Impact of Blind versus Non-Blind Channel Estimation on the BER Performance of GSM Receivers

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Abstract

We investigate in this paper whether the HOS-based blind channel estimation method EVI (EigenVector approach to blind Identification) can compete with the non-blind cross-correlation-based scheme used in state-of-the-art GSM receivers (Global System for Mobile comm.). For blind, non-blind, and ideal estimates of COST-207[2] *mobile radio channels, we give simulated bit error rates (BER) after Viterbi detection in terms of the mean signal-to-noise ratio* $(\overline{\mathrm{SNR}})$. *Averaged over three COST-207 propagation environments, EVI leads to an* $\overline{\mathrm{SNR}}$ *loss of 1.2dB only, while it saves the 22% overhead in GSM data rate due to the transmission of training sequences. Since just 142 samples are used for channel estimation, we consider this performance outstanding for an approach based on HOS.*

1. Linear time-variant GSM channel model

CONSIDER[1] the equivalent baseband representation of a GSM communication system in Figure 1, where source and channel coding are omitted to enhance clarity. In the transmitter, coded information bits $d(k)$ together with reference bits $f(k)$ are assembled into *bursts* of 142 bits, where a value from $\{-1, 1\}$ is taken each bit period $T = 48/13\,\mu\mathrm{s} \approx 3.7\,\mu\mathrm{s}$. Each burst is encoded differentially to facilitate demodulation. Then, it is modulated by *Gaussian Minimum Shift Keying (GMSK)* with $f_{3\,\mathrm{dB}} = 0.3/T$ and transmitted over the multipath radio channel.

In a mobile scenario, the physical multipath radio channel is *time-variant* with a baseband impulse response depending on the time difference τ between the observation and excitation instants as well as the observation time t. We adopt the stochastic *Gaussian Stationary Uncorrelated Scattering (GSUS)* model leading to the following impulse response of the channel including the receive filter [1]

$$h_c(\tau, t) = \frac{1}{\sqrt{N_e}} \sum_{\nu=1}^{N_e} e^{j(2\pi f_{d,\nu} t + \Theta_\nu)} \cdot g_{Rc}(\tau - \tau_\nu)\,, \quad (1)$$

where N_e is the no. of elementary echo paths, $g_{Rc}(\tau)$ denotes the receive filter impulse response, and the subscript in $h_c(\cdot)$ suggests its continuous-time property. 3D sample impulse responses can easily be determined from (1) by independently drawing N_e Doppler frequencies $f_{d,\nu}$, N_e initial phases Θ_ν, and N_e echo delay times τ_ν from random variables with Jakes, uniform, and piecewise exponential probability density functions, respectively. As for the echo delay times τ_ν, we use standard COST-207[2] *Typical Urban (TU)*, *Bad Urban (BU)* and *Hilly Terrain (HT)* profiles.

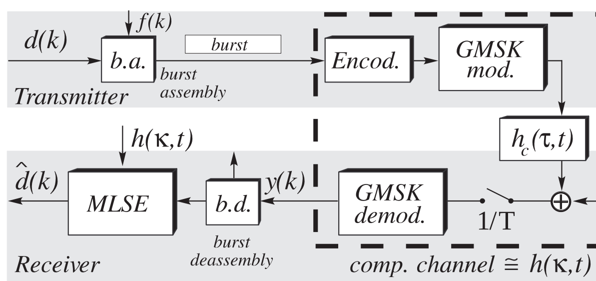

Figure 1: GSM communication system

According to Figure 1, each received burst is corrupted by additive Gaussian noise $r(t)$ which is colored by the 5th order Butterworth receive filter $g_{Rc}(\tau)$ with cut-off frequency at $75\,\mathrm{kHz}$. We use this filter throughout the paper for reasons of adjacent channel suppression (GSM carrier spacing is $200\,\mathrm{kHz}$). Upon symbol-rate sampling, a simple derotation demodulator can be used to obtain $y(k)$.

Maximum Likelihood Sequence Estimation (MLSE) represents the optimum procedure to remove intersymbol interference from a received digital communication signal such as $y(k)$. However, it assumes the "composite channel",

[1]This research is supported by the German NSF (DFG). *Matlab* programs and compressed postscript files of our publications are also available from our WWW server `http://www.comm.uni-bremen.de`.

[2]European Cooperation in the field of Scientific and Technical research.

i.e. the equivalent symbol-rate system between $d(k)$ and $y(k)$ (see the dashed frame in Figure 1), to be *(R1)* linear with finite impulse response which *(R2)* must be known. Moreover, *(R3)* the noise component of $y(k)$ is supposed to be white. Strictly speaking, none of the requirements *(R1)* to *(R3)* is met in typical GSM systems.

However, as for *(R1)*, it can be shown that the composite channel can be approximated by the *linear* model

$$h(\kappa, t) = \begin{cases} j^{-\kappa} \cdot g_0(\kappa T, t) & \text{for } \kappa \in \mathcal{K} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

with

$$g_0(\tau, t) \triangleq c_0(\tau) * h_c(\tau, t), \tag{3}$$

where $c_0(\tau)$ represents the Laurent approximation [2] of the (non-linear) GMSK modulator, "$*$" denotes the convolution operator and the factor $j^{-\kappa}$ takes the combined effects of differential encoding and derotation demodulation into account. In order to obtain a (time-variant) FIR model, let $h(\kappa, t)$ be limited to the range $\mathcal{K}$ of "relevant" indices κ.

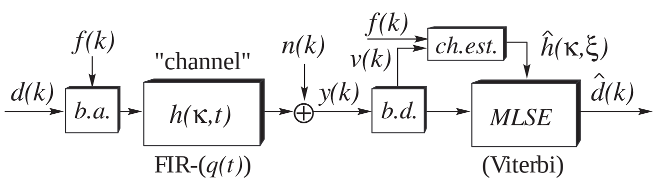

Figure 2: GSM system with linear time-variant channel model

Using the linear model (2) for the composite channel, Figure 1 can be redrawn according to Figure 2. In the sequel, $h(\kappa, t)$ will simply be termed "channel". Note that for any fixed value t_0, the channel slice $h(\kappa, t_0)$ may be mixed-phase. Let $q(t_0)$ denote its effective order, which may vary from slice to slice due to time selective fading. The noise sequence $n(k)$ emerges from $r(t)$ in Figure 1 by symbol-rate sampling and demodulation. Although $n(k)$ is colored due to the Butterworth receive filter, this is tolerated in typical GSM receivers (c.f. requirement *(R3)*).

According to *(R2)*, MLSE requires the knowledge of $h(\kappa, t)$ in order to equalize a block of the demodulated sequence $y(k)$. As this knowledge is not available, the problem of *channel estimation* arises. This is indicated in Fig. 2 by a box termed "ch.est.". Let $\hat{h}(\kappa, \xi)$ denote the estimate which will be used to equalize the ξ-th burst.

In the following section, we focus on two algorithms to calculate estimates $\hat{h}(\kappa, \xi)$. They will be compared in section 3 in terms of their channel estimation quality as well as the resulting bit error performance after MLSE.

2. Blind and non-blind channel estimation

Channel estimation is a particular form of *system identification*. All methods we apply in this paper suppose the system to be mixed-phase, linear and to have a finite impulse response. With time-variance being relatively slow in both GSM-900 and DCS-1800 applications ($f_{d,max}^{-1} = (226\,\mathrm{Hz})^{-1} \approx 4.4\,\mathrm{ms} \gg T$), the channel can also be assumed *piecewise (quasi) time-invariant*, i.e. time-invariant over a certain number Δk of bit periods T

$$h(\kappa, t) \approx h(\kappa, t_0) \quad \text{for} \quad |t - t_0| \leq \Delta k\, T/2\ . \tag{4}$$

If at most Δk consecutive samples of $y(k)$ are observed at once, $y(k)$ can be considered quasi stationary so that the respective channel estimation algorithm may assume a mixed-phase linear time-invariant FIR system $h(\kappa, t_0)$. Generally, a system can be identified with or without reference data (training sequences).

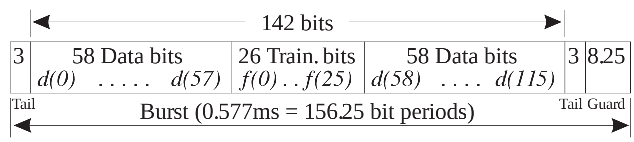

Figure 3: GSM "normal" burst

As an example for a channel estimation approach based on training sequences, consider the cross-correlation (CC) scheme used in state-of-the-art GSM receivers. According to Figure 3, each "normal" burst contains a training sequence $f(k)$ of 26 bits surrounded by two packets of 58 data bits emerging from the information sequence $d(k)$. On the assumption of time-invariance over $\Delta k = 21$ bit periods, channel estimates $\hat{h}(\kappa, \xi)$ can be derived from the received sequence sampled at symbol-rate by using the sample cross-correlation between the demodulated (corrupted) and the stored (ideal) training sequences $v(k)$ and $f(k)$, respectively (see Fig. 2). For any given order $\hat{q}$ of the FIR system to be estimated, the estimate is given by

$$\begin{bmatrix} \hat{h}(0, \xi) \\ \vdots \\ \hat{h}(\hat{q}, \xi) \end{bmatrix} = \mathbf{F} \cdot \begin{bmatrix} v(5) \\ \vdots \\ v(20 + \hat{q}) \end{bmatrix}, \tag{5}$$

where $\mathbf{F}$ denotes the $(\hat{q} + 1) \times (\hat{q} + 16)$ Toeplitz matrix containing the orthogonal part of the training sequence

$$\mathbf{F} \triangleq \frac{1}{16} \begin{bmatrix} f(5) & \cdots & f(20) & & 0 \\ & \ddots & \ddots & \ddots & \\ 0 & & f(5) & \cdots & f(20) \end{bmatrix}. \tag{6}$$

N.B.: Although the above channel estimation scheme supposes time-*in*variance of the channel over a period of $\Delta k = 21$ training bits only, the resulting estimate is used for MLSE on the adjacent data fields. As the channel coefficients might already have changed in the data fields, there is an implicit assumption of *quasi time-invariance over one burst* ($\Delta k = 142$) in this concept.

The repeated transmission of training sequences leaves a GSM system with an overhead capacity of $26/116 = 22.4\%$. This capacity could be used for other purposes if the channel was estimated from the received signal only (*blind* system identification). As just one symbol-rate sampled received sequence is available in GSM mobile units, in the downlink, this can only be accomplished by exploiting HOS of the (quasi) stationary demodulated sequence $y(k)$.

Remark: If the sampling period was a fraction of T, or alternatively, the symbol-rate sampled signals received by several antennae were interleaved (in the uplink, e.g.), the resulting demodulated sequence is (quasi) *cyclostationary*. Generally, *Second Order Cyclostationary Statistics* (SOCS) are sufficient to retrieve the channel. However, there are "singular" channel classes which can *not* be identified [3]. We have shown in [4] that singular channels represent a severe limitation to SOCS-based methods because estimation performance from few samples is heavily affected if subchannel zeros are just "close" to each other. As this can *not* be prevented in a mobile environment, SOCS-based algorithms are neglected in this paper.

In summary, we have used the following criteria for the selection of a blind channel estimation algorithm:

(1) **Reliable channel estimates must be obtained from 142 samples of the demodulated sequence $y(k)$ only.**
(2) This should apply to arbitrary channels (even if there are zeros on/close to the complex plane's unit circle).
(3) As the effective channel order is unknown (and time-variant), an order misfit must not represent a problem.
(4) The estimates should be as robust as possible with respect to stationary additive Gaussian noise.

We have selected the EIGENVECTOR APPROACH TO BLIND IDENTIFICATION (EVI) by Kammeyer, Jelonnek, and Boss [5, 6], which is based on 4th (and 2nd) order statistics, because recent simulation results suggested that it meets the above criteria. For linear modulation with raised cosine transmit and receive filtering, we have demonstrated in [7] that EVI can blindly estimate COST-207 channels from 142 samples within a normalized mean square error bound of about 5 per cent (at a constant SNR of $7\,\mathrm{dB}$). EVI's estimation performance was also compared with methods based on SOCS [4] and HOS [7]: On the above conditions, EVI's estimation performance was found to be superior in both cases. Finally, for GSM data transmission over COST-207 mobile channels, we have compared EVI with the optimum non-blind least squares (LS) scheme in terms of the resulting bit error rate (BER) after MLSE. We have demonstrated that EVI entails an average $\overline{\mathrm{SNR}}$ loss of $1.1\,\mathrm{dB}$ only [8]. While the channel used for those simulations was time-*in*variant within each burst, we apply true time-variant filtering in this paper and investigate the effects on channel estimation as well as BER performance.

3. Simulation results

Fig. 4 shows the magnitude impulse response $|g_0(\tau,t)|$ according to (3), where $h_c(\tau,t)$ is a sample COST-207 *Hilly Terrain (HT)* channel obtained from (1) with $N_e = 100$. Both time axes are normalized to the GSM bit period $T \approx 3.7\,\mu\mathrm{s}$. The max. Doppler frequency is set to $f_{d,max} = 200\,\mathrm{Hz}$. For GSM-900 and DCS-1800, this corresponds to velocities of the mobile unit in the ranges $225\ldots243\,\mathrm{km/h}$ and $115\ldots126\,\mathrm{km/h}$, respectively. For Fig. 4, eq. (3) is evaluated over a t range covering 3 min. Doppler periods $T_{d,min} = 1/f_{d,max} = 5\,\mathrm{ms}$, i.e. $15\,\mathrm{ms}$, or equivalently, 4062 bit periods or 26 burst periods. The surface lines are obtained by sampling $|g_0(\tau,t)|$ four times each bit period on the τ axis and once each burst period ($156.25\,T$, c.f. Fig. 3) on the t axis. Note that the magnitude impulse response $|h(\kappa,t)|$ of the linear model (2), which is the time-variant FIR system to be estimated, can also be seen at $\tau = \kappa T$.

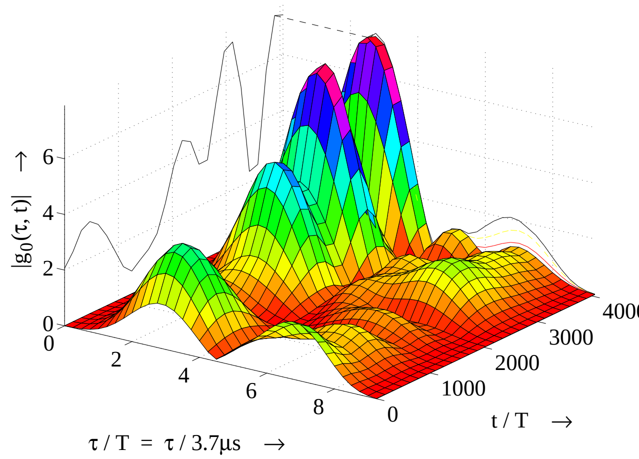

Fig. 4: Linear model of a sample *Hilly Terrain* composite channel

For the simulation of a GSM data transmission link according to Fig. 1, $h_c(\tau,t)$ is required rather than $g_0(\tau,t)$. However, $|h_c(\tau,t)|$ looks pretty much like Fig. 4 because of the bell-shaped form of the Laurent impulse $c_0(\tau)$. The "hills" are just slightly narrower (in direction of τ).

In order to ensure *(i)* meaningful measurements of BER and *(ii)* a satisfactory approximation of a GSUS channel by a sample impulse response $h_c(\tau,t)$, we use a range of t for the simulations which is much wider than the one displayed in Fig. 4. Let D_ξ denote the ξ-th burst containing 156 bits with values from $\{-1,1\}$. For the results given below, a total of 2000 bursts $D_0,\cdots,D_{1999}$ is transmitted, i.e. 232000 information bits. Since at most each 4th burst is sent to/from the same mobile station, this covers a t range of 8000 burst periods, i.e. $4.6\mathrm{s}$ or 924 min. Doppler periods. To take the channel's time-variance within each burst period into account, $h_c(\tau,t)$ is sampled seven times per burst at

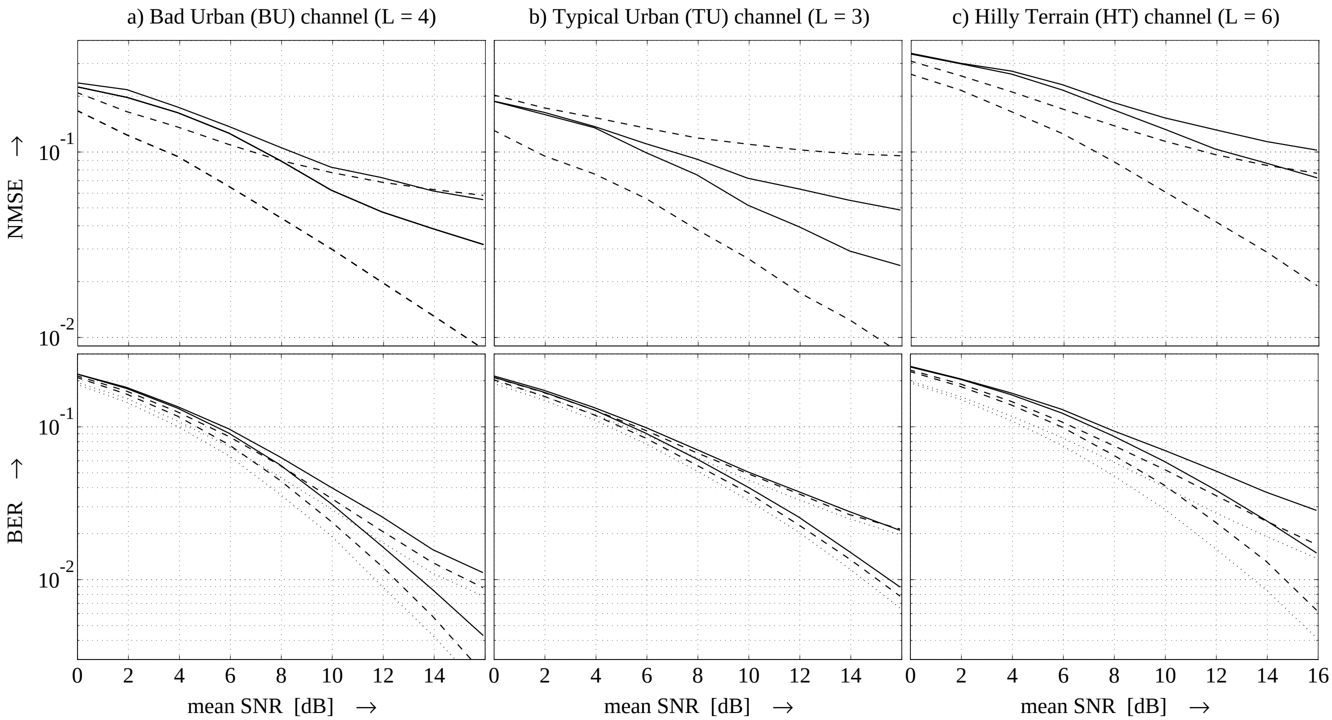

Figure 5: NMSE (above) and BER (below) in terms of $\overline{\mathrm{SNR}}$ for the estimates of three sample COST-207 GSUS channels
solid: blind EVI channel estimation, dashed: non-blind CC channel estimation, dotted: "ideal" non-blind channel estimation

$t_{\xi,\mu} = (4 \cdot 156.25\,T)\,\xi + (26\,T)\,\mu$ with $\mu = 0, \cdots, 6$ and then interpolated linearly.

For the following description of the simulation procedure, assume that a sample impulse response $h_c(\tau, t)$ and a value of the mean signal-to-noise ratio $\overline{\mathrm{SNR}}$ was selected. Referring to Fig. 1, the burst D_ξ is encoded, modulated, and then propagated through the time-variant channel $h_c(\tau, t)$, where $t_{\xi,0} \le t \le t_{\xi,6}$. Gaussian noise $r(t)$, colored by the receive filter, is added according to $\overline{\mathrm{SNR}}$. Upon symbol-rate sampling and demodulation, 148 samples of $y(k)$ are obtained. Then, both the non-blind cross-correlation (CC) and the blind EVI channel estimation schemes are applied to $y(k)$, where the former utilizes the training midamble (see eq. (5)) while the latter uses 142 samples[3] of $y(k)$. Both approaches are given the number $L = \hat{q}+1$ of channel coefficients to be estimated, where L is calculated from the effective length of the sample power delay spectrum of the GSUS channel. Note that the actual mean effective length $\bar{q}+1$ of the channel $h(\kappa, t)$ may well be shorter due to time selective fading (see Fig. 4 at $t \approx 1200T$, e.g.). The resulting channel estimates $\hat{h}(\kappa, \xi)$ are then passed to the Viterbi detector to obtain the estimated bursts $\hat{D}_\xi$.
In the frame of Monte-Carlo runs, this procedure is executed for 2000 bursts $D_0, \cdots, D_{1999}$ and $\overline{\mathrm{SNR}}$ values in the range from 0 to $16\,$dB. Finally, BER is calculated from all bursts $\hat{D}_\xi$ and D_ξ transmitted at a given $\overline{\mathrm{SNR}}$.

[3]For EVI, the training sequence was replaced with 26 additional data bits.

For each of the seven true channel slices $h(\kappa, t_{\xi,\mu})$ per burst ($\mu = 0, \cdots, 6$), the *Normalized Mean Square Error (NMSE)* of the associated estimate $\hat{h}(\kappa, \xi)$ is calculated[4]. Averaging over all values of ξ delivers the quality measure

$$\mathrm{NMSE}(\mu) \triangleq \frac{1}{2000} \sum_{\xi=0}^{1999} \frac{\sum_\kappa |\hat{h}(\kappa,\xi) - h(\kappa, t_{\xi,\mu})|^2}{\sum_\kappa |h(\kappa, t_{\xi,\mu})|^2} \quad (7)$$

at seven instants $26T\mu$ per burst. Finally, a weighted average over all values of μ is calculated to obtain an overall measure (denoted NMSE) for each value of $\overline{\mathrm{SNR}}$.

Figure 5: For three sample COST-207 GSUS channels, this figure displays the performance of both the channel estimation algorithms and the Viterbi detector in terms of $\overline{\mathrm{SNR}}$. The upper row of subplots displays the NMSE values on a logarithmic scale, while the bottom row shows the bit error rates (BER). For the Figures 5a, b, and c, a *Bad Urban (BU)*, a *Typical Urban (TU)*, and a *Hilly Terrain (HT)* sample channel is used, where the number of coefficients of each estimate is set to $L = 4$, 3, and 6, respectively. The solid lines refer to EVI, while the dashed lines indicate the performance of the CC approach. All subplots contain two lines of either type, where the bottom one is obtained

[4]As all blind system identification methods can *not* estimate one complex factor, each estimate was multiplied with the optimum constant ($\Rightarrow$ min. Euclidean distance from the true channel) before NMSE was calculated. To ensure fairness of the comparison, this is done for CC's estimates, too.

by suppressing time-variance of the channel within each burst (although this is in accordance with the assumption made by the channel estimation and Viterbi algorithms, it is quite unrealistic and meant for reference only). Note that all curves suffer from the linearity assumption for the GMSK modulator as well as from the order misfit between L and the mean effective length $\bar{q}+1$ of the current channel.
From the bottom CC and EVI lines in the upper subplots, we realize that for EVI, NMSE is 1.3 to three times as high as for CC, where the factor 1.3 holds for low values of $\overline{\mathrm{SNR}}$, while 3 applies above $14\,\mathrm{dB}$. From the upper solid and dashed lines, however, we realize that CC's estimates are heavily affected by time-variance within each burst period while those of EVI degrade moderately so that estimation quality becomes rather similar. This is due to the fact that non-blind approaches have no information about the channel coefficients outside the training midamble, while blind approaches estimate some sort of *mean* channel slice. Note that for the *TU* channel, EVI even outperforms CC.
With the bottom subplots of Fig. 5, dotted lines are added for comparison. They are obtained when using "ideal" non-blind channel estimates for MLSE, i.e. $\hat{h}(\kappa,\xi) = h(\kappa, t_{\xi,3})$ for $\kappa = 0,\cdots,L-1$. For the channels with suppressed time-variance within each burst (bottom three lines), we realize that at $\mathrm{BER} \approx 2\%$, blind EVI channel estimation requires about $1\,\mathrm{dB}$ *(BU)*, $0.4\,\mathrm{dB}$ *(TU)*, and $2.2\,\mathrm{dB}$ *(HT)* more in $\overline{\mathrm{SNR}}$ than the non-blind CC scheme. If the quasi time-invariance assumption over each burst is violated (upper three lines) BER levels off for high values of $\overline{\mathrm{SNR}}$. For the *BU* channel, again, EVI requires about $1\,\mathrm{dB}$ more in $\overline{\mathrm{SNR}}$ than CC. While this loss falls to zero for the *TU* channel, it increases to about $2.5\,\mathrm{dB}$ for the sample *HT* channel. Averaging over the three sample channels results in a mean loss of about $1.2\,\mathrm{dB}$.

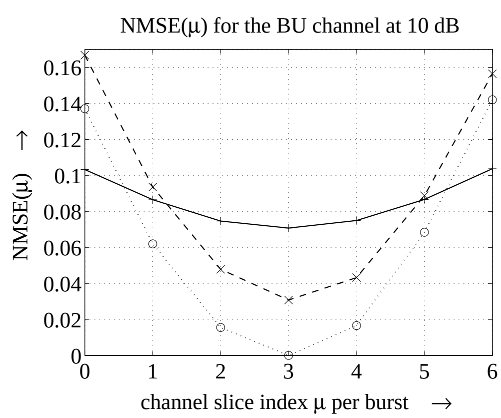

Figure 6: NMSE in terms of the channel slice index μ per burst for the EVI (solid), CC (dashed) and "ideal" (dotted) estimates of the *BU* channel at $\overline{\mathrm{SNR}} = 10$ dB

Figure 6: For the *Bad Urban (BU)* simulation results at $10\,\mathrm{dB}$, this figure provides a detailed view of channel estimation quality in terms of the bit index per burst. It shows the evolution of NMSE as a function of the channel slice index μ. Obviously, "ideal" estimates (dotted line) based on the training midamble have a vanishing value of $\mathrm{NMSE}(\mu)$ in the center of each burst only ($\mu = 3$), while it increases as the burst boundaries are approached. At higher values, CC's estimates (dashed line) reveal a similar behavior. Although EVI's estimation quality is inferior at the center, it degrades just slightly towards the beginning and the end of the burst, as can be seen from the solid line.

4. Conclusions

We have demonstrated that for GSM data transmission over mobile comm. channels, the HOS-based blind channel estimation method EVI entails an $\overline{\mathrm{SNR}}$ loss of $1.2\,\mathrm{dB}$ only (averaged over three COST-207 propagation environments) compared with the non-blind cross-correlation scheme. As just 142 samples of the demodulated sequence can be used for channel estimation, we consider these results quite remarkable for an algorithm based on HOS.

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