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Single-Channel Blind Source Separation of Co-frequency Overlapped GMSK Signals under Constant-Modulus Constraints

Chuanlong Wu, Zheng Liu, Xiang Wang, Wenli Jiang, and Xiaohu Ru

***Abstract*—As for the single-channel overlapped signals, in which the modulation parameters of the component signals are identical or similar, it is still a great challenge to extract the component signals. Aiming at this problem, a single-channel blind source separation (SCBSS) algorithm, constrained by constant modulus, is proposed for the overlapped Gaussian minimum shift keying (GMSK) signals and this algorithm also applies to other overlapped signals with constant modulus (CM), such as binary phase shift keying (BPSK) signals and quadrature phase shift keying(QPSK) signals. By tracking the channels of the component signals, the algorithm can effectively estimate their symbol sequences. Simulation results show that the proposed algorithm can work more robustly and reduce the frame error rate (FER), comparing with the competing algorithms.**

***Index Terms*—Constant modulus, GMSK, per-survivor processing, single-channel blind source separation.**

I. Introduction

THE separation problems of single-channel overlapped signals in time-frequency domain have drawn attention for many years[1], [2]. So far, a large variety of algorithms have been proposed to separate these overlapped signals and some of them have been used in many applications, such as speech signal processing [3] and electroencephalogram (EEG) signal processing [4], with great success. Depending on the cyclostationarity of the digital modulation signals, a blind adaptive FRESH filter is constructed to extract the source signals successfully [5]. The single-channel co-frequency overlapped signals exist extensively in current systems, such as the space-based Automatic Identification System (AIS) [6]. Meanwhile, the identical or similar modulation parameters add up the difficulties to separate the overlapped signals. As the difference of the modulation parameters is not very significant, it is almost impossible to use the above algorithms to solve the practical problems.

After further research, particle filtering was utilized to separate the co-frequency multiple phase shift keying (MPSK) signals [7], but the high complexity impeded its application. Per-survivor processing (PSP) provides a general framework for the approximation of maximum likelihood sequence estimation (MLSE) algorithm. Based on channel adaptation, PSP algorithm is used for single-channel separation of two QPSK signals [8] and its complexity is much lower than that of particle filtering. Wan *et al.* [9] then enhanced the algorithm's robustness by the oversampling technology, but this greatly increases the algorithm complexity. Combined with the interaction between maximum a posteriori (MAP) decoding and likelihoods, the iterative decoding algorithms are proposed for single-channel co-frequency signals [10]-[12] and yield better than PSP algorithm. Nevertheless, in some application systems, it is unreliable to make use of coding information to improve the separation performance. Here, aiming at further improving the robustness and enhancing algorithm's ability to adapt to the uncertain environment, a novel SCBSS algorithm, constrained by constant modulus, is proposed without considering the coding information. When tracking the channels of overlapped GMSK signals, the proposed algorithm performs more stablely than the conventional PSP (CPSP) algorithm.

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II. Overlapped GMSK Signals Model

We assume that the bandwidths and the symbol periods of the GMSK signals $s_i(t),\ i = 1 \,\text{and}\, 2,$ are identical. The baseband single-channel overlapped signals, which consist of two GMSK signals, can be modeled as

$$\begin{aligned} y(t) &= x_1(t) + x_2(t) + n(t) \\ &= A_1 e^{j2\pi\Delta f_1 t + \theta_1} s_1(t) + A_2 e^{j2\pi\Delta f_2 t + \theta_2} s_2(t) + n(t), \end{aligned} \tag{1}$$

where A_i are the amplitudes, Δf_i are the frequency shifts, θ_i are the initial phases, $n(t)$ is additive white Gaussian noise (AWGN). According to amplitude modulated pulses (AMP) decomposition [13], $s_i(t)$ are defined as

$$s_i(t) = \sum_{N=-\infty}^{\infty} J^{Z_N^i} R_i(t - NT + \tau_i), \tag{2}$$

$$Z_N^i = \sum_{n=-\infty}^{N} a_n^i, \tag{3}$$

where $0 \le \tau_i < T$ are the relative time delays, which keep the local clock for reference; Z_N^i are the sum of the first N bits, which are independent and identically distributed random sequences with unit energy; $R_i(t)$ are the pulse responses of

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channel filters; T is the symbol period; $J = \exp(jh\pi)$, where h is the modulation index.

Sampling the overlapped signals at symbol rate 1/*T*, (1)-(3) can be rewritten respectively as

$$\begin{aligned} y_k =& A_1 e^{j2\pi\Delta f_1 kT+\theta_1} s_1(kT) \\ &+ A_2 e^{j2\pi\Delta f_2 kT+\theta_2} s_2(kT) + n_k, \end{aligned} \tag{4}$$

$$s_i(kT) = \sum_{N=1-M_1}^{M_2} c_{k+N}^i R_i(-NT+\tau_i), \tag{5}$$

$$c_{k+n}^i = J^{Z_{-M_1}^i + \sum_{m=1-M_1}^{n} a_m^i}, \tag{6}$$

where $M = M_1 + M_2$ is the finite durations of the equivalent channel filters. Assuming that

$$\mathbf{c}_k^i = \left[c_{k-M_1+1}^i, c_{k-M_1+2}^i, \cdots, c_{k+M_2}^i\right]^{\mathrm{T}}, \tag{7}$$

$$\mathbf{R}_k^i = \left[R_i\left((M_1-1)T+\tau_i\right), \cdots, R_i\left((-M_2)T+\tau_i\right)\right]^{\mathrm{T}}, \tag{8}$$

where $[\cdot]^{\mathrm{T}}$ denotes transpose operator, y_k can be modified as

$$\begin{aligned} y_k &= A_1 e^{j2\pi\Delta f_1 kT+\theta_1} \left(\mathbf{R}_k^1\right)^{\mathrm{T}} \mathbf{c}_k^1 \\ &\quad + A_2 e^{j2\pi\Delta f_2 kT+\theta_2} \left(\mathbf{R}_k^2\right)^{\mathrm{T}} \mathbf{c}_k^2 + n_k \\ &= \left(\mathbf{R}'^1_k\right)^{\mathrm{T}} \mathbf{c}_k^1 + \left(\mathbf{R}'^2_k\right)^{\mathrm{T}} \mathbf{c}_k^2 + n_k, \end{aligned} \tag{9}$$

where $\mathbf{R}'^i_k = A_i e^{j2\pi\Delta f_i kT+\theta_i} \mathbf{R}_k^i$. The next work is to estimate two sequences a_m^i of the component signals $s_i(t)$.

III. The SCBSS Algorithm Based on CPSP

Defining the channel state at epoch k as $\mu_k = \left[\mathbf{c}_k^1, \mathbf{c}_k^2\right]$, when $\left[c_{k+M_2+1}^1 \; c_{k+M_2+1}^2\right]$ becomes available, a state will transfer from μ_k to μ_{k+1}. The state transition is defined as

$$\mu_k \xrightarrow[y_{k+1}]{\left[c_{k+M_2+1}^1 \; c_{k+M_2+1}^2\right]} \mu_{k+1}. \tag{10}$$

At the epoch *k*, the branch metrics of all the state transitions may be calculated as

$$\lambda\left(\mu_k \to \mu_{k+1}\right) = \left|e\left(\mu_k \to \mu_{k+1}\right)\right|^2, \tag{11}$$

where

$$\begin{aligned} e\left(\mu_k \to \mu_{k+1}\right) =& y_k - \left(\mathbf{R}'^1_k\right)^{\mathrm{T}} \mathbf{c}_k^1\left(\mu_k \to \mu_{k+1}\right) \\ & - \left(\mathbf{R}'^2_k\right)^{\mathrm{T}} \mathbf{c}_k^2\left(\mu_k \to \mu_{k+1}\right). \end{aligned} \tag{12}$$

Denoting $\Gamma(\mu_k)$ as the survivor metrics, for all the successor states, the accumulated metrics $\Gamma(\mu_{k+1})$ are calculated by selecting the minimization over the different branch metrics of the current states μ_k and then $\Gamma(\mu_{k+1})$ can be expressed as

$$\Gamma\left(\mu_{k+1}\right) = \min_{\mu_k}\left(\Gamma\left(\mu_k\right) + \lambda\left(\mu_k \to \mu_{k+1}\right)\right). \tag{13}$$

Ultimately, the survivors, which satisfy (13), will be extended to the subsequent states. The channel estimates are then updated based on the classic LMS algorithm

$$\hat{\mathbf{R}}'_{k+1} = \hat{\mathbf{R}}'_k + \beta \cdot e\left(\mu_k \to \mu_{k+1}\right) \mathbf{c}_k^*\left(\mu_k \to \mu_{k+1}\right), \tag{14}$$

where the constant β is the step size and $[\cdot]^*$ denotes usual complex conjugate; $\hat{\mathbf{R}}'_k = \left[\begin{array}{cc} \hat{\mathbf{R}}'^1_k & \hat{\mathbf{R}}'^2_k \end{array}\right]$, $\mathbf{c}_k = \left[\begin{array}{cc} \mathbf{c}_k^1 & \mathbf{c}_k^2 \end{array}\right]$.

As shown in Fig. 1, the channel memory *M* is 3, the modulation index *h* is 0.5, there are 16 initial states and each current state has 4 branches. The main steps of the SCBSS algorithm based on CPSP (CPSP-SCBSS) algorithm are summarized as follows.

1) Initialization: initialize the channels by estimating all the parameters of the overlapped GMSK signals.

2) Extending branches: aiming at 16 current states, extend every current state to 4 branches by incorporating the transition and select the optimal branch, whose branch metric is minimum.

3) Updating channels: update the channels by using (14) and return step *2)*.

4) Selecting optimal sequence: according to 16 accumulated metrics, select optimal sequence, whose accumulated metric is minimum.

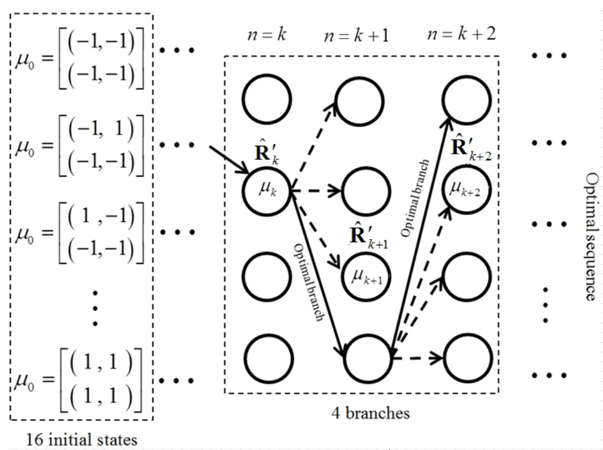

Fig. 1. The state transition of CPSP-SCBSS algorithm (*M* = 3).

IV. Modified Algorithm

A. LMS algorithm constrained by constant modulus

The received signal can be expressed as

$$y(k) = x(k) + n(k), \tag{15}$$

where $x(k)$ is GMSK signal. The GMSK signal estimates $\hat{x}(k)$ are

$$\begin{aligned} \hat{x}(k) = \mathbf{R}_k^{\mathrm{T}} \mathbf{c}_k &= A e^{j2\pi\Delta f k+\theta_0} \sum_{n=0}^{M-1} R'(n)\, c(k-n) \\ &= \sum_{N=1-M_1}^{M_2} R(n)\, c(k-n), \end{aligned} \tag{16}$$

where $R(n) = A e^{j2\pi\Delta f k+\theta_0} R'(n)$ is the channel filter. At epoch *k*, the error is calculated as

$$e(k) = x(k) - \hat{x}(k) = x(k) - \sum_{n=0}^{M-1} R(n)\, c(k-n). \tag{17}$$

The cost function can be written as

$$\begin{aligned} J(\mathbf{R}_k) &= E\left[e(k)e^*(k)\right] \\ &= E\left[|x(k)|^2\right] - \sum_{n=0}^{M-1} R\left(n\right) E\left[c\left(k-n\right)x^*(k)\right] \\ &\quad - \sum_{n=0}^{M-1} R^*\left(n\right) E\left[c^*\left(k-n\right)x(k)\right] \\ &\quad + \sum_{n=0}^{M-1}\sum_{l=0}^{M-1} R^*\left(n\right) R\left(l\right) E\left[c^*\left(k-n\right)c\left(k-l\right)\right]. \end{aligned} \tag{18}$$

Assuming that $\sigma_x^2 = E\left[|x(k)|^2\right], p(-n) = E\left[c\left(k-n\right)x^*(k)\right], p^*(-n) = E\left[c^*\left(k-n\right)x(k)\right], q(l-n) = E\left[c^*\left(k-n\right)c\left(k-l\right)\right]$, the cost function can be rewritten as

$$J(\mathbf{R}_k) = \sigma_d^2 - \mathbf{R}_k^{\mathrm{T}}\mathbf{p}_k - \mathbf{p}_k^{\mathrm{T}}\mathbf{R}_k - \mathbf{R}_k^{\mathrm{T}}\mathbf{Q}_k\mathbf{R}_k^*, \tag{19}$$

where σ_x^2 denotes the square error of the desired signal $x(k)$; $\mathbf{p}_k = [p(0), p(-1), \cdots, p(1-M)]^{\mathrm{T}}$; $\mathbf{Q}_k = E\left[\mathbf{c}_k^*\mathbf{c}_k^{\mathrm{T}}\right]$.

According to (16), the amplitude of the desired GMSK signal $x(k)$ is A. On account of GMSK's characteristic of constant modulus, the cost function $J(\mathbf{R}_k)$, using Lagrange multiplier λ, can be described as

$$\begin{cases} \mathbf{R}_{k-opt} = \min\left\{J(\mathbf{R}_k)\right\}, \\ s.t. \quad \left|\mathbf{R}_k^{\mathrm{T}}\mathbf{c}_k\right|^2 = A^2\,, \end{cases} \tag{20}$$

$$L(\mathbf{R}_k, \lambda) = J(\mathbf{R}_k) + \lambda\left(\left|\mathbf{R}_k^{\mathrm{T}}\mathbf{c}_k\right|^2 - A^2\right), \tag{21}$$

where $|\cdot|$ denotes the signal module. The instantaneous estimates of $\mathbf{Q}_k$ and $\mathbf{p}_k^*$ are respectively defined as follows

$$\hat{\mathbf{Q}}_k = \mathbf{c}_k^*\mathbf{c}_k^{\mathrm{T}}, \tag{22}$$

$$\hat{\mathbf{p}}_k^* = \mathbf{c}_k x^*(k). \tag{23}$$

The complex gradient of (21) for $\mathbf{R}_k$ is calculated as

$$\begin{aligned} \nabla_{\mathbf{R}_k} L(\mathbf{R}_k, \lambda) &= \frac{\partial J(\mathbf{R}_k)}{\partial \mathbf{R}_k} + \lambda \frac{\partial\left(\left|\mathbf{R}_k^{\mathrm{T}}\mathbf{c}_k\right|^2 - A^2\right)}{\partial \mathbf{R}_k} \\ &= -2\mathbf{p}_k^* + 2\mathbf{Q}_k\mathbf{R}_k^* + \lambda\mathbf{c}_k{\mathbf{c}_k}^{\mathrm{H}}\mathbf{R}_k^* \\ &= -2\hat{\mathbf{p}}_k^* + 2\hat{\mathbf{Q}}_k\mathbf{R}_k^* + \lambda\hat{\mathbf{Q}}_k\mathbf{R}_k^*, \end{aligned} \tag{24}$$

where $[\cdot]^{\mathrm{H}}$ denotes Hermitian transpose operator.

Assuming that $\nabla_{\mathbf{R}_k} L(\mathbf{R}_k, \lambda) = 0$, we can get

$$\hat{\mathbf{Q}}_k\mathbf{R}_k^* = \frac{2\hat{\mathbf{p}}_k^*}{2+\lambda}. \tag{25}$$

According to (20), (22), (23), and (25), we have the following solution

$$\lambda = 2\left(\frac{\hat{x}(k)x^*(k)}{A^2} - 1\right). \tag{26}$$

And then, the LMS algorithm constrained by constant modulus (LMSCCM) is updated as

$$\begin{aligned} \hat{\mathbf{R}}_{k+1} &= \hat{\mathbf{R}}_k + \beta\left(2\mathbf{c}_k x^*(k) - 2\mathbf{c}_k^*\mathbf{c}_k^{\mathrm{T}}\mathbf{R}_k\right. \\ &\quad \left. -2\left(\frac{\hat{x}(k)x^*(k)}{A^2} - 1\right)\mathbf{c}_k^*\mathbf{c}_k^{\mathrm{T}}\mathbf{R}_k\right) \\ &= \hat{\mathbf{R}}_k + \beta\left(2\mathbf{c}_k^* e(k) - 2\left(\frac{\hat{x}(k)x^*(k)}{A^2} - 1\right)\mathbf{c}_k^*\mathbf{c}_k^{\mathrm{T}}\mathbf{R}_k\right). \end{aligned} \tag{27}$$

B. SCBSS algorithm based on PSPCCM

By replacing LMS algorithm with LMSCCM algorithm, we can get the novel PSP algorithm constrained by constant modulus (PSPCCM) algorithm. As for the overlapped signals, the channel update of the component signals is represented as

$$\begin{aligned} \hat{\mathbf{R}}'_{k+1} &= \hat{\mathbf{R}}'_k + \beta\cdot\left(2\mathbf{c}_k^*\left(\mu_k \to \mu_{k+1}\right)e\left(\mu_k \to \mu_{k+1}\right)\right. \\ &\quad - \left[\lambda_1\left(\mathbf{c}_k^1\right)^*\left(\mu_k \to \mu_{k+1}\right)\left(\mathbf{c}_k^1\right)^{\mathrm{T}}\left(\mu_k \to \mu_{k+1}\right)\hat{\mathbf{R}}'^1_k \right. \\ &\quad \left.\left. \lambda_2\left(\mathbf{c}_k^2\right)^*\left(\mu_k \to \mu_{k+1}\right)\left(\mathbf{c}_k^2\right)^{\mathrm{T}}\left(\mu_k \to \mu_{k+1}\right)\hat{\mathbf{R}}'^2_k\right]\right), \end{aligned} \tag{28}$$

where $\hat{\mathbf{R}}'_k = \left[\begin{array}{cc} \hat{\mathbf{R}}'^1_k & \hat{\mathbf{R}}'^2_k \end{array}\right]$, $\mathbf{c}_k = \left[\begin{array}{cc} \mathbf{c}_k^1 & \mathbf{c}_k^2 \end{array}\right]$ and

$$\begin{cases} \lambda_1 = 2\left(\frac{\hat{x}_1(k)x_1^*(k)}{\hat{A}_1^2} - 1\right), \\ \lambda_2 = 2\left(\frac{\hat{x}_2(k)x_2^*(k)}{\hat{A}_2^2} - 1\right), \end{cases} \tag{29}$$

where $\hat{A}_i$ are the component signal amplitudes, $x_i(k)$ are the desired signals. For the received overlapped signals, $x_i(k)$ can't be obtained accurately, (29) is approximately written as

$$\begin{cases} \lambda_1 = 2\left(\frac{|\hat{x}_1(k)|^2}{\hat{A}_1^2} - 1\right), \\ \lambda_2 = 2\left(\frac{|\hat{x}_2(k)|^2}{\hat{A}_2^2} - 1\right). \end{cases} \tag{30}$$

V. Simulation Results

The signal-to-noise ratio is set as 5 dB and the amplitude is 1.8. From Fig. 2, it is found that the proposed LMSCCM algorithm has a much faster convergence speed. The root mean square error (RMSE) of LMSCCM algorithm converges to 0, but for LMS algorithm, it converges to -0.2. The LMS algorithm needs approximately 30 iterations to converge to minimum value, whereas the LMSCCM algorithm only requires approximately 15 iterations. It is as expected that the performance of LMS algorithm can be improved by additionally exploiting the constant-modulus constraints.

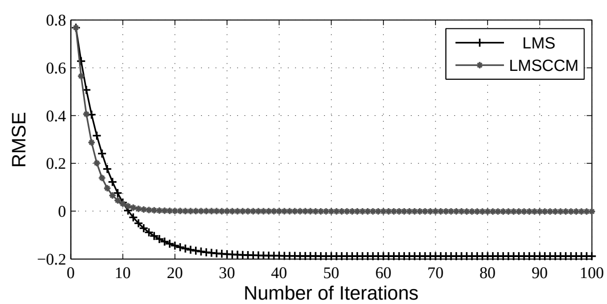

Fig. 2. Convergence analysis of LMS algorithm and LMSCCM algorithm.

To reproduce a typical communications environment, an AIS frame is assumed, which is a block of binary symbols including a known preamble, some information symbols, and a known tail as shown in Fig. 3. Each frame consists of 250 bits and the bit rate is 9.6 *kbps*. Depending on (1), the overlapped signals are produced.

Assuming that the relative time delay of the component signals $x_1\left(t\right)$ and $x_2\left(t\right)$ is 0 *ms*, the carrier-to-noise ratio (CNR) is 16 dB and A_1 is equal to A_2. When used to update the channel, the iteration numbers of CPSP-SCBSS algorithm and

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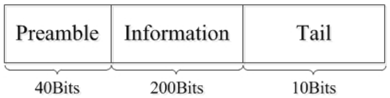

Fig. 3. Frame format of AIS signal.

PSPCCM-SCBSS algorithm are all 5. The channel memory M is 3. Fig. 4 shows the RMSE of tracking channel. The PSPCCM-SCBSS algorithm shows the robustness when used to track the channel parameters. Nevertheless, the RMSE of the CPSP-SCBSS algorithm jumps up near the 50*th* symbol interval.

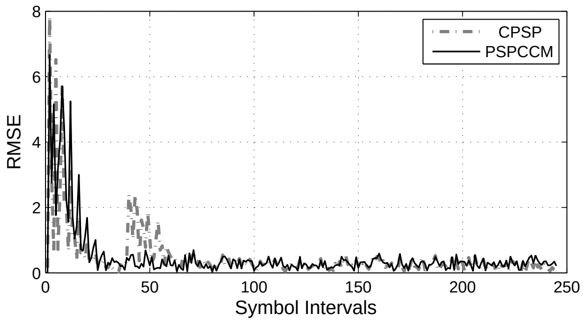

Fig. 4. Tracking RMSE for CPSP-SCBSS algorithm and PSPCCM-SCBSS algorithm (M = 3).

Considering that the other modulation parameters are identical, the time delays have an important impact on sequence estimation of the component signals. In order to better investigate the performance of sequence estimation under different CNRs, the different relative time delays ($0T$, $0.2T$, $0.4T$) have been assumed in Fig. 5. The FER curves of single-sampling CPSP-SCBSS (SSCPSP-SCBSS) algorithm, double-sampling CPSP-SCBSS (DSCPSP-SCBSS) algorithm, and PSPCCM-SCBSS algorithm are obtained with a value of the step size $\beta = 0.005$ and the number of Monte Carlo simulation is 500. When CNR varies from 0dB to 16 dB, the FER of PSPCCM-SCBSS algorithm is much lower than that of the other two algorithms.

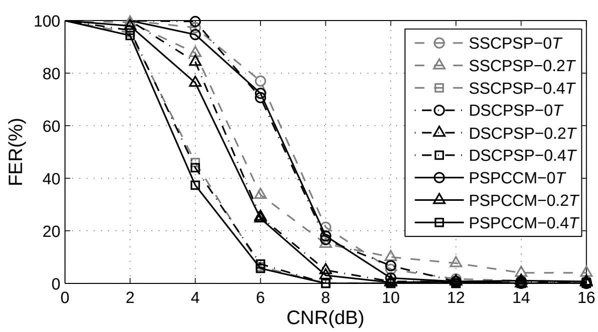

Fig. 5. FER versus CNR for all the algorithms under different relative time delays ($0T$, $0.2T$, $0.4T$).

VI. Conclusions

An approach to the SCBSS problem has been presented, resulting in a robust novel method under constant modulus constraints. The presented results show that the LMSCCM algorithm has better convergence rate and precision. Based on PSPCCM algorithm, the PSPCCM-SCBSS algorithm is proposed for the overlapped GMSK signals. The algorithm's performance excels that of SSCPSP-SCBSS algorithm and the DSCPSP-SCBSS algorithm over the different relative time delays. In addition, the complexity of the novel algorithm is much lower. It is worth mentioning that because the amplitudes of the component signals need to be estimated, the estimation error maybe affect the performance of the PSPCCM-SCBSS algorithm.

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