5. Exercise Sheet – Brain-Inspired Computing (WS 15/16)

Due date 16.11.16.

Name(s): Group: Points: ___/___/___/___

5.1 Generation of Poisson spike trains (35 Points)

a) In the lecture, it was discussed that the rate of a stochastic point process is equal to the inverse of the mean inter-event interval. Prove that this holds in the particular case of the Poisson process using its exponential ISI distribution given in the lecture.

b) Additionally, prove that for the Poisson process, the following relationship holds:

$$\mathbb{E}[N] = \mathrm{Var}[N] = \lambda\,\Delta t \tag{1}$$

where N is the number of spikes in a time window Δt and λ is the average firing rate.

c) You are given `numpy.random.rand`, a pseudo-random number generator producing numbers uniformly distributed in the interval $[0, 1)$. You can use it in the following way:

```python
import numpy.random as npr
# provide a consistent seed to make experiments repeatable
npr.seed(42)
# draw a single random number
print npr.rand()
# draw an array of random numbers
print npr.rand(100)
```

Using no other source of randomness, your task is to implement your own Poisson generator.

1) A very crude approach is to go to very small time bins:

$$p(N > 0, \mathrm{d}t) = 1 - p(N = 0, \mathrm{d}t) = 1 - e^{-\lambda \mathrm{d}t} \approx 1 - (1 - \lambda \mathrm{d}t) = \lambda \mathrm{d}t \tag{2}$$

If the time bin size $\mathrm{d}t$ is chosen small enough, at most one spike will fall into each bin.

Implement a Poisson generator based on this simple thresholding problem. Plot the resulting path $N(t)$, as well as the ISI distribution. What can you easily read from $N(t)$? How well does the ISI distribution match the theoretical prediction for varying rates and time bin sizes $\mathrm{d}t$?

2) Instead of computing the probability of a spike occurring in each time bin, you can draw ISIs instead by mapping the CDF (cumulative distribution function) of the uniform distribution on $[0, 1)$ onto the CDF of the ISI distribution: For each random value drawn $\in [0, 1)$, a corresponding ISI can be calculated.

Implement a Poisson generator that generates spike trains in this fashion. Plot the resulting path $N(t)$, as well as the ISI distribution. How well does the ISI distribution match the theoretical prediction for varying rates now?

d) Is there a significant difference in run time between the two implementations?

5.2 High conductance state (35 Points)

The free membrane potential time course of the CUBA LIF neuron has been given in the lecture as:

$$u(t) = E_\mathrm{l} + \frac{I^\mathrm{ext}}{g_\mathrm{l}} + \sum_{\mathrm{syn}\,k} \sum_{\mathrm{spk}\,s} \frac{\tau_k^\mathrm{syn} w_k}{g_\mathrm{l}(\tau_k^\mathrm{syn} - \tau_m)} \Theta(t - t_s) \left[\exp(-\frac{t - t_s}{\tau_k^\mathrm{syn}}) - \exp(-\frac{t - t_s}{\tau_m}) \right], \tag{3}$$

where the sum on the r.h.s is is done over PSPs.

a) Use the expression for the CUBA LIF neuron to derive an approximation of the PSP shape of the COBA LIF neuron in the high-conductance state (HCS). Hint: In the numerator of the PSP, you will have a term that depends on the membrane potential. Replace it with $\langle u_{\text{eff}} \rangle$. Assuming that, in the HCS, $g^{\text{tot}} \approx \langle g^{\text{tot}} \rangle$, find a closed-form expression for $\langle u_{\text{eff}} \rangle$.

b) Calculate the membrane potential time course of the COBA LIF neuron with the PSP shape you obtained from a) under Poisson stimulus. You will need to replace the offset $E_l + I^{\text{ext}}/g_l$ in the CUBA solution with $u_{\text{eff}}{}^0 = \frac{g_l}{\langle g^{\text{tot}} \rangle} \left(\langle u_{\text{eff}} \rangle - E_l - \frac{I^{\text{ext}}}{g_l} \right)$. Compare your result with a PyNN simulation under different rates of Poisson sources. You can express the quality of the analytic approximation by computing the distance (L2 norm) between the calculated and the simulated membrane potential. Use the following neuron parameters:

c_{m}	$0.1\,\text{nF}$
τ_{m}	$20\,\text{ms}$
V_{rest}	$-50\,\text{mV}$
$E_{\text{e}}^{\text{rev}}$	$0\,\text{mV}$
$E_{\text{i}}^{\text{rev}}$	$-100\,\text{mV}$
ϑ	$70\,\text{mV}$
τ^{syn}	$10\,\text{ms}$
I^{ext}	$0\,\text{nA}$

Use $\text{dt} = 0.01\,\text{ms}$ or lower (why?), a synaptic weight of $w = 0.01\,\mu\text{S}$ and the total Poisson rates $\nu_{\text{exc}} = \nu_{\text{inh}}$ (balanced excitation and inhibition) from 10 to 5000 Hz for the L2 norm simulation!

5.3 The AdEx model (30 Points)

a) Prove that ExLIF (AdEx without the Ad) converges to LIF in the limit of $\Delta_{\text{T}} \to 0$.

b) Prove that for $u > \theta$, the latter being the minimum voltage needed for the neuron to spike, u diverges *in finite time.*

Hints:

- Prove that for any starting value $u_0 > \theta$, any finite value $u' > \theta$ is reached in finite time.
- For large enough u', the linear term in the ExLIF ODE can be neglected. Use this to find a closed-form solution for the ExLIF ODE in this regime. Use this solution to prove that u goes from u' to ∞ in finite time.