

# Simple Coherent Receivers for Partial Response Continuous Phase Modulation

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**Abstract**—By using a pulse amplitude modulation representation of the binary continuous phase modulation signals, we develop a new optimum Viterbi sequence detector and a near optimum Viterbi receiver with low complexity. Also, for the modulation index 0.5 where a linear receiver can be used, a minimum mean-squared error linear receiver filter is derived. Their performance is analyzed. The Gaussian minimum shift keying signal (GMSK) is used for illustration. It is shown that a GMSK receiver consisting of two matched filters and a four-state Viterbi algorithm performs with less than 0.24 dB degradation compared with the optimal receiver. The linear receiver is optimum for all values of  $E_b/N_0$  (bit energy to noise one-sided spectral density ratio). A design method for its receiver filter is given. The filter is equivalent to a cascade of a matched filter and a Wiener filter estimator. Both an upper and a lower bounds for the bit-error probability are calculated. Simulation results which confirm the analysis are also given.

## I. INTRODUCTION

THE binary continuous phase modulation (CPM) signals presenting attractive spectra and good error probability often require a maximum-likelihood sequence detector implemented using the Viterbi algorithm (VA) [1]. The receiver consists of a filter bank followed by a Viterbi processor, where the number of filters and states can be quite large. The total number of states in the trellis is  $p2^{L-1}$  where  $p$  is the number of phase states and  $L$  is the duration, in symbol intervals, of the impulse response of the filter at the modulator input. Several authors have tried to reduce this complexity and have proposed to modify the VA to obtain simpler receivers with some loss in performance. In [1, Section 8.1] the phase tree of the transmitted signals is approximated by a phase tree based on a shorter impulse response and the Viterbi receiver for this approximate tree is used instead. The receivers proposed here are based on the representation, developed by Laurent [3], of the binary CPM signal as a sum of pulse amplitude modulation (PAM) signals. Such a decomposition helps to simplify the receiver design because the signal then has a linear form.

In the first part of this paper, new optimum and reduced-complexity Viterbi detectors are presented. It is shown that near-optimal performance can be obtained with a Viterbi receiver in which the number of matched filters

and states is appreciably reduced. The key is to approximate CPM signals by a sum of few PAM signals and use the correlation of these approximate signals with the observation to calculate survivor metrics needed by the VA. An application to the case of the Gaussian minimum shift-keying signal (GMSK) [8] leads to a receiver composed of two matched filters and four-state VA. The performance of the simplified receiver is then evaluated by calculating the degradation in performance with respect to the optimum receiver. It is shown that in the GMSK case the degradation is less than 0.24 dB. Simulation results which confirm the analysis are also given.

However, for general binary partial response CPM signals with modulation index equal to 0.5, it is possible to construct a simple minimum shift-keying (MSK) type receiver as in Fig. 1 [1], [4], [7]. The MSK signal is a particular case of binary Continuous-phase modulation (CPM). It has an equivalent linear offset quadrature modulation which allows the construction of a simple optimal coherent receiver. A complex model of this system is shown in Fig. 1. The information symbols  $\alpha_k$  are differentially encoded into complex symbols  $a_k$  according to the relation  $a_k = j\alpha_k a_{k-1}$ , and then the encoded symbols modulate the amplitude of a real and even pulse. The receiver filter, which has an impulse response  $h(t)$ , is matched to the modulator filter. The decisions on symbols  $a_{2k}$  are taken at times  $2kT$  using samples on real (or in-phase) arm and on symbols  $a_{2k+1}$  at times  $(2k+1)T$  using samples on imaginary (or quadrature) arm. Finally, a differential decoder delivers the information symbols  $\alpha_k$ . The main design problem of such a receiver for the general partial response CPM is to find an optimum receiver filter  $h(t)$  following a specific criterion.

In the second part of this paper, we present an optimum coherent linear receiver for the general partial response CPM with index 0.5 based on the minimum mean square error (MMSE) criterion and give a design method of the optimum receiver filter. Performances are evaluated; we give the MMSE and calculate an upper and a lower bounds for the bit error probability. We then apply these results to the GMSK signal and we report numerical results concerning the eye diagram, filter shape, and error probability bounds. Bit error rates obtained from simulation are also presented.

Several authors [4]–[7] previously tried to define the receiver filter. A survey of published design methods is given in [1, Section 8.2]. An empirical filter was first used

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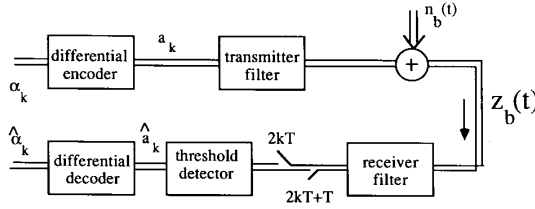


Fig. 1. Baseband model for MSK system.

in [7] for the tamed frequency modulation (TFM). Similarly, an optimized Gaussian-shaped filter is used in [8] for the GMSK signal. An interesting approach based on the minimum probability of error criterion was proposed by Galko and Pasupathy, summarized in [4], and then modified by Svensson and Sundberg [5]. In these works, algorithms are proposed to give solutions for the optimum filter in the asymptotic cases of vanishingly small  $E_b/N_0$  (bit energy to noise one-sided spectral density ratio) and of arbitrary large  $E_b/N_0$ . However, no solution has been found for intermediate values of  $E_b/N_0$ . Our method gives a precise solution for all values of  $E_b/N_0$ . An approximate MMSE solution is given in [6] and another solution based on an approximate pulse amplitude modulation (PAM) model can be found in [4]. Our solution is based on an exact PAM representation of CPM [3].

We present in Section II the signal model for the partial response binary CPM. In Section III, the CPM signal with general modulation index is developed into a sum of PAM signals. The optimum VA receiver is then deduced for rational modulation index in Section IV and its simplified version is presented in Section V. In order to illustrate the method, we apply it in Section V-B to the case of the GMSK signal. Performance analysis is made in Section V-C. Section VI is devoted to the linear reception of CPM signals with index 0.5. The derivation of an MMSE optimum linear receiver filter is made in Section VI-A. We then calculate in Section VI-B an upper and a lower bounds of the probability of error. In Section VII numerical and simulation results are given for both simplified Viterbi receiver and linear receiver. Finally, based on [3], the Appendix presents the PAM representation of CPM.

Notations: In the following  $x_b(t)$  denotes the baseband (complex envelope) of a passband signal  $x(t)$ , \* complex conjugate and  $E[\cdot]$  the expectation. Also

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-y^2/2) dy,$$

$$(x, y) \triangleq \int_{-\infty}^{+\infty} x(t) y^*(t) dt, \quad \|x\|^2 \triangleq (x, x).$$

## II. SIGNAL MODEL

A general binary partial response CPM signal has the expression [1]

$$s_i(t) = \text{Re} \{ s_{bi}(t) e^{j2\pi f_0 t} \} \quad (1a)$$

$$s_{bi}(t) = \sqrt{\frac{2E_b}{T}} e^{j\phi_i(t)} \quad (1b)$$

$$\phi_i(t) = \pi h \int_{-\infty}^t \sum_{n=0}^{N-1} \alpha_n^i f(\tau - nT) d\tau \quad (1c)$$

$$= \pi h \sum_{n=0}^{N-1} \alpha_n^i g(t - nT) \quad (1d)$$

where  $f_0$  is a reference frequency,  $E_b$  is the energy per bit,  $\alpha_k^i$  are independent symbols which take their values in the set  $\{1, -1\}$  with equal probabilities. The index  $i$  designates a transmitted message  $i$  (composed of  $N$  consecutive symbols) among  $2^N$  possible messages. To simplify the notation, this index is not often written.  $T$  is the symbol time interval,

$$g(t) = \int_{-\infty}^t f(\tau) d\tau, \quad (2)$$

$f(t)$  is the impulse response of a smoothing filter limited to the interval  $[0, LT]$  and satisfying the following two properties:

$$f(t) = f(LT - t) \quad (3a)$$

$$g(LT) = 1, \quad (3b)$$

and  $h$  is the modulation index which takes on rational values  $2k/p$ ;  $k$  and  $p$  are integers.

## III. PAM REPRESENTATION

As Laurent [3] showed, the baseband signal  $s_b(t)$  can be written as a sum of  $2^{L-1}$  PAM signals, i.e.,

$$s_b(t) = \sqrt{2E_b} \sum_{k=0}^{2^{L-1}-1} \sum_{n=0}^{N-1} a_{k,n} h_k(t - nT) \quad (4)$$

where, see Appendix,

$$a_{0,n} = \exp \left( j\pi h \sum_{k=0}^n \alpha_k \right) = a_{0,n-1} J^{\alpha_n}, \quad (5)$$

$$J \triangleq e^{j\pi h}, \quad (6)$$

and the other  $a_{k,n}$  are also complex symbols which have the following general expression:

$$a_{k,n} = a_{0,n-L} \prod_{i \in I_k} J^{\alpha_{n-i}}, \quad k = 1, 2, \dots, 2^{L-1} - 1. \quad (7)$$

In this equation,  $I_k$  is a nonempty subset of the set  $\{0, 1, \dots, L-1\}$ . For instance, for the special case  $L = 4$  specified in the Appendix,  $I_1$  is the subset  $\{0, 2, 3\}$ . In fact, by using (5) in (A.19) we get

$$a_{1,n} = a_{0,n-4} J^{\alpha_{n-3}} J^{\alpha_{n-2}} J^{\alpha_n} \quad (8)$$

The time pulses  $\{h_k(t)\}$  in (4) are real and equal to the product of  $L$  shifted versions of a basic function  $c(t)$  defined by (A.3).

$$c(t) \triangleq \begin{cases} \frac{\sin[\pi h - \pi h g(t)]}{\sin \pi h}, & t \in [0, LT) \\ c(-t), & t \in (-LT, 0] \\ 0, & |t| \geq LT \end{cases}$$

Most of the signal energy is carried by a PAM signal corresponding to the main pulse  $h_0(t)$  of (4) above. This  $h_0(t)$  is defined as

$$h_0(t) = \begin{cases} \frac{1}{\sqrt{T}} \prod_{k=1}^L c(t - kT); & t \in [0, (L+1)T] \\ 0; & \text{otherwise.} \end{cases} \quad (9)$$

For the case  $L = 4$ , expressions for  $h_k(t)$  are given in the Appendix.

#### IV. OPTIMUM RECEIVER

The received signal to be processed in a coherent receiver is

$$z(t) = s_i(t) + n(t) \quad (10)$$

where  $n(t)$  is a realization of a zero-mean second-order Gaussian noise, independent of the signal, and with double-sided power spectral density  $N_0/2$  over the bandwidth of  $s_i(t)$ .

Since all the possible transmitted signals have equal energy and equal *a priori* probability, the optimum receiver which minimizes the message error probability decides that message  $i$  is transmitted if and only if  $i$  maximizes the following metric [2]:

$$\Lambda_i \triangleq 2(z, s_i) = \text{Re}(z_b, s_{b_i}). \quad (11)$$

Substituting (4) in (11) yields

$$\Lambda_i = \sqrt{2E_b} \sum_{n=0}^{N-1} \lambda_i(n), \quad (12)$$

$$\lambda_i(n) \triangleq \text{Re} \sum_{k=0}^{2^{L-1}-1} r_{k,n} a_{k,n}^i, \quad (13)$$

$$r_{k,n} \triangleq \int z_b(t) h_k(t - nT) dt. \quad (14)$$

Relation (13) shows that  $\{r_{k,n}\}$  are sufficient statistics for the decision, and (14) indicates that they can be obtained by sampling at times  $nT$  the outputs of  $2^{L-1}$  matched filters  $\{h_k(-t); k = 0, 1, \dots, 2^{L-1} - 1\}$  simultaneously fed by the complex input  $z_b(t)$ , as shown in Fig. 2.

The calculation of all the possible  $\lambda_i(n)$  requires the knowledge, at every time  $nT$ , of all the possible  $\{a_{k,n}; k = 0, 1, \dots, 2^{L-1} - 1\}$ . These in turn depend upon the symbol  $\alpha_n$  and a state defined by the vector  $\{a_{0,n-L}, \alpha_{n-L+1}, \dots, \alpha_{n-2}, \alpha_{n-1}\}$ , see (5) and (7). If  $h = 2k/p$  ( $k, p$  integers),  $a_{0,n-L}$  takes  $p$  discrete values, and therefore the state vector takes  $p2^{L-1}$  values. The decision rule can be now implemented by using the Viterbi algorithm (VA). The complexity of the VA is proportional to the number of states  $p2^{L-1}$ .

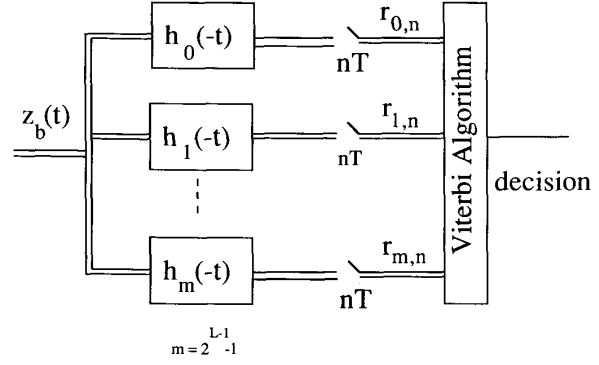


Fig. 2. Optimum receiver.

Although the optimum Viterbi receiver presented here is different from the one described in [1], its complexity is nearly the same. However, the complexity of our receiver can be easily reduced by the method given in the following section.

#### V. SIMPLIFIED VITERBI RECEIVER

##### A. Decision Rule

The complexity of the optimum receiver is caused by the fact that the transmitted signal is composed of a relatively large number of PAM components. The complexity can be reduced if the receiver is designed to decide on approximate signals composed of a smaller number of PAM components. Let us then decompose the baseband signal (4) as

$$s_b(t) = \bar{s}_b(t) + \epsilon_b(t) \quad (15)$$

$$\bar{s}_b(t) = \sqrt{2E_b} \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} a_{k,n} h_k(t - nT) \quad (16)$$

where  $\epsilon_b(t)$  is a negligible term generated by the pulses  $\{h_k(t); K \leq k \leq 2^{L-1} - 1\}$  having very small energy. The received signal (10) can now be written as

$$z_b(t) = \bar{s}_b(t) + \epsilon_b(t) + n_b(t)$$

where  $n_b(t)$  is a realization of a zero-mean second-order complex Gaussian noise, independent of the signal, and with power spectral density  $2N_0$  over the bandwidth of the baseband signal  $s_b(t)$ . A low-complexity receiver has to maximize this simpler metric

$$\bar{\Lambda}_i = 2(z, \bar{s}_i) = \text{Re}(z_b, \bar{s}_{b_i}) \quad (18a)$$

$$= \sqrt{2E_b} \sum_{n=0}^{N-1} \bar{\lambda}_i(n) \quad (18b)$$

with

$$\bar{\lambda}_i(n) \triangleq \text{Re} \sum_{k=0}^{K-1} r_{k,n} a_{k,n}^i. \quad (19)$$

Since  $\{r_{k,n}; k = K, K+1, \dots, 2^{L-1} - 1\}$  are considered as irrelevant, the number of matched filters needed is reduced to  $K$ . In addition, a great reduction in the VA

complexity is achieved since  $a_{k,n}$ ;  $k = K, K+1, \dots, 2^{L-1} - 1$  are not considered by the algorithm.

We show in the next section how to employ this simplified receiver for GMSK.

### B. Application to GMSK

To illustrate these results we consider the case of GMSK for which  $h = 0.5$ . In such a case, (5) and (8) become

$$a_{0,n} = \prod_{k=0}^n j\alpha_k = j\alpha_n a_{0,n-1} \quad (20a)$$

$$a_{0,2n} \in \{1, -1\}, \quad a_{0,2n+1} \in \{j, -j\}. \quad (20b)$$

and

$$a_{1,n} = j\alpha_n a_{0,n-2} \quad (21a)$$

$$a_{1,2n} \in \{j, -j\}, \quad a_{1,2n+1} \in \{1, -1\}. \quad (21b)$$

For the GMSK signal the smoothing filter  $f(t)$  specified in Section II is given by

$$f\left(t + \frac{LT}{2}\right) = \frac{1}{T} \{Q(\sigma t_1) - Q(\sigma t_2)\}$$

where

$$t_1 = t - \frac{T}{2}, \quad t_2 = t + \frac{T}{2}, \quad \sigma = \frac{2\pi B}{\sqrt{\ln 2}}.$$

and  $L$  is chosen such that  $f(t)$  can be considered, with good approximation, limited to the interval  $[0, LT]$ . For the particular case when  $BT = 0.25$ , we take  $L = 4$ . The calculation of  $g(t)$  defined by (2) for GMSK yields

$$g\left(t + \frac{LT}{2}\right) = 1 + \frac{1}{T} \{t_1 Q(\sigma t_1) - t_2 Q(\sigma t_2)\} - \frac{1}{\sigma\sqrt{2\pi}} \{e^{-(\sigma^2/2)t_1^2} - e^{-(\sigma^2/2)t_2^2}\}.$$

Computer calculations based on formulas (A.3), (A.11)–(A.12) show that a fraction 0.991944 of the signal energy is contained in the first PAM component carried by the pulses  $\{h_0(t - kT)\}$ , and that a fraction 0.00803 of the energy is carried by the pulses  $\{h_1(t - kT)\}$ . Only a fraction  $2.63 \times 10^{-5}$  of the signal energy ( $-46$  dB) is contained in  $\epsilon_b(t)$  generated by modulating the remaining six pulses. Values of the same order are obtained for TFM.

By taking  $K = 2$  in the decomposition (15)–(16), only two matched filters  $h_0(-t)$  and  $h_1(-t)$  are needed. The resulting receiver is depicted in Fig. 3. From (19) the branch metric is

$$\bar{\lambda}_i(n) = \text{Re} \{r_{0,n} a_{0,n}^* + r_{1,n} a_{1,n}^*\}. \quad (22)$$

This expression can be simplified by introducing the real symbols  $\bar{a}_n$  which take values 1 and  $-1$ ,

$$\bar{a}_n = \begin{cases} a_{0,n}; & n \text{ even} \\ -ja_{0,n}; & n \text{ odd} \end{cases} \quad (23)$$

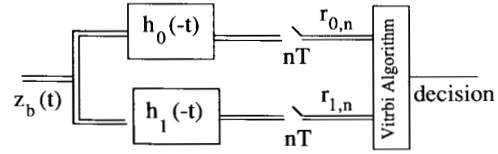


Fig. 3. Simplified Viterbi receiver.

and by using (21). We thus get

$$\bar{\lambda}_i(2n) = \text{Re} \{r_{0,2n}\} \bar{a}_{2n}^i - \text{Im} \{r_{1,2n}\} \cdot \bar{a}_{2n-2}^i \bar{a}_{2n-1}^i \bar{a}_{2n}^i \quad (24a)$$

$$\bar{\lambda}_i(2n-1) = \text{Im} \{r_{0,2n-1}\} \bar{a}_{2n-1}^i - \text{Re} \{r_{1,2n-1}\} \bar{a}_{2n-3}^i \bar{a}_{2n-2}^i \bar{a}_{2n-1}^i. \quad (24b)$$

The state vector at time  $nT$  is defined by  $\{\bar{a}_{n-2}, \bar{a}_{n-1}\}$ . Therefore, four states are needed by the VA. Fig. 4 shows the corresponding trellis and branch metrics. No multiplication is used in calculating the metrics. The VA gives decisions on symbols  $\bar{a}_n$  and then a different decoder gives the related  $\alpha_n$  symbols by applying the following rule:

$$\alpha_{2n} = -\bar{a}_{2n} \bar{a}_{2n-1} \quad (25a)$$

$$\alpha_{2n+1} = \bar{a}_{2n+1} \bar{a}_{2n}. \quad (25b)$$

Other trellis configurations which save differential decoding are possible.

### C. Performance Analysis

It is well known [1] that the bit error probability  $P_b$  of the optimum receiver is given by

$$P_b \leq C \cdot Q\left[\frac{d_{\min}}{\sqrt{2N_0}}\right] \quad (26)$$

where  $C$  depends upon the number of nearest neighbors to the transmitted signal, and  $d_{\min}$  is the minimum Euclidean distance between transmitted signals, i.e.,  $d_{\min} = \min_{i,j} \|s_i - s_j\|$ ,  $i \neq j$ . Often, in order to save the tedious calculation of the multiplying factor  $C$ , the parameter  $d_{\min}$  is considered as a performance measure. The term  $Q(\cdot)$  in (26) is the pairwise probability that the VA final survivor in the trellis leaves the correct path and follows an incorrect one where these two paths correspond to two signals separated by  $d_{\min}$ . In order to evaluate the performance of the proposed simplified receiver, it is sufficient to calculate the probability of this same event when using the simplified receiver and deduce the degradation with respect to the optimum one.

Let us consider an error which lasts  $M$  branches in the trellis associated with the simplified receiver. Let  $s_1(t)$  be the transmitted signal during this event and  $s_2(t)$  the signal corresponding to the deviation. Both signals are defined for the same interval and their duration is  $MT$ . It is clear that

$$\|s_1\|^2 = \|s_2\|^2 = ME_b. \quad (27)$$

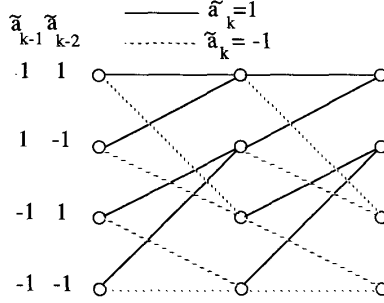


Fig. 4. Four-state trellis for a GMSK simplified Viterbi receiver.

Given the correct path due to  $s_1(t)$  and another path through the trellis corresponding to  $s_2(t)$ , with corresponding metrics  $\bar{\Lambda}_1$  and  $\bar{\Lambda}_2$ , respectively, the probability that the incorrect path causes an error is

$$P_{\text{simpl}} = \Pr \{ (Z, \bar{s}_1) < (Z, \bar{s}_2) / s_1 \}$$

where (18a) was used. Substituting  $Z(t)$  by  $s_1(t) + N(t)$ , we get

$$\begin{aligned} P_{\text{simpl}} &= \Pr \{ N, \bar{s}_2 - \bar{s}_1 > (s_1, \bar{s}_1 - \bar{s}_2) \} \\ &= \Pr \{ (N, s_2 - s_1 - \epsilon_2 + \epsilon_1) > (s_1, s_1 - s_2 + \epsilon_2 - \epsilon_1) \}. \end{aligned}$$

Using the identity resulting from (27),

$$\|s_1 - s_2\|^2 = 2(s_1, s_1 - s_2)$$

we get

$$P_{\text{simpl}} = Q \left( \frac{\|s_1 - s_2\|^2 + 2(s_1, \epsilon_2 - \epsilon_1)}{\sqrt{2N_0} \|s_1 - s_2 + \epsilon_2 - \epsilon_1\|} \right). \quad (28)$$

In order to obtain an upper bound of (28), we calculate as follows a lower bound on the argument of  $Q(\cdot)$ .

Let us introduce  $\beta \triangleq (1/\|s_1\|) \max \{ \|\epsilon_1\|, \|\epsilon_2\| \}$ . We then write

$$\begin{aligned} (s_1, \epsilon_2 - \epsilon_1) &\geq -(s_1, \epsilon_2) - (s_1, \epsilon_1) \\ &\geq -\|s_1\| \cdot \|\epsilon_2\| - \|s_1\| \cdot \|\epsilon_1\| \\ &\geq -2\beta \|s_1\|^2 \end{aligned}$$

and

$$\begin{aligned} \|s_1 - s_2 + \epsilon_1 - \epsilon_2\| &\leq \|s_1 - s_2\| + \|\epsilon_1\| + \|\epsilon_2\| \\ &\leq \|s_1 - s_2\| + 2\beta \|s_1\|. \end{aligned}$$

With the above bounds, (28) can be written as

$$P_{\text{simpl}} \leq Q \left( \frac{\|s_1 - s_2\|}{\sqrt{2N_0}} \cdot \gamma \right) \quad (29)$$

where

$$\gamma \triangleq \frac{1 - 4\beta \frac{\|s_1\|^2}{\|s_1 - s_2\|^2}}{1 + 2\beta \frac{\|s_1\|}{\|s_1 - s_2\|}}. \quad (30)$$

We can also see that for an optimum receiver the probability of the same event is obtained from (28) with  $\epsilon_1(t) = \epsilon_2(t) = 0$ , i.e.,

$$P_{\text{opt}} = Q \left( \frac{\|s_1 - s_2\|}{\sqrt{2N_0}} \right). \quad (31)$$

As shown by (26) we are interested particularly in  $\|s_1 - s_2\| = d_{\min}$ . Therefore, from (29) and (31) we deduce that when using the simplified receiver instead of the optimum one, the degradation is at most  $-20 \log_{10} \gamma$  dB. This analysis corroborates the intuitive result that the probability of error decreases when receiver complexity increases. In fact, we deduce from (15), (16) and the definition of  $\beta$  that  $\beta$  decreases with  $K$ , the number of PAM components considered by the receiver. Also, since  $\beta$  is positive and smaller than 1, (30) indicates that  $\gamma$  decreases with  $\beta$ . We, therefore, conclude that  $\gamma$  increases with  $K$ .

Let us apply this result to the case of GMSK with  $BT = 0.25$  received by the 4-state simplified VA receiver described in Section IV-B. The minimum distance according to reference [8, Fig. 5] is given by  $d_{\min}^2 \approx 3.4E_b$ . It corresponds to the distance between signals generated by  $\alpha_k$ -sequences  $(\dots, 1, -1, \dots)$  and  $(\dots, -1, 1, \dots)$ . The corresponding  $\bar{a}_k$ -sequences of the simplified trellis are  $(\dots, 1, \dots)$  and  $(\dots, -1, \dots)$  which represent a path deviation of length  $M = 3$  branches. Our calculation for the neglected fraction of energy contained in  $\epsilon_b(t)$  gives  $\beta^2 = 2.63 \times 10^{-5}$ . Using these values and (27) in (30), we get a performance degradation of less than 0.24 dB with respect to the optimum receiver.

## VI. LINEAR RECEIVER

We present now the MSK-type linear receiver having the structure depicted in Fig. 1. The complex envelope (4) of the signal can be written as

$$s_b(t) = \sqrt{2E_b} \sum_{n=0}^{N-1} a_{0,n} h_0(t - nT) + v(t) + \epsilon_b(t) \quad (32)$$

$$v(t) = \sqrt{2E_b} \sum_n a_{1,n} h_1(t - nT). \quad (33)$$

The term  $\epsilon_b(t)$  accounts for the remainder of the signal and represents inherent interference signals originating from symbols  $a_{k,n}$ ,  $k = 2, 3, \dots, 2^{L-1} - 1$ . The energy of  $\epsilon_b(t)$  is a very small fraction of the energy of  $s_b(t)$ ; as it was reported in Section V-B for the case of GMSK with  $BT = 0.25$  and  $L = 4$ . Only  $2.63 \times 10^{-5}$  of the signal energy ( $-46$  dB) is contained in  $\epsilon_b(t)$ . Usually the noise level is much higher than the energy of  $\epsilon_b(t)$ . Henceforth, in order to simplify calculations, we will drop the

term  $\epsilon_b(t)$ . The signal  $v(t)$  is also an interference term caused by symbols  $a_{1,n}$ . Its energy is determined by  $h_1(t)$ . Fig. 5 shows the pulses  $h_0(t)$  and  $h_1(t)$  for GMSK with  $BT = 0.25$ .

The output of the receiver filter is sampled at times  $kT$ . The samples are then used by a threshold detector to give decisions  $\hat{a}_{0,n}$  on symbols  $a_{0,n}$ . Finally, the differential decoder delivers the received data symbols  $\hat{\alpha}_n$  by using the decoding rule

$$\hat{\alpha}_n = \text{Im} \{ \hat{a}_{0,n} \hat{a}_{0,n-1}^* \}.$$

Notice that the model (32) is valid for any modulation index  $h$ . The limitation of the linear receiver to  $h = 0.5$  is explained by the fact that the performance of a symbol-by-symbol detector which gives a binary decision on the symbol  $a_{0,n}$  depends on the distance between the binary values. It is clear from expressions (5) and (6), that this distance is maximum when the modulation index is  $h = 0.5$ . In this case,  $a_{0,n}$  and  $a_{1,n}$  are given by (20a)–(20b). It is easy to show that the symbols  $a_{0,n}$  and  $a_{1,n}$  are mutually uncorrelated. In fact, it is sufficient to prove, by using (21), the following typical relation (with  $m > 0$ )

$$E \{ a_{0,k} a_{1,k-m}^* \} = E \{ j\alpha_k j\alpha_{k-1}^* \cdots j\alpha_{k-m+1}^* \cdot j\alpha_{k-m-1} j\alpha_{k-m-2}^* \} = 0$$

since symbols  $\alpha_k$  are zero-mean and independent.

#### A. Receiver Filter

Neglecting  $\epsilon_b(t)$ , the received signal has a complex envelope

$$z_b(t) = \sqrt{2E_b} \sum_{n=0}^{N-1} a_{0,n} h_0(t - nT) + v(t) + n_b(t) \quad (34)$$

where the noise  $n_b(t)$  is specified as in (17).

Although  $v(t)$  contains information on the message, we neglect the statistics it carries since our receiver is restricted to the class of linear MSK-type receivers of Fig. 1. Hence, we match the receiver filter to the PAM signal of highest energy to generate these statistics [2]

$$\begin{aligned} \text{Re} \int_{-\infty}^{\infty} z_b(t) \sum_k a_{0,k}^* h_0(t - kT) dt &= \text{Re} \sum_k a_{0,k}^* r_{0,k} \\ &= \sum_k a_{0,2k} \text{Re} \{ r_{0,2k} \} + \text{Im} \{ a_{0,2k+1} \} \text{Im} \{ r_{0,2k+1} \} \end{aligned} \quad (35)$$

where

$$r_{0,k} \triangleq \int_{-\infty}^{\infty} z_b(t) h_0(t - kT) dt. \quad (36)$$

Relation (35) indicates that for linear reception the needed statistics are  $\text{Re} \{ r_{0,2k} \}$  and  $\text{Im} \{ r_{0,2k+1} \}$ . Relation (36) shows that they can be obtained by means of a real matched filter  $h_0(-t)$  followed by a sampler acting at times  $kT$ . By using (33) and (34) in (36) we obtain the

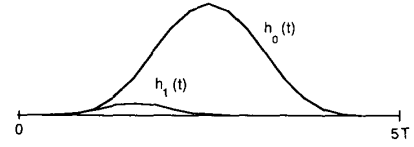


Fig. 5. Pulses  $h_0(t)$  and  $h_1(t)$ , case of GMSK,  $BT = 0.25$ .

statistics

$$\begin{aligned} \text{Re} \{ r_{0,2k} \} &= \sqrt{2E_b} \left[ \sum_m p_{00}(2mT) a_{0,2k-2m} \right. \\ &\quad \left. + \sum_m p_{10}(2mT - T) a_{1,2k-2m+1} \right] \\ &\quad + \text{Re} \{ w_{2k} \} \end{aligned} \quad (37)$$

$$\begin{aligned} \text{Im} \{ r_{0,2k+1} \} &= \sqrt{2E_b} \left[ \sum_m p_{00}(2mT) \right. \\ &\quad \left. + \sum_m p_{10}(2mT - T) \text{Im} \{ a_{1,2k-2m+2} \} \right] \\ &\quad + \text{Im} \{ w_{2k+1} \} \end{aligned} \quad (38)$$

where

$$p_{00}(t) \triangleq \int h_0(\tau) h_0(\tau - t) d\tau \quad (39)$$

$$p_{10}(t) \triangleq \int h_1(\tau) h_0(\tau - t) d\tau \quad (40)$$

$$w_k \triangleq \int n_b(t) h_0(t - kT) dt. \quad (41)$$

Expressions (37) and (38) show that the statistics are perturbed by noise and intersymbol interference (ISI) from some  $a_{0,k}$  and  $a_{1,k}$  symbols. Fig. 6 shows the eye diagram at the output of the matched filter for the signal GMSK with  $BT = 0.25$ .

Before delivering the statistics (37) and (38) to the threshold detector, the variance of perturbation can be reduced by inserting in the receiver filter a Wiener estimator which delivers to the threshold detector an optimum estimate of the symbols  $a_{2k}$  and  $\text{Im} \{ a_{2k+1} \}$  based on the minimum mean square error criterion (MMSE) [2, ch. 6]. It is easy to deduce that the filter is real and is the same for both in-phase samples (37) and quadrature samples (38).

Let  $\{c_k; -N \leq k \leq N\}$  be the filter coefficients. The filter output is

$$y_n = \sum_{k=-N}^N c_k r_{0,n-2k}. \quad (42)$$

Only  $\text{Re} \{ y_{2k} \}$  and  $\text{Im} \{ y_{2k+1} \}$  are delivered to the detector. The optimum coefficients are obtained from the

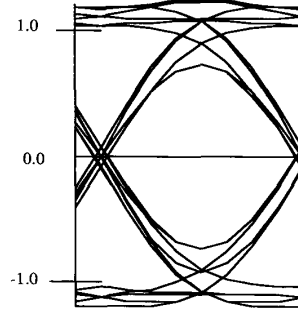


Fig. 6. The eye diagram at the output of the matched filter, case of GMSK,  $BT = 0.25$ .

orthogonality conditions

$$\text{Re} \{E[(a_{0,2n} - y_{2n}) r_{0,2n-2i}]\} = 0; -N \leq i \leq N \quad (43)$$

$$\text{Im} \{E[(a_{0,2n+1} - y_{2n+1}) r_{0,2n+1-2i}]\} = 0; -N \leq i \leq N. \quad (44)$$

The relations (43) or (44) lead separately to the following Wiener-Hopf equations:

$$\sum_{k=-N}^N \Psi_{ik} c_k = \frac{1}{\sqrt{2E_b}} p_{00}(-2i); -N \leq i \leq N \quad (45)$$

with

$$\begin{aligned} \Psi_{ik} = & \sum_m p_{00}(2mT) p_{00}[2(m+k-i)T] \\ & + \sum_m p_{10}(2mT - T) p_{10}[2(m+k-i)T - T] \\ & + \frac{N_0}{2E_b} p_{00}[2(k-i)T]. \end{aligned} \quad (46)$$

The solution of (45) gives the filter

$$C(e^{j2\pi f 2T}) = \sum_{k=-N}^N c_k e^{-j2\pi f k 2T}. \quad (47)$$

The resulting MMSE is

$$\text{MMSE} = 1 - \sqrt{2E_b} \sum_{k=-N}^N c_k p_{00}(-2kT). \quad (48)$$

When the filter length approaches infinity, the filter transfer function can be written as

$$C_\infty(e^{j2\pi f 2T}) = \frac{1}{\sqrt{2E_b}} \frac{P_{00}(e^{j2\pi f 2T})}{D(e^{j2\pi f 2T})} \quad (49)$$

and the resulting MMSE becomes

$$\begin{aligned} \text{MMSE}_\infty = & 2T \int_{-1/4T}^{1/4T} \frac{|P_{10}(e^{j2\pi f 2T})|^2 + \frac{N_0}{2E_b} P_{00}(e^{j2\pi f 2T})}{D(e^{j2\pi f 2T})} df \\ & \cdot \end{aligned} \quad (50)$$

where

$$\begin{aligned} D(e^{j2\pi f 2T}) \triangleq & |P_{00}(e^{j2\pi f 2T})|^2 + |P_{10}(e^{j2\pi f 2T})|^2 \\ & + \frac{N_0}{2E_b} P_{00}(e^{j2\pi f 2T}) \end{aligned}$$

$$P_{00}(e^{j2\pi f 2T}) = \frac{1}{2T} \sum_m \left| H_0\left(f - \frac{m}{2T}\right) \right|^2 \quad (51)$$

$$P_{10}(e^{j2\pi f 2T}) = \frac{1}{2T} \sum_m P_{10}\left(f - \frac{m}{2T}\right). \quad (52)$$

The optimum receiver filter is then formed by combining the matched and Wiener filters as seen in Fig. 7. Its transfer function is

$$H(f) = H_0^*(f) C_\infty(e^{j2\pi f 2T}) \quad (53)$$

and its impulse response

$$h(t) = \sum_{k=-\infty}^{\infty} c_k h_0(-t + 2kT). \quad (54)$$

This filter can be implemented either as a cascade composed of a matched filter, a sampler, and a digital FIR filter  $C(e^{j2\pi f 2T})$ , or as a FIR fractionally spaced transversal digital filter. In this case, the coefficients are optimized by deriving equations similar to (43) and (44).

#### B. Probability of Error

The error probability  $P[\epsilon]$  on the transmitted symbols  $\alpha_k$  is related to  $P_a$ , the probability of error when deciding on symbols  $a_{0,2k}$  or  $a_{0,2k+1}$ , by the relation

$$P[\epsilon] = 2(P_a - P_a^2). \quad (55)$$

To calculate  $P_a$ , it is sufficient to consider the real arm of the receiver. To get a convenient expression for the decision variable  $y_{2k}$ , we define the following two digital filters having coefficients  $\beta_k$  and  $\gamma_k$  given by

$$\beta_k \triangleq \sum_{i=-N}^N c_i p_{10}(2kT - 2iT - T) \quad (56)$$

$$\gamma_k \triangleq \sum_{i=-N}^N c_i p_{00}(2kT - 2iT). \quad (57)$$

Introducing (56), (57), and (37) in (42) gives the decision variable

$$y_{2k} = \sqrt{2E_b} (\gamma_0 a_{0,2k} + \beta_0 a_{1,2k+1} + \Delta) + U \quad (58)$$

where  $U$  is a real Gaussian noise of zero-mean and variance  $N_0 \|h\|^2$  with

$$\|h\|^2 \triangleq \int h^2(t) dt,$$

$\beta_0$  is the largest coefficient in the filter  $\{\beta_k\}$ . It represents the main contribution to the residual ISI.  $\Delta$  accounts for the rest of the residual ISI, i.e.,

$$\Delta \triangleq \sum_{i,i \neq 0} \gamma_i a_{2k-2i} + \sum_{i,i \neq 0} \beta_i b_{2k-2i+1}. \quad (59)$$

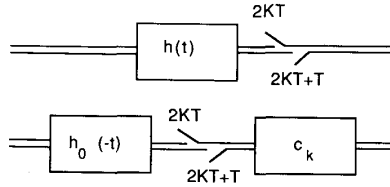


Fig. 7. Two equivalent systems.

Since the values of symbol  $a_{2k}$  have equal probabilities and are symmetric with respect to the decision threshold 0, we have

$$P_a = \Pr \{ y_{2k} < 0 / a_{0,2k} = 1 \} \\ = \Pr \{ \sqrt{2E_b} [\gamma_0 + \beta_0 a_{1,2k+1} + \Delta] + U < 0 \}.$$

Averaging over the binary random variable  $a_{1,2k+1}$  gives

$$P_a = \frac{1}{2} \Pr \{ \sqrt{2E_b} [\gamma_0 + \beta_0 + \Delta] + U < 0 \} \\ + \frac{1}{2} \Pr \{ \sqrt{2E_b} [\gamma_0 - \beta_0 + \Delta] + U < 0 \}. \quad (60)$$

In order to calculate  $P_a$ , we have to average (60) over the random variable  $\Delta$ . This can be easily accomplished by using (59). However, since the values taken by  $\Delta$  are much less than 1, we can simplify the calculation by bounding  $P_a$  by an upper bound and a lower bound.

We observe that the random variable  $\Delta$  takes discrete values in the interval  $[-\delta, \delta]$  where

$$\delta \triangleq \sum_{i,i \neq 0} |\gamma_i| + \sum_{i,i \neq 0} |\beta_i|.$$

The probabilities in (60) can now be bounded in the following way:

$$\Pr \{ \sqrt{2E_b} [\gamma_0 \pm \beta_0 + \Delta] + U < 0 \} \\ < \Pr \{ \sqrt{2E_b} [\gamma_0 \pm \beta_0 - \delta] + U < 0 \} \\ = Q \left( \sqrt{\frac{2E_b}{N_0}} \frac{\gamma_0 \pm \beta_0 - \delta}{\|h\|} \right)$$

and similarly

$$\Pr \{ \sqrt{2E_b} [\gamma_0 \pm \beta_0 + \Delta] + U < 0 \} \\ > \Pr \{ \sqrt{2E_b} [\gamma_0 \pm \beta_0 + \delta] + U < 0 \} \\ = Q \left( \sqrt{\frac{2E_b}{N_0}} \frac{\gamma_0 \pm \beta_0 + \delta}{\|h\|} \right).$$

Inserting these bounds in (60) and then in (55), and after neglecting squared terms, we get the lower bound

$$P[\epsilon] > Q \left( \sqrt{\frac{2E_b}{N_0}} \frac{\gamma_0 + \beta_0 + \delta}{\|h\|} \right) \\ + Q \left( \sqrt{\frac{2E_b}{N_0}} \frac{\gamma_0 - \beta_0 + \delta}{\|h\|} \right) \quad (61)$$

and the upper bound

$$P[\epsilon] < Q \left( \sqrt{\frac{2E_b}{N_0}} \frac{\gamma_0 + \beta_0 - \delta}{\|h\|} \right) \\ + Q \left( \sqrt{\frac{2E_b}{N_0}} \frac{\gamma_0 - \beta_0 - \delta}{\|h\|} \right). \quad (62)$$

## VII. NUMERICAL RESULTS

The receivers proposed in this paper have been simulated to receive a GMSK signal with  $BT = 0.25$ . The signal is simulated according to (1.b) and (1.d). Four samples per bit time interval are generated. Noisy samples are then processed by a simplified Viterbi receiver and also by a linear receiver.

A simplified Viterbi receiver is used as described in Section V-B. It is composed of two matched filters, a sampler and a four-state VA. The bit error rate obtained is reported in Fig. 10 together with the bit error probability of optimum minimum shift keying (MSK) system as a reference. In order to confirm the simulation results, we have also simulated MSK with the same symbols and noise samples. The results are also reported in Fig. 7. Recall that for MSK  $d_{\min} = 2\sqrt{E_b}$  which is 0.7 dB better than GMSK. As it is demonstrated by theoretical analysis, Figure 10 shows that the difference in performances between MSK and the simulated GMSK is less than  $0.7 + 0.24$  dB.

The simulated linear receiver uses a filter given by (54). The number of coefficients  $c_k$  is 11. The impulse response of the receiver filter is shown in Fig. 8. Fig. 9 shows the eye diagram at the output of the receiver filter  $h(t)$ . This eye should be compared to that of Fig. 6 obtained at the output of the matched filter. We observe that at the sampling time, the eye shows only four levels. Equation (58) can explain this observation; the term  $\Delta$  is small and therefore the filter output corresponds to the four possible values of the couple  $(a_{2k}, b_{2k+1})$ . For the particular case where  $N_0 = 0$  in (26),  $\delta = 0.0114$  and  $\beta = 0.0866$ . Noise enhancement by the filter is small; we get  $\|h\|^2 = 1.0078$ .

The bit error rate obtained with the linear receiver is given in Fig. 10 together with the lower and upper bounds (61) and (62). This shows that using the proposed receiver, GMSK performances are very close to those of MSK. The filter sensitivity to the choice of the noise spectral density has been tested by using a filter calculated with  $N_0 = 0$ . The difference in performance was negligible.

The four-state simplified VA receiver has better performance than the linear receiver because the second PAM component  $v(t)$  is considered as relevant by the former and as noise by the latter. However, for the simulated case the performance difference is small because the second PAM component has small energy. The utility of the simplified Viterbi receiver is more appreciated when the (rational) modulation index is different from 0.5.



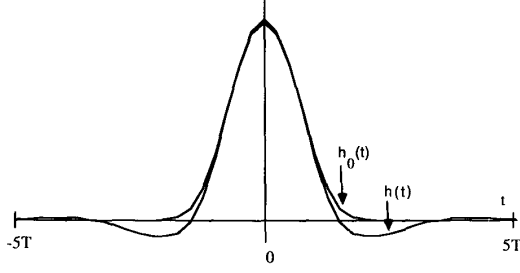


Fig. 8. Impulse response of the optimum receiver filter  $h(t)$  compared to the matched filter  $h_0(-t)$ .

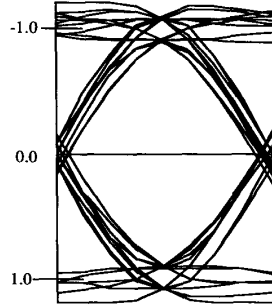


Fig. 9. The eye diagram at the output of the optimum receiver filter, case of GMSK,  $BT = 0.25$ .

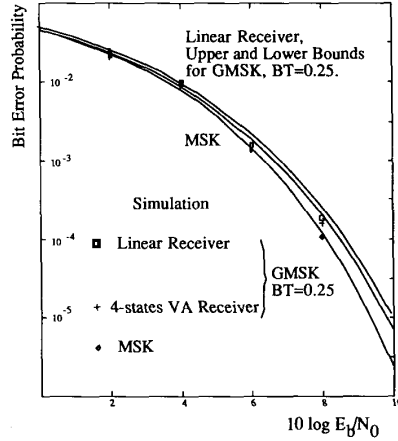


Fig. 10. Error Probabilities and simulation results.

### VIII. CONCLUSIONS

A new optimum Viterbi receiver for the binary partial response CPM with rational index is presented. A near-optimum Viterbi receiver with low complexity is then deduced. A bound for its degradation of performance with respect to the optimum receiver is calculated. Also, an MMSE optimum linear receiver is derived for the modulation index 0.5 and for all values of  $E_b/N_0$ . A design method for the receiver filter is given. The MMSE and upper and lower bounds for the bit error probability are calculated. Explicit expressions are given for all needed parameters. The signal GMSK is used as an illustration.

### APPENDIX

Following [3], we expand here the phase-modulated signal (1a)–(1d) into a sum of pulse amplitude modulation (PAM) signals given by (4).

Using (1d), we get

$$e^{j\phi(t)} = \exp \left( j\pi h \sum_{k=0}^{n-L} \alpha_k \right) \prod_{k=n-L+1}^n \exp [j\pi h \alpha_k g(t - kT)], \quad t \in [nT, nT + T]. \quad (\text{A.1})$$

Let us introduce

$$J \triangleq e^{j\pi h} \quad (\text{A.2})$$

and the even function

$$\begin{aligned} & \frac{\sin [\pi h - \pi h g(t)]}{\sin \pi h}, \quad t \in [0, LT) \\ c(t) & \triangleq c(-t), \quad t \in (-LT, 0]. \\ & 0, \quad |t| \geq LT \end{aligned} \quad (\text{A.3})$$

We first notice that

$$J^{\alpha_k} = j\alpha_k \sin(\pi h) + \cos(\pi h)$$

using this relation, the product terms in (A.1) can be expressed as

$$\begin{aligned} & \exp [j\pi h \alpha_k g(t - kT)] \\ & = j\alpha_k \sin [\pi h g(t - kT)] + \cos [\pi h g(t - kT)] \\ & = J^{\alpha_k} \frac{\sin [\pi h g(t - kT)]}{\sin(\pi h)} \\ & \quad + \frac{\sin [\pi h - \pi h g(t - kT)]}{\sin(\pi h)}. \end{aligned} \quad (\text{A.4})$$

We also notice that conditions (3a) and (3b) imply that

$$1 - g(t) = g(LT) - g(t) = g(LT - t). \quad (\text{A.5})$$

Relations (A.3) and (A.5) give

$$\frac{\sin [\pi h g(t)]}{\sin \pi h} = c(t - LT). \quad (\text{A.6})$$

Using (A.1) to (A.6), (1b) can be written as

$$\begin{aligned} s_b(t) & = \sqrt{\frac{2E_b}{T}} a_{0,n-L} \prod_{k=n-L+1}^n \\ & \quad \cdot [J^{\alpha_k} c(t - kT - LT) + c(t - kT)] \end{aligned} \quad (\text{A.7})$$

where the complex symbols  $a_{0,n}$  are related to the symbols  $\alpha_n$  by the following differential rule:

$$a_{0,n} = \exp \left( j\pi h \sum_{k=0}^n \alpha_k \right) = a_{0,n-1} J^{\alpha_n}. \quad (\text{A.8})$$

Expanding the right-hand side of (A.7) results in  $2^L$  terms. Each term is composed of a time pulse multiplied by a product of symbols  $a_{0,n-L}$  by some  $\alpha_k$ ;  $k = n, n-1, \dots, n-L+1$ . Time pulses are obtained by the product of  $L$  shifted versions of  $c(t)$ . Laurent [3] noticed that there are only  $2^{L-1}$  different pulses.

To clarify this expansion, let us examine  $L = 4$  as special case. Equation (A.7) can be written as

$$s_b(t) = \sqrt{\frac{2E_b}{T}} a_{0,n-4} \prod_{i=0}^3 [J^{\alpha_{n-i}} c(t - nT + iT - 4T) + c(t - nT + iT)] \quad (\text{A.9})$$

Effecting the multiplication results in the following 16 terms:

$$\begin{aligned} s_b(t) = \sqrt{2E_b} [ & a_{0,n} h_0(t - nT) + a_{0,n-1} h_0(t - nT + T) \\ & + a_{0,n-2} h_0(t - nT + 2T) \\ & + a_{0,n-3} h_0(t - nT + 3T) \\ & + a_{0,n-4} h_0(t - nT + 4T) + a_{1,n} h_1(t - nT) \\ & + a_{1,n-1} h_1(t - nT + T) \\ & + a_{1,n-2} h_1(t - nT + 2T) \\ & + a_{2,n} h_2(t - nT) + a_{2,n-1} h_2(t - nT + T) \\ & + a_{3,n} h_3(t - nT) + a_{3,n-1} h_3(t - nT + T) \\ & + a_{4,n} h_4(t - nT) + a_{5,n} h_5(t - nT) \\ & + a_{6,n} h_6(t - nT) + a_{7,n} h_7(t - nT)] \quad (\text{A.10}) \end{aligned}$$

where (A.8) and the following relations are used:

$$h_0(t) \triangleq \frac{1}{\sqrt{T}} c(t - 4T) c(t - 3T) c(t - 2T) c(t - T) \quad (\text{A.11})$$

$$h_1(t) \triangleq \frac{1}{\sqrt{T}} c(t - T) c(t - 2T) c(t - 4T) c(t + T) \quad (\text{A.12})$$

$$h_2(t) \triangleq \frac{1}{\sqrt{T}} c(t - T) c(t - 3T) c(t - 4T) c(t + 2T) \quad (\text{A.13})$$

$$h_3(t) \triangleq \frac{1}{\sqrt{T}} c(t - T) c(t - 4T) c(t + T) c(t + 2T) \quad (\text{A.14})$$

$$h_4(t) \triangleq \frac{1}{\sqrt{T}} c(t - 2T) c(t - 3T) c(t - 4T) c(t + 3T) \quad (\text{A.15})$$

$$h_5(t) \triangleq \frac{1}{\sqrt{T}} c(t - 4T) c(t - 2T) c(t + 3T) c(t + T) \quad (\text{A.16})$$

$$h_6(t) \triangleq \frac{1}{\sqrt{T}} c(t - 4T) c(t - 3T) c(t + 3T) c(t + 2T) \quad (\text{A.17})$$

$$h_7(t) \triangleq \frac{1}{\sqrt{T}} c(t - 4T) c(t + 3T) c(t + 2T) c(t + T) \quad (\text{A.18})$$

$$\begin{aligned} a_{1,n} &\triangleq a_{0,n-2} J^{\alpha_n} \\ a_{2,n} &\triangleq a_{0,n-3} J^{\alpha_{n-1}} J^{\alpha_n}, & a_{3,n} &\triangleq a_{0,n-3} J^{\alpha_n} \\ a_{4,n} &\triangleq a_{0,n-4} J^{\alpha_{n-2}} J^{\alpha_{n-1}} J^{\alpha_n}, & a_{5,n} &\triangleq a_{0,n-4} J^{\alpha_{n-2}} J^{\alpha_n} \\ a_{6,n} &\triangleq a_{0,n-4} J^{\alpha_{n-1}} J^{\alpha_n}, & a_{7,n} &\triangleq a_{0,n-4} J^{\alpha_n}. \end{aligned} \quad (\text{A.19})$$

For general values of  $L$ , (A.10) and (A.11) become (4) and (9), respectively.

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