1. a) assume $\rho(x,t) = T(t)X(x)$

$$\frac{T'}{DT} = \frac{X''}{X} + \frac{C}{D} = -k.$$

Then, we have $T_k = e^{-Dkt}$

$X'' = -(k + \frac{C}{D})X$. $X(x) = \sin(\frac{n\pi}{L}x)$

and, $k = (\frac{n\pi}{L})^2 - C/D$

For Large time, $n \to \infty$, $k > 0$, $t \to \infty$.

Thus, $T(t) = e^{-Dkt} \approx 0$.

Then, $\rho(x,t) = T(t) \cdot X(x) = 0$, So there is no critical mass

(b) $$\frac{\partial \rho}{\partial t} = D \cdot \frac{\partial^2 \rho}{\partial x^2} + C\rho$$

If we assume $H = D \cdot \frac{\partial^2}{\partial x^2} + C$.

$$\frac{\partial \rho}{\partial t} = H \cdot \rho \Rightarrow \frac{\rho_j^{n+1} - \rho_j^n}{\tau} = H \cdot \rho^{n+1}.$$

If we apply C-N algorithm.

$$\frac{\rho_j^{n+1} - \rho_j^n}{\tau} = \frac{1}{2} H (\rho_j^n + \rho_j^{n+1}).$$

$$(1 - \frac{\tau}{2}H)\rho^{n+1} = (1 + \frac{\tau}{2}H)\rho^n$$

$$\boxed{\rho^{n+1} = (\mathbb{I} - \frac{\tau}{2}H)^{-1}(\mathbb{I} + \frac{\tau}{2}H)\rho^n.}$$

Boundary condition : $\rho(L/2, t) = \rho(-L/2, t)$
$= 0$.