a) For polar coordinates. $r=\sqrt{x^2+y^2}$ $\theta = \tan^{-1}\left(\frac{x}{y}\right)$.

$$\frac{dr}{dt} = \frac{dr}{dx}\cdot\frac{dx}{dt} + \frac{dr}{dy}\cdot\frac{dy}{dt}$$

$$= x(x^2+y^2)^{-\frac{1}{2}}(ax+y-x(x^2+y^2)) + y(x^2+y^2)^{-\frac{1}{2}}(-x+ay-y(x^2+y^2))$$

$$= a(x^2+y^2)^{\frac{1}{2}} - (x^2+y^2)^{\frac{3}{2}} = \boxed{ar-r^3}$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dx}\cdot\frac{dx}{dt} + \frac{d\theta}{dy}\cdot\frac{dy}{dt}$$

$$= \frac{1}{(1+(\frac{x}{y})^2}\cdot\frac{1}{y}\cdot(ax+y-x(x^2+y^2)) + \frac{1}{(1+(\frac{x}{y})^2}\cdot\left(\frac{-x}{y^2}\right)\cdot(-ax+ay-y(x^2+y^2))$$

$$= \left(\frac{1}{1+(\frac{x}{y})^2}\cdot\left(1+\frac{x^2}{y^2}\right)\right) = 1$$

Then we have, $\boxed{\frac{dr}{dt} = ar-r^3 \quad \frac{d\theta}{dt} = 1}$

for $a<0$ $\frac{dr}{dt}<0$. $r_{n+1} < r_n$, thus they fall into origin.

for $a>0$, We have $r(a-r^2)=0$

$a=r^2 \Rightarrow r=\sqrt{a}$.

2 a) $L = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & & \lambda_n \end{bmatrix}$

Since $M\vec{v_i} = \lambda\vec{v_i}$, then $\sum_{i=1}^{n} M\vec{v_i} = \sum_{i=1} \lambda\vec{v_i}$

it can be expressed in Matrix

$$M\begin{bmatrix} (v_1)_1 & v_2 & v_3 \cdots v_n \\ (v_1)_n & \cdots & \cdots \end{bmatrix} = \begin{bmatrix} (v_1)_1 & v_2 & v_3 \cdots v_n \\ (v_1)_2 & & \\ (v_1)_n & \cdots & \end{bmatrix}\begin{bmatrix} \lambda_1 & & 0 \\ 0 & \lambda_2 & \\ 0 & & \ddots \lambda_n \end{bmatrix}$$

$MV = VL$

If we multiply both sides by V^{-1}

$MVV^{-1} = VLV^{-1} \Rightarrow M = VLV^{-1}$

b) for $n=2$ $M^2 = VLV^{-1}\cdot VLV^{-1} = VL^2V^{-1}$.

for n, $M^n = VL^nV^{-1}$

and the eigvalues of $M^n = \lambda_i^n$

$M^n b = VL^nV^{-1}b = (VL^n)\cdot(V^{-1}b)$

assume $V^{-1}b = c$

$$VL^n = [v_1 \cdots v_n]\begin{bmatrix} \lambda_1^n & & 0 \\ & \lambda_2^n & \\ 0 & & \ddots \lambda_n \end{bmatrix}$$

$$VL^n c = [v_1 \cdots v_2 \cdots v_n]\begin{bmatrix} \lambda_1^n & & 0 \\ & \lambda_2^n & \\ 0 & & \ddots \lambda_n \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

$$M^n b = [v_1 \cdots v_2 \cdots v_n]\begin{bmatrix} c_1\lambda_1^n \\ \vdots \\ c_n\lambda_n^n \end{bmatrix}$$

$$= \boxed{c_1\lambda_1^n v_1 + c_2\lambda_2^n v_2 \cdots c_n\lambda_n^n v_n}.$$

This equation can be rewrite as.

$$M^n b = C_m \lambda_{max}^n \left[\frac{C_1}{C_m} \left(\frac{\lambda_1}{\lambda_{max}}\right)^n v_1 + \cdots\cdots \frac{C_n}{C_m} \left(\frac{\lambda_n}{\lambda_{max}}\right)^n v_N \right]$$

Sin $\frac{\lambda_i}{\lambda_{max}} < 1$ $\quad \lim_{n\to\infty} \left(\frac{\lambda_i}{\lambda_{max}}\right)^n \approx 0.$

Thus, $\lim_{n\to\infty} M^n b = C_m \lambda_{max}^n v_{max} \propto v_{max}.$