1.

(a) $U_x = \alpha Re(1-y^2)/2$, $U_y = U_z = 0$

$$\dot{u} = \left(\frac{\partial U_x}{\partial t}, \frac{\partial U_y}{\partial t}, \frac{\partial U_z}{\partial t}\right) = (0, 0, 0)$$

$$(\vec{U}\cdot\nabla)\vec{u} = (\nabla\cdot\vec{u})\cdot\vec{u}$$

$$\vec{\nabla}\cdot\vec{u} = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z} = 0$$

$$(\vec{u}\cdot\nabla)\vec{u} = (0, 0, 0)$$

$$-\nabla p = -\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right) = (\alpha, 0, 0)$$

$$\nabla^2 u = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(U_x, U_y, U_z)$$

$$\frac{\partial^2 U_x}{\partial y^2} = -\alpha Re. \Rightarrow \nabla^2 u = (-\alpha Re, 0, 0)$$

$$Re^{-1}\nabla^2 u = (-\alpha, 0, 0)$$

Thus, $\dot{\vec{u}} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\nabla p + Re^{-1}\nabla^2\vec{u}$.

Satisfy the N-S Equation.

For Boundary Condition.

when $y = \pm 1$, $u = \alpha Re(1-(\pm 1)^2)/2 = 0$.

Satisfy the boundary condition.

b) $\vec{\omega} = \nabla \times \vec{u} = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}, \frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}, \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right)$

$= (0, 0, \omega)$

$$\omega = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}.$$

$$\nabla^2 \omega = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$$

$$\dot{\omega} = \frac{\partial}{\partial t}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = \frac{\partial^2 u_y}{\partial x \partial t} - \frac{\partial^2 u_x}{\partial y \partial t}.$$

c) No,