BIT SYNCHRONIZATION AND TIMING SENSITIVITY IN ADAPTIVE VITERBI EQUALIZERS FOR NARROWBAND-TDMA DIGITAL MOBILE RADIO SYSTEMS

A. Baier, G. Heinrich, U. Wellens

Philips Kommunikations Industrie AG, Nuremberg
Federal Republic of Germany

ABSTRACT

In this paper, an adaptive Viterbi-MLSE equalizer for narrowband-TDMA digital mobile radio systems is presented which uses a straightforward channel sounding technique for fast periodic equalizer start-up. The equalizer is specifically tailored for the application with h = 0.5 partial-response CPM modulation schemes such as GMSK or GTFM. Exploiting the features of the channel-sounding Viterbi equalizer, a simple sample timing and bit synchronization technique is developed which gets along with a free-running sampling clock. Finally, the timing sensitivity of a digital mobile radio receiver based on the presented equalization and synchronization concepts is studied by means of computer simulations.

I. INTRODUCTION

In narrowband-TDMA digital mobile radio (DMR) systems, typically m = 8 ... 12 users (i.e. traffic channels carrying speech data) share one common carrier frequency using a time division multiplex (TDM) technique /1/. The TDM technique usually is based on frames of a duration T_{Fr} which are subdivided into m time slots of duration $T_{TS} = T_{Fr}/m$ each, one for each user, cf. Fig. 1. The gross bit rate of the data to be transmitted over the mobile radio channel typically is in the order of R = 200 ... 400 kbit/s. As to the modulation format, partial-response-type continuous phase modulation schemes (CPM) /2/ such as GMSK /3/ or GTFM /4/ are favourite candidates because of their high bandwidth efficiency and their constant envelope feature.

An important example of a narrowband-TDMA DMR system as outlined above is the future Pan-European GSM system /5/. In the GSM standard, m = 8, T_{Fr} = 4.615 ms, T_{TS} = 0.5769 ms, and R = 270.83 kbit/s is chosen. The modulation is GMSK with BT = 0.3 which will be used with a carrier spacing of 200 kHz.

Data transmission with bit rates in the order of R = 200 ... 400 kbit/s is seriously handicapped by the time-varying multipath propagation characteristics of the mobile radio channel /6/. Depending on the actual propagation conditions, the transmitted signals may be affected by flat fading or frequency-selective fading. In the frequency-selective fading case, the width of the multipath delay profile exceeds the bit period T = 1/R which results in a time-varying intersymbol interference (ISI) spanning several bits. A powerful adaptive equalizer is required in the DMR receiver in order to cope with this ISI and to ensure an acceptable transmission quality in all propagation conditions /1, 5/.

A promising concept for adaptive equalization in narrowband-TDMA DMR receivers is based on the combination of the 'maximum likelihood sequence estimation' (MLSE) principle using the Viterbi algorithm /7/ and the 'channel sounding' technique known from /8/, i.e. measuring the instantaneous channel impulse response by means of training signals or preambles transmitted once in every time slot of the TDM frame /9/.

In Section II of this paper, a narrowband-TDMA DMR receiver comprising a matched filter based channel sounding unit and an adaptive Viterbi-MLSE equalizer will be presented which is specifically tailored for the application with partial-response CPM schemes with modulation index h = 0.5 such as GMSK or GTFM.

In Section III, attention is focussed onto the problem of bit synchronization and sample timing in the adaptive Viterbi-MLSE receiver. It will be

CH2622-9/88/0000-0377 $1.00 © 1988 IEEE

shown that the energy detection feature of the Viterbi-MLSE equalizer in conjunction with the channel sounding technique allows for a free-running sampling clock at the receiver input as long as the sampling rate is chosen in accordance with the sampling theorem. Since there is no need for a clock recovery loop, receiver implementation is simplified substantially.

Finally, in Section IV, the sampling and bit synchronization technique of Section III is verified and investigated by computer simulations for the case of a 270 kbit/s GMSK modulation. The timing sensitivity of an adaptive 16-state Viterbi-MLSE receiver in terms of BER versus sampling phase is studied for sampling rates of 1, 2, and 4 times the bit rate assuming both, a static AWGN and a fading multipath channel model, the latter with path delay differences of up to 5 bit periods (18.5 µs).

II. CHANNEL SOUNDING VITERBI-MLSE EQUALIZER

The most powerful equalization technique for channels with severe amplitude and phase distortion is MLSE equalization using the Viterbi algorithm /10, 11/. A detailed description of the principles of Viterbi-MLSE equalization (short: Viterbi equalization) can be found e.g. in /7/ or /10/.

If the width of the instantaneous channel impulse response is about

$$T_{Ch} \approx (K+1)T \qquad (1)$$

each transmitted symbol of duration T is interfered by K preceeding symbols. Consequently, the 'constraint length' or 'memory' of the channel in multiples of the symbol period T is K, and the number of states in the discrete finite state model of the channel is 2^K for binary signalling. A Viterbi equalizer with a fixed constraint length K or equivalently a fixed number of 2^K states can cope with channel impulse responses or multipath delay profiles having a width T_{Ch} of up to (K+1)T, cf. equ. (1). However, we have to take into account that the partial-response modulation introduces additional ISI and, hence, increases the effective constraint length of the transmission channel. Since the complexity of the Viterbi equalizer is proportional to 2^K, a real-time hardware implementation seems to be practical as long as K is less than 5 or 6. For example, this is the case in the GSM narrowband-TDMA DMR system mentioned in Section I.

An essential prerequisite for the proper operation of the Viterbi equalizer is the knowledge of the instantaneous channel impulse response $h_{Ch}(t)$. As $h_{Ch}(t)$ is not known beforehand, we have to provide a more or less accurate estimate $\tilde{h}_{Ch}(t)$. Due to the bursty nature of the TDMA signalling and the rapid time fluctuations of the mobile radio channel, iterative channel estimation techniques as described in /10/ are not adequate in DMR applications, unless additional measures are taken for fast initial equalizer start-up (aquisition) in each time slot. The appropriate technique to achieve this fast equalizer start-up is the channel sounding technique presented in /8/.

Channel sounding is accomplished by transmitting a known pseudo-random training signal or bit sequence of duration $N_T \cdot T$ once in every time slot of the TDM frame as indicated in Fig. 1. In the receiver, matched filter correlation of corresponding sections of the input signal and the known training signal yields an estimate $\tilde{h}_{Ch}(t)$ of the current channel response $h_{Ch}(t)$, provided the pseudo-random training signal has a dirac-pulse like autocorrelation /8/.

The basic structure of a digital quadrature-type narrowband-TDMA receiver with channel sounding and adaptive Viterbi equalization is shown in Fig. 2. After sampling and A/D conversion, the I and Q quadrature signals corresponding to a selected time slot are buffered in a RAM which can be accessed as required by the channel sounding unit and the Viterbi equalizer. Once an initial channel response estimate $\tilde{h}_{Ch}(t)$ has been determined for the current time slot, the Viterbi equalizer can start operation. If high vehicle speeds are envisaged, it may be necessary to supply the Viterbi equalizer with an additional iterative channel estimator according to /10/ which tracks the rapid channel variations within the time slot.

In /9/ an adaptive Viterbi equalizer for CPM modulated data signals has been proposed where the impulse response $h_{Ch}(t)$ of the pure radio channel is estimated via channel sounding with a chirp-type training signal. As a drawback of this technique, the nonlinear partial-response CPM modulation has to be incorporated into the metric calculations of the Viterbi equalizer which results in a heavy computational burden. If, on the other hand, channel sounding is accomplished with binary training sequences inserted into the time slot data prior to CPM modulation, cf. Fig. 1, the influence of the partial-response CPM is automatically taken into account during channel estimation which significantly reduces the complexity of the Viterbi equalizer /12, 13/. In the following, this technique will be described in more detail.

For a binary bit stream represented by the symbols $b_k = +/- 1$, $k = \ldots, 0, 1, \ldots$, the CPM modulated data signal in complex lowpass notation is given by

$$s_0(t) = A\cdot\exp\left[j\pi h\cdot\sum_k b_k\int_{-\infty}^{t-kT} g(\tau)d\tau + \Phi_0\right], \quad (2)$$

cf. /2/. A and Φ_0 denote amplitude and relative carrier phase, h is the modulation index, and g(t) is the partial-response frequency pulse defining the phase modulation. In the following, the modulation index will be restricted to h = 0.5. For this important modulation index, $s_0(t)$ can be well approximated by the linear partial-response QAM signal

$$\tilde{s}_0(t) = A\cdot\exp(j\Phi_0)\cdot\sum_k a_k\tilde{g}(t-kT), \quad (3)$$

where the terms a_k are complex-valued data symbols and $\tilde{g}(t)$ is a real-valued partial-response pulse shaping function /14/. The symbols a_k directly originate from the binary data symbols b_k and take on the values +1, -1, +j, and -j only. As demonstrated in /14/, the approximation $\tilde{s}_0(t)$ is very close to $s_0(t)$ if the pulse shaping function $\tilde{g}(t)$ is chosen properly. As an example, $\tilde{g}(t)$ is shown in Fig. 3 for a GMSK modulation with BT = 0.3.

The approximation (3) can be used to set up a joint linear model for the h = 0.5 CPM modulation and the mobile radio channel as indicated in Fig. 4. Starting from the approximation (3), the signal $\tilde{s}_{Ch}(t)$ at the output of the radio channel with impulse response $h_{Ch}(t)$ is given by

$$\begin{aligned}\tilde{s}_{Ch}(t) &= \tilde{s}_0(t) * h_{Ch}(t)\\ &= A\cdot\exp(j\Phi_0)\cdot\sum_k a_k\tilde{g}(t-kT) * h_{Ch}(t) \quad (4)\end{aligned}$$

or equivalently

$$\tilde{s}_{Ch}(t) = \sum_k a_k h(t-kT) \quad (5)$$

with

$$h(t) = A\cdot\exp(j\Phi_0)\cdot\tilde{g}(t) * h_{Ch}(t). \quad (6)$$

Noise and interference has been ignored in (4) and (5). As a consequence of equ. (5), the transmission system can be regarded as a linear QAM system with transmitted data symbols $a_k \in \{+1, -1, +j, -j\}$ and an overall impulse response h(t) which includes both, the pure channel impulse response $h_{Ch}(t)$ and the characteristics of the modulation, i.e amplitude A, carrier phase Φ_0, and the partial-response pulse shaping function $\tilde{g}(t)$. This has previously been observed in /12/ and /13/.

Based on the linear QAM model of the CPM transmission system, a Viterbi equalizer whose metric calculation is much simpler than the metric calculation in /9/ can be designed using the conventional QAM approach /10, 11/. Since the data symbols a_k originate from the binary data b_k, a simple trellis with binary tansitions and 2^K states can be set up where K is now the constraint length of the overall impulse response h(t). Hence, the complexity of the h = 0.5 CPM Viterbi equalizer will be similar to the complexity of a Viterbi equalizer for binary PAM or PSK.

To demonstrate how the overall impulse response h(t) can be estimated using a matched filter based channel sounding unit, we assume a known training bit sequence $\hat{b}_k$ of length N_T being transmitted through the joint linear channel model shown in Fig. 4. In the matched filter correlator, the bits $\hat{b}_k$, k = 0, ..., N-1, with $N \leq N_T$, will be applied in form of the corresponding complex data symbols $\hat{a}_k$, k = 0, ..., N-1, i.e. the signal

$$y(t) = \sum_{m=0}^{N-1} \hat{a}_m^*\cdot x(t+mT) \quad (7)$$

is calculated. x(t) denotes the received signal in complex lowpass notation. For simplicity, a non-causal notation has been chosen in (7). Assuming that the training sequence has perfect autocorrelation properties, i.e.

$$\sum_{m=0}^{N-1} \hat{a}^*_{m-k}\hat{a}_m = \begin{cases} N \text{ for } k=0 \\ 0 \text{ for } k\neq 0, \end{cases} \quad (8)$$

and that $x(t) = \tilde{s}_{ch}(t)$ (ignoring noise and interference), we obtain from (5) and (7)

$$y(t) = \sum_{m=0}^{N-1} \sum_{k} \hat{a}^*_m \hat{a}_k \cdot h(t-kT+mT)$$
$$= N \cdot h(t). \quad (9)$$

In practice, $y(t)$ is an estimate of $h(t)$ since $x(t)$ is affected by noise and interference and since the finite-length training sequence produces autocorrelation side lobes which can not be neglegted. In a discrete-time notation assuming J samples per bit period T and sampling instants

$$t_i = t_0 + iT/J, \quad i = 0, 1, \ldots, \quad (10)$$

the channel sounding operation (7) resp. (9) can be rewritten as

$$y_i = \sum_{m=0}^{N} \hat{a}^*_m \cdot x_{i+mJ}$$
$$= N \cdot h_i, \quad (11)$$

where $x_i = x(t_i)$, $y_i = y(t_i)$, and $h_i = h(t_i)$. Fig. 5 shows a simple FIR filter implementation of the discrete-time matched filtering used in (11). To illustrate the described channel sounding technique, Fig. 6 shows the matched filter output signal $y(t)$ for a simple 2-ray channel response $h_{ch}(t)$ and a GMSK modulation with BT = 0.3. The two propagation paths have identical amplitude and phase and a delay difference of three bit periods T. The estimate $y(t)$ of $h(t)$ is exact, since the channel has been assumed noise-free and a long training sequence with vanishingly small autocorrelation side lobes has been used. Fig. 6 clearly indicates that the measured impulse response $h(t)$ is the convolution of $h_{ch}(t)$ and the partial-response pulse shaping function $\tilde{g}(t)$ in the linear QAM approximation for GMSK, cf. Fig. 3.

III. SAMPLE TIMING AND BIT SYNCHRONIZATION

In addition to reducing the complexity of the Viterbi equalizer, the channel sounding and equalization technique presented in Section II has major advantages with respect to the problem of sample timing and bit synchronization in discrete-time signal processing. To demonstrate this we assume sampling instants t_i, $i = 0, 1, \ldots$, as defined by (10), where t_0 represents a free timing phase parameter. As indicated in equ. (11), sampling the received signal $x(t)$ at the instants t_i directly corresponds to sampling the overall impulse response $h(t)$ (resp. its estimate $y(t)$) at equivalent instants t_i and vice versa, cf. Fig. 6. Since the operation of a MLSE equalizer is based on a discrete-time replica of the linear transmission channel which is determined by the measured impulse response samples $h_i = h(t_i)$, the Viterbi equalizer automatically adapts to the actual timing phase t_0.

In spite of this adaptation, the performance of the channel sounding Viterbi equalizer may depend on t_0. However, if the sampling rate with respect to the bandwidth of the overall impulse response $h(t)$ does not violate the sampling theorem, the equalizer performance will be independent of t_0, because the MLSE equalizer represents an energy detector and

$$E_h = T/J \cdot \sum_{i=-\infty}^{\infty} |h(t_i)|^2 \quad (12)$$

is independent of t_0. Via the pulse shaping function $\tilde{g}(t)$ in (6), the bandwidth of $h(t)$ is limited to the bandwidth of the narrowband partial-response CPM signal and, hence, a rather low sampling rate of J = 1 ... 4 samples per bit period will be sufficient in practice, cf. Section IV.

As a consequence of the timing phase independent equalizer performance, we can allow for a free-running sampling clock at the receiver input, provided the actual sampling rate f_s is close enough to the desired sampling rate J/T such that the timing phase t_0 remains constant at least for the duration T_{TS} of the current time slot. In the next time slot to be processed (usually located in the next frame), the timing phase t_0 may have slipped by an arbitrary amount and the equalizer is re-adapted via channel sounding. In order to identify typical stability requirements on the free-running clock generator we set up the relation

$$\Delta t_{max}/T = N_{TS} \cdot \Delta f_s/f_s \quad (13)$$

between the maximum timing phase offset Δt_{max} in a N_{TS} bit time slot and the frequency stability

$\Delta f_s/f_s$ of the clock generator. Allowing a negligible maximum timing offset of $\Delta t_{max} = 10^{-2} \cdot T$ only and assuming a time slot length of $N_{TS} \leq$ 200 bits typical for TDMA DMR systems, we end up with a frequency stability of 50 ppm. This can be guaranteed with very low cost quartz oscillators and impressively demonstrates the feasibility of the free-running clock sampling technique. In practice, the absence of a clock recovery loop significantly simplifies receiver implementation.

The channel sounding, equalization, and sample timing technique presented above has been derived on the tacit understanding that the overall impulse response h(t) is not longer than (K+1)T, where K is the fixed constraint length of the Viterbi equalizer. In reality, the width of h(t) may exceed this bound which results in excess ISI the Viterbi equalizer can no longer cope with. If additional signal processing such as pre-filtering shall be avoided, the proper technique in that situation is to determine the discrete-time estimate $y_i = y(t_i)$ of h(t) in a window of width W·T with W > K+1 and select a gliding window of width (K+1)T, i.e. (K+1)J samples, starting at an instant $t_{\hat{m}}$ such that

$$E_m = \sum_{i=m}^{m+(K+1)J-1} |h_i|^2, \quad m = 0, 1, \ldots \tag{14}$$

is maximized for $m = \hat{m}$. The samples inside the selected window make up the channel estimate

$$\tilde{h}_i = y_{\hat{m}+i} \quad ,i=0,\ldots,(K+1)J-1 \tag{15}$$

passed on to the Viterbi equalizer. In doing so, the impulse response energy that can be exploited by the Viterbi equalizer is maximized and the excess ISI mentioned above is minimized. In principle, the gliding window search to select $\tilde{h}_i$ according to (14) and (15) can be regarded as a bit synchronization technique which has the coarse resolution of the sampling period T/J. Due to the energy detection feature of the Viterbi equalizer, this coarse bit synchronization on basis of a free-running sampling clock is sufficient for proper operation of the receiver.

IV. COMPUTER SIMULATION RESULTS

In order to verify and investigate as well the channel sounding and Viterbi equalization technique of Section II as the sampling and bit synchronization technique of Section III, a computer simulation of an appropriately tailored narrowband-TDMA DMR transmission system has been set up. In the simulations, a signalling format according to the GSM standard has been adopted, i.e. firstly, a basic time slot of T_{TS}= 0.577ms duration comprising a N_T = 26 bit training sequence (N = 16) and 116 data bits and, secondly, a R = 270 kbit/s differentially encoded GMSK modulation with BT = 0.3. As lowpass receive filter a raised-cosine filter with roll-off 1.0 and a 6 dB cut-off frequency of R/2 = 135 kHz has been assumed.

Fig. 7 shows the simulated BER performance of an adaptive channel-sounding 16-state Viterbi equalizer for a static AWGN channel. For a N = 255 bit training sequence (dotted line) and, hence, a very accurate channel sounding, the performance is very close to the theoretical BPSK bound (straight line). Application of the shorter GSM training sequence (N = 16) results in a degradation of 1 dB (dashed line), which is due to the increased influence of noise and autocorrelation side lobes on the channel estimate y(t). Finally, the dashed and dotted curve indicates that an additional degradation of about 0.3 dB occurs if autonomous bit synchronization via the gliding window search of Section III is incorporated.

To demonstrate the feasibility of the free-running clock sampling technique proposed in Section III, we have studied the timing sensitivity of the channel-sounding 16-state Viterbi equalizer with gliding window bit synchronization for sampling rates f_s of 1, 2, and 4 times the bit rate R. In all simulations, the results for f_s = 4R were identical with those for f_s = 2R, and therefore only f_s = R and f_s = 2R will be considered in the following. For both sampling rates, Figs. 8 and 9 show the BER performance achieved with four different values of the timing phase parameter t_0 introduced in equ. (10). Fig. 8 holds for the static AWGN channel and Fig. 9 holds for a multipath channel with two independent Rayleigh fading propagation paths of equal average power. The relative delay of the paths is τ/T = 0, 1, ..., 5. For the given bit rate, the maximum delay of τ= 5T corresponds to 18.5 µs. The average E_b/N_0 assumed in Fig. 9 is 18 dB.

As revealed by Figs. 8 and 9, the equalizer performance is virtually independent of the timing phase t_0, even in the case f_s = R. This is due to the very compact frequency spectrum of GMSK with BT = 0.3. Merely for the cases τ = 4T and τ = 5T in Fig. 9, where the Viterbi equalizer suffers from a degradation due to excess ISI in the overall impulse response h(t), the BER performance varies slightly with t_0 if only one sample per bit period is taken. In that particular situation, a gliding window bit synchronization with the resolution of T/2 is superior because excess ISI can be minimized with more accuracy if the optimum window position is selected with a higher resolution in time, cf. Fig. 6.

The simulation results show that the proposed free-running clock sampling and bit synchronization technique is feasible for partial-response CPM schemes such as GMSK and that it allows sampling rates as low as 1 or 2 times the bit rate. Hence, this technique is very attractive for an application in narrowband-TDMA DMR receivers.

V. REFERENCES

/1/ J. Uddenfeldt and B. Persson, "A narrowband TDMA system for a new generation cellular radio", in Proc. 37th IEEE Vehicular Tech. Conf., Tampa, 1987, pp. 286-292.

/2/ C.-E. Sundberg, "Continuous phase modulation", IEEE Commun. Mag., vol. 24, pp. 25-38, April 1986.

/3/ K. Murota and K. Hirade, "GMSK modulation for digital mobile radio telephony", IEEE Trans. Commun., vol. COM-29, pp. 1044-1050, July 1981.

/4/ K.-S. Chung, "Generalized tamed frequency modulation and its application for mobile radio communications", IEEE Trans. Vehicular Tech., vol. VT-33, pp. 103-113, Aug. 1984.

/5/ A. Maloberti, "Definition of the radio subsystem for the GSM Pan-European digital mobile communication system", in Proc. Int. Conf. on Digital Land Mobile Radio Commun., Venice, 1987, pp. 37-47.

/6/ W.C.Y Lee, Mobile Communications Engineering. New York: McGraw-Hill, 1982.

/7/ G.D. Forney, "Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference", IEEE Trans. Inform. Theory, vol. IT-18, pp. 363-378, May 1972.

/8/ G.L. Turin, "Introduction to spread-spectrum antimultipath techniques and their application to urban digital radio", Proceedings IEEE, vol. 68, pp. 328-353, March 1980.

/9/ T. Maseng and O. Trandem, "Adaptive digital phase modulation", in Proc. 2nd Nordic Seminar on Digital Land Mobile Radio Commun., Stockholm, 1986.

/10/ J.G. Proakis, Digital Communications. New York: McGraw-Hill, 1983.

/11/ S.U.H. Qureshi, "Adaptive Equalization", Proceedings IEEE, vol. 73, pp. 1349-1387, Sept. 1985.

/12/ J.L. Dornstetter, "The digital cellular SFH 900 system", in Proc. 2nd Nordic Seminar on Digital Land Mobile Radio Commun., Stockholm, 1986.

/13/ R. D'Avella, L. Moreno, M. Sant'Agostino, "Adaptive equalization in TDMA mobile radio systems", in Proc. 37th IEEE Vehicular Tech. Conf., Tampa, 1987, pp. 385-392.

/14/ P.A. Laurent, "Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (AMP)", IEEE Trans. Commun., vol. COM-34, pp. 150-160, Feb. 1986.

ILLUSTRATIONS

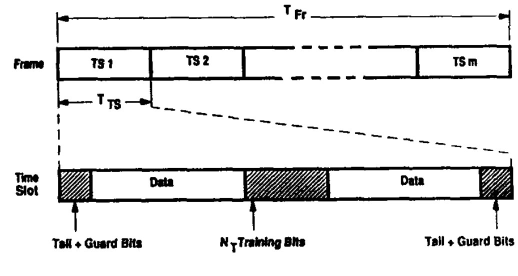


Fig. 1: Typical narrowband-TDMA signalling format

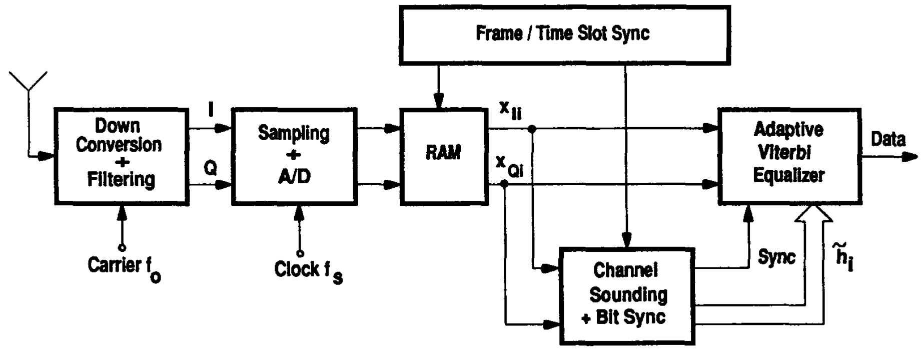


Fig. 2: Narrowband-TDMA DMR receiver with channel sounding and adaptive Viterbi equalization

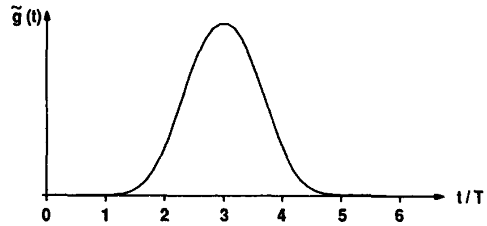


Fig. 3: Linear QAM pulse shaping function $\tilde{g}(t)$ for GMSK with BT = 0.3

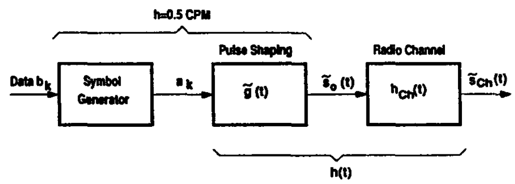


Fig. 4: Joint linear model für h = 0.5 CPM and radio channel

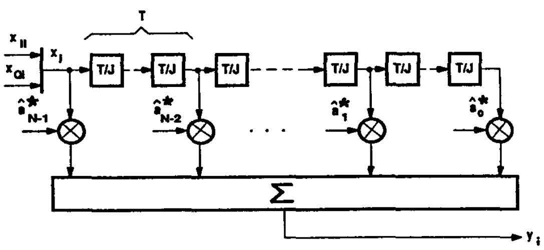


Fig. 5: Matched filter correlator for channel sounding

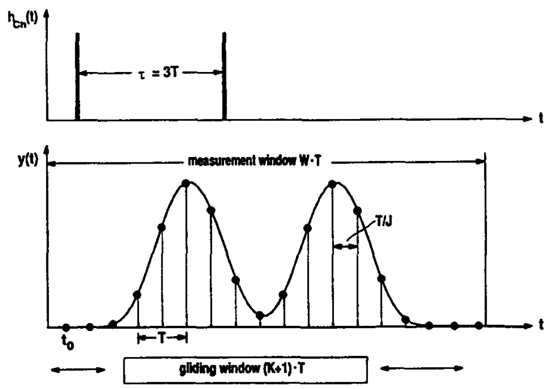
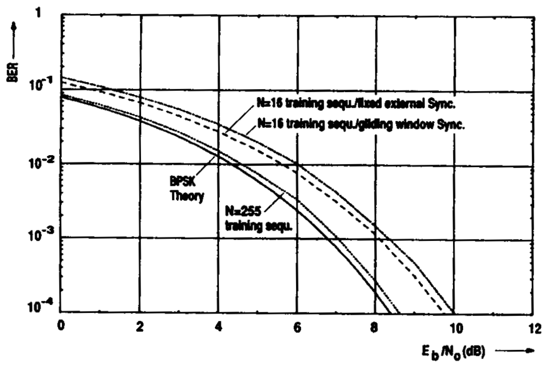
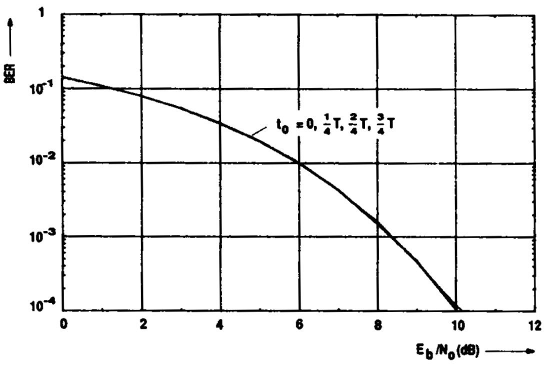


Fig. 6: Result y(t) of channel sounding for a particular 2-ray channel impulse response $h_{Ch}(t)$ and GMSK with BT = 0.3

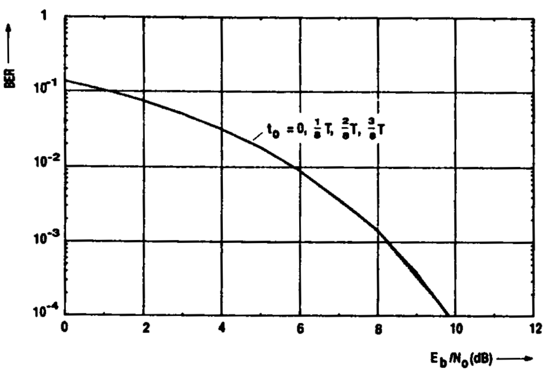


<u>Fig. 7:</u> Performance of a channel-sounding 16-state Viterbi equalizer in terms of BER versus E_b/N_0 for BT = 0.3 GMSK and a non-fading AWGN channel

a)



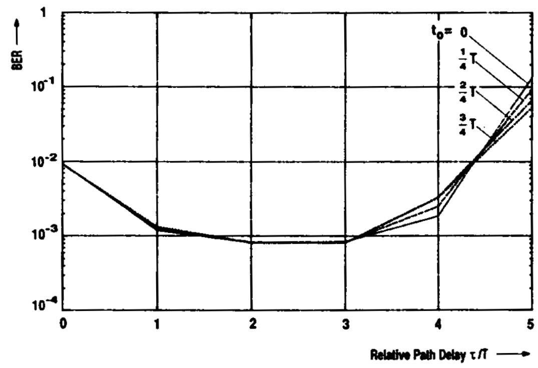
b)



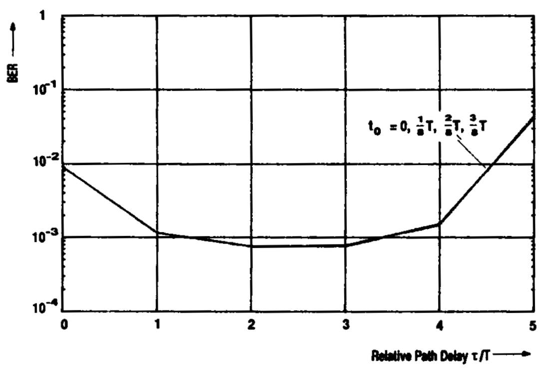
<u>Fig. 8:</u> Timing sensitivity of a channel-sounding 16-state Viterbi equalizer for BT = 0.3 GMSK and a non-fading AWGN channel: BER versus E_b/N_0 for different timing phases t_0

a) $f_s = R$ b) $f_s = 2R$

a)



b)



<u>Fig. 9:</u> Timing sensitivity of a channel-sounding 16-state Viterbi equalizer for BT = 0.3 GMSK and a 2-ray Rayleigh fading channel with AWGN: BER versus relative path delay $\tau/T=1,2,\ldots,5$ for different timing phases t_0 (average E_b/N_0 = 18 dB)

a) $f_s = R$ b) $f_s = 2R$