On the Modeling of GMSK Binary Transmission over Time-Variant TDMA/FDD Macrocellular Radio Channels with Derotation-Based Coherent Demodulation

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Abstract

GMSK is an efficient modulation technique for digital mobile communications. In this paper a simple solution to the modeling and simulation of GMSK systems impaired by fading channels and additive white Gaussian noise is given, and its effects on the channel characteristics are also considered. The modeling is based on the well-known linearized representation of phase-modulated signals. The analysis is not only useful for modeling and simulation purposes, but also gives a tool for the design of simple receivers.

1. Introduction

Since [2] Gaussian filtered Minimum Shift Keying (GMSK) is considered an efficient modulation technique for digital mobile telephony and is employed in the Pan-European Cellular Communication System (GSM). A common problem is to simulate the system in a simple way, being GMSK a non linear modulation technique. A linear model gives significant simplifications, and a lot of work has been carried out in this direction. The approximation of Continuous Phase Modulated (CPM) signals by a superposition of amplitude modulated pulses was first developed in [3] and then particularized for GMSK in [4], where it was used to design linear receivers yielding near-optimum performance. Linearized GMSK signals can be modeled as QAM signals with a rotational signal structure or, equivalently, as offset QPSK. In [8] a generalized derotation technique that abolishes this rotational structure has been derived, thus leading to very simple linear receivers.

The aim of this paper is to analyze the effects of linearization and derotation on the channel characteristics; in particular, the TDMA/FDD macrocellular radio channel is considered. Starting from the very general scheme of Fig.1, a complex discrete-time tapped-delay-line equivalent model for the digital channel is derived and its statistical properties are fully characterized. The effect of a frequency offset possibly present is also considered.

2. The Complex Representation of CPM

GMSK signals represent a sub-class of binary partial-response CPM signals with modulation index $h_F = 0.5$ and Gaussian-type pulse shape. The complex envelope of an amplitude-normalized CPM signal is $s(t,\underline{b}) = exp\big(j\Phi(t,\underline{b})\big)$ (see [1]). The phase $\Phi(t,\underline{b})$ depends on the N-long data sequence $\underline{b}=<b(1),b(2),...,b(N)>$ as:

$$\Phi(t,\underline{b}) = 2\pi\, h_F \sum_{i=1}^{N} b(i)q(t-iT_b)\,, \qquad (1)$$

where $q(t)=\int_{-\infty}^{t} g(\tau)d\tau$ and the pulse shape $g(t)$ is a real function of length LT_b (i.e., it is zero outside $[0,Lt_b]$) with normalized amplitude (i.e., $\int_{-\infty}^{+\infty} g(\tau)d\tau=\frac{1}{2}$).

The Gaussian-type pulse shape $g(t)$ is defined as [2]:

$$g(t)=\frac{1}{2T_b}\left[Q(2\pi B_b(\frac{t-T_b/2}{\sqrt{ln2}}))-Q(2\pi B_b(\frac{t+T_b/2}{\sqrt{ln2}}))\right], \quad (2)$$

where $Q(t)=\int_{t}^{\infty}\frac{1}{\sqrt{2\pi}}exp(-\tau^2/2)d\tau$, B_b is the bandwidth of the pulse shaping filter and $B_N = B_b T_b$ is the normalised bandwidth. Small values of B_N give GMSK power spectra with narrow main-lobe and low side-lobes levels, thus being attractive for data transmission in bandwidth-limited environments. However, this is obtained at the expense of

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considerable ISI in the transmitted signal (i.e., $L>>1$), thus increasing the complexity of the equaliser located at the receiver. The GSM standard assumes $B_N=0.3$, so that the pulse shape $g(t)$ spans approximately 3 signalling periods. For $B_N \to \infty$ the standard full-response binary MSK is obtained [2].

3. Linearized representation of the Binary GMSK Complex-Envelope

As a consequence of the phase-modulation operation, $s(t,\underline{b})$ exhibits a nonlinear dependence from the information stream $\underline{b}$. From [3] the following linearized representation for the complex-envelope $s(t,\underline{b})$ of a binary GMSK signal can be derived [7]:

$$s(t,\underline{b}) \cong \sum_{i=1}^{N} c(i)(j)^i\, p(t-iT_b) \qquad (3)$$

where the binary antipodal differentially-encoded sequence $c(i)$ is defined as follows:

$$c(i) = \prod_{k=1}^{i} b(k) = c(i-1)b(i),\ i \geq 2\ ,\ \text{with } c(1) = b(1). \qquad (4)$$

The real-valued "main-pulse" $p(t)$ spans the time interval $[0,(L+1)T_b]$ and is related to $g(t)$ as in Eq.(4.9) of [3] with $h_F=0.5$, C=0, S=1, $\Phi=\pi/2$.

The linearization procedure of the GMSK data-stream then gives rise not only to a differential encoding of the transmitted data stream but also to a time-varying phase rotation of consecutive multiples of $\pi/2$.

It is easily proved that, when the input antipodal stream $\{b(i)\}$ is constituted by equiprobable independent symbols, the same holds for the sequence $\{c(i)\}$.

An interesting interpretation of binary GMSK modulation formats was made in [2], where an equivalent BPSK-like signal set has been derived. This analysis leads to the following expression for the bit error rate of a GMSK modulation system for the case of one-shot coherent detection:

$$P_e \cong \frac{1}{2} erfc\left(\frac{d_{min}}{2\sqrt{N_o}}\right) = \frac{1}{2} erfc\left(\sqrt{\alpha \frac{E_b}{N_o}}\right) \qquad (5)$$

where α is a function of B_N and d_{min} is the minimum value of the signal distance between mark and space in the Hilbert space. The residual ISI is taken into account considering a modified BPSK-like signal constellation where the distance between mark and space in the Hilbert space is less than $2\sqrt{E_b}$.

The value of 2α versus B_N is plotted in Fig.5 of [2]. According to this figure, the performance of GMSK with B_N =0.2, 0.25 is predicted by Eq.(9) and α=0.75, 0.85, with a resulting performance loss (with respect to ideal antipodal transmission) of 1.2 dB, 0.7 dB respectively. Computer simulation [2,Fig.13] have shown that the real performance loss is of about 3dB, 1.7 dB, and that the real values of α would be 0.52, 0.68. This mismatch between theoretical and simulation results means that the residual ISI after the linear receiver is larger than the expected; to the author's knowledge, this mismatch has not been clearly pointed out in literature. If an equalising filter is used to remove the ISI introduced by Gaussian filter, the performance degradation previously described can be drastically reduced. In [4] it is shown that a one-shot coherent GMSK (B_N =0.25) followed by a discrete-time Wiener filter exhibits a performance degradation of 0.94 dB with respect to coherent BPSK and this is 0.8 dB better than the case without equalising filter. The use of a linear receiver followed by an equalising filter gives a performance loss of only 0.24 dB with respect to the optimum GMSK receiver, constituted by a bank of $2^{(L-1)}$ filters matched to the pulses of the Laurent decomposition and followed by a Viterbi decoder.

4. A Discrete-Time Tapped-Delay Line Equivalent Representation of the Modulation Channel

The additive complex noise $\{n(t)=n_c(t)+jn_s(t) \in C^1\}$ introduced by the receiver is assumed to be a baseband zero-mean white complex Gaussian process, with bilateral power spectral density equal to $N_o/2$ per component and with mutually uncorrelated components.

Although the optimal receiver would require a front-end filter matched to the cascade of the transmitter and radio channel filters, in practice the channel response is unknown and fastly time-variant so that time-invariant receiver filters matched to the transmitter filter only are generally employed [5,Chap.7].

Denoting as $g_r(t;\eta)$ the complex input delay-spread function of the linear time-variant channel [6,Sect.III.B, Eq.(7)], it is useful to introduce an equivalent time-variant linear filter (modulation channel) with complex input delay-spread function $g_{eq}(t;\xi)$ including the ordered cascade of the time-invariant shaping filter $p(t)$, the time-variant radio channel $g_r(t;\eta)$ and the time-invariant receiver filter $h_r(t)$. Transmitting the phase-rotated differentially-encoded data sequence $\{c(i)(j)^i\}$ over the modulation channel yields the following received signal:

$$y(t) = \sum_{l} c(l)(j)^l g_{eq}(t;\xi=t-lT_b) + v(t). \qquad (6)$$

For a large class of impulse responses (e.g., for the commonly employed square-root raised-cosine responses [5,Sect.6.2.1]) it can be assumed that the filtered noise sequence $\{v(i) = n(t)*h_r(t) = v_c(i)+jv_s(i)\}$ is a stationary zero-mean white Gaussian sequence with variance equal to $N_o/2$ per component and with uncorrelated components.

The baud-rate sampled version of the received signal can be expressed as follows:

$$y(i) = y(t=iT_b) = \sum_l c(l)(j)^l g_{eq}(t=iT_b;\xi=(i-l)T_b) + v(iT_b) = \\ = \sum_m g_{eq}(t=iT_b;\xi=mT_b)c(i-m)(j)^{i-m} + v(iT_b). \quad (7)$$

Now, according to [6,Sect.7.5] a causal modulation channel of length LT_b is assumed, i.e.:

$$g_{eq}(t;\xi) = 0, \text{ for } \xi < 0 \text{ and } \xi > LT_b = T_m \quad (8)$$

where T_m is the multipath spread of the modulation channel. Eq.(7) is then rewritten as

$$y(i) = \sum_{m=0}^{L-1} g_{eq}(i;m)c(i-m)(j)^{i-m} + v(i) \quad (9)$$

Following [8] and introducing the derotation function $(-j)^i$, the derotated received samples are

$$y_d(i) = (-j)^i y(i) = \sum_{m=0}^{L-1} (-j)^i g_{eq}(i;m)c(i-m) + (-j)^i v(i) = \\ = \sum_{m=0}^{L-1} g_d(i;m)c(i-m) + v_d(i), \quad (10)$$

where $g_d(i;m) \equiv (-j)^i g_{eq}(i;m)$ is the sampled input delay-spread function of the derotated modulation channel and $v_d(i) \equiv (-j)^i v(i)$ is the additive noise sequence acting on the derotated modulation channel. The complex discrete-time T_b-spaced tapped-delay-line equivalent model of Eq.(10) is shown in Fig.2; this can be viewed as a form of the 'canonical models' originally developed in [6,Sects.III-VI]. Such a kind of discrete-time equivalent circuit is largely employed in computer simulation for modelling randomly fast time-variant data communication systems [5,Chap.7],[6],[8,Sects.2.5,6.2.8.3].

5. Second Order Statistical Description of the Modulation-Channel

The propagation of the electromagnetic wave between the Mobile Station (MS) and the Base Station (BS) is impaired by several independent random perturbations due to tropospheric scattering and diffraction from natural and artificial obstacles, so that the received signals exhibit a randomly time variant fast-fading component mainly originated by the movement of the MSs [8,Chaps.1,2]. The rapid random fluctuations superimposed to the received signal are commonly modeled as Wide Sense Stationary (WSS) complex Gaussian statistics, so that a fairly complete statistical description of the modulation channels is obtained by means of the WSS channel correlation functions [8,Chap.1-2]. In a macrocellular land-mobile environment the fast random fluctuations exhibited by the envelope of the received signal can be suitably modelled as a (stationary) Rayleigh-type distribution, with phases uniformly distributed [8,Chaps.1,2,6]. Moreover, for TDMA macrocellular systems the assumption of Uncorrelated-Scattering (US) fading process seems generally quite appropriate [3,Chap.7].

It then follows that the complex bi-dimensional discrete-index random field $\{g_{eq}(i;m)\}$ defined in Eqs.(7)-(9), can be modeled as a stationary zero-mean complex Gaussian process with uncorrelated identically-distributed component processes, thus implying a stationary Rayleigh distribution for the envelope sequence $\{|g_{eq}(i;m)|\}$ and a uniform distribution for the phase sequence $\{\Phi(i;m) = \text{Arg}(g_{eq}(i;m))\}$. From the US assumption the L taps of the discrete-channel $\{g_{eq}(i;m)\}$ constitute a set of i-indexed random sequences mutually uncorrelated with respect to m. From the Wide Sense Stationary Uncorrelated Scattering (WSSUS) assumption it follows that the time-variant random fluctuations exhibited by the fading channel-process can be fully described by the so-called Doppler-power-spectra of the L fading processes:

$$P_{eq}^{(m)}(f) \equiv \mathfrak{F}\left\{R_{eq}^{(m)}(k)\right\}, \; 0 \le f \le \frac{1}{2T_b} \quad (11)$$

where $R_{eq}^{(m)}(k) = E\left\{g_{eq}(i;m)g_{eq}^{*}(i+k;m)\right\}$ are the autocorrelation functions (ACFs) (in the i-index) of the channel-tap L complex sequences.

Several models have been proposed in literature for the ACFs [9]. However, analytical and experimental results (obtained on the basis of measurements conducted on TDMA macrocellular environments) seem to confirm that the following model for the ACF behaviour is quite appropriate [1,Chap.1,2]:

$$R_{eq}^{(m)}(k) = \sigma_m^2 J_o(2\pi B_d T_b k), \; 0 \le m \le L-1, \; k \ge 0 \quad (12)$$

where $B_d = vf_c / c$ is the doppler spread (v is the mobile speed, c the speed of light, f_c the radio frequency carrier) and $\sigma_m^2 = E\{|g_{eq}(i;m)|^2\}$, $0 \le m \le L-1$ is the m-th sample of the power-delay-profile of the fading process.

6. A Discrete-Time Tapped-Delay Line Equivalent Representation of the Derotated Modulation-Channel

It can be easily shown that the 2D complex discrete random field $\{g_d(i;m)\}$ constitutes a zero-mean complex Gaussian WSSUS random process of length L, with the same first and second order statistics (including the Doppler power spectrum) as the underotated field $\{g_{eq}(i;m)\}$; in fact, the following relationships hold:

$$|g_d(i;m)| = |(-j)^m||g_d(i;m)| = |g_d(i;m)|, \tag{13}$$

$$arg\{g_d(i;m)\} = arg\{g_{eq}(i;m)\} - (m\frac{\pi}{2})\Big|_{mod\ 2\pi} \tag{14}$$

$$E\{g_d(i;m)g_d^*(i;n)\} = (-j)^m(j)^n E\{g_{eq}(i;m)g_{eq}^*(i;n)\} = = E\{|g_{eq}(i;m)|^2\}\delta(m,n) \tag{15}$$

$$R_d^{(m)}(k) \equiv E\{g_d(i;m)g_d^*(i+k;m)\} = = (-j)^m(j)^m E\{g_{eq}(i;m)g_{eq}^*(i+k;m)\} = = E\{g_{eq}(i;m)g_{eq}^*(i+k;m)\} = R_{eq}^{(m)}(k) \tag{16}$$

(in (15) $\delta(m,n)$ is unity for $m=n$ and otherwise is zero). As shown above, the de-rotation operation acts on the delay-parameter of the channel-response, i.e. it does not modify the statistical properties of the channel-response with respect to the observation-time parameter.

The derotated noise sequence $\{v_d(i)\}$ exhibits the same first and second order statistics as the underotated sequence $\{v(i)\}$, i.e. it is a stationary zero-mean white Gaussian sequence with variance equal to $N_o/2$ per component and with uncorrelated components.

From a statistical point of view the channel responses $g_d(i;m)$ and $g_{eq}(i;m)$ must be considered equivalent (at least, up to the second-order description), and the same holds for the noises $\{v_d(i)\}$ and $\{v(i)\}$. The complex discrete-time tapped-delay-line equivalent model for the derotated modulation-channel of Fig.2 is shown in Fig.3, where a frequency-offset modulator (described in the next Section) is also included.

7. Frequency-Offset Effects on the Received Signal and Equivalent Tapped-Delay-Line Circuit-Model

The effects of a frequency-offset Δf_o (Hz) possibly present in the intermediate frequency demodulation procedure at the receiver side appears as a time-varying residual carrier-phase that contributes to the time-variations of the response of the modulation-channel. When a frequency-offset is present, the received sequence $\{y_d(i)\}$ can be expressed as follows:

$$y_o(i) \equiv exp(j2\pi\Delta f_o T_b i)y_d(i) = \sum_{m=0}^{L-1} g_o(i;m)c(i-m) + v_o(i) \tag{17}$$

where

$$g_o(i;m) \equiv exp(j2\pi\Delta f_o T_b i)g_d(i;m),\ i \ge 1,\ 0 \le m \le L-1 \tag{18}$$

$$v_o(i) \equiv exp(j2\pi\Delta f_o T_b i)\,v_d(i),\ i \ge 1 \tag{19}$$

The resulting model is shown in Fig.3. The main effect of the frequency-offset is to modify the Doppler power-spectrum of the resulting channel response according to the following relationship:

$$R_o^{(m)}(k) \equiv E\{g_o(i;m)g_o^*(i+k;m)\} = = exp(-j2\pi\Delta f_o T_b k)R_d^{(m)}(k) = = exp(-j2\pi\Delta f_o T_b k)\sigma_m^2 J_o(2\pi B_d T_b k), \tag{20}$$

having exploited eq.(16). The above equations show that a frequency-offset introduces a frequency-shift in the corresponding Doppler-power-spectrum. The offset noise sequence $\{v_o(i)\}$ is again a complex zero-mean Gaussian white sequence, with variance equal to $N_o/2$ per component and with uncorrelated components.

8. Conclusions

A discrete-time low-pass complex equivalent model has been derived for the digital-channel embedded into the band-pass continuous-time data communication system linking the MS to the BS in the TDMA/FDD macrocellular land-mobile radio-system. It takes explicitly into account the GMSK-type mo/demodulation procedure, the impairments introduced by the randomly time-variant time-dispersive Rayleigh-faded noisy radio-channel and the effects of a frequency-offset possibly present in the demodulation procedure.

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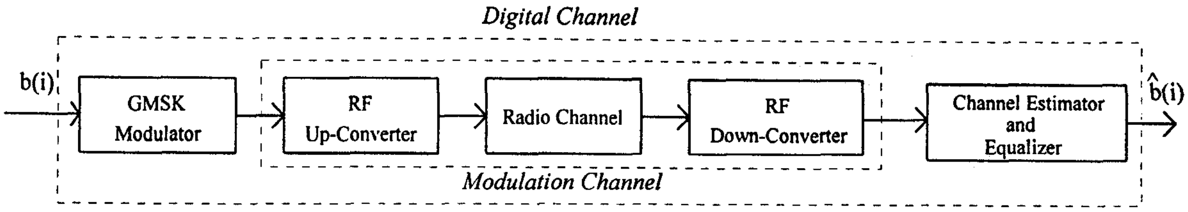

Fig.1 - General block diagram of a discrete communication channel.

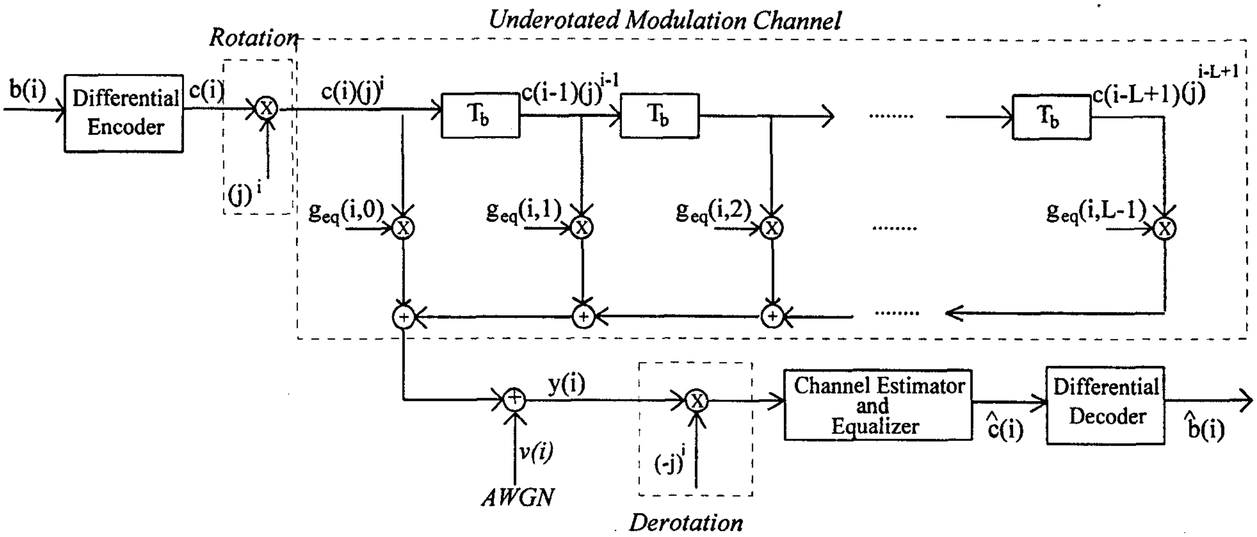

Fig.2 - Complex discrete-time T_b-spaced tapped-delay-line equivalent model of the digital channel; the underotated modulation channel is also shown.

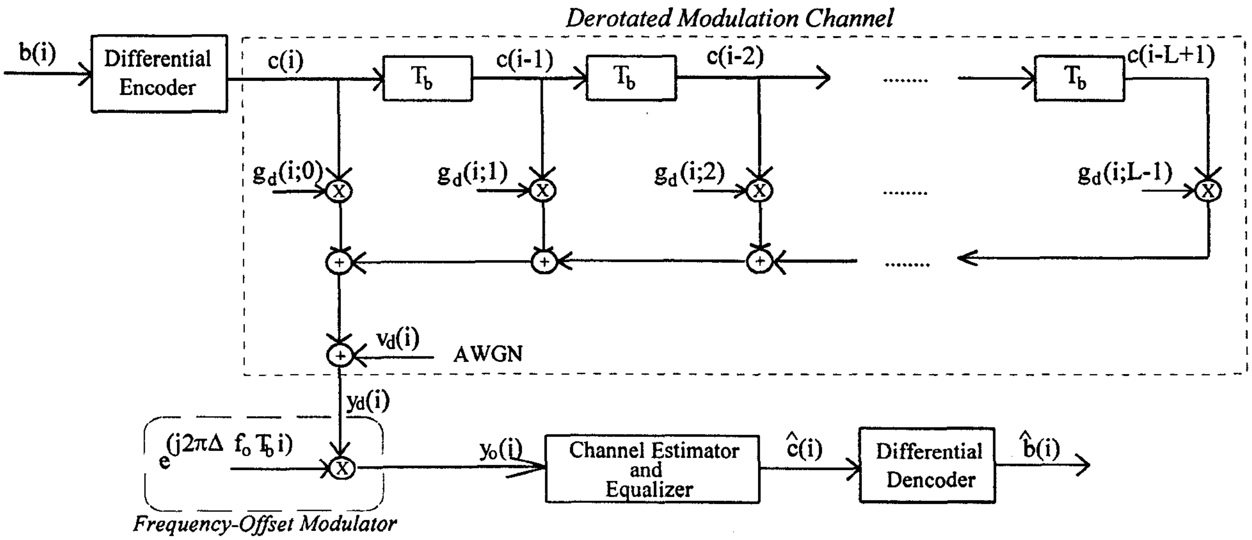

Fig.3 - Tapped-delay-line complex equivalent model for the derotated modulation channel with the frequency offset taken into acount.