Projects in Wireless Communication, Part 1
Lecture 1

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Lund University, Sweden

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Outline

- Introduction to the course
- Basics of digital communications
- Discrete-time implementations
- Carrier transmission

Introduction

Lecturer and course responsible: Fredrik Rusek, E:2377
4-5 scheduled lectures

Teaching assistant: Meifang Zhu,
4 scheduled labaratory lessons, time-slots to be decided today.

Email: {fredrik.rusek,meifang.zhu}@eit.lth.se

Introduction

Ultimate goal for PWC1 + PWC2:
Two computers should communicate via speaker/microphones

We aim at a file-transfer and/or a conversation via the keyboards
Some form of advanced system should be implemented, e.g. MIMO, OFDM, Turbo coding etc

The projects should be performed in groups of TWO students (HARD LIMIT)

PWC 1

In PWC 1 we only work in software. For a passing grade you must solve three tasks:

1. A digital baseband BPSK system should be implemented in C++ and its performance should be measured and verified against theoretical results

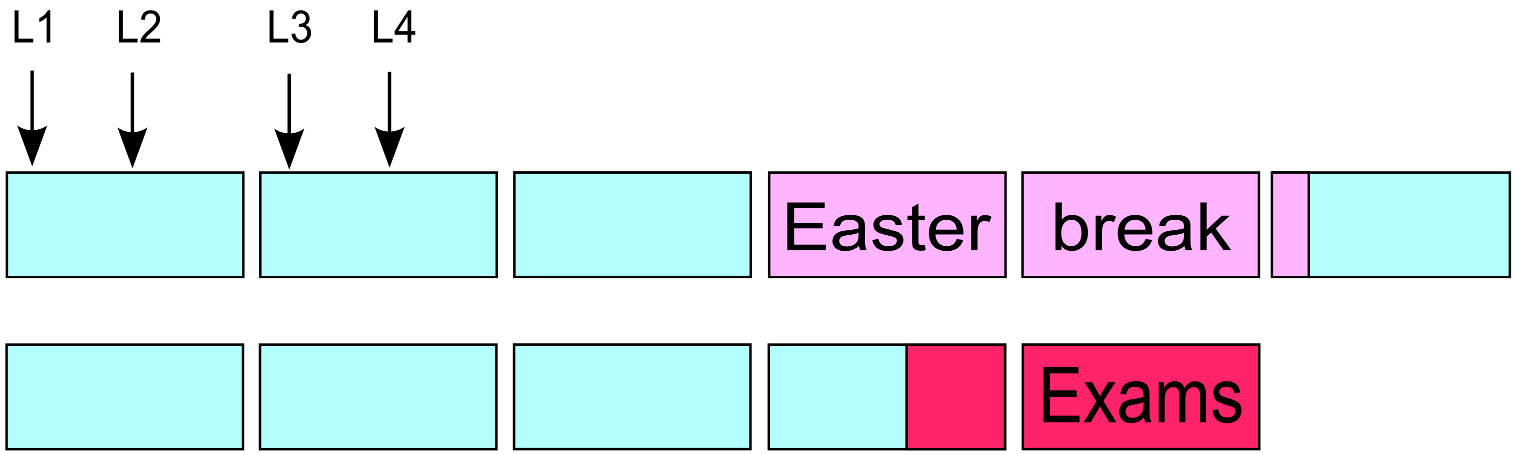
$$P_e = \mathcal{Q}\left(\sqrt{d_{\min}^2 \frac{E_b}{N_0}}\right)$$

2. In PWC2 you will encounter physical passband signals at the input of the microphone. In PWC1, we will provide each group with one such signal; the bits carried by the signals correspond to the ASCII code of a secret password. If you can decode the signals and provide me with the password, you have passed task 2.

3. Same as 2 but with OFDM transmission and convolutional code.

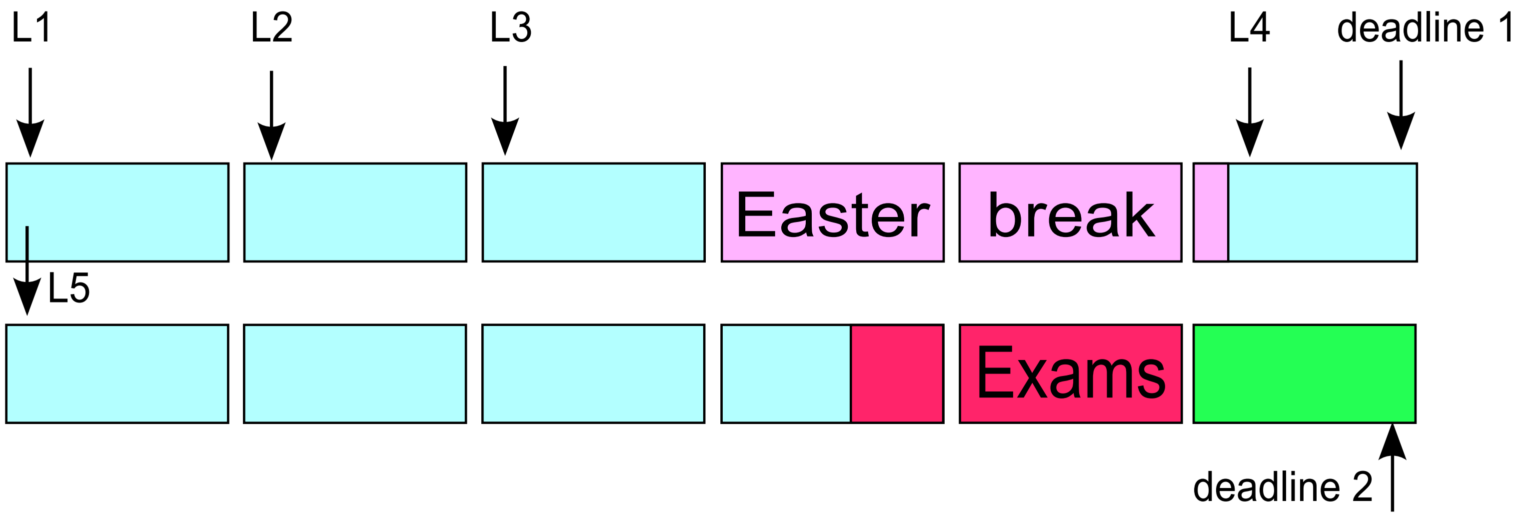
Schedule

Schedule according to LTH.



Schedule

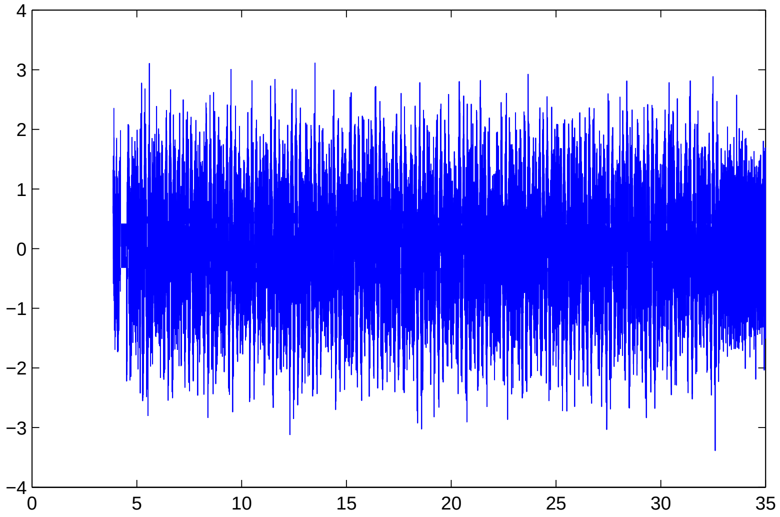
Proposed schedule
Lecture 1-3 deals with Tasks 1 and 2 (deadline 1), while lecture 4-5 deals with task 3



(OFDM, deadline 3).
In addition, Meifang will have help sessions in the lab.

Example

Assume that you receive the following noisy signal



You must remove the noise...

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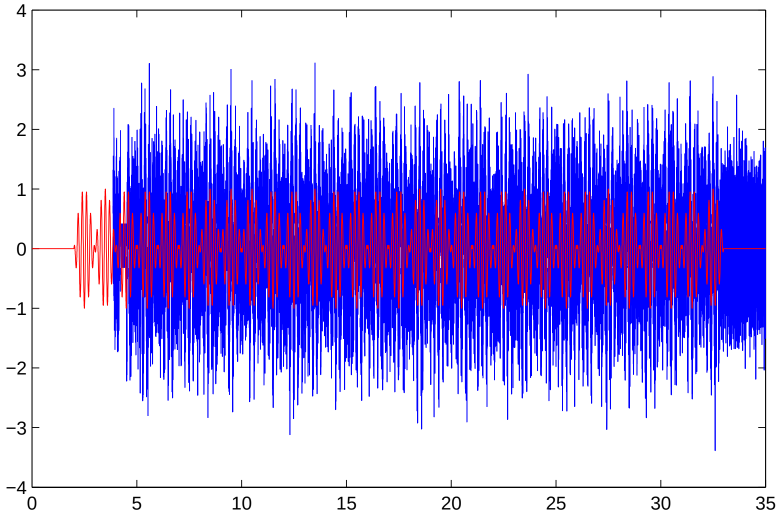

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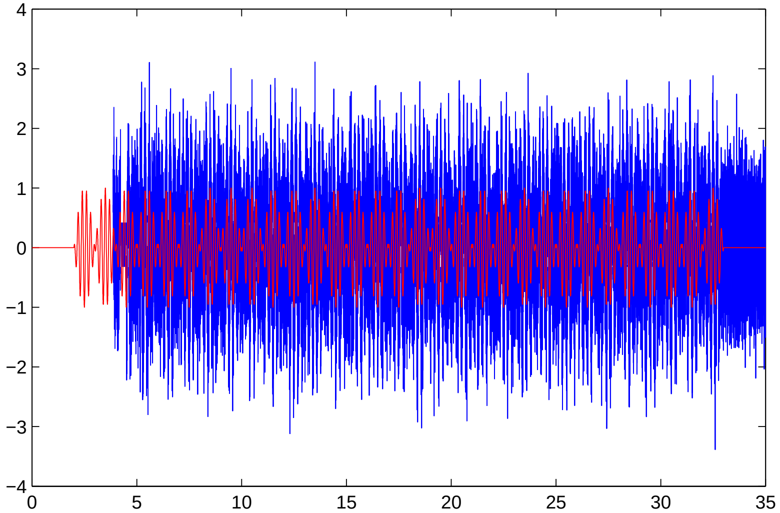

You must remove the noise...Done!
Decode the bits:

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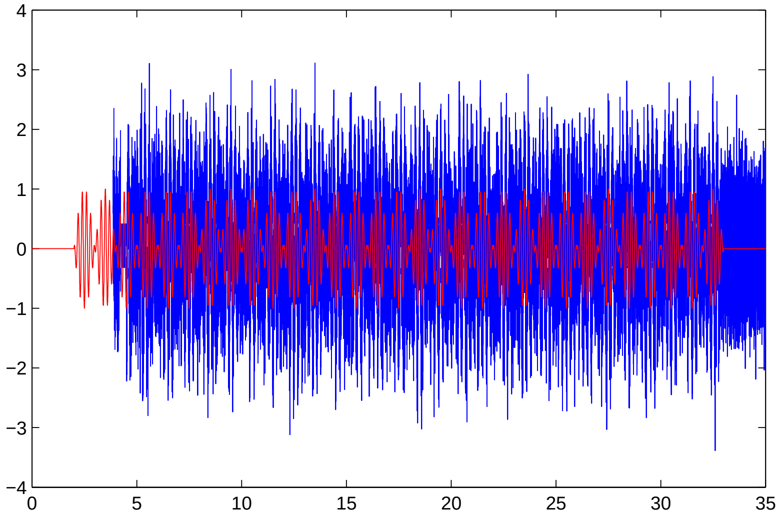

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Decode the bits: 1 1 1 0 0 0 0 0 1 1 0 1 0 0 1 1 1 0 1.....

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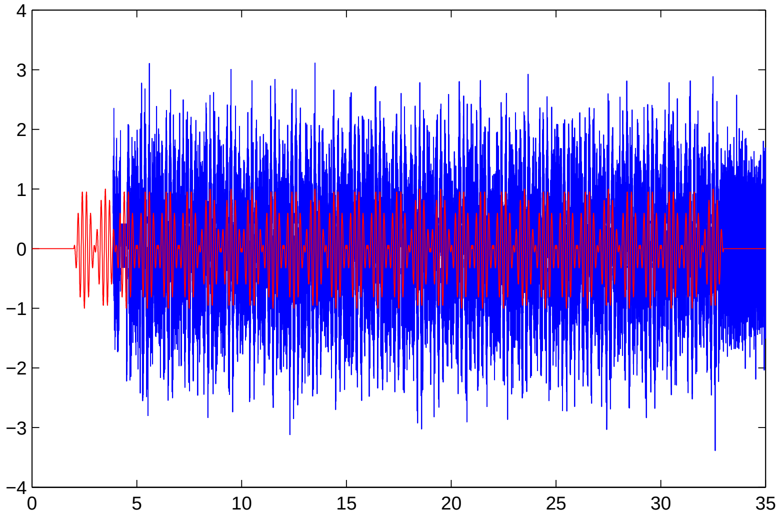

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Convert to ASCII:

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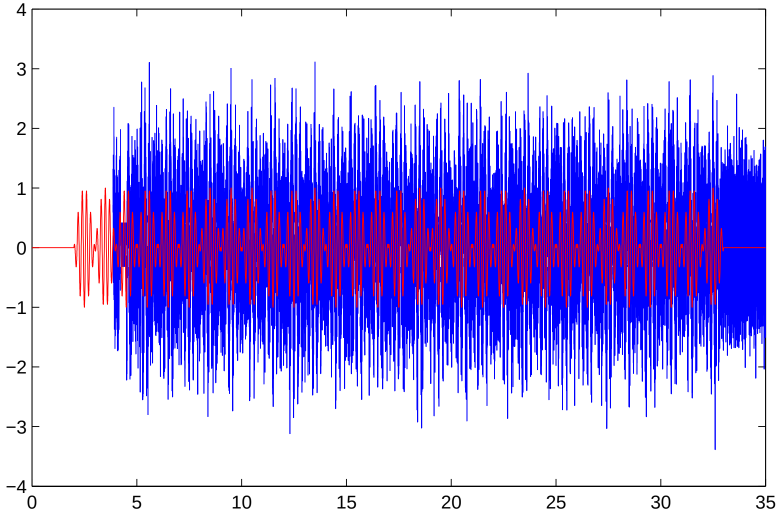


You must remove the noise...Done!

Decode the bits: 1 1 1 0 0 0 0 0 1 1 0 1 0 0 1 1 1 0 1.....

Convert to ASCII: You have passed PWC1, congratulations......

Introduction

Formal descriptions of the tasks can (soon) be found online.

Basics of Digital Communications

This is a recall of baseband digital communications....

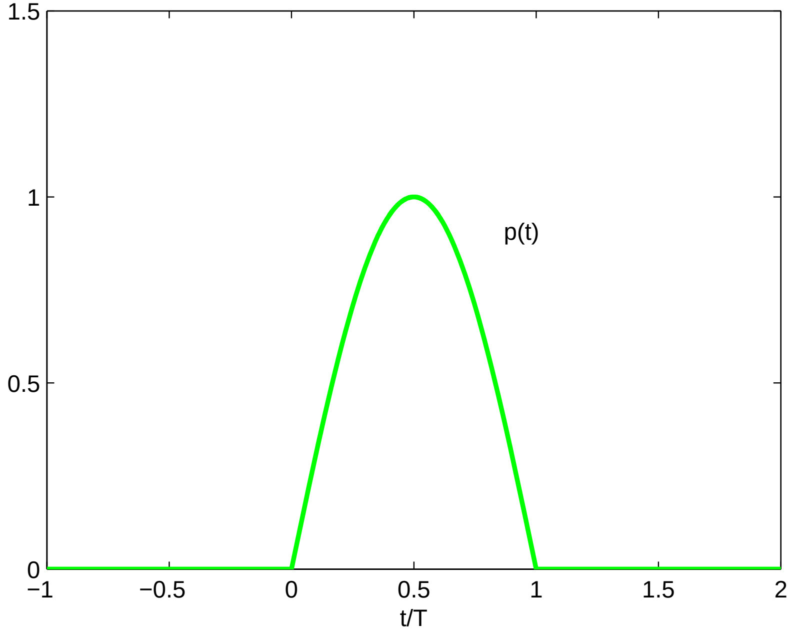
We need to transmit a bit sequence $\{u_k\} = 0111010.....$
Map to symbols $\{a_k\}$

$$\text{BPSK}: \quad a_k = \begin{cases} 1, & u_k = 0 \\ -1, & u_k = 1 \end{cases}$$

$$\text{QPSK}: \quad a_k = \begin{cases} 1, & u_{2k}u_{2k+1} = 00 \\ i, & u_{2k}u_{2k+1} = 01 \\ -1, & u_{2k}u_{2k+1} = 10 \\ -i, & u_{2k}u_{2k+1} = 11 \end{cases}$$

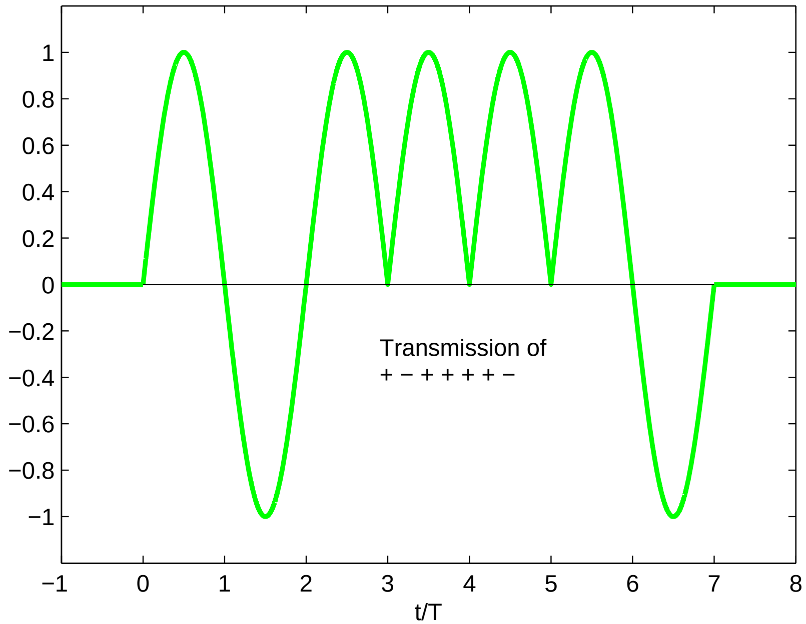
Basics of Digital Communications

Each symbol is carried by a base pulse $p(t)$ of length T, e.g. the half-cycle sinus



Basics of Digital Communications

So the transmission of bits 0 1 0 0 0 0 1 generates the pulse train $y(t)$



Mathematically we have

$$y(t) = \sum_k a_k p(t - kT_s)$$

Note that T_s is the symbol time while T is the duration of the base pulse $p(t)$.

Basics of Digital Communications

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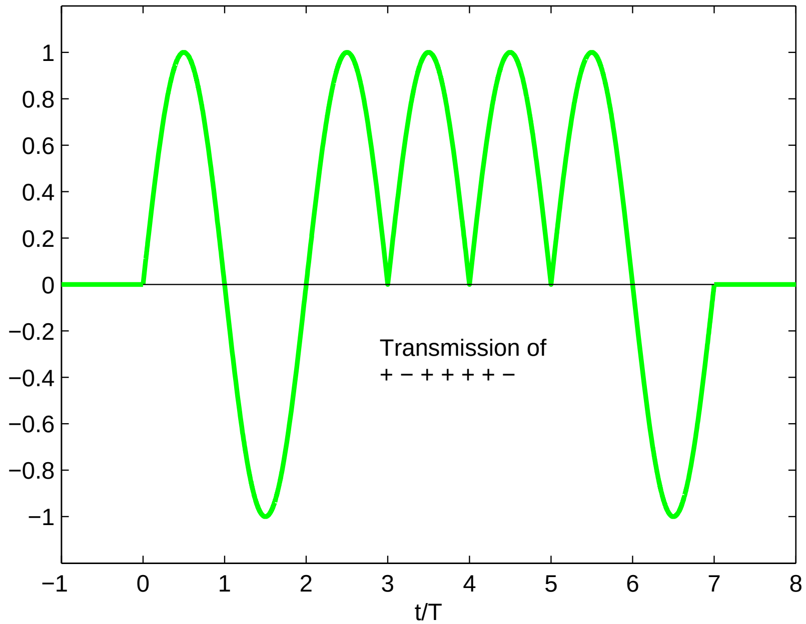

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How does T and T_s relate in this example?

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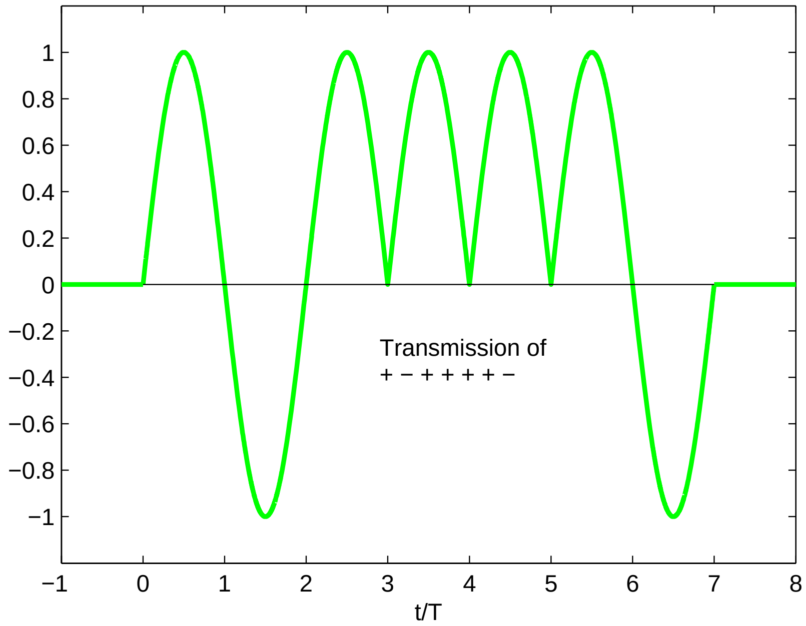


Mathematically we have

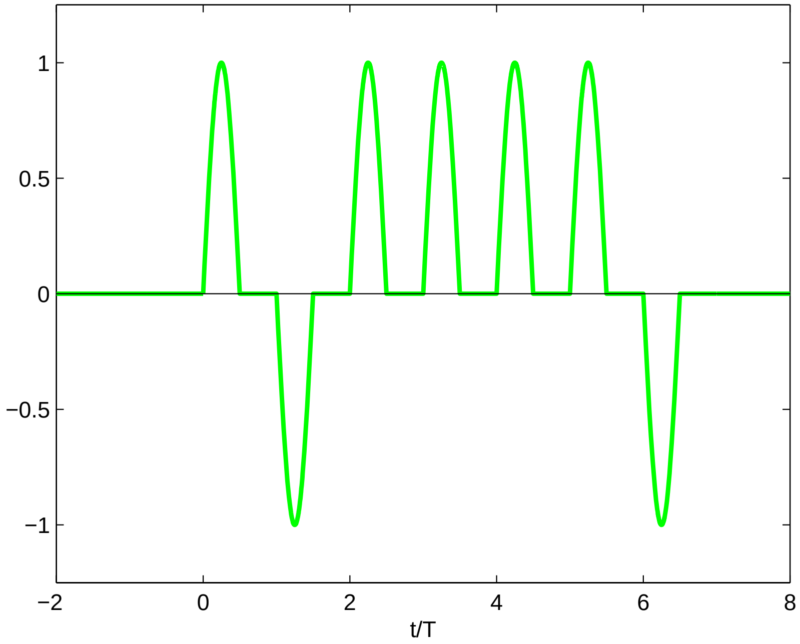
$$y(t) = \sum_k a_k p(t - kT_s)$$

Note that T_s is the symbol time while T is the duration of the base pulse $p(t)$.

How does T and T_s relate in this example? $T = T_s$

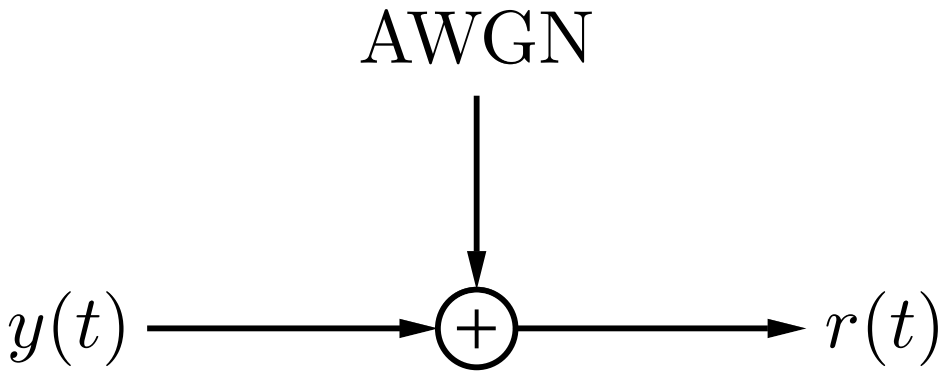
Basics of Digital Communications

To avoid intersymbol interference one can use $T < T_s$

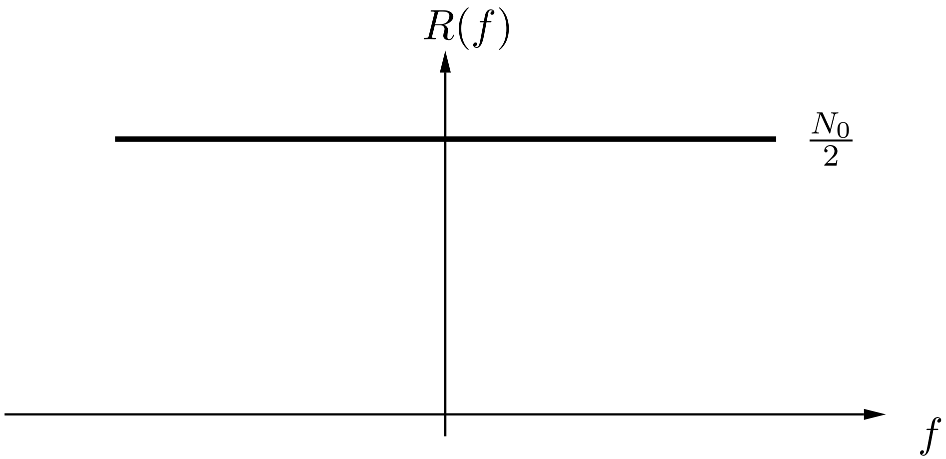


In this example we have $T = T_s/2$

The channel model assumed in this review is a pure AWGN channel



Where the noise $n(t)$ satisfies $\mathcal{E}\{n^*(t)n(t+\tau)\} = \delta(\tau)N_0/2$; such a noise process must have power spectral density



Basics of Digital Communications

What does WGN look like?
Can we show an example?

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Consider the power of the process

$$P = \int R(f)\mathrm{d}f$$

What does WGN look like?
Can we show an example?

Consider the power of the process

$$P = \int R(f)\mathrm{d}f$$

$n(t)$ has infinite power!

Thus, not possible to show an example of WGN

Basics of Digital Communications

Explanation: Every signal we ever see in reality has been filtered by some low-pass filter.

Basics of Digital Communications

Mathematically, in what way should the receiver process the received signal $r(t)$.

In other words

$$\hat{\mathbf{a}} = ...?$$

Basics of Digital Communications

Mathematically, in what way should the receiver process the received signal $r(t)$.

Maximum-likelihood detection is the answer!

$$\hat{\mathbf{a}} = \arg\max_{\mathbf{a}} Prob\{r(t)|\mathbf{a}\}$$

Basics of Digital Communications

Mathematically, in what way should the receiver process the received signal $r(t)$.

Maximum-likelihood is equivalent to minimum Euclidean distance detection

$$\hat{\mathbf{a}} = \arg\min_{\mathbf{a}} \int_{-\infty}^{\infty} \left| r(t) - \sum_k a_k p(t - kT_s) \right|^2 dt$$

Basics of Digital Communications

To decode the (complex valued) signal $r(t)$, we pass $r(t)$ through a matched filter $z(t)$

$$z(t) = p(-t)$$

For symmetric pulses $p(t)$, we get

$$z(t) = p(t)$$

Let

$$\begin{aligned} x(t) &= r(t) \star p(t) \\ &= \sum_k a_k g(t - kT_s) + \eta(t) \end{aligned}$$

where $\eta(t)$ is $n(t) \star p(t)$ and $g(t) = p(t) \star z(t)$. Take samples every T_s seconds: $x_k = x(kT_s)$. Then

$$x_k = E_p a_k + \eta_k$$

where η_k is a complex Gaussian random variable with variance $E_p N_0$, that is $E_p N_0/2$ per dimension!

Basics of Digital Communications

Energy computations and error probability:

The energy per transmitted symbol E_s is given by: $E_s = \underbrace{\int p^2(t)\mathrm{d}t}_{E_p}$ while the energy per transmitted bit is

$$E_b = \begin{cases} E_s, & \text{BPSK} \\ E_s/2, & \text{QPSK} \end{cases}$$

The physical minimum Euclidean distance is

$$D_{\min}^2 = \begin{cases} 4E_p, & \text{BPSK} \\ 2E_p, & \text{QPSK} \end{cases}$$

In both cases we end up with a normalized distance $d_{\min}^2 = 2$. The error probability is given by

$$P_e \approx \mathcal{Q}\left(\sqrt{2\frac{E_b}{N_0}}\right)$$

Discrete-Time Implementations

In a computer-based package such as Matlab or C/C++, we cannot represent the signals $y(t)$ as continuous time signals. Hence we must work with sampled versions.

Let f_s be the sample rate in samples/second and N be the number of samples per symbol.

In PWC2, $f_s = 44100$ samples/second

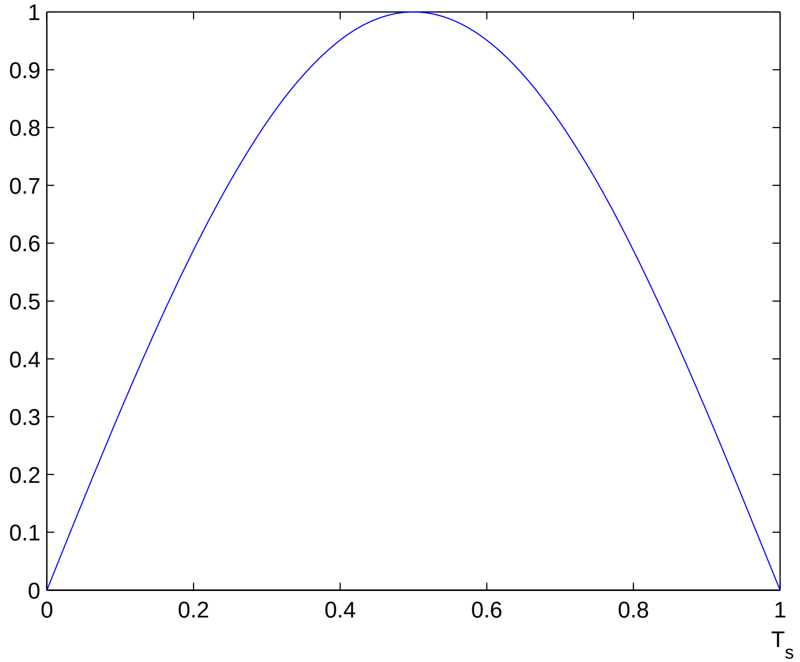
We get that $T_s = \frac{N}{f_s}$

The symbol rate becomes

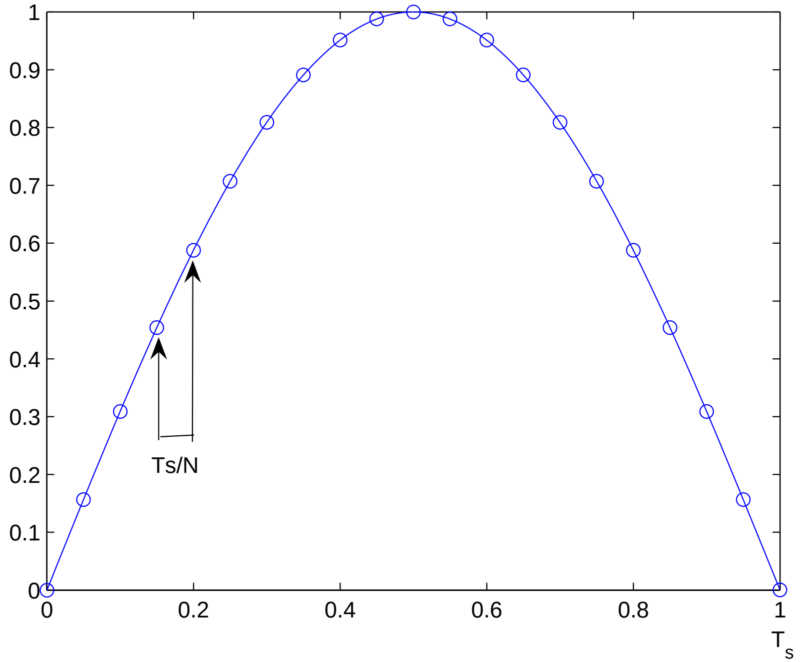
$$R_s = \frac{f_s}{N}$$

Discrete-Time Implementations

We must sample the base pulse $p(t)$.

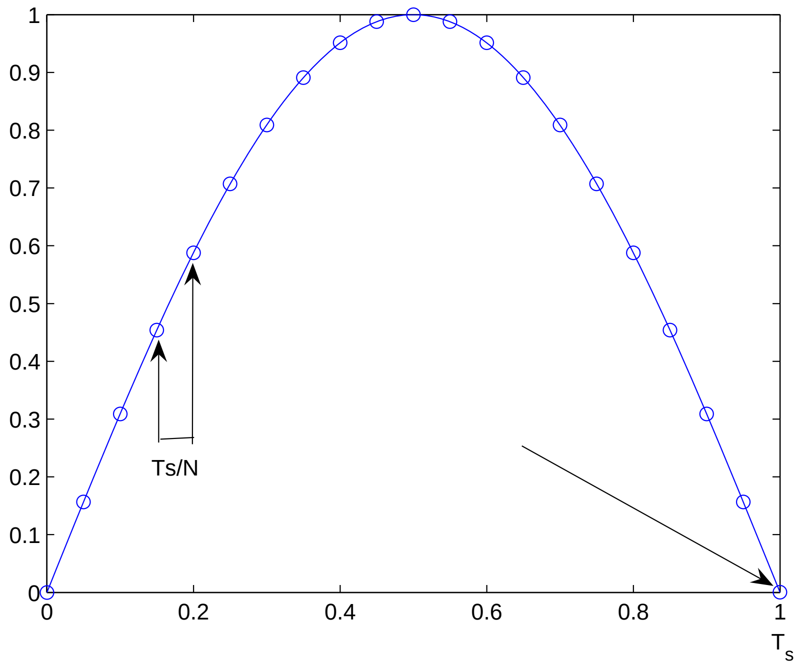


We must sample the base pulse $p(t)$. Assume a sample interval of T_s/N seconds



Discrete-Time Implementations

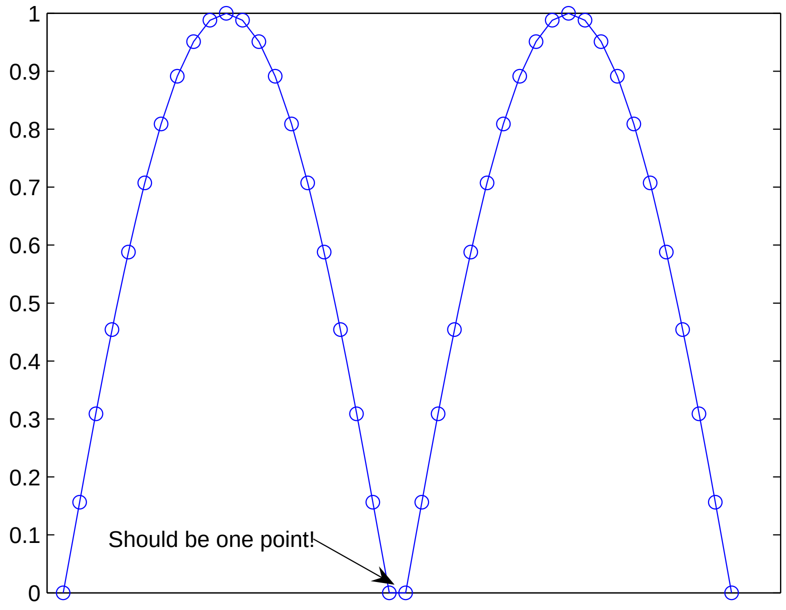
We must sample the base pulse $p(t)$. N+1 samples per symbol implies sample interval of T_s/N seconds



This is wrong!

Discrete-Time Implementations

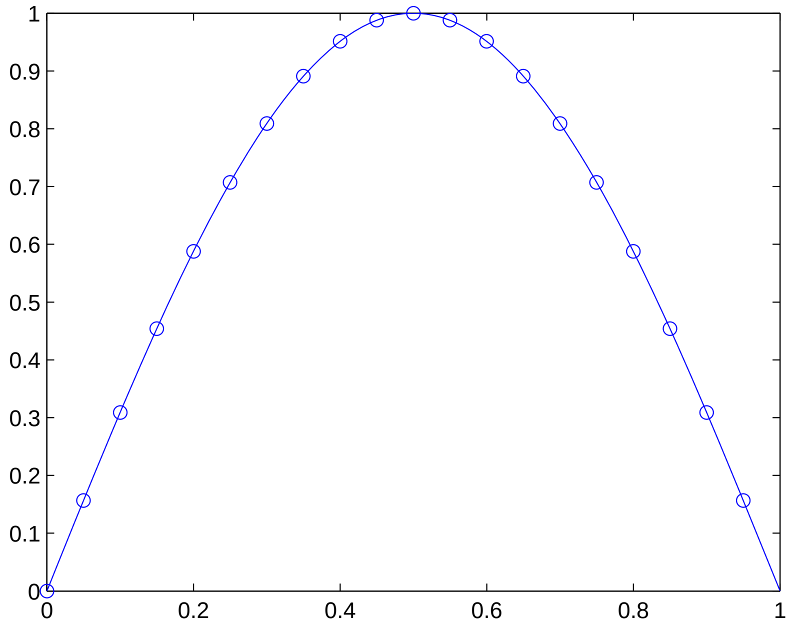
Explanation: Plot two consecutive pulses.



There should only be one point.

Discrete-Time Implementations

Correct sampling!

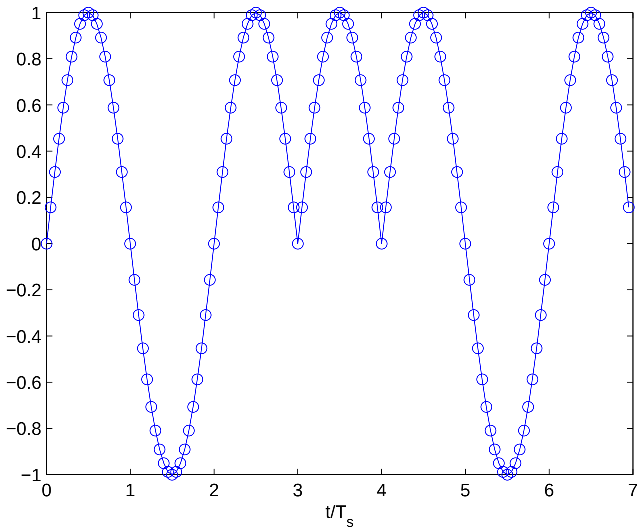


Represent the samples in a vector

$$\boldsymbol{p} = [0\ 0.159\ .309\ \ldots]$$

Discrete-Time Implementations

A sampled transmission signal of $+ - + + + - +$



Slightly harder mathematical representation. Let $\{b_k\}$ be a zero-padded version of $\{a_k\}$

$$\boldsymbol{b} = [a_1 \ \underbrace{00\ldots0}_{N-1} \ a_2 \ \underbrace{00\ldots0}_{N-1} \ a_3 \ \underbrace{00\ldots0}_{N-1} \ a_4 \ldots]$$

Then,

$$y_k = \sum_{\ell} b_\ell p_{k-\ell} \qquad \text{or simply } \boldsymbol{y} = \boldsymbol{b} \star \boldsymbol{p}$$

Discrete-Time Implementations

Convolutions in discrete-time:

A convolution of $x(t)$ and $y(t)$ in continuous time is carried out as

$$\int x(\tau)y(t-\tau)\mathrm{d}\tau \tag{1}$$

Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be sampled version of $x(t)$ and $y(t)$; the sampling rate is f_s. The discrete time version of (??) is

$$\frac{1}{f_s}\sum_{\ell} x_\ell y_{k-\ell}$$

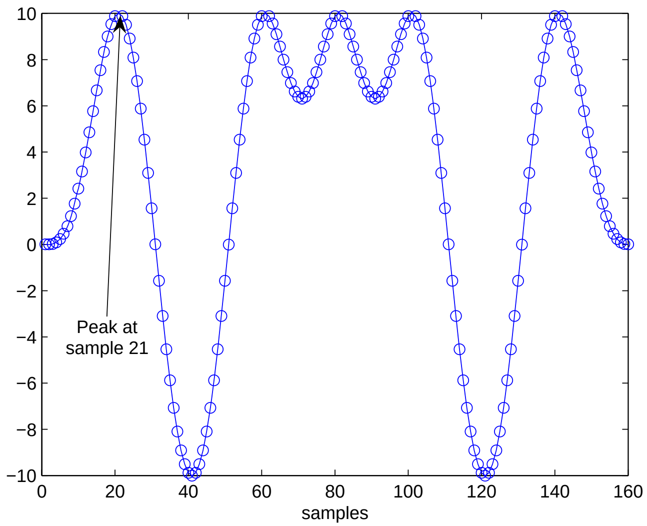
The discrete time convolution must be scaled by the sampling rate!. $1/f_s$ works as $\mathrm{d}\tau$ in (??).

The energy of the pulse $p(t)$ must be approximated as

$$E_p = \frac{1}{f_s}\sum_k p_k^2$$

Matched filters in discrete-time:

The pulse train $\boldsymbol{p}$ should be filtered by a discrete-time matched filter. For symmetric pulses, we can take this mathched filter as $\boldsymbol{z} = \boldsymbol{p}$ where $\boldsymbol{p}$ includes the last sample!, i.e. the length of $\boldsymbol{p}$ is $N+1$. (This is however not crucial.) Then the output of the matched filter is ($N = 20$)



The number of samples in $\boldsymbol{y}$ is $N \times \text{number of symbols}$ and the length of the filter output is $N + N \times \text{number of symbols}$. The peak occurs at samples $1 + kN$, $k = 1, 2, 3 \ldots$.

Discrete-Time Implementations

If there is a guard band ($T < T_s$), then the pulse is not symmetric and we can not take $\boldsymbol{z} = \boldsymbol{p}$. We must then use

$$z_k = p_{N+2-k}, \quad k = 1...N+1$$

It is still true that the peaks occur at samples $1 + kN$, $k = 1, 2, 3....$

Implementation of discrete-time AWGN:

Until now we have constructed a modulation signal $\boldsymbol{y}$ in discrete time. We now seek a noise vector $\boldsymbol{n}$ to be added to $\boldsymbol{y}$ that represents continuous time AWGN (that has $\inf$ power).

We have that samples of

$$\eta(t) = n(t) \star z(t)$$

are zero-mean and have variance $E_p N_0/2$.

In discrete-time, a sample of the filtered noise process is given by

$$\eta_k = \frac{1}{f_s} \sum_{\ell} n_\ell z_{k-\ell}$$

Assume that the variance of each n_k is σ_n^2. From probability theory it follows that η_k has variance $\sigma_n^2 \sum z_k^2 / f_s^2$.

Since

$$\sigma_n^2 \sum_k z_k^2 / f_s^2 = E_p \frac{N_0}{2}$$

Discrete-Time Implementations

we get that

$$\sigma^2 = E_p \frac{N_0}{2} \frac{f_s^2}{\sum_k z_k^2} = \frac{N_0}{2} f_s$$

Thus, The sampling rate affects the variance of the discrete time representation of continuous AWGN

Carrier Transmission

The transmitted signal is $y(t) = \sum_k a_k h(t - kT)$. What is the bandwidth? More generally, what is its Fourier transform?

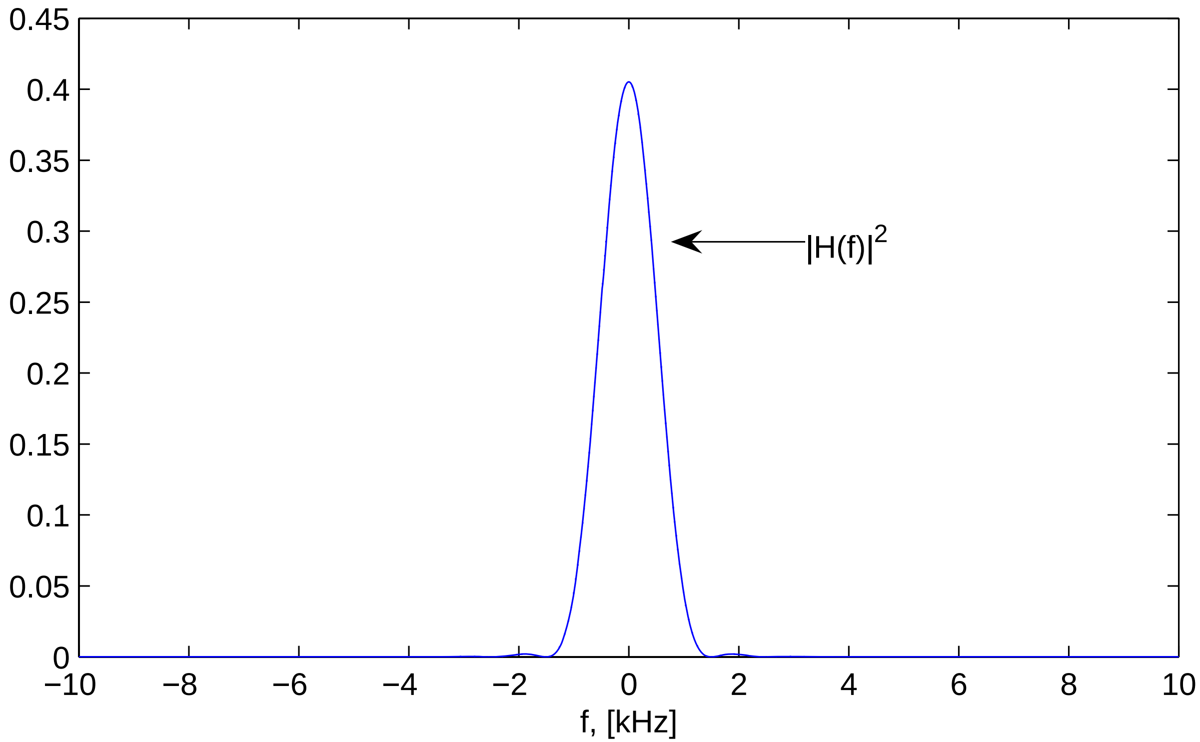
Carrier Transmission

Table 2.3 Properties of the Fourier transform

1.	Linearity	$ax_1(t) + bx_2(t) \leftrightarrow aX_1(f) + bX_2(f)$
2.	Inverse	$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j\omega t}\, df$
3.	Translation (time shift)	$x(t - t_0) \leftrightarrow X(f)\, e^{-j\omega t_0}$
4.	Modulation (frequency shift)	$x(t)\, e^{j\omega_0 t} \leftrightarrow X(f - f_0)$ $x(t) \cos \omega_0 t \leftrightarrow \frac{1}{2} X(f + f_0) + \frac{1}{2} X(f - f_0)$
5.	Time scaling	$x(at) \leftrightarrow \frac{1}{\lvert a \rvert} X(f/a)$
6.	Differentiation in time	$\frac{d}{dt} x(t) \leftrightarrow j\omega X(f)$
7.	Differentiation in frequency	$tx(t) \leftrightarrow -\frac{1}{j2\pi} \frac{d}{df} X(f)$
8.	Integration in time	$\int_{-\infty}^{t} x(\tau)\, d\tau \leftrightarrow \frac{1}{j\omega} X(f)$
9.	Duality	$X(t) \leftrightarrow x(-f)$
10.	Conjugate functions	$x^*(t) \leftrightarrow X^*(-f)$
11.	Convolution in time	$x_1(t) * x_2(t) \leftrightarrow X_1(f) X_2(f)$
12.	Multiplication in time	$x_1(t) x_2(t) \leftrightarrow X_1(f) * X_2(f)$
13.	Parseval's formulas	$\int_{-\infty}^{\infty} x_1(t) x_2^*(t)\, dt = \int_{-\infty}^{\infty} X_1(f) X_2^*(f)\, df$ or, when $x_1(t) = x_2(t)$, $\int_{-\infty}^{\infty} \lvert x(t) \rvert^2\, dt = \int_{-\infty}^{\infty} \lvert X(f) \rvert^2\, df$

Carrier Transmission

The baseband signal is $y(t) = \sum_k a_k h(t - kT)$. The power spectral density of the transmission is $\propto |H(f)|^2$



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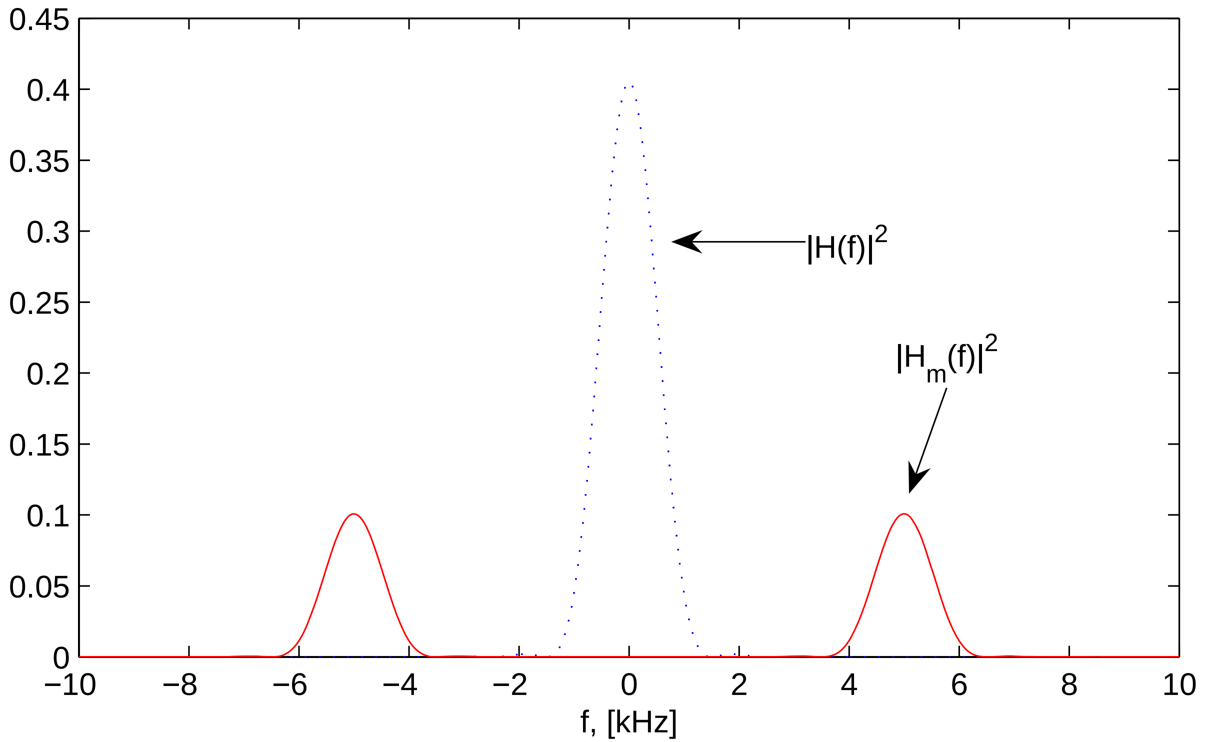

The carrier modulated signal is $y_m(t) = y(t)\cos(2\pi t f_c)$

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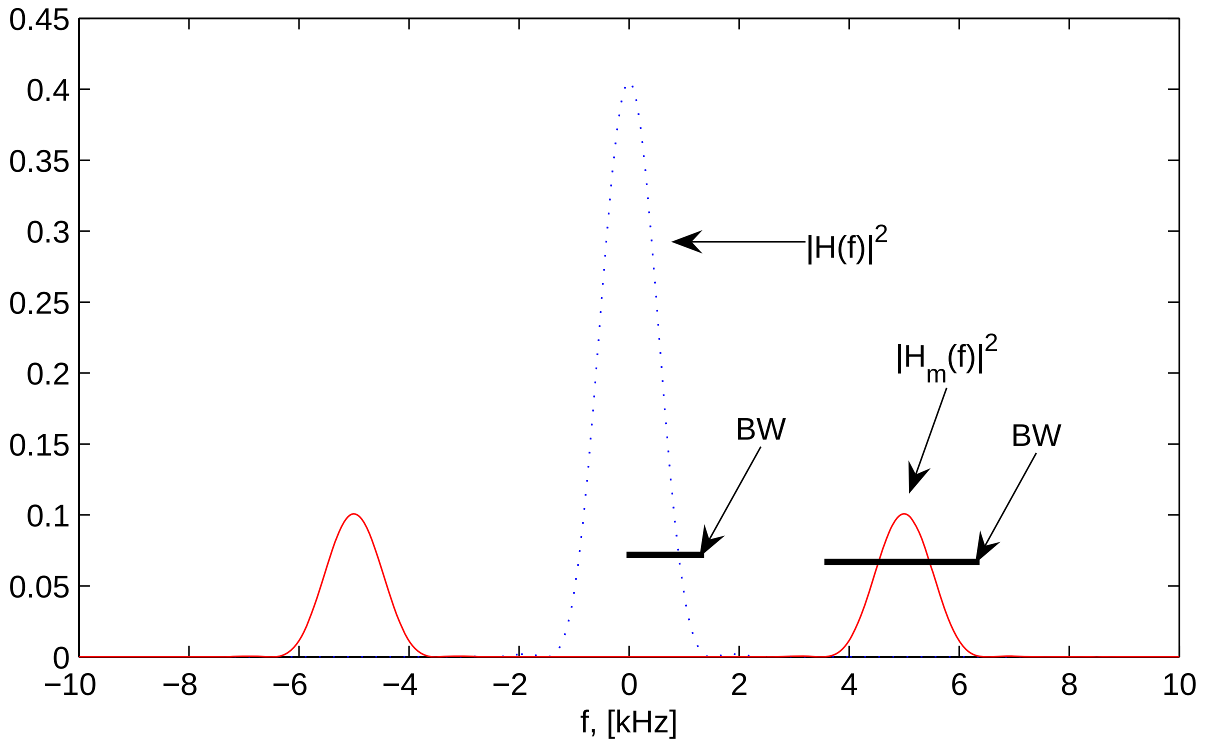

The carrier modulated signal is $y_m(t) = y(t)\cos(2\pi t f_c)$

But bandwidth gets twice as large!

Carrier Transmission

Where did the energy go?

Basic Fourier relations:

$$\cos(2\pi f_c t)h(t) \longleftrightarrow \frac{1}{2}H(f-f_c)+\frac{1}{2}H(f+f_c)$$

$$\sin(2\pi f_c t)h(t) \longleftrightarrow \frac{1}{2}H(f-f_c)-\frac{1}{2}H(f+f_c)$$

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The $1/2$ factor corresponds to a $1/4$ of the energy. Since there are two terms, $1/2$ of the energy is preserved.

Carrier Transmission

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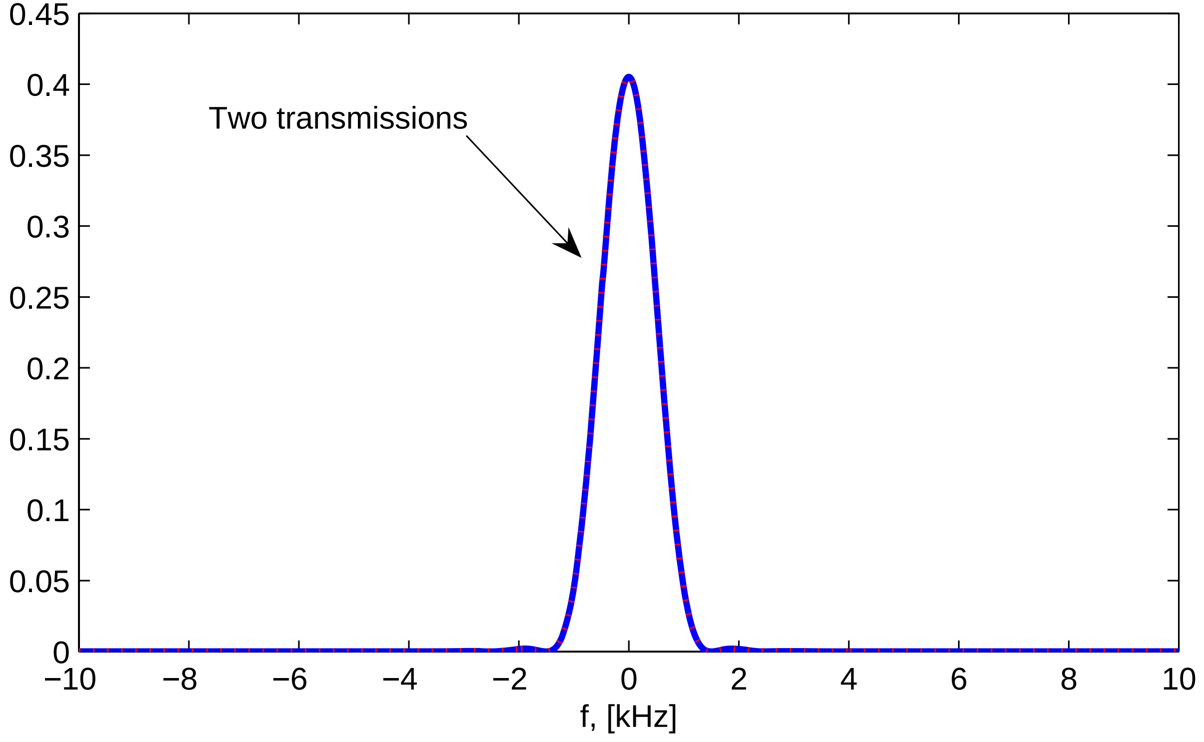
What about the increased bandwidth?

Important

Carrier Transmission

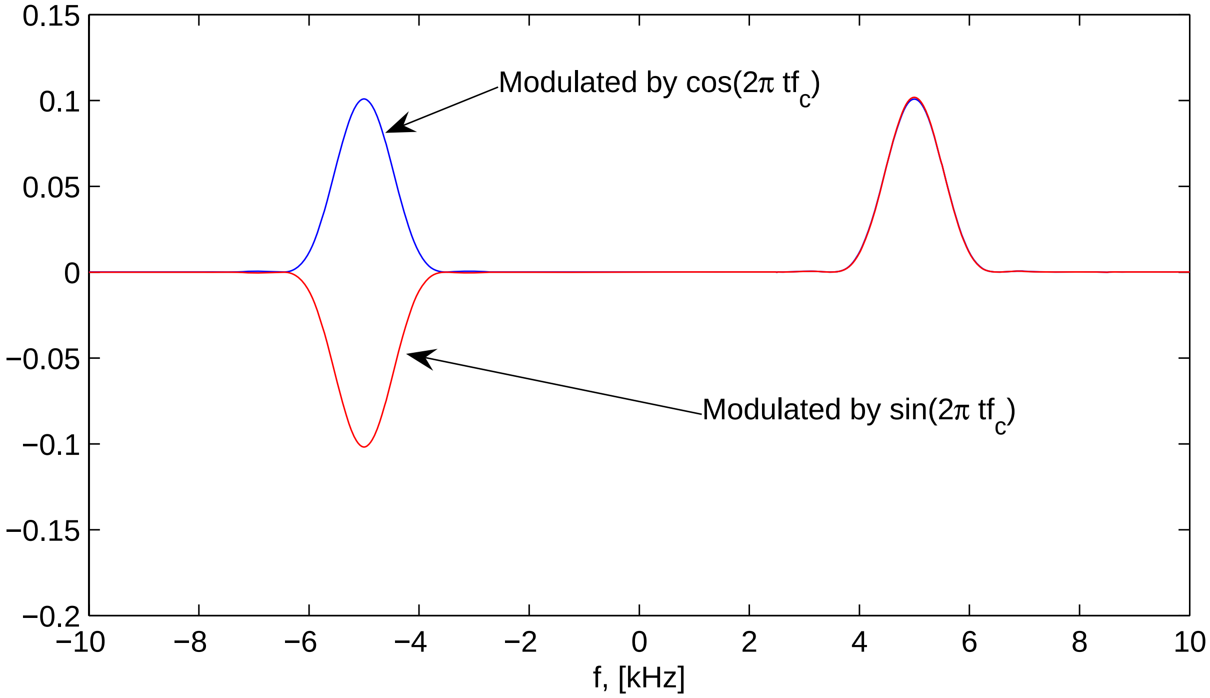
Assume two independent baseband transmissions



Carrier Transmission

Assume two independent baseband transmissions

After modulation with $\cos(2\pi t f_c)$ and $\sin(2\pi t f_c)$ we get



Carrier Transmission

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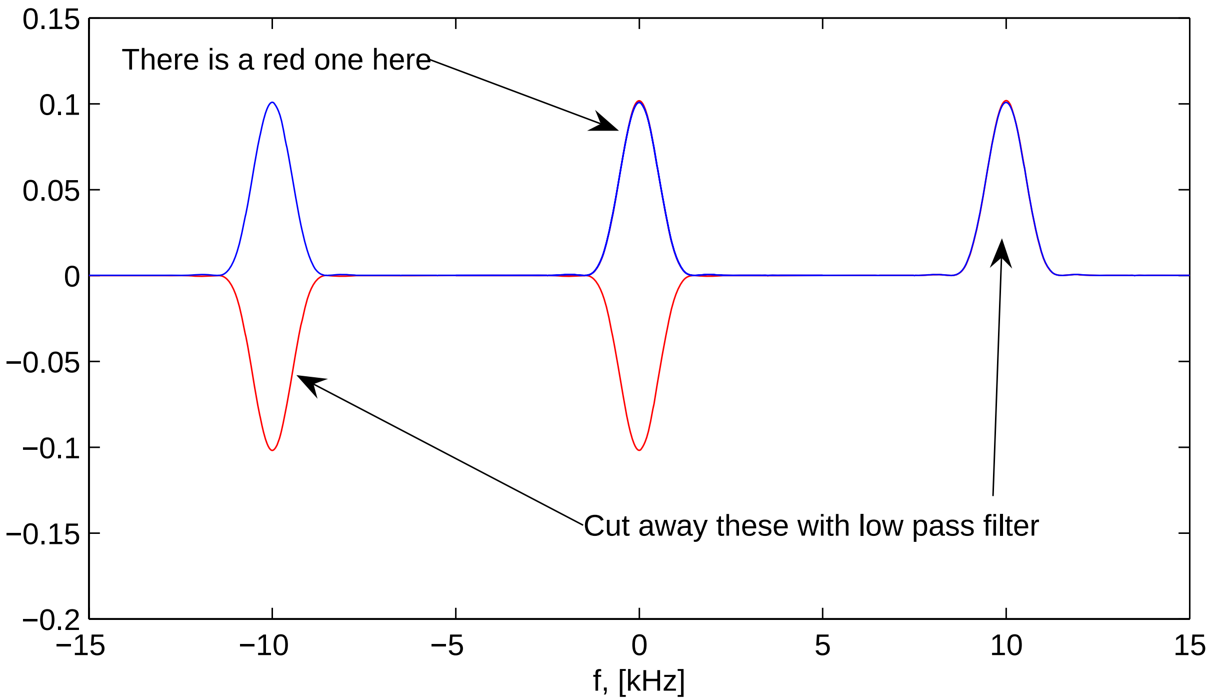
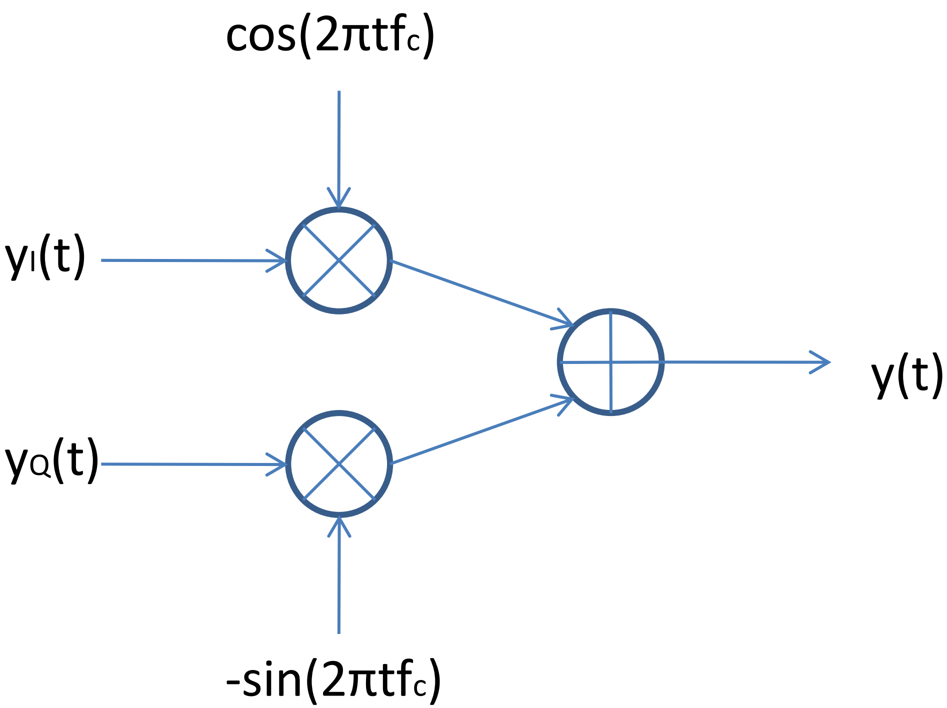


The red spectras cancel out, thus, we can detect the blue independently from the red

Similar for demodulation with $\sin(2\pi t f_c)$

Carrier Transmission

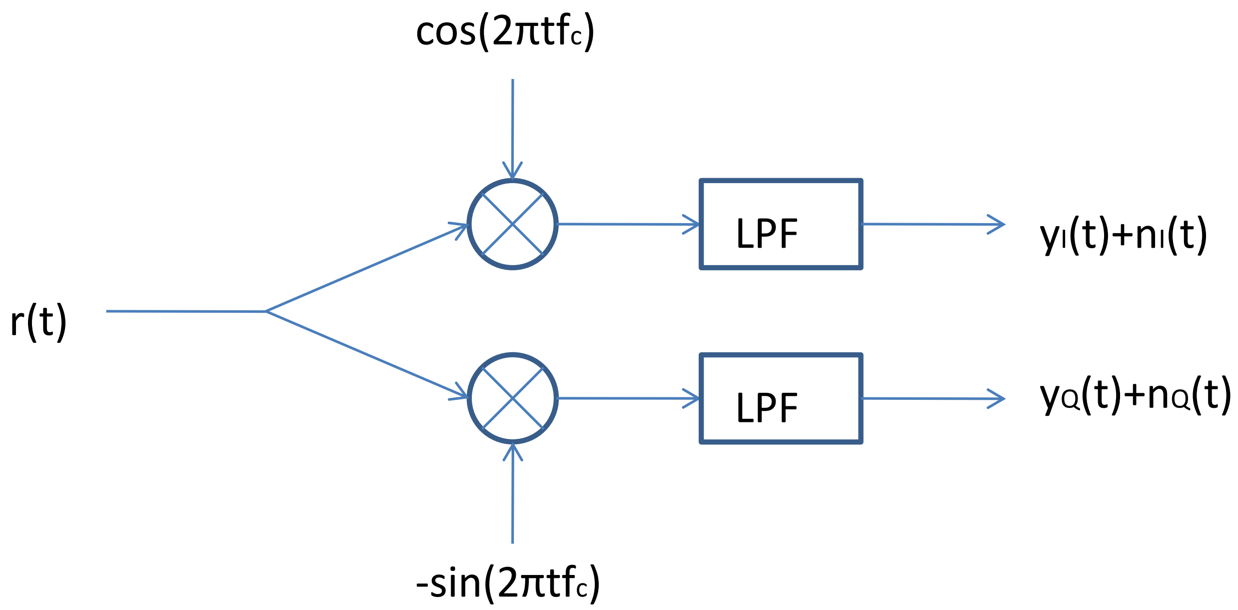
The block diagram of the transmitter is



$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t)$$

Carrier Transmission

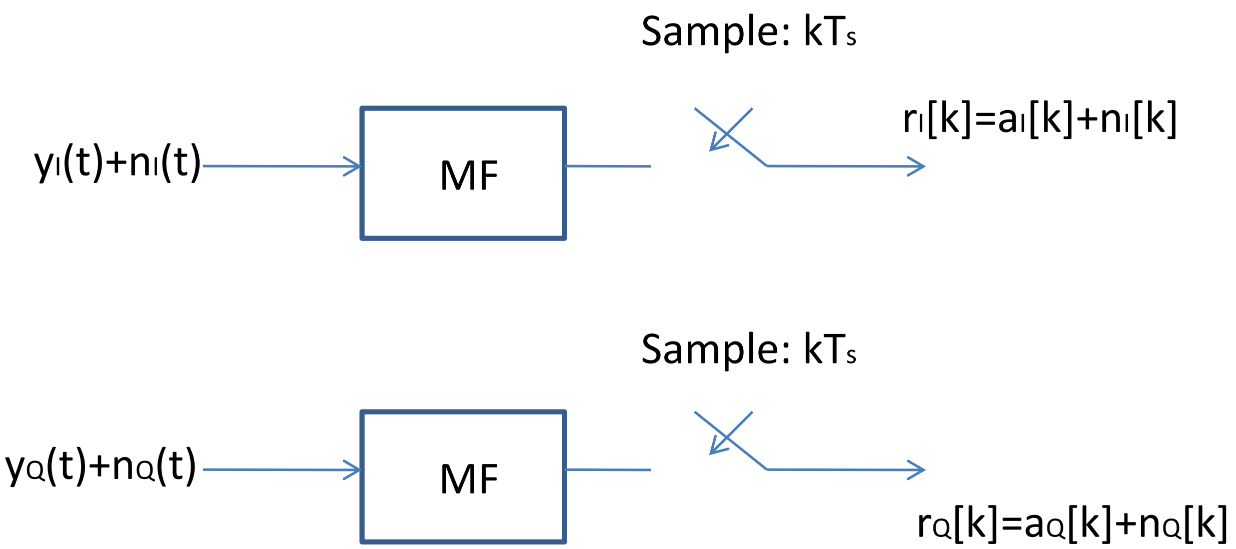
The block diagram of the receiver is



The in-phase and the quadrature components can be independently detected!
The LPF (low pass filters) can be taken as a matched filter to $h(t)$

Carrier Transmission

The signals at both rails are baseband signals, and conventional processing follows: matched filter $\longrightarrow$ sampling every T_s second $\longrightarrow$ decision unit



Carrier Transmission

What is a complex-valued symbol $1 + i$?

In QPSK, we transmit complex valued symbols. In one symbol interval, we have

$$y(t) = \underbrace{h(t)}_{y_I(t)} \cos(2\pi f_c t) - \underbrace{h(t)}_{y_Q(t)} \sin(2\pi f_c t)$$

Carrier Transmission

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Real part goes here

and imaginary here

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We can alternatively express the signal $y(t)$ as

$$\begin{aligned} y(t) &= y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t) \\ &= e(t)\cos(2\pi f_c t + \theta(t)) \end{aligned}$$

where $e(t)$ is the envelope and $\theta(t)$ is the phase

For QPSK, $e(t) = \sqrt{2}h(t)$ and $\theta(t) \in \{0, \pi/2, \pi, 2\pi/2\}$

We can further manipulate $y(t)$ into

$$\begin{aligned} y(t) &= \mathrm{Re}\{(y_I(t) + iy_Q(t))\mathrm{e}^{2\pi f_c t}\} \\ &= \mathrm{Re}\{\tilde{y}(t)\mathrm{e}^{i2\pi f_c t}\} \end{aligned}$$

where

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Carrier Transmission

Example

Assume that we have two bits to transmit, say $+1$ and -1.

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or as

$$y(t) = \sqrt{2}\mathrm{Re}\{(1-i)h(t)\mathrm{e}^{2\pi f_c t}\}$$

Carrier Transmission

In the last representation, we can change the receiver processing into