LISTA DE EXERCICIOS Nº 04

Pede-se determinar pelo método dos elementos finitos, considerando elementos T3, as cargas hidráulicas e velocidades de fluxo nos pontos nodais da malha da figura. Admitir um coeficiente de permeabilidade isotrópico do solo de fundação k = 1 x 10-4 cm/s.

Para efeitos de coordenadas dos nós, considerar a origem dos eixos cartesianos no nó inferior esquerdo. Condições de contorno em relação ao NR na interface solo-rocha impermeável:

a) Linha equipotencial máxima h = 13m.
b) Contorno vertical esquerdo, admitindo ausência de fluxo h = 13m.
c) Linha equipotencial sob o centro da barragem, devido à simetria do problema h = 10,5m.

Os elementos T3 da malha são todos iguais, representados por triângulos retângulos com catetos de 4m.

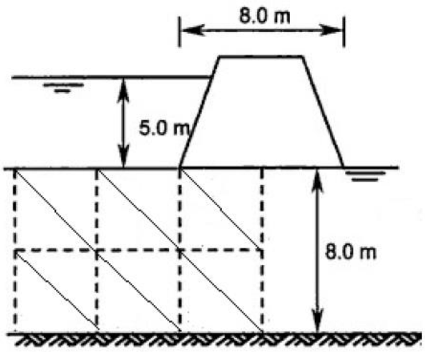


Figura 01. - Barragem de concreto e malha de elementos finitos T3 em metade da geometria do problema.

Solução.

1. **Divisão o continuo em elementos finitos 2D.**

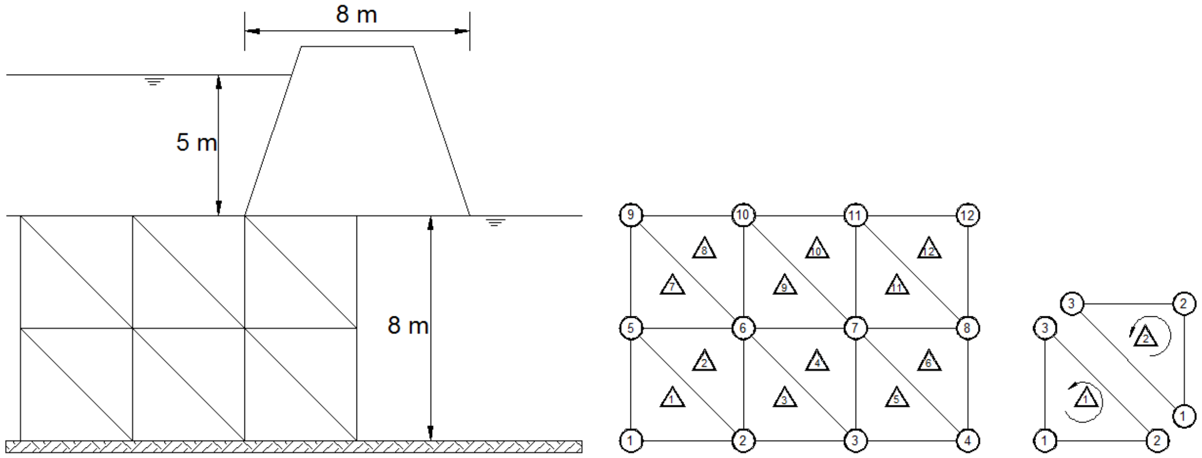


Figura 02- Esquematização do sistema em coordenadas globais e locais.

2. **Formulação das propriedades de cada elemento.**

$$\Omega = \frac{1}{2}\int_A \{q\}^T [B]^T [R][B]\{q\} dA - \int_A \overline{Q}\{q\}^T [N]^T dA - \int_{S1} \overline{q}\{q\}^T [N]^T dS$$

$$\partial\Omega = \frac{\partial\Omega}{\partial\{q\}}\partial\{q\} = 0$$

$$\frac{\partial\Omega}{\partial\{q\}} = \int_A [B]^T [R][B]\{q\} dA - \int_A \overline{Q}[N]^T dA - \int_{S1} \overline{q}[N]^T dS = 0$$

$$\frac{\partial\Omega}{\partial\{q\}} = \left(\int_A [B]^T [R][B] dA\right)\{q\} = \int_A \overline{Q}[N]^T dA + \int_{S1} \overline{q}[N]^T dS$$

Como

$$[k]\{q\} = \{Q\}$$

Então

$$[k] = \int_A [B]^T [R][B] dA = [B]^T [R][B] A$$

$$\{Q\} = \int_A \overline{Q}[N]^T dA + \int_{S1} \overline{q}[N]^T dS$$

Onde

$$[B] = \frac{1}{2A}\begin{bmatrix} y_{23} & y_{31} & y_{12} \\ x_{32} & x_{13} & x_{21} \end{bmatrix}$$

Então

$$[B]_I = \frac{1}{2A}\begin{bmatrix} -4 & 4 & 0 \\ -4 & 0 & 4 \end{bmatrix} \text{ e } [B]_{II} = \frac{1}{2A}\begin{bmatrix} 0 & 4 & -4 \\ -4 & 4 & 0 \end{bmatrix}$$

$$[R] = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

Para o solo isotrópico de permeabilidade k

$$[R]=\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}=k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

a. Calculando a matriz $[k]$ para os elementos 1, 3, 5, 7, 9 e 11.

$$[k]_I = A[B]_I^T[R][B]_I = A\left(\frac{1}{2A}\begin{bmatrix} -4 & -4 \\ 4 & 0 \\ 0 & 4 \end{bmatrix}\right)\left(k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\left(\frac{1}{2A}\begin{bmatrix} -4 & 4 & 0 \\ -4 & 0 & 4 \end{bmatrix}\right)$$

$$[k]_I = \frac{4k}{A}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

b. Calculando a matriz $[k]$ para os elementos 2, 4, 6, 8, 10 e 12.

$$[k]_{II} = A[B]_{II}^T[R][B]_{II} = A\left(\frac{1}{2A}\begin{bmatrix} 0 & -4 \\ 4 & 4 \\ -4 & 0 \end{bmatrix}\right)\left(k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\left(\frac{1}{2A}\begin{bmatrix} 0 & 4 & -4 \\ -4 & 4 & 0 \end{bmatrix}\right)$$

$$[k]_{II} = \frac{4k}{A}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

c. Calculando a matriz $[Q]$ para os elementos.

Como não tem fluxos nodais prescritos então

$$\{Q\}=0$$

Então.

$$[k]\{q\}=0$$

3. Montagem da matriz de rigidez global [K]

GRAU DE LIBERDADE												
GLOBAL	**LOCAL**											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	--	--	--	--	--	--	--	--	--	--	--
2	2	1	1	--	--	--	--	--	--	--	--	--
3	--	--	2	1	1	--	--	--	--	--	--	--
4	--	--	--	--	2	1	--	--	--	--	--	--
5	3	3	--	--	--	--	1	--	--	--	--	--
6	--	2	3	3	--	--	2	1	1	--	--	--
7	--	--	--	2	3	3	--	--	2	1	1	--
8	--	--	--	--	--	2	--	--	--	--	2	1
9	--	--	--	--	--	--	3	3	--	--	--	--
10	--	--	--	--	--	--	--	2	3	3	--	--
11	--	--	--	--	--	--	--	--	--	2	3	3
12	--	--	--	--	--	--	--	--	--	--	--	2

$$
[k]=\begin{bmatrix}
k_{11}^{1} & k_{12}^{1} & 0 & 0 & k_{13}^{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
k_{21}^{1} & k_{22}^{1}+k_{11}^{2}+k_{11}^{3} & k_{12}^{3} & 0 & k_{23}^{1}+k_{13}^{2} & k_{12}^{2}+k_{13}^{3} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & k_{21}^{3} & k_{22}^{3}+k_{11}^{4}+k_{11}^{5} & k_{12}^{5} & 0 & k_{23}^{2}+k_{13}^{4} & k_{12}^{4}+k_{13}^{5} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & k_{21}^{5} & k_{22}^{5}+k_{11}^{6} & 0 & 0 & k_{23}^{5}+k_{13}^{6} & k_{12}^{6} & 0 & 0 & 0 & 0\\
k_{31}^{1} & k_{32}^{1}+k_{31}^{2} & 0 & 0 & k_{33}^{1}+k_{33}^{2}+k_{11}^{7} & k_{32}^{2}+k_{12}^{7} & 0 & 0 & k_{13}^{7} & 0 & 0 & 0\\
0 & k_{21}^{2}+k_{31}^{3} & k_{22}^{3}+k_{31}^{4} & 0 & k_{23}^{2}+k_{21}^{7} & k_{22}^{2}+k_{33}^{3}+k_{33}^{4}+k_{22}^{7}+k_{11}^{8}+k_{11}^{9} & k_{32}^{4}+k_{12}^{9} & 0 & k_{23}^{7}+k_{13}^{8} & k_{12}^{9}+k_{13}^{9} & 0 & 0\\
0 & 0 & k_{21}^{4}+k_{31}^{5} & k_{32}^{5}+k_{31}^{6} & 0 & k_{23}^{4}+k_{21}^{9} & k_{22}^{4}+k_{33}^{5}+k_{33}^{6}+k_{22}^{9}+k_{11}^{10}+k_{11}^{11} & k_{32}^{6}+k_{12}^{11} & 0 & k_{23}^{9}+k_{13}^{10} & k_{12}^{10}+k_{13}^{11} & 0\\
0 & 0 & 0 & k_{21}^{6} & 0 & 0 & k_{23}^{6}+k_{21}^{11} & k_{22}^{6}+k_{22}^{11}+k_{11}^{12} & 0 & 0 & k_{23}^{11}+k_{13}^{12} & k_{12}^{12}\\
0 & 0 & 0 & 0 & k_{31}^{7} & k_{32}^{7}+k_{31}^{8} & 0 & 0 & k_{33}^{7}+k_{33}^{8} & k_{32}^{8} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & k_{21}^{8}+k_{31}^{9} & k_{32}^{9}+k_{21}^{10} & 0 & k_{23}^{8} & k_{22}^{8}+k_{33}^{9}+k_{33}^{10} & k_{32}^{10} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & k_{21}^{10}+k_{31}^{11} & k_{32}^{11}+k_{31}^{12} & 0 & k_{23}^{10} & k_{22}^{10}+k_{33}^{11}+k_{33}^{12} & k_{32}^{12}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{21}^{12} & 0 & 0 & k_{23}^{12} & k_{22}^{12}
\end{bmatrix}
$$

$$
[k]=\frac{4k}{A}\begin{bmatrix}
2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-1 & 4 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 4 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
-1 & 0 & 0 & 0 & 4 & -2 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & -2 & 0 & 0 & -2 & 8 & -2 & 0 & 0 & -2 & 0 & 0\\
0 & 0 & -2 & 0 & 0 & -2 & 8 & -2 & 0 & 0 & -2 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & -2 & 4 & 0 & 0 & 0 & -1\\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -1 & 4 & -1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -1 & 4 & -1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 2
\end{bmatrix}
$$

4. Aplicar os carregamentos conhecidos.

Então.

$$[k]\{q\}=0$$

$$\frac{4k}{A}\begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 4 & -2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & -2 & 8 & -2 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & -2 & 8 & -2 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -2 & 4 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 2 \end{bmatrix} x \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \\ h_{11} \\ h_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

5. Condições de contorno.

a) Linha equipotencial máxima h = 13m, então.

$$h_9 = h_{10} = h_{11} = 13m$$

b) Contorno vertical esquerdo, admitindo ausência de fluxo h = 13m, então.

$$h_1 = h_5 = h_9 = 13m$$

c) Linha equipotencial sob o centro da barragem, devido à simetria do problema h = 10,5m, então.

$$h_4 = h_8 = h_{12} = 10.5m$$

$$\frac{4k}{A}\begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 4 & -2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & -2 & 8 & -2 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 & -2 & 8 & -2 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & -2 & 4 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 2 \end{bmatrix} x \begin{Bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \\ h_{11} \\ h_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\frac{4k}{A}\begin{bmatrix} 4 & -1 & -2 & 0 \\ -1 & 4 & 0 & -2 \\ -2 & 0 & 8 & -2 \\ 0 & -2 & -2 & 8 \end{bmatrix} x \begin{Bmatrix} h_2 \\ h_3 \\ h_6 \\ h_7 \end{Bmatrix} = \frac{4k}{A}\begin{Bmatrix} 0-(-1)h_1-(0)h_4-(0)h_5-(0)h_8-(0)h_9-(0)h_{10}-(0)h_{11}-(0)h_{12} \\ 0-(0)h_1-(-1)h_4-(0)h_5-(0)h_8-(0)h_9-(0)h_{10}-(0)h_{11}-(0)h_{12} \\ 0-(0)h_1-(0)h_4-(-2)h_5-(0)h_8-(0)h_9-(-2)h_{10}-(0)h_{11}-(0)h_{12} \\ 0-(0)h_1-(0)h_4-(0)h_5-(-2)h_8-(0)h_9-(0)h_{10}-(-2)h_{11}-(0)h_{12} \end{Bmatrix}$$

$$\frac{4k}{A}\begin{bmatrix} 4 & -1 & -2 & 0 \\ -1 & 4 & 0 & -2 \\ -2 & 0 & 8 & -2 \\ 0 & -2 & -2 & 8 \end{bmatrix} x \begin{Bmatrix} h_2 \\ h_3 \\ h_6 \\ h_7 \end{Bmatrix} = \frac{4k}{A}\begin{Bmatrix} 0-(-1)h_1 \\ 0-(-1)h_4 \\ 0-(-2)h_5-(-2)h_{10} \\ 0-(-2)h_8-(-2)h_{11} \end{Bmatrix} = \frac{4k}{A}\begin{Bmatrix} 0-(-1)(13) \\ 0-(-1)(10.5) \\ 0-(-2)(13)-(-2)(13) \\ 0-(-2)(10.5)-(-2)(13) \end{Bmatrix}$$

$$\begin{bmatrix} 4 & -1 & -2 & 0 \\ -1 & 4 & 0 & -2 \\ -2 & 0 & 8 & -2 \\ 0 & -2 & -2 & 8 \end{bmatrix} x \begin{Bmatrix} h_2 \\ h_3 \\ h_6 \\ h_7 \end{Bmatrix} == \begin{Bmatrix} 13 \\ 10.5 \\ 52 \\ 47 \end{Bmatrix}$$

6. Resolver o sistema de equações

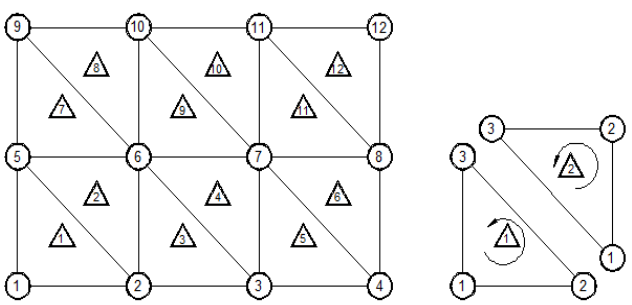
$$\begin{Bmatrix} h_2 \\ h_3 \\ h_6 \\ h_7 \end{Bmatrix} == \begin{Bmatrix} 12.488 \\ 11.727 \\ 12.612 \\ 11.960 \end{Bmatrix} m$$

7. Calculo das quantidades secundarias (velocidade de fluxo)

$$\{g\}=[B]\{q\}$$
$$\{v\}=-[R][B]\{q\}=-k[B]\{q\}$$

A área e igual a $8m^2$ então,

$$[B]_I = \frac{1}{4}\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \text{ e } [B]_{II} = \frac{1}{4}\begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$



Elemento 1.

$$\{g\}=\frac{1}{4}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{Bmatrix}13\\ 12.488\\ 13\end{Bmatrix}=\begin{Bmatrix}-0.128\\ 0\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}1.28*10^{-7}\\ 0\end{Bmatrix}\frac{m}{seg}$$

Elemento 2.

$$\{g\}=\frac{1}{4}\begin{bmatrix}0 & 1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{Bmatrix}12.488\\ 12.612\\ 13\end{Bmatrix}=\begin{Bmatrix}-0.097\\ 0.031\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}9.70*10^{-8}\\ -3.10*10^{-8}\end{Bmatrix}\frac{m}{seg}$$

Elemento 3.

$$\{g\}=\frac{1}{4}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{Bmatrix}12.488\\ 11.727\\ 12.612\end{Bmatrix}=\begin{Bmatrix}-0.190\\ 0.031\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}1.90*10^{-7}\\ -3.10*10^{-8}\end{Bmatrix}\frac{m}{seg}$$

Elemento 4.

$$\{g\}=\frac{1}{4}\begin{bmatrix}0 & 1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{Bmatrix}11.727\\ 11.960\\ 12.612\end{Bmatrix}=\begin{Bmatrix}-0.163\\ 0.058\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}1.63*10^{-7}\\ -5.83*10^{-8}\end{Bmatrix}\frac{m}{seg}$$

Elemento 5.

$$\{g\}=\frac{1}{4}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{Bmatrix}11.727\\ 10.50\\ 11.960\end{Bmatrix}=\begin{Bmatrix}-0.307\\ 0.058\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}3.07*10^{-7}\\ -5.83*10^{-8}\end{Bmatrix}\frac{m}{seg}$$

Elemento 6.

$$\{g\}=\frac{1}{4}\begin{bmatrix}0 & 1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{Bmatrix}10.5\\ 10.5\\ 11.960\end{Bmatrix}=\begin{Bmatrix}-0.365\\ 0\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}3.65*10^{-7}\\ 0\end{Bmatrix}\frac{m}{seg}$$

Elemento 7.

$$\{g\}=\frac{1}{4}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{Bmatrix}13\\ 12.612\\ 13\end{Bmatrix}=\begin{Bmatrix}-0.097\\ 0\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}9.70*10^{-8}\\ 0\end{Bmatrix}\frac{m}{seg}$$

Elemento 8.

$$\{g\}=\frac{1}{4}\begin{bmatrix}0 & 1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{Bmatrix}12.612\\ 13\\ 13\end{Bmatrix}=\begin{Bmatrix}0\\ 0.097\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}0\\ -9.70*10^{-8}\end{Bmatrix}\frac{m}{seg}$$

Elemento 9.

$$\{g\}=\frac{1}{4}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{Bmatrix}12.612\\ 11.960\\ 13\end{Bmatrix}=\begin{Bmatrix}-0.163\\ 0.097\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}1.63*10^{-7}\\ -9.70*10^{-8}\end{Bmatrix}\frac{m}{seg}$$

Elemento 10.

$$\{g\}=\frac{1}{4}\begin{bmatrix}0 & 1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{Bmatrix}11.960\\ 13\\ 13\end{Bmatrix}=\begin{Bmatrix}0\\ 0.260\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}0\\ -2.60*10^{-7}\end{Bmatrix}\frac{m}{seg}$$

Elemento 11.

$$\{g\}=\frac{1}{4}\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}\begin{Bmatrix}11.960\\ 10.5\\ 13\end{Bmatrix}=\begin{Bmatrix}-0.365\\ 0.260\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}3.65*10^{-7}\\ -2.60*10^{-7}\end{Bmatrix}\frac{m}{seg}$$

Elemento 12.

$$\{g\}=\frac{1}{4}\begin{bmatrix}0 & 1 & -1\\ -1 & 1 & 0\end{bmatrix}\begin{Bmatrix}10.5\\ 10.5\\ 13\end{Bmatrix}=\begin{Bmatrix}-0.625\\ 0\end{Bmatrix}\Rightarrow v=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}6.25*10^{-7}\\ 0\end{Bmatrix}\frac{m}{seg}$$

Então graficamente.

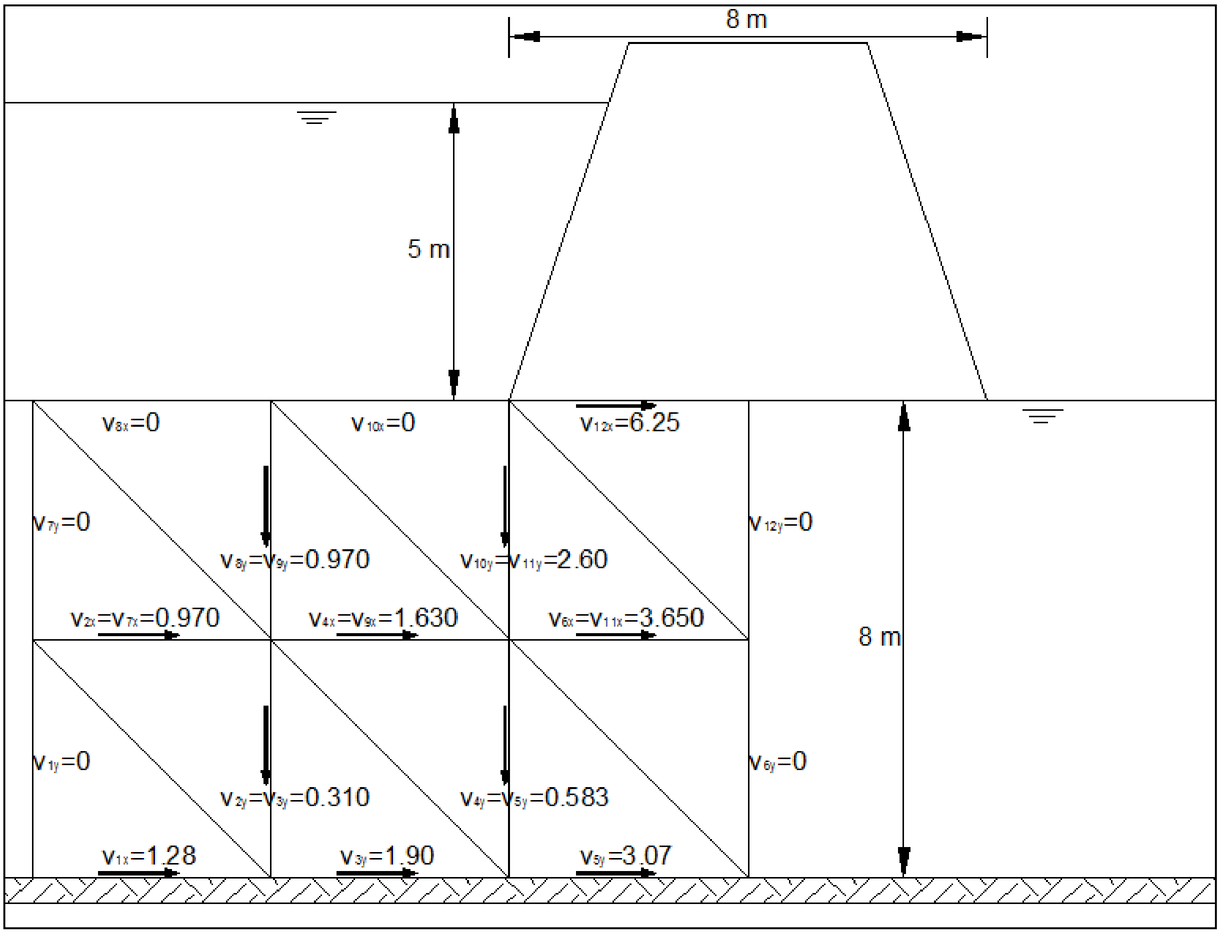


Figura 03- Esquematização das velocidades em (10^{-7}) m/seg.