# Lecture 9: Code acquisition and tracking - advanced strategies, positioning oriented

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# Outline

- Code acquisition:
  - Search strategies
  - Detection strategies
  - Threshold choice, Performance measures
- Multipath propagation and impact on code synchronization
- Code tracking:
  - Basic principles, definition of an S-curve
  - Feedback solutions for multipath mitigation: narrow correlator, double-delta correlators, ...
  - Feedforward solutions: deconvolution algorithms, non-linear operators, peak tracking, ....
  - Performance measures
- Few implementation issues
- Conclusions

# Introduction

- Code shift and Doppler frequency acquisition are needed for reliable performance of any CDMA system, such as UMTS/WCDMA, GPS and Galileo.
- The code synchronization task is typically split into coarse synchronization (or acquisition stage) and fine synchronization (or code tracking stage).
- Previously, you were explained the basic acquisition and tracking concepts. This lecture focuses on more detailed description of each stage, as well as on the challenges encountered in wireless transmissions (e.g., multipath presence).

- ◇ There are $2$ main parameters to consider when designing an acquisition system:
  - frequency uncertainty
  - time uncertainty: clock uncertainties $+$ range uncertainties.
- ◇ The larger the time or frequency uncertainty is $\Rightarrow$ the greater time will take to achieve acquisition, or the greater receiver complexity is required for a given acquisition time requirement.
- ◇ Another important design parameter is the receiver front-end bandwidth.

# Acquisition structures

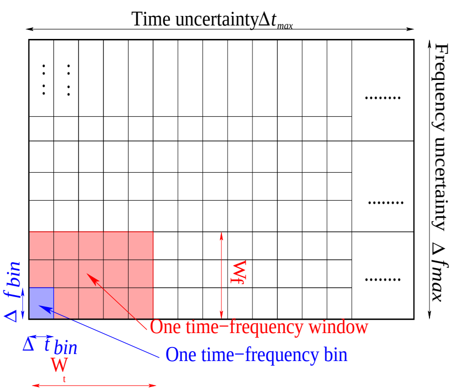
◇ Typically, there are two stages in the acquisition structure:

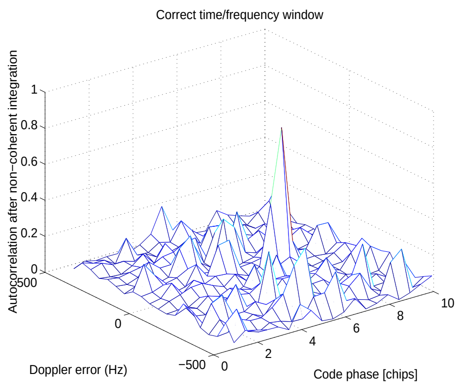
- *Searching stage* - correlation $\Rightarrow$ build the time-frequency window with various delay and frequency candidates.
- *Detection stage* - form a decision variable and compare it with a threshold in order to detect whether the signal is present or absent.

◇ Threshold choice is also an important step in designing a good acquisition stage. Either constant or variable (e.g., according to SNR) thresholds can be used, as it will be discussed later on.

# Basic acquisition model - searching stage

◇ The search space consists of several time-frequency windows of size $W_t \times W_f$; each window has one or several time-frequency bins of size $\Delta t_{bin} \times \Delta f_{bin}$ (chips $\times$ Hz), E.g., typically for GPS, $\Delta t_{bin} = 0.5$ chips and $\Delta f_{bin} = 1$ kHz. Example: Left plot: search space division; right plot: time-frequency window with $W_t = 10$ chips and $W_f = 1$ kHz:



# Acquisition: Search strategy

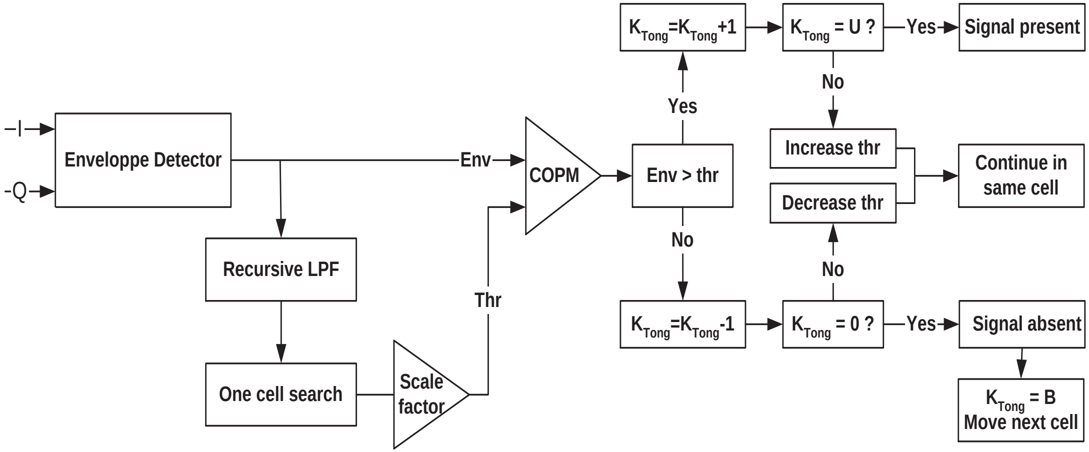
◇ The search strategy can be

- serial: there is only one bin per window $\Rightarrow$ low complexity (only one complex correlator needed), high acquisition time (many bins to be searched; acquisition time is proportional with the code epoch length, i.e. $1023$ chips for standard GPS, up to $10230$ for Galileo and modernized GPS).
- hybrid: there are several bins per window and there are several windows in the whole search space $\Rightarrow$ tradeoff between complexity and acquisition time; general case.
- parallel: there are more than one bin per window and there is only one window in the whole search space $\Rightarrow$ high complexity (need many correlators), low acquisition time.

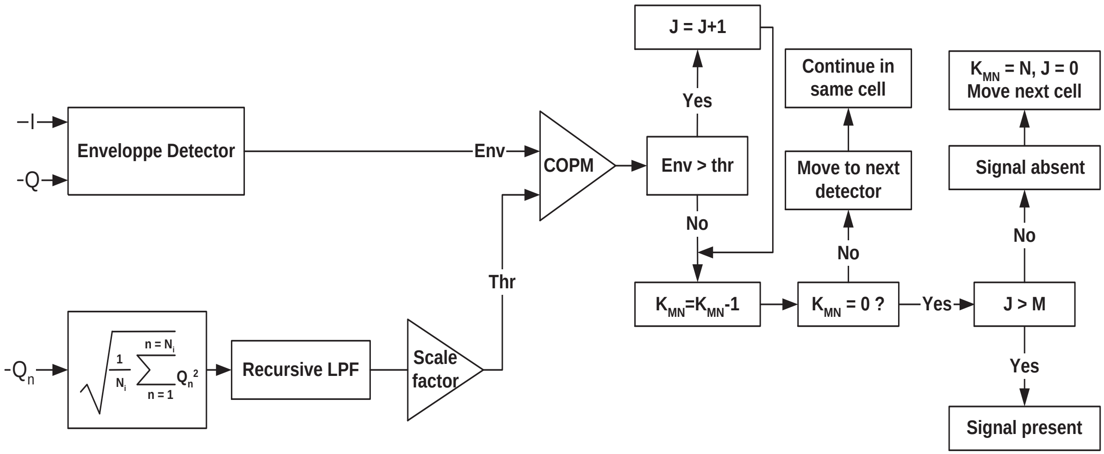
# Acquisition: Detection strategy (I)

- The detection strategy can be classified into: variable dwell time detector (also called sequential detector) and fixed dwell time detector.
  - Examples of detector structures for GPS:
    1. Sequential detector: Tong search detector - it uses a counter variable $K_{Tong}$ and a confirmation threshold $U$. If $K_{Tong} = U \Rightarrow$ acquisition. At each cell, the counter is initialized to $B > 0$ value. Tong detector is sub-optimum, but more efficient than a fixed dwell time detector.
    2. Fixed dwell detector: M of N search detector - N correlation envelopes are compared with a threshold; if at least M of them exceed the threshold $\Rightarrow$ acquisition.

# Tong search detector block diagram

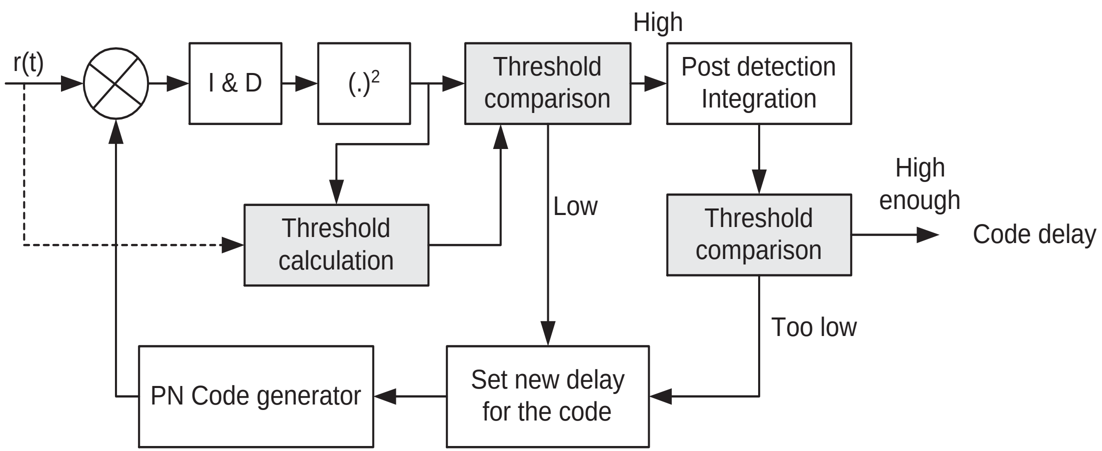


# M of N search detector block diagram



# Acquisition: Detection strategy (III)

- The detection strategy can also be: single-dwell (detection is taken in one step) or multiple-dwell (see figure for an example of a two-dwell detector)




- Multiple-dwell structures are typically used to decrease Mean Acquisition Times (MAT) (tradeoff with increased complexity).

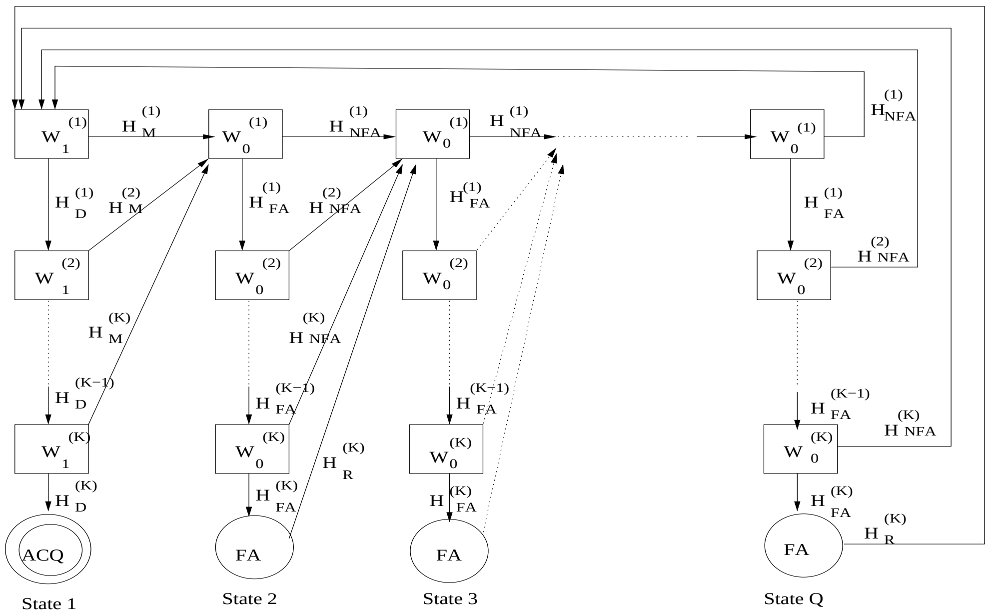
# Acquisition: Mean Acquisition Time (MAT) - simplified example

- Assume that the time uncertainty corresponds to $N$ pseudorandom chips (or $NT_c = \Delta T$ seconds, where $T_c$ = chip duration. Assume that there is no carrier frequency uncertainty (e.g., assisted acquisition)
- Assume also that detection probability at the correct hypotheses is $P_d = 1$ and the false alarm probability at incorrect hypotheses is $P_{fa} = 0$ (ideal case).
- What is the MAT time for a dwell time $\tau_D$ if the timing search update is in half-chip increments ($0.5T_c$)?
- Answer: there are $Q = 2N$ timing positions (hypotheses, bins) to be search and the time to search one time bin is $\tau_D \Rightarrow T_{acq} = 2N\tau_D$
- If all hypotheses are equally probable $\Rightarrow$ the mean acquisition time can be approximated by half of $T_{acq}$: $\bar{T}_{acq} \approx N\tau_D$

# Acquisition: Flow chart model for MAT derivation in multiple dwell approach - Notations

| | |
|---|---|
| $W_1^{(k)}/W_0^{(k)}$ | Correct/Incorrect window at $k$-th stage of a multiple-dwell structure |
| $P_d^{(k)}/P_{fa}^{(k)}$ | Detection/False alarm probability at $k$-th stage |
| $\tau_k$ | the dwell time of the $k$-th stage |
| $K_{penalty}$ | penalty factor associated with a false alarm |
| $N_c$ | Coherent integration time (code epochs) |
| $N_{nc}$ | Non-coherent integration time (blocks of $N_c$ code epochs) |
| $N$ | Number of bins per window |
| $H_D(\cdot)$ | transfer function corresponding to a Detection situation |
| $H_{FA}(\cdot)$ | transfer function corresponding to a False Alarm situation |
| $H_M(\cdot)$ | transfer function corresponding to a Miss situation |
| $H_{NFA}(\cdot)$ | transfer function corresponding to a Non False Alarm situation |

# Acquisition: State diagram of a generic $K$-dwell acquisition structure



# Acquisition: Steps in the derivation of Mean Acquisition Time (MAT)

- Find the equivalent transfer function $H(z)$ of the $K$-dwell state diagram. Note: there are $Q$ states (windows) in the state diagram: one corresponding to the signal presence (correct window $W_1$) and $Q-1$ incorrect windows ($W_0$).
- Find MAT (i.e., $\mathbf{E}[T_{ACQ}]$) as the first derivative of $H(z)$:

$$\mathbf{E}[T_{ACQ}] = \left.\frac{dH(z)}{dz}\right|_{z=1}. \tag{1}$$

# Acquisition: Transfer functions

- Step-by-step transfer functions:

$$\begin{cases} H_D^{(k)}(z) = P_D^{(k)} z^{\tau_k} \\ H_M^{(k)}(z) = (1 - P_D^{(k)}) z^{\tau_k} \\ H_{NFA}^{(k)}(z) = (1 - P_{fa}^{(k)}) z^{\tau_k} \\ H_{FA}^{(k)}(z) = P_{fa}^{(k)} z^{\tau_k} \\ H_R(z) = z^{K_{penalty} \tau_K}. \end{cases} \tag{2}$$

- We need to know the distribution of the initial states: typically, uniform distribution is assumed $\Rightarrow$:

$$H(z) = \frac{H_D(z) \sum_{i=1}^{Q} \left[H_T(z)\right]^{Q-i}}{Q\left(1 - H_M(z)\left[H_T\right]^{Q-1}\right)}, \tag{3}$$

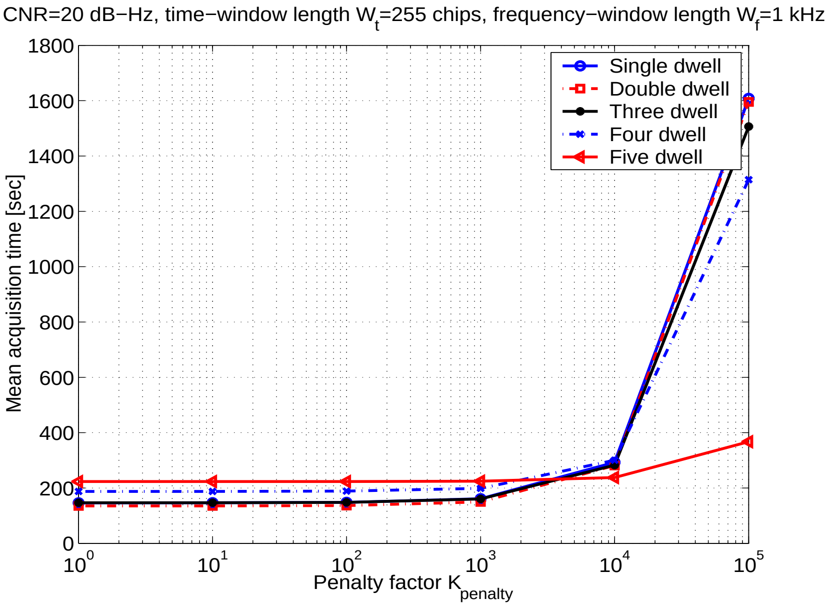
# Acquisition: Particular case

- For single dwell case, we obtain the same formula as derived in Lecture 3:

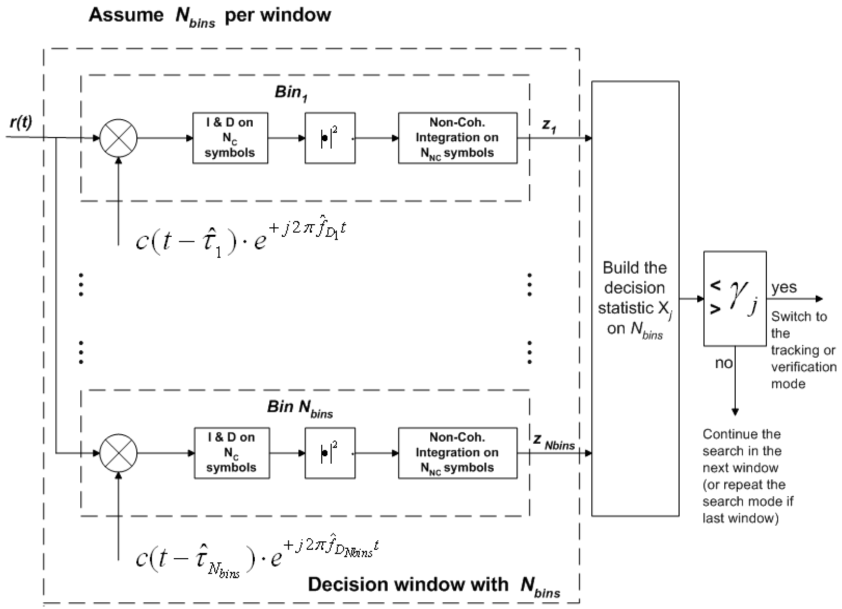
$$\bar{T}_{acq} = (Q-1)\frac{2-P_d}{2P_d}(T_d + P_{fa}T_{fa}) + \frac{T_d}{P_d} \tag{4}$$

- Above, $T_d = \tau_1$ is the dwell time and $T_{fa} = K_{penalty}\tau_1$ is the penalty time associated with a false alarm.
- For the simplified example from few slides before ($P_d = 1$, $P_{fa} = 0$, $K_{penalty} = 1$ and $\tau_1 = \tau_D$) $\Rightarrow$ $\bar{T}_{acq} = \frac{Q}{2}\tau_D - \frac{\tau_D}{2} \approx \frac{Q}{2}\tau_D$ (the approximation holds for a large number of states).
- For multiple dwell cases, the closed-form formulas are rather heavy, but their Matlab implementation is straightforward.

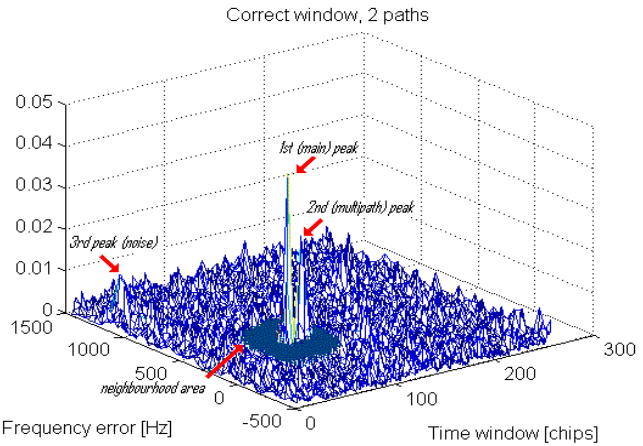
# Acquisition: Example - MAT values for various K-dwell structures



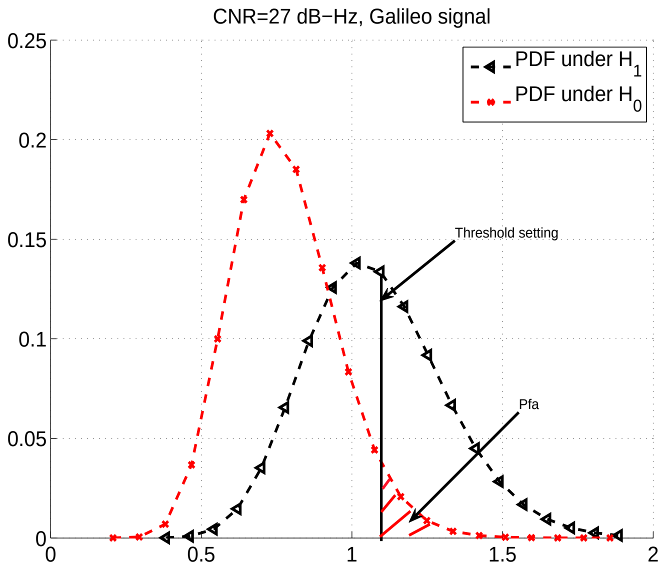
# Acquisition: Threshold choice



- Acquisition problem is in fact a detection problem: detect signal in noise, or, equivalently, separate between hypothesis $H_1$ (signal plus noise are present) and hypothesis $H_0$ (noise only is present).
- The Probability Distribution Functions (PDFs) under each hypothesis are derived according to the test statistic, which can be, for example, the maximum among all correlation envelopes in a window or the ratio between the maximum and the noise floor (e.g., in the plot below).

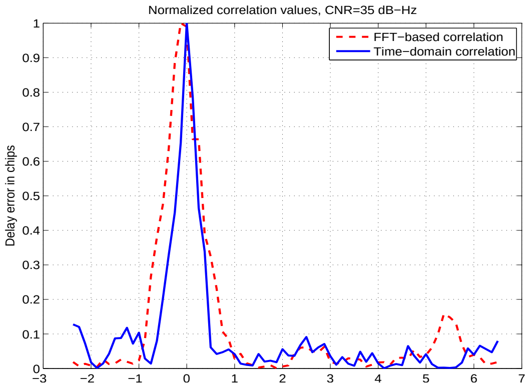


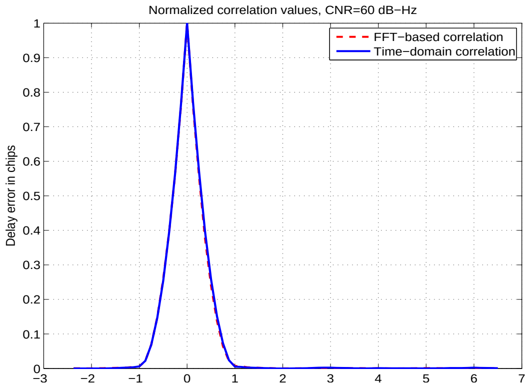
- Threshold can be chosen as a tradeoff between a good $P_d$ and a sufficiently low $P_{fa}$ ($P_d$ increases when $P_{fa}$ increases).



# Acquisition: Time-domain versus FFT-based correlation

- Time-domain correlation between the received signal and the reference code is equivalent with frequency-domain multiplication
- FFT-based correlation means that we apply FFT on the incoming signal and on the reference code, then the outputs of FFT are multiplied, and IFFT is applied.



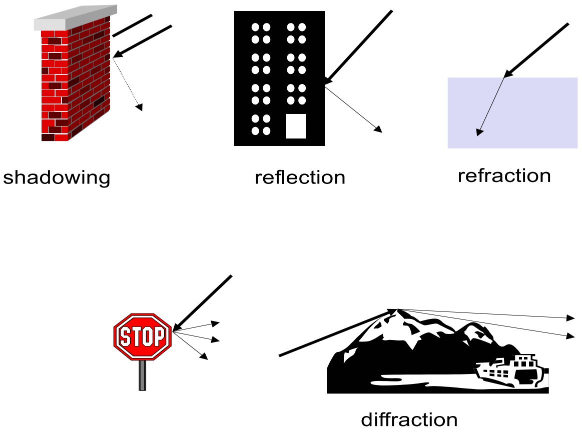



# Acquisition: Performance measures

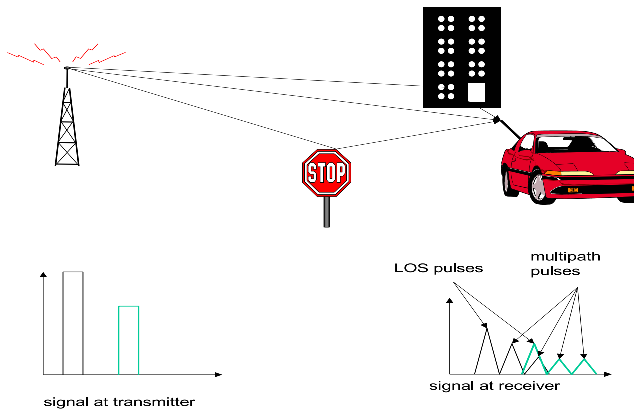
- Detection probability $P_d$ at a certain false alarm probability ($P_d$ is defined within a certain region, typically within $\pm 1$ chip error
- Mean acquisition time (MAT) $\bar{T}_{acq}$
- Time To First Fix (TTFF): time required by a GPS receiver to acquire the satellite signals and navigation data and to calculate a position solution. It is related to MAT.

# Multipath propagation and impact on code synchronization

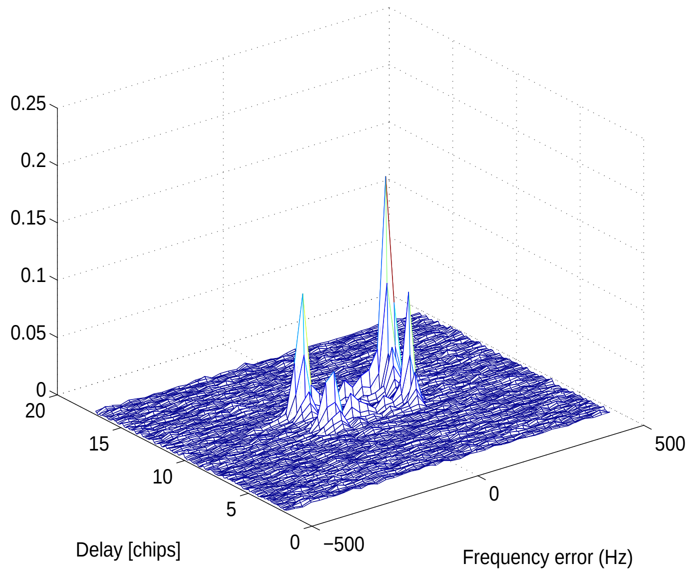
- Received signal power always influenced by temporal/spatial variations of the wave:



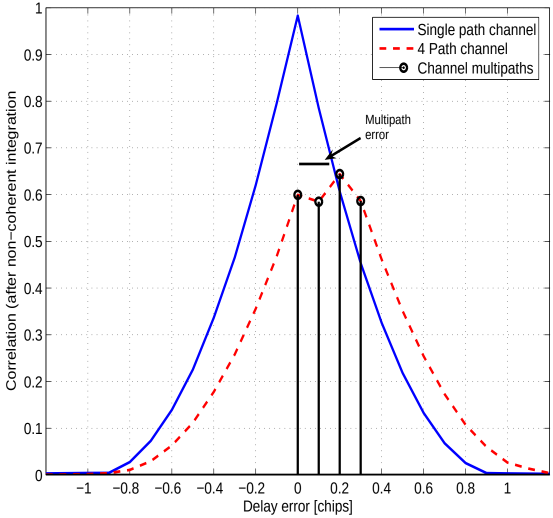
- Signal can take many different paths between transmitter and receiver due to reflection, scattering and diffraction



- Impact on code acquisition: multiple correlation peaks; possibility to acquire an NLOS path $\Rightarrow$ more than one chip apart from true LOS. Below: example of Rayleigh fading channel with 2 paths (additional spread in frequency is due to Doppler spread):



- Impact on code tracking: incorrect lock-in state $\Rightarrow$ multipath errors:

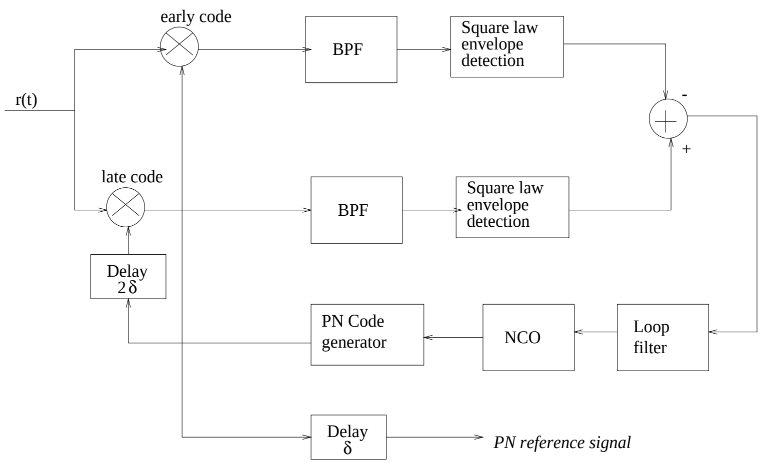


## Code tracking: Introduction

- The purpose of a tracking loop is to reduce the estimation error of the delay between the received signal and the replica code at the receiver. A coarse delay estimate is obtained from the acquisition stage; the tracking loop is further refining/improving this estimate and trying to keep track of delay changes (e.g., due to the relative receiver-transmitter movement).
- In practice, we need both carrier tracking (e.g., achieved with a PLL) and code phase tracking; here we focus only on the code tracking part.

# Code tracking

- Basic code tracking loop is the Delay Locked Loop with 1 chip early-late spacing (or wideband early-minus-late correlator). Both coherent and non-coherent (see below) DLLs exist. Below, the block diagram is for complex signals:



# Tracking: DLL basic discriminator types

| Type | Discriminator or S-curve | Characteristics |
|---|---|---|
| Coherent | $I_E - I_L$ | Simplest of all; do not require Q branches, but requires a good carrier tracking loop |
| Noncoherent (Early-Minus-Late) Power) | $(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)$ | un-normalized |
| | $\frac{(I_E^2+Q_E^2)-(I_L^2+Q_L^2)}{(I_E^2+Q_E^2)+(I_L^2+Q_L^2)}$ | normalized (better behaviour in noise) |
| Dot-Product | $I_P(I_E - I_L) + Q_P(Q_E - Q_L)$ | uses all 6 correlator outputs |

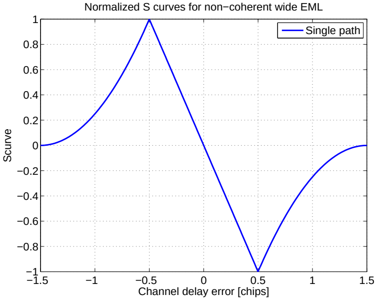
- $I_E, I_P, I_L$ = early, prompt and late in-phase correlations
- $Q_E, Q_P, Q_L$ = early, prompt and late quadrature correlations

# Code tracking: S-curve

- The S-curve or discriminator curve translates the correlation values into a delay error index. For example:

$$D(\tau) = (I_E^2(\tau) + Q_E^2(\tau)) - (I_L^2(\tau) + Q_L^2(\tau))$$

- The zero-crossings from above of the S-curve signal the presence of a channel path $\Rightarrow$ they signal the delay error. Note: if late-minus-early (instead of early-minus-late) curve $\Rightarrow$ need to consider the zero crossings from below



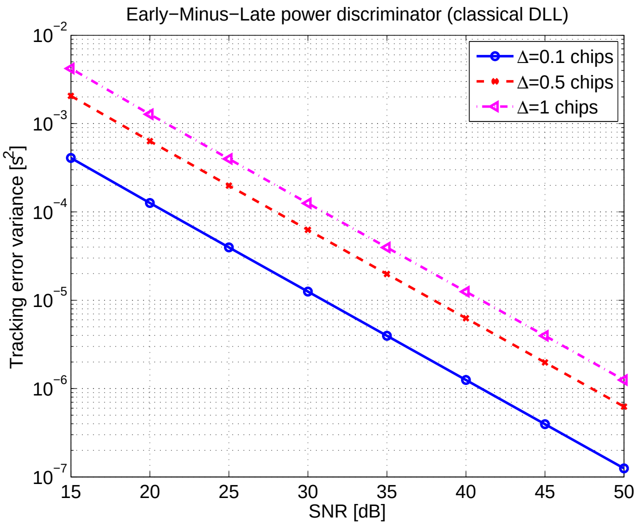
# Tracking: Narrow correlator

- Early-late spacing ($\Delta = 2\delta$) is smaller than half of the main lobe of the correlation envelope. E.g., for BPSK and QPSK modulation, this means: $\Delta < 1$ chip
- Introduced by Dierendonck and Fenton in 1992, for GPS in order to reduce the code tracking error in the presence of noise and multipaths.
- Closed-loop variance of delay tracking error for a non-coherent early-minus-late power DLL is:

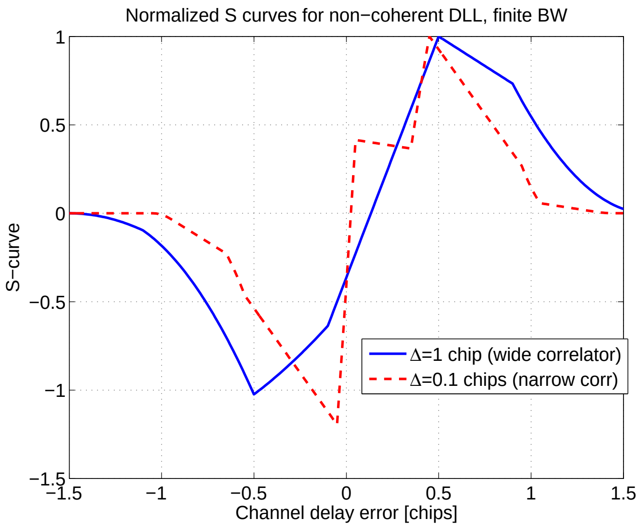
$$\sigma^2 = \frac{B_L \Delta}{2S/N}\left(1 + \frac{2}{(2-\Delta)(S/N)T_{coh}}\right) \tag{5}$$

- Above, $B_L =$ loop bandwidth, $S/N =$ Signal to Noise ratio.

- Example 1: tracking error variances for different early-late spacing, a loop bandwidth $B_L = 4$ MHz and $T_{coh} = 1$ ms, early-minus-late power discriminator (or classical DLL). Note that, when $\Delta$ decreases $\Rightarrow$ tracking loop error variance decreases.

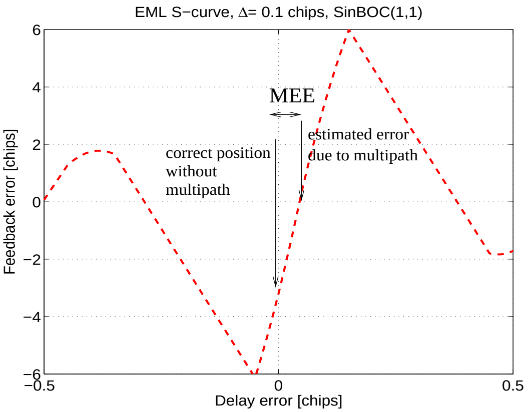


- Example 2: multipath behaviour for wide and narrow correlator; 2 in-phase paths spaced at $0.4$ chips apart, second path 3 dB smaller than the first, early-minus-late power discriminator. Note that, when $\Delta$ decreases $\Rightarrow$ multipath error decreases.

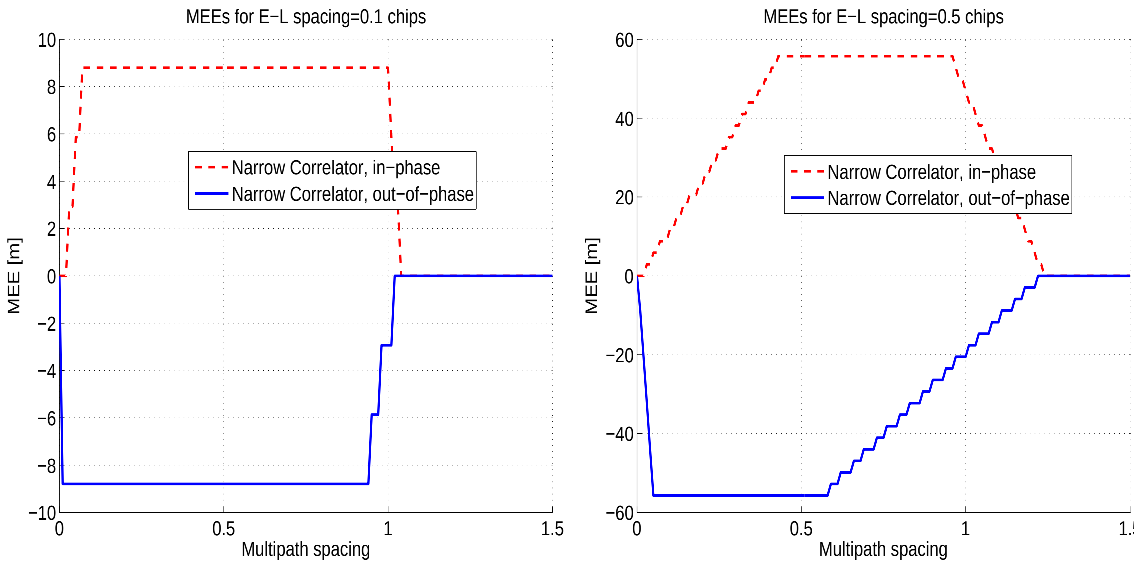


# Code tracking: Multipath Error Envelope

- Typical performance criterion in multipath channels is the Multipath Error Envelope (MEE). MEE is usually computed in the absence of noise, with only 2 paths, in 2 cases: in-phase paths and out-of-phase paths (180 degree phase difference). An example below shows how the multipath error is computed, starting from the S-curve.



## Example: Multipath Error Envelope for narrow correlator, $0.1$ chips spacing (left) and $0.5$ chips spacing (left)



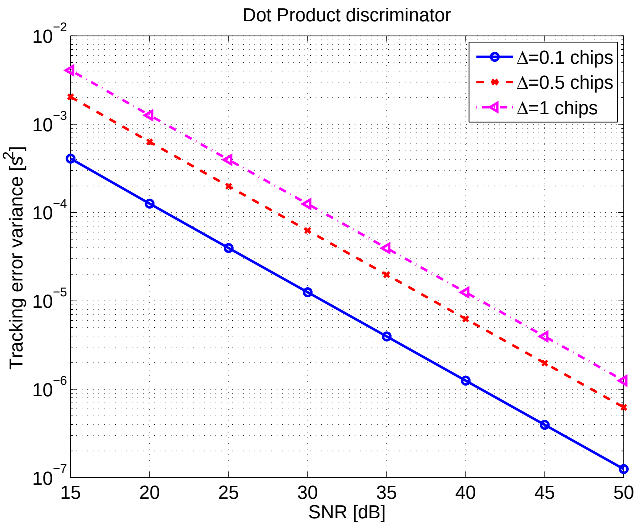
# Tracking: Dot-product discriminator

- S-curve output (discriminator): $I_P(I_E - I_L) + Q_P(Q_E - Q_L)$
- Closed-loop variance of delay tracking error for a dot-product DLL is:

$$\sigma^2 = \frac{B_L \Delta}{2S/N}\left(1 + \frac{1}{(S/N)T_{coh}}\right) \tag{6}$$

- In general, carrier phase tracking is better with respect to noise in the dot-product mode because of the availability of a punctual power estimate, which has a greater signal strength in the presence of noise.
- However, the advantages of Dot Product discriminator versus early-minus-late power discriminator are sometimes controverted, especially in multipath fading channels.

- Example 3: tracking error variances for different early-late spacing, a loop bandwidth $B_L = 4$ MHz and $T_{coh} = 1$ ms, dot-product discriminator. Note that, when $\Delta$ decreases $\Rightarrow$ tracking loop error variance decreases.



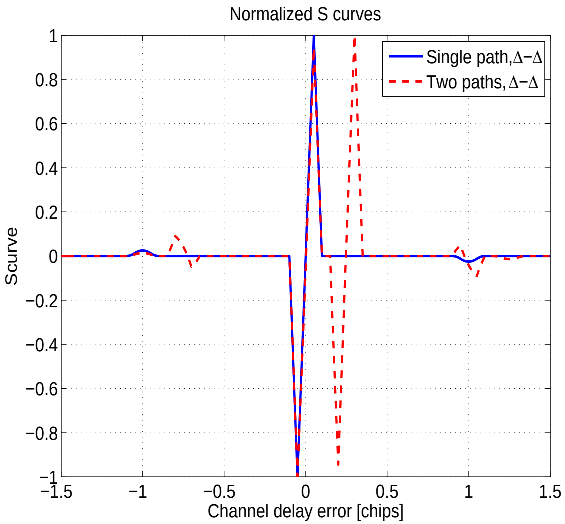
# Tracking: Double-delta ($\Delta\Delta$)correlators

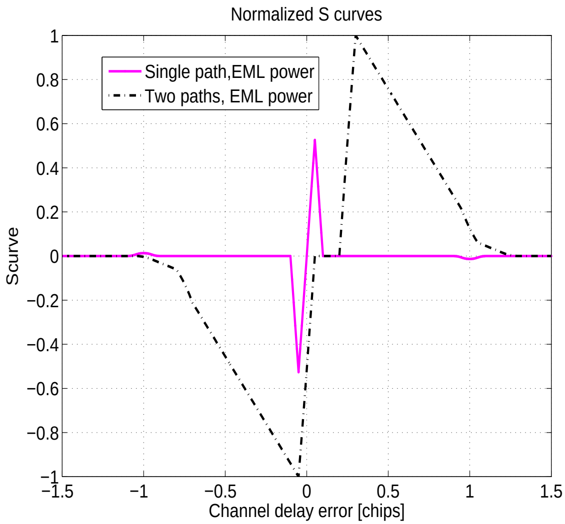
- Add 2 extra (complex) correlators: very early (VE) and very late (VL) (or, equivalently, 4 extra real correlators)
- Form the discriminator output via:

$$(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2) - a * \left((I_{VE}^2 + Q_{VE}^2) - (I_{VL}^2 + Q_{VL}^2)\right)$$

- The factor $a$ above is typically set to $0.5$
- If the spacing between early and late correlators is $\Delta$, then the VE-VL spacing is $2\Delta$ (that's why the name of 'double delta' or $\Delta\Delta$ correlators)
- Also known under other names: Pulse Amplitude Correlator (PAC), Very Early-Very Late correlator, High Resolution Correlator (HRC)

- Example 4: multipath behaviour for double-delta correlator; 2 in-phase paths spaced at $0.25$ chips apart, second path 3 dB smaller than the first. Left: double-delta; right: early-minus-late power. Narrow correlation spacing of $0.1$ chips. Note that, Double-delta correlator behaves better in multipaths than narrow early-minus-late power correlator.



# Tracking: Multipath Estimating Delay Locked Loop (MEDLL)

- Maximum Likelihood (ML)-based approach
- Received signal in multipaths: $r(t) = \sum_{l=1}^{L} a_l e^{j\theta_l} s(t - \tau_l) + n(t)$
- After correlation with the reference code:

$$\mathcal{R}(\tau) = \sum_{l=1}^{L} a_l e^{-j\theta_l} \mathcal{R}_{ref}(\tau - \tau_l) + \tilde{n}(\tau)$$

- $\mathcal{R}(\tau)$ is the I/Q downconverted correlation function and $\mathcal{R}_{ref}(\cdot)$ is the reference correlation function (e.g., the triangular shaped function for ideal pseudorandom codes and BPSK modulation)

- In MEDLL, $L$ (number of paths) is considered known
- The amplitudes, phases and delays of each paths are estimated via:

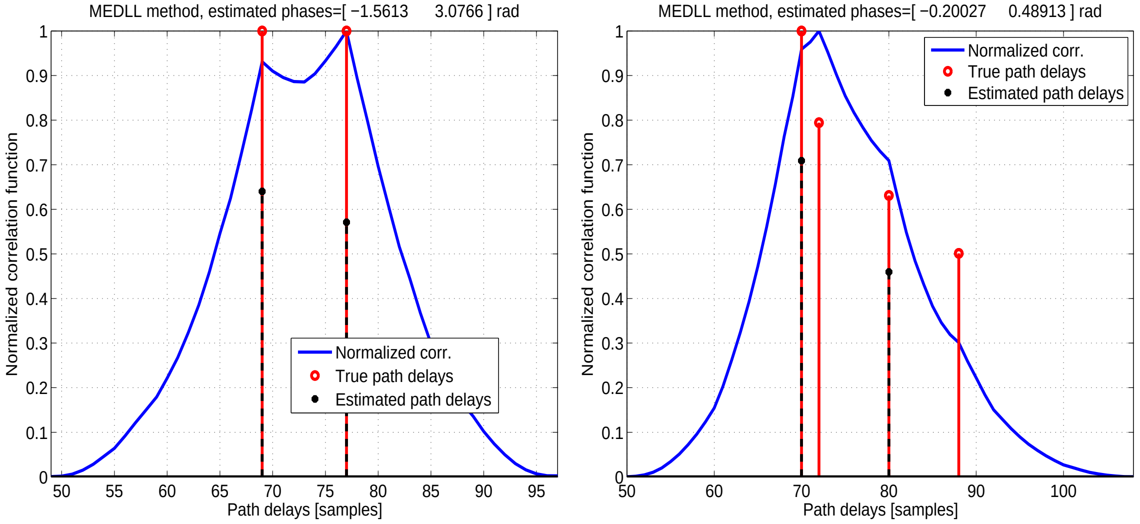
$$\widehat{\tau}_m = arg\max_{\tau} Real\left\{ \left(\mathcal{R}(\tau) - \sum_{l\neq m, l=1}^{L} \widehat{a}_l e^{-j\widehat{\theta}_l} \mathcal{R}_{ref}(\tau - \widehat{\tau}_l)\right) e^{-j\widehat{\theta}_i} \right\}$$

$$\widehat{a}_m = Real\left\{ \left(\mathcal{R}(\tau) - \sum_{l\neq m, l=1}^{L} \widehat{a}_l e^{-j\widehat{\theta}_l} \mathcal{R}_{ref}(\tau - \widehat{\tau}_l)\right) e^{-j\widehat{\theta}_i} \right\}$$

$$\widehat{\theta}_m = arg\left\{ \left(\mathcal{R}(\tau) - \sum_{l\neq m, l=1}^{L} \widehat{a}_l e^{-j\widehat{\theta}_l} \mathcal{R}_{ref}(\tau - \widehat{\tau}_l)\right) e^{-j\widehat{\theta}_i} \right\}$$

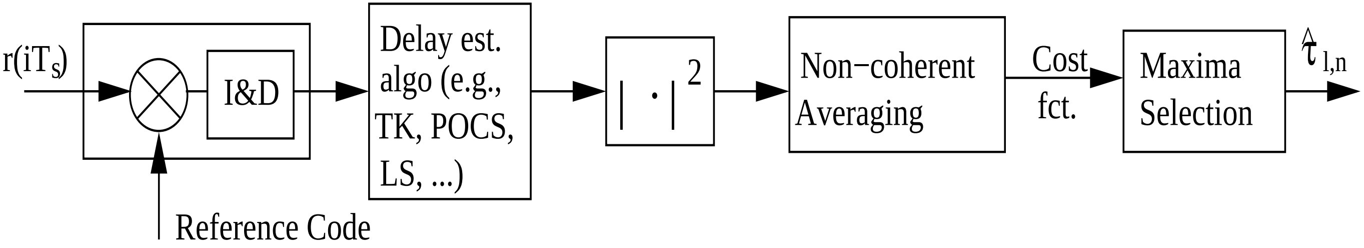
- In practice, $L$ is not known and should be estimated as well (e.g., via some thresholding)

- Example: MEDLL estimates for 2 paths and 4 path channels. BPSK modulation, Carrier-to-Noise ratio (CNR)=$50$ dB-Hz



# Tracking: Feedforward estimators (1/3)

- Generic principle: delays are estimates directly from the correlation function; no feedback loop; higher number of correlators needed.
- Block diagram is shown below. Delay estimation block can be a linear or non-linear operator, as it will be explained next, e.g., Teager-Kaiser (TK), Least Squares (LS), Projection Onto Convex Sets (POCS), etc.



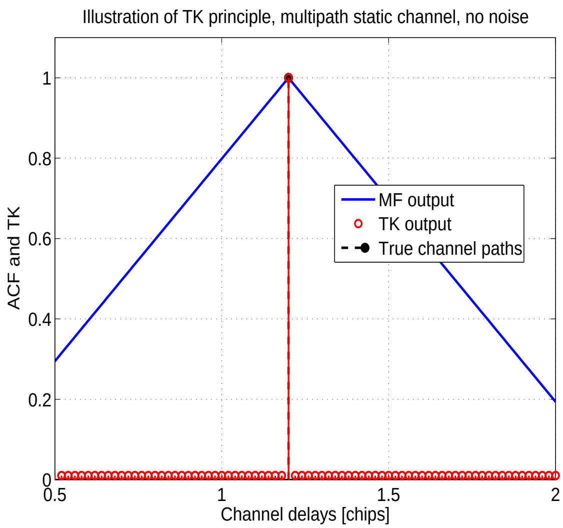
## Tracking: Feedforward estimators (2/3)

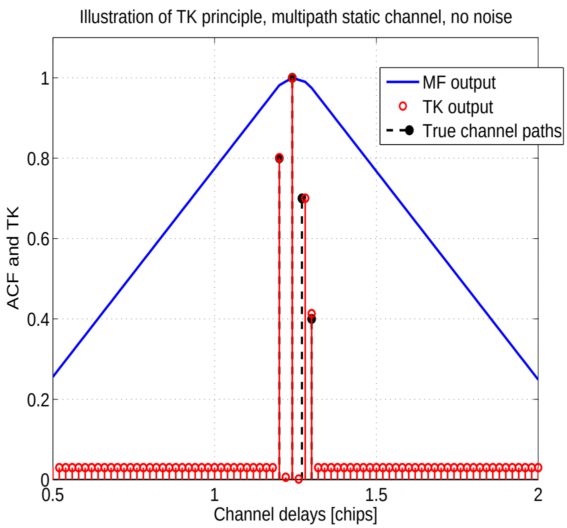
- Teager-Kaiser based: apply a non-linear operator, namely Teager-Kaiser on the complex correlation function:

$$\Psi_d\big(z(n)\big) = z(n-1)z^*(n-1) - \frac{1}{2}\Big(z(n-2)z^*(n) + z(n)z^*(n-2)\Big).$$

- Here, $z(n)$ is the discrete-sampled version of the correlation function $\mathcal{R}(\tau)$

- Example: Left: TK in single-path channel. Right: TK applied on the correlation function for 4 path channel, BPSK modulation, no noise:



# Tracking: Feedforward estimators (3/3)

- Deconvolution-based: the received signal is the result of the convolution between the (satellite) transmitted signal $s(t)$ and the channel impulse response $h(t)$:

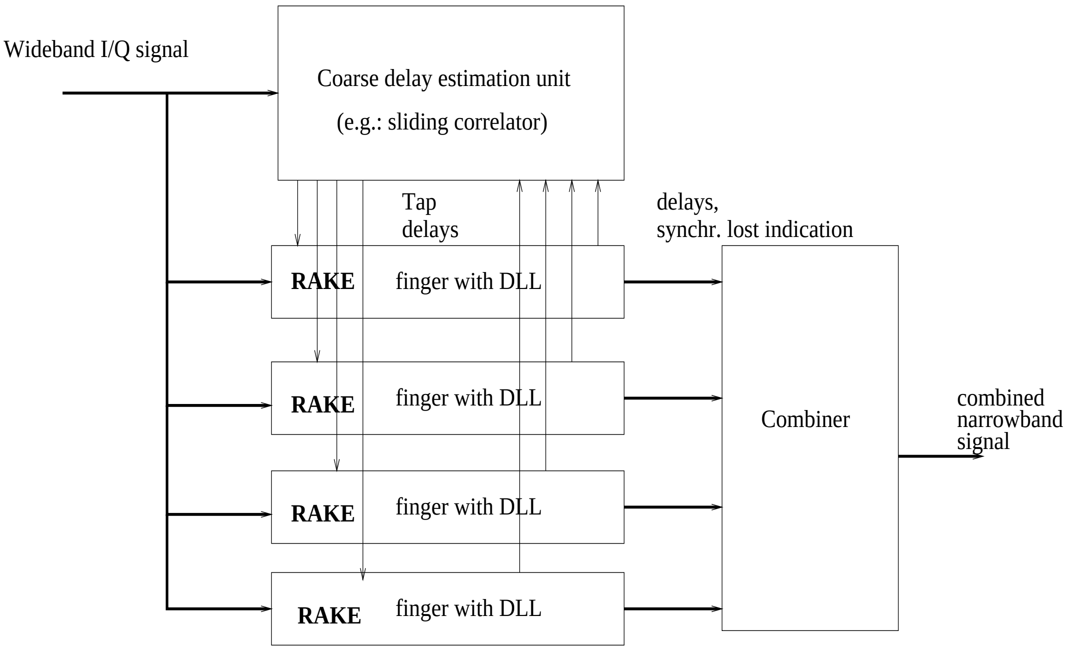
$$r(t) = s(t) * h(t) + n(t) \Rightarrow R(f) = S(f)H(f) + N(f)$$

- Deconvolution attempts to estimate the channel response $\widehat{h}(t)$, invert it and then convolve it with the incoming signal, to estimate the transmitted signal. Usually, solved iteratively.
- Examples of deconvolution algorithms: Least Squares (LS), Least Squares with constraints (e.g., Projection Onto Convex Sets algorithm), etc.
- Both TK and deconvolution-based algorithms are quite sensitive to noise.

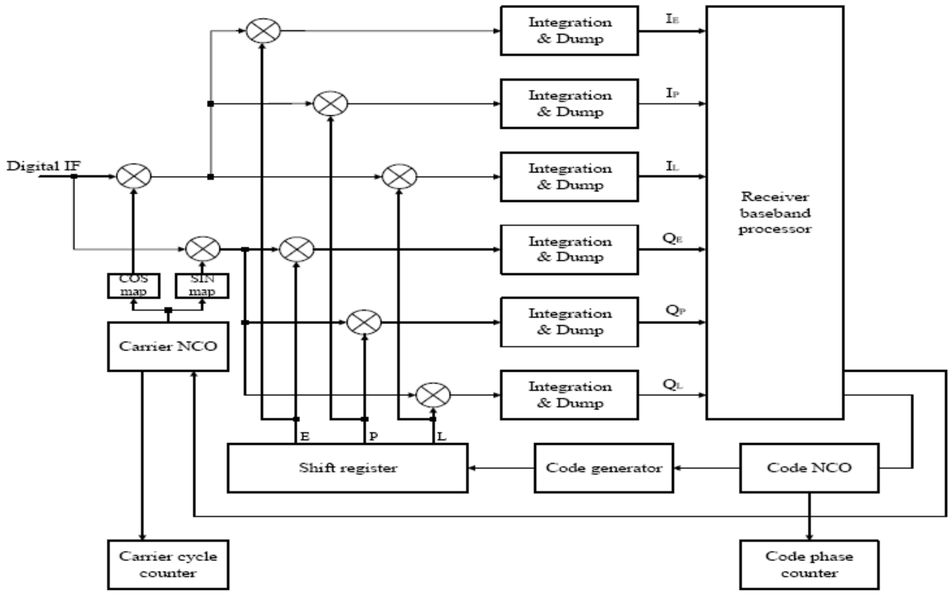
# Code tracking: Performance measures

- Tracking loop error variance
- Delay error (mean error, root mean square error, etc)
- Multipath range of tracking errors or multipath error envelope (MEE)
- Mean Time to Lose Lock (MTLL): time until a loss of lock occur and acquisition should be re-started. The larger the MTLL, the better the loop.
- All these parameters are dependent on the Carrier To Noise ratio (CNR) and loop bandwidth.

# Implementation aspects: Rake receiver with code tracking



# Implementation aspects: Carrier and code tracking



# Conclusions

- Acquisition problem can be seen as a detection problem; detection theory and flow graph theory can be used to analyze the acquisition performance and to design the best acquisition structures (e.g., single dwell versus multiple dwell, threshold choice optimization, etc.)
- Delay tracking can be done in closed loop (feedback algorithms) or in open-loop (feedforward).
- Feedback algorithms typically require less number of correlators and are quite robust; they are employed nowadays in commercial receivers. However, they can suffer of feedback error propagation and are not very accurate in closely-spaced multipaths.

- Feedforward solutions may be more adequate for accurate delay estimation; however, their implementation constraints are still an issue.
- In communication receivers (e.g., WCDMA receiver), it is important to track several channel multipaths $\Rightarrow$ Rake receiver
- In navigation receivers, only the first, Line-Of-Sight, path is of interest. However, knowledge about the other multipaths can help in reducing the multipath interference.