

## Training Sequence Design Based On Channel Estimation

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**Abstract.** Channel estimation is an important research direction in wireless communications; channel estimation based on the training sequence is the most commonly used method. Different training sequence is very different from the performance of channel estimation, in order to improve channel estimation accuracy. This paper analyzes the common m sequence and from the optimal channel estimated performance point design of applicable in any given length of sequence. The sequence, respectively, under the traditional channel estimated algorithm and based on compressed sensing to channel estimation algorithm has the criterion of Minimum Mean Square Error. Meanwhile, the sequence obtained after the demodulated training sequence through different modulation systems are different, so as to have greater flexibility.

### Introduction

In modern mobile communication system, channel is usually not desirable, the transmission process in the presence of fading, and changes in the channel having the uncertainty. The premise of giving full play to the advantages of wireless communication system is able to obtain accurate channel state information, and obtain channel state information need channel estimation techniques to complete. In order to complete the channel estimation accurately, and tracks the change of the radio channel timely, most of the current wireless mobile communication system by means of a pilot symbol. For such channel estimation, the training sequence design is one of the cores, it plays a decisive role in estimation accuracy and implementation complexity. Document[1] pointed out that the performance of channel estimation by the decision of channel correlation matrix, and the training sequence. Thus for a given environment and system, improve the accuracy of channel estimation method is to design the reasonable training sequence as much as possible to improve the accuracy of channel estimation.

Study of sequences with good properties can be traced back to the 1950s, the groundbreaking work is Golomb and Zierler study of m sequence[2]. In 1967, Gold designed Gold sequence, it also has good correlation properties. At present, the m sequence and Gold sequence has been applied to the TD-SCDMA and GSM system. Due to good autocorrelation characteristics of m training sequence which has been widely used, but not from the channel estimation performance quality considerations. This article from the perspective of the optimal channel estimation performance presented a new approach for the design of training sequence. Design process is based on the modulated digital signals, respectively designed three kinds of different distributed sequence of real Numbers: Bernoulli sequence, Gaussian distribution, Uniform distribution of sequences, and applied to channel estimation. Finally, after different modulation system we can obtain new training sequences.

### Common Training Sequence

Based on training sequence channel estimation is made by sending training sequences, repassing unknown wireless channel, arriving at the receiver, the receiver through known training sequence and the receiver signal to estimate the value of channel or direct feedback equalizer. Many practical communication systems are relying on the training sequence to estimate the channel, such as the GSM system, TD-SCDMA system. Since the m-sequence with good correlation properties, it is commonly used as a training sequence.

M sequence is known as the maximum length of linear feedback shift register sequence, assuming that  $m$  is the length of  $L_s = 2^k - 1$ , the sequence has the number of 0 and 1 are  $2^{k-1}$  and  $2^{k-1} - 1$  within a cycle.

Here are 4 linear feedback shift register m sequence generated, as shown in Fig.1.

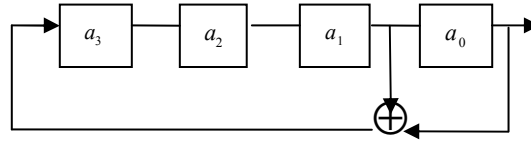


Fig.1. 4 linear feedback shift register

The initial state of the shift register are not all 0, i.e., "0,0,0,0". Otherwise, the shift register will remain unchanged after cyclic shift state, so there are only  $2^{k-1} - 1$  state. For the 4-bit shift register, and the 15 kinds of available states are: "1 0 0 0", "1 1 0 0", "1 1 1 0", "1 1 1 1", "0 1 1 1", "1 0 1 1", "0 1 0 1", "1 0 1 0", "1 1 0 1", "0 1 1 0", "0 0 1 1", "1 0 0 1", "0 1 0 0", "0 0 1 0", "0 0 0 1". Ideally m sequence has good autocorrelation properties, autocorrelation function in a cycle is:

$$C_p(i) = \begin{cases} 1 & i=0 \\ -1/L_s & 1 \leq i \leq L_s - 1 \end{cases} \quad (1)$$

Where  $i$  is the number of M sequence cyclic shift bits.

M sequence through different modulation with digital signal into the channel transmission, in the process of channel estimation, modulation of m sequence after cyclic shift obtain channel matrix  $M$  and Get the channel impulse response estimation based on the channel estimation algorithm. But different training sequence channel estimation precision is also different, based on the above consideration, this article from the perspective of the optimal channel estimation performance design the training sequence.

## New training sequence design

### System model

In order to convenient, only consider noise pollution channel. Channel model[3] as shown in Fig.2, the digital signal  $a$  transmitted in multipath fading channels, noise as an ideal additive white Gaussian noise(AWGN), expressed as  $n$ . The task of the receiver is detected the transmission information  $a$  from the received signal  $y$ . In addition, the detector also need channel vector  $\hat{h}$ , this need by channel estimation algorithm.

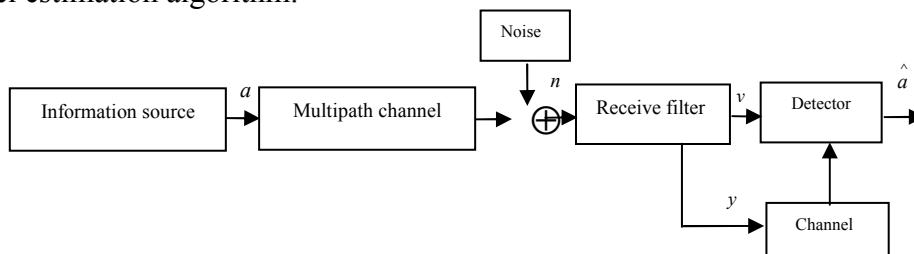


Fig.2. Channel estimation system block diagram

The received signal  $y$  can be represented as:

$$y = Mh + n \quad (2)$$

Where  $h$  is the channel impulse response, expressed as:

$$h = [h_0, h_1, \dots, h_n]^T \quad (3)$$

$n$  is the noise sampling,  $P$  is the reference length,  $L$  is the protection length.

Channel matrix  $M$  can be expressed as:

$$M = \begin{bmatrix} m_L & \cdots & m_1 & m_0 \\ m_{L+1} & \cdots & m_2 & m_1 \\ \vdots & \vdots & \vdots & \vdots \\ m_{L+P-1} & \cdots & m_P & m_{P-1} \end{bmatrix} \quad (4)$$

In each data packet, the transmitter transmits a training sequence. The training sequence is represented as:

$$m = [m_0, m_1, \dots, m_{P+L-1}]^T \quad (5)$$

Least Squares Estimation (LS) channel estimation algorithm is to make the following squared error minimum:

$$\hat{h} = \arg \min \|y - Mh\|^2 \quad (6)$$

Only considered the white Gaussian noise, then the above equation can be expressed as:

$$\hat{h}_{LS} = (M^H M)^{-1} M^H y \quad (7)$$

**The optimal design of sequences of real number based on Channel estimation parameter model.** The Eq.5 and Eq.6 shows that the performance of the channel estimation is determined by the correlation matrix of channel and the using of a training sequence. Clearly, in a certain environment and system, the method of improving accuracy of channel estimation is to design the training sequence which can match the information of current channel. We should reduce channel estimation errors as much as possible. We should determine the channel matrix  $M$  firstly before the algorithm of channel estimation. The matrix  $M$  of the algorithm is a Toeplitz matrix generated by the training sequence based on nature of deconvolution. In common channel estimate based on training sequence, the training sequences we selected are not processed by modulator. In the process of performance comparison with the commonly used m-sequence of Conventional channel estimation, due to the m-sequences usually has two values, in order to maintain the consistency of the variable, here it should be convert into real number sequences through different modulation systems. This article designs a random sequences of real number which obey different distributions, they are Gaussian distribution, Uniform distribution and the Bernoulli distribution.

① The random sequence of Gaussian distribution

It selects random sequence of Gaussian distribution  $\{m_i\}_{i=1}^N$ , and all elements are independent and identically distributed random variables, the average value is 0, the variance is  $1/\sqrt{K}$ .

② The random sequence of Uniformly distribution

It selects random sequence of uniformly distribution  $\{m_i\}_{i=1}^N$ , and all elements are independent and identically distributed random variables, the value of each element is in the range of (-1,1),and it obeys Uniform distribution.

③ The random sequence of Bernoulli distribution

It selects random sequence of Bernoulli distribution  $\{m_i\}_{i=1}^N$ , and all elements are independent and identically distributed random variables, the value of every element is  $+1/\sqrt{K}$  or  $-1/\sqrt{K}$  according to the probability of 0.5, and it obeys Bernoulli distribution.

Every element of the training sequence is one-to-one relationship with  $m$  of the model of the channel estimation, and after the operation of modulator it adds the random sequence which obeys Gaussian distribution, Uniform distribution and the Bernoulli distribution as training sequence to estimate sparse channel, then we compare the estimated performance with the estimated performance of commonly used m sequence in conventional channel estimation.

**Representation of binary sequence.** The three kinds of random sequence of training above all are the modulated sequence of real numbers. New binary training sequences can be generated by different demodulation methods. Here, the three kinds of random sequence generate a new training sequence through the non-coherent demodulating of the GMSK and BPSK.

**GMSK Demodulation.** There are several methods of non-coherent demodulation of GMSK, here we take the method of difference demodulation of 1-bit delay. Suppose the received signal is:

$$s(t) = s_{GMSK}(t) = A(t) \cos[\omega_c t + \theta(t)] \quad (8)$$

Here,  $A(t)$  is time-varying envelope caused by the fading of channel. Receiver divides  $s(t)$  into two ways, one through 1-bit-delay and  $\pi/2$  phase shift, then it multiply with  $s(t)$  of another one, and we get  $x(t)$  :

$$x(t) = s(t)W(t) = A(t)A(t-T_b) \frac{1}{2} \{ \sin[\theta(t) - \theta(t-T_b) + \omega_c T_b] - \sin[2\omega_c t - \omega_c T_b + \theta(t) + \theta(t-T_b)] \} \quad (9)$$

After the processing of low-pass filtered, at the same time taking into account of equation  $\omega_c T_b = 2n\pi$ . We can get as follow:

$$y(t) = \frac{1}{2} A(t)A(t-T_b) \sin[\theta(t) - \theta(t-T_b) + \omega_c T_b] = \frac{1}{2} A(t)A(t-T_b) \sin[\Delta\theta(t)] \quad (10)$$

In the Eq.10,  $\Delta\theta(t) = \theta(t) - \theta(t-T_b)$  is a phase increment of symbol. In due to  $A(t)$  is envelope, so  $A(t)A(t-T_b) > 0$ . At the time of  $t = (k+1)T_b$ , we sample for  $y(t)$ , and the get  $y[(k+1)T_b]$ , its symbol depends on the symbol of  $\Delta\theta[(k+1)T_b]$ . Based on the analysis of  $\Delta\theta(t)$  path, we can make a decision.

**BPSK Demodulation.** BPSK demodulation adopt two-way quadrature demodulator of I / Q. Received BPSK signal is:

$$s(t) = \cos[2\pi f_0 t + \varphi m(t)] \quad (11)$$

$\varphi$  is the index of phase modulation. If we take  $\varphi$  as  $\pi$ , then in BPSK signal, if  $m(t)=0$ , so  $\varphi m(t) = \pi \times 0 = 0$ ; if  $m(t)=1$ , so  $\varphi m(t) = \pi$ .

BPSK signal after two-way mixers:

$$s_{1I}(t) = \cos[2\pi f_0 t + \varphi m(t)] \cdot \cos(2\pi f_0 t) = \cos[4\pi f_0 t + \varphi m(t)] + \cos[\varphi m(t)] \quad (12)$$

$$s_{1Q}(t) = \cos[2\pi f_0 t + \varphi m(t)] \cdot \sin(2\pi f_0 t) = \sin[4\pi f_0 t + \varphi m(t)] + \sin[\varphi m(t)] \quad (13)$$

The baseband signal of BPSK can be recovered after low-pass filter:

$$s_{2I}(t) = \cos[\varphi m(t)] \quad s_{2Q}(t) = \sin[\varphi m(t)] \quad (14)$$

## Analysis if simulation results

**LS channel estimation algorithm different sequence effect.** Select single antenna frequency fading channel, only consider multipath delay spread in the channel characteristics, without consider the time-varying characteristic of channels, we choose the channel that length is 256 and contains 10 multipath component. The multipath components of arbitrary distribute at different time delays and noise is Gaussian white noise.

Send Gaussian distribution, Uniform distribution, Bernoulli random sequence and m sequence after GMSK, BPSK modulation, the simulation results can be concluded that the designed training sequence and modulated m sequence of channel estimation results are obtained. Channel estimation mean square error is defined as:

$$MSE = E \left[ \sum_k \left| h(k) - \hat{h}(k) \right|^2 \right] \quad (15)$$

Fig.3 shows the five kinds of the sequences, in the same channel, the channel estimation error can be seen that with the improvement of the signal-to-noise ratio, estimation accuracy of the three sequences of the above design is higher than the estimation accuracy of the m sequences after different modulation. Bernoulli sequence has the highest precision.

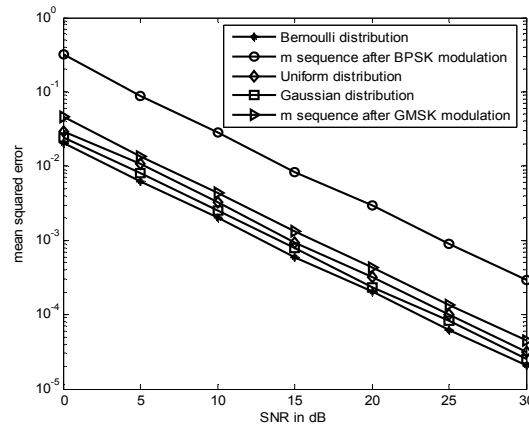


Fig.3. 5 sequence channel estimation error

**Channel estimation effect of different sequence under OMP algorithm.** Frequency selective fading channel performance sparse channel multipath channel with only a few relatively small numbers of nonzero coefficients, LS algorithm does not use the low dimension feature of sparse channel, the estimated price is large. Compressed sensing theory[6] showed that when the signal is sparse or compressible, it can use far below the Nyquist rate of the frequency of the sampling signal to obtain a small amount of the signal projection value and achieve the accuracy or similar reconfigurable. The emergence of the theory offers a new way for broadband sparse channel estimation. If we see broadband channel sparse impulse response as sparse signal in compressed sensing theory, then put the training sequence and channel convolution function as the sampling observation process, then the compressed sensing theory can significantly reduce the size of the observation matrix, with less cost to obtain better estimation result.

Select the same channel that length is 256 and contains different number of significant multipath component, transmit Gaussian distribution, uniform distribution, Bernoulli distribution random sequence as well as GMSK, BPSK modulation of the m-sequence as the training sequence. In the process of compressed sensing reconstruction OMP algorithm[7] can effectively reduce the number of iterations and obtain better reconstruction effect, this experiment OMP algorithm is used to estimate of the sparse channel. Due to the training sequence is generated randomly, it is not the same as the precision of the experiment for every time, therefore, for each measurement value  $m$  is used to run 1,000 times by seeking an average value of the relative error method to compare the estimated performance and the relative error is defined as follows:

$$Error = \frac{\|\hat{h} - h\|_2}{\|h\|_2} \quad (16)$$

In the Eq.16,  $\hat{h}$  represents channel impulse response estimated value of  $h$ . Respectively estimate sparse channel which the length is 256 and sparsity are 10,15,25,50. The results are as follows:

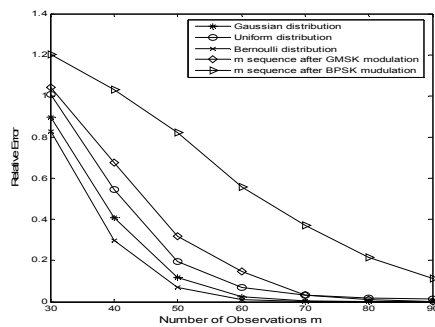


Fig.4. Sparsity 10 channel estimation result

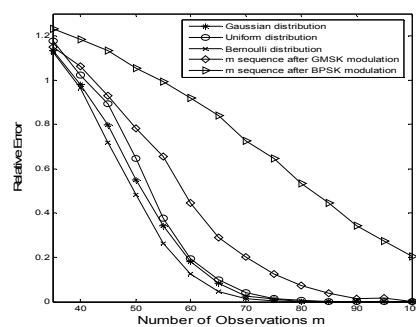


Fig.5. Sparsity 15 channel estimation result

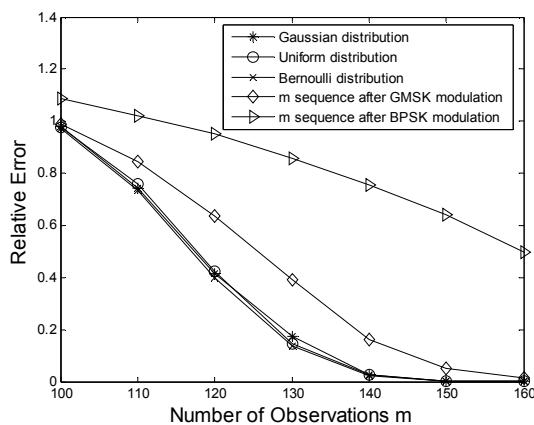


Fig.6. Sparsity 25 channel estimation result

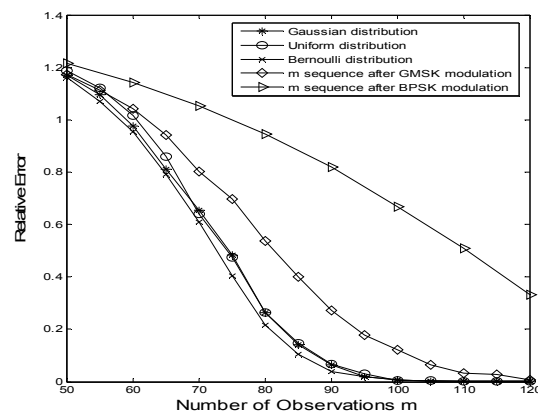


Fig.7. Sparsity 50 channel estimation result

By Fig.4 to Fig.7 shows the estimation error of sparse channel which contain different multipath component of five kinds training sequence, the results show that compared with the BPSK and GMSK modulated m-sequence, the application of the design of three kinds of training sequence in channel estimation can effectively lower the relative error of estimated income value, improve the performance of channel estimation. By figure can be seen, random sequences of Bernoulli distribution on channel estimation the relative error is minimal.

**Generate new sample training sequence.** This paper from the perspective of the optimal channel estimation performance design three kinds of real number sequence, respectively by GMSK demodulation and BPSK demodulation binary sequence can be obtained, so as to generate new training sequence. we selected wherein the length of the 26 sequences, as shown in the following tables:

Table.1. After GMSK demodulation obtain new training sequence

Sequence	Binary						
Gaussian sequence	01	0110	0101	0100	1011	0100	0010
Uniform sequence	10	1001	0110	0100	0101	0010	0001
Bernoulli sequence	00	1001	1000	1010	0110	0110	0100

Table.2. After BPSK demodulation obtain new training sequence

Sequence	Binary						
Gaussian sequence	01	0100	1001	0000	0011	0100	0100
Uniform sequence	00	0001	0100	0111	0100	0011	0011
Bernoulli sequence	00	0100	1100	1000	0101	0101	0111

**Related performance analysis.** M sequence because of its good autocorrelation properties[8] is used as training sequences for channel estimation, but not from the channel estimation optimization guidelines to improve the channel estimation accuracy. Based on channel estimation optimization we design the new training sequence, and from above simulation results can be concluded that the designed three sequences can obtain better effect of channel estimation, and a sequence of Bernoulli distribution on work best. Below we analysis related performance of the new training sequence, compare autocorrelation properties between the Bernoulli sequence after demodulation and m-sequence, as shown in Fig.8 and Fig.9.

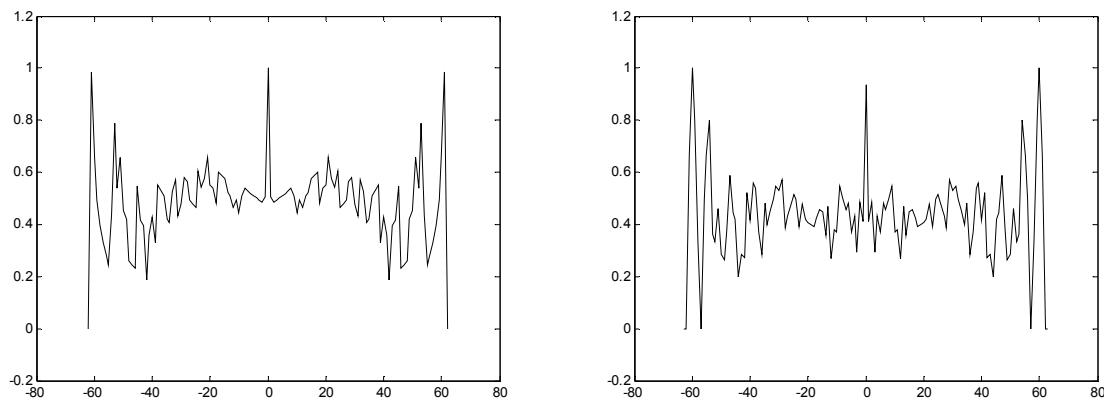


Fig.8.The autocorrelation function of m sequence Fig.9.The autocorrelation function of Bernoulli sequence

As can be seen from Fig.8 and Fig.9, compared with the m-sequence, the Bernoulli distribution sequence also has good auto-correlation properties.

## Conclusion

The paper presents system model of Channel estimation, analyzes the common training sequence, and design three kinds of random sequence of real Numbers from the Angle of the optimal channel estimation performance. Respectively, in the traditional channel estimated algorithm and channel estimated algorithm based on compressed sensing, simulation experiment is done. The simulation results show that in the same channel environment, the traditional LS algorithm and the OMP algorithm based on compressed sensing channel estimation, the design of the real number sequence compared with after different modulation of m sequence, reduces the channel estimation error, the estimation performance is also improved. Finally, three random sequences of real numbers, respectively, after GMSK demodulation and BPSK demodulation, generate new training sequence. Be self-correlation analysis, compared with m training sequence the new training sequence is also having good autocorrelation characteristics.

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