1.1 The power of the brain

a) Average energy consumption in human brain per action potential:

$$E = \frac{P_{Brain}}{N_{Neu}\nu} = 2 \cdot 10^{-10}\,J$$

b) Average energy consumption in human brain per synaptic event:

$$E = \frac{P_{Brain}}{N_{Syn}\nu} = 2 \cdot 10^{-14}\,J$$

c) Energy consumption per spike and synaptic event in simulation:

$$E_{spike} = \frac{P_K t}{N_{Neu,K}} = 24.6J$$

$$E_{syn} = \frac{P_K t}{N_{Neu,K} \cdot N_{partner}} = 0.004J$$

d) Scale up of simulations:

$$P_{scale} = 100 \cdot P_K = 1.26GW$$

e) Power after speed up the K computer:

$$P_{speed} = \frac{t_{simul}}{t_{real}} P_{scale} = 3024GW$$

The nuclear power plant Sedai has two reactors, each with 890 MW output.

$$N_{PowerPlants} = \frac{P_{Speed}}{P_{NPP}} = 1700$$

1.2 Stimulus Currents

Simplified equivalent circuit:

$$C_m \frac{du}{dt} = g_l(u - E_l) + I_{ext}$$

a) Dirac delta function: $I(t) = I_0 \cdot \frac{C_m}{g_l}\delta(t - t_0)$

$$C_m \frac{du}{dt} = g_l(u - E_l) + I_0 \cdot \frac{C_m}{g_l}\delta(t - t_0)$$

Look for solution of homogenous equation:

$$C_m \frac{du}{dt} = g_l u$$

$$\frac{du}{u} = \frac{g_l}{C_m} dt \Rightarrow u = De^{\frac{g_l}{C_m}t}$$

$D \to D(t)$:

$$C_m \dot{u} = C_m \dot{D}(t) e^{\frac{g_l}{C_m}t} + D(t) g_l e^{\frac{g_l}{C_m}t} = g_l D(t) e^{\frac{g_l}{C_m}t} - g_l E_l + I_0 \frac{C_m}{g_l}\delta(t - t_0)$$

$$\frac{dD}{dt} = e^{-\frac{g_l}{C_m}t}\left(\frac{I_0}{g_l}\delta(t - t_0) - \frac{g_l}{C_m}E_l\right)$$

$$D(t) - D_0 = \int e^{-\frac{g_l}{C_m}t}\frac{I_0}{g_l}\delta(t - t_0)dt - \int e^{-\frac{g_l}{C_m}t}\frac{g_l}{C_m}E_l dt = \frac{I_0}{g_l}e^{-\frac{g_l}{C_m}t_0}\theta(t - t_0) + E_l e^{-\frac{g_l}{C_m}t}$$

Putting everything together yields:

$$u(t) = \left(D_0 + \frac{I_0}{g_l}e^{-\frac{g_l}{C_m}t_0}\theta(t - t_0) + E_l e^{-\frac{g_l}{C_m}t}\right)e^{\frac{g_l}{C_m}t} = D_0 e^{\frac{g_l}{C_m}t} + \frac{I_0}{g_l}e^{\frac{g_l}{C_m}(t - t_0)}\theta(t - t_0) + E_l$$

b) Step current: $I(t) = I_0 \cdot \Theta(t - t_0)$

The homogenous solution is the same as before, so:

$$D - D_0 = \frac{1}{C_m}\int e^{-\frac{g_l}{C_m}t} I_0\Theta(t-t_0)dt - \int e^{-\frac{g_l}{C_m}t}\frac{g_l}{C_m}E_l dt$$

$$= \frac{\frac{I_0}{C_m}C_m}{g_l}\theta(t-t_0)\left(e^{-\frac{g_l}{C_m}t_0} - e^{-\frac{g_l}{C_m}t}\right) + E_l e^{-\frac{g_l}{C_m}t}$$

Putting everything together yields:

$$u(t) = D_0 e^{\frac{g_l}{C_m}t} + E_l + \frac{I_0}{g_l}\theta(t-t_0)(e^{\frac{g_l}{C_m}(t-t_0)} - 1)$$

c) Exponential current: $I(t) = I_0 \cdot \Theta(t-t_0)\cdot \frac{C_m}{\tau_s g_l}\cdot \exp\left(-\frac{t-t_0}{\tau_s}\right)$

Using the same as before:

$$D - D_0 = \frac{1}{C_m}\int e^{-\frac{g_l}{C_m}t} I_0\Theta(t-t_0)\frac{C_m}{\tau_s g_l}\exp\left(-\frac{t-t_0}{\tau_s}\right)dt - \int e^{-\frac{g_l}{C_m}t}\frac{g_l}{C_m}E_l dt$$

$$= \frac{1}{\tau_s g_l}e^{\frac{t_0}{\tau_s}}I_0\int_{t_0}^{\infty}\exp\left(-\left(\frac{g_l}{C_m}+\frac{1}{\tau_s}\right)t\right)dt + E_l e^{-\frac{g_l}{C_m}t}$$

$$= \frac{1}{\tau_s g_l}e^{\frac{t_0}{\tau_s}}I_0\frac{1}{\frac{g_l}{C_m}+\frac{1}{\tau_s}}\left(\exp\left(-\left(\frac{g_l}{C_m}+\frac{1}{\tau_s}\right)t_0\right) - \exp\left(-\left(\frac{g_l}{C_m}+\frac{1}{\tau_s}\right)t\right)\right)\theta(t - t_0) + E_l e^{-\frac{g_l}{C_m}t}$$

$$= \frac{I_0}{g_l}\frac{1}{\frac{g_l\tau_s}{C_m}+1}\left(\exp\left(-\frac{g_l}{C_m}t_0\right) - \exp\left(-\left(\frac{g_l}{C_m}+\frac{1}{\tau_s}\right)t\right)\right)\theta(t-t_0) + E_l e^{-\frac{g_l}{C_m}t}$$

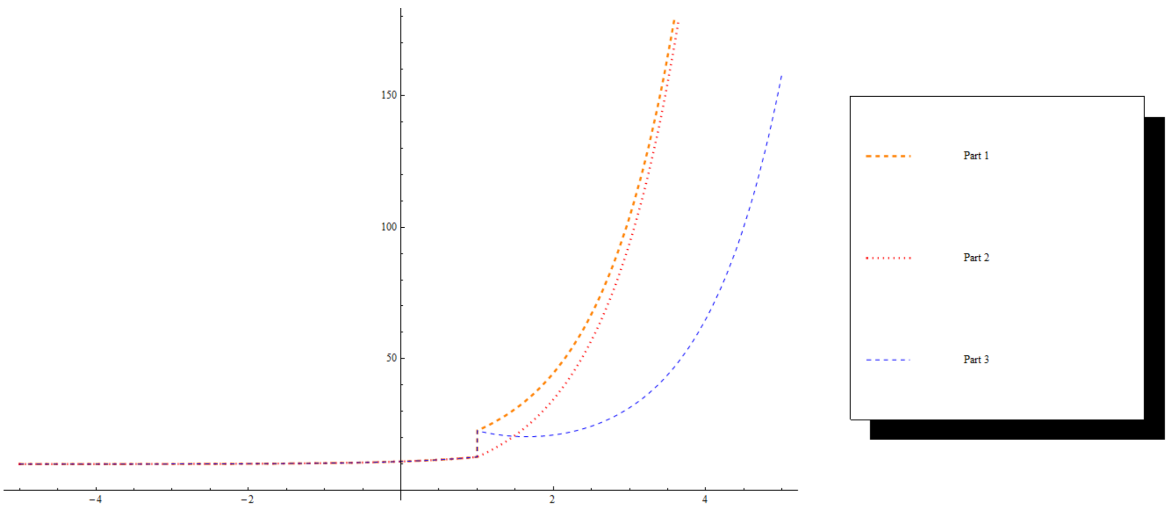
Putting everything together:

$$u(t) = E_l + D_0 e^{\frac{g_l}{C_m}t} + \frac{I_0}{g_l}\frac{1}{\frac{\tau_s}{C_m}+1}\theta(t-t_0)\left(e^{-\frac{g_l}{C_m}(t-t_0)} - \exp\left(-\frac{t}{\tau_s}\right)\right)$$

d) In the limit of very short synaptic time constants $\tau_s \to 0$ we get:

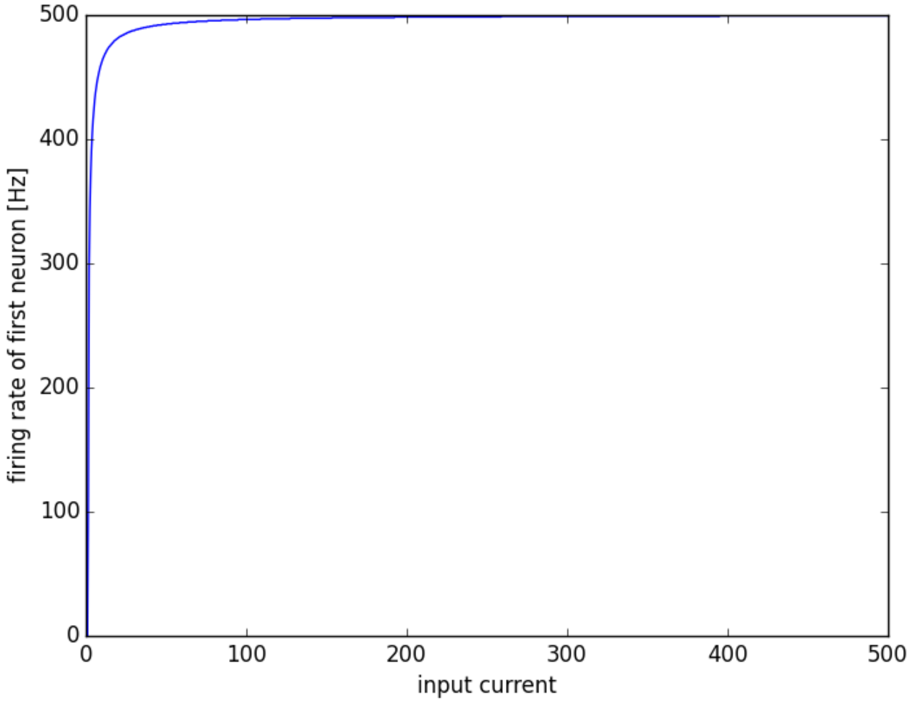
$$\lim_{\tau_s\to 0} u(t) = E_l + D_0 e^{\frac{g_l}{C_m}t} + \theta(t-t_0)\left(\frac{I_0}{g_l}e^{-\frac{g_l}{C_m}(t-t_0)}\lim_{\tau_s\to 0}\frac{1}{\frac{\tau_s}{C_m}+1} - \frac{I_0}{g_l}\lim_{\tau_s\to 0}\frac{1}{\frac{\tau_s}{C_m}+1}\exp\left(-\frac{t}{\tau_s}\right)\right)$$

$$= E_l + D_0 e^{\frac{g_l}{C_m}t} + \theta(t-t_0)\left(\frac{I_0}{g_l}e^{-\frac{g_l}{C_m}(t-t_0)} - \frac{I_0}{g_l}\right)$$



1.3 Facets Live!

Two-neuron demo: output firing rate of first neuron as a function if its input current:



Firing rate of second neuron as a function of its input firing rate: