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Project 2: Racing Shells

Math Modeling
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2.3.5

Problem: If you have been to a rowing regatta, you may have observed that the more oarsmen there are in a boat, the faster the boat travels. Investigate whether there is a mathematical relationship between the speed of a boat and the number of crew members. Consider the following assumptions (partial list) in formulating a model:

1. The total force exerted by the crew is constant for a particular crew throughout the race.
2. The drag force experienced by the boat as it moves through the water is proportional to the square of the velocity times the wet surface area of the hull.
3. Work is defined as force times distance. Power is defined as work per unit ti

Hint: When additional oarsmen are added to a shell, it is not obvious whether the amount of force is proportional to the number in the crew or the amount of power is proportional to the number in the crew. Which assumption appears the most reasonable? Which yields a more accurate model?

Data

Crew	Distance (m)	Race 1 (sec)	Race 2 (sec)	Race 3 (sec)	Race 4 (sec)	Race 5 (sec)	Race 6 (sec)
1	2500	20:53	22:21	22:49	26:52		
2	2500	19:11	19:17	20:02			
4	2500	16:05	16:42	16:43	16:47	16:51	17:25
8	2500	9:19	9:29	9:49	9:51	10:21	10:33

Table 1: Taken from text (distance m, time min:sec)

Assumptions

We are assuming the total force of the crew is constant, meaning each member will exert the same amount of force (power) in each race. We will also be assuming that the total force is proportional to the amount of power exerted by each member. It is also important to note that in our calculations we are not considering a change in weather, water resistance, wind resistance, and tiredness. All of which are assumed to be uniform throughout each race. We will assume that each crew member weigh (weight density) the same.

Method

From the stated problem, we know that power is work per unit time, work is force times distance, and the forward force is $k \cdot v^2 \cdot SA_{hull}$. Putting all of this together, we get:

power=$\frac{\text{work}}{\text{time}}$

power $= \frac{\text{force * distance}}{\text{time}}$

power $= \frac{k \cdot (\text{distance}^2/\text{time}^2) \cdot SA_{hull} \cdot \text{distance}}{\text{time}}$

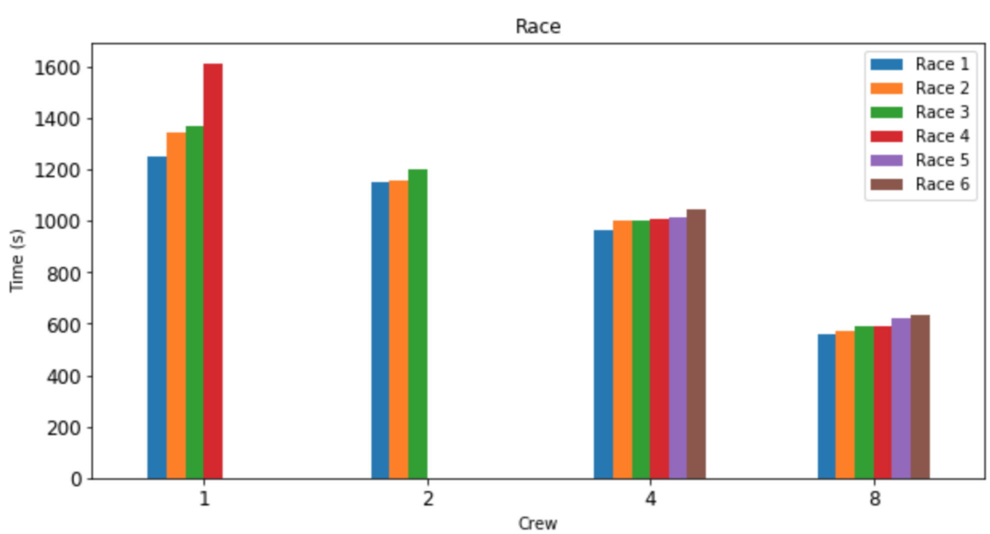
power $= \frac{k \cdot \text{distance}^3 \cdot SA_{hull}}{\text{time}^3}$

Some new assumptions we need to note are: k will be equal to 1 because we have limiting data to actually calculate for k. SA_{hull} will amount to the number of crew. The surface area of the boat requires data on the measurement of the boat which we don't have. We would also need to consider the buoyancy, density, etc of the water; however, with little data, we will use what is provided. Note, this will most likely amount to a not-so-accurate realistic analysis due to the limited data.

Analysis

General data analysis

Thus, we will concentrate the focus of our analysis on crew numbers. To begin, I created a bar chart of the table above, examining only the races, crew numbers, and times.

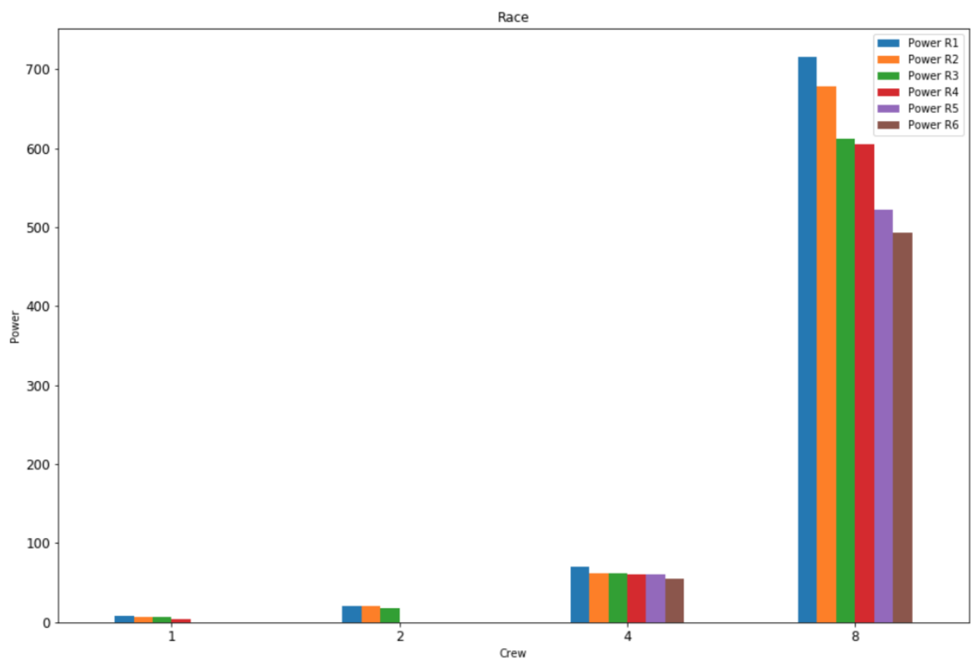


It is obvious that as the crew members increase the race times decrease tremendously. The race times of all numbers of crews seem to be quite close except for races with only one crew.

Analysis of power

The following table and graph represents the power model we created above with the appropriate data:

	Crew	Power R1	Power R2	Power R3	Power R4	Power R5	Power R6
1	1	7.94268	6.47938	6.08989	3.73014	NaN	NaN
2	2	20.4939	20.1767	17.9944	NaN	NaN	NaN
3	4	69.5502	62.1265	61.9409	61.2057	60.4821	54.7685
4	8	715.607	678.536	611.736	605.547	521.958	492.832

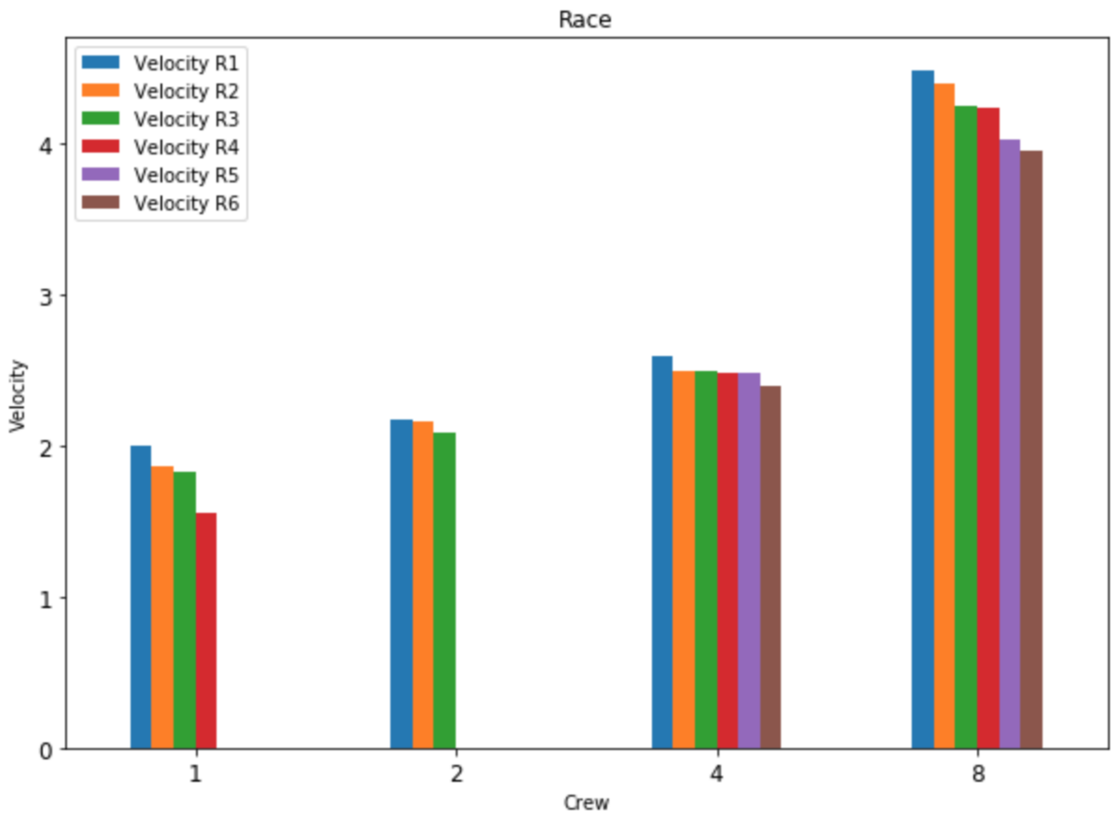


Logically, this makes sense. The more crew members there are, the more power is exerted on the boat.

Analysis of velocity

The following table and graph represents the velocity (m/s) of our data.

	Crew	Velocity R1	Velocity R2	Velocity R3	Velocity R4	Velocity R5	Velocity R6
1	1	1.99521	1.86428	1.82615	1.55087	NaN	NaN
2	2	2.17202	2.16076	2.07987	NaN	NaN	NaN
3	4	2.59067	2.49501	2.49252	2.48262	2.4728	2.39234
4	8	4.47227	4.39367	4.24448	4.23012	4.02576	3.94945



Again, this makes sense. The more crew members there are, the faster the boat will go.

Conclusion

From the analysis of power and velocity based on the number of crew members, we see an increase similarity between power and velocity. However, it is not a strong correlation. There is an approximately 88% jump from a race with one crew member vs. a race with 8 crew members. However, the velocity of the boats according to the number of crew members have an increase of about 2.5%. Our goal was to examine the correlation between speed and number of crew members. It is evident that there is a correlation. We see that the more crew members there are, the faster the boat goes.

Limitations

I do want to reiterate the fact that we are very limited on data. We would have a better overall conclusion if we had more data on the boat size, buoyancy of the water, how much power each individual exerts, and lastly consideration of fatigueness.