

5(a) For three body problem.

$$\ddot{\mathbf{r}}_1 = -GM \left(\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3} \right)$$

$$\ddot{\mathbf{r}}_2 = -GM \left(\frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3} - \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3} \right)$$

$$\ddot{\mathbf{r}}_3 = -GM \left(\frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - \frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3} \right)$$

If we apply $\frac{1}{GM} \frac{\partial \mathbf{R}}{\partial \mathbf{r}_i} = \frac{\partial^2 \mathbf{r}_i}{\partial t^2}$, then we have

$$\ddot{\mathbf{r}}_1 = -\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3} \quad \ddot{\mathbf{r}}_2 = -\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3} - \frac{\mathbf{r}_2 - \mathbf{r}_3}{|\mathbf{r}_2 - \mathbf{r}_3|^3}$$

$$\ddot{\mathbf{r}}_3 = -\frac{\mathbf{r}_3 - \mathbf{r}_1}{|\mathbf{r}_3 - \mathbf{r}_1|^3} - \frac{\mathbf{r}_3 - \mathbf{r}_2}{|\mathbf{r}_3 - \mathbf{r}_2|^3}$$

Velocity of 1 km/s :

$$\frac{1}{GM} \cdot R = r$$

$$v = \frac{R}{t} = 1 \text{ km/s}$$

$$\frac{r \cdot GM}{t} = 10^3 \text{ m/s} \Rightarrow \frac{r}{t} = v = \frac{10^3}{GM}$$

$$5(b). \quad \ddot{\mathbf{r}}_1 = -\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} - \frac{\mathbf{r}_1 - \mathbf{r}_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3}$$

We have $\mathbf{r}_1 = (0, 0)$ $\mathbf{r}_2 = -\mathbf{r}_3$

$$\ddot{\mathbf{r}}_1 = -\frac{(-\mathbf{r}_2)}{|\mathbf{r}_2|^3} - \frac{(-\mathbf{r}_3)}{|\mathbf{r}_3|^3}$$

$$= \frac{\mathbf{r}_2}{|\mathbf{r}_2|^3} + \frac{\mathbf{r}_3}{|\mathbf{r}_3|^3}, \text{ since } \mathbf{r}_2 = -\mathbf{r}_3$$

$$\frac{\mathbf{r}_2}{|\mathbf{r}_2|^3} + \frac{\mathbf{r}_3}{|\mathbf{r}_3|^3} = 0 \Rightarrow \ddot{\mathbf{r}}_1 = 0.$$

thus, it is remain origin for all time.