SIMPLIFIED VERSIONS OF THE MAXIMUM-LIKELIHOOD FREQUENCY DETECTOR

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ABSTRACT

We investigate several simplifications of the maximum-likelihood frequency detector (MLFD) proposed by Gardner to extend the acquisition range of carrier recovery loops. The basic simplification concerns the implementation of the frequency-matched filter (FMF) required by this detector. Another simplification consists of replacing the original MLFD by its polarity-type version. It is shown that these simplifications do not reduce the frequency range over which the average detector output has the same sign as the frequency offset. We also investigate the effect of reducing the oversampling factor in digital implementation of the detector. Our results show that the complexity of the MLFD can be substantially reduced without significantly degrading the loop acquisition performance.

INTRODUCTION

One of the basic tasks of the receiver in digital communication systems is to regenerate a local carrier and demodulate the received signal. The carrier recovery loop takes the form of a phase-lock loop (PLL) which, in its simplest form, comprises a phase detector (PD), a loop filter, and a voltage-controlled oscillator (VCO). The two basic requirements for the carrier recovery loop are: (a) a small phase jitter compatible with the steady-state performance requirements, and (b) a large acquisition (pull-in) range compatible with the maximum frequency offset of the oscillators used.

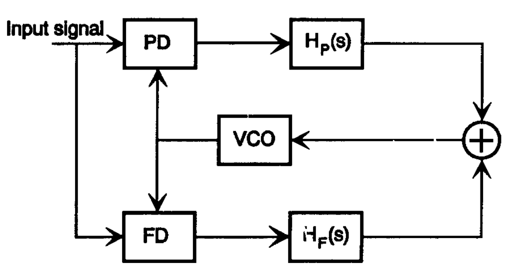

Fig. 1. Acquisition-aiding technique using a frequency detector

In many applications, these two requirements are incompatible, and the loop acquisition range must be extended using an acquisition-aiding technique such as frequency sweeping, switching of the loop filter, and the use of an auxiliary frequency detector (FD) [1]. The latter technique, illustrated in Fig. 1, turns out to be the most effective one, as it leads to the smallest acquisition times. The loop acquisition behavior is dictated by the FD and the filter of transfer function $H_F(s)$, whereas its steady-state performance is determined by the PD and the filter of transfer function $H_P(s)$. An even more elegant solution for aided-acquisition is to combine the PD and FD functions in a single circuit, called phase and frequency detector (PFD). The loop then comprises a single arm (instead of 2), and the detector behaves as an FD during acquisition and as a PD in the steady-state. Such PFD's were proposed in [2] and [3] and used today in many digital radio systems. Roughly speaking, this technique allows to cope with frequency offsets not exceeding 10% of the baud rate, which is usually the case in high- and medium-capacity digital radio systems.

Unfortunately, these PFD's are not adequate for low-capacity radios where the carrier frequency offset may be as large as the baud rate. To achieve a normalized acquisition range of that magnitude, classical frequency detectors such as the quadricorrelator remains the best alternative. The quadricorrelator was presented in [4] and [5], and further analyzed in [6]. It was shown in [6] that the quadricorrelator tracks the center-of-gravity of the received signal spectrum.

Another FD is the maximum-likelihood (ML) frequency estimator recently developed by Gardner [7]. The use of this detector in the loop leads to an acquisition range which may be in excess of the baud rate. Its structure surprisingly resembles that of the quadricorrelator. The sole difference is that time-derivatives in the quadricorrelator are replaced by frequency-derivatives. The ML frequency detector (MLFD) easily lends itself to a digital implementation, but in addition to the conventional matched filter, it also involves a frequency matched filter.

The purpose of this paper is to investigate some simplifications of the MLFD which make it easier to implement in practice. Before presenting these simplifications, we briefly recall in the next section the derivation of the MLFD and give its characteristics in the presence of additive noise.

MAXIMUM-LIKELIHOOD FREQUENCY DETECTOR

We assume an M^2-state QAM signal transmitted over an additive white Gaussian noise (AWGN) channel. The received complex signal is of the form

$$r(t) = \sum_k d_k h(t - kT - \tau) e^{j(2\pi f_c t + \theta)} + n(t) \qquad (1)$$

where $h(t)$ is the transmit filter impulse response, the d_k's are the transmitted data symbols, and $n(t)$ is the additive noise. The symbol period T, the timing phase τ, the carrier frequency f_o, and the carrier phase θ are assumed unknown from the receiver.

For a given set of parameters $(\{\tilde{d}_k\}, \tilde{\tau}, \tilde{f}_o, \tilde{\theta})$, the log-likelihood function is

$$\mathcal{L} = \frac{2}{N_o} \int_{T_o} Re\{r(t) s^*(t)\} dt \qquad (2)$$

where N_o is the two-sided noise spectral density, T_o is the observation interval, and $s(t)$ is the signal given by

$$s(t) = \sum_k \tilde{d}_k h(t - kT - \tilde{\tau}) e^{j(2\pi \tilde{f}_o t + \tilde{\theta})}. \qquad (3)$$

Writing the observation interval as $T_o = NT$, and substituting (3) into (2), we can write

$$\mathcal{L} = \frac{2}{N_o} \sum_{k=0}^{N-1} [Re(d_k) Re(p_k) - Im(d_k) Im(p_k)] \qquad (4)$$

where p_k is the instantaneous output of the matched filter with impulse response $h(-t)$.

The data dependence of (4) is removed by averaging the log likelihood function over all possible values of d_k. Assuming that the real and imaginary parts of the complex symbols are independent and equally likely, and making some mathematical approximations that are valid at low signal-to-noise ratio (SNR) values, we obtain [7]

$$\mathcal{L} = \frac{M^2 - 1}{6} \sum_{k=0}^{N-1} \left[\left[\frac{2}{N_o} Re(p_k) \right]^2 + \left[\frac{2}{N_o} Im(p_k) \right]^2 \right]. \qquad (5)$$

Taking now the derivative of this function with respect to $\tilde{f}_o$, we obtain

$$\frac{\partial \mathcal{L}}{\partial \tilde{f}_o} \cong \frac{M^2 - 1}{3} \frac{2}{N_o} \sum_{k=0}^{N-1} [Re(p_k) Re(\dot{p}_k) + Im(p_k) Im(\dot{p}_k)] \qquad (6)$$

where $\dot{p}_k$ is the instantaneous output of the frequency matched filter with input $r(t)$. Dropping the multiplicative terms and using $N = 1$ in (6), we obtain a control signal

$$\varepsilon_k = Re(p_k \dot{p}_k^*) \qquad (7)$$

which represents the MLFD output at time $kT + \tilde{\tau}$. A block diagram of this detector is shown in Fig. 2. The real and imaginary parts of p_k are obtained by passing the in-phase (I) and quadrature (Q) components of the received signal to the matched filter whose impulse response is $h(-t)$. Similarly, the real and imaginary parts of $\dot{p}_k$ are obtained by passing the same components in a frequency-matched filter of transfer function $C(\omega) = \frac{\partial H(\omega)}{\partial \omega}$, or equivalently, of impulse response $c(t) = -j2\pi t h(-t)$.

After low-pass filtering, the MLFD output is used to control a local oscillator and synchronize it with the carrier frequency of the incoming signal. Of course, this detector can not be used for phase synchronization, and therefore an additional PD and loop filter are required in a parallel arm as shown in Fig. 1.

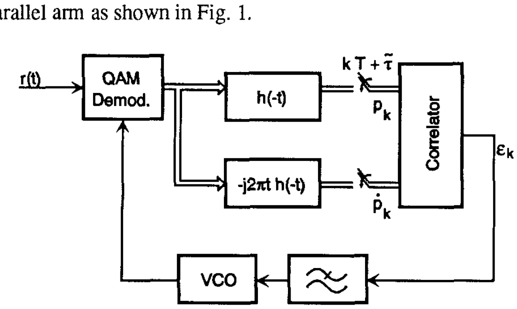

Fig. 2. Block diagram of the MLFD

The characteristic of an FD is its average output $E(\varepsilon_k)$ plotted as a function of the frequency offset $\Delta f = f_o - \tilde{f}_o$. For a noiseless channel, the FD characteristic of the MLFD is explicitly given in [7] and will not be repeated here for brevity. We only give the main result: The frequency range over which the average detector output has the correct sign extends up to $|\Delta f T| = 1 + \alpha$, where α is the roll-off factor of the Nyquist filter which represents the transmit and receive filters in cascade.

Now we will show that the results given in [7] for the noiseless case are also valid in the presence of noise. In other words, the FD characteristics are independent of the additive noise. To demonstrate this, we will write the detector output as

$$\varepsilon_n = \varepsilon_n^o + \varepsilon_n^b \qquad (8)$$

where ε_n^b is the contribution of noise. The terms ε_n^o and ε_n^b are respectively given by

$$\varepsilon_n^o = Re(p_n \dot{p}_n^*) \qquad (9)$$

and

$$\varepsilon_n^b = Re(p_n v_n^* + \dot{p}_n^* u_n + u_n v_n^*) \qquad (10)$$

where p_n and $\dot{p}_n$ are noiseless signals, and u_n and v_n are the noise terms at the outputs of the MF and FMF, respectively. Assuming that the additive noise is uncorrelated with the transmitted data symbols, we obtain

$$E(\varepsilon_n^b) = Re\{E(u_n v_n^*)\}$$
$$= Re\{R_{uv}(\theta)|_{\theta=0}\} \qquad (11)$$

where $R_{uv}(\theta)$ is the cross-correlation function of $u(t)$ and $v(t)$.

In the frequency domain, we can write

$$S_{uv}(f) = 2N_o H(f) C^*(-f) \tag{12}$$

and using this expression in (11), it follows that

$$E(\varepsilon_n^b) = 2N_o \int_{-\infty}^{\infty} H(f) C^*(-f)\, df. \tag{13}$$

Finally, since $H(f)$ is an even function and $C(f)$ is an odd function, we obtain $E(\varepsilon_n^b) = 0$, which completes the proof.

DIGITAL IMPLEMENTATION

In this section, we discuss the digital implementation of the MLFD. The oversampling factor will be denoted λ, i.e., the sampling frequency f_s is of the form $f_s = \lambda/T$. To avoid aliasing of the input signal spectrum, we must have

$$\lambda \geq 1 + \alpha + 2\Delta f \cdot T \tag{14}$$

and assuming that the frequency offset does not exceed $1/T$, we obtain $\lambda \geq 3 + \alpha$. It is, therefore, desirable to choose $\lambda = 4$ for digital implementation of the MLFD. In practical applications, we usually have $\alpha \leq 0.5$, and consequently, the oversampling factor $\lambda = 4$ ensures that there will be no spectral aliasing for frequency offsets up to $1.25/T$.

Unfortunately, this oversampling factor involves an excessive complexity in the implementation of the MF and FMF. To limit the complexity of this synchronizer, it is desirable to reduce the oversampling factor to $\lambda = 2$. In this work, we investigate the effect of the spectrum folding corresponding to $\lambda = 2$ on the MLFD characteristic for $0 \leq \alpha \leq 0.5$. The average detector output is given by the following expression:

$$\frac{1}{K}E(\varepsilon_n) =$$

$$\begin{cases} \frac{1}{2}\left[1 - \cos\frac{\pi}{\alpha}\Delta fT\right] + \frac{\pi}{4}\left[1 - \frac{\Delta fT}{\alpha}\right]\left[1 + \cos 2\pi\frac{\Delta\tau}{T}\right] \times \sin\frac{\pi}{\alpha}\Delta fT, & 0 \leq |\Delta fT| \leq \alpha \\ 1, & \alpha \leq |\Delta fT| \leq 1-\alpha \\ \frac{1}{2}\left[1 - \cos\frac{\pi}{\alpha}(\Delta fT - 1)\right] + \frac{\pi}{4\alpha}[\Delta fT - 1 + \alpha]\left[1 - \cos 2\pi\frac{\Delta\tau}{T}\right] \times \sin\frac{\pi}{\alpha}(1 - \Delta fT), & 1-\alpha \leq |\Delta fT| \leq 1 \end{cases} \tag{15}$$

where K is a constant related to the data variance and the raised-cosine filter dc-gain, and $\Delta\tau = \tau - \tilde{\tau}$.

Using $\Delta\tau/T = 0$, this function is plotted in Fig. 3 for different values of α. These results indicate that an acquisition range on the order of the baud rate may be achieved even with $\lambda = 2$. Next, using $\alpha = 0.5$ and $\Delta\tau/T = 0$, we compared the characteristics obtained with $\lambda = 2$ and $\lambda = 4$. The results are given in Fig. 4. They show that the two curves are identical over the range $|\Delta f \cdot T| \leq 1 - \alpha$. Beyond this value, the detector output is significantly reduced with $\lambda = 2$.

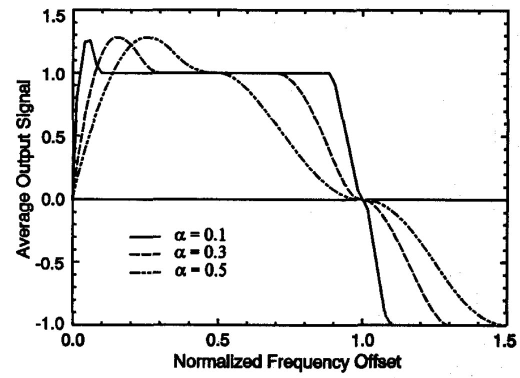

Fig. 3. MLFD characteristics with $\lambda = 2$

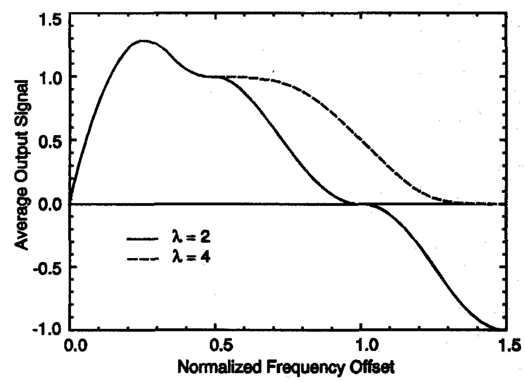

Fig. 4. MLFD characteristics with $\lambda = 2$ and $\lambda = 4$ for $\alpha = 0.5$ and $\Delta\tau/T = 0$

The results of this section demonstrate that digital implementation of the MLFD can be substantially simplified while still guaranteeing a useful frequency range on the order of the baud rate.

FURTHER SIMPLIFICATIONS

In this section, we turn our attention to the implementation of the FMF. We propose a modification of the FMF which makes it extremely simpler to implement. The impulse response of the modified FMF is

$$c'(t) = -j\,\mathrm{sgn}(t)\, h(-t) \tag{16}$$

where sgn (.) stands for the mathematical sign function.

Assuming that the MF is implemented using an FIR filter with $2L + 1$ taps whose tap-gains are denoted $h_{-L}, h_{-L+1}, \ldots, h_0, \ldots, h_{L-1}, h_L$, a slight modification of this filter makes it possible to compute both the p_n and $\dot{p}_n$ signals required to implement the MLFD. This is sketched in Fig. 5, where the MF output is

$$p_n = \gamma_n^{-1} + \gamma_n^0 + \gamma_n^1 \tag{17}$$

with

$$\gamma_n^{-1} = \sum_{i=-1}^{-L} h_i x_{n-i},$$

$$\gamma_n^0 = h_0 x_n,$$

$$\gamma_n^1 = \sum_{i=1}^{L} h_i x_{n-i},$$

and where x_n represents the received baseband signal.

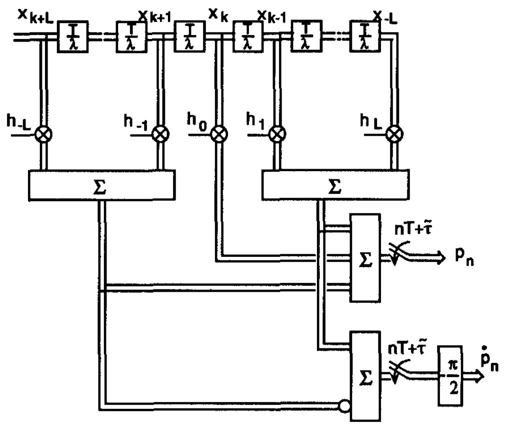

Fig. 5. Block diagram of the modified MLFD

Once the partial products γ_n^{-1}, γ_n^0, and γ_n^1 are computed, the FMF output $\dot{p}_n$ is computed as

$$\dot{p}_n = j(\gamma_n^1 - \gamma_n^{-1}). \tag{18}$$

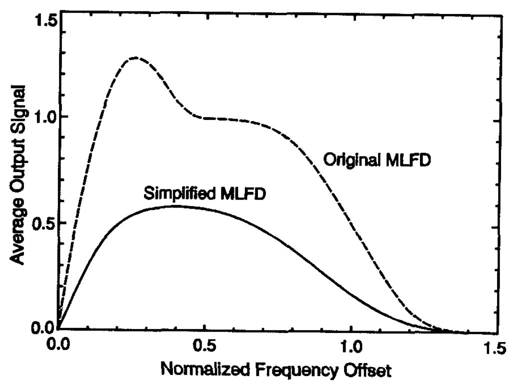

Fig. 6. Simplified MLFD characteristic

Performance of the modified MLFD was evaluated by means of computer simulations. A QPSK system with a roll-off factor $\alpha = 0.5$ and 17-tap matched filters were used in these simulations. The FD characteristic is plotted in Fig. 6 and compared to that of the original MLFD. It turns out that this simplification of the MLFD does not significantly affect its characteristic.

Finally, we investigate the performance of a polarity-type MLFD whose output is

$$\varepsilon_n = \mathrm{sgn}\,(Re\,(p_n))\,\mathrm{sgn}\,(Re\,(\dot{p}_n)) + \mathrm{sgn}\,(Im\,(p_n))\,\mathrm{sgn}\,(Im\,(\dot{p}_n)) \tag{19}$$

This version of the MLFD substantially reduces the complexity of the correlator needed to compute ε_n. It also reduces the complexity of the digital loop filter, because its input ε_n is a 3-state signal which takes its values from the set {+2,0,-2}.

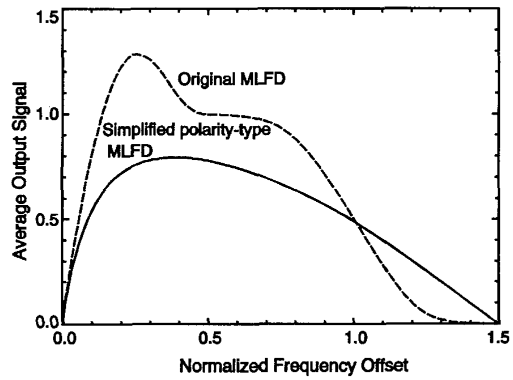

Fig. 7. Simplified polarity-type MLFD characteristic

Combining the two simplifications presented in this section results in a substantially reduced complexity. The FD characteristic of this detector is shown in Fig. 7 for $\alpha = 0.5$. These results clearly indicate that the useful frequency range remains equal to $1 + \alpha$ as in the original MLFD.

SUMMARY AND CONCLUSIONS

We have investigated several simplifications of the MLFD proposed by Gardner [7]. First, we have examined the effect of reducing the oversampling factor at the input of the MF and the FMF required by this synchronizer. It was shown that with an oversampling factor of 2, the useful frequency range is $0 \le |\Delta f| \le 1/T$.

Next, we examined a simplification of the FMF which makes its output easily computed from the partial products used to generate the output of the MF placed in parallel with this filter. This modification of the MLFD substantially reduces its implementation complexity without significantly degrading its characteristics.

Finally, we investigated the performance of the polarity-type version of the MLFD which only uses the sign information of the detector output. It was found that this simplification does not degrade the FD characteristics. It thus turns out that the MLFD complexity can be substantially reduced while still maintaining a large frequency range over which the average detector output has the correct sign.

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