3. Exercise Sheet – Brain-Inspired Computing (WS 15/16)

Due date 2.11.16.

Name(s): Group: Points: ___/___/___/___

3.1 (30 Points)

a) Show that the equations

$$F_u + G_w < 0,$$
$$F_u \cdot G_w - F_w \cdot G_u > 0$$

define the necessary and sufficient stability conditions for the two-dimensional system of differential equations defined in the lecture.

b) For the FitzHugh-Nagumo model defined by

$$F = u - u^3/3 - w + I,$$
$$G = \varepsilon \cdot (a + b \cdot u - w)$$

use the values $\varepsilon = 0.1$, $a = 15/8$, $b = 3/2$, and calculate whether stable fixed points exist for $I = 0$ and $I = 15/8$.

Hint: You may use cubic equation solvers.

3.2 Piecewise linear nullclines (40 Points)

The piecewise linear FitzHugh-Nagumo model is defined as:

$$\frac{du}{dt} = f(u) - w + I,$$
$$\frac{dw}{dt} = \varepsilon(bu - w)$$

with $f(u) = au$ for $u < 0.5$, $f(u) = a(1 - u)$ for $0.5 < u < 1.5$ and $f(u) = c_0 + c_1 u$ for $u > 1.5$, where $a, c_1 < 0$ are parameters and $c_0 = -0.5u - 1.5c_1$. Furthermore, $b > 0$ and $0 < \varepsilon \ll 1$.

a) Draw on the phase plane a schematic representation of the nullclines and flow for the model with parameters $a = c_1 = -1$ and $b = 2$, and mark the stable fixed point, suppose $I = 0$.

b) A hyperpolarizing current is introduced very slowly and increased up to a maximal value of $I = -2$. Calculate the new value of the stable fixed point. Draw the nullclines and flow for $I = -2$ on a different phase plane.

c) The hyperpolarizing current is suddenly removed. Use the phase planes in a) and b) to find out what will happen. Draw schematically the evolution of the neuron's state as a membrane potential time-series and as a trajectory in the phase plane. Use $\varepsilon = 0.1$.

Hint: the resting state from b) is the initial value of the trajectory in c).

3.3 Exploring the FitzHugh-Nagumo model (30 Points)

Use the Forward Euler algorithm from Exercise 2.3 to integrate the FitzHugh-Nagumo model equations

$$\dot{u} = u - \frac{u^3}{3} - w + I,$$
$$\dot{w} = \varepsilon\,(a + bu - w)$$

with $\varepsilon = 0.1$, $a = 15/8$, $b = 3/2$.

a) Measure and plot the activation function (spiking rate ν as function of input current I). You need to find a definition for "spike" and write a detection function.

b) Plot the nullclines and an example trajectory for

- some point of the activation function where $\nu = 0$,
- another point where $\nu \neq 0$.

Choose an initial condition for the trajectories not too far away from both nullclines.