through a trajectory of subsequent microstates, then we should obtain a deployment with lowest cost.

The Metropolis Criterion

Simulated annealing essentially runs on a trial-and-error (Monte-Carlo) scheme, aided by a clever choice of "allowed moves" from each microstate. At each iteration, an allowed move is randomly selected and a decision is made whether or not to make that move. We apply the *Metropolis criterion*:

$$p(\sigma \to \tau) = \begin{cases} \exp\left(-\dfrac{\Delta H}{k_B T}\right), & \text{if } \Delta H > 0; \\ 1 & \text{otherwise.} \end{cases}$$

Since our problem is nonphysical, we replace $k_B T$ in the formula with a control parameter T in units of money.

To lower temperature, a logarithmic process is generally used. We update the temperature after each attempted configuration step with the rule $T_{\text{new}} = \alpha T_{\text{old}}$, where α is usually taken to be in the range $(0.8, 0.999)$ [Bentner 2001].

Choosing a Starting Temperature

Case 1

The idea is to start at a temperature sufficiently high that initially the trajectory is free to move across the configuration space. In this case, if α is close enough to 1 and the set of allowable transitions is chosen well, then the algorithm should slowly settle very near to the global minimum of the hamiltonian. The initial temperature can be chosen by performing a random walk across the configuration space with the defined set of configuration transitions (i.e., iterating the annealing algorithm from the initial condition with T fixed at ∞). Multiply the observed maximum of the ΔH by 10 to obtain an initial temperature T_s very likely to be sufficiently high to meet the above description.

Case 2

If a preprocessing step is added (i.e., use a naive algorithm to presort jobs among escorts), then it may no longer be desirable to start with so high a temperature. If there is a reason to be confident that the start is in the general vicinity of an optimum, then a good starting temperature may be the dollar value of the initial condition or even half that.

Our Hamiltonian

We define the hamiltonian in terms of an algorithm that takes a given configuration as an input. This algorithm can be viewed in the flowchart in **Figure 3**. We define some of the objects found in the chart.

We define two time functions that act on job nodes.

- The first one acts on just a single job node:

$$t_{\text{dep}}\big((1,i)\big) = \frac{d(x_{i1}, x_{i2})}{v_{\text{wheel}}}.$$

  This simply defines the amount of time that it takes for an escort to perform a job from pickup to drop-off.

- The second one acts on a pair of job nodes:

$$t\big((1,i),(1,j)\big) = \frac{d(x_{i1}, x_{i2})}{v_{\text{wheel}}} + \frac{d(x_{i2}, x_{j1})}{v_{\text{walk}}}.$$

  This measures the time to perform the job on the first node from pickup to drop-off and then further to walk to the arrival gate for the second job.

The algorithm starts at the $k = 1$ box. Where multiple arrows leave a box, all of those actions are performed, right to left (or up to down for horizontal arrows).

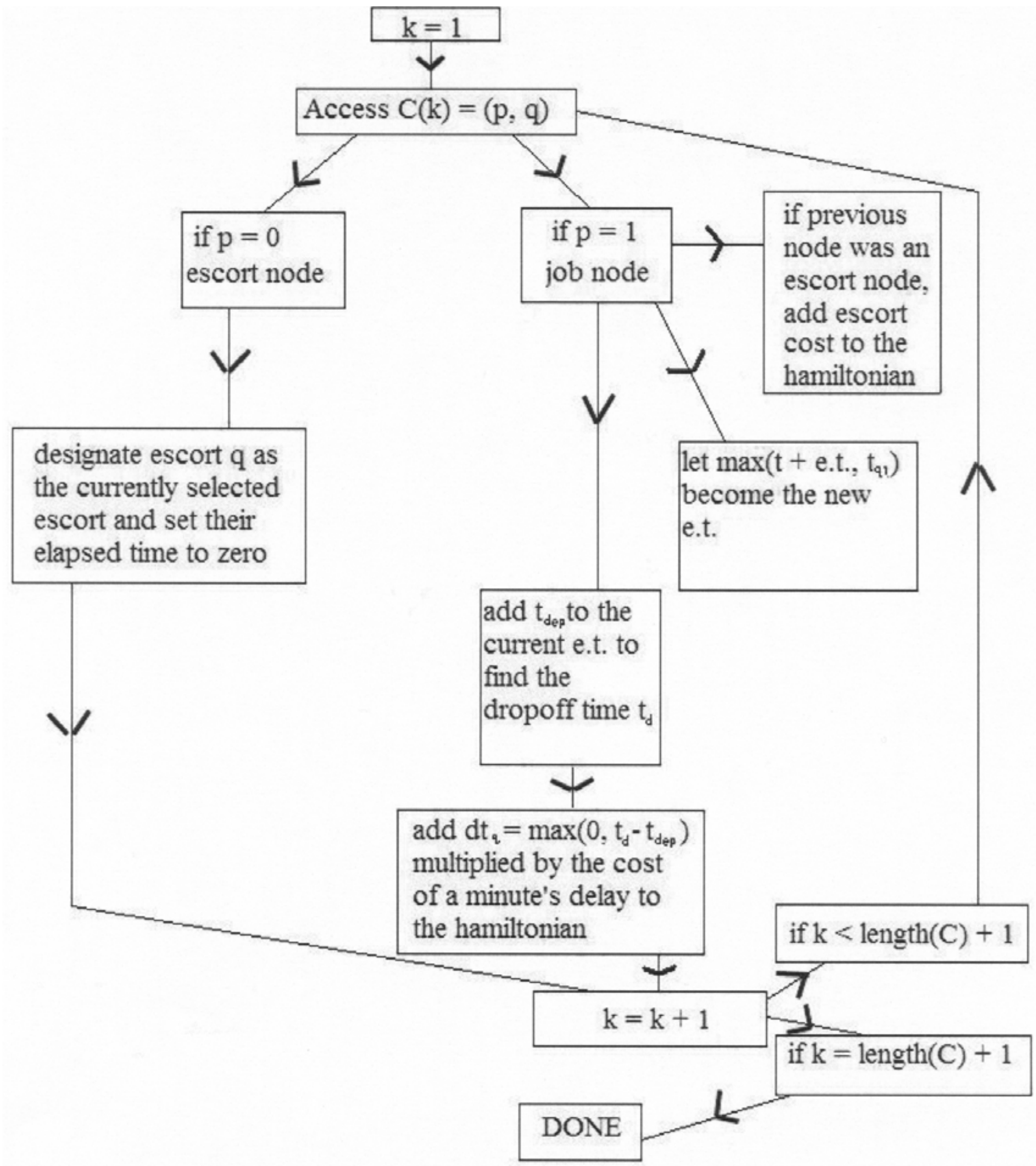

Figure 3. Flowchart outlining the algorithm for determining the hamiltonian value (cost) for a given input configuration.

The job node branches require a little more explanation. On the rightmost branch, when the job node was preceded by an escort node, we add the cost of an escort to the total cost incurred by the configuration. The cost of an escort is not just salary P_E but $(P_E + P_W + R_A A_W)$, since an escort requires a wheelchair, with its maintenance and storage costs.

On the middle branch ("let $\max \ldots$ "), there are two cases. To explain this, we note that the initial conditions for our escorts are defined with random trivial job nodes (i.e., job nodes with the same gate as both beginning and end). Now, say that the previous node was an escort node. Then t is $t\big((1, \text{ic}), (1, q)\big)$, where $(1, \text{ic})$ represents the trivial job node initial condition; that is, we take into account the time for the escort to get to the the first job from the initial location. When the previous node was not an escort node, then t is simply $t\big((1, q-1), (1, q)\big)$, i.e., the time from when the escort began transporting the WP on the previous job until arrival at the gate for the next job. The maximum over the previous elapsed time updated for additional travel time and the next job's arrival time takes into account that the escort cannot leave for the next job until both events have occurred (reached the arrival gate and the flight has arrived).

Finally, on the leftmost branch, where t_{dep} is added to the elapsed time to compute the drop-off time t_d, the function t_{dep} takes the node $(1, q)$ as its argument. This step does not update the elapsed time. Continuing along that branch, the cost for a single day is defined differently for long-term analysis vs. short-term.

Determining Neighboring Configurations

We seek allowed moves that provide for fast traversal of the configuration space but with relatively modest energy (cost) differences among neighbors. Our criteria are:

- It's best to keep things simple (i.e., move only a few around nodes in any transition), to avoid propagation effects.
- It's best to leave escort nodes in place and move job nodes around them.
- Only one escort ought to be added or deleted at a time, for salary considerations; this feature is enforced by our move types.

To avoid large changes in time ordering and propagation effects, only job nodes with departure times within a threshold of each other (we used 40 min) may be swapped. Likewise, a job node may be transferred only to a position where the job node ahead of it has a departure time within 40 min of its own.

Model Testing: Simulations and Discussion

We present a naive test algorithm that simplistically assigns escorts, as a way to demonstrate our model's relative superiority.

Given an ordered list of ordered escorts and a list of temporally-ordered transfer jobs, the naive algorithm treats the jobs as a queue. It takes a job off the queue and begin checking at the least-ordered escort. If the escort is free (has dropped off the last transfer or has had no job yet), the algorithm assigns the job to the escort. Otherwise, it checks if the next escort in order is otherwise free. If no escort is free for a given job, it assigns the job to an escort at random.

We ran simulations in a large airport environment and compared performance of our simulated annealing solution to this naive algorithm. We ran 100 random shifts over a range of numbers of escorts (using the starting condition provided by the naive algorithm), while modifying the hamiltonian so that the cost of an escort becomes zero. Essentially, we are simulating to determine how many employees should be hired so that (on an average basis) costs are minimized. We then reconstruct the full costs before plotting them and determining a minimum. From **Figure 4**, our simulated annealing solution (corner points on upper line) significantly outperforms the naive algorithm (lower line) at all numbers of escorts.

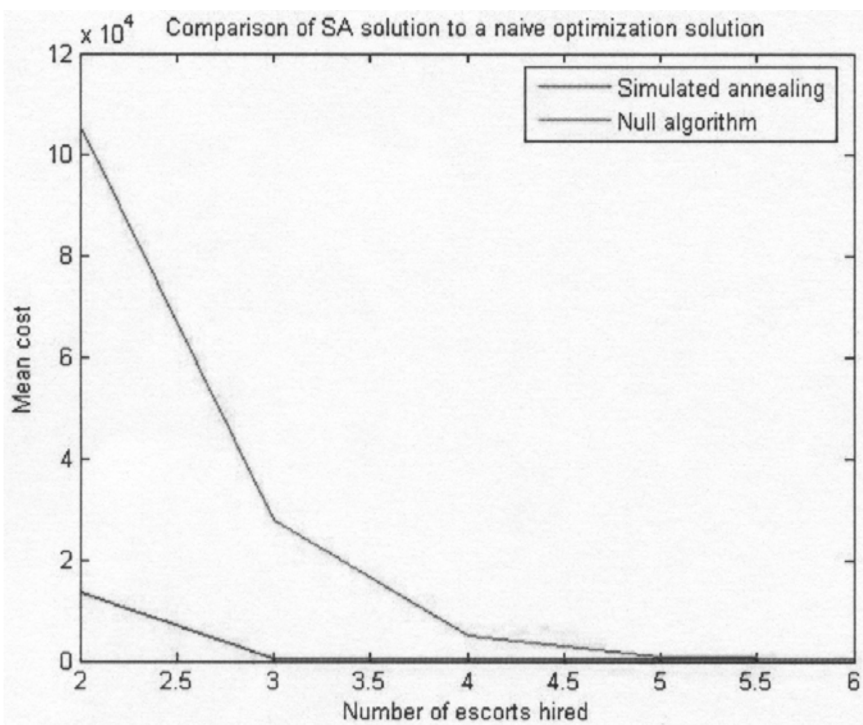

Figure 4. Trials for four concourses of 25 gates each, half the gates occupied by planes, 0.4% wheelchair passengers, and short-term cost analysis. For any number of escorts, the simulated annealing solution (corner points on lower line) yields a lower cost than the naive algorithm (upper line) because it is more efficient in allocating jobs. The apparent mean costs of 0 are an artifact of the large scale (1 vertical unit = $10,000); the optimal number of escorts in the simulated annealing solution is 4, with average cost about $475.

To check our model's response to the dynamic changes, we ran our algorithm for

- increasing concourses per terminal (airport size);
- increasing average gate occupancy (air traffic);
- increasing S_{WC}, simulating a future increase in life expectancy and thus an increase in abundance of elderly among air travelers;
- short-term vs. long-term.

We find that

- the optimal number of escorts increases from 1 in the small single-concourse terminal to 2 for the two-concourse terminal and to 4 for the large four-concourse terminal;
- as terminal traffic increases, the required number of escorts increases.
- increasing the percentage of passengers requiring wheelchair assistance increases the number of escorts needed;
- when there are enough escorts so that there is negligible delay time, the short-term and long-term costs are essentially the same—but when there is delay, its cost is exaggerated in the long-term model. The optimal solution tends to hire too many escorts for the average day.

Finally, we address the stability of our algorithm. We take the same initial configuration, and run our full algorithm on it (starting from a random initial configuration) 50 times in two different scenarios: stressed and unstressed. Ideally, this would create very tight distributions in both cases, but the distributions in the stressed case are wider. This observation suggests that from a random initial configuration we need to use more time steps to ensure repeatability than from a preprocessed initial condition.

Weaknesses and Strengths of the Model

Weaknesses

- Every time a new piece of information is acquired (delayed flight, notification of a WP), we must perform a new optimization.
- Another weakness involves the hamiltonian function. There is a range of configurations in which delay time is zero (and the number of escorts is the same). When the hamiltonian is constant across a large set, the annealing algorithm is designed to take any proposed move that keeps it in the set, essentially a random walk. In many cases, we have the ability to specify a

preference between two configurations with a delay time of zero; we prefer as many escorts as possible to be as early as possible on their deliveries.

- We do not have an optimal method to step through configuration space.

Strengths

- Our algorithm finds good allocations even for few escorts, and effectively reduces cost better than a reasonable alternative algorithm.
- Our model is robust. It easily accommodates any airport, and any number of wheelchair passengers per flight, and does so with speed. As a result, our model can easily be applied to any one of Epsilon Airline's fine terminals.
- Another feature of our model is that the coarseness of the optimization can be tuned with the logarithmic parameter α (i.e. by increasing the number of steps taken, α is made successively closer to 1, making our simulated annealing more and more adiabatic).

Conclusion

We have found a solution to the problem of wheelchair transfer escort allocation by employing the techniques of simulated annealing. We present a robust method to determine the optimal number of escorts, under a variety of traffic and population conditions. We have also mapped out short- and long-term budget effects. Our algorithm assigns escorts to jobs—on a real time and cost-effective basis.

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Application of Min-Cost Flow to Airline Accessibility Services

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Summary

We formulate the problem as a network flow in which vertices are the locations of escorts and wheelchair passengers. Edges have costs that are functions of time and related to delays in servicing passengers. Escorts flow along the edges as they proceed through the day. The network is dynamically updated as arrivals of wheelchair passengers are announced.

We solve this min-cost flow problem using network flow techniques such as price functions and the repeated use of Dijkstra's algorithm. Our algorithm runs in an efficient polynomial time. We prove a theorem stating that to find a no-delay solution (if one exists), we require advance notice of passenger arrivals only equal to the time to traverse the farthest two points in the airport.

We run our algorithm on three simulated airport terminals of different sizes: linear (small), Logan A (medium), and O'Hare 3 (large). In each, our algorithm performs much better than the greedy "send-closest-escort" algorithm and requires fewer escorts to ensure that all passengers are served.

The average customer wait time under our algorithm with a 1-hour advance notice is virtually the same as in the full-knowledge optimal solution. Passengers giving only 5-min notice can be served with only minimal delays.

We define two levels of service, Adequate and Good. The number of escorts for each level scales linearly with the number of passengers.

One hour of advance notice is more than enough. Epsilon Airlines can make major improvements by using our algorithm instead of "send-closest-escort" ; it should hire a number of escorts somewhere between the numbers for Adequate and Good service.

The UMAP Journal 27 (3) (2006) 367–385.

Introduction

Annually, 2.2 million disabled people over the age of 65 travel [Sweeney 2004]. Major airlines provide wheelchairs and escorts (attendants) for disabled passengers but struggle to manage costs. We consider how few escorts are needed and how best to route them through the day. The algorithm should be flexible enough to deal with incomplete information about the arrival times of passengers or with an unexpected arrival. We recast the problem as a min-cost flow problem, the daily schedule of the escorts being the network flow.

Terminology and Conventions

- **Job.** The process of an escort picking up a wheelchair passenger *(WP)* at an arrival gate and taking them to their connecting departure gate.
- **Job starting time.** The time when a WP arrives at the airport. If an escort begins a job after the starting time, the WP had to wait before being helped.
- **Job ending time.** The time at which a WP needs to be at the departure gate to board in a timely manner. If an escort cannot finish a job after the ending time, the passenger misses their plane entirely!
- **Vertex** and **Edge**. Our airport is set up as a graph that is used for distance calculations. Our algorithm is set up as a min-cost flow problem on a network that uses a completely unrelated graph.
- **Price**, **Cost**, and **Effective Cost**. We use the standard graph-theory definitions for these words, not business definitions. Price and cost are not the same thing: Cost is the penalty for performing jobs (which is negative, since we want people to perform jobs), while price is an artificial construct that we need to run Dijkstra's algorithm.

Assumptions and Assumption Justifications

About Airports and Flights

- **In our airport graphs, each edge has uniform density of gates**. This is not always the case in real life, but the mathematics of our model would not change if this assumption were removed, only the maps used in its implementation.
- **The passengers' flight connections go between random gates.** Since passengers may be connecting a wide variety of places, the airline cannot optimize for all the distances between departure and arrival gates.

- **Arrival times and gates do not change less than 1 h before arrival; departure times and gates do not change less than 3 h before departure.** We permit planes to be delayed or have gates changed but not while an escort is helping a WP or moving to pick up a WP. This assumption is not fully realistic, but it should not affect our end results too much and it makes the model easier to implement. We can still have the unexpected arrival of WPs.

- **All passengers are making connections, not arriving at their final destinations.** This assumption is unrealistic, but our model could easily be changed to handle final destinations as well, by adding a gate to represent the baggage claim and having passengers depart from this gate.

- **Effective layover times range from 45 min to 132 min**, linearly biased towards shorter layover times. Effective layover time means that we take into consideration that WPs are usually the last to deboard and the first to board. These times are arbitrary, not based on empirical data.

- **WP arrivals are random and uncorrelated.** There is no tendency for such passengers to travel in groups. Although this assumption might be false around the time of the Special Olympics or other specific events, for a typical day it is reasonable.

About Escorts

- **Escorts receive orders via computer.** Communication of orders to escorts is fast and robust, and a computer runs the algorithm.

- **Escorts all walk at the same pace.** A typical modeling assumption.

- **Escorts travel half as fast when a person occupies the wheelchair as with an empty wheelchair**. The factor of two is arbitrary, in the absence of data.

- **The level of traffic in the airport does not affect travel times within the airport.** We make this assumption so that when we analyze our results we avoid confounding the effects of increasing the number of WPs in the airport with the effects of changing the airport layout.

- **Escorts do not abandon WPs.** Regulations prohibit airlines from leaving a passenger in a wheelchair unattended for more than 30 min [U.S. Department of Transportation n.d.]. Such a passenger may still require transportation to restrooms. The escort remains on hand until the passenger boards, when the flight crew takes over. So each escort pushes a single wheelchair that never leaves their sight (hence less concern about loss of equipment).

Cost Structure

We tabulate statistics on the delays in picking up and dropping off WPs for a given traffic load and number of escorts. We graph the delay statistics vs. number of escorts and find the "sweetspot" at which the marginal utility of adding another escort declines.

The term *cost* in this paper is a network-flow definition associated with being late in picking up a passenger, not a direct monetary expenditure. We assign a cost of 1 unit for each minute that a passenger must wait to be picked up upon arriving at the airport. If the passenger cannot be taken to their departure gate at least 15 min before flight time, they miss preboarding and a 30-unit penalty is charged for delaying takeoff of the plane. If the passenger misses a flight, a penalty of 100,000 units is charged.

Simulated Airport Structure

We model the physical structure of the airport as a graph. An edge represents a corridor and a vertex is where corridors meet. Each edge is assigned an amount of time that it takes to transverse it (in minutes, rounded up). We do not distinguish between the left and right sides of a terminal. Some edges are designated as filled with gates from which planes arrive and depart. Escorts can walk only part way down an edge, taking the appropriate fraction of traversal time to do so.

We randomly generate the arrival and departure locations of WPs at arbitrary points on the edges, corresponding to our assumption that gates are spread uniformly along the concourses.

We simulate the following airports: a single straight line, Boston Logan Terminal A (two concourses connected by an underground walkway), and Chicago O'Hare Terminal 3 (Concourses G, H, K, and L).

We precompute the shortest path lengths between each pair of vertices, using the Floyd-Warshall algorithm [Floyd 1962]. This algorithm repeatedly computes $d_k(i,j)$, the minimum path length from i to j passing through only vertices in the set $\{0, 1, \ldots, k\}$, for all pairs of vertices i, j. The key insight is that

$$d_k(i,j) = \min\big(d_{k-1}(i,j), d_{k-1}(i,k) + d_{k-1}(k,j)\big).$$

The runtime of this algorithm on a graph with n vertices and m edges is $O(n^3)$.

We use this algorithm for ease of implementation. Later we perform shortest-distance computations on a different network using Dijkstra's algorithm.

Network Flow Definitions and Principles

We give a brief summary of flows [Blum 2005; Demaine and Karger 2003].

Definition. A *network* is a directed graph where each edge is given a positive real valued *capacity* and where two vertices are labeled *source* and *sink*.

Definition. A *flow* is an assignment of nonnegative real numbers to edges such that the number for an edge is at most its capacity; for all vertices other than the source or sink, the sum of values on edges from the vertex equals the sum of values on edges to the vertex. The *size* of a flow is the sum of values on edges from the source minus the sum of on edges to the source.

Definition. A *max-flow* is a flow of maximum possible size for a given network.

We can also assign real-valued *costs* per unit flow to the edges, so that we can talk about the cost of a flow.

Definition. The *cost* of a flow is the sum over all edges of the cost of the edge times the value that the flow assigns to that edge.

Definition. A *min-cost-max-flow* is a max-flow with the smallest possible cost of any max-flow.

If all the edge capacities are integers, there is always a solution that assigns an integer flow to each edge. We will create a data structure to dynamically find such solutions to min-cost-max-flow to decide which escorts should handle which job.

Definition. For a network and a flow on it, the *residual network* is another network on the same vertex set whose edges have capacities equal to the amount of extra flow that could be sent through that edge. Since the flow across some edges can now be decreased, the residual graph has some edges that are the reverse of edges in the original graph. More precisely, the capacity of an edge (i, j) in the residual network is the capacity of (i, j) in the original network plus the flow from j to i minus the flow from i to j. In the residual graph, the cost of (i, j) should be the negative of the cost of (j, i).

The residual graph is useful because *solving min-cost-max-flow on the original graph is equivalent to solving it on the residual graph.*

Definition. A *circulation* is a flow in which the total flow into all vertices (including source and sink) equals total flow out of them.

Definition. A *min-cost circulation* in a network is a circulation of minimum cost.

If f is already a max-flow, the residual graph can accept no more flow. *Hence the problem we must solve is to find a min-cost circulation.*

Definition. A *price function* is a function that associates a price to each vertex.

Definition. The *effective cost* of an edge (i, j) is $c_{i,j} + p_i - p_j$, where $c_{i,j}$ is the cost of (i, j), and p_i and p_j are the prices associated with i and j respectively.

Lemma 1. *The cost of a circulation does not change if we replace costs by effective costs.*

Let $f_{i,j}$ be the flow through edge (i,j) for circulation f. Let $c_{i,j}$ be the cost of (i,j). Let p_i be the price at i. Then the cost of f using effective costs is

$$\begin{aligned}\sum_{i,j} f_{i,j}(c_{i,j} + p_i - p_j) &= \sum_{i,j} f_{i,j}c_{i,j} + \sum_i p_i \left(\sum_j f_{i,j} - \sum_j f_{j,i} \right) \\ &= \sum_{i,j} f_{i,j}c_{i,j} + \sum_i p_i \cdot 0 = \sum_{i,j} f_{i,j}c_{i,j},\end{aligned}$$

which is the cost of the circulation f. □

Hence solving a min-cost circulation problem is equivalent to solving the same problem using effective costs. If all costs are nonnegative, the empty circulation is the min-cost circulation.

Suppose that all of the graph's vertices are reachable from some particular vertex v and all edges have nonnegative cost. We compute shortest paths from v to all other vertices using costs as edge weights, for the following reason: Let w and w' be arbitrary other vertices. If we then *introduce prices* $p_w = d(v,w)$, meaning that the price at w is the distance between v and w, then:

- Since $d(v,w) \leq d(v,w') + c_{w',w}$, the effective cost from w' to w is $c_{w,w'} + d(v,w') - d(v,w)$, which is nonnegative.
- Since there is a path from v to w along which the above inequality is strict, there is a path from v to w of effective cost 0 for all w.

Dijkstra's Algorithm

Our algorithm uses Dijkstra's algorithm every time a new WP arrives or is scheduled to arrive. Dijkstra's algorithm [1959] computes single-source shortest paths in a graph with nonnegative edge weights (i.e., for some vertex v, it computes $d(v,w)$ for all w). The idea is to determine the distances from v in order of increasing distance (a breadth-first search). We use Dijkstra instead of Floyd-Warshall because Dijkstra is more efficient for single-source shortest paths. We approximate $d(v,w)$ and $\min_u(d(v,u)+l_{u,w})$, where $l_{u,w}$ is the length of the edge from u to w and the minimum is over neighbors u of w with known distance from v. The key insight is that the shortest approximate distance is actually correct.

Algorithm Overview

We associate a cost to picking up a job after its starting time, equal to the number of minutes late the job starts, with 30 min more penalty for not transporting a WP to a gate at least 15 min early for preboarding. We associate a cost of effectively infinite magnitude to completely failing to perform a job.

We want at any given time to be able to schedule escorts to complete the currently known jobs at minimal cost. We add jobs to network as the arrival or scheduled arrival of WPs becomes known.

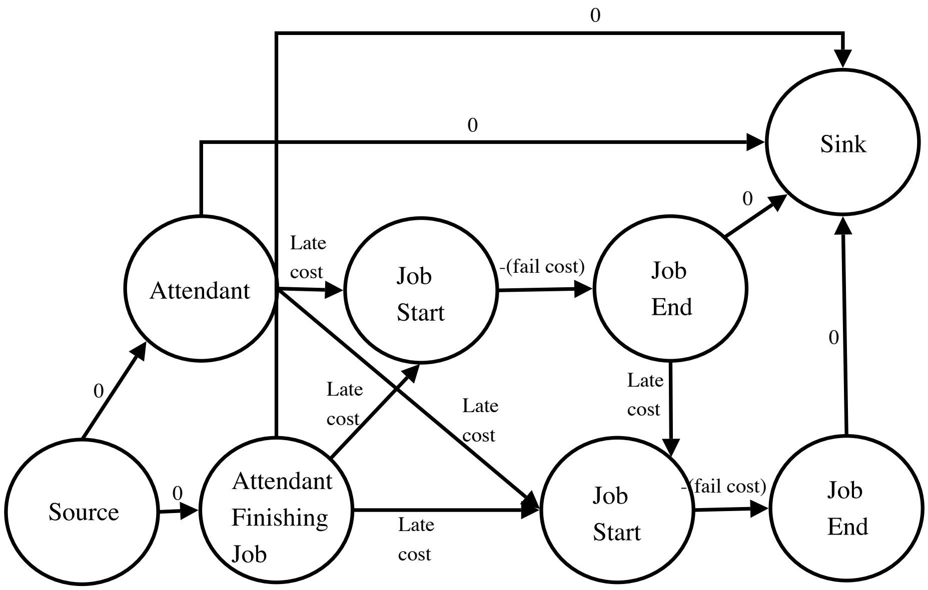


Figure 1. The graph for which we want to solve min-cost flow. Vertices represent jobs, and flow along the edges represents the movement of escorts. Delays are represented as costs, and the edges are labeled with these costs.

We restate the problem as a min-cost-max-flow problem (**Figure 1**). The escorts' schedule is a network flow—of escorts, not WPs. The source is escorts arriving at the start of the day, the sink is them leaving at the end of the day. The vertices are unbusy escorts and the beginning and end of known jobs.

In a min-cost-max-flow problem, we set costs for traversing edges. However, an escort neglecting to pick up a WP involves *not traversing* an edge; we cannot charge for this directly. We equivalently charge a large negative cost of $-100{,}000$ for taking the job. At the end of the day, we use this large negative cost to check that all WPs were transported to their flights.

If we knew exactly when each passenger would arrive, we could simply solve min-cost-max-flow on the graph to find the optimal schedule. Instead, we add jobs to our graph as we learn of impending arrivals. We also must delete edges for jobs already performed or that can no longer be performed. We maintain our data structure under the following updates.

Updates in the Algorithm

Time Passing: As time passes, an escort may no longer be able to make it to a job in time; we *update the lateness costs* (edge costs) for this escort. An edge in the current flow corresponds to a job that an escort is trying to get to on time, so its cost does not decrease. For a job that an escort is walking to, the passage of

1 min of time makes the edge cost 1 min more expensive but moves the escort 1 min closer, so these effects cancel. Hence, this update does not decrease any effective cost in our residual graph and maintains our invariants (**Figure 2**).

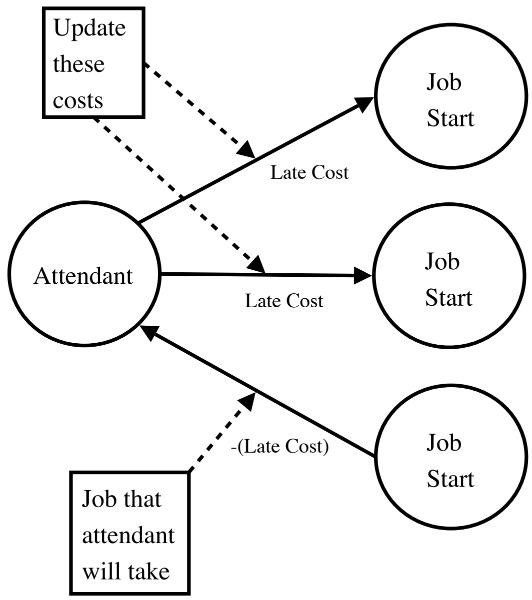


Figure 2. When time passes, we update edge costs.

Taking up a job: When an escort starts work on a job, we *delete the vertices corresponding to the escort and to the start of the job* (along with adjacent edges) and connect the source to the vertex corresponding to the end of the job. Doing this makes for a corresponding change in the residual graph but does not produce negative effective prices (**Figure 3**).

Finishing a job: When an escort finishes a job, the vertex corresponding to the end of that job should be *relabeled* to correspond to the current location of the escort (**Figure 4**). If a new job is assigned, the escort walks toward the job; an escort with no job scheduled does not move.

A new job is announced: A new job is announced when the arrival or predicted arrival time of a WP becomes known. We *create the vertices u and v corresponding to the beginning and end* of the job. Next, we create the edges pointing to u from every end of job and every escort who could make it to this job in time, and edges from v to the sink and to the starts of jobs that could be reached after completing the job given by edge uv. Lastly, we create the edge E from u to v. All edges are given the appropriate costs based on the current time. Assign a high enough price to v and a low enough price to u so that the effective prices of all edges into u or out of v are positive. Then in the residual graph minus E, all effective prices are nonnegative. Our entire graph is accessible from v, because from v we can reach the sink and trace back each of the escorts' schedules to the source. Each job that someone is scheduled to complete can be reached

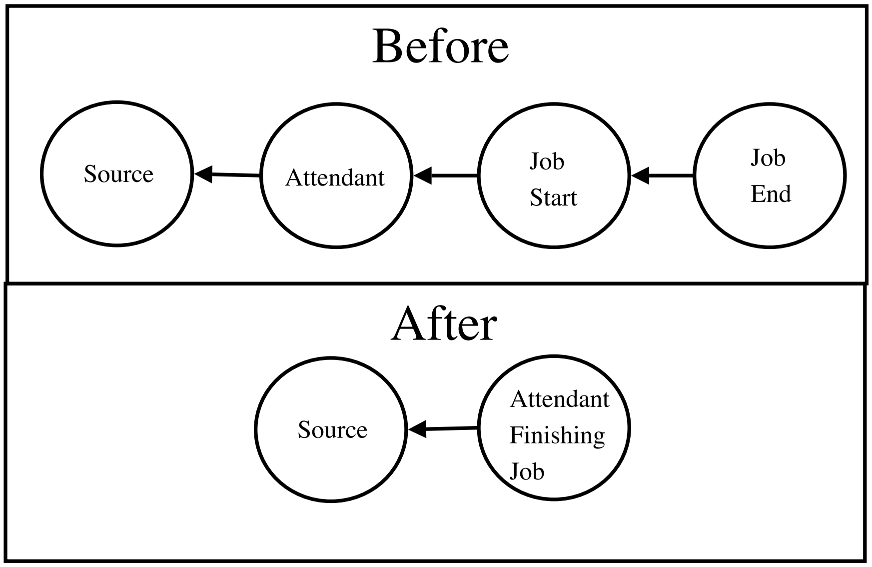


Figure 3. When an escort starts a new job we make sure other escorts don't take the same job.

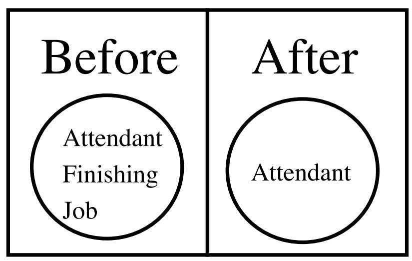


Figure 4. When a job ends, we relabel the job end vertex as an escort vertex.

by tracing back that escort's schedule from the sink. All job start vertices that could be accomplished, but are not scheduled to be, can be reached by getting to the vertex corresponding to an escort who can reach the job and following the edge from to the job's beginning.

Applying the technique above, and adding the size of the smallest cost path from v to w to the price at w, we get nonnegative values for all of these effective costs and a path P from v to u in the residual graph consisting of edges with zero effective cost. If the effective cost of E, is nonnegative, we are done; otherwise, we augment our flow by the cycle PE. This negates the effective cost of all these edges in the residual graph and changes their direction, and hence makes all the effective costs nonnegative (**Figure 5**).

Algorithm Performance

This data structure is fairly efficient. Suppose that there are A escorts and J jobs. The runtime of time passing is $O(AJ)$; that of taking up or finishing a

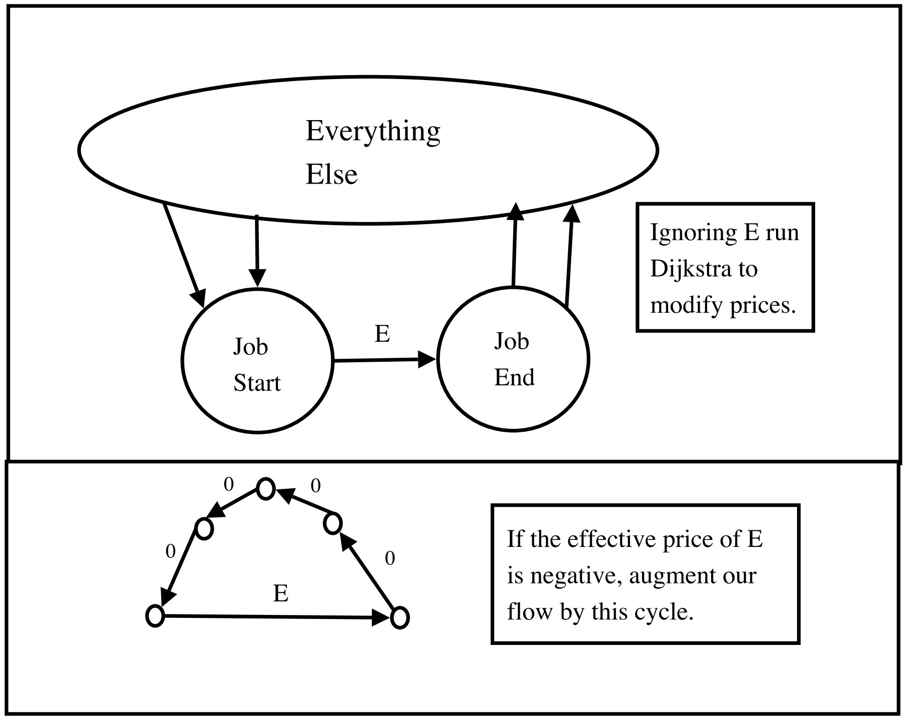


Figure 5. We add new jobs as they are announced.

job is $O(1)$; and learning about a new job depends on the runtime of Dijkstra's algorithm and is bounded by $O\big((A+J)^2\log(A+J)\big)$ [Dijkstra 1959].

Our Algorithm Finds Optimal Solutions

We present a theorem that states what classes of algorithms find a solution that transports all passengers to their flights without delays, provided such a solution exists.

We formulate a more general version of the problem as follows. We have some number of escorts; they arrive in the airport at predetermined times of day in predetermined places, but particular escorts are not guaranteed to leave at a particular time. We are required to complete jobs, each requiring that an escort be in a particular location at a given time and remain occupied until released at another known location and time; that end location can be reached from the start location in the time allotted.

All jobs are announced enough ahead of time that an escort could reach the starting location from anywhere in the airport from the time when it was announced. This time is bounded by the time to traverse between the farthest two points in an airport (e.g., 18 min in Chicago O'Hare Terminal 3 with an empty wheelchair; for any airport, advance notice of 30–60 min for arrival of a WP is more than enough).

We stipulate:

- Escorts are interchangeable.
- An escort finishing a job is equivalent to a new escort appearing at a given time and place.
- An escort taking up a job or leaving work is equivalent to an escort disappearing at a given time and place.

Algorithm: When a new job is announced:

- Considering all escorts in the airport and all escorts who will arrive in the airport (or finish jobs), and considering all known jobs that have not started yet, determine which escorts can do which jobs.
- Find (if one exists) a pairing that associates an escort to each job.
  - If none exists, the jobs cannot all be completed;
  - if such a pairing does exist, tell the escorts to start to their assigned jobs (or do so once they appear).

Theorem 2. *This algorithm works if the problem admits a solution.*

[EDITOR'S NOTE: We omit the authors' proof, which uses the marriage lemma [Halmos and Vaughan 1950.]

Our algorithm has an additional property:

If it is possible not to be late for any job nor miss any job, our algorithm will produce such a solution.

Comparison against a Greedy Algorithm

We implemented also a "send-closest-escort" algorithm, which greedily assigns escorts to the closest job according to four rules:

1. When a job becomes known, assign the closest available escort.
2. When a new escort becomes available, assign that escort to the closest available job.
3. Never unassign an escort from a job.
4. Escorts who have nothing to do stay put where they are.

For rules 1 and 2, jobs are not assigned until a fixed time before the arrival time of the passenger (the time to traverse the farthest two points).

Experimental Setup

We use three maps as our test airports: a single straight line, Boston Logan Terminal A (two concourses connected by an underground walkway), and Chicago O'Hare Terminal 3 (Concourses G, H, K, and L).

For each simulation, we ran an 8-h shift with time increments of 1 min. For each terminal and varied numbers of escorts, we ran the simulation 10 times with the same passenger arrival times and gates.

We suppose that 95% of WPs give 1 h notice and 5% show up with only 5 min before penalties accrue.

We ran each airport terminal at two loads of traffic, heavy and light. Heavy traffic is 84 wheelchair passengers in the 8-hour period at the single-line terminal, 155 at Logan Terminal A, and 550 at O'Hare Terminal 3; light traffic is half as many. We arrive at the heavy traffic number for O'Hare from an estimate of Thanksgiving traffic [Chicago Department of Aviation 2001], scaling, and assuming that 1% of passengers need wheelchairs; the numbers for Logan and the straight-line terminal are scaled from the number of gates.

Results

In **Figures 6–7**, the numbers of escorts are plotted as data points; each data point is the average of 10 simulations. The average wait times are the total wait time divided by the number of passenger served. Missed passengers do not affect the average wait time, since their wait time is effectively all day. Results in the lower left-hand corner are more desirable. Any deviation from zero on the x-axis is bad because some passenger was outright ignored!

The results corresponding to the perfect-knowledge solution are almost exactly covered by the corresponding results for our algorithm.

Although the "send-closest-escort" algorithm is frequently competitive for average wait times, it is significantly worse in terms of jobs missed. It scales roughly linearly in balancing delay time and number of jobs missed; it does not perform well without a large number of escorts.

We define two service levels, with corresponding numbers of escorts:

- *Adequate:* The average number of missed passengers in each scenario is lower than 1.
- *Good:* Everyone is served, with an average wait time under 15 min.

In **Table 1**, we summarize the minimum number of escorts needed to reach each service level.

For each airport, the difference between the Good and Adequate service levels is roughly a factor of two, with slightly increasing returns to scale; with larger scales, the staff are spread more uniformly, so it is less likely that a job will crop up with nobody close enough to take it.

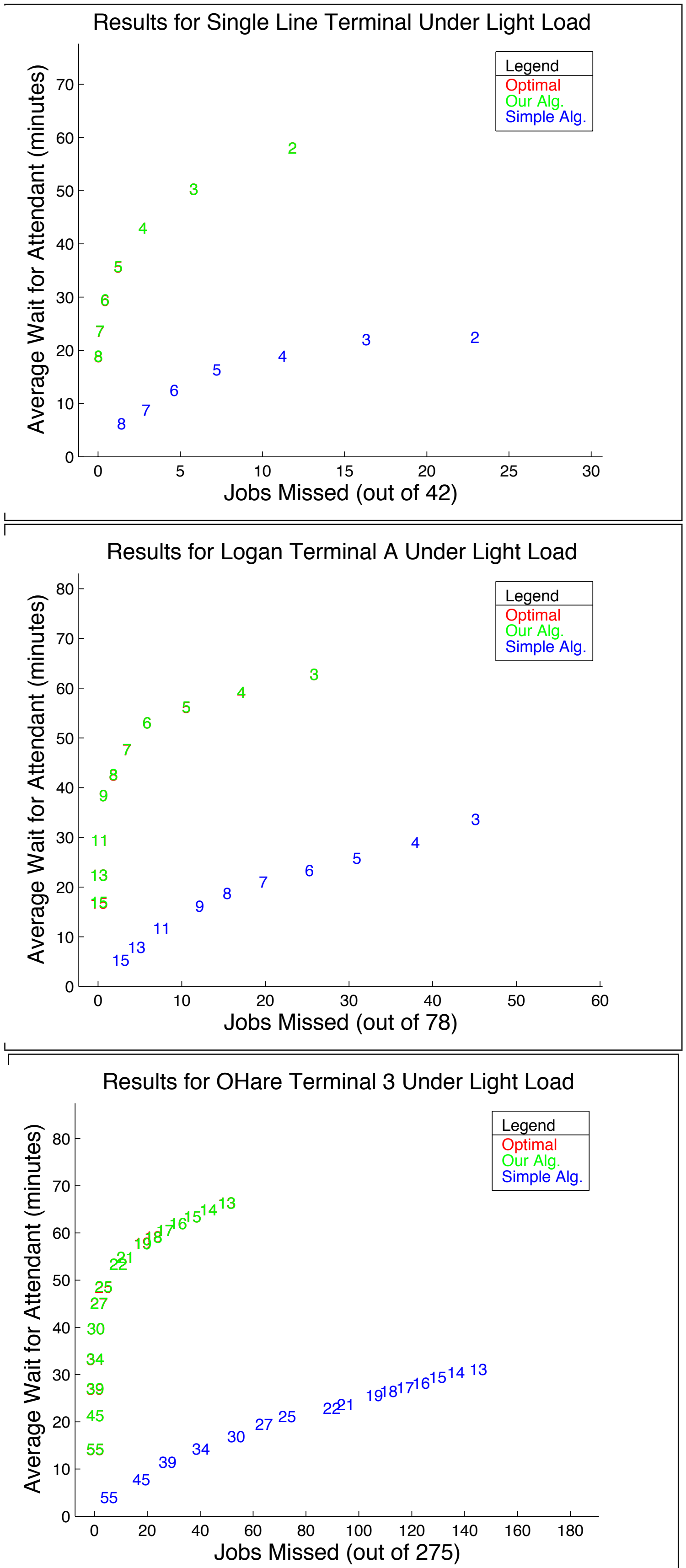


Figure 6. Light traffic: Passengers missed and average delay for each number of escorts, for each airport.

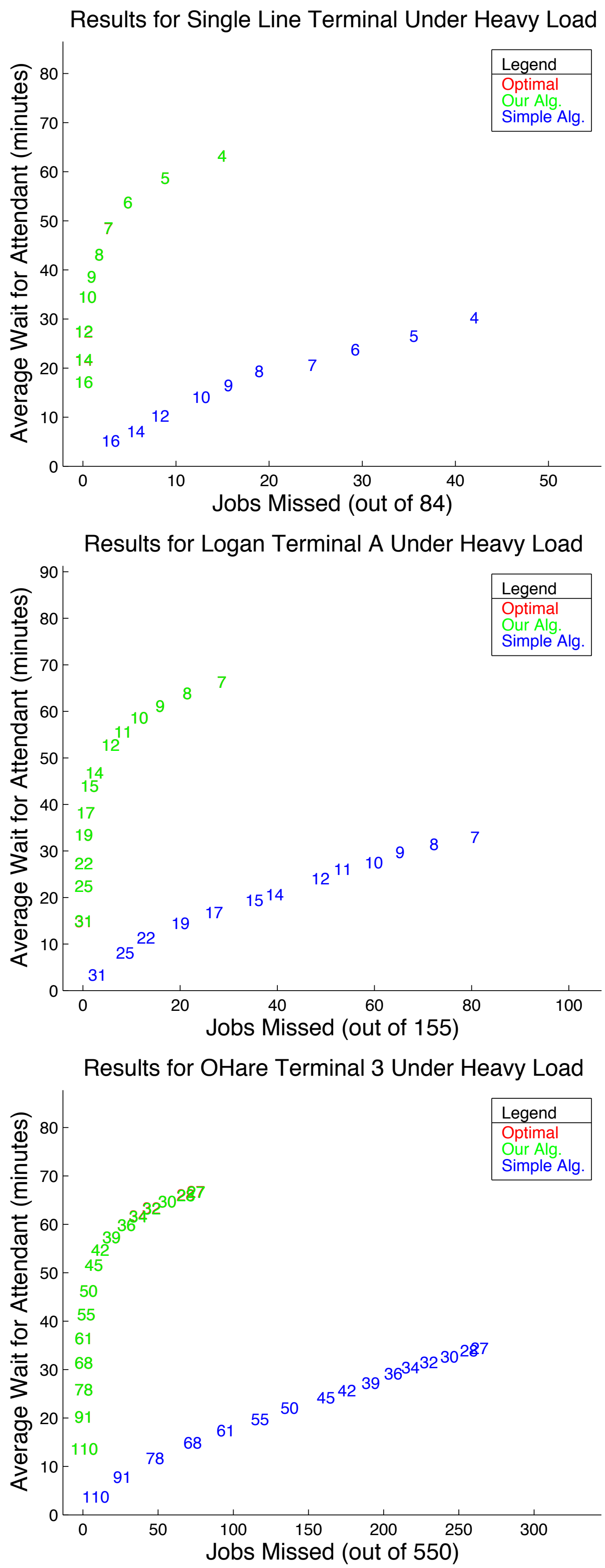


Figure 7. Heavy traffic: Passengers missed and delays for each number of escorts, for each airport.

Table 1.
Numbers of escorts needed to achieve service levels.

Airport	Traffic	Passengers served	Number of escorts	
			Adequate Service	Good Service
Line	light	42	6	9
	heavy	84	10	18
Logan	light	78	9	17
	heavy	155	15	31
O'Hare	light	275	27	56
	heavy	550	47	106

Sensitivity to Parameters

Short-Notice Passengers

We varied the percentage of short-notice passengers from 0% and 100% using the Logan airport map and heavy traffic. The algorithm is very insensitive to this percentage. Why? Because 5 min is enough to cover a lot of ground.

Airport Geometries

For either 155 WPs or 515 (5% of whom give short notice), our algorithm does roughly equally well in all airports. Geometry mainly affects time wasted traveling from one job to another—which should not be significant, since the maximum diameters of our airports are 11, 12, and 18 min.

Load Scaling

Sensitivity to load scaling is important because the proportion of WPs is expected to increase. We tested the Logan airport map using passenger loads from 50% to 150% of the heavy traffic load. The numbers of escorts for Adequate and for Good service scale close to linearly (**Figure 8**).

Strengths and Weaknesses

Strengths

- *Algorithm optimal with perfect knowledge.*
- *Performs well with only modest advanced notice.* Advance notice of 1 h is more than enough. Even if many passengers show up with no advance warning, our algorithm performs well.

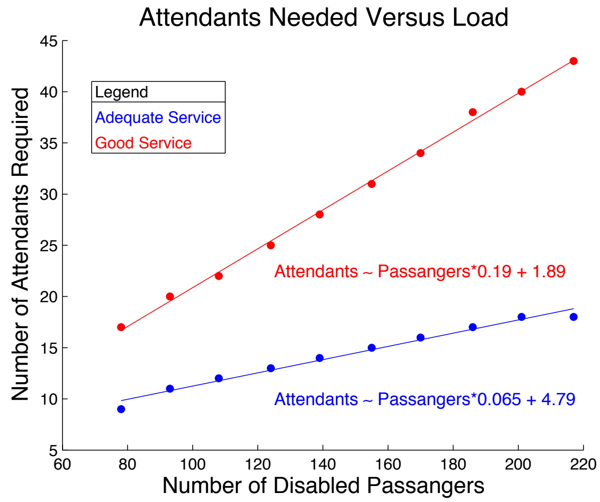


Figure 8. Scaling of escorts as WPs increase.

- *Proved an interesting theorem.* Given sufficient advance notice, our algorithm handles all passengers with no delay if doing so is possible. In practice, a company might stop hiring escorts when delays are small rather than hire until delay is nonexistent.
- *Efficient algorithm runtime.* Our algorithm runs in real time with modest computational resources.

Weaknesses

- *Requires a computer.*
- *Requires a job end time to be specified.* For the algorithm to compute when an escort will no longer be occupied and assign orders into the future, we must set a fixed job end time before starting the job. If an end time is unspecified, the problem becomes NP-complete, because it can be used to solve the hamiltonian path problem.
- *Algorithm uses only factual knowledge of today, no statistical projections.* Our algorithm plans based on current knowledge of scheduled arrivals but makes no attempt to guess where a WP may appear.
- *Hard to explain algorithm to nonmathematicians.*

Conclusion

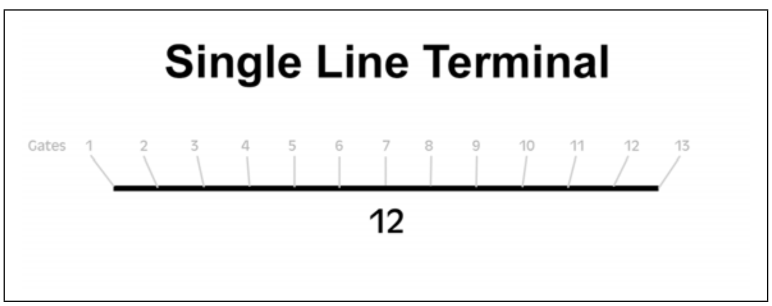
Our model envisions the problem in terms of flow of escorts through the work day. We present an algorithm with compelling theoretical basis for producing good results.

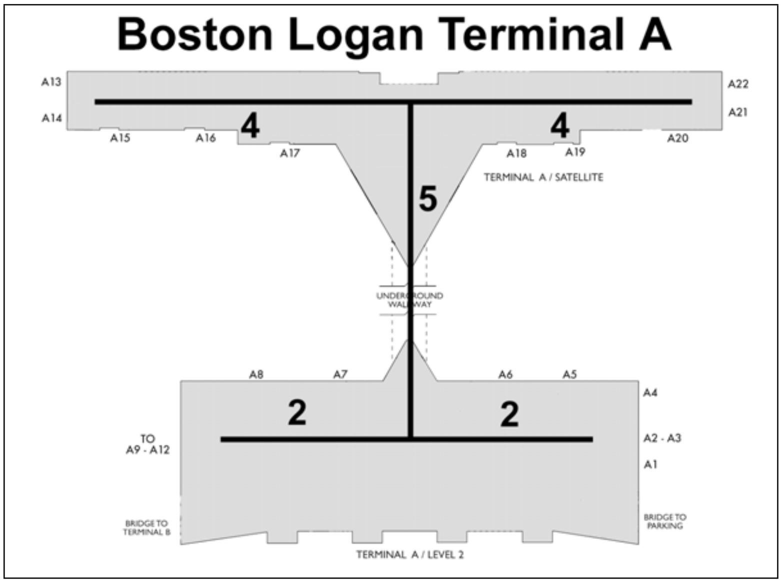
We make the following recommendations:

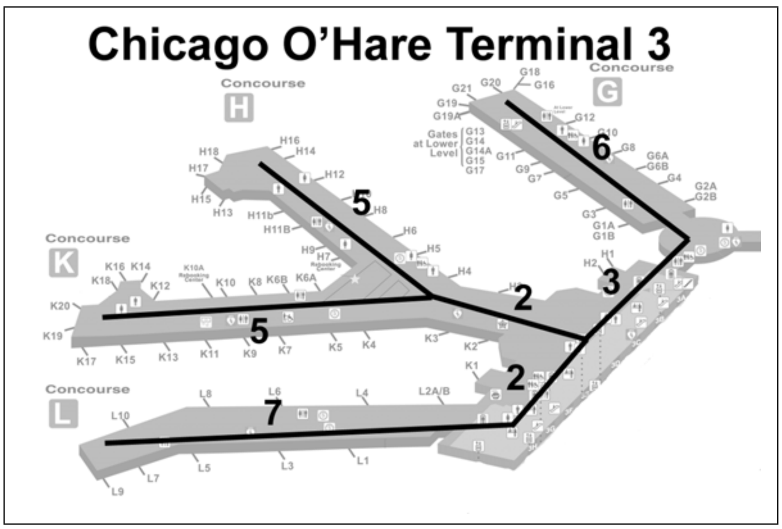
- *WPs do not need to provide much advance notice.* Requesting assistance at first check-in is more than enough notice for all connecting flights.
- *The "send-closest-escort" algorithm is not good.*
- *The number of escorts for a given level of service scales linearly with the number of passengers.*
- *Airlines should hire a number of escorts somewhere between the numbers required for Good and Adequate service levels.* Fewer escorts than for Adequate service results in severe problems; more escorts than needed for Good service produces diminishing returns.

Appendix A: Terminal Maps

These are the graphs for our small, medium, and large simulated airports. A large boldface numbers on an edge is the length of time to traverse the edge; these numbers should be doubled for an occupied wheelchair. The Logan and O'Hare graphs are superimposed on digitally edited versions of the official terminal maps [Massachusetts Port Authority n.d.; City of Chicago n.d.]. The gate positions in the single-line terminal are those used in our simulation; those on the other two maps are the gate positions in real life. In our simulation, gates occur every minute of walking distance along the edges except for edges that are transportation between concourses.







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Pp. 387–412 can be found on the *Tools for Teaching* 2006 CD-ROM.

Cost Minimization of Providing a Wheelchair Escort Service

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Summary

Epsilon Airlines provides a wheelchair escort service to passengers who require aid. We use an optimized earliest-due-date-first (EDD) algorithm to minimize the overall cost. Our algorithm is broad enough to accommodate various airport concourses, flight schedules, and flight delays. In addition, it allows for wheelchair escorts to perform other tasks beneficial to the airline, such as provide information at a kiosk, to help reduce the overall cost. Moreover, it creates schedules for each employee.

A naive strategy would be to employ the minimum number of escorts to guarantee that all passengers reach their gates on time. We show that this strategy is not optimal but can be improved by assigning different numbers of escorts to shifts based on expected traffic. For example, if Delta Airlines were to utilize the naive strategy at Atlanta International Airport, the cost would be over \$5 million/yr, whereas our strategy reduces this cost to under \$4 million/yr. A similar reduction in cost could be expected for Epsilon Airlines.

Assumptions

- The original problem can be adequately modeled with a numerical simulation that uses a discrete time step Δt and discrete length step Δd, provided that Δt, Δd are small compared to actual dimensions.
- The layout of the airport concourse(s) is known, along with positions of gates. The concourse(s) can be on one or two levels.

- There is a kiosk or information desk in the concourse that can communicate with escorts, for example, by using walkie-talkies.
- The movement of escorts is constrained to a rectangular lattice.
- The escort transports a wheelchair passenger *(WP)* from an incoming flight gate to a connecting flight gate.
- The number of WPs on a flight is small compared to the total number of passengers on the flight [Backman et al. 2004]. This assumption allows for tracking individual passengers rather than flights.
- Most WPs are known to the airline in advance, but some arrive unexpectedly.
- Both incoming flights and connecting flights can be delayed.
- When escorting a WP, the wheelchair and the escort stay together.
- An unoccupied escort performs another job (such as providing information at the information desk) [Backman et al. 2004].
- The goal is to minimize the cost of escorting WPs.

Approach

Our objective is to provide cost-minimizing staffing and inventory recommendations, as well as an algorithm to generate optimal schedules for wheelchair escorts.

We model the geometry of the airport and simulate arriving and departing WPs according to a fixed schedule. We add unscheduled WP arrivals and allow for random unscheduled flight delays. We set out rules to govern the behavior of an escort at each time step throughout their shift. These rules are comparisons between choices that the escort can make; the algorithm predicts which choice will result in the least cost to the airline, based on an objective cost function.

Formulating the Optimization Criteria

The costs of operating an escort service are associated with:

- flight delays due to WPs arriving late to connecting flights;
- WPs having to wait for service (i.e., long-term decrease in ticket sales as a result of damage to airline reputation);
- employing escorts;

- depreciation, maintenance, and storage of wheelchairs; and
- reassigning idle employees to alternative tasks (a "negative cost").

We adopt the following notation, where the length of one time step Δt in the simulation is 1 min:

$$\begin{aligned}
w &= \text{number of escorts on the shift} \\
T &= \text{length of a single shift (min)} \\
K &= \text{wage of a worker (\$/min)} \\
W_0 &= \text{costs associated with wheelchairs} \\
\omega(t) &= \text{number of escorts employed at an alternative job at time } t. \\
\psi\left(t_{\text{fd}}\right) &= \text{number of flights delayed } t_{fd} \text{ min} \\
c_\psi\left(t_{\text{fd}}\right) &= \text{cost associated with delaying a flight by } t_{\text{fd}} \text{ min (\$)} \\
\phi\left(t_{\text{pw}}\right) &= \text{number of WPs made to wait } t_{\text{pw}} \text{ min for an escort.} \\
c_\phi\left(t_{\text{pw}}\right) &= \text{cost associated with having a passenger wait } t_{\text{pw}} \text{ min (\$)}
\end{aligned}$$

Quantities are measured at discrete points in time (at each minute). We can now mathematically express the costs listed above:

$$\begin{aligned}
\text{Flight Delay Cost} &= \sum_{t_{\text{fd}}=1}^{\infty} \psi\left(t_{\text{fd}}\right) c_\psi\left(t_{\text{fd}}\right), && \textbf{(1)} \\
\text{Passenger Waiting Cost} &= \sum_{t_{\text{pw}}=1}^{\infty} \phi\left(t_{\text{pw}}\right) c_\phi\left(t_{\text{pw}}\right), && \textbf{(2)} \\
\text{Employment Cost} &= wKT, && \textbf{(3)} \\
\text{Wheelchair Cost} &= W_0. && \textbf{(4)}
\end{aligned}$$

Note that W_0 depends only on the airport and the number and type of wheelchairs, so is constant with respect to the number of escorts on a particular shift, which can be varied.

Multitasking Employees

A key feature of our model is that we allow employees to take on a secondary task. This task can be any other job the escort can perform when not actively assisting a WP, such as providing information at a kiosk. This cuts down on inefficiency associated with low volume points during a shift.

Guided by microeconomic theory, we assume that the extra benefit to the airline from adding a worker to perform a task is inversely proportional to the number of workers already contributing to that task [Wikipedia 2006a]. Mathematically, this implies that the benefit from n employees performing a certain task is approximately proportional to $\ln(n+1)$. Since the airline would have to hire another worker to perform this secondary task if no escorts were available, the benefit to the airline from a single employee working a secondary job should equal the employee's wage. This implies that the net negative cost for a single shift is

$$\frac{-K}{\ln 2} \sum_{t=0}^{T} \ln\bigl(\omega(t) + 1\bigr). \tag{5}$$

Assumptions About Waiting Costs

We assume that the average total cost associated with delaying a flight is \$44/min, and the average total cost associated with delaying a single passenger is approximately \$0.25/min [Federal Aviation Administration 2000]. Since the marginal cost of waiting an additional minute independent of the total delay, it follows that

$$c_\phi\left(t_{\text{pw}}\right) = 0.25 t_{\text{pw}}, \qquad c_\psi\left(t_{\text{fd}}\right) = 44 t_{\text{fd}}.$$

Since the expression $\psi(t_{\text{fd}})t_{fd}$ is the sum of the delay times of all planes delayed exactly t_{fd} minutes, the aggregate flight delay time is

$$T_{\text{fd}} = \sum_{t_{\text{fd}}=1}^{\infty} \psi(t_{\text{fd}}) t_{\text{fd}}.$$

Similarly, the aggregate passenger wait time is

$$T_{\text{pw}} = \sum_{t_{\text{pw}}=1}^{\infty} \phi(t_{\text{pw}}) t_{\text{pw}}.$$

These gives alternative expressions for the costs defined in **(1)** and **(2)**,

$$\text{Flight Delay Cost} = 44 T_{\text{fd}}, \qquad \text{Passenger Waiting Cost} = 0.25 T_{\text{pw}}.$$

The Cost Function for a Shift

Combining the results in **(1)–(5)**, the total cost is

$$C = 44 T_{\text{fd}} + 0.25 T_{\text{pw}} + wKT + W_0 - \frac{K}{\ln 2} \sum_{t=0}^{T} \ln\bigl(\omega(t) + 1\bigr).$$

The Cost Function for the Year

The objective is to minimize cost not only on a day-to-day basis, but also over the entire year. We assume that each of the 1,095 8-hour shifts during a year fall into one of the three following categories of air traffic:

- Light: These are the shifts from 4 P.M. to 12 A.M. and from 12 A.M. to 8 A.M. on days of the year in the bottom 90% (329 days) of air traffic days. These shifts comprise 60% (657) of all 1095 shifts during the year. We estimate that mean traffic on these days is approximately one-half of total mean traffic.
- Heavy: We define these shifts as the top 10% 8-hour shifts by air traffic (top 110 shifts). We estimate that these shifts have a mean traffic 2.5 times total mean traffic.
- Medium: Those shifts not falling under the above two definitions (328 remaining shifts). We estimate that these shifts have a mean traffic 1.5 times total mean traffic.

The cost function for the year is a weighted sum of the cost functions for each type of day, which we denote as C_l , C_m, C_h for light, medium, and heavy traffic days:

$$C_{\text{annual}} = 657C_l + 328C_m + 110C_h. \tag{6}$$

Algorithm Implementation

We implement an algorithm that dynamically assigns escorts. The overall behavior is depicted in the flow chart on page 392. The algorithm is divided into six main parts.

Input

The algorithm requires the layout of the concourse(s) and the flight schedule of known WPs. The layout of the concourse(s) includes the positions of the information kiosk, the arrival and departure gates, and an elevator if the concourse has two floors. The flight schedule includes the incoming and connecting flight gates and times.

Main Simulation Loop

In each time step the algorithm queues WPs, assigns escorts, and moves escorts to their optimum destinations.

Queueing WPs

The first task of the algorithm is to consider the possibility of the arrival of an unexpected WP. The average number of unexpected arrivals is taken to be a fixed percentage of the number of expected WPs, which can be specified in the algorithm; we take it to be 5%.

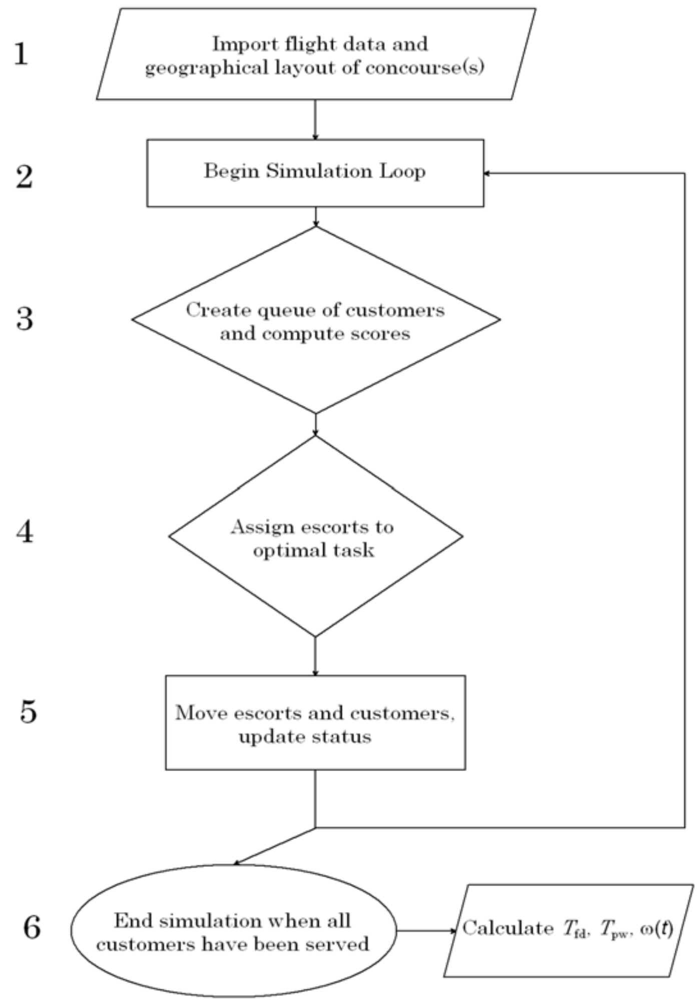

Figure 1. Flow chart outlining the algorithm used in the numerical simulations. The output of the simulation gives the aggregate flight delay time T_{fd}, the aggregate passenger waiting time T_{pw}, and $\omega(t)$, the number of escorts employed in an alternative job at time t.

Each WP (previously known or new) is assigned a score, the amount of time before their connecting flight departs. The algorithm then reorders the queue, inserting newly-known WPs who have not yet been assigned an escort. WPs with a lower score are served first. This score function reduces the total flight delay time t_{fd} as much as possible, since this is the most costly delay for the airline.

Escort Assignment

The nearest available escort is assigned to the WP at the head of the queue using the "city block" distance $|\Delta x| + |\Delta y|$ instead of the Euclidean distance $\sqrt{(\Delta x)^2 + (\Delta y)^2}$.

An escort assigned to a WP may be able to return to (or remain at) the information kiosk and perform an alternative job for some time before picking up the assigned WP.

Motion and Status of Escorts

After the assignment of escorts, escorts who are not at their destination are moved toward it. In each time step Δt, an escort can move one lattice distance Δd. This defines a natural speed, $\Delta d/\Delta t$. We take $\Delta t = 1$ min and the speed of escorts to be 2.5 feet/second (reasonable, considering that a concourse may have obstacles including other passengers). It follows that $\Delta d = 150$ ft.

When an escort reaches the incoming or connecting gate for a WP, the escort's status is updated accordingly.

However, when an escort reaches the incoming flight gate of a WP, the WP may not have arrived yet. We denote the probability of a flight being delayed by p_{delay}; in fact, $p_{\text{delay}} \approx 0.29$ [Mueller and Chatterji 2002]. For delayed flights, we take the length of the delay to be distributed exponentially [Wikipedia 2006b].

Output

The main simulation loop continues until all WPs on the input flight schedule have been escorted to connecting flights. Once this condition is met, the algorithm outputs the aggregate WP waiting time T_{pw}, aggregate flight delay time T_{fd} caused by a WP not arriving for their connecting flight on time, and $\omega(t)$, the number of escorts employed in an alternative task at time t.

Case Studies

We did case studies of Delta Airlines concourses in three airports; O'Hare International Airport (Chicago, IL), John F. Kennedy International Airport (New York, NY), and Atlanta International Airport (Atlanta, GA).

We take the wage of an escort to be $K = \$3.50$/h, an average of the \$7/h paid by some airlines and \$0 paid to volunteers.

In each case study, we generated a simulated schedule of passengers over an 8-h shift that approximates the actual frequency of incoming and connecting Delta Airlines flights in each airport [City of Chicago 2005; Schumacher 1999]. Using the mean time interval between arrivals and departures, the airline's number of terminals, and the number of planned passengers from airline flight and concourse data, we modeled the number of expected WP arrivals per interval as a Poisson process. Unexpected WPs are not included in this schedule, but are accounted for in the simulation. Also, passengers are assumed to connect to Delta Airlines flights, which allows for analysis of the Delta Airlines concourse in isolation.

For each airport, we obtained satellite images of the concourse(s) from Google Earth in order to find distances between gates and kiosks. We found gate information at Delta Airlines' Website [2006]. We translated this information into a grid layout of the concourse, to serve as input to the simulation. Gates much less than the lattice spacing ($\Delta d = 150$ ft) apart are assigned a common point on this grid. For the JFK concourse, a point was also assigned for an elevator; this concourse has arrivals and departures on different floors, so every route from an arrival gate to a departure gate includes a trip on the elevator (assumed to take one time step $\Delta t = 1$ min).

JFK International Airport

For a medium-size two-concourse airport terminal, we consider the Delta Airlines concourses at JFK International Airport, with 20 gates. The low, medium, and high traffic flows are 10, 30, and 50 incoming flights per 8-h period [Delta Airlines 2006]. Even though the JFK concourses are larger than those at O'Hare, many fewer Delta Airlines planes fly into JFK than into O'Hare. The predicted cost of numbers of escorts for these 8-h shifts are shown in **Figure 2**. The costs for small numbers of escorts are astronomically high because of the cost of delayed planes and missed flights, since there are too few escorts for the demand.

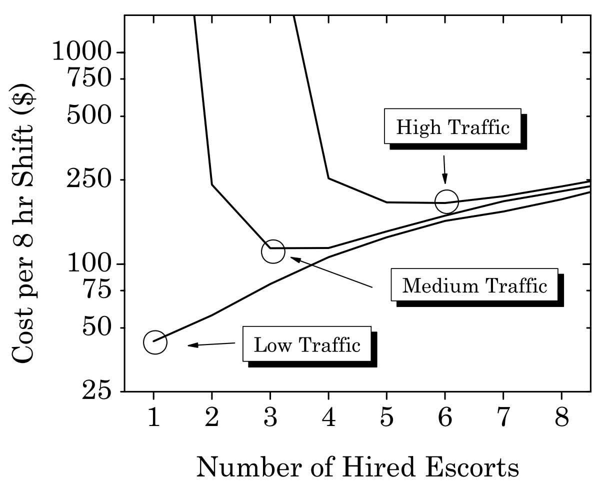

Figure 2. Simulated cost curves under low, medium, and high traffic flow in 2005 for the Delta Airlines JFK International Airport concourses.

The least-cost numbers of escorts for each traffic flow rate are 1, 3, and 6, at costs $40, $115, and $190 per shift. From **(6)**, the minimum total annual cost of escorts is $121,000.

O'Hare International Airport

The Delta Airlines concourse at O'Hare International Airport has only five main gate areas. The low, medium, and high traffic flows are 43, 129, and 215 incoming flights per 8-h period [Delta Airlines 2006]. The predicted cost of numbers of escorts for these 8-h shifts are shown in **Figure 3**.

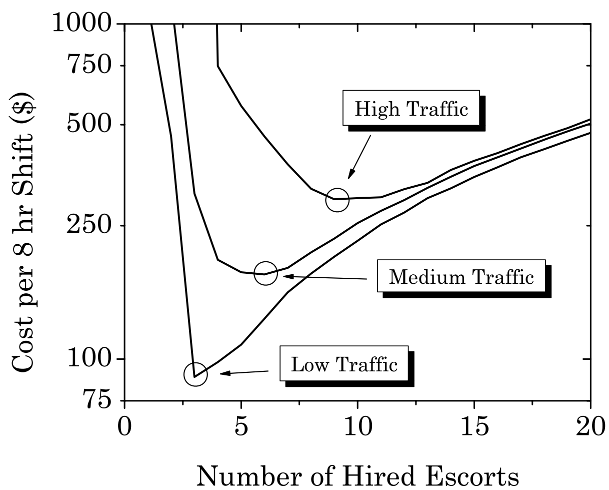

Figure 3. Simulated cost curves under low, medium, and high traffic flow in 2005 for the Delta Airlines Chicago O'Hare concourse.

The least-cost numbers of escorts for each traffic flow rate are 3, 6, and 9, at costs \$90, \$180, and \$300 per shift. From **(6)**, the minimum total annual cost of escorts is \$152,000.

Atlanta International Airport

Delta Airlines has its headquarters in Atlanta, with a large four-concourse terminal with 20 main gate areas. Even though JFK has the same number of gates, the gates in Atlanta are spread much farther apart and handle much more traffic. The low, medium, and high traffic flows are 107, 321, and 535 incoming flights per 8-h period [Delta Airlines 2006] The predicted cost of numbers of escorts for these 8-h shifts are shown in **Figure 4**.

The least-cost numbers of escorts are 32, 40, and 70, at costs of \$1,180, \$3,880, and \$6,430 per shift. From **(6)**, the minimum total annual cost is \$3,977,000. This figure is much larger than for O'Hare or JFK because the concourses at Atlanta are roughly four times as large and handle between three and ten times the traffic of the other airports. The main component of the cost for Atlanta is the cost of flight delays. To compensate for the larger concourses, we set the average time between connecting flights in Atlanta to 75 min (in agreement with actual flight schedules), as opposed to 45 min for the other airports.

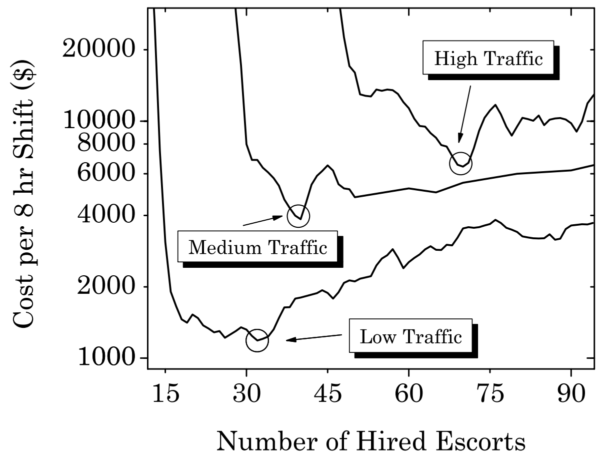

Figure 4. Simulated cost curves under low, medium, and high traffic flow in 2005 for the Delta Airlines Atlanta International Airport concourses.

Predictions of Future Escort Needs

Those who are most likely to require wheelchair assistance at airports are senior citizens. The senior population will increase 7% over the next 5 years, 87% over the next 25 years, and 112% over the next 45 years [Department of Health and Human Services 2006]. Assuming that flying patterns for this age group remain unchanged, and that the layout of airports does not significantly change, our algorithm can predict the change in cost to the airline. The number of flight arrivals to the JFK Delta concourse under medium traffic will increase from 50 to about 100 over the next 30 years. Using our simulation, we find that the optimal number of escorts will increase from 6 to 9, with a corresponding increase $600 per 8-h shift. These results are depicted in **Figure 5**, with predictions of costs also for low traffic and high traffic.

Model Strengths and Weaknesses

- The simplicity of the model makes it very versatile. Parameters such as the layout, the speed of WP and escort, costs, and other parameters can be specified with minimal modifications to the algorithm.
- Simulation times are quite small for even the busiest airport.
- The lattice and the use of a "city block" distance is more natural and realistic for the interior of buildings.
- The algorithm is simple and intuitive, making it easy to communicate and justify.

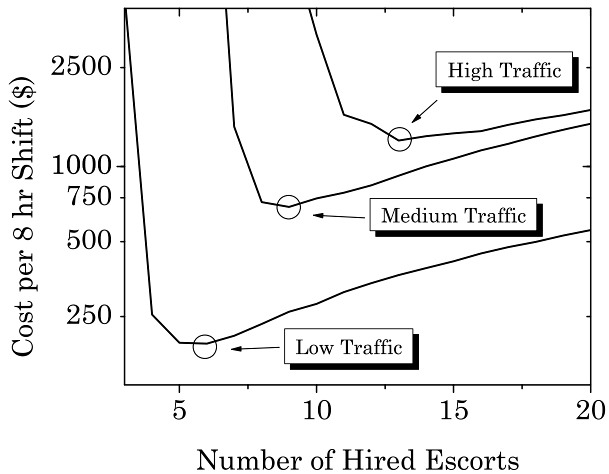

Figure 5. Simulated cost curves under predicted future low, medium, and high traffic flows in 2036 for the Delta Airlines JFK International Airport concourses.

- The algorithm is also practical, outputting a schedule for each escort.

Conclusion

We have presented a systematic study of how to administer a wheelchair escort service at the least cost to an airline. Our algorithm can predict the costs under different concourse layouts, flight schedules, arrival of unexpected WPs, flight delays, and reallocation of escorts to a secondary task to reduce cost. We applied our approach and presented results for case studies of Delta Airlines terminals in Chicago, New York, and Atlanta.

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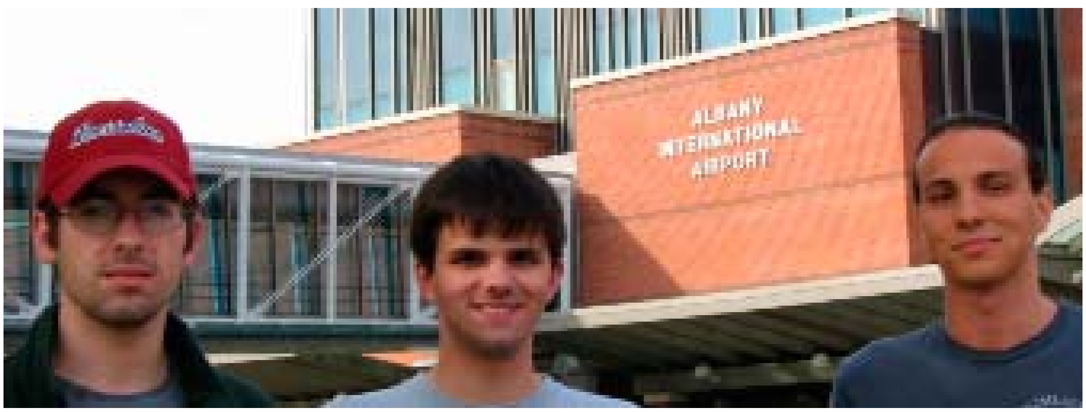

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A Simulation-Driven Approach for a Cost-Efficient Airport Wheelchair Assistance Service

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Summary

Although roughly 0.6% of the U.S. population is wheelchair-bound, the strain of travel is such that more than twice that amount relies on wheelchairs in airports [Haseltine Systems Corp. 2006].

Two issues have the greatest impact on the cost and effectiveness of this service: the number of wheelchairs and how they should be deployed. The proper number of escorts and wheelchairs is not only a question of the airport but of the volume of passengers, which can vary greatly. If escorts determine their own movements within the airport, lack of coordination could result in areas being unattended; however, fluctuation in requests could be so great that a territory-based plan could overwork some escorts and underwork others.

We present an algorithm for scheduling of the movement of escorts that is both simple in implementation and effective in maximizing the use of available time in each escort's schedule. Then, given the implementation of this algorithm, we simulate the scheduling of requests in a given airport to find the number of wheelchair/escort pairs that minimizes cost.

Methods And Assumptions

We propose a stochastic simulation-driven optimization procedure. We partition the problem into three categories: pre-simulation processing, simulation rules and dynamics, and optimization. The pre-simulation phase generates the necessary inputs for the simulation phase, such as the airport layout and a master passenger request list containing the wheelchair assistance requests for a single day. The simulation phase consists of a continuous event-based model of passenger arrival/departure and wheelchair/escort movement. Finally, we minimize costs over the number of escorts.

Pre-Simulation

Airport Layout

The simulation is customized for the layout of each airport. We represent an airport as a bidirectional graph, in which nodes indicate gates, entrance/exit points, or other places of similar interest. Edges between nodes indicate travel paths, usually through the main hallway of a concourse. We constructed the graphs with the aid of satellite images [Google.com n.d.].

Passengers must travel through long corridors to reach departure gates. A typical concourse has gates on either side of the main corridor. We assume that the time required to travel from any gate to any gate is substantial; that is, even if two gates are adjacent or on opposite sides of the main corridor, we assess the travel time between the two gates.

The graphical representation of the airport is encoded in an $n \times n$ adjacency matrix A, with entry $A_{i,j}$ denoting the travel time between location i and location j. We determine travel times by figuring the actual distance divided by a walking speed of 3 mph. The shortest possible travel times (calculated using Dijkstra's algorithm [Wang 2006]) from every location to every other location is referenced in matrix D, with entry D_{ij} denoting the shortest travel time between nodes i and j.

We assume that the escorts know the shortest path between any two gates, because they are familiar with the airport environment. In our simulations we do not consider the distance between the gate and the airplane.

Wheelchairs And Escorts

We treat a wheelchair and its escort as a single traveling entity, the "escort." The airline may have additional wheelchairs on hand in the event of a malfunction, and we incorporate the cost of the additional wheelchairs into the maintenance and storage costs of wheelchairs in operation. The escort's job is to travel to the arrival gate of the passenger and transfer the passenger to the departure gate.

An important assumption is that the number of escorts remains constant throughout the simulation period. In reality, escorts rotate in shifts; but with a simulation period of one day, we assume that escorts starting a shift immediately replace escorts ending one. Similarly, during the simulation we do not allow hiring or firing of workers, nor buying or breaking of wheelchairs; instead, we represent the costs associated with these actions with a sunken cost term in the total cost function.

Passenger Request List

Given a terminal, we create a flight schedule for one day. We look at the total number of passengers who pass through the airport in one day. We estimate the average load of a plane to be 125 passengers, which we use to estimate the number of flights arriving or departing at the terminal in one day. Observation of departing flight information at a busy airport [City of Atlanta 2006] confirms the information in another source [Neufville 1976]: There is regular activity between 6 A,M, and 10 P.M. and relatively few flights at night. We therefore space our departures evenly between these times, and then perturb these values by a random shift of less than an hour, so that we are certain not to test our algorithm against just one schedule. Subsequently, these flights are assigned to specific gates.

Next, we create the requests for the day. We generate the number of requests based on the passenger volume that we are trying to mimic and the percentage of the population that requires wheelchair assistance. For different runs, this value was either 0.6% or 1.2% [Haseltine Systems Corp. 2006]. Each request is assigned an arriving flight and a departing flight, with the assumption that no layover of less than half an hour should be attempted. We assume that a certain percentage of wheelchair passengers have phoned the airline ahead of time; time of request is set to 0. For the remainder, we generate random request times, varying from more than five hours to a half hour before arrival of the wheelchair passenger. We sort this list by request time, so that when the algorithm descends the list, it mimics a dispatcher's receiving the requests at varying times throughout the day, including the wheelchair needs that occur with little notification.

Different daily scenarios can be modeled by altering the generation of the request list. The request list models the passenger traffic load throughout the simulation, since it contains a greater concentration of requests during peak hours of operation. Furthermore, request frequency throughout the day can be increased to reflect operation during holiday-travel seasons at hub airports, or yearly peak-travel at airports in popular vacation destinations.

Scheduling Plan

We assume that escorts communicate with a dispatcher via a walkie-talkie, and that the dispatcher has a schedule for each escort.

An escort who has completed a task calls the dispatcher to find out the next. We assume that the dispatcher knows how long it takes an escort to get between two points x and y, which we call δ_{xy}.

A dispatcher receives requests at varying times throughout the day. Each request contains four pieces of information: the time and location of the passenger's arrival, t_a and a, and the time and location of departure, t_d and d. For the passenger to reach the destination in time, the escort must arrive at a location by some final time,

$$t_f = t_d - \delta_{ad}.$$

The algorithm's version of "first come, first served" is that one task cannot replace another on a schedule if its final time is later. This keeps every schedule compact: If a switch is made, the only result is the new task starting sooner after the previous one (switching will be discussed shortly).

To determine which escort should be assigned a passenger, the dispatcher first finds out if anyone can complete the task. From those who can, he selects one who can complete it in an optimal way, as follows.

The worst way in which a request can be fulfilled is for the escort to bring the passenger to his destination late yet within the window δ_w in which the flight is delayed. To determine if escort e can do this, the dispatcher looks at what location l_e the escort will be at completion of the last job before the final time of the request, and when that job will be completed, t_e. We need

$$t_e + \delta_{ea} < t_f + \delta_w.$$

An escort who meets this requirement is in group O_1.

A better way in which a request can be fulfilled is if an escort can bring the passenger to the destination on time but by removing another passenger from his schedule later. The condition for this one is the same as above, but without the delay term:

$$t_e + \delta_{ea} < t_f.$$

An escort who meets this requirement is in group O_2.

Because we want to do as little reshuffling of schedules as possible, an even better situation is for an escort to be able to take on a request but have to push back the time of completion of later tasks without forcing any late departures. An escort who meets this requirement is in group O_3.

Finally, the ideal situation is when the dispatcher can assign a request to an escort without rescheduling that escort's later tasks. If the next task is to start at time t_s at location s, then escort e must be able to complete the request with enough time to go from d to s by t_s,

$$t_e + \delta_{ds} < t_s.$$

An escort who meets this requirement is in group O_4.

The dispatcher determines the most optimal group that is not empty and chooses an escort in it for the request whose previous task brings that escort closest to the arrival location of the passenger in the new request. If a new request bumps out one or more queued requests, they must be rescheduled before the dispatcher advances to to the next request on the list.

If a request cannot be scheduled, every escort will either be busy with another request or will be too far away to arrive in time. In such a case, the passenger must be scheduled for a later flight and/or compensated for missing the flight, and the situation falls out of the algorithm.

Simulation Algorithm

We envision the problem as a temporal "packing" problem—the dispatcher must fit as many tasks onto the escorts' schedules as possible.

Algorithm : $\textsc{MainSimulation}(D, R, N_E)$

create escort task array E
$I = \textsc{FindIndexUnfulfilledRequest}(R)$
while $I > 0$
do $\begin{cases} O = \textsc{MakeOptimalityMat}(D, R_I, E) \\ \textbf{comment: } \text{Now execute request} \\ (R, E) = \textsc{ExecuteRequest}(D, R, E, I, O) \\ I = \textsc{FindIndexUnfulfilledRequest}(R) \end{cases}$
$totalmissed = \textsc{Sum}(R_{\text{missed flights}})$
$totaldelay = \textsc{Sum}(delaytime)$
return $(totalmissed, totaldelay)$

$\textsc{MainSimulation}()$ handles the entire simulation. We input the shortest-travel-time matrix D, list of requests R, and number of escorts N_E. Entry E_j of the task array is escort j's task schedule. The two main routines within the simulation are $\textsc{MakeOptimalityMat}()$ and $\textsc{ExecuteRequest}()$. Together, they determine which escort is most suited to be assigned the request at hand, and how the current schedule of the escort will be changed to accommodate the new request. $\textsc{MakeOptimalityMat}()$ makes a $N_E \times 4$ matrix, with row j representing the inclusion in or exclusion from the optimality groups of escort e_j.

Each row of $\textsc{MakeOptimalityMat}()$ is generated by $\textsc{OptimalityCheck}()$, shown on the next page.

For each request, $\textsc{OptimalityCheck}()$ assigns escorts into optimality groups. $\textsc{OptimalityCheck}()$ creates a row of an optimality matrix O, with entry $O_{i,j}$ denoting whether escort i is *group j-optimal*—that is, i is in group j.

Finally, $\textsc{ExecuteRequest}()$ (shown on p. 401) assigns an escort, if possible.

As the groups descend in optimality, the dispatcher undertakes more and more actions in attempting to assign the request at hand.

Algorithm : $\textsc{OptimalityCheck}(D, R_I, e_j)$

initialize $O_j = (0, 0, 0, 0)$
if Escort j is Group 1 Optimal (can fulfill R_I with delay)
  then $O_{j,1} = 1$
if Escort j is Group 2 Optimal(can fulfill R_I w/o delay)
  then $O_{j,2} = 1$
if Escort j is Group 3 Optimal(can fulfill R_I w/o removing tasks)
  then $O_{j,3} = 1$
if Escort j is Group 4 Optimal(can fulfill R_I w/o shifting tasks)
  then $O_{j,4} = 1$
return (O_j)

Deployment Plan

We measure costs in United States Dollars (USD). There are two cost categories:

- costs dependent on the number of wheelchair/escort pairs, specifically, wheelchair maintenance/storage costs and escorts' wages, and
- costs dependent on the number of delayed flights and passengers who miss their flights.

All wheelchair and escort costs are considered fixed throughout the (one-day) duration of a simulation.

In our cost function, N_E is the number of escorts, R is the daily request list, D is the airport layout, X is the number of missed flights, and Y is total amount of time that flights that must be held at the gate due to late passengers. The objective function is thus:

$$[X, Y] = \textsc{MainSimulation}(D, R, N_E),$$
$$C(X, Y) = \text{Cost}_{\text{fixed}} + \text{Cost}_{\text{miss}} X + \text{Cost}_{\text{delayed}} Y$$

The average cost per hour of a flight held at the gate is assumed to be \$1,018, the average cost in 1986 [Nathan L. Kleinman, Stacy D. Hill 1998] adjusted for inflation [Friedman 2006]. The cost of missing a flight is \$500, the price of ticket reimbursement and/or lodging if necessary. Our fixed costs include the maintenance costs of the wheelchairs and wages of escorts. We assume that wheelchairs cost \$130/yr/chair [Alexander 2006] to maintain, store, and replace as needed, and that escort wages are \$10/h. [Avjobs.com 2006]. The

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Algorithm : ExecuteRequest(D, R, E, I, M)

---

if column 4 of O contains a 1

then
- Find escort with closest previous task e^*
- Insert task into schedule of e^*
- Mark request R_I as fulfilled

else if column 3 of O contains a 1

then
- Find escort with closest previous task e^*
- Determine insertion spot s for task
- Move jobs after s back farthest possible
- Insert task into schedule of e^*
- Mark request R_I as fulfilled
- Move jobs after s forward farthest possible

else if column 2 of O contains a 1

then
- Find escort with closest previous task e^*
- Determine insertion spot s for task
- Move jobs after s back farthest possible
- Insert task into schedule of e^*
- Mark request R_I as fulfilled
- Remove overlapping tasks in schedule
- Mark corresponding requests as unfulfilled
- Move remaining jobs after s forward farthest possible

else if column 1 of O contains a 1

then
- Find escort with closest previous task e^*
- Determine insertion spot s with minimum delay
- Move jobs after s back farthest possible
- Insert task into schedule of e^*
- Mark request R_I as fulfilled
- Remove overlapping tasks in schedule
- Mark corresponding requests as unfulfilled
- Move remaining jobs after s forward farthest possible
- Log delay of R_I

else
- Mark task as fulfilled
- Log R_I as missed flight

return (E, R)

$\text{Cost}_{\text{fixed}}$ term is thus given by

$$\text{Cost}_{\text{fixed}} = \left(10 + \frac{130}{365 \times 24}\right) H N_E$$

where H is the simulation period in hours.

Results and Analysis

Test Protocol

Before delving into the results of test simulations, we clarify certain protocols used.

- Each data point on every plot represents 10 trials.
- Error bars are one standard deviation.
- Half of all requests are known ahead of passenger arrival (typical).

Optimizing Schedules

We demonstrate the effectiveness of adaptive scheduling, that is, the allowing of escorts to be placed in optimality groups 1, 2, and 3. We simulated a small one-concourse setting with moderate traffic (15,000 passengers/day) and successively removed placing escorts in optimality groups 1, 2, and 3, in that order. In other words, the simplest algorithm is for a request to be missed if no escort is in O_4, the next simplest algorithm is for a request to be missed if no escort is in O_4 or O_3, etc. These simulations were carried out assuming that 1.2% of the passengers require wheelchair assistance.

There are large gains from shifting around existing tasks in an escort's schedule, that is, the inclusion of group O_3. The effect is substantial—we avoid hiring five or six escorts, resulting in savings of about \$1,000/d (**Figure 1**). Savings would increase with passenger traffic and airport size. With several airports, Epsilon Airlines may see total savings on the order of \$10,000/d, merely by adopting our adaptive scheduling.

Performance/Sensitivity across Concourses

We use Chicago O'Hare terminals as test locations for examining performance in different-sized settings. We ran simulations for one-, two- and four-concourse settings, corresponding to **Figures 2—4**. Each set of simulations spans three levels of terminal traffic, assuming that 0.6% of passengers require wheelchair assistance. Comparing costs vs. number of escorts, we observe

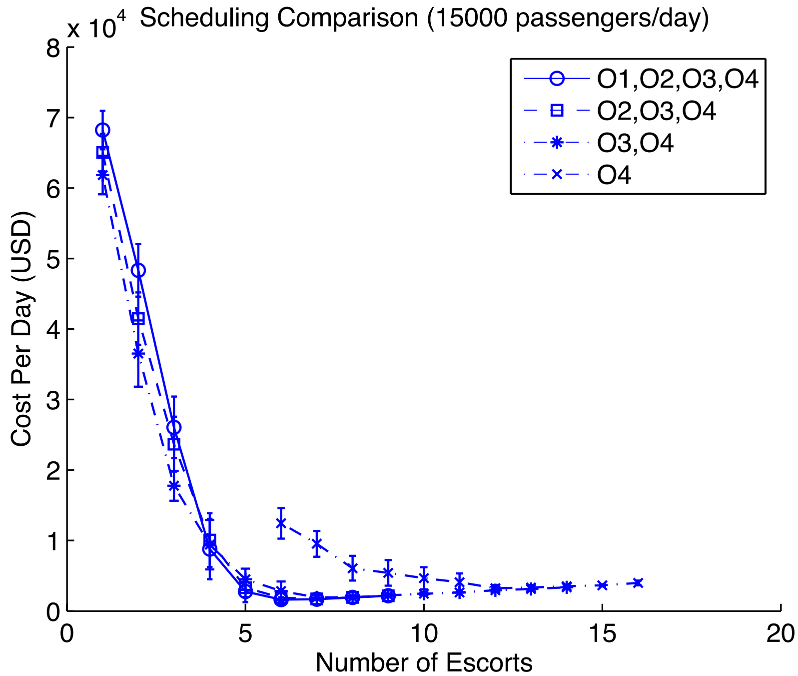


Figure 1. Comparison of scheduling methods.

an increase in the optimal number of escorts roughly proportional to the increase in the number of concourses. Furthermore, as airport traffic increases, we generally see a rise in the number of escorts needed to maintain optimal cost. Figure 4 shows that in a single-concourse setting, increased traffic density barely increases the optimal number of escorts, whereas **Figures 3** and **4** show substantial increases in escort demand within the two- and four-concourse settings, due to longer travel times between gates.

Performance/Sensitivity across Airports

Our algorithm has the flexibility to address any airport configuration, since we use satellite images of an airport to convert it to its node-edge representation.

We perform a comparison analysis on three airports. We examine two-concourse sections from New York's LaGuardia (LGA), Chicago O'Hare (ORD), and Dallas/Fort Worth (DFW) airports, at two separate traffic levels, assuming that 1.2% of passengers require wheelchair assistance. From the satellite images, we hypothesized that DFW would fare the worst, due to passengers having to walk from one side of the circular terminals to the other (almost a mile), and that Chicago O'Hare would perform best, since it has a more compact layout, with passengers able to cross through a triangular intersection among concourses.

However, **Figure 5** shows that algorithm performance is statistically equivalent, despite different airport geometries, over the wide range of numbers of passengers per day, and over 10 averaged trials for each data point. Our al-

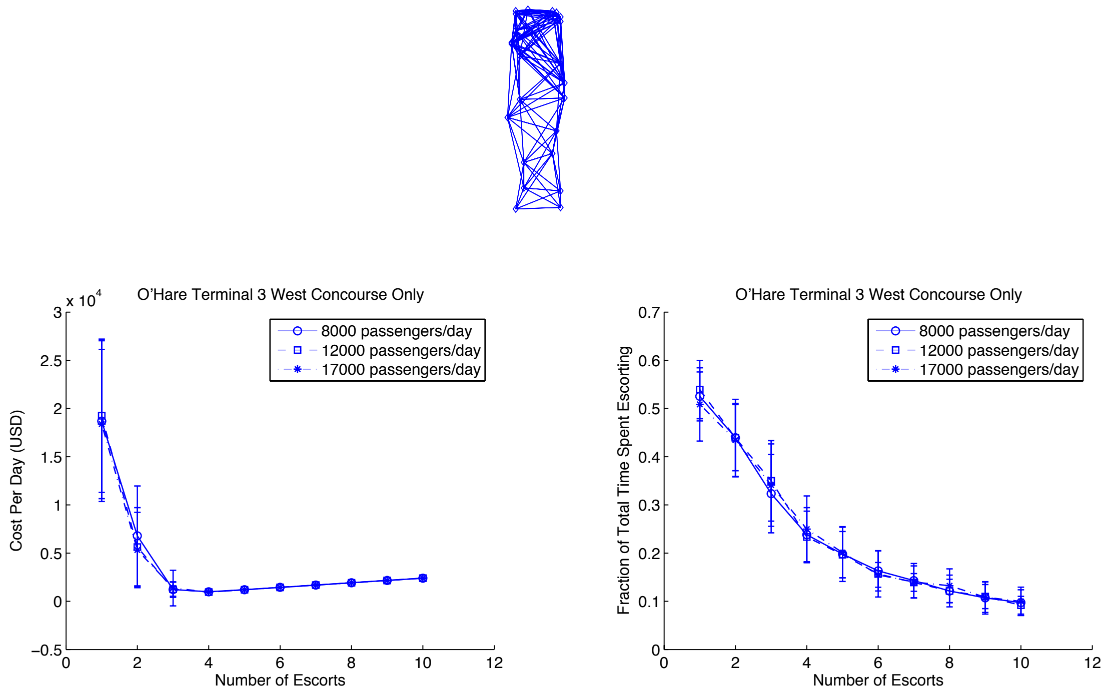


Figure 2. Chicago O'Hare Terminal 3 West Concourse only.

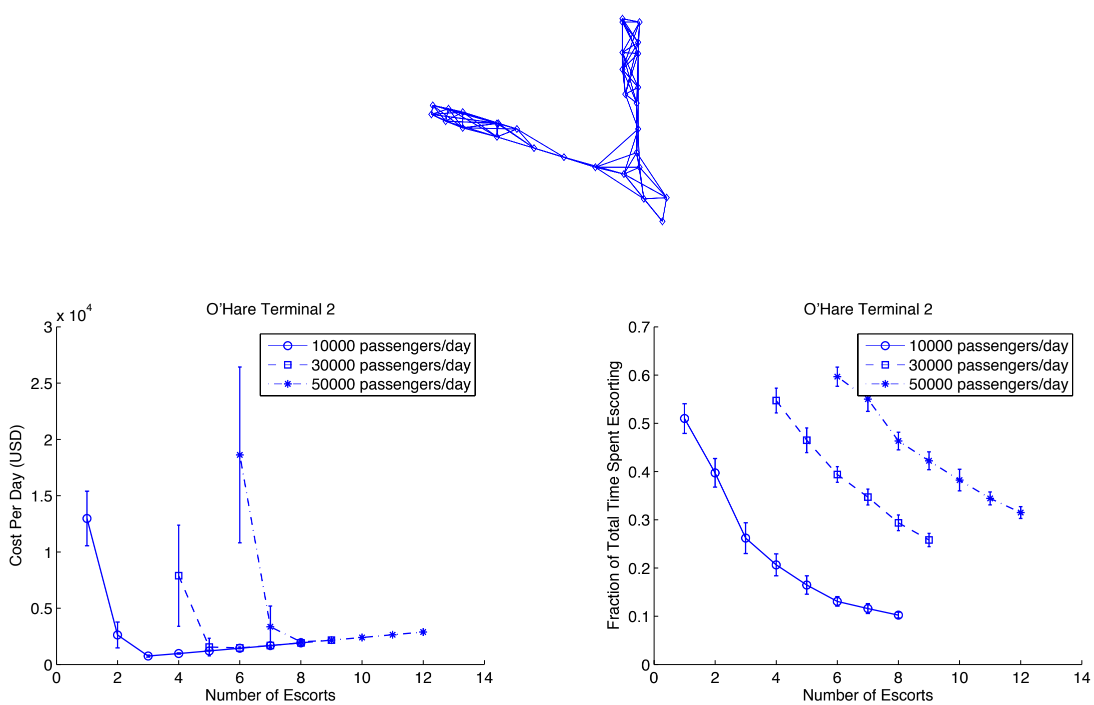
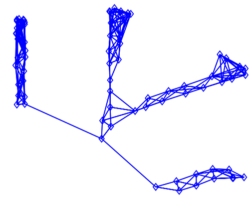


Figure 3. Chicago O'Hare Terminal 2.



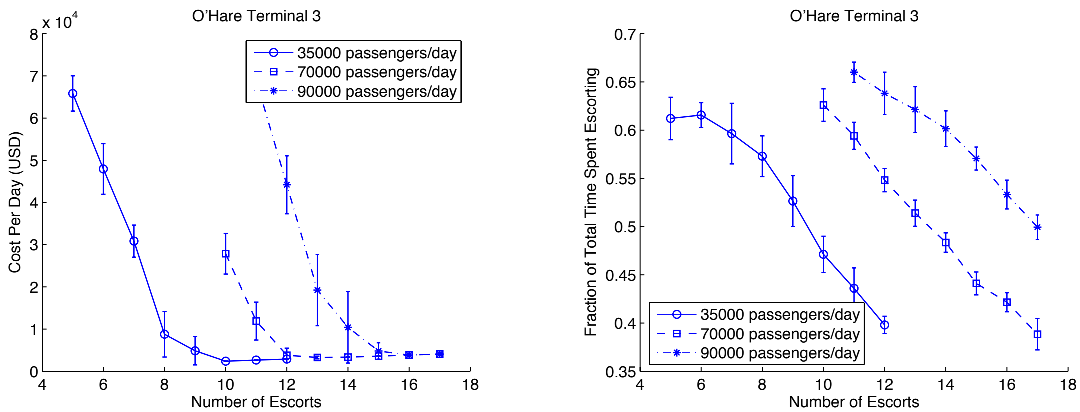


Figure 4. Chicago O'Hare Terminal 3.

gorithm performs equally well regardless of airport configuration, though for higher traffic levels we might see differences among airport geometries.

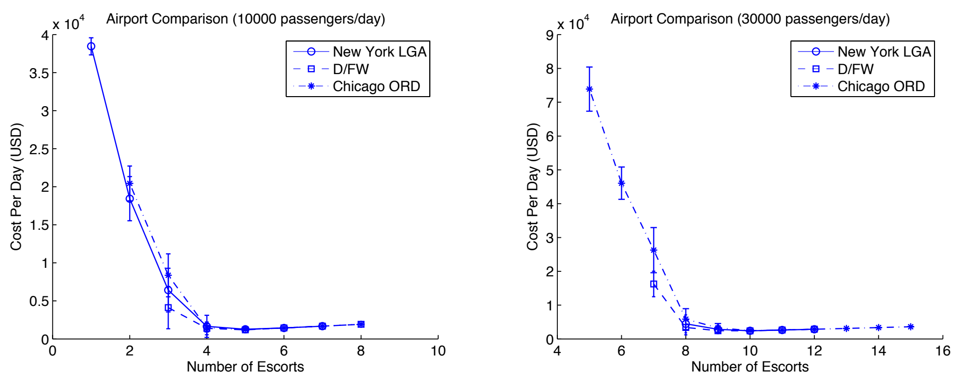


Figure 5. Comparison of algorithm performance at different airports.

Predicting For An Aging Population

The demand for wheelchair assistance will increase in the future. The question is, What does this entail? Should an increase of 10% in the requests per capita be treated the same as a 10% increase in total volume of passengers? The answer from our simulations appears to be that it should be treated as more. A 10% percent increase in population would increase the number of scheduled flights per day; a 10% increase in requests per capita would not. In other words, as the average passenger ages, the result is not just more requests but more requests per plane.

We ran two series of simulations for a two-concourse airport.

- We increased the number of passengers (and thus correspondingly the number of flights) from 33,000 to 42,000.
- We held the number of passengers constant at 30,000 and increased the request percentage from 1.32% to 1.68%.

In each series, the number of requests is the same, but the average minimum cost is greater when the percentage increases (**Figure 6**).

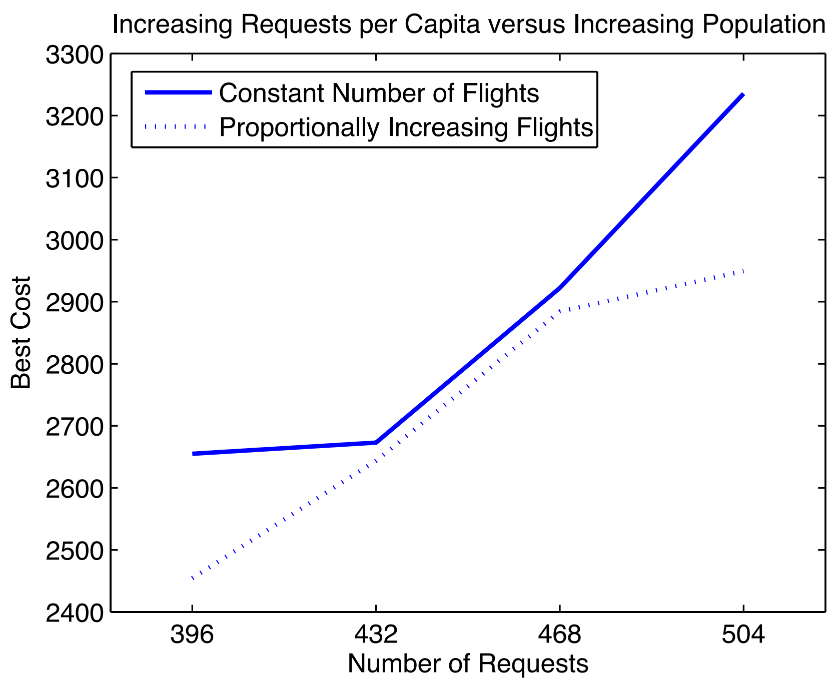


Figure 6. Comparison of increase in requests.

Conclusions

A weakness of our approach is that we do not specifically optimize towards minimizing damages. For example, an escort may have five short tasks (in terms of time from arrival gate to departure gate) queued, when a lengthy request is received. The dispatcher may end up striking three or four tasks from the escort's schedule in order to fulfill the one new task—but the removed tasks' passengers may miss their departure times. We sacrifice many passengers for the sake of one. However, this situation is rare, since we try to minimize the chance of any passenger missing his/her flight. If this type of situation is encountered frequently, then we have probably not optimized the number of escorts.

Our method also assumes no delayed arrivals or departure delays due to other causes.

The dispatcher has the difficult task of managing escorts' schedules, but this is feasible with a computer.

These weaknesses are outweighed by the demonstrated robustness and improvements of our approach. We have shown in a variety of settings that not only does our algorithmic approach work but it outperforms simpler algorithms by substantial margins.

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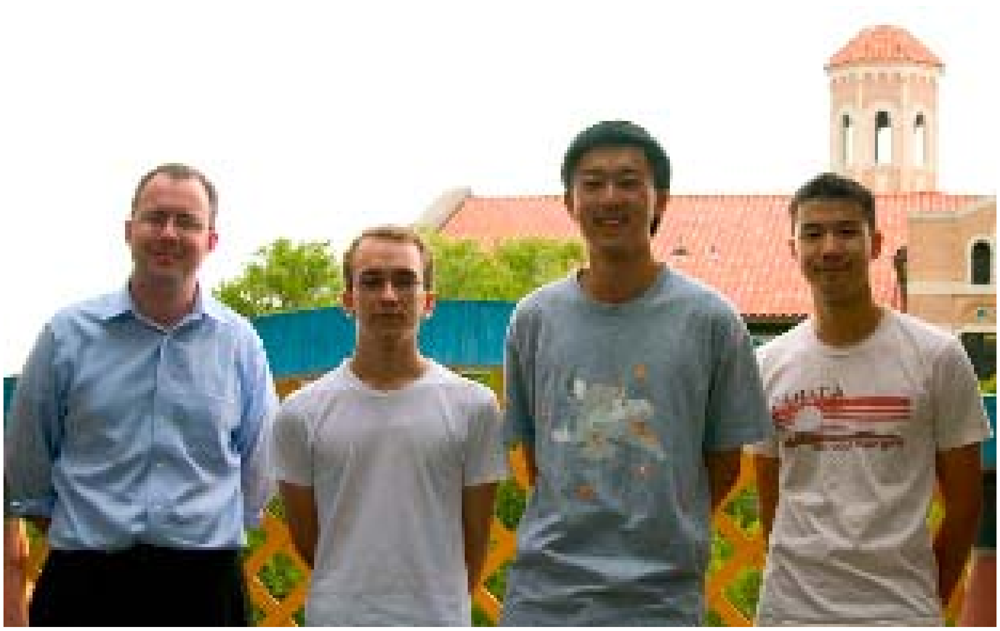

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Judges' Commentary: The Fusaro Award Wheelchair Paper

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The Ben Fusaro Award for the 2006 B problem went to the team from Maggie L. Walker Governor's School in Richmond, VA. Their paper fell just short of the Outstanding designation due to a slightly less sophisticated level of mathematics than could have been used. However, the paper exemplified some outstanding characteristics:

- It presented a high-quality application of the complete modeling process;
- it demonstrated noteworthy originality and creativity in their modeling effort; and
- was well-written, in a clear expository style making it a pleasure to read.

Addressing real-world problems involves formulating a mathematical description of the problem, solving the mathematical model, interpreting the mathematical solution, and critically evaluating the model. Before a team could formulate a mathematical description of the problem, it was necessary to do research to estimate reasonable values for parameters to be used.

The Maggie L. Walker team began by getting current statistics on the number of wheelchair passengers and how airlines and airports serve their needs. In addition, they looked at the Department of Transportation Congressional Report on disability-related airline complaints. From their assumptions, it was clear that the team considered many issues. Certain assumptions—for example, wheelchairs are always functional, an important issue—treated issues that

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might seem superfluous but are otherwise not tractable in a model. The team justified their assumption that the pattern of calls for wheelchairs follows a Poisson process.

The team considered how many escorts and wheelchairs to place at each airport, and the most efficient way for escorts and wheelchairs to move around. The team considered small and large airports and linear, pier, satellite, and curvilinear-shaped concourses.

Their model consisted of three parts:

- an algorithm for finding the number of escorts that an airport should hire, based on the need to balance costs and recognizing that the primary costs are the salaries of escorts;
- establishing that the best wheelchair-to-escort ratio is one-to-one;
- showing that wheelchair service is most efficient when escorts have a central hub, whose location depends on the concourse type.

To test the efficiency of their model, the team used a spreadsheet to simulate wheelchair service in small, medium, and large airports. They recognized that some of their assumptions—for example, that all escorts are perfectly efficient and all passengers are completely cooperative—weaken their model by ignoring the human element. However, they demonstrated the flexibility in their model, allowing for changes as the airline industry grows and the traveling population ages.

This paper is a fine example of the fact that mathematical modeling can be done at many levels. The team is to be congratulated on their thoroughness, their clarity, and their utilization of the mathematics that they knew to create their own model and solve the problem at hand. The judges felt that the model itself was both reasonable and well thought out.

About the Authors

Marie Vanisko is in her fifth year of teaching at Cal State Stanislaus. Prior to that, she taught for 31 years at Carroll College in Montana and was a visiting professor at the U.S. Military Academy at West Point. She chairs a College Board committee for the SAT Subject Tests in Mathematics and serves on a national joint committee for the NCTM and MAA. For each of the past two years, Marie has co-directed an MAA Tensor Foundation grant project for high school girls, entitled Preparing Women for Mathematical Modeling, with the hope of encouraging more young women to select careers that involve mathematics. She serves as a judge for the COMAP MCM and HiMCM has also been active in the MAA PMET (Preparing Mathematicians to Educate Teachers) project.

UNITED STATES POSTAL SERVICE

Statement of Ownership, Management, and Circulation (All Periodicals Publications Except Requester Publications)

1. Publication Title	2. Publication Number	3. Filing Date
The UMAP Journal	0 1 9 7 _ 3 6 2 2	9/20/2006
4. Issue Frequency	**5. Number of Issues Published Annually**	**6. Annual Subscription Price**
Quarterly	4	$104.00

7. Complete Mailing Address of Known Office of Publication *(Not printer) (Street, city, county, state, and ZIP+4®)*

COMAP, Inc.
175 Middlesex Tpk, Suite 3B, Bedford, MA 01730

Contact Person
Kevin Darcy

Telephone *(Include area code)*
781/862-7878 x131

8. Complete Mailing Address of Headquarters or General Business Office of Publisher *(Not printer)*

Same

9. Full Names and Complete Mailing Addresses of Publisher, Editor, and Managing Editor *(Do not leave blank)*

Publisher *(Name and complete mailing address)*

Solomon Garfunkel
175 Middlesex Tpk, Suite 3B, Bedford, MA 01730

Editor *(Name and complete mailing address)*

Paul Campbell
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Consortium for Mathematics and	175 Middlesex Tpk., Suite 3B
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☐ Has Not Changed During Preceding 12 Months
☐ Has Changed During Preceding 12 Months *(Publisher must submit explanation of change with this statement)*

13. Publication Title	14. Issue Date for Circulation Data	
The UMAP Journal	11/15/2006	
15. Extent and Nature of Circulation	**Average No. Copies Each Issue During Preceding 12 Months**	**No. Copies of Single Issue Published Nearest to Filing Date**
a. Total Number of Copies *(Net press run)*	725	740
b. Paid Circulation *(By Mail and Outside the Mail)* (1) Mailed Outside-County Paid Subscriptions Stated on PS Form 3541 *(Include paid distribution above nominal rate, advertiser's proof copies, and exchange copies)*	650	675
(2) Mailed In-County Paid Subscriptions Stated on PS Form 3541 *(Include paid distribution above nominal rate, advertiser's proof copies, and exchange copies)*	0	0
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c. Total Paid Distribution *(Sum of 15b (1), (2),(3), and (4))*	680	710
d. Free or Nominal Rate Distribution *(By Mail and Outside the Mail)* (1) Free or Nominal Rate Outside-County Copies Iincluded on PS Form 3541	0	0
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(4) Free or Nominal Rate Distribution Outside the Mail *(Carriers or other means)*	0	0
e. Total Free or Nominal Rate Distribution *(Sum of 15d (1), (2), (3) and (4)*	12	18
f. Total Distribution *(Sum of 15c and 15e)*	692	728
g. Copies not Distributed *(See Instructions to Publishers #4 (page #3))*	8	14
h. Total *(Sum of 15f and g)*	700	742
i. Percent Paid *(15c divided by 15f times 100)*	98	98

16. Publication of Statement of Ownership

☐ If the publication is a general publication, publication of this statement is required. Will be printed in the Third issue of this publication.

☐ Publication not required.

17. Signature and Title of Editor, Publisher, Business Manager, or Owner	Date
Sol Garfunkel	11/15/2006

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