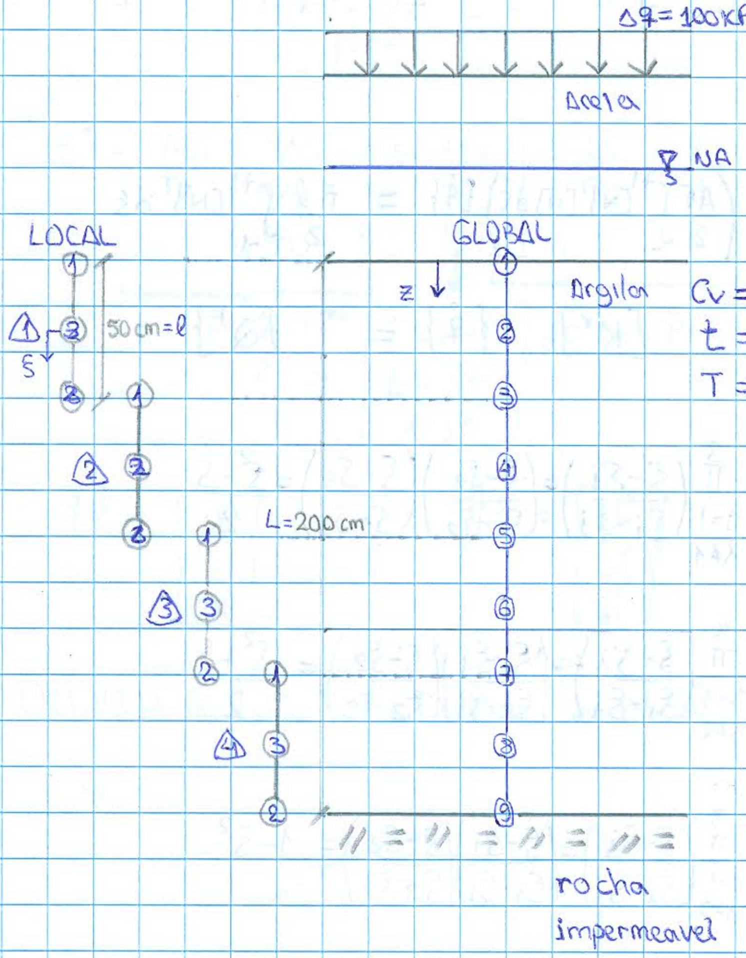
Lista de Exercícios Nº 03

Estudando: Celso Salvador Villalobos

Materia: Métodos Numéricos em Engenharia Civil



$C_v = 10^{-2}\ cm^2/s$

$t = 18{,}5$ dias

$T = \frac{C_v t}{H^2} = 0{,}40$

Formulação Variacional:

$$\Omega = \int_V \left[\frac{1}{2} C_v \left(\frac{\partial U_e}{\partial z}\right)^2 + \frac{\partial U_e}{\partial t} . U_e\right] dV - \int_{z_1}^{z_2} \bar{P}\ U_e\ dz$$

Onde $\bar{P} = \bar{P}(z)$ é excesso de presão neutra prescrito por unidad de comprimento

$$U_e = [N]\{q\} \qquad \frac{\partial U_e}{\partial z} = \frac{\partial}{\partial z}[N]\{q\} = [B]\{q\}$$

$$\frac{\partial We}{\partial t} = \frac{\partial}{\partial t}[N]\{q\} = [N]\{\dot{q}\} \qquad dV = A\,dz = A\frac{l}{2}d\xi$$

Substituindo no funcional:

$$\Omega = \frac{A}{2}\int_{-1}^{+1}\{q\}^T[B]^T[R][B]\{q\}\frac{l}{2}d\xi + A\int_{-1}^{+1}\{q\}^T[N]^T[N]\{\dot{q}\}\frac{l}{2}d\xi - \int_{-1}^{+1}\{q\}^T[N]^T\bar{p}\,\frac{l}{2}d\xi$$

$$\delta\Omega = 0$$

$$\underbrace{\left(\frac{Al}{2}\int_{-1}^{+1}[B]^T[R][B]d\xi\right)}_{[K']}\underbrace{\{q\}}_{\{q\}} + \underbrace{\left(\frac{Al}{2}\int_{-1}^{+1}[N]^T[N]d\xi\right)}_{[K^*]}\underbrace{\{\dot{q}\}}_{\{\dot{q}\}} = \underbrace{\frac{\bar{p}\,l}{2}\int_{-1}^{+1}[N]^T d\xi}_{\{Q'\}}$$

$$N_i = \prod_{\substack{j=1\\ i\neq j}}^{n}\left(\frac{\xi-\xi_j}{\xi_i-\xi_j}\right) \qquad N_1 = \prod_{\substack{j=1\\ j\neq 1}}^{3}\left(\frac{\xi-\xi_j}{\xi_i-\xi_j}\right)_{i=1} = \left(\frac{\xi-\xi_2}{\xi_1-\xi_2}\right)\left(\frac{\xi-\xi_3}{\xi_1-\xi_3}\right) = \frac{\xi^2-\xi}{2}$$

$$N_2 = \prod_{\substack{j=1\\ j\neq 2}}^{3}\left(\frac{\xi-\xi_j}{\xi_i-\xi_j}\right)_{i=2} = \left(\frac{\xi-\xi_1}{\xi_2-\xi_1}\right)\left(\frac{\xi-\xi_2}{\xi_2-\xi_3}\right) = \frac{\xi^2+\xi}{2}$$

$$N_3 = \prod_{\substack{j=1\\ j\neq 3}}^{3}\left(\frac{\xi-\xi_j}{\xi_i-\xi_j}\right)_{i=3} = \left(\frac{\xi-\xi_1}{\xi_3-\xi_1}\right)\left(\frac{\xi-\xi_2}{\xi_3-\xi_2}\right) = 1-\xi^2$$

$$N = [N_1 \; N_2 \; N_3] = \left[\frac{\xi^2-\xi}{2},\; \frac{\xi^2+\xi}{2},\; 1-\xi^2\right]$$

$$B = \frac{dN}{d\xi}\cdot\frac{d\xi}{dz} = \frac{\partial}{\partial\xi}\left[\frac{\xi^2-\xi}{2},\; \frac{\xi^2+\xi}{2},\; 1-\xi^2\right]\cdot\frac{2}{l} = \frac{1}{l}\left[2\xi-1,\; 2\xi+1,\; -4\xi\right]$$

$$K' = \frac{A\ell}{2}\int_{-1}^{+1} \frac{1}{\ell}\begin{bmatrix} 2\xi-1 \\ 2\xi+1 \\ -4\xi \end{bmatrix} C_v \frac{1}{\ell}\left[2\xi-1,\ 2\xi+1,\ -4\xi\right] d\xi = \frac{AC_v}{6\ell}\begin{bmatrix} 14 & 2 & -16 \\ 2 & 14 & -16 \\ -16 & -16 & 32 \end{bmatrix}$$

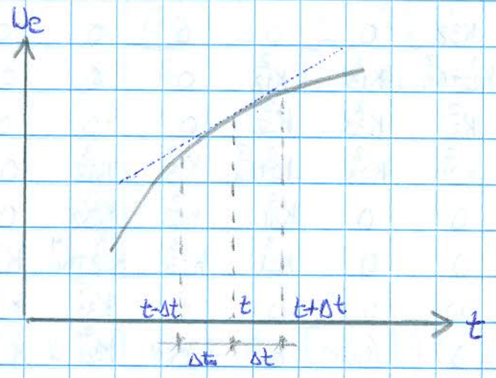
$$K^* = \frac{A\ell}{2}\int_{-1}^{+1} \begin{bmatrix} (\xi^2-\xi)/2 \\ (\xi^2+\xi)/2 \\ 1-\xi^2 \end{bmatrix}\left[\frac{\xi^2-\xi}{2},\ \frac{\xi^2+\xi}{2},\ 1-\xi^2\right] d\xi = \frac{A\ell}{2}\int_{-1}^{+1}\begin{bmatrix} \frac{1}{4}\xi^2(\xi-1)^2 & \frac{1}{4}\xi^2(\xi^2-1) & \frac{1}{2}\xi(\xi-1)(1-\xi^2) \\ & \frac{1}{4}\xi^2(\xi+1)^2 & \frac{1}{2}\xi(\xi+1)(1-\xi^2) \\ & & (1-\xi^2)^2 \end{bmatrix} d\xi$$

$$K^* = \frac{A\ell}{2}\begin{bmatrix} 4/15 & -1/15 & 2/15 \\ -1/15 & 4/15 & 2/15 \\ 2/15 & 2/15 & 16/15 \end{bmatrix} = \frac{A\ell}{30}\begin{bmatrix} 4 & -1 & 2 \\ -1 & 4 & 2 \\ 2 & 2 & 16 \end{bmatrix}$$

$$Q' = \bar{P}\frac{\ell}{2}\int_{-1}^{+1}\begin{bmatrix} \frac{1}{2}\xi(\xi-1) \\ \frac{1}{2}\xi(\xi+1) \\ 1-\xi^2 \end{bmatrix} d\xi = \bar{P}\frac{\ell}{2}\begin{bmatrix} 1/3 \\ 1/3 \\ 4/3 \end{bmatrix} = \frac{\bar{P}\ell}{6}\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$$

$$\{q\} = \begin{Bmatrix} U_e^1 \\ U_e^2 \\ U_e^3 \end{Bmatrix} \qquad \{\dot{q}\} = \begin{Bmatrix} \dot{U}_e^1 \\ \dot{U}_e^2 \\ \dot{U}_e^3 \end{Bmatrix}$$

Derivada no tempo do excesso da pressão neutra:



Diferença finita central: $\dot{U}_e = \lim_{\Delta t \to 0} \dfrac{U_e(t+\Delta t) - U_e(t+\Delta t)}{2\Delta t}$

No tempo t,

$$[K']\{q\}_t + [K^*]\{\dot{q}\}_t = \{Q'\}_t$$

$$[K']\{q(t)\} + [K^*]\{\dot{q}(t)\} = \{Q(t)\}$$

Pode ser re-escrita, considerando-se

$$\{q_{(t)}\} = (1-\theta)\{q\}_{t-\Delta t} + \theta\{q\}_t$$

$$\{\dot{q}_{(t)}\} = \frac{\{q\}_t - \{q\}_{t-\Delta t}}{\Delta t}$$

$$\{Q'_{(t)}\} = (1-\theta)\{Q'\}_{t-\Delta t} + \theta\{Q'\}_t$$

$$[K'][(1-\theta)\{q\}_{t-\Delta t} + \theta\{q\}_t] + [K^*]\left[\frac{\{q\}_t - \{q\}_{t-\Delta t}}{\Delta t}\right] = (1-\theta)\{Q'\}_{t-\Delta t} + \theta\{Q'\}_t$$

$$(\Delta t\,\theta[K'] + [K^*])\{q\}_t = ([K^*] - (1-\theta)\Delta t[K'])\{q\}_{t-\Delta t} + \Delta t((1-\theta)\{Q'\}_{t-\Delta t} + \theta\{Q'\}_t)$$

$\theta = 1/2$ (Metodo Implicito, Crank-Nicolson)

$$\left([K^*] + \frac{\Delta t}{2}[K']\right)\{q\}_t = \left([K^*] - \frac{\Delta t}{2}[K']\right)\{q\}_{t-\Delta t} + \frac{\Delta t}{2}\left(\{Q'\}_{t-\Delta t} + \{Q'\}_t\right)$$

	LOCAL			
Global	1	2	3	4
1	1			
2	3			
3	2	1		
4		3		
5		2	1	
6			3	
7			2	1
8				3
9				2

$$\begin{bmatrix}
K^1_{11} & K^1_{13} & K^1_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\
K^1_{31} & K^1_{33} & K^1_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\
K^1_{21} & K^1_{23} & K^1_{22}+K^2_{11} & K^2_{13} & K^2_{12} & 0 & 0 & 0 & 0 \\
0 & 0 & K^2_{31} & K^2_{33} & K^2_{32} & 0 & 0 & 0 & 0 \\
0 & 0 & K^2_{21} & K^2_{23} & K^2_{22}+K^3_{11} & K^3_{13} & K^3_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & K^3_{31} & K^3_{33} & K^3_{32} & 0 & 0 \\
0 & 0 & 0 & 0 & K^3_{21} & K^3_{23} & K^3_{22}+K^4_{11} & K^4_{13} & K^4_{12} \\
0 & 0 & 0 & 0 & 0 & 0 & K^4_{31} & K^4_{33} & K^4_{32} \\
0 & 0 & 0 & 0 & 0 & 0 & K^4_{21} & K^4_{23} & K^4_{22}
\end{bmatrix}$$

Para $t = 18{,}5$ dias $= 1598400$ seg, 10 intervalos de t então $\Delta t = 159\,840$ seg

$$\Delta T = \frac{10^{-2}\ \text{cm}^2/\text{seg} \times 159\,840\ \text{seg}}{(50\ \text{cm})^2} = 0{,}63936$$

OFISCOOL

$$K^{*} = \begin{bmatrix} 4 & 2 & -1 & & & & & & \\ 2 & 16 & 2 & & & & & & \\ -1 & 2 & 8 & 2 & -1 & & & & \\ & & 2 & 16 & 2 & & & & \\ & & -1 & 2 & 8 & 2 & -1 & & \\ & & & & 2 & 16 & 2 & & \\ & & & & -1 & 2 & 8 & 2 & -1 \\ & & & & & & 2 & 16 & 2 \\ & & & & & & -1 & 2 & 4 \end{bmatrix} \qquad \frac{A\ell}{30} = [M]\frac{A\ell}{30}, \quad A = 1\,cm^2$$

$$K' = \begin{bmatrix} 14 & -16 & 2 & & & & & & \\ -16 & 32 & -16 & & & & & & \\ 2 & -16 & 28 & -16 & 2 & & & & \\ & & -16 & 32 & -16 & & & & \\ & & 2 & -16 & 28 & -16 & 2 & & \\ & & & & -16 & 32 & -16 & & \\ & & & & 2 & -16 & 28 & -16 & 2 \\ & & & & & & -16 & 32 & -16 \\ & & & & & & 2 & -16 & 14 \end{bmatrix} \qquad \frac{A C_v}{6\ell} = [N]\frac{A C_v}{6\ell}, \quad A = 1\,cm^2$$

$$Q' = \begin{bmatrix} 1 \\ 4 \\ 2 \\ 4 \\ 2 \\ 4 \\ 2 \\ 4 \\ 1 \end{bmatrix} \qquad \frac{\bar{P}\ell}{6} = [0]\frac{\bar{P}\ell}{6}$$

$$\left(\frac{\ell}{30}[M] + \frac{\Delta t}{2}\frac{C_v}{6\ell}[N]\right)\{q\}_t = \left(\frac{\ell}{30}[M] - \frac{\Delta t}{2}\frac{C_v}{6\ell}[N]\right)\{q\}_{t-\Delta t} + \frac{\Delta t}{2}\left(\frac{\bar{P}\ell}{2}[0]_{t-\Delta t} + \frac{\bar{P}\ell}{2}[0]_t\right)$$

$$\times(12/\ell) \quad \left(\frac{2}{5}[M] + \Delta T[N]\right)\{q\}_t = \left(\frac{2}{5}[M] - \Delta T[N]\right)\{q\}_{t-\Delta t} + 3\Delta t\,\bar{P}\left([0]_{t-\Delta t} + [0]_t\right)$$

$\bar{P} = 0$, $\quad t_1 = t_0 + \Delta t = 159840$ seg <> 1,85 dias

$$[X] = \frac{2}{5}[M] + \Delta T\,[N] = \begin{bmatrix} 10{,}55 & -9{,}43 & 0{,}88 & & & & & & \\ -9{,}43 & 26{,}86 & -9{,}43 & & & & & & \\ 0{,}88 & -9{,}43 & 21.10 & -9{,}43 & 0{,}88 & & & & \\ & & -9{,}43 & 26{,}86 & -9{,}43 & & & & \\ & & 0{,}88 & -9{,}43 & 21{,}10 & -9{,}43 & 0.88 & & \\ & & & & -9{,}43 & 26{,}86 & -9{,}43 & & \\ & & & & 0{,}88 & -9{,}43 & 21{,}10 & -9{,}43 & 0{,}88 \\ & & & & & & -9{,}43 & 26{,}86 & -9{,}43 \\ & & & & & & 0{,}88 & -9{,}43 & 10{,}55 \end{bmatrix}$$

$$[Y] = \frac{2}{5}[M] - \Delta T\,[N] = \begin{bmatrix} -7{,}35 & 11{,}03 & -1{,}68 & & & & & & \\ 11{,}03 & -14{,}06 & 11{,}03 & & & & & & \\ -1{,}68 & 11{,}03 & -14{,}70 & 11{,}03 & -1{,}68 & & & & \\ & & 11{,}08 & -14{,}06 & 11{,}03 & & & & \\ & & -1{,}68 & 11{,}03 & -14{,}70 & 11{,}03 & -1{,}68 & & \\ & & & & 11{,}03 & -14{,}06 & 11{,}03 & & \\ & & & & & 11{,}03 & -14{,}70 & 11{,}03 & -1{,}68 \\ & & & & & & 11{,}03 & -14{,}06 & 11{,}03 \\ & & & & & & -1{,}68 & 11{,}03 & -7{,}35 \end{bmatrix}$$

$$[X]\,\{q\}_t = [Y]\,\{q\}_{t-\Delta t}$$

$t_1 = 1{,}85$ dias

$t_1 - \Delta t = 1{,}85 - 1{,}85 = 0$ dias

$$[X]\,\{q\}_{1{,}85} = [y]\,\{q\}_0$$

$$[X]\begin{Bmatrix} U_e^1 \\ U_e^2 \\ U_e^3 \\ U_e^4 \\ U_e^5 \\ U_e^6 \\ U_e^7 \\ U_e^8 \\ U_e^9 \end{Bmatrix}_{1{,}85} = [y]\begin{Bmatrix} U_e^1 \\ U_e^2 \\ U_e^3 \\ U_e^4 \\ U_e^5 \\ U_e^6 \\ U_e^7 \\ U_e^8 \\ U_e^9 \end{Bmatrix}_0 = [y]\begin{Bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{Bmatrix}_0 = \begin{bmatrix} 200 \\ 800 \\ 400 \\ 800 \\ 400 \\ 800 \\ 400 \\ 800 \\ 200 \end{bmatrix}$$

$$\begin{bmatrix} U_e^1 \\ U_e^2 \\ U_e^3 \\ U_e^4 \\ U_e^5 \\ U_e^6 \\ U_e^7 \\ U_e^8 \\ U_e^9 \end{bmatrix}_{1,85} = \begin{bmatrix} 0 \\ 58,822 \\ 82,710 \\ 92,879 \\ 97,008 \\ 98,763 \\ 99,468 \\ 99,750 \\ 99,821 \end{bmatrix}$$

$t_2 = 3,7$ dias
$t_2 - \Delta t = 1,85$ dias

$$[X]\,[U_e]_{3,7} = [Y]\,[U_e]_{1,85} = \begin{bmatrix} 509,854 \\ 85,254 \\ 294,452 \\ 676,411 \\ 381,735 \\ 778,523 \\ 386,746 \\ 795,673 \\ 199,493 \end{bmatrix} \Rightarrow [U_e]_{3,7} = \begin{bmatrix} 0 \\ 21,916 \\ 53,385 \\ 74,414 \\ 86,842 \\ 93,425 \\ 96,707 \\ 98,194 \\ 98,609 \end{bmatrix}$$

$t_3 = 5,55$ dias
$t_3 - \Delta t = 3,7$ dias

$$[X]\,[U_e]_{5,55} = [Y]\,[U_e]_{3,7} = \begin{bmatrix} 152,047 \\ 280,698 \\ 131,866 \\ 500,443 \\ 322,532 \\ 710,990 \\ 380,407 \\ 773,728 \\ 195,836 \end{bmatrix} \Rightarrow [U_e]_{5,55} = \begin{bmatrix} 0 \\ 24,638 \\ 40,412 \\ 58,674 \\ 73,643 \\ 84,128 \\ 90,585 \\ 93,958 \\ 94,990 \end{bmatrix}$$

$t_4 = 7{,}4$ dias

$t_4 - \Delta t = 5{,}55$ dias

$$[X]\,[Ue]_{7,4} = [Y]\,[Ue]_{5,55} = \begin{bmatrix} 203{,}865 \\ 99{,}334 \\ 201{,}155 \\ 433{,}070 \\ 272{,}479 \\ 628{,}595 \\ 349{,}386 \\ 725{,}843 \\ 185{,}937 \end{bmatrix} \Rightarrow [Ue]_{7,4} = \begin{bmatrix} 0 \\ 16{,}926 \\ 37{,}678 \\ 52{,}093 \\ 64{,}778 \\ 75{,}191 \\ 82{,}735 \\ 87{,}201 \\ 88{,}672 \end{bmatrix}$$

$t_5 = 9{,}25$ dias

$t_5 - \Delta t = 7{,}4$ dias

$$[X]\,[Ue]_{9,25} = [Y]\,[Ue]_{7,4} = \begin{bmatrix} 123{,}395 \\ 177{,}609 \\ 98{,}586 \\ 397{,}662 \\ 249{,}412 \\ 569{,}883 \\ 317{,}183 \\ 664{,}573 \\ 171{,}093 \end{bmatrix} \Rightarrow [Ue]_{9,25} = \begin{bmatrix} 0 \\ 17{,}346 \\ 30{,}572 \\ 46{,}061 \\ 58{,}457 \\ 68{,}168 \\ 75{,}275 \\ 79{,}656 \\ 81{,}138 \end{bmatrix}$$

$t_6 = 11{,}1$ dias

$t_6 - \Delta t = 9{,}25$ dias

$$[X]\,[Ue]_{11,1} = [Y]\,[Ue]_{9,25} = \begin{bmatrix} 139{,}965 \\ 93{,}324 \\ 151{,}763 \\ 334{,}372 \\ 222{,}805 \\ 516{,}622 \\ 289{,}437 \\ 605{,}272 \\ 155{,}779 \end{bmatrix} \Rightarrow [Ue]_{11,1} = \begin{bmatrix} 0 \\ 13{,}889 \\ 29{,}664 \\ 41{,}290 \\ 52{,}487 \\ 61{,}676 \\ 68{,}402 \\ 72{,}472 \\ 73{,}839 \end{bmatrix}$$

$t_7 = 12{,}95$

$t_7 - \Delta t = 11{,}1$

$$[X]\,[Ue]_{12,95} = [Y]\,[Ue]_{11,1} = \begin{bmatrix} 103{,}360 \\ 131{,}915 \\ 84{,}385 \\ 325{,}588 \\ 199{,}405 \\ 466{,}241 \\ 261{,}915 \\ 549{,}962 \\ 141{,}734 \end{bmatrix} \Rightarrow [Ue]_{12,95} = \begin{bmatrix} 0 \\ 13{,}655 \\ 24{,}906 \\ 37{,}562 \\ 47{,}558 \\ 55{,}812 \\ 61{,}972 \\ 65{,}975 \\ 67{,}057 \end{bmatrix}$$

$t_8 = 14{,}8$

$t_8 - \Delta t = 12{,}95$

$$[X]\,[Ue]_{14,8} = [Y]\,[Ue]_{12,95} = \begin{bmatrix} 108{,}773 \\ 82{,}724 \\ 118{,}938 \\ 271{,}156 \\ 184{,}858 \\ 423{,}399 \\ 237{,}563 \\ 498{,}393 \\ 128{,}516 \end{bmatrix} \Rightarrow [Ue]_{14,8} = \begin{bmatrix} 0 \\ 11{,}515 \\ 24{,}025 \\ 33{,}653 \\ 43{,}077 \\ 50{,}616 \\ 56{,}197 \\ 59{,}628 \\ 60{,}791 \end{bmatrix}$$

$t_9 = 16{,}65$

$t_9 - \Delta t = 14{,}8$

$$[X]\,[Ue]_{16,65} = [Y]\,[Ue]_{14,8} = \begin{bmatrix} 86{,}648 \\ 103{,}095 \\ 72{,}666 \\ 266{,}974 \\ 161{,}482 \\ 383{,}331 \\ 215{,}397 \\ 452{,}008 \\ 116{,}472 \end{bmatrix} \Rightarrow [Ue]_{16,65} = \begin{bmatrix} 0 \\ 11{,}044 \\ 20{,}525 \\ 30{,}808 \\ 38{,}916 \\ 45{,}810 \\ 50{,}917 \\ 54{,}050 \\ 55{,}105 \end{bmatrix}$$

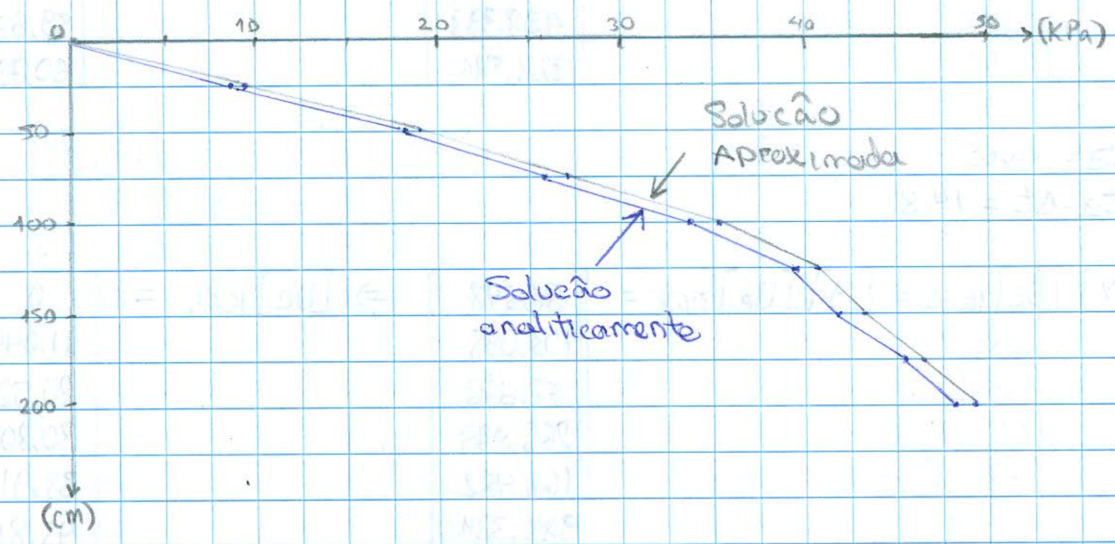
$t_{10} = 18{,}5$

$t_{10} - \Delta t = 16{,}65$

$$[X]\,[Ue]_{18,5} = [Y]\,[Ue]_{16,65} = \begin{bmatrix} 87{,}333 \\ 71{,}112 \\ 94{,}531 \\ 222{,}474 \\ 153{,}009 \\ 346{,}769 \\ 195{,}021 \\ 409{,}480 \\ 105{,}609 \end{bmatrix} \Rightarrow [Ue]_{18,5} = \begin{bmatrix} 0 \\ 9{,}526 \\ 19{,}592 \\ 27{,}591 \\ 35{,}405 \\ 41{,}537 \\ 46{,}134 \\ 48{,}974 \\ 49{,}937 \end{bmatrix}$$

Comparando resultados com os valores da tabela abaixo. no tempo t = 18,5 dias

Z (cm)	0	25	50	75	100	125	150	175	200
Ue (analiticamente)	0	9,27	18,18	26,40	33,59	39,50	43,88	46,58	47,50
Ue (aproximada)	0	9,52	19,59	27,59	35,40	41,53	46,13	48,97	49,93



Comparando resultados com a solução gráfica T = 0,40

H = 200 $\quad$ Uo = 100 KPa

z	Z	Ue	Uz
0	0	0	1
25	0,125	9,526	0,904
50	0,25	19,592	0,804
75	0,375	27,591	0,724
100	0,5	35,405	0,625
125	0,625	41,537	0,584
150	0,75	46,134	0,538
175	0,875	48,974	0,510
200	1	49,937	0,500

T = 0,4

Z	Uz
0	1
0,2	0,86
0,35	0,76
0,45	0,7
0,6	0,62
0,7	0,58
0,85	0,54
1	0,53