1.

1. Using Euler method for pendulum. $\frac{d\omega}{dt} = \alpha(\theta)$, $\frac{d\theta}{dt} = \omega$

$\theta_{n+1} = \theta_n + \tau\omega_n$

$\omega_{n+1} = \omega_n + \tau\alpha(\theta_n)$

$E_{n+1} = \frac{mL^2}{2}\omega_{n+1}^2 + \frac{mgL}{2}\theta_{n+1}^2 - mgL$

$$\Delta E = E_{n+1} - E_n = \frac{mL^2}{2}[(\omega_n + \tau\alpha(\theta_n))^2 - \omega_n^2] + \frac{mgL}{2}[(\theta_n + \tau\omega_n)^2 - \theta_n^2]$$

$$= \frac{mL^2}{2}(\tau^2\alpha^2(\theta_n) + 2\omega_n\tau\alpha(\theta_n)] + \frac{mgL}{2}[\tau^2\omega_n^2 + 2\theta_n\tau\omega_n]$$

Since we have $\alpha(\theta) = -(g/L)\theta$; $E = -mgL\cos\theta_m$ (maximum angle).

$\Delta E = mg\tau^2(\cos\theta - \cos\theta_m)$ $\theta < \theta_m$, $\cos\theta > \cos\theta_m \rightarrow \Delta E > 0$.

Thus, E increase with time.

2.

2(a)

increase with time.

2. $x(\tau) = x_0 \cos(\tau) + \frac{v_0}{\omega}\sin(\tau)$ $\ddot{x}(\tau) = -x(\tau)$

$\tau = \omega t$, $d\tau = \omega dt$. $\frac{d\tau^2}{dt^2} = \omega^2$ $\ddot{x}(t) = \frac{d^2x}{d\tau^2} \cdot \frac{d\tau^2}{dt^2} = -\omega^2 x$.

$v = \frac{dx(\tau)}{dt} = -x_0 \sin(\tau) + \frac{v_0}{\omega}\cos(\tau)$, for $t=0$, $\tau=0$,

$v(0) = \frac{v_0}{\omega}$ (Initial velocity).

3(a) (b)

$v = \frac{dx}{dt} = -x_0 \sin(\tau) + \frac{\quad}{\omega}\cos(\tau)$, for $t=0$, $\tau=0$,

$v(0) = \frac{v_0}{\omega}$ (Initial velocity).

3. (a). $\cancel{\frac{\partial u}{\partial t} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial t} = \frac{\partial[v(x-ct)]}{\partial t} = -c}$

$y = x - ct \qquad \frac{\partial u}{\partial t} = \frac{\partial u}{\partial v}\cdot\frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial t} = (x-ct)\cdot v'(y)\cdot(-c)$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial v}\cdot\frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial x} = (x-ct)\cdot v'(y)$$

$-cv'(y) + y6v(y)v'(y) + y^2 v'''(y) = 0$, Thus, can reduce to ODE.

$$-cv'(y) + 6v(y)v'(y) + v'''(y) = 0$$

(b) $\cancel{\frac{\partial v}{\partial y} = \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial y} = c^{\frac{3}{2}} w(z)}$. $\quad \frac{\partial v}{\partial y} = \frac{\partial v}{\partial w}\cdot\frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial y} = c\cdot w(z)\cdot c^{-\frac{1}{2}}$

$$= c^{\frac{1}{2}} w'(z)$$

$$-cv'(y) + 6v(y)v'(y) + v'''(y) = 0$$

$$= -c^{\frac{3}{2}} w'(z) + 6c^{\frac{3}{2}} w(z)w'(z) + c^{\frac{3}{2}} w'''(z) = 0$$

$$= \boxed{-w'(z) + 6w(z)w'(z) + w'''(z) = 0} \Rightarrow [6w(z) - 1]w'(z) + w'''(z) = 0$$