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# DEROTATION TECHNIQUES IN RECEIVERS FOR MSK-TYPE CPM SIGNALS

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## 1. INTRODUCTION

Binary continuous phase modulations (CPM) with a modulation index of $h$=0.5 represent an important class of constant-envelope modulation schemes with fast spectral roll-off [1]. Starting from the classical full response Minimum Shift Keying (MSK) [2], a multitude of MSK-type partial response CPM schemes has been derived in the past. Amongst these are well-known modulations such as Raised-Cosine MSK, Gaussian MSK (GMSK), Tamed Frequency Modulation (TFM), and Generalized TFM (GTFM) [1].

In this paper, a signal processing technique called '*derotation*' will be presented which simplifies the structure and reduces the complexity of receivers for MSK-type CPM signals.

Section 2 leads off with a brief review of MSK-type CPM signals. By applying a linear approximation introduced in [3], it will be shown in Section 3 that these signals can be modelled as quadrature amplitude modulated (QAM) signals with a rotational signal structure. In Section 4, a generalized derotation technique is derived which abolishes this rotational signal structure and, thus, results in a simplified linear transmission model. Easy-to-implement versions of the derotation technique are presented in Section 5.

In Section 6, we will demonstrate how the derotation technique can be applied to different types of receivers for MSK-type CPM signals. In particular, a simple coherent threshold receiver, a noncoherent matched filter receiver for MSK-type spread spectrum signals, and an adaptive maximum likelihood Viterbi receiver for signals transmitted via intersymbol interference (ISI) channels will be considered.

## 2. MSK-TYPE CPM SIGNALS

A CPM signal has the general form

$$\tilde{s}(t) = A \cos\,[2\pi f_0 t + \Phi(t,\underline{b}) + \Phi_0] \qquad (1)$$

where $f_0$ is the carrier frequency, $A$ is an amplitude factor assumed to be 1, and $\Phi_0$ is a constant phase offset assumed to be 0 in the following. With the data sequence to be transmitted denoted by $\underline{b} = (\ldots, b_i, \ldots)$, the modulating phase function is given by [1]

$$\Phi(t,\underline{b}) = 2\pi h \sum_{i=-\infty}^{\infty} b_i \int_{-\infty}^{t-iT} g(\tau)\mathrm{d}\tau \,. \qquad (2)$$

$h$ is the modulation index, $T$ is the symbol period, and $g(t)$ is a smooth pulse shaping function over a finite time interval $0 \le t \le LT$ and zero outside. $g(t)$ determines the smoothness of the continuous phase frequency modulation and, hence, has a great influence on the spectral properties of the CPM signal [1].

Following common conventions, we use '*MSK-type CPM signals*' as synonym for CPM signals characterized by a modulation index of $h$ = 0.5 and by binary data symbols $b_i$ taking on the values +1 and -1 [1], [3]. Furthermore, we will present all signals in complex baseband notation. With $\mathrm{j} = \sqrt{-1}$ the CPM signal in baseband notation is

$$s(t) = \exp\,[\mathrm{j}\Phi(t,\underline{b})] \qquad (3)$$

## 3. LINEAR QAM SIGNAL MODEL

According to [3] a $h$=0.5 CPM signal $s(t)$ can be closely approximated by the linear QAM signal

$$v(t) = \sum_{i=-\infty}^{\infty} \exp\,(\mathrm{j}\frac{\pi}{2} \sum_{k=-\infty}^{i} b_k)\; p(t-iT) \,. \qquad (4)$$

$p(t)$ is a real-valued pulse shaping function spanning the time interval $0 \le t \le (L+1)T$. If $p(t)$ is properly matched to the frequency pulse shape $g(t)$ of equ. (2), the approximation error is negligible [3]. As an example, $p(t)$ is plotted in Fig. 1 for a GMSK signal with $BT$=0.3 and with $g(t)$ truncated to a length of $5T$ [1].

For MSK-type CPM signals with $b_i = \pm 1$, $v(t)$ can be further simplified to

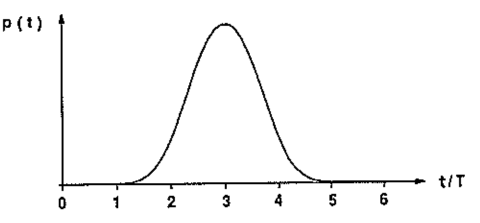


Fig. 1 Pulse shape $p(t)$ for GMSK ($BT$=0.3, $L$=5)

$$v(t) = \sum_{i=-\infty}^{\infty} c_i\, j^i\, p(t-iT) \qquad (5)$$

where the modified data symbols $c_i = \pm 1$ are determined by the recursion

$$c_i = c_{i-1}\, b_i \,. \qquad (6)$$

Obviously, the data sequence $\underline{c} = (\ldots, c_i, \ldots)$ is obtained from $\underline{b}$ by differential encoding assuming that the symbols +1 and -1 represent the logical 0 and 1 states, respectively. It is worth noting that an equivalent version of (5) exists in which j is substituted by -j and $b_i$ in (6) is substituted by $-b_i$.

According to (5) and (6), the MSK-type CPM signal may be regarded as a linear partial response QAM signal with differentially encoded data symbols $c_i j^i$ which are phase-rotated in the complex plane by consecutive multiples of $\pi/2$. Fig. 2 illustrates this QAM signal model.

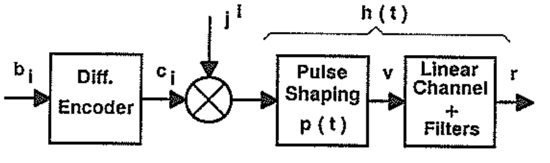


Fig. 2 Linear QAM model of MSK-type CPM signal transmission

Note that alternative MSK-type CPM schemes may be defined in which the data bits are not directly mapped onto the instantaneous frequency as in (2) but are mapped as in the Offset QPSK case [2]. For these modulations, the differential encoder in Fig. 2 has to be omitted and the symbol sequence $\underline{c}$ directly represents the data stream to be transmitted.

In Fig. 2, the QAM model has been extended to cover the effect of linear transmission channels and receiver filters. The received signal $r(t)$ of Fig. 2 is given by

$$r(t) = \sum_{i=-\infty}^{\infty} c_i\, j^i\, h(t-iT) \qquad (7)$$

where $h(t)$ is the overall complex impulse response of the linear transmission system including the pulse shaping filter $p(t)$.

## 4. GENERALIZED DEROTATION TECHNIQUE

In the following, we will formulate the derotation technique for continuous-time signals. However, a discrete-time formulation may easily be derived using an equivalent approach.

Let $q(t)$ be a complex function of time which fulfills the condition

$$q(t+iT) = q(t)(-j)^i \quad \text{for every } i. \qquad (8)$$

Due to the rotation factor $(-j)^i$ which exhibits the opposite sense of rotation with respect to the rotation factor in (7), $q(t)$ will be called '*derotation function*' henceforth.

Applying $q(t)$ of (8), we now define the derotated received signal

$$r'(t) = r(t)q(t) = \sum_{i=-\infty}^{\infty} c_i\, j^i\, h(t-iT)q(t)$$
$$= \sum_{i=-\infty}^{\infty} c_i\, j^i(-j)^i\, h(t-iT)q(t-iT)\,. \qquad (9)$$

By introducing the derotated impulse response

$$h'(t) = h(t)q(t)\,, \qquad (10)$$

$r'(t)$ can finally be expressed as

$$r'(t) = \sum_{i=-\infty}^{\infty} c_i\, h'(t-iT)\,. \qquad (11)$$

In (11), the derotated received signal $r'(t)$ is represented as a binary pulse amplitude modulated (PAM) signal, i.e. the rotational signal structure of $r(t)$ has been abolished. As a consequence, applying the derotation technique will result in simplified receiver structures.

## 5. PRACTICAL DEROTATION FUNCTIONS

The two most practical derotation functions $q(t)$ are

$$q_1(t) = \exp\left[-j\frac{2\pi}{4T}(t-t_o)\right]\,, \qquad (12)$$

$$q_2(t) = (-j)^i \quad \text{for } iT \le (t-t_o) < (i+1)T \qquad (13)$$

where $t_o$ is an optional time offset. Obviously, both $q_1(t)$ and $q_2(t)$ obey the condition (8).

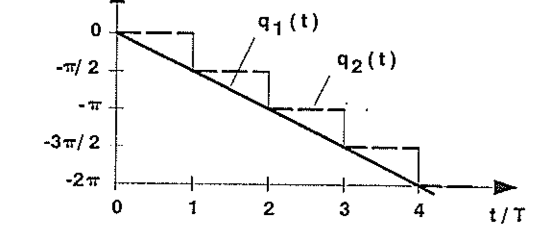


Fig. 3 Phase of derotation functions $q_1(t)$ and $q_2(t)$ for $t_o$=0

The derotation function $q_1(t)$ simply corresponds to a negative frequency shift by a quarter of the symbol rate $1/T$. $q_2(t)$, on the other hand, represents a stepwise derotation function which takes on the values ±1 and ±j only, cf. Fig. 3. This property makes the stepwise derotation function well suited for a low-complexity implementation in digital signal processing.

## 6. MSK-TYPE CPM RECEIVERS WITH DEROTATION

### 6.1. Coherent Threshold Receiver

In the following sections, we will demonstrate how the presented derotation technique can be utilized to derive simplified receiver structures. In the first example, we will consider a coherent threshold receiver with perfect carrier phase and symbol synchronization [1].

After having passed an appropriate receiver filter [1], the received signal $r(t)$ is sampled at optimum sampling instants $iT+t_1$ and derotated using the discrete-time derotation function $q_i = (-j)^i$, cf. Fig. 4. The derotation function must be synchronized with $r(t)$ to enable a coherent demodulation. The real part (inphase component) of the derotated signal $r_i'$ is fed to a sign detector which recovers the data symbols $c_i$. Finally, the symbols $c_i$ may be differentially decoded to obtain the data symbols $b_i$.

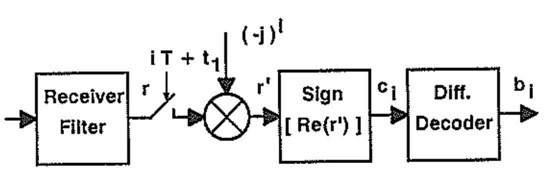


Fig. 4 Coherent threshold receiver with derotation

Alternatively, the derotation could be performed prior to sampling by means of the frequency shifting derotation function $q_2(t)$ according to (12). This leads us to the serial MSK receiver proposed by Amoroso [4].

### 6.2 Noncoherent Matched Filter Receiver

Next we consider a noncoherent matched filter (MF) receiver for finite-length MSK-type spread spectrum signals which is realized as quadrature receiver in the baseband [5].

Let $\underline{b} = (b_0,\ldots,b_{N-1})$ be a binary pseudo-noise (PN) sequence of lenght $N$ which is used to generate an MSK-type CPM modulated spread spectrum signal $s(t)$ according to (2) and (3). $T$ now represents the '*chip period*'. $\underline{b}$ is assumed to be chosen such that the differentially encoded sequence $\underline{c} = (c_0,\ldots,c_{N-1})$ defined by (6) has good autocorrelation properties, i.e.

$$\sum_{i=0}^{N-1} c_i c_{i+k} \cong \begin{cases} N & \text{for } k=0 \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The function of the noncoherent MF receiver is to decide whether the spread spectrum signal $s(t)$ is present in the received signal or not [5]. Note that the PN chips $b_i$ and $c_i$ are not explicitly recovered by the MF receiver.

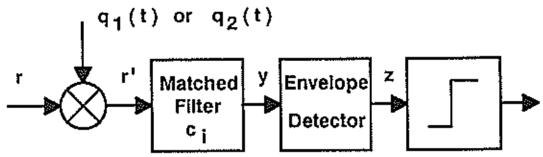


Fig. 5 Noncoherent MF receiver with derotation

In the noncoherent MF receiver of Fig. 5, the received signal $r(t)$ is derotated using the derotation function $q_1(t)$ or $q_2(t)$. $r'(t)$ is fed to a MF followed by an envelope detector and a threshold decider. The MF has a tapped-delay-line structure with a tap spacing of $T$ and with the binary tap weights $c_i = \pm 1$. This MF produces the output signal

$$y(t) = \sum_{i=1}^{N} c_{N-i}\, r'(t-iT) \ . \tag{15}$$

Using (11) and (14), it can be proven that

$$y(t) \cong N\, h'(t-NT) \ . \tag{16}$$

As $|q(t)| = 1$ for the considered derotation functions, the output signal $z(t)$ of the envelope detector in Fig. 5 becomes

$$z(t) = |y(t)| \cong N\, |h(t-NT)| \ . \tag{17}$$

Hence, the output signal of the noncoherent MF directly corresponds to the magnitude of the impulse response $h(t)$ or, if the effects of the transmission channel and receiver filter can be neglected, to the pulse shape $p(t)$ of the MSK-type CPM.

It should be pointed out that due to the envelope detection according to (17), the derotation function $q(t)$ does not need to be phase or time synchronized with the received signal, i.e. the time offset $t_0$ introduced in (12) and (13) may be chosen arbitrarily in this kind of application.

An examination of the MF structure presented in [5] for MSK-type signals makes apparent that the MF structure is substantially simplified by introducing the derotation technique since now only real-valued binary MF tap weights are required.

### 6.3. Adaptive MLSE Viterbi Receiver

In the third example, an adaptive maximum likelihood sequence estimation (MLSE) Viterbi receiver with derotation will be presented. This receiver takes into account ISI caused by a time-dispersive transmission channel and, thus, acts as an adaptive equalizer [6], [7].

Let us again start from an MSK-type CPM signal $s(t)$ according to (2) and (3). The effect of the time-dispersive transmission channel is assumed to be included in the impulse response $h(t)$ introduced in (7). Based on the linear QAM model (7), an MLSE receiver without derotation may be designed using the matched filter approach presented in [6]. In order to recover the data sequence $\underline{b}$ or $\underline{c}$ hidden in the received signal $r(t)$, the non-derotation MLSE receiver calculates the metrics

$$M(\underline{c}) = \sum_{i=-\infty}^{\infty} \mathrm{Re}\,[c_i j^{-i}(x_i - \sum_{k\geq 1} R_k c_{i-k} j^{(i-k)})] \quad (18)$$

with

$$x_i = \int_{-\infty}^{\infty} r(\tau+iT)h^*(\tau)\mathrm{d}\tau \quad (19)$$

$$R_i = \int_{-\infty}^{\infty} h(\tau+iT)h^*(\tau)\mathrm{d}\tau \quad (20)$$

for any possible sequence $\underline{c}$ and searches for that particular sequence which maximizes $M(\underline{c})$. The asterisk in (19) and (20) denotes complex conjugation. The search for the optimum sequence $\underline{c}$ is efficiently carried out by means of the Viterbi algorithm [6]. As to techniques for estimating the impulse response $h(t)$ required for the metric calculation refer to [7].

The metric calculation in the described MLSE Viterbi receiver is rather complicated due to the rotation factors occuring in (18). This can be avoided and complexity can be reduced if the MLSE receiver operates on the derotated signal $r'(t)$. Starting from the binary PAM model (11), the metric

$$M'(\underline{c}) = \sum_{i=-\infty}^{\infty} \mathrm{Re}\,[c_i(x_i' - \sum_{k\geq 1} R_k' c_{i-k})] \quad (21)$$

with

$$x_i' = \int_{-\infty}^{\infty} r'(\tau+iT)h'^*(\tau)\mathrm{d}\tau \quad (22)$$

$$R_i' = \int_{-\infty}^{\infty} h'(\tau+iT)h'^*(\tau)\mathrm{d}\tau \quad (23)$$

is obtained. Using eqs. (8), (9), and (10), it can be verified that $M(\underline{c})$ of (18) and $M'(\underline{c})$ of (21) are identical provided $|q(t)| = 1$.

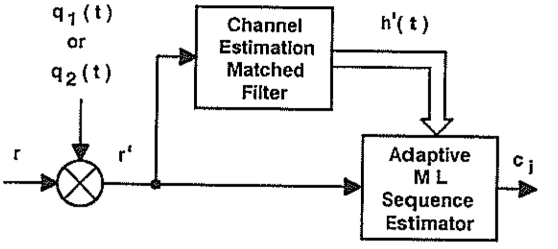


Fig. 6 Adaptive MLSE Viterbi receiver with derotation

The derotated impulse response $h'(t)$ required in (22) and (23) can be directly estimated in the receiver by observing the derotated signal $r'(t)$. For example, consider a MF which is matched to a finite-length pseudo-random sequence of training symbols $c_i$ which are included in the transmitted signal and are known in the receiver [7]. From Section 6.2 it is evident that such a MF operating on $r'(t)$ directly produces an estimate of $h'(t)$, cf. (16).

Fig. 6 illustrates the structure of an adaptive MLSE Viterbi receiver with derotation comprising a channel estimation MF. Owing to the estimation of $h'(t)$ and the receiver adaptation, the derotation function $q(t)$ does not need to be synchronized with the received signal $r(t)$.

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