LINEAR MMSE CHANNEL ESTIMATION FOR GSM

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Abstract

A new channel estimation algorithm capable of exploiting a-priori knowledge about modulation and receive filters and the power delay profile of the physical channel is presented. In order to incorporate the a-priori knowledge into the linear minimum mean square error (LMMSE) channel estimator, a Bayesian approach is applied to an appropriate GSM system model. The performance of the new algorithm is compared to a conventional least squares (LS) channel estimator by means of analysis and simulations. Simulations are carried out for the standard GSM channel profiles (TU50, HT100, RA250). Information about the shape of the power delay profile is assumed to be unavailable to the receiver. The merit of the new channel estimator is confirmed by yielding an MSE approximately 1 - 2 dB lower than the LS estimator. The lower MSE translates into a BER advantage of approximately 0.25 dB. Exploiting additional knowledge about the shape of the power delay profile results in further BER performance improvements of up to 0.3 dB.

1 Introduction

The "classical" channel estimator that is normally employed in GSM receivers is based on the Least Squares (LS) approach and does not exploit any available a-priori information about the channel impulse response (CIR). However, it is highly desirable to incorporate a-priori knowledge into the channel estimator, because this will improve the quality of the channel estimate and translate directly into better BER performance of the receiver. The effective CIR that is "seen" by the equaliser of the GSM receiver consists, roughly speaking, of the convolution of the transmit filter, the physical channel $p(t)$, and the receive filters. The characteristics of the transmit filter and any digital receive filter are known to the receiver, and can thus be exploited. In 1997, Khayrallah et al. [4] presented a constrained LS solution in order to incorporate the a-priori information into the channel estimator. However, their approach suffers from a drawback in that a symbol spaced physical channel is assumed (which is not to be confused with the symbol spaced effective channel). Consequently, the correct sampling instance within the symbol period has to be estimated by using multiple sets of estimators, each one corresponding to a different sampling time. Here, we resort to a Bayesian approach in order to incorporate the a-priori knowledge into the channel estimator. In other words, it is necessary to model the CIR taps as random variables with a given prior PDF. With this approach it is possible to find a linear estimator which is optimal in a mean square error (MSE) sense with respect to the assumed prior PDF.

2 Signal Model and LS channel estimation

In a digital communication system, such as GSM, information symbols $\{a_k\}$ are sent at rate $1/T$ over a known effective channel $\mathbf{h}$ with inter-symbol interference (ISI). This is a reasonable assumption which follows the concept of synchronised detection [1] for which a channel estimate $\hat{\mathbf{h}}$ must be formed and subsequently used for detection as if it were the true known channel. Furthermore, the channel is assumed to be time-invariant for the duration of a burst. In GSM systems, the received signal is approximately bandlimited to $1/(2T)$ and therefore symbol rate sampling guarantees sufficient statistics for the tasks of detection and synchronisation. The GSM standard provides midamble training sequences of length 26 that can be exploited to calculate a symbol spaced estimate of $\mathbf{h}$. Denote the training sequence as $\mathbf{a} = (a_0, \ldots, a_{25})^T$. Furthermore, if the channel memory is L then write the $(L+1) \times 1$ T-spaced CIR vector $\mathbf{h} = (h_0, \ldots, h_L)^T$. In matrix notation, the corresponding linear transmission model is given by

$$\begin{aligned} \mathbf{r}_i &= \begin{pmatrix} a_{L+i} & a_{L-1+i} & \cdots & a_i \\ a_{L+1+i} & a_{L+i} & \cdots & a_{1+i} \\ \vdots & \vdots & \ddots & \vdots \\ a_{L+15+i} & a_{L+14+i} & \cdots & a_{15+i} \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ \vdots \\ h_L \end{pmatrix} + \mathbf{n}_i \\ &= \mathbf{A}_i \mathbf{h} + \mathbf{n}_i \end{aligned} \quad (1)$$

The received signal samples are stored in the 16×1 vector $\mathbf{r}_i$, and the subscript i determines the exact position of the samples within a received burst, usually by means of a so-called coarse timing estimate. The exact procedure is not within the scope of this paper, and suffice to say that 16

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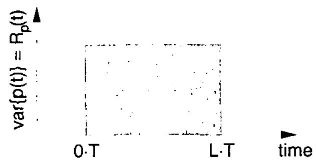

Figure 1: Power delay profile of physical channel.

received signal samples are used, because this facilitates an easy implementation of the conventional LS channel estimator. If, for ease of notation, the subscript i is omitted, then the LS channel estimator for the above linear transmission model is given by

$$\begin{aligned}\hat{\mathbf{h}}_{LS} &= \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{r} \\ &= \mathbf{X}_{LS}\mathbf{r} \qquad (2)\end{aligned}$$

Note, that the linear signal model is only an approximation, because the GSM system employs a non-linear modulation format called Gaussian Minimum Shift Keying (GMSK) [2]. However, it is possible to find a close linear approximation to GMSK [5]. Thus it becomes possible to treat GMSK as a linear modulation with transmit filter $C_0(t)$, without noticeably compromising the performance of the receiver. For the LMMSE channel estimator that is presented next, an even closer approximation is feasible that includes some of the non-linearities inherent in GMSK.

3 Linear Minimum Mean Square Error (LMMSE) Channel Estimation

In order to incorporate the a-priori knowledge, the CIR vector $\mathbf{h}$ is modelled as a random vector with mean $E(\mathbf{h}) = 0$ and known covariance matrix $\mathbf{C}_{hh}$. Each tap in $\mathbf{h}$ is assumed rayleigh faded. The noise vector is Gaussian with PDF $\mathcal{N}(\mathbf{0}, \mathbf{C}_n)$. In that case, the optimal linear minimum mean square error (LMMSE) estimator is given by [3]

$$\begin{aligned}\hat{\mathbf{h}}_{LMMSE} &= \mathbf{C}_{hr}\left(\mathbf{C}_{rr}+\mathbf{C}_n\right)^{-1}\mathbf{r} \\ &= \mathbf{X}_{LMMSE}\mathbf{r} \qquad (3)\end{aligned}$$

From the above equation it is seen that in order to find the LMMSE estimator and assess its performance, we have to determine the covariance matrices $\mathbf{C}_{hr}$, $\mathbf{C}_{rr}$, and $\mathbf{C}_n$. How this can be accomplished will be shown in the following. Firstly, we assume that we do not have any prior knowledge about the power delay profile of the physical channel, apart from the channel memory L. In other words, the delays of the multipath rays are, on average, uniformly distributed in an interval $[0, LT]$, and the powers of the rays are, on average, equal. The power delay profile of the resulting time-continous physical channel $p(t)$ is shown in Figure 1. For wide sense stationary uncorrelated scattering we have

$$E\{p^*(t)p(\tau)\} = \begin{cases} R_p(t) & t = \tau \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

It is now possible to build a model of the discrete-time effective channel $\mathbf{h}$ (see Figure 2). Basically it consists of three parts. The first part is an approximation of the GMSK modulation as described in [5]. This approximation is equivalent to a PAM modulation using $\pi/2$-phase rotated BPSK symbols and transmit pulses $C_0(t)$ and $C_1(t)$. We define

$$a_{0,k} = a_k e^{j\frac{\pi}{2}k} \qquad (5)$$

$$a_{1,k} = \prod_{i=-1}^{1} a_{k+i} e^{j\frac{\pi}{2}k} \qquad (6)$$

The symbols $\{a_{0,k}\}$ are simply the rotated original BPSK symbols $\{a_k\}$ which are subsequently filtered with the pulse $C_0(t)$. This in itself constitutes the optimal linear approximation to GMSK. The symbols $\{a_{1,k}\}$ are rotated and nonlinearly combined BPSK symbols which are then filtered with the $C_1(t)$ pulse. Both resulting signals are then added to form a reasonably accurate approximation to the GMSK modulation. In theory it is possible to approximate the GMSK modulation ever more closely by including more higher-order terms, but in practice this is not necessary, because these higher-order terms are negligibly small ($\approx -60dB$). The second part is the discrete-time physical channel $\mathbf{p}$. The discrete channel results from filtering the noise corrupted output of the continous time channel $p(t)$ with an anti-aliasing filter (AAF) of cutoff frequency $1/2T$ and then sampling with a clock period of T. Note, that for simplicity we consider the AAF as part of the physical channel. This can be justified, because normally the following digital receive LP filter dominates the overall frequency response, and incorporating the a-priori knowledge about the AAF does not result in further performance improvements. Therefore, the a-priori information that we would like to exploit is contained in the first part, the GMSK approximation, and the third part, the digital LP filter g_k. Apart from the channel memory L, the physical channel is unknown. From now on, the symbols $\{a_{0,k}\}$ and $\{a_{1,k}\}$ are treated as deterministic quantities, calculated from the known midamble sequence. Time-continous versions $a_0(t)$, $a_1(t)$, and $g(t)$ of the symbols $\{a_{0,k}\}$, $\{a_{1,k}\}$, and the digital LP filter, respectively, can be created by expanding $\{a_{0,k}\}$, $\{a_{1,k}\}$ and $\{g_k\}$ with an ∞-fold expander. Now we make the following definitions

$$f_r(t) = (a_0(t) \star C_0(t) + a_1(t) \star C_1(t)) \star g(t) \qquad (7)$$

$$f_h(t) = C_0(t) \star g(t) \qquad (8)$$

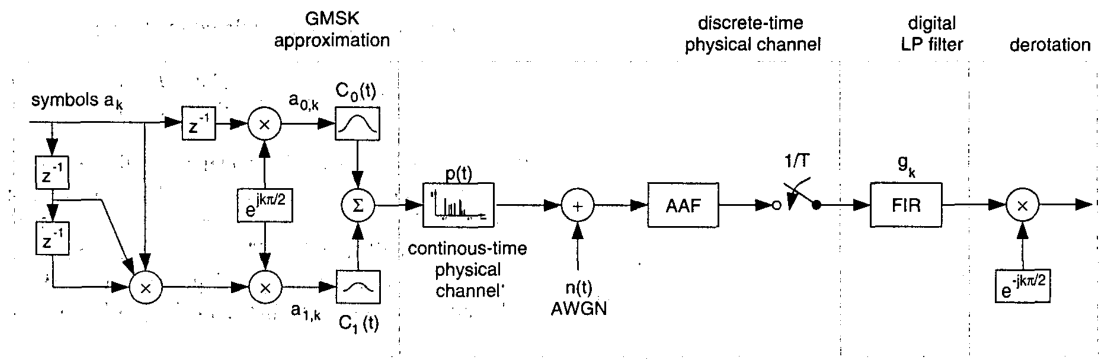

Figure 2: Channel model.

where $\star$ stands for convolution. Both of these time-continous functions $f_r(t)$ and $f_h(t)$ contain the a-priori knowledge which is available at the receiver. Time-continous functions are required here, because in order to calculate the covariance matrices, it is necessary to convolve these functions with the power delay profile $R_p(t)$. Now, with the above definitions in mind, it is possible to calculate the covariance matrices $\mathbf{C}_{hh}$, $\mathbf{C}_{rr}$, $\mathbf{C}_{hr}$. Firstly, the sampled received midamble signal which is not subject to AWGN is written as

$$\tilde{r}(k) = e^{-jk\pi/2} \int_{-\infty}^{\infty} p(t) f_r(kT - t)dt \tag{9}$$

Then the covariance matrix $\mathbf{C}_{rr}$ is given by

$$[\mathbf{C}_{rr}]_{kl} = E\{\tilde{r}^*(k)\tilde{r}(l)\} = e^{\frac{j(k-l)\pi}{2}} \int_{-\infty}^{\infty} R_p(t) f_r^*(kT - t) f_r(lT - t)dt \tag{10}$$

The covariance matrix $\mathbf{C}_{hh}$ can be calculated using $f_h(t)$ as

$$[\mathbf{C}_{hh}]_{kl} = e^{\frac{j(k-l)\pi}{2}} \int_{-\infty}^{\infty} R_p(t) f_h^*(kT - t) f_h(lT - t)dt \tag{11}$$

and, similarly the cross-covariance $\mathbf{C}_{hr}$ is given by

$$[\mathbf{C}_{hr}]_{kl} = e^{\frac{j(k-l)\pi}{2}} \int_{-\infty}^{\infty} R_p(t) f_h^*(kT - t) f_r(lT - t)dt \tag{12}$$

Following the same reasoning as before, the digital LP filter $\{g_k\}$ is taken to dominate the combined frequency response of both, the AAF and the digital filter. In that case, the noise covariance matrix of $\mathbf{C}_n$ is determined only by the impulse response of the digital LP filter $\{g_k\}$, i.e.

$$[\mathbf{C}_n]_{kl} = e^{j(k-l)\pi/2} \cdot \sigma_n^2 \cdot \sum_{j=0}^{L_g-1} g_{j-k}^* g_{j-l} \tag{13}$$

where the digital LP has L_g coefficients, and σ_n^2 is the power of the lowpass-filtered AWGN process $n(t)$. Of course, σ_n^2 is unknown to the receiver and has to be replaced in practice by either an estimate or some desired SNR operating (design) point. The latter option has the great advantage that the estimation matrix $\mathbf{X}_{LMMSE}$ can be pre-computed and additional on-line computations are not necessary. Note, that the resulting estimator is very insensitive to any mismatch between the true SNR and the design SNR and a near optimal performance gain is therefore feasible over a wide range.

4 Performance Analysis of LMMSE channel estimation

In order to assess the performance of the channel estimation algorithm, it is necessary to define a performance measure ε. We define ε as the mean squared error (MSE) between the channel $\mathbf{h}$ and its estimate $\hat{\mathbf{h}}$. The MSE can be written as the sum of a noise attenuation factor λ_n weighted with the noise power P_n and a self-noise factor λ_s weighted with the signal power P_s, i.e. we have

$$\begin{aligned} \varepsilon &= E\{|\mathbf{h} - \hat{\mathbf{h}}|^2\} \\ &= \lambda_n P_n + \lambda_s P_s \end{aligned} \tag{14}$$

Remember that both, the LS and the LMMSE channel estimators, can be put into the general linear form $\hat{\mathbf{h}} = \mathbf{X}\mathbf{r}$, where $\mathbf{X}$ can be the estimation matrix of one or the other approach. In order to calculate the noise attenuation λ_n, the estimation matrix $\mathbf{X}$ is simply applied to the received noise sample vector $\mathbf{n}$, and we get

$$\begin{aligned} \lambda_n &= \mathrm{tr}\left(E\left\{\mathbf{X}\mathbf{n}\mathbf{n}^H\mathbf{X}^H\right\}\right) \\ &= \mathrm{tr}\left(\mathbf{X}\mathbf{C}_n\mathbf{X}^H\right) \end{aligned} \tag{15}$$

The self-noise factor λ_s is given by the mean squared error between the channel $\mathbf{h}$ and its estimate $\hat{\mathbf{h}}_0$, where the estimate $\hat{\mathbf{h}}_0$ is computed from received signal samples $\mathbf{r}$ which are **not** corrupted by the noise vector $\mathbf{n}$ (hence self-noise). If we define

$$\mathbf{f}_h(\tau) = [f_h(-\tau), f_h(T-\tau), \ldots, f_h(LT-\tau)]^T \qquad (16)$$

and similarly $\mathbf{f}_r(\tau)$ using 16 samples of the function $f_r(\tau)$, then λ_s can be written as

$$\lambda_s = \int_{-\infty}^{\infty} R_p(\tau) \left| \mathbf{f}_h(\tau) - \mathbf{X}\mathbf{f}_r(\tau) \right|^2 d\tau \qquad (17)$$

Using ε, it is possible to compare different channel estimation algorithms. The most convenient way to perform the comparison is by calculating the ratio of two ε's belonging to different channel estimation algorithms, i.e.

$$\rho = 10 \log_{10} \left(\frac{\varepsilon_1}{\varepsilon_2} \right) \qquad (18)$$

Figure 3 shows ρ plotted against the *SNR* for the LS and LMMSE channel estimation algorithms. The ratio shown in the Figure is computed with respect to the LS channel estimator. It is seen that the LMMSE channel estimation algorithm easily outperforms the LS channel estimation algorithm. In the low SNR region (from $0dB$ to $10dB$) the mean square error of the LMMSE channel estimate is approximately $1dB$ better than that of the *LS* channel estimate. In the higher SNR regions (up to $30dB$) this advantage increases up to approximately $2.5dB$, indicating that the LMMSE channel estimator has a much lower self-noise factor than the LS channel estimator. Note, that a trade-off between noise suppression and self noise is possible by designing the estimator for a specific SNR via the noise covariance matrix of equation (13).

5 Simulation Results

The LS and LMMSE channel estimation algorithms are simulated with a GSM receiver under various conditions. The reference sensitivity test was performed for the TU50, HT100, and RA250 channel profiles. All the tests were performed for the class II (uncoded) bits. The simulation results are shown in Figures 4 to 6. For the TU50 and RA250 channel profiles, the LMMSE channel estimator results in a BER advantage of approximately $0.25dB$, whereas for the HT100 channel profile the advantage is $0.15dB$. One possibility to improve the quality of the LMMSE channel estimates even further is to include more a-priori knowledge in the power delay profile $R_p(t)$. For example, all three channel models used in the reference sensitivity tests (TU50, HT100, RA250) have the energy concentrated towards the first taps, and this knowledge can be incorporated into the power delay profile $R_p(t)$ by shaping it accordingly. An exponential power delay profile was chosen as indicated by way of example in Figure 7, so that the taps from all channels (TU50, HT100, RA250) lie underneath thus guaranteeing that performance does not decrease for any possible channel realisation. The simulation results confirmed an additional BER performance improvement of $0.3dB$ for the TU50 and RA250 channels, and $0.2dB$ for the HT100 channel.

6 Conclusion

A new LMMSE channel estimation algorithm is presented that easily outperforms the "classical" LS-type channel estimator. This is accomplished by incorporating a-priori information about known modulation and receive filters. The merit of the new channel estimator is confirmed by yielding an MSE approximately 1 - 2 dB higher than the LS estimator. For the GSM system, the improved channel estimate translates into BER improvements of up to $0.25dB$. On one hand, this improvement is not enormous, but on the other hand the computational complexity required by the LMMSE channel estimator is comparable to that of the LS channel estimator. Much more complex channel tracking algorithms that try to accurately estimate the time-variations of the channel within one burst achieve BER improvements in the same ballpark. If additional knowledge about the shape of the power delay profile is available at the receiver, further BER improvements of up to $0.3dB$ are possible. The LMMSE approach is not limited to GSM systems and can be applied to any communication system that uses a training sequence assisted channel estimator.

Acknowledgment

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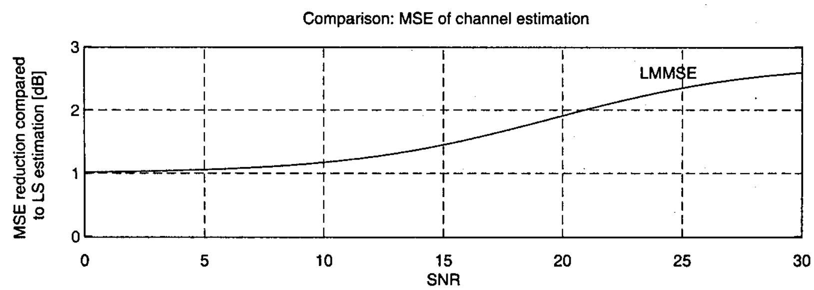

Figure 3: Performance comparison of LS and LMMSE channel estimation schemes.

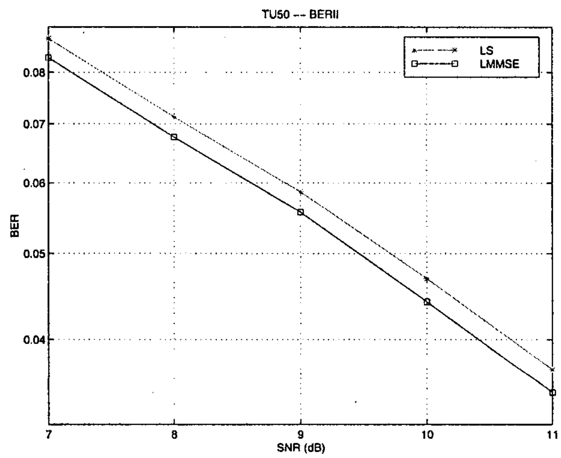

Figure 4: Reference sensitivity test, class II bits, TU50 channel.

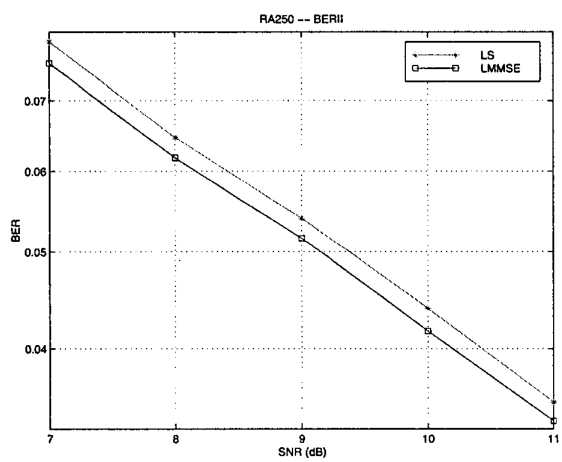

Figure 5: Reference sensitivity test, class II bits, RA250 channel.

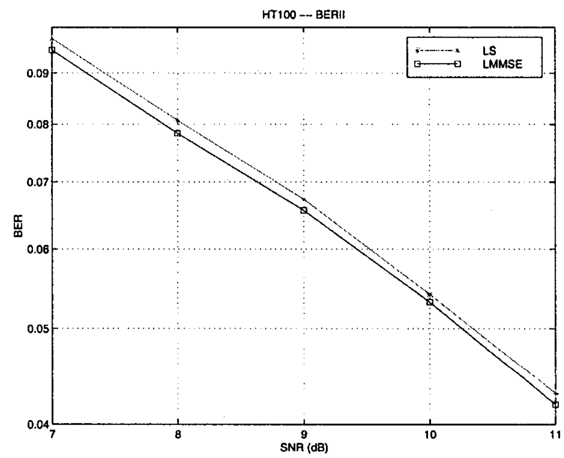

Figure 6: Reference sensitivity test, class II bits, HT100 channel.

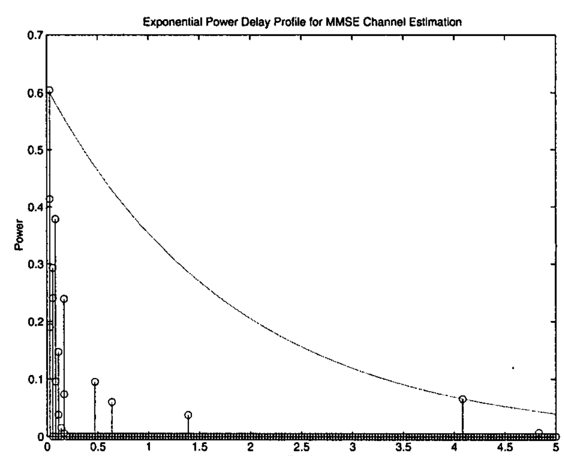

Figure 7: Exponential Power Delay Profile.