

2. Subset Component

You are given an array with n 64-bit integers: $d[0], d[1], \dots, d[n-1]$.

$\text{BIT}(x, i) = (x \gg i) \& 1$, where $B(x, i)$ is the i^{th} lower bit of x in binary form. If we regard every bit as a vertex of a graph G , there is an undirected edge between vertices i and j if there is a value k such that $\text{BIT}(d[k], i) == 1 \ \&\& \ \text{BIT}(d[k], j) == 1$.

For every subset of the input array, how many [connected-components](#) are there in that graph?

A connected component in a graph is a set of nodes which are accessible to each other via a path of edges. There may be multiple connected components in a graph.

Example

$d = \{1, 2, 3, 5\}$

In the real challenge, there will be 64 nodes associated with each integer in d represented as a 64 bit binary value. For clarity, only 4 bits will be shown in the example but all 64 will be considered in the calculations.

Decimal	Binary	Edges	Node ends
$d[0] = 1$	0001	0	
$d[1] = 2$	0010	0	
$d[2] = 3$	0011	1	0 and 1
$d[3] = 5$	0101	1	0 and 2

Consider all subsets:

Subset	Edges		Connected components
	Count	Nodes	
{1}	0		64
{2}	0		64
{3}	1	0-1	63
{5}	1	0-2	63
{1, 2}	0		64
{1, 3}	1	0-1	63
{1, 5}	1	0-2	63
{2, 3}	1	0-1	63
{2, 5}	1	0-2	63
{3, 5}	2	0-1-2	62
{1, 2, 3}	1	0-1	63
{1, 2, 5}	1	0-2	63
{1, 3, 5}	2	0-1-2	62
{2, 3, 5}	2	0-1-2	62
{1, 2, 3, 5}	2	0-1-2	62
Sum			944

The values 3 and 5 have 2 bits set, so they have 1 edge each. If a subset contains only a 3 or 5, there will be one connected component with 2 nodes, and 62 components with 1 node for a total of 63.

If both 3 and 5 are in a subset, 1 component with nodes 0, 1 and 2 is formed since node 0 is one end of each edge described. The other 61 nodes are solitary, so there are 62 connected components total.

All other values have only 1 bit set, so they have no edges. They have 64 components with 1 node each.

Function Description

Complete the `findConnectedComponents` function in the editor below.

`findConnectedComponents` has the following parameters:

- `int d[n]`: an array of integers

Returns

- `int`: the sum of the number of connected components for all subsets of d

Input Format

The first row contains the integer n , the size of d .

The next row has n space-separated integers, $d[i]$.

Constraints

$1 \leq n \leq 20$

$0 \leq d[i] \leq 2^{63} - 1$

Sample Input 0

```
3
2 5 9
```

Sample Output 0

```
504
```

Explanation 0

There are 8 subset of $\{2, 5, 9\}$.

$\{\}$

=> We don't have any number in this subset => no edge in the graph => Every node is a component by itself => Number of connected-components = 64.

$\{2\}$

=> The Binary Representation of 2 is 00000010. There is a bit at only one position. => So there is no edge in the graph, every node is a connected-component by itself => Number of connected-components = 64.

$\{5\}$

=> The Binary Representation of 5 is 00000101. There is a bit at the 0th and 2nd position. => So there is an edge: (0, 2) in the graph => There is one component with a pair of nodes (0,2) in the graph. Apart from that, all remaining 62 vertices are independent components of one node each (1,3,4,5,6...63) => Number of connected-components = 63.

$\{9\}$

=> The Binary Representation of 9 is 00001001. => There is a 1-bit at the 0th and 3rd position in this binary representation. => edge: (0, 3) in the graph => Number of components = 63

$\{2, 5\}$

=> This will contain the edge (0, 2) in the graph which will form one component

=> Other nodes are all independent components

=> Number of connected-component = 63

$\{2, 9\}$

=> This has edge (0,3) in the graph

=> Similar to examples above, this has 63 connected components

$\{5, 9\}$

=> This has edges (0, 2) and (0, 3) in the graph

=> Similar to examples above, this has 62 connected components

$\{2, 5, 9\}$

=> This has edges(0, 2) (0, 3) in the graph. All three vertices (0,2,3) make one component => Other 61 vertices are all independent components

=> Number of connected-components = 62

$S = 64 + 64 + 63 + 63 + 63 + 63 + 62 + 62 = 504$