STATISTICAL PROPERTIES OF JAKES' FADING CHANNEL SIMULATOR

Matthias Pätzold and Frank Laue

Technical University of Hamburg-Harburg
Department of Digital Communication Systems
Denickestr. 17, D-21071 Hamburg, Germany
phone/fax: +49 40-7718-3149/2941
e-mail: paetzold@tu-harburg.d400.de

Abstract — Jakes' method is often used for the design of effective (Rayleigh) fading channel simulators, but only less is known about the achievable degree of accuracy. Our paper fills this gap by studying the statistical properties of Jakes' simulator. Therefore, analytical expressions are presented for the autocorrelation and cross-correlation function of inphase and quadrature components, as well as for the probability density function of envelope and phase. Moreover, an analytical investigation of the second order statistics (level-crossing rate and average duration of fades) is also a topic of this paper. All these expressions are fundamental for the investigation of inevitable degradation effects due to a limited number of low-frequency oscillators or due to inherent insufficiencies of Jakes' method.

I. INTRODUCTION

One of the most famous fading channel simulators was proposed by Jakes in his meanwhile classical and recently reissued book [1]. The so-called Jakes simulator models the received complex low-pass envelope of a stationary frequency non-selective mobile fading channel under isotropic scattering conditions, whereby it is assumed that the line-of-sight component is absent. An appropriate analytical model for such a channel is a zero-mean complex Gaussian noise process with uncorrelated inphase and quadrature components. Jakes' method allows an effective approximation of the desired analytical model by using a finite number of low-frequency oscillators.

Other methods are closely related to the procedure proposed by Jakes, e.g. the method of exact Doppler spread [2], the L_p-norm method [3], the mean square error method [4], and the Monte-Carlo method [4, 5], only to name a few. All these procedures have in common with Jakes' method that they enable an approximation of coloured Gaussian noise processes by a sum of specifically dimensioned low-frequency oscillators. This principle is fundamental and can be traced back historically to Rices' sum of sinusoids [6, 7]. Clearly, for an infinite number of harmonic oscillators, each of the above mentioned methods results in (a sample function of) an ideal (complex) Gaussian noise process with given correlation properties. But for a practical relevant number of harmonic oscillators, say seven or eight, all methods enable only more or less an approximation of the aspired Gaussian noise process, and the unavoidable degradation effects should be studied before implementing the designed fading simulator in software or hardware.

In this paper we study the statistical properties of the popular Jakes simulator. Therefore, Section II recapitulates the underlying theoretical (reference) model. Section III presents the simulation model proposed by Jakes. Section IV analyses its statistical properties. Of special interest are the autocorrelation and cross-correlation function of inphase and quadrature components, probability density function of envelope and phase, level-crossing rate, and average duration of fades. Finally, Section V concludes the paper with a summary of the main results.

II. THE REFERENCE MODEL

In order to guarantee a fair judgement of the statistical properties of Jakes' simulation model a stochastic reference model is required. An appropriate stochastic reference model for the received complex low-pass envelope is a (coloured) complex Gaussian noise process

$$\mu(t) = \mu_1(t) + j\mu_2(t) \; , \tag{1}$$

where $\mu_1(t)$ and $\mu_2(t)$ are uncorrelated zero-mean real Gaussian noise processes with identical variances[1] $\sigma_0^2 = E\{\mu_i^2(t)\}$ $(i = 1, 2)$. On the assumption that we have an isotropic scattering environment, then the autocorrelation function (ACF) of the stochastic processes $\mu(t)$ and $\mu_i(t)$ $(i = 1, 2)$ are given by

$$r_{\mu\mu}(\tau) = 2\sigma_0^2 J_0(2\pi f_{max}\tau) \tag{2a}$$
$$r_{\mu_i\mu_i}(\tau) = \sigma_0^2 J_0(2\pi f_{max}\tau) \; , \; i = 1, 2 \; , \tag{2b}$$

respectively, where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, and f_{max} denotes the maximum Doppler frequency. The absolute value of (1) is a Rayleigh process

$$\zeta(t) = |\mu(t)| = |\mu_1(t) + j\mu_2(t)| \; , \tag{3}$$

[1] The operator $E\{\cdot\}$ denotes statistical average

with probability density function (PDF)

$$p_\zeta(z) = \begin{cases} \frac{z}{\sigma_0^2} e^{-\frac{z^2}{2\sigma_0^2}} & , \quad z \geq 0 \quad , \\ 0 & , \quad z < 0 \quad . \end{cases} \tag{4}$$

The phase of (1), denoted by $\vartheta(t)$, is uniformly distributed over the interval $(-\pi, \pi]$, i.e.

$$p_\vartheta(\theta) = \frac{1}{2\pi} \; , \quad -\pi < \theta \leq \pi \; . \tag{5}$$

It is a common practice to describe the second order statistics of Rayleigh processes $\zeta(t)$ by the level-crossing rate (LCR), $N_\zeta(r)$, and the average duration of fades (ADF), $T_{\zeta_-}(r)$, which can be expressed by [1]

$$N_\zeta(r) = \sqrt{\frac{\beta}{2\pi}} p_\zeta(r) \qquad , \qquad T_{\zeta_-}(r) = \frac{P_{\zeta_-}(r)}{N_\zeta(r)} \; , \tag{6a,b}$$

respectively. Thereby, the quantity β is defined by $\beta = -\ddot{r}_{\mu_i\mu_i}(0)$, and $P_{\zeta_-}(r)$ denotes the cumulative distribution function of $\zeta(t)$, i.e. $P_{\zeta_-}(r) = \int_0^r p_\zeta(z)\; dz$. From (2b) it follows that β can be expressed by $\beta = 2(\pi f_{max}\sigma_0)^2$.

A profound insight into the statistical behaviour of Rayleigh processes $\zeta(t)$ can be obtained by investigating the distribution of the fading time intervals τ_-, $p_{0_-}(\tau_-; r)$, that is the conditional PDF that the Rayleigh process $\zeta(t)$ crosses a certain level r for the first time in the interval $(t_1 + \tau_- \;, t_1 + \tau_- + d\tau_-)$ with positive slope, given a crossing downward through r at time $t = t_1$. The derivation of an exact analytical expression for $p_{0_-}(\tau_-\,; r)$ is still an open problem, but fortunately a good approximation exists for $p_{0_-}(\tau_-\,; r)$ at deep fades ($r \ll 1$) [8]

$$p_{0_-}(\tau_-\,; r) \approx \frac{2\pi u^2 e^{-u}}{T_{\zeta_-}(r)} \left[I_0(u) - \left(1 + \frac{1}{2u}\right) I_1(u) \right] \; , \tag{7}$$

where $u = 2[T_{\zeta_-}(r)/\tau_-]^2/\pi$, and $I_n(\cdot)$ denotes the nth-order modified Bessel function of the first kind.

III. THE JAKES SIMULATOR

A simple network transformation allows us to present the structure of Jakes' fading simulator [1, p. 70] in the form as shown in Fig. 1.

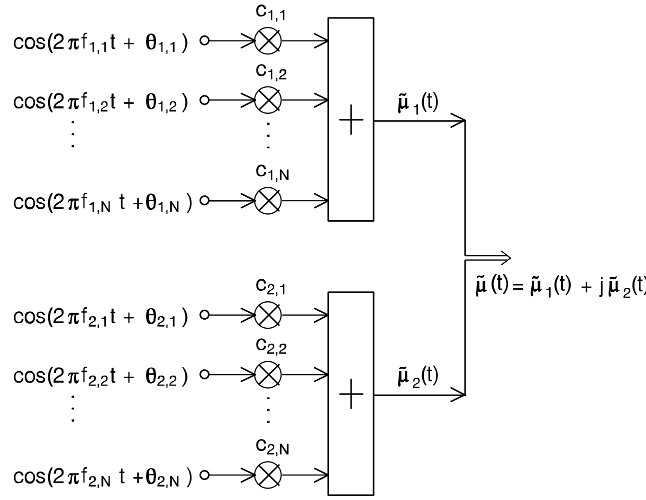


Fig. 1: Jakes' fading simulator that generates a complex low-pass envelope by using a number of N weighted low-frequency oscillators

Thereby, the received complex low-pass envelope (1) is modeled by

$$\tilde{\mu}(t) = \tilde{\mu}_1(t) + j\tilde{\mu}_2(t) \; , \tag{8}$$

where

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}) \; , \quad i = 1, 2 \; , \tag{9}$$

represents a superposition of N weighted low-frequency oscillators. The expression (9) corresponds to Rice's sum of sinusoids [6, 7], which enables for a given number of sinusoids $N < \infty$ an approximation of coloured Gaussian noise processes. The methods proposed by Rice and Jakes differ only in the computation of the model parameters ($c_{i,n}$, $f_{i,n}$, $\theta_{i,n}$). According to Jakes' method, the gains $c_{i,n}$, discrete Doppler frequencies $f_{i,n}$, and phases $\theta_{i,n}$ are given by:

$$c_{i,n} = \begin{cases} \frac{2\sigma_0}{\sqrt{N-\frac{1}{2}}} \sin\left(\frac{\pi n}{N-1}\right), n = 1, \ldots, N\text{-}1, & i = 1, \\ \frac{2\sigma_0}{\sqrt{N-\frac{1}{2}}} \cos\left(\frac{\pi n}{N-1}\right), n = 1, \ldots, N\text{-}1, & i = 2, \\ \frac{\sigma_0}{\sqrt{N-\frac{1}{2}}} \; , \qquad\qquad n = N, & i = 1, 2, \end{cases} \tag{10}$$

$$f_{i,n} = \begin{cases} f_{max} \cos\left(\frac{n\pi}{2N-1}\right), n = 1, \ldots, N\text{-}1, & i = 1, 2, \\ f_{max} \qquad\qquad\qquad , n = N, & i = 1, 2, \end{cases} \tag{11}$$

$$\theta_{i,n} = 0, \quad n = 1, 2, \ldots, N, \quad i = 1, 2 \; . \tag{12}$$

From the above equations (10)-(12) we realize, that the model parameters ($c_{i,n}$, $f_{i,n}$, $\theta_{i,n}$) are constant quantities. Consequently, the complex envelope $\tilde{\mu}(t) = \tilde{\mu}_1(t) + j\tilde{\mu}_2(t)$ is a completely deterministic function. Important characteristic quantities of such a pseudo-random function can therefore widely be analysed on the basis of time averages instead of statistical averages, as we will see in the next section.

IV. Analysis of Jakes' Method

In this section, we analyse the Jakes fading simulator, and we compare its statistical properties with those of the underlying reference model.

A. Mean Value and Mean Power

From (11) we realize that $f_{i,n} \neq 0$ for all $n = 1, 2, \ldots, N$ and $i = 1, 2$. Consequently, the mean value of the deterministic function $\tilde{\mu}_i(t)$ – defined by [2] $\tilde{m}_{\mu_i} =< \tilde{\mu}_i(t) >$ – is zero as prescribed by the reference model, i.e.

$$\tilde{m}_{\mu_i} =< \tilde{\mu}_i(t) >= E\{\mu_i(t)\} = 0 \; . \tag{13}$$

Note that we have normalized the gains $c_{i,n}$ [see (10)] such that the mean power of the deterministic function $\tilde{\mu}_i(t)$ – defined by $\tilde{\sigma}^2_{\mu_i} =< \tilde{\mu}^2_i(t) >$ – is equal to the variance of the stochastic process $\mu_i(t)$, i.e.

$$\tilde{\sigma}^2_{\mu_i} =< \tilde{\mu}^2_i(t) >= E\left\{\mu^2_i(t)\right\} = \sigma^2_0 \; . \tag{14}$$

Both, the mean value and mean power of the simulated complex envelope are in exact conformity with the corresponding quantities of the stochastic reference model.

B. Autocorrelation and Cross-Correlation Functions

The deterministic behaviour of $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ enables us to investigate the correlation properties of these functions by computing the following time average

$$\tilde{r}_{\mu_i\mu_j}(\tau) :=< \tilde{\mu}_i(t) \cdot \tilde{\mu}_j(t+\tau) > \tag{15}$$

for all combinations $i, j = 1, 2$. Substituting (9) in (15) leads to the following analytical expressions

$$\tilde{r}_{\mu_i\mu_i}(\tau) = \sum_{n=1}^{N} \frac{c^2_{i,n}}{2} \cos(2\pi f_{i,n}\tau) \; , \; i = 1, 2 \; , \tag{16a}$$

$$\tilde{r}_{\mu_1\mu_2}(\tau) = \tilde{r}_{\mu_2\mu_1}(\tau) = \sum_{n=1}^{N} \frac{c_{1,n}c_{2,n}}{2} \cos(2\pi f_{1,n}\tau) \; . \tag{16b}$$

First, we concentrate our investigations on the ACF $\tilde{r}_{\mu_i\mu_i}(\tau)$. By substituting (10) and (11) in (16a) and using $N = 10$, we find for $\tilde{r}_{\mu_1\mu_1}(\tau)$ and $\tilde{r}_{\mu_2\mu_2}(\tau)$ the results presented in Fig. 2(a) and Fig. 2(b), respectively. The desired behaviour (2b) is also shown in these figures. Obviously, the approximations $r_{\mu_1\mu_1}(\tau) \approx \tilde{r}_{\mu_1\mu_1}(\tau)$ and $r_{\mu_2\mu_2}(\tau) \approx \tilde{r}_{\mu_2\mu_2}(\tau)$ are poor, even for small values of τ.

Let $N \to \infty$, then the substitution of (10) and (11) in (16a) results in the expressions

$$\begin{aligned}\lim_{N\to\infty} \tilde{r}_{\mu_1\mu_1}(\tau) &= \frac{2\sigma^2_0}{\pi} \int_0^{\pi/2} [1 - \cos(4z)] \cos(2\pi f_{max}\tau \cos z) dz \\ &= \sigma^2_0 \left[J_0(2\pi f_{max}\tau) - J_4(2\pi f_{max}\tau)\right]\end{aligned} \tag{17}$$

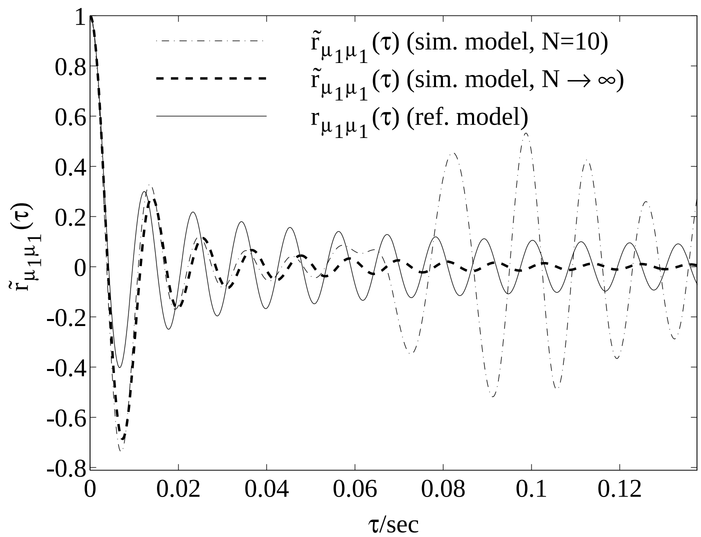
[2]The operator $< \cdot >$ denotes time average

and

$$\begin{aligned}\lim_{N\to\infty} \tilde{r}_{\mu_2\mu_2}(\tau) &= \frac{2\sigma^2_0}{\pi} \int_0^{\pi/2} [1 + \cos(4z)] \cos(2\pi f_{max}\tau \cos z) dz \\ &= \sigma^2_0 \left[J_0(2\pi f_{max}\tau) + J_4(2\pi f_{max}\tau)\right] \; .\end{aligned} \tag{18}$$

Thus, $\tilde{r}_{\mu_i\mu_i}(\tau)$ does not tend to $r_{\mu_i\mu_i}(\tau)$ if $N \to \infty$ $(i = 1, 2)$. For completeness the ACFs (17) and (18) are also plotted in Figs. 2(a) and 2(b), respectively.

(a)



(b)

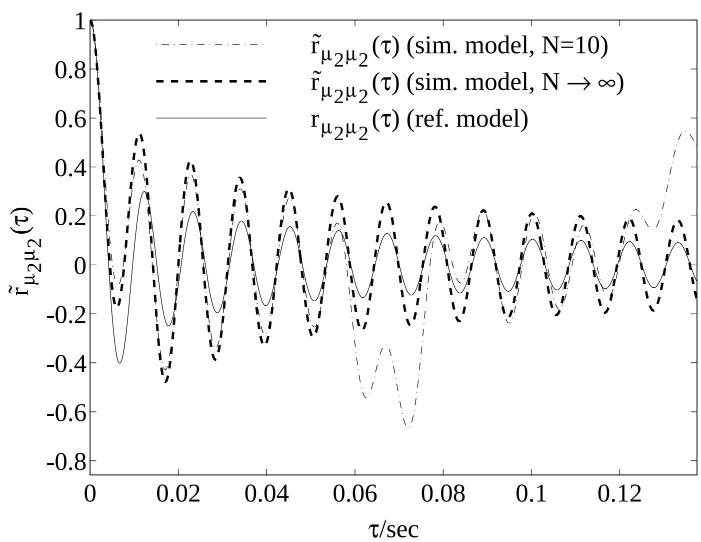


Fig. 2: Comparison of $\tilde{r}_{\mu_i\mu_i}(\tau)$ (sim. model) with $r_{\mu_i\mu_i}(\tau) = \sigma^2_0 J_0(2\pi f_{max}\tau)$ (ref. model) for (a) $i = 1$ and (b) $i = 2$ $(\sigma^2_0 = 1, \; f_{max} = 91$ Hz$)$

Next, we consider the CCF $\tilde{r}_{\mu_1\mu_2}(\tau)$ [see (16b)]. The substitution of (10) and (11) in (16b) results for $N \to \infty$ in the integral

$$\lim_{N\to\infty} \tilde{r}_{\mu_1\mu_2}(\tau) = \frac{2\sigma_0^2}{\pi} \int_0^{\pi/2} \sin(4z)\cos(2\pi f_{max}\tau\cos z)dz \ , \quad (19)$$

which has to be solved numerically. A comparison of $\tilde{r}_{\mu_1\mu_2}(\tau)$ and $r_{\mu_1\mu_2}(\tau)$ is presented in Fig. 3.

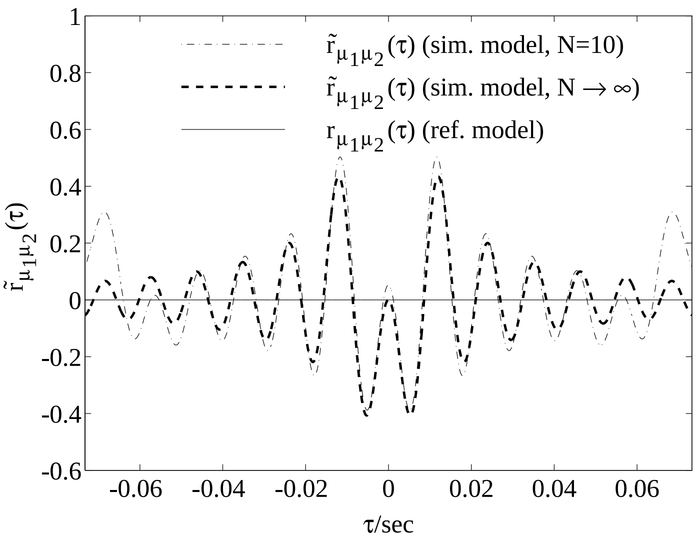


Fig. 3: Comparison of $\tilde{r}_{\mu_1\mu_2}(\tau)$ (sim. model) with $r_{\mu_1\mu_2}(\tau) = 0$ (ref. model) ($\sigma_0^2 = 1$, $f_{max} = 91$ Hz)

One problem with Jakes' method is that the cross-correlation between the inphase and quadrature components are significantly different from zero. In [9] a modified method has been suggested to solve this problem. But the proposed procedure only guarantees $\tilde{r}_{\mu_1\mu_2}(\tau) = 0$ at $\tau = 0$. In order to assure $\tilde{r}_{\mu_1\mu_2}(\tau) = 0$ for all τ, we have to design the deterministic processes $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ by using disjunct sets $\{f_{1,n}\}$ and $\{f_{2,n}\}$ (cf. [3]).

For the ACF of the complex low-pass envelope $\tilde{\mu}(t) = \tilde{\mu}_1(t) + j\tilde{\mu}_2(t)$, we obtain

$$\begin{aligned} \tilde{r}_{\mu\mu}(\tau) &:= <\mu(t)\mu^*(t+\tau)> \\ &= \tilde{r}_{\mu_1\mu_1}(\tau) + \tilde{r}_{\mu_2\mu_2}(\tau) + j(\tilde{r}_{\mu_1\mu_2}(\tau) - \tilde{r}_{\mu_2\mu_1}(\tau)) \\ &= \tilde{r}_{\mu_1\mu_1}(\tau) + \tilde{r}_{\mu_2\mu_2}(\tau) \ . \end{aligned} \quad (20)$$

Thereby, we have used the property $\tilde{r}_{\mu_1\mu_2}(\tau) = \tilde{r}_{\mu_1\mu_2}(-\tau) = \tilde{r}_{\mu_2\mu_1}(\tau)$ [see (16b)]. It is typical for the Jakes method that $\tilde{r}_{\mu\mu}(\tau)$ tends to $r_{\mu\mu}(\tau) = 2\sigma_0^2 J_0(2\pi f_{max}\tau)$ if $N \to \infty$, as can be seen immediately after substituting (17) and (18) in (20). For a limited number of oscillators N, the approximation quality $r_{\mu\mu}(\tau) \approx \tilde{r}_{\mu\mu}(\tau)$ is excellent if τ is within the interval $[0, N/(2f_{max})]$. Fig. 4 shows an example for the ACF $\tilde{r}_{\mu\mu}(\tau)$, in case the number of oscillators is 10. For the reason of comparison, the ACF of the reference model $r_{\mu\mu}(\tau)$ [see (2a)] is also presented in that figure.

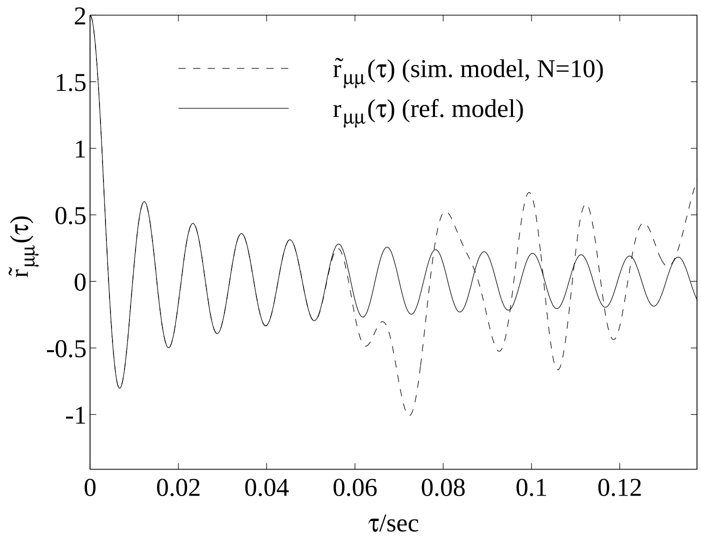


Fig. 4: Comparison of $\tilde{r}_{\mu\mu}(\tau)$ (sim. model) with $r_{\mu\mu}(\tau) = 2\sigma_0^2 J_0(2\pi f_{max}\tau)$ (ref. model) ($\sigma_0^2 = 1$, $f_{max} = 91$ Hz)

C. Probability Density Function of Envelope and Phase

Let us now consider the time variable t as a uniformly distributed random variable. In that case, $\tilde{\mu}(t) = \tilde{\mu}_1(t) + j\tilde{\mu}_2(t)$ describes a complex random variable, whose statistical properties are determined by the primary model parameters $(c_{i,n}, f_{i,n})$ and the number of low-frequency oscillators N. General analytical expressions for the PDF of the envelope $\tilde{\zeta}(t) = |\tilde{\mu}(t)|$ and phase $\tilde{\vartheta}(t) = \arg\{\tilde{\mu}(t)\}$ of such a complex random variable have been derived in [3]. In that paper the following results can be found

$$\tilde{p}_\zeta(z) = z \int_{-\pi}^{\pi} \tilde{p}_{\mu_1}(z\cos\theta)\tilde{p}_{\mu_2}(z\sin\theta)d\theta \ , \ z \geq 0, \quad (21a)$$

$$\tilde{p}_\vartheta(\theta) = \int_0^\infty z\tilde{p}_{\mu_1}(z\cos\theta)\tilde{p}_{\mu_2}(z\sin\theta)dz \ , \ |\theta| \leq \pi, \quad (21b)$$

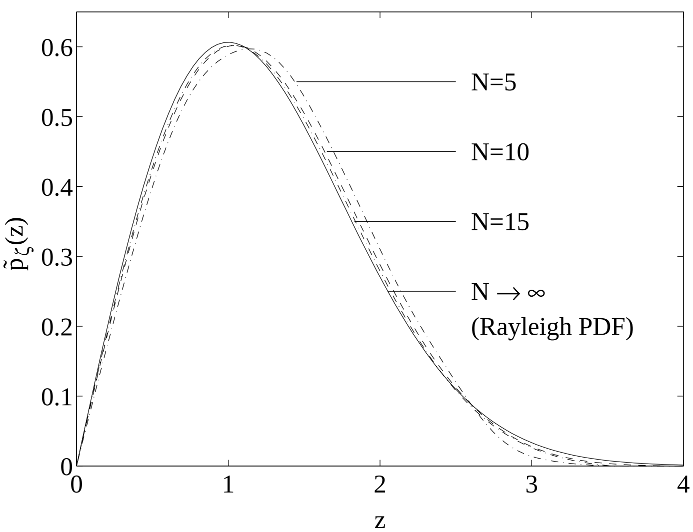
where

$$\tilde{p}_{\mu_i}(x) = 2\int_0^\infty \left[\prod_{n=1}^N J_0(2\pi c_{i,n}\nu)\right] \cos(2\pi\nu x)d\nu \ , \quad (22)$$

expresses the PDF of $\tilde{\mu}_i(t)$ [see (9)] for $i = 1, 2$ as a function of the gains $c_{i,n}$ and the number of low-frequency oscillators N. By making use of (10), it is not surprising (central limit theorem) that (22) tends in the limit $N \to \infty$ to a Gaussian density with zero mean and variance σ_0^2. Thus, if $N \to \infty$ then it follows from (21a) and (21b) that $\tilde{p}_\zeta(z)$ approaches the Rayleigh distribution (4) and $\tilde{p}_\vartheta(\theta)$ approaches the uniform distribution (5). However, for finite values of N we get $\tilde{p}_\zeta(z) \approx p_\zeta(z)$ and $\tilde{p}_\vartheta(\theta) \approx p_\vartheta(\theta)$. Fig. 5(a) and Fig. 5(b) illustrate us that the approximation errors are small if N is not less than 10.

A further increasing of N does not lead to a significant improvement of the approximation quality. Fig. 5(b) shows that the PDF $\tilde{p}_{\vartheta}(\theta)$ [see (21b)] is an equiripple approximation of the uniform distribution (5). The maximum relative error of $\tilde{p}_{\vartheta}(\theta)$ is 5.16% for $N = 5$ (2.06% for $N = 10$ and 1.31% for $N = 15$).

(a)



(b)

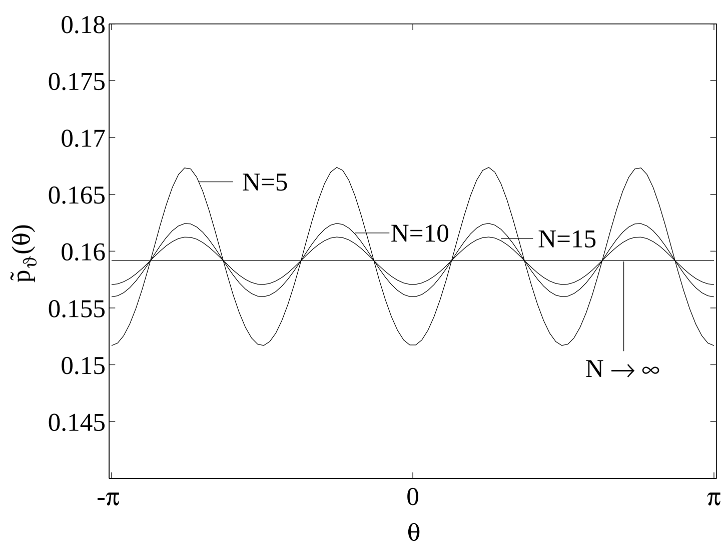


Fig. 5: Probability density functions of (a) envelope $\tilde{p}_{\zeta}(z)$ and (b) phase $\tilde{p}_{\vartheta}(\theta)$ if $N \in \{5, 10, 15, \infty\}$ ($\sigma_0^2 = 1$)

A deeper insight into the performance of Jakes' fading simulator can be achieved by considering the mean square error

$$E_{p_{\mu_i}} := \int_{-\infty}^{\infty} \left(p_{\mu_i}(x) - \tilde{p}_{\mu_i}(x)\right)^2 dx \ , \tag{23}$$

that is an appropriate measure for the error of the approximation $p_{\mu_i}(x) \approx \tilde{p}_{\mu_i}(x)$. Substituting the ideal Gaussian density $p_{\mu_i}(x) = 1/(\sqrt{2\pi}\sigma_0)\exp(-x^2/(2\sigma_0^2))$ and $\tilde{p}_{\mu_i}(x)$ according to (22) in (23) and using (10) gives the results shown in Fig. 6. This figure illustrates the convergence behaviour $\tilde{p}_{\mu_i}(x) \to p_{\mu_i}(x)$ for increasing N. Now, let the gains $c_{i,n}$ are given by $c_{i,n} = \sigma_0\sqrt{2/N}$, then the optimal approximation of the ideal Gaussian density $p_{\mu_i}(x)$ given the power constraint ($\tilde{\sigma}_{\mu_i}^2 = \sigma_0^2$) is achieved [3]. Hence, by choosing $c_{i,n} = \sigma_0\sqrt{2/N}$ a lower bound for $E_{p_{\mu_i}}$ can be computed [see Fig. 6]. From the discussion above and from Fig. 6 it follows that the Jakes method results for any given $N < \infty$ in a non-optimal approximation of $p_{\mu_i}(x)$ ($p_{\zeta}(z)$, $p_{\vartheta}(\theta)$).

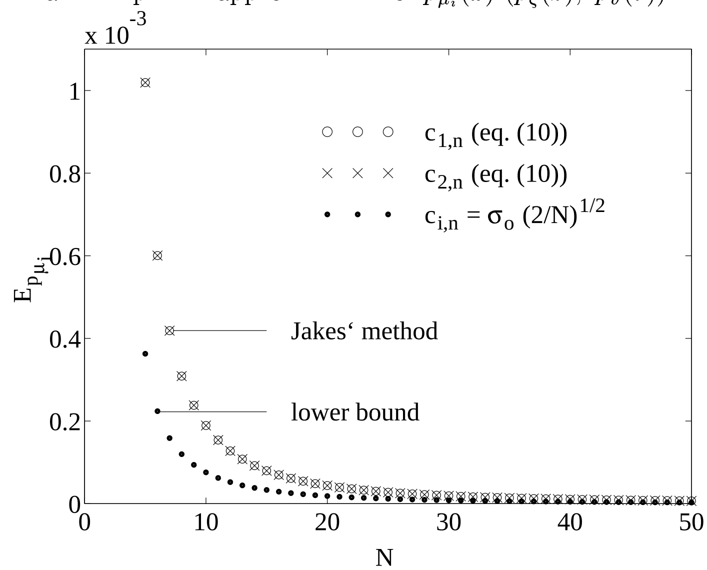


Fig. 6: Mean square error $E_{p_{\mu_i}}$ as function of N ($\sigma_0^2 = 1$)

D. Level-Crossing Rate and Average Duration of Fades

The level-crossing rate (LCR) of the envelope $\tilde{\zeta}(t) = |\tilde{\mu}(t)|$, denoted here by $\tilde{N}_{\zeta}(r)$, can be derived in a similar way as the envelope PDF of deterministic simulation models has been derived in [3]. Here, we only present the final result, which is given by [10]

$$\begin{aligned}\tilde{N}_{\zeta}(r) = & 8r \int_0^{\infty}\int_0^{\pi/2} \tilde{p}_{\mu_1}(r\cos\theta)\tilde{p}_{\mu_2}(r\sin\theta)\cdot \\ & \int_0^{\infty} j_1(z,\theta)j_2(z,\theta)\dot{z}\cos(2\pi z\dot{z})dz d\theta d\dot{z} \ ,\end{aligned} \tag{24}$$

where

$$j_1(z,\theta) = \prod_{n=1}^{N} J_0\left[(2\pi)^2 c_{1,n} f_{1,n} z\cos\theta\right] \ , \tag{25a}$$

$$j_2(z,\theta) = \prod_{n=1}^{N} J_0\left[(2\pi)^2 c_{2,n} f_{2,n} z\sin\theta\right] \ , \tag{25b}$$

and $\tilde{p}_{\mu_i}(\cdot)$ is the density of $\tilde{\mu}_i(t)$ ($i = 1, 2$) as introduced by (22). Note that the LCR $\tilde{N}_{\zeta}(r)$ depends on the parameters

$c_{i,n}$, $f_{i,n}$, and N. It can be shown that $\tilde{N}_\zeta(r)$ approaches $N_\zeta(r)$ [see (6a)] if $N \to \infty$. Results for the normalized LCR $\tilde{N}_\zeta(r)/f_{max}$ by using a finite (small) number of low-frequency oscillators are presented in Fig. 7. This figure shows that the LCR of Jakes' simulator is close to the LCR of the reference model if N is equal or greater than 10.

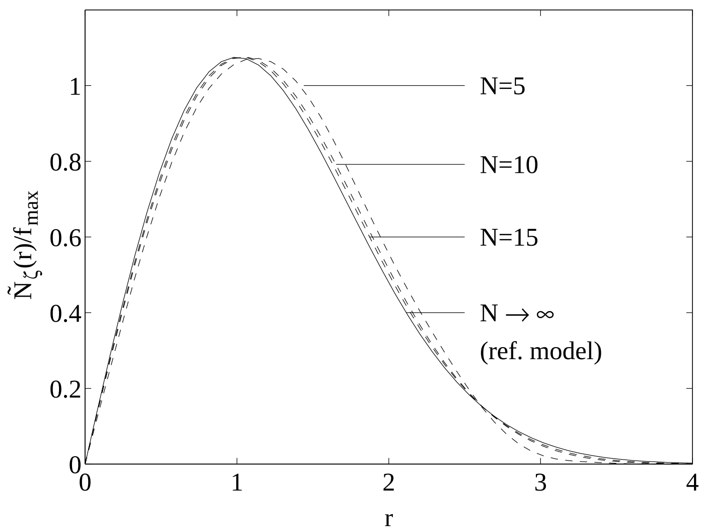


Fig. 7: Normalized LCR of Jakes' simulator $\tilde{N}_\zeta(r)/f_{max}$ ($\sigma_0^2 = 1, f_{max} = 91$ Hz)

By analogy with (6b), we can express the average duration of fades (ADF) of Jakes' simulator by

$$\tilde{T}_{\zeta_-}(r) = \frac{\tilde{P}_{\zeta_-}(r)}{\tilde{N}_\zeta(r)} , \tag{26}$$

where $\tilde{P}_{\zeta_-}(r)$ is the cumulative distribution function of $\tilde{\zeta}(t)$ that can be expressed after performing some algebraic manipulations and using (21a) as follows

$$\tilde{P}_{\zeta_-}(r) = \int_0^r \tilde{p}_\zeta(z)dz = 4r \int_0^\infty J_1(2\pi r z) \cdot \int_0^{\pi/2} \prod_{n=1}^{N} \left[J_0(2\pi c_{1,n} z \cos\theta) J_0(2\pi c_{2,n} z \sin\theta)\right] d\theta dz \, . \tag{27}$$

Thus, the substitution of (24) and (27) in (26) enables us to analyse the ADF of Jakes' fading simulator analytically.

E. Density of Fading Time Intervals

Finally, we analyse for the Jakes simulator the resulting PDF of fading time intervals. This PDF, denoted by $\tilde{p}_{0-}(\tau_-;r)$, describes the conditional probability density that the envelope $\tilde{\zeta}(t)$ crosses a certain level r with positive slope for the first time in the interval $(t_1 + \tau_-, t_1 + \tau_- + d\tau_-)$ given a level-crossing downward through r at time $t = t_1$. For short, we only analyse the PDF $\tilde{p}_{0-}(\tau_-;r)$ at deep fades, i.e. $r \ll 1$. An approximation for $\tilde{p}_{0-}(\tau_-;r)$ is obtained from (7) by substituting $\tilde{T}_{\zeta_-}(r)$ for $T_{\zeta_-}(r)$, i.e.

$$\tilde{p}_{0-}(\tau_-;r) \approx \frac{2\pi \tilde{u}^2 e^{-\tilde{u}}}{\tilde{T}_{\zeta_-}(r)} \left[I_0(\tilde{u}) - \left(1 + \frac{1}{2\tilde{u}}\right) I_1(\tilde{u}) \right] , \tag{28}$$

where $\tilde{u} = 2[\tilde{T}_{\zeta_-}(r)/\tau_-]^2/\pi$, and $\tilde{T}_{\zeta_-}(r)$ is given by (26). An example for the graph of (28) at level $r = 0.1$ is illustrated in Fig. 8 for $N = 10$. The corresponding simulation results for $\tilde{p}_{0-}(\tau_-;r)$ are also presented in that figure. These results have been obtained after evaluating 10^6 fading time intervals τ_- of the simulated channel envelope sequence $\tilde{\zeta}(kT_s)$ at level $r = 0.1$, where k denotes an integer value and T_s is the sampling interval that was equal to $T_s = 0.02$ msec.

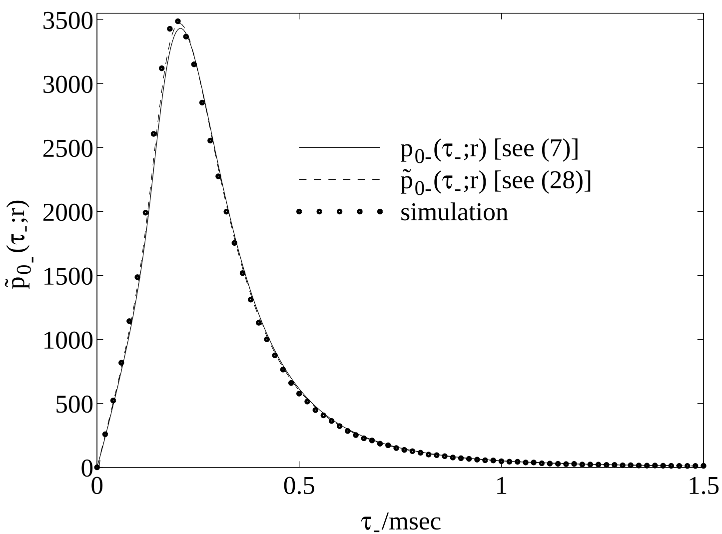


Fig. 8: PDF of fading time intervals $\tilde{p}_{0-}(\tau_-;r)$ at level $r = 0.1$ for $N = 10$ (Jakes' simulator with $\sigma_0^2 = 1$ and $f_{max} = 91$ Hz)

V. CONCLUSION

In the present paper, we have analysed the statistical properties of Jakes' fading simulator. Analytical expressions have been derived especially for the ACF and CCF of the inphase and quadrature components, the PDF of envelope and phase, the LCR and ADF, as well as an approximative solution has been presented for the distribution of fading time intervals at low signal levels. It has been pointed out that Jakes' method suffers under two main disadvantages. First, the inphase and quadrature components are correlated. Second, for a given number of low-frequency oscillators the inphase and quadrature components are non-optimal Gaussian distributed. Nevertheless, our investigations have shown that the performance loss is small and can be compensated by a slight increase of the number of low-frequency oscillators N. Sufficient results can be expected by choosing N equal to or greater than 10. All in all, Jakes' method is quite useful for the design of (frequency non-selective) Rayleigh fading channel models.

REFERENCES

[1] W.C. Jakes, Ed., *Microwave Mobile Communications.* New Jersey: IEEE Press, 1993.

[2] M. Pätzold, U. Killat, F. Laue, and Y. Li, "A new and optimal method for the derivation of deterministic simulation models for mobile radio channels," in *Proc. IEEE 46th Veh. Technol. Conf.*, Atlanta, Georgia, USA, Apr./May 1996, pp. 1423–1427.

[3] M. Pätzold, U. Killat, F. Laue, and Y. Li, "On the statistical properties of deterministic simulation models for mobile fading channels," *IEEE Trans. Veh. Technol.*, submitted and accepted for publication.

[4] M. Pätzold, U. Killat, and F. Laue, "A deterministic digital simulation model for Suzuki processes with application to a shadowed Rayleigh land mobile radio channel," *IEEE Trans. Veh. Technol.*, vol. VT-45, no. 2, pp. 318–331, May 1996.

[5] P. Höher, "A statistical discrete-time model for the WSSUS multipath channel," *IEEE Trans. Veh. Technol.*, vol. VT-41, no. 4, pp. 461–468, Nov. 1992.

[6] S.O. Rice, "Mathematical analysis of random noise," *Bell Syst. Tech. J.*, vol. 23, pp. 282–332, July 1944.

[7] S.O. Rice, "Mathematical analysis of random noise," *Bell Syst. Tech. J.*, vol. 24, pp. 46–156, Jan. 1945.

[8] S.O. Rice, "Distribution of the duration of fades in radio transmission: Gaussian noise model," *Bell Syst. Tech. J.*, vol. 37, pp. 581–635, May 1958.

[9] P. Dent, G.E. Bottomley, and T. Croft, "Jakes fading model revisited," *Electronics Letters*, vol. 29, no. 13 pp. 1162–1163, June 1993.

[10] M. Pätzold and F. Laue, "Level-crossing rate and average duration of fades of deterministic simulation models for Rice fading channels," *IEEE Trans. Veh. Technol.*, submitted and accepted for publication.