

INITIAL ACQUISITION

Author's note (Oct. 2017)

- What discussed in this presentation has been expanded by including more details and additional materials and included in the newly published book “Synchronization in digital communications’ by Cambridge University Press. More information can be found in:

<http://www.cambridge.org/ec/academic/subjects/engineering/communications-and-signal-processing/synchronization-digital-communication-systems?format=HB#iHKrFm2hAfGePhsO.97>

and

https://www.amazon.com/Synchronization-Digital-Communication-Systems-Fuyun/dp/110711473X/ref=asap_bc?ie=UTF8

OUTLINE

- Introduction and System Model
- Quantitative Analysis
- Using Pre- and Post-detection Integration and Optimality
- Practical Design Considerations
- Examples of Initial Acquisition in Wireless Communications
- Summary

INTRODUCTION AND SYSTEM MODEL

Receiver Synchronization Process

- Initial Acquisition
- Timing initial acquisition
- Fine initial acquisition
- Time tracking
- Frequency tracking
- Setting other receiver functions related to synchronization

Note: This presentation will focus on initial acquisition, the carrier and timing synchronizations will be covered in subsequent presentations

Objectives of Initial Acquisition

- Find out if a desired signal exists
- Determine the starting point of a data block (frame synch)
- Determine signal level
 - AGC Initialization
- Determine coarse timing
 - Timing tracking loop initialization
- Determine coarse frequency offset
 - Frequency tracking loop initialization

Signal Detection in Initial Acquisition

- Generally it can be modeled as detection of a known signal in additive, white Gaussian noise (AWGN)
- The timing of the signal to be detected is unknown, in general
- The phase of the channel is usually unknown
- The channel may be static or time variant, single path or multipath
 - The static single path channel is most important
- It is a typical problem in detection theory

Signal Model in Initial Acquisition

- Consider a sampled digital system, at time kT , the received signal sample can be expressed as

$$r_k = h_k e^{j\phi_k} s_k + z_k$$

- Assume that the complex channel gain $h_k e^{j\phi_k}$ in a given time interval does not change
- s_k is a known, usually BPSK or QPSK, sequence
- SNR of r_k is equal to $|h_k|^2 / \sigma_z^2$, where σ_z^2 is the variance of z_k , assuming $|s_k|^2 = 1$
- Multiple samples are (coherently) combined by correlating $\{s_k\}$ with $\{r_k\}$ to form a detected symbol with no-loss of information
- Detected symbols can be non-coherent combined
- Both are for improving the detection reliability

The Detection Process

- (1) Correlating s_k with r_k , assuming h and $e^{j\phi}$ do not change, we have

$$d_n = \sum_{k=n}^{n+K-1} s_k^* r_k = \sum_{k=n}^{n+K-1} h_{c,k} e^{j\phi_k} |s_k|^2 + \sum_{k=n}^{n+K-1} s_k^* z_k = Kh_c e^{j\phi} + z'$$

The SNR of d_n is K times of the SNR of r_k ($\gamma_d = K\gamma_r$)

- (2) Forming the decision variable

- In most cases, ϕ is unknown: none-coherent detection

$$D_n = |Kh_c e^{j\phi} + z'|^2 = |d_n|^2$$

- (3) Comparing D_n with threshold T

- $D_n < T : \theta = \theta_0 \Rightarrow$ No Signal
- $D_n \geq T : \theta = \theta_1 \Rightarrow$ Signal sequence $s_n \dots s_{n+k}$ detected!

The Detection Process (cont.)

(4) If no signal, go to samples starting at T_{n+1} or $T_{n+0.5}$, until the detection of signal sequence successful

- Post detection (non-coherent) combining
 - For a time variant channel, the value of K is limited by the channel coherent time
 - Performance can be improved by summing multiple D_n 's to form a composite decision valuable

$$D_{n,L} = \sum_{l=0}^L |d_{n+lK}|^2$$

- $D_{n,L}$ is compared to a threshold to perform the same detection as above

QUANTITATIVE ANALYSIS

Decision variables' pdfs – without post-detection combining

- D_n above has the following pdfs:
 - No signal ($\theta = \theta_0$) – Central chi-square distribution with 2 degrees of freedom:

$$p_0(D) = \frac{1}{\sigma_{z'}^2} e^{-\frac{D}{\sigma_{z'}^2}}, \quad D \geq 0$$

- Signal exists ($\theta = \theta_1$) – Non-central chi-square distribution with 2 degrees of freedom:

$$p_1(D) = \frac{1}{\sigma_{z'}^2} e^{-(D+\lambda^2)/\sigma_{z'}^2} I_0\left(\frac{2|\lambda|\sqrt{D}}{\sigma_{z'}^2}\right), \quad D \geq 0$$

$$\lambda = E[a_n] = K|h_c|e^{j\phi}$$

Decision variables' pdfs – with post-detection combining

- $D_{n,L}$ given above has the following pdfs:
 - No signal ($\theta = \theta_0$) – Central chi-square distribution with $2L$ degrees of freedom:

$$p_{0,L}(D) = \frac{1}{(L-1)! (\sigma_{z'}^2)^L} D^{L-1} e^{-\frac{D}{\sigma_{z'}^2}}, \quad D \geq 0$$

- Signal exists ($\theta = \theta_1$) – Non-central chi-square distribution with $2L$ degrees of freedom:

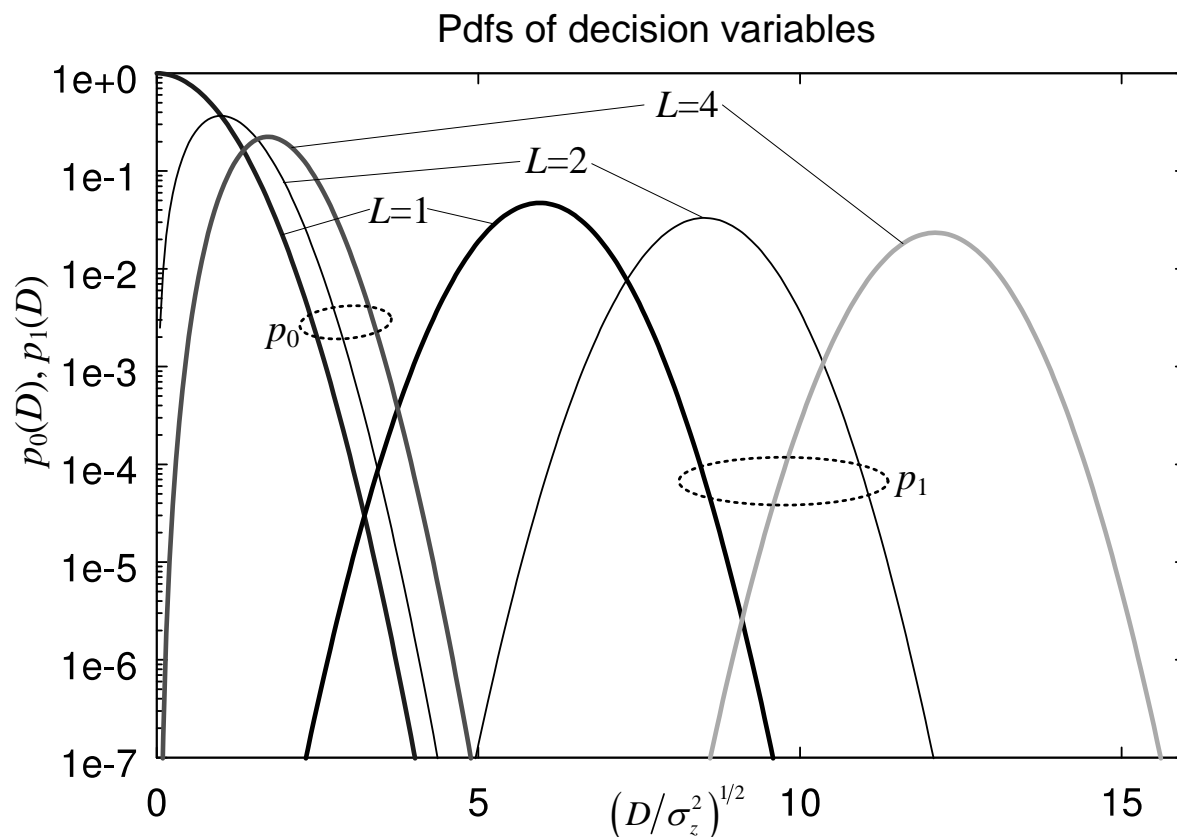
$$p_{1,L}(D) = \frac{1}{\sigma_{z'}^2} \left(\frac{D}{s^2} \right)^{\frac{L-1}{2}} e^{-(D+s^2)/\sigma_{z'}^2} I_{L-1} \left(\frac{2s\sqrt{D}}{\sigma_{z'}^2} \right), \quad D \geq 0$$

$$s = \sqrt{\sum_{k=0}^{L-1} |\lambda_k|^2} \quad \lambda_k = E[a_k] = K |h_{c,k}| e^{j\phi_k}$$

I_{L-1} – $(L-1)^{\text{th}}$ order Modified Bessel function of the first kind

Decision variables' pdfs (cont.)

Assuming all λ_k 's are equal



Detection and false Probability (P_D & P_F)

- False Probability (P_F):

$$P_F = \int_{Th}^{\infty} p_{0,L}(u) du = \int_{Th}^{\infty} \frac{1}{(L-1)! (\sigma_z^2)^K} u^{L-1} e^{-\frac{u}{\sigma_z^2}} du = e^{-Th} \sum_{k=0}^{L-1} \frac{(Th)^k}{k!}$$

– where $Th = Th / \sigma_z^2$, – Normalized threshold

- Detection Probability (P_D):

$$P_D = \int_{Th}^{\infty} p_{1,L}(u) du = \int_{Th}^{\infty} \frac{1}{\sigma_z^2} \left(\frac{u}{s^2} \right)^{\frac{L-1}{2}} e^{-(u+s^2)/\sigma_z^2} I_{L-1} \left(\frac{2s\sqrt{u}}{\sigma_z^2} \right) du$$

$$= \int_{Th}^{\infty} \left(\frac{v}{\tilde{s}^2} \right)^{\frac{K-1}{2}} e^{-(v+\tilde{s}^2)/\sigma_z^2} I_{L-1} (2\tilde{s}\sqrt{v}) dv$$

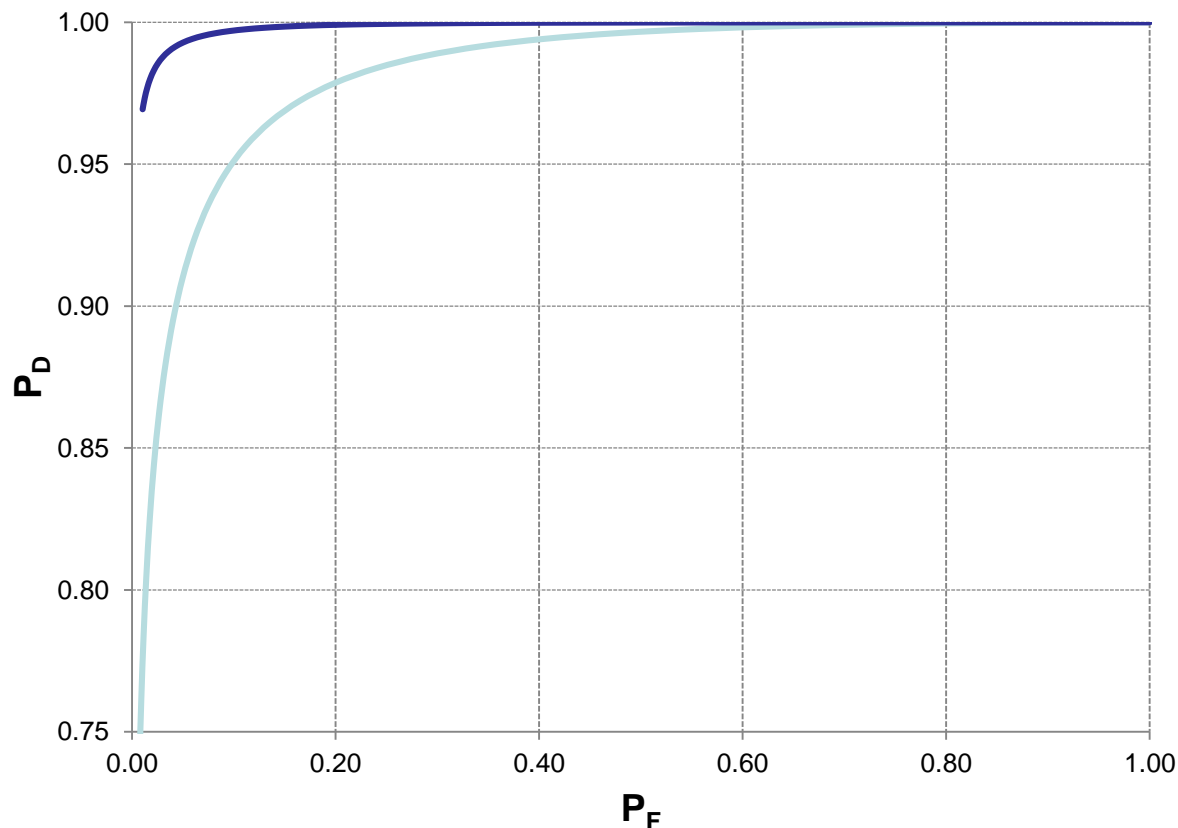
– where $Th = Th / \sigma_z^2$, $\tilde{s} = s / \sqrt{\sigma_z^2}$

P_D , P_{miss} & P_F : Discussion

- P_F only depends on the variance of noise and interference, since there is no signal
- It is more convenient to set threshold based on a predetermined constant P_F
- For the same P_F , the P_D will be larger for a larger L at the same SNR
 - For the same P_D and P_F , double L reduces required SNR by 1.8 – 2.5 dB (the gain is larger at high SNR, see examples below)
- The miss probability is defined as 1 minus the Detection probability ($P_{\text{miss}} = 1 - P_D$)

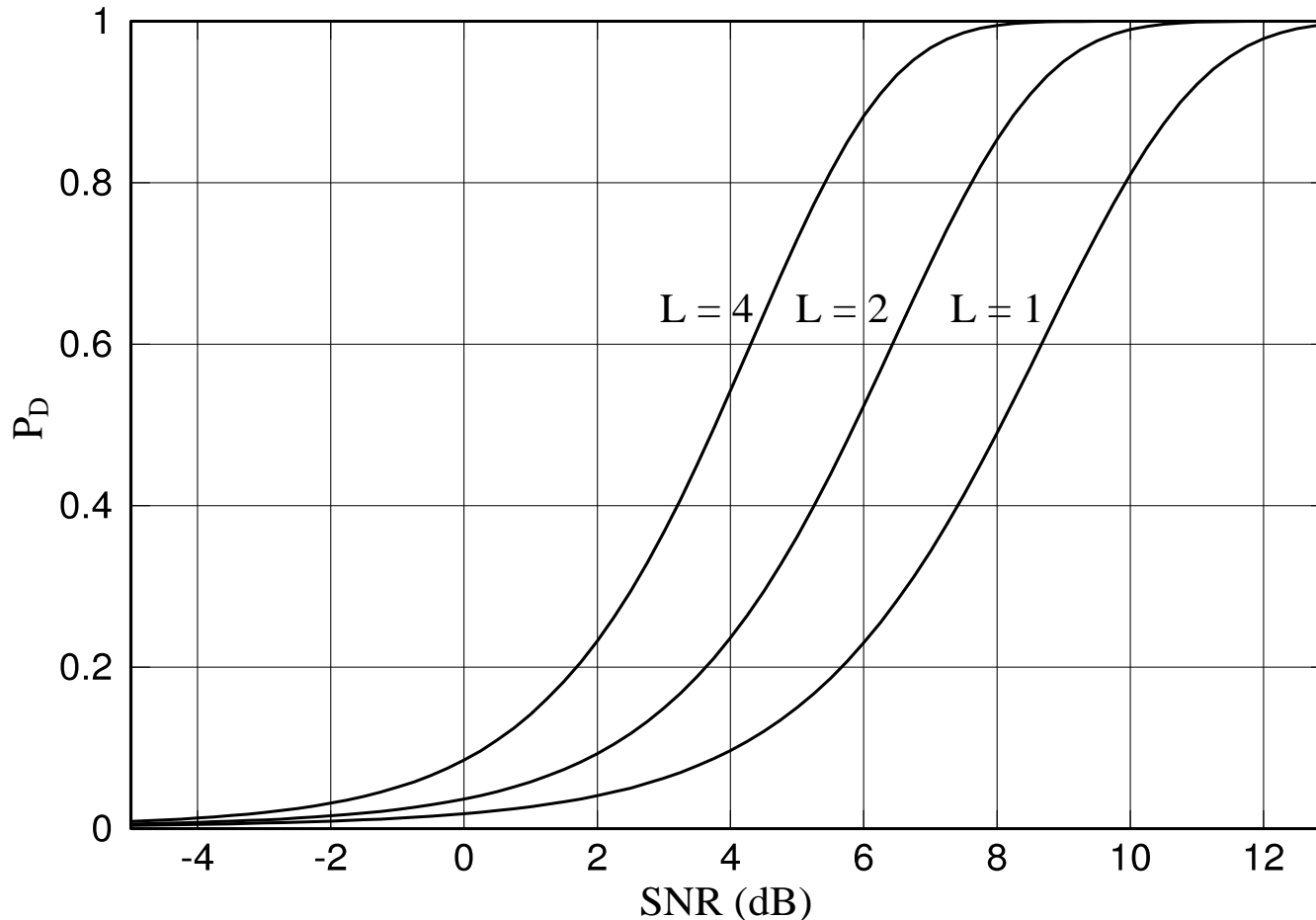
P_D , P_{miss} & P_F – Examples

- P_D vs. P_F $L = 1$ and 2 for the same SNR (5dB)



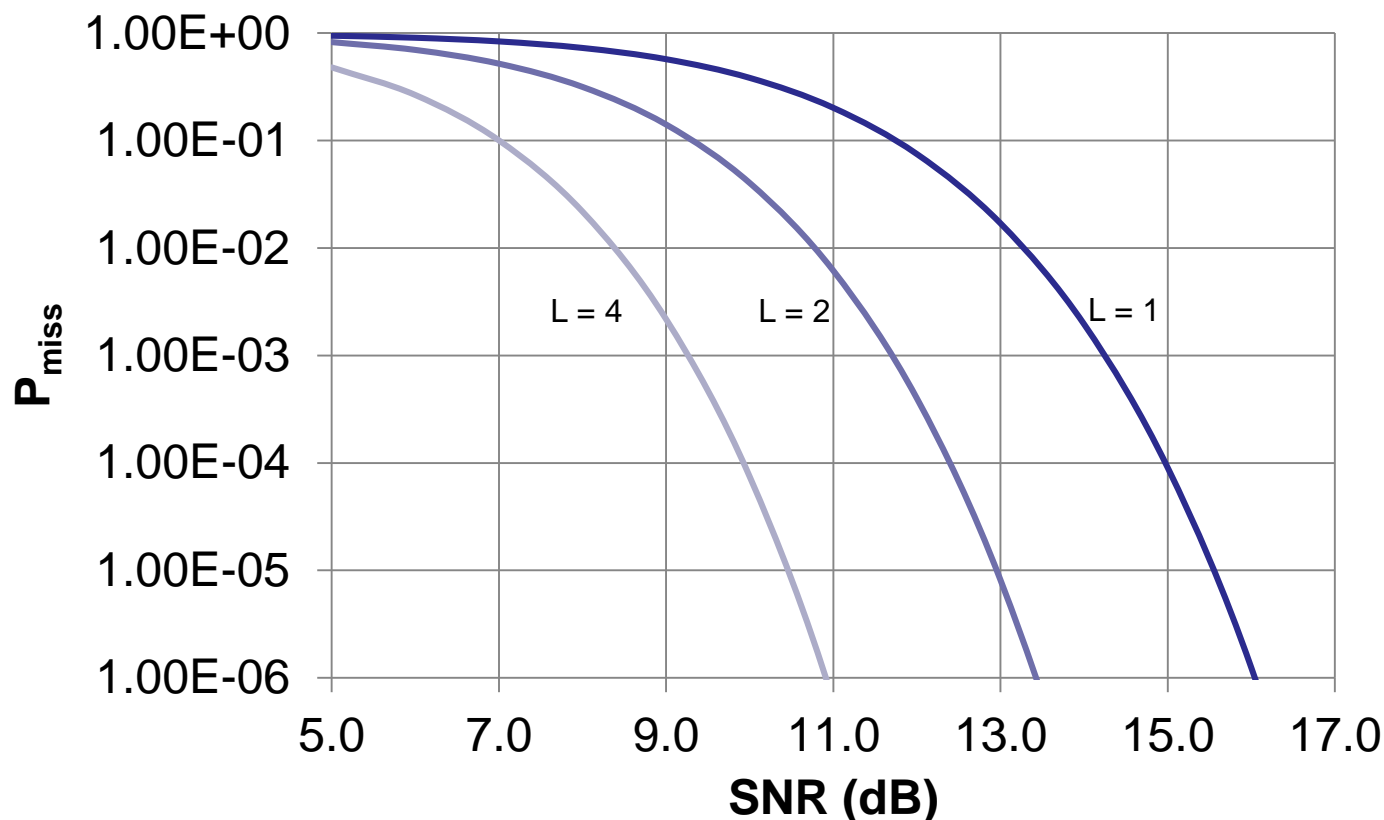
P_D , P_{miss} & P_F – Examples (cont.)

P_D for $L = 1, 2, 4$, at $P_F = 0.001$



P_D , P_{miss} & P_F – Examples (cont.)

- P_{miss} for $L = 1, 2, 4$ @ $P_F = 0.0001$ (high SNR)



USING PRE- AND POST- DETECTION INTEGRATION AND THEIR OPTIMARITY

Pre-detection integration

- Pre-detection coherent integration (correlation)

$$a_n = \sum_{k=n}^{n+K-1} s_k^* r_k = \sum_{k=n}^{n+K-1} h_{c,k} e^{j\phi_k} |s_k|^2 + \sum_{k=n}^{n+K-1} s_k^* z_k = Kh_c e^{j\phi} + z'$$

- Assuming phase and magnitude do not change: Coherent combining
- 3 dB gain when the samples in integration double
- For time-varying channel the maximum number of samples in coherent integration are limited
- In the simplest case, if there is a frequency offset
 - the integrated signal energy will reduced (combining loss)
 - The lost energy becomes an additional interference
- The combining gain is less than 3 dB for doubling samples

Pre-detection integration (cont.)

- The coherent integration operation can be viewed as signal passing through a rectangular MA filter with frequency response:

$$H(f) = \frac{\sin(\pi K T f)}{\pi K T f} = \text{sinc}(\pi K T f)$$

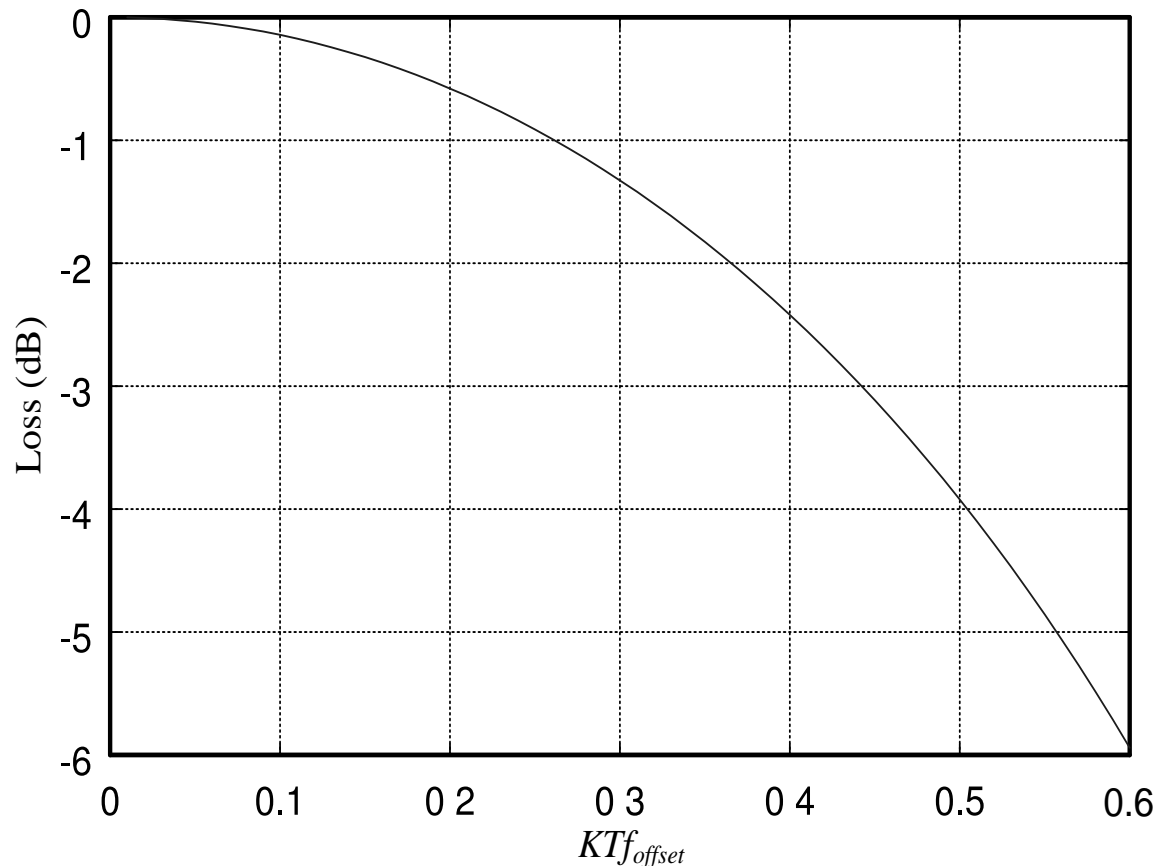
- Assuming frequency offset is f_{offset} the combined energy is equal to

$$K^2 |h_c|^2 \text{sinc}^2(\pi K T f_{\text{offset}})$$

- Loss is equal to $10\log[\text{sinc}^2(\pi K T f_{\text{offset}})]$
- The interference due to the reduced energy is equal to

$$K^2 |h_c|^2 \times (1 - \text{sinc}^2(\pi K T f_{\text{offset}}))$$

Loss in coherent integration due to frequency offset



Post-Detection Integration

- As shown above, the combining gain is about 2 dB for post-detection integration if the number of samples doubles in the integration at relatively low SNR
- At high SNR, the gain can be up to 2.5 dB
- It is necessary to determine the parameters for post- and integration based on the channel coherent time and/or frequency offset

Optimal Detection for Unknown Channel Phase

- Above we described initial acquisition procedure by comparing the squared value of the coherently integrated descrambled samples (with or without non-coherent integration) to a threshold.
- This procedure is not only convenient but also optimal when the channel phase is unknown but uniformly distributed between 0 and 2π .
- In order for it to be optimal, we need to show that this procedure satisfies the Neyman-Pearson lemma with likelihood testing
- We can prove this in three steps

Can We Argue It Is Optimal?

(1) Neyman-Pearson lemma for binary hypothesis test:

$$\Lambda(y) = \frac{p_1}{p_0} \begin{cases} > \eta : \theta = \theta_1 \\ \leq \eta : \theta = \theta_0 \end{cases}$$

- $\Lambda(y)$: Likelihood ratio, p_1, p_0 , pdfs (likelihood functions) of received signal with/without desired signal, η : threshold
- It is optimal in the sense if $\Lambda(y) \leq \eta$ we have P_F equals to a given probability, then P_D is maximized
- Does the detection procedure discussed satisfies the N-P lemma? Two questions need to be answered:
 - (1) Are $p_0(D)$ and $p_1(D)$ used in detection the likelihood functions based on the observed received signal ?
 - (2) Does comparing D to a threshold equivalent to compare $\Lambda(y)$ to η ?

Can We Argue It Is Optimal? (cont.)

(2) The coherent integration output when signal exists is

$$d_n = K |h_c| e^{j\phi_n} + z'$$

- with ϕ uniformly distributed between 0 and 2π , the pdf of d_n averaged over ϕ_n is

$$p_1(d_n) = \frac{1}{\sigma_{z'}^2} e^{-(D+|\lambda|^2)/\sigma_{z'}^2} I_0 \left(\frac{2|\lambda|\sqrt{D}}{\sigma_{z'}^2} \right), \text{ where } (D = |d_n|^2)$$

- If signal does not exist, i.e., the noise only case, the pdf is:

$$p_0(d_n) = e^{-D/\sigma_{z'}^2} / \sigma_{z'}^2$$

- Conclusion: $p_0(D)$ and $p_1(D)$ are the averaged pdfs (likelihood functions) of the coherent integrator outputs (before squaring!) with and without signal, respectively

Can We Argue It Is Optimal?(cont.)

- (3) The likelihood ratio Λ is a monotonically increasing function of D
- Th can be selected such that $D <> Th$ is equivalent to $\Lambda <> h$
 - Proof of detection with post-detection integration is similar

Notes:

- We have shown in what sense the detection process is optimal
- However, the optimality in the area of statistics is always arguable 😊

PRACTICAL DESIGN CONSIDERATIONS

Noise Variance Estimation

- Noise variance estimate is the key for setting accurate detection thresholds
 - Accuracy of the estimate determines if the detection performance meets design expectation
- Examples of noise estimation
 - Very low SNR environment, e.g., CDMA voice systems (IS-95, IS-2000)
 - Total noise power is approximately equal to total power
 - With AGC, the total power is approximately a constant
 - => The noise power is a constant
 - => Threshold determined by AGC setting

Noise Variance Estimation (cont.)

- Examples of noise estimation (cont.)
 - Medium to high SNR Environment
 - Total power at AGC output is equal to signal power plus noise power ($P_s + P_n = A$)
 - After correlation with the expected sequence:
 - With no desired signal: $E[D_n] = KE[|a_n|^2] = KP_n$
 - With desired signal:
$$E[D_n] = KE[|a_n|^2] = K^2 P_s + KP_n = B$$
 - Then $P_n = \left(\frac{AK^2 - B}{K(K-1)} \right)$ if $K \gg 1$, $P_n \approx A - \frac{B}{K^2}$
 - Comments:
 - » Estimation of A is usually more accurate due to averaging
 - » With post detection integration, the estimate can be improved by averaging multiple D_n 's

Noise Variance Estimation (cont.)

- Further discussions:
 - For detection, the accuracy of estimation of noise variance has ultimate importance.
 - Noise variance estimation can be improved if there are known orthogonal spaces in which only contains noise but no known signal, e.g.,
 - Different frequencies
 - Different Walsh code spaces
 - “Blank out” time intervals
 - Need to make sure the noises in these orthogonal spaces has the same variance as that we want to estimate for using the hypothesis testing

P_D and P_F Parameter Selection

- The target of P_F usually selected to be much smaller than P_{miss} , i.e., $1 - P_D$
 - Many possible false events would occur for one detection event, for example:
 - In CDMA-2000 initial acquisition, for each θ_I , there could be 32767 offsets (or 65534 for half chip sampling) to result θ_0
 - For WCDMA need to try many false hypotheses to find the true P-SCH and S-SCH in addition to time offset hypotheses
 - Many effort have been made to reduce the search effort (mainly hierarchical cell-search). However there are still a lot of hypotheses to test.
 - LTE also uses hierarchical cell-search approach (PSS and SSS). Thus, there are also a lot of hypotheses to test

P_D and P_F Selection (cont.)

- On the other hand, rejecting a false alarm event is usually easier than correcting a miss event
 - False alarm event may be verified by additional correlations of the same sequences with different offset
 - However, false event with additional processing may cause missing the opportunity of acquiring the signal
 - To acquired the right detection again, the acquisition need to be performed one more cycle

P_D and P_F and Acquisition Performance

- The acquiring process can be expressed by a state machine
- The acquisition performance depends on:
 - The costs of verifying a wrong test results
 - The costs needed before next successful acquisition
 - Costs include the needed time and processing power consumption
- The average power and time for a successful acquisition can be computed based on P_D , P_F and the associated cost.
- Parallel processing can reduce acquisition time but not power consumption

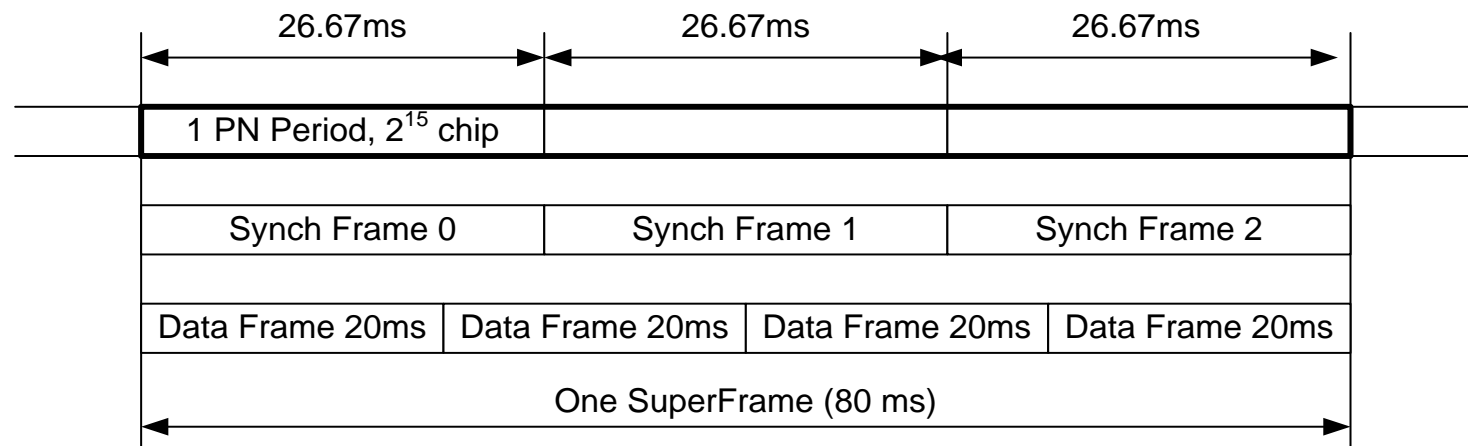
Dealing with Large Initial Frequency Offset

- It would be desirable to use (low cost) XOs with high initial frequency error, which will impact initial acquisition performance
- Shorter coherent integration length can be used followed by longer non-coherent post-detection integration
 - Acquisition time will be longer, or P_D and/or P_F will be larger
 - Coherent integration length cannot be too short due to aperiodic integration loss
- Another method is to use multiple initial frequency hypotheses
 - Setting initial frequencies to be $0, \pm\Delta F_h, \pm2\Delta F_h, \dots$ reduces the maximum initial frequency offset to $\pm0.5\Delta F_h$
 - This will increase computational complexity of acquisition
 - May also increase initial acquisition time

EXAMPLES OF INITIAL ACQUISITION IN WIRELESS COMMUNICATIONS

CDMA-2000

- It's pilot channel is used for initial acquisition
- Pilot channel signal is periodic with period (26.6667 ms long) of 32768 with QPSK modulation (with a pair of augmented $2^{15}-1$ long PN sequences known to receiver)
- Three period of the pilot channel constitutes an 80 ms Superframe, which contains 3 26.67 ms Synch frames and 4 20 ms data frames



CDMA-2000 (cont.)

- To detect the pilot channel, the receiver correlates the received signal with one or more segments of the sequence.
- The received signal should be sampled more than once per sample (chip) interval, usually, 0.5 chip interval, i.e., 2 correlations per one chip interval is appropriate
- For parallel processing and/or post-detection combining, segments of the PN sequence can be used to correlate with the same or different signal sequences
- Once a match is found, the receiver knows the beginning of the PN sequence
- A PN sequence period aligns with a frame of sync channel

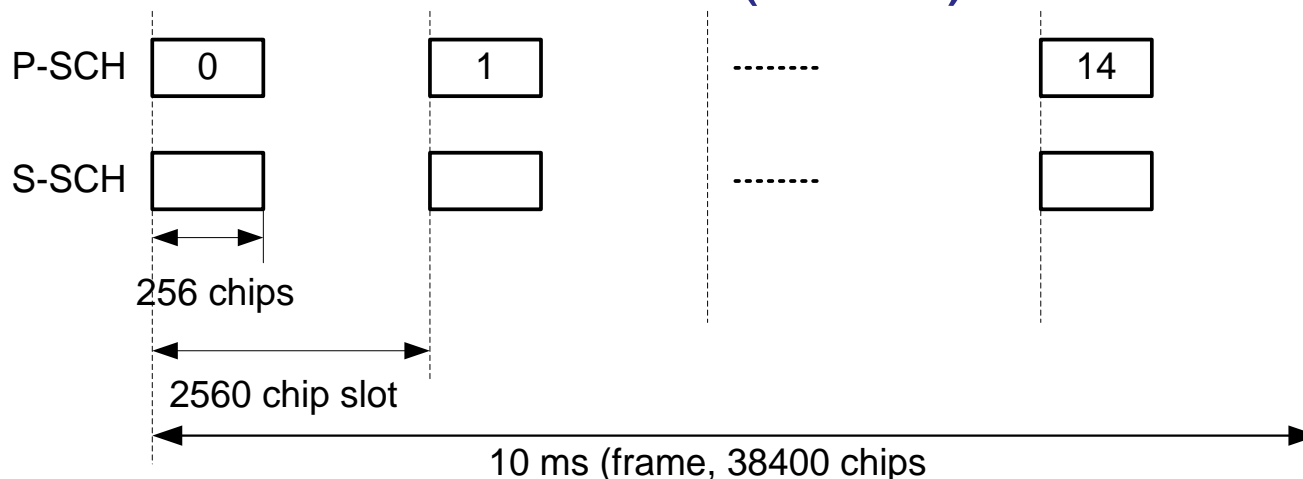
CDMA-2000 (cont.)

- Once the beginning of PN sequence, the receiver demodulate the synch channel to determine the start of the Superframe and information for demodulating other forward link channels
- Selection of coherent integration length:
 - Assume we have initial frequency accuracy of 2ppm (2×10^{-6})
 - For 800 MHz band, the frequency offset is 1600 Hz
 - The CDMA2000 chip rate is 1.2288 Mchips/sec
 - If we choose to integrate 100 chips, we will have a loss of
$$10\log\left[\text{sinc}^2(\pi * 100 * 1.2288 * 10^{-6} * 1600)\right] = -0.64\text{dB}$$
 - For 1.9G band, the integration will be 40 chips to have the same loss
- Multiple such coherent integrated outputs could be non-coherently combined

WCDMA

- WCDMA employs a two stage initial acquisition procedure with two Synchronization Channels:
 - Primary Synchronization Channel (P-SCH)
 - Utilize a 256 chip spreading sequence
 - Same for all of the cells
 - Secondary Synchronization Channel (S-SCH)
 - Each cell transmits one out of 64 possible S-SCH codes
 - Each S-SCH codes is a combination of 15 different sequences, each of which is from a group of 16 256 chip sequences
- WCDMA SCH channel structure is as shown below

WCDMA (cont.)



- Acquisition process:
 - First search for P-SCH
 - 256 chip (@ 3.84 MHz = 66.67 μ s) coherent integration
 - Non-coherent combining of coherent integration output may be used.
 - Successful P-SCH search determines slot boundary

WCDMA (cont.)

- Acquisition process (cont.)
 - Search for S-SCH
 - Determine S-SCH code sequences at slot boundaries by correlating all 16 possible 256 S-SCH chip sequences
 - The largest peaks at the correlator output determines the S-SCH chip sequences transmitted by the cell
 - Match the determined chip sequences to one of the 64 possible S-SCH code words (may need to check 15 start positions)
 - Once the S-SCH code word is determined, the receiver knows the frame boundary
 - Search for primary scrambling codes of the pilot and data channels (Each S-SCH code word corresponding to a code group with 8 scrambling codes)

LTE

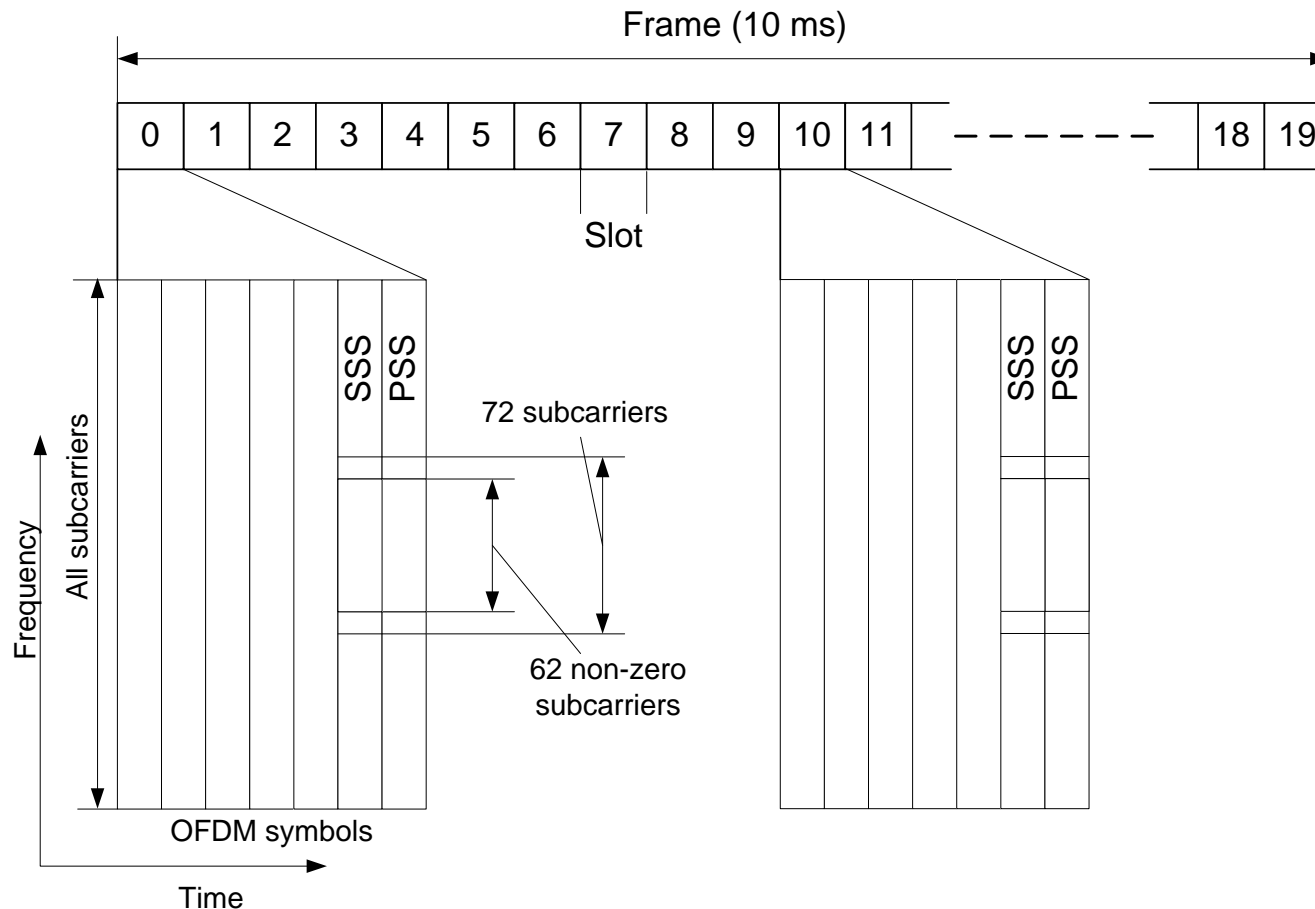
- LTE has 6 possible transmission signal bandwidths: 1.4, 3, 5, 10, 15 and 20
- LTE has FDD (discussed here) and TDD modes
- To facilitate the initial acquisition, the synch channel bandwidth is 1.4 MHz (72 subcarriers, 62 non-zero data).
 - For wider bandwidth transmission, the synch channels occupy the center 72 subcarriers (73 including the zero subcarrier).
- Similar to WCDMA, LTE employs a two stage initial acquisition procedure using two Synch Channels:
 - Primary Synchronization Channel (P-SCH or PSS)
 - Secondary Synchronization Channel (S-SCH or SSS)
 - PSS and SSS occupies one OFDM symbol each
 - A pair of PSS and SSS symbols are transmitted every 5 ms

LTE (Cont.)

- PSS OFDM Symbol
 - It is generated from a frequency domain Zadoff-Chu sequence with two 32 long segments
 - Constant Amplitude (low peak to average power ratio)
 - Impulse (time domain) autocorrelation
 - There are three such PSS symbols ($N_{ID}^{(2)} = 0, 1, 2$)
 - It is the last OFDM symbol in the 0th and 10th slot
- SSS OFDM Symbol:
 - Consists of 2 interleaved length-31 m-sequences, each of which has a different cyclic shift
 - There are 168 such SSS OFDM symbols ($N_{ID}^{(1)} = 0, 1, \dots, 167$), with combinations of different shifts
 - scrambled according to $N_{ID}^{(2)}$
 - There are a total of 504 cell ID's: $N_{ID}^{cell} = 3N_{ID}^{(2)} + N_{ID}^{(1)}$
 - It is the OFDM symbol proceeding PSS

LTE (Cont.)

- PSS and SSS in time and frequency



LTE (Cont.)

- PSS Acquisition:
 - Even though PSS is defined in frequency domain, the detection is most efficient done in time domain.
 - The total data bandwidth of PSS is 945KHz, the signal can be first filtered to between 0.945 to 1.08 MHz and down sampled
 - The down sampled signals are correlated the three possible sample sequences of the time domain representations of PSS
 - OFDM symbol length is $67\mu\text{s}$ so coherent combining can be used
 - The correlator output are squared, likely non-coherently combined with multiple such output spaced by 5 ms, and compared to a threshold.
 - Such hypothesis testing need to performed every 0.5 sample interval until a success is declared
 - This determines the half frame boundary and the cell ID $N_{ID}^{(2)}$

LTE (Cont.)

- SSS Detection:
 - The scramble code of SSS is known with $N_{ID}^{(2)}$ from PSS detection
 - The received signal corresponding to SSS position is correlated with the 168 time domain representations of SSS
 - The largest correlation outputs determines the cell ID
 - The frame boundary is determined by based on SSS
 - The zeroth and 10th SSS has the same two 31 sequences but swapped in place
 - Non-ideal cross-correlation property between SSS sequences may require extra steps to reduce the false probability
- Cell ID (PSS and SSS) determines the scrambling sequences of other channels

Further Discussions

- In a multipath channel a moving average filter of the squared detection outputs as the decision variable may provide a better detection probability
- The received signal sample sequence that passed the detection hypotheses can provide a rough timing estimate
 - In LTE, the timing will need to be refined if the data bandwidth is wider than the PSS/SSS bandwidth, i.e., 1.4 MHz
- The phase differences between the consecutive correlation outputs can provide an estimate of the carrier phase shift between them and thus frequency offset
- These conclusions can be used for all of the three examples discussed above
 - Due to the large number of possible bands for LTE deployment, pre-PSS frequency scanning may be necessary to determine which bands contain valid LTE signals

SUMMARY

- Initial acquisition is usually done by the receiver trying to find a known sequence sent at regular interval by Tx
- The detection is done by correlating a known transmitted sequence with the received signal samples and the squared outputs are compared to a threshold for detection
- The theoretical pre-detection (coherent) and post-detection (non-coherent) characteristics were derived
- The coherent and non-coherent combining performances were shown and their trade-offs were discussed
 - It is shown numerically, post-detection (non-coherent) combining has about 2 dB gain or higher when samples are doubled
 - In theory pre-detection (coherent) combining has a 3 dB gain or when samples are doubled. However, the gain is reduced when channel is time-varying

- It is shown the square detection metrics are optimal in statistical sense
- Practical design considerations were discussed
- Three examples of wireless communication initial acquisitions were presented, including CDMA2000, WCDMA and LTE
- Initial acquisition can provide initial estimates for receiver frequency, timing and AGC blocks
- Only the simplest case of single path static AWGN channel was discussed, because:
 - It can be used base-line for system design and accurate verification of receiver performance in simulation and lab testing
 - It is the foundation of system initial acquisition for more complex system models
 - Special considerations must be given for specific systems, e.g. LTE, especially TDD LTE