A High-Performance Reduced-Complexity GMSK Demodulator

Naofal Al-Dhahir and Gary Saulnier

***Abstract*—A four-state adaptive maximum-likelihood sequence estimation (MLSE) Gaussian minimum-shift keying (GMSK) demodulator for modulation $(BT = 0.3)$ on AWGN channels is analyzed and simulated. This demodulator uses the linear representation of GMSK signals and achieves near-optimum BER performance. The channel-impulse response used in the MLSE demodulator is initialized to the highest energy component in the linear representation, and then adapted in a decision-directed mode to offset any performance losses incurred by initially ignoring other lower energy (and time-varying) components in the linear representation. The number of MLSE states is reduced to two, at about 0.1-dB performance loss, by implementing a whitening matched filter which concentrates most of the GMSK pulse energy in its two leading samples.**

***Index Terms*—Gaussian minimum-shift keying, linear approximation, Viterbi algorithm.**

I. Introduction

GAUSSIAN minimum-shift keying (GMSK) has been adopted as the digital modulation scheme for the European global system for mobile communications (GSM) standard due to its spectral efficiency and constant-envelope property [7], [8]. These two characteristics result in superior performance in the presence of adjacent channel interference (ACI) and nonlinear amplifiers.

Since a GMSK signal is obtained from an MSK signal by prefiltering it with a narrow-band Gaussian filter, it can likewise be interpreted as a special case of continuous-phase frequency-shift keying (CPFSK) with a modulation index of 0.5 or as filtered offset quadrature phase-shift keying (OQPSK). Therefore, a GMSK signal can be demodulated either differentially or coherently [2] depending on the performance/complexity requirements. In this letter, we consider the problem of designing a low-complexity GMSK demodulator for a satellite communication system where the constraints on satellite power, antenna size, and information bit rate are such that the available E_b/N_0 is between 1 and 3 dB. The GMSK demodulator is followed by powerful decoders that reduce the high channel bit-error rates (BER) to levels acceptable for reliable voice and data communications.

The Gaussian prefilter in GMSK modulation introduces intersymbol interference (ISI) that spreads over several bit intervals, thus degrading performance from MSK when coherent symbol-by-symbol detection is used [7]. Maximum-likelihood sequence estimation (MLSE) using the Viterbi algorithm is well known to achieve optimal performance in the presence of ISI [4]. The optimal MLSE demodulator for GMSK requires $4(2^{L-1})$ states [2], [8] on AWGN channels, where L is the ISI duration in bit intervals. The presence of severe multipath fading and narrow-band receive filtering (to reduce ACI) further increases the number of states, making implementation complexity prohibitively high.

In this letter, we present an MLSE GMSK demodulator that requires 2^{L-1} states and achieves essentially the same BER performance as MSK. Following [5], we utilize a linear representation of GMSK signals in terms of basic pulse amplitude modulation (PAM) signals, that was derived in [6]. Our linearized MLSE GMSK demodulator has the following several attractive features.

- We use a standard off-the-shelf Viterbi algorithm (VA), that requires 2^{L+1} additions for updating state metrics and 2^L comparisons to select survivor path, with one sample per bit, whereas the VA branch metric computation in [5] is nonstandard and uses 4 samples per bit which implies more computations per bit period.
- We show how to reduce the number of MLSE states to 2, at around 0.1-dB performance loss from MSK, by implementing a whitening matched filter before the Viterbi demodulator. This represents factors of 2 and 8 reduction in complexity over [5] and [8], respectively.
- Our 4-state GMSK demodulator achieves better performance than that of [5] on AWGN channels by using the adaptive decision-directed least mean-square (LMS) algorithm to get a better overall channel impulse response (CIR) estimate taking the effect of other terms in the linear representation into account, without incurring training overhead.
- Because of its linearity, our GMSK demodulator can be readily applied to channels with multipath fading simply by increasing the number of states according to the multipath delay spread.

Other reduced-complexity CPFSK receivers that do not use the linear approximation approach we follow here are described in [2, Chaps. 8, 9] and the references therein.

II. Linear Approximation of GMSK Signal

It was shown in [6] that any binary continuous phase modulation (CPM) signal with a modulation filter duration L can be expressed as the sum of 2^{L-1} PAM signals. For a GMSK signal with $BT = 0.3, L = 3$, hence it can be represented as the sum of four PAM-modulated signals as

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follows:

$$s(t) = \sqrt{\frac{2E_b}{T}} \sum_{k=0}^{3} \sum_{n} j^n a_{k,n} h_k(t - nT)$$
$$\stackrel{\text{def}}{=} \sqrt{\frac{2E_b}{T}} \sum_{k=0}^{3} \sum_{n} b_{k,n} h_k(t - nT) \tag{1}$$

where E_b is the input energy per bit, $j = e^{j(\pi/2)}$ corresponds to a 90° phase shift, $a_{k,n} \in \{\pm 1\}$, and the complex symbols $b_{k,n} \stackrel{\text{def}}{=} j^n a_{k,n}$ belong to the constellation $\{\pm 1, \pm j\}$.

An accurate linear approximation for a GMSK signal (with $BT = 0.3$) can be derived by ignoring the negligible contributions of $h_2(t)$ and $h_3(t)$ in (1). To arrive at this approximation, we shall derive an expression relating $b_{1,n}$ and $b_{0,n}$. Our starting point is the following two expressions for $b_{0,n}$ and $b_{1,n}$ given in [5], (5) and (7):

$$b_{1,n} = b_{0,n-3} \prod_{t \in \{0,2\}} j^{\alpha_{n-t}} = j\alpha_n j^{\alpha_{n-2}} b_{0,n-3} \tag{2}$$

and

$$b_{0,n} = j^{\alpha_n} b_{0,n-1} = j^{\alpha_n} j^{\alpha_{n-1}} j^{\alpha_{n-2}} b_{0,n-3}. \tag{3}$$

Combining (2) and (3), we get

$$b_{1,n} = (j\alpha_n) b_{0,n} (-j\alpha_{n-1})(-j\alpha_n) = -j\alpha_{n-1} b_{0,n}. \tag{4}$$

To delineate the structure of the ISI associated with $h_1(t)$, we relate the information symbols α_{n-1} to the precoded sequence $\{b_0\}$ as follows:

$$\alpha_{n-1} = a_{0,n-1} a_{0,n-2} = \left(\frac{b_{0,n-1}}{j^{n-1}}\right)\left(\frac{b_{0,n-2}}{j^{n-2}}\right).$$

Therefore,

$$-j\alpha_{n-1} = (-1)^{n-1} b_{0,n-1} b_{0,n-2}. \tag{5}$$

From (4) and (5), we get

$$b_{1,n} = (-1)^{n-1} b_{0,n} b_{0,n-1} b_{0,n-2}. \tag{6}$$

We can also express $b_{1,n}$ in terms of the sequence $\{a_0\}$ as follows:

$$b_{1,n} = (j)^{n-1} a_{0,n} a_{0,n-1} a_{0,n-2}. \tag{7}$$

This results in the following representation of a GMSK signal (with $BT = 0.3$)

$$s_k = \sqrt{\frac{2E_b}{N_0}} \sum_n (b_{0,n} h_{0,k-n} + b_{1,n} h_{1,k-n})$$
$$= \sqrt{\frac{2E_b}{N_0}} \sum_n j^n a_{0,n} (h_{0,k-n} - j a_{0,n-1} a_{0,n-2} h_{1,k-n})$$
$$\stackrel{\text{def}}{=} \sqrt{\frac{2E_b}{N_0}} \sum_n j^n a_{0,n} h^n_{eff,k-n} \tag{8}$$

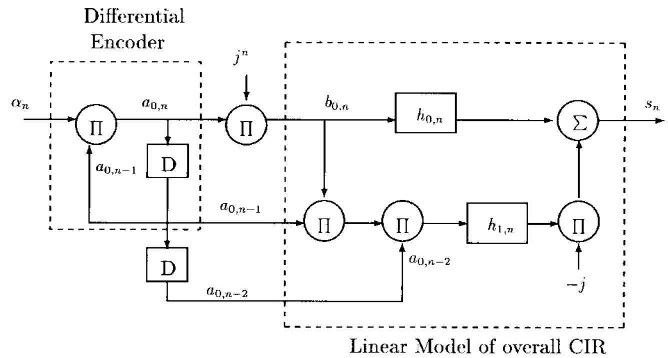

Fig. 1. Linear approximation of GMSK modulator.

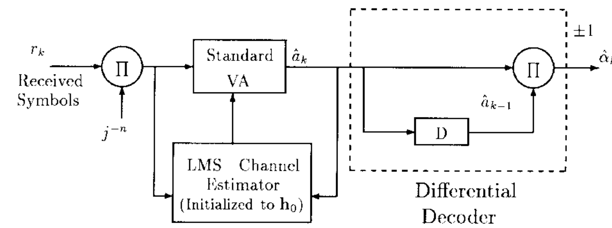

Fig. 2. Proposed GMSK demodulator. The differential encoder can be removed by differentially encoding the GMSK signal, resulting in a factor of 2 improvement in BER.

where $\{h^n_{\text{eff}}\}$ is the equivalent overall CIR in the linear GMSK representation. It is complex-valued and time-varying due to its dependence, at time n, on the encoded symbols $a_{0,n-1}$ and $a_{0,n-2}$. This time variance demonstrates the need for an adaptive channel estimator in a linear GMSK demodulator (even on an AWGN channel) to track the time-varying ISI component due to $h_1(t)$. The linearized GMSK modulator model is depicted in Fig. 1.

III. Linear GMSK Demodulator

The block diagram of the proposed linearized GMSK demodulator is shown in Fig. 2. The Viterbi demodulator is preceded by a *derotation* operation to undo the rotation performed by the GMSK modulator (cf. Fig. 1). For an AWGN channel, a good initial estimate of the overall CIR is given by the T-spaced samples of $h_0(t)$ given by $\boldsymbol{h}_0 \stackrel{\text{def}}{=} [0.2605 \;\; 0.9268 \;\; 0.2605]$. In this letter, we assume no training overhead, therefore, the Viterbi demodulator updates its CIR estimate (from the initial value of $\boldsymbol{h}_0$ to take the effect of $h_1(t)$ into account) using its previous decisions. More precisely, referring to (8), the two previous decisions $\hat{a}_{0,n-1}$ and $\hat{a}_{0,n-2}$ are substituted for $a_{0,n-1}$ and $a_{0,n-2}$ in estimating the overall CIR. Finally, the output (hard) decisions of the Viterbi demodulator are differentially decoded to obtain the estimated information sequence $\hat{\alpha}_n$.

A. Differential Encoding

A factor of two improvement in BER can be achieved by precoding the information sequence before passing it through the GMSK modulator. This allows us to remove the differential decoder at the receiver (cf. Fig. 2) and make decisions based on the output of the VA, $\hat{a}_{0,n}$, directly.

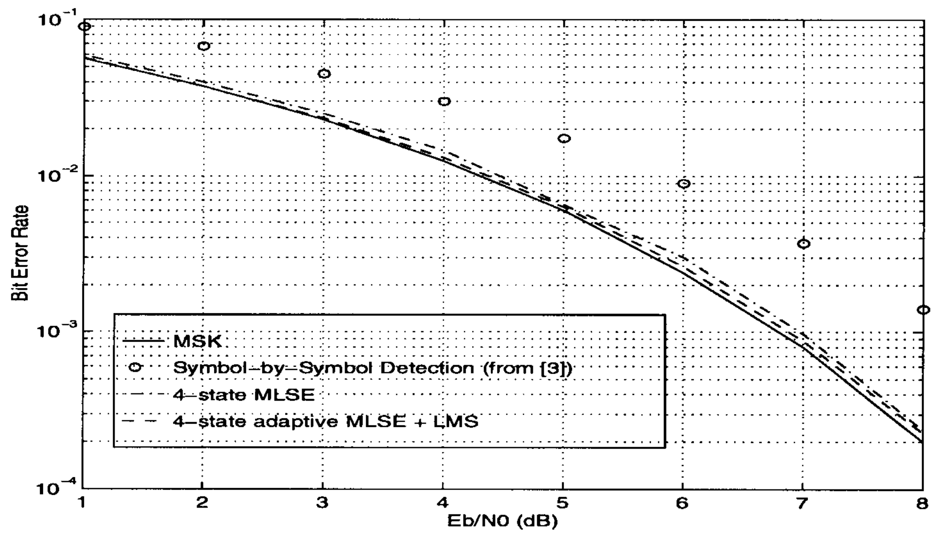

Fig. 3. BER performance comparison between two implementations of the linearized GMSK demodulator (with and without adaptation) and both coherent symbol-by-symbol detection and ideal MSK.

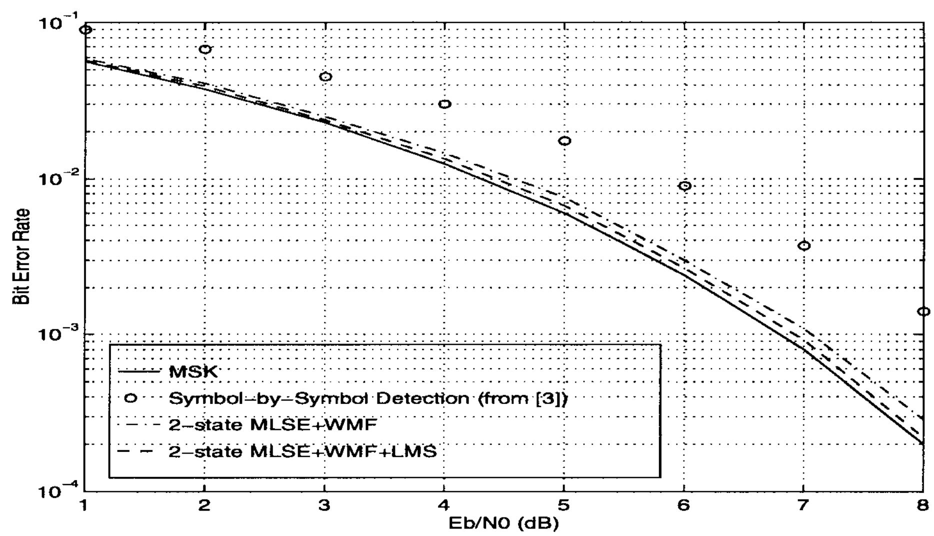

Fig. 4. BER performance comparison between two implementations of the 2-state linearized GMSK demodulator with the whitened matched filter (with and without adaptation) and both coherent symbol-by-symbol detection and ideal MSK.

It was shown in [6] and [5] that

$$\begin{aligned} b_{0,n} &= j^{\Sigma_{k=0}^{n}\alpha_k} = j^{\alpha_n} b_{0,n-1} \\ &= j\alpha_n b_{0,n-1}: \qquad \text{since } \alpha_n \in \{\pm 1\} \\ \Rightarrow \alpha_n &= \Im\{b_{0,n} b_{0,n-1}^*\} \end{aligned} \tag{9}$$

where $\Im(\cdot)$ denotes the imaginary part. This shows the *double-error* characteristic of GMSK modulation. To convert double errors to single errors, the precoding rule $\alpha_n = a_{0,n}a_{0,n-1}$ is employed at the transmitter. Using (9) we have

$$\begin{aligned} b_{0,n} &= ja_{0,n}a_{0,n-1}b_{0,n-1} \\ &= (ja_{0,n}a_{0,n-1})(ja_{0,n-1}a_{0,n-2}b_{0,n-2}) \\ &= (ja_{0,n}a_{0,n-1})(ja_{0,n-1}a_{0,n-2})\cdots(ja_{0,1}a_{0,0}b_{0,0}) \\ &= j^n a_{0,n}a_{0,0}b_{0,0} = \pm j^n a_{0,n}. \end{aligned}$$

Therefore, an estimate of the transmitted data sequence, $\hat{a}_{0,n}$, is available at the output of the Viterbi demodulator, and we can eliminate the differential decoder.

B. Whitening Matched Filter

It was shown in [4] that the maximum-likelihood receiver for any digital amplitude-modulated sequence corrupted by ISI and noise consists of a *whitening matched filter* (WMF) whose samples (taken at the symbol rate) are a sufficient statistic for the VA. The signal at the output of the WMF consists of the input signal filtered by an equivalent canonical (causal, monic, and minimum phase) CIR and corrupted by AWGN. The WMF, denoted by $w^*(D^{-1})$ where D and $(\cdot)^*$ denote unit delay and complex conjugate, respectively, is computed from the *spectral factorization*[1] $h_0(D)h_0^*(D^{-1}) = \gamma_0^2 g(D)g^*(D^{-1})$ as follows [4]:

$$w^*(D^{-1}) = \frac{1}{\gamma_0}\,\frac{h_0^*(D^{-1})}{g^*(D^{-1})} \tag{10}$$

[1] This WMF is time invariant and based on $h_0(t)$ only, i.e., it neglects the effect of $h_1(t)$.

where $h_0(D) \stackrel{\text{def}}{=} \Sigma_k \, h_0(kT)D^k, g(D)$ is the canonical equivalent of $h_0(D)$, and γ_0^2 is a positive scalar. It is clear from (10) that the WMF is an anticausal all-pass filter (phase equalizer) and can be realized with finite delay. For $h_0(D) = 0.2605 + 0.9268D + 0.2605D^2$, it can be readily checked that most of the WMF impulse response energy is concentrated in its first six samples, therefore, we shall use the following 6-tap FIR approximation:

$$\begin{aligned} w^*(D^{-1}) &= 3.2501 \frac{(1+0.3077D)}{(1+3.2501D)} \\ &\approx D^{-5}(0.009 - 0.0264D + 0.0857D^2 \\ &\quad - 0.2786D^3 + 0.9054D^4 + 0.3077D^5) \end{aligned}$$

which is realized with a delay of five bit periods. The overall CIR seen by the Viterbi demodulator is

$$\begin{aligned} &w^*(D^{-1})h_0(D) \\ &\quad = D^5\gamma_0 g(D) = D^5(0.8466 + 0.521D + 0.08D^2). \end{aligned}$$

In the next section, we simulate a 2-state Viterbi demodulator based on the first two samples which contain 99.36% of the overall CIR energy.

IV. Simulation Results

The proposed GMSK demodulator was simulated using SPW© [1] to evaluate its performance. It is worth emphasizing that in our simulations, we used an actual GMSK modulator, not its linear approximation depicted in Fig. 1. First, we simulated the 4-state linearized GMSK demodulator in AWGN under two scenarios. In the first scenario, the CIR estimate was *fixed* at $h_0(t)$, while in the second scenario, it was updated using a decision-directed LMS algorithm with a *fixed* step size of 0.001. As mentioned in the Introduction, the intended operating range for E_b/N_0 is 1–3 dB, however, we present BER results in Fig. 3 for E_b/N_0 up to 8 dB, for the interest of the general readership. For comparison, we have also included BER curves for coherent symbol-by-symbol detection (taken from [3]) and for MSK. The nonadaptive MLSE demodulator exhibits about 0.1–0.2 dB performance loss from MSK since it is based on $h_0(t)$ only. As shown in Fig. 3, this performance loss is eliminated at low E_b/N_0 by implementing an adaptive MLSE demodulator where the CIR estimate takes into account ISI due to $h_1(t)$. The slight performance degradation (less than 0.1 dB) of this adaptive MLSE demodulator from MSK at higher E_b/N_0 is due to the time-varying ISI effect of $h_1(t)$. We have found through simulations that this loss can be reduced by running more iterations of the LMS channel estimator and optimizing its step size. Such a degradation is not present at lower E_b/N_0 levels since the effect of $h_1(t)$ is less significant there.

Fig. 4 shows the simulated BER results of a 2-state adaptive Viterbi demodulator preceded by a WMF. It can be seen that this demodulator exhibits a loss of around 0.1 dB only from MSK, at a factor of 8 reduction in complexity from the GMSK Viterbi demodulator described in [8]. This small performance loss is due to ignoring the effects of $h_1(t)$ and the third tap of $g(D)$ in designing the (fixed) WMF and the (adaptive) MLSE demodulator, respectively.

V. Conclusions

We described and simulated a 4-state symbol-spaced adaptive MLSE GMSK demodulator that achieves near-optimum BER performance on AWGN channels. An accurate linear CIR estimate is derived analytically and used to initialize MLSE branch metric calculations, thus avoiding any training overhead. Performance optimization is achieved by adapting this initial CIR estimate, in a decision-directed mode, to model time-varying linear ISI components of the GMSK signal not accounted for by the initial CIR estimate. Finally, we show that preceding the demodulator by a WMF allows the use of a 2-state adaptive MLSE at a performance loss of around 0.1 dB from MSK.

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