PERFORMANCE OF VITERBI EQUALISERS FOR THE GSM SYSTEM

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INTRODUCTION

The forthcoming Pan-European Digital Cellular Radio System (usually known as the *GSM system*) will employ TDMA with eight channels per carrier, at an aggregate bit rate of about 270 kb/s. Such a bit rate requires a channel coherence bandwidth which is often not available in the mobile radio environment. In particular, echo delay spreads in urban areas can span 6-8 μs, and up to 16 μs and above in mountainous terrain. For the GSM bit period of about 3.7 μs, such dispersion will result in significant intersymbol interference, which may only be combatted by using *equalisers*.

In addition, the channel characteristics are liable to change very quickly, particularly when the handset or its surroundings are subject to fast displacements (e.g. at 60 km/h a vehicle will go through one wavelength in about 20 ms). In the GSM TDMA format, slots for each of the eight channels are separated by about 4.6 ms, and hence channel identification is required in each TDMA slot in order to correctly operate the equaliser. The channel characteristics may of course change during each TDMA slot, so that it is desirable to have a slot duration as short as possible. However, the fixed overhead required for channel identification purposes will then become a significant part of the slot, and a suitable compromise has been chosen for the GSM system. Each slot lasts for 557 μs, and the slot structure is shown in Fig.1. A 26 bit training/synchronisation sequence is located in the middle of the slot, and 58+58 bits are available for information transmission. Note in particular that the positioning of this sequence in the middle of the slot leads to a total delay of only about 250 μs between channel estimation and the last information bit on either side, thereby reducing the changes in channel characteristics for a given vehicle speed. Adaptive equalisation techniques may still be required at high speeds when the changes in each burst become significant.

The purpose of this paper is to present performance figures validating the choice of slot parameters and channel estimation procedures. In doing so, comparisons will also be made between two types of equaliser, both of which are based on the Viterbi Algorithm (VA), performing maximum-likelihood sequence estimation of the incoming data sequence. Computer simulation has been used to characterise the performance of both structures under a range of static and dynamic channel conditions, and typical results are presented for these.

RECEIVER STRUCTURE

The modulation used is Gaussian Minimum Shift Keying (GMSK) with a low-pass filter BT product of 0.3, and the slot structure is as shown in Fig.1, although for simplicity only the right-hand side information block has been examined. A possible receiver structure is shown in Fig.3 (not including the front end). The GMSK signal is treated as a form of offset QAM, with coherent quadrature detection resulting in I and Q baseband signals. These are then digitised (the sampling frequency being at least equal to the system bit rate) and subsequently filtered by a Butterworth 5th order phase equalised low-pass filter with variable bandwidth. During reception of the training sequence, the output samples are processed by the channel estimator, which provides information for synchronisation (i.e. decimation if required and time alignment) and for operation of any further processing (matched filter and Viterbi algorithm).

CHANNEL ESTIMATION PROCEDURE

The training sequence used is one of several specified by GSM [1]. The estimation procedure involves essentially the convolution of the received signal with the stored central 16 bits of the sequence. Let the slot I and Q data be represented by a_k and b_k respectively, with the preamble data being a_1 to a_{13} and b_1 to b_{13}. Then the received signal is, using complex notation:

$$s(t) = \sum_{k=-N}^{M} a_k\, h(t-2kT) + j \sum_{k=-N}^{M} b_k\, h(t-2kT-T) \quad (1)$$

where the data rate is 1/T and N and M are set by the length of the slot. The shaping function h(t) is generally complex, and can be considered to represent the combined effect of all dispersive effects in the signal path (e.g., modulation, channel frequency selectivity and both transmitter and receiver filters).

The received signal is then convoluted with the central 16 bits of the training sequence and it can easily be shown that the result is:

$$y(t) = h(t) \otimes R(t) \quad (2)$$

where $\otimes$ denotes convolution and R(t) approximates the circular autocorrelation function of the central 16 bit sequence. It is found in fact that R(t) = 0 for $t = \pm T$, $\pm 2T$, $\pm 3T$ and $\pm 4T$, due essentially to the padding provided by the extra 10 bits. Outside the interval

-4T < t < +4T, the function R(t) becomes unpredictable since it is dependent on the information bits. However, its properties inside the interval ensure that there is a sufficiently wide window for which y(t) essentially identifies with the shaping function h(t).

It should be noted that this window closes as h(t) becomes longer, and that if the width of h(t) is greater than about 4T, then the channel estimate will be distorted irrespective of other system parameters. Further, the estimate will also be significantly corrupted by noise, since the training sequence is relatively short. Fig.2 shows examples of channel estimates obtained with this method for a two ray channel with both rays of equal amplitude and exactly opposite phase, for two delay values (5 and 10 μs). Note in particular that the actual points found would correspond to samples of the curves shown, and that the overall shape of each 'ray' is determined by the GMSK modulation and any filters, and is therefore fairly wide (up to 2T width). The distortion at the edge of the estimate can also be clearly seen.

VITERBI BASED EQUALISATION ALGORITHMS

Two different algorithms have been used in the work described here, which will be referred to as VE1 and VE2. Each has in fact been used with a different synchronisation technique, which will also be described below.

Algorithm VE1 corresponds to the well known technique proposed in [2], with some tailoring as required for the specific modulation used, and its overall structure is shown in Fig.3. This algorithm essentially attempts to minimise the function

$$\int_{-\infty}^{+\infty} |s(t) - s_m(t)|^2 \, dt \qquad (3)$$

where $s_m(t)$ is the reconstructed complex baseband signal for the *mth* possible transmitted sequence. It can be shown [2] that the above function can be simplified and also calculated recursively. Further, applying the resulting likelihood function to the case of offset binary quadrature modulation, it is found that the most likely sequence is that which maximises:

$$\Lambda_{mn} = \Lambda_{m(n-1)} + \mathrm{Re}\left[(\alpha_n^m)^* \, [y_n - \sum_{k=1}^{L} \alpha_{n-k}^m x_k]\right] \qquad (4)$$

where α_n^m is the *nth* data sample of the *mth* sequence, taking values ± 1 or $\pm j$, y_n is a sample taken at time nT at the output of a filter matched to the overall impulse response h(t) when its input is the received signal s(t), and x_k is a sample of the same filter output when its input is h(t).

The above metric is the basis of the modified VA depicted in Fig.3. Also shown in Fig.3 is the inherent matched filter. The VA will have 2^L states, for an effective h(t) width of LT, and in what follows L will be taken to be 4 (i.e., 16 states, with a width of just under 15 μs). For this algorithm, the channel estimation procedure involves finding the sequence of 5 complex channel samples (taken every T) with maximum energy, by sliding a window through the estimate. If the sampling rate is a multiple of 1/T, this also involves searching through all possible offset points (i.e., the window increment will be T/n, where n is the oversampling factor).

The second algorithm considered (VE2) differs significantly from the above in that the function to be minimised is :

$$\sum_{k=-\infty}^{+\infty} [p(kT) - p_m(kT)]^2 \qquad (5)$$

where p(kT) is the real part of s(kT) when k is even, and its imaginary part when k is odd (and similarly for $p_m(kT)$). This means that the I and Q signals are sampled alternately, each at rate 1/2T (the data rate for each quadrature signal). Otherwise eqn.5 represents a standard VA metric [3], but without any matched filtering before the equaliser. The structure is shown in Fig.4, and it may be seen that it is effectively left to the fixed low-pass filter to approximate the action of the MF. Such a procedure is obviously sub-optimal, but it may be used to reduce the processing involved in the equaliser. Whether such a reduction is significant depends on factors such as the exact formulation of each algorithm (and the number of VA states) and the processor architecture.

The channel estimation procedure is similar to that followed for VE1 - however from eqn.5 only $L+1$ alternate real and imaginary samples of h(t) are required for signal reconstruction and metric computation. Therefore alternate real/imaginary parts are disregarded when choosing the optimum window position. For correct operation, the receiver must also be phase synchronised to the largest sample of $|h(t)|$ *before* the window is chosen.

SIMULATION RESULTS : STATIC CHANNELS

A simple two-path channel model has been used to test the performance of the algorithms. The equivalent low-pass transfer function is :

$$H(f) = 1 - b \exp[-j2\pi(f - f_o)\tau] \qquad (6)$$

where f_o is the frequency offset of the notch from the carrier frequency, τ is the relative delay between the two paths, and b is the amplitude of the second path normalised to that of the first. In all tests below, $f_o = 0$ and $b = 1$, so that the channel has a notch of infinite depth at the carrier frequency. The sensitivity of the two algorithms to the bandwidth of the low-pass filter has been examined, and is shown in Figs.5 and 6.

In the case of VE1, the curves for the various delay values are similar, with optimum values about $BT = 0.4$ and in fact a value of 0.45 was fixed (corresponding to a

3 dB bandwidth of about 120 kHz). This value enables a reduction of the noise power before the matched filter while not affecting its operation. The results for VE2 however show a slight increase in the bit error rate as well as a more irregular dependence on the BT product. For most of the cases shown, a very narrow bandwidth seems to be desirable, and in fact the BT product was set to 0.25 (approximately 67.5 kHz) for this receiver. As shown in Fig.6, this choice causes degradation when τ is small since H(f) is then a high pass function within the frequency range of the signal.

Complete bit error rate (BER) curves have also been obtained for the same channel types with VE1 and VE2. It has been found that in an ideal channel performance degrades by just over 0.5 dB from that of coherent GMSK, due to the noisy channel estimate and the assumption of linearity for GMSK. A further 1-2 dB is lost when the channel becomes frequency selective (from mis-match and distorted estimates), and breakdown occurs when τ increases beyond 4T. An unequalised receiver would however exhibit irreducible values of BER for any τ.

RAYLEIGH FADING TWO - PATH CHANNELS

A more realistic indication of performance in the mobile radio environment is obtained when the two paths undergo independent Rayleigh fading. If the 'vehicle' speed is low enough (i.e., $v \rightarrow 0 km/h$), such fading will be unnoticed during each TDMA burst, but the signal level and channel characteristics will be uncorrelated from burst to burst. This *burst-stationary fading* gives a good indication of the equaliser performance for handsets or slow moving vehicles.

A set of BER curves is shown in Fig.7 using algorithm VE1, for different values of τ and where the two paths have the same average power. These curves show that the equaliser performance under these conditions is almost independent of the value of τ, until this becomes excessive for the 16-state VA (i.e., over 15 μs). Another interesting result is the fact that the receiver gives better performance when the channel is frequency selective, since it is unlikely that both paths will fade together. These conclusions are confirmed by Fig.8 which depicts the estimated BER for an average Eb/No of 14 dB for different values of τ and for both VE1 and VE2. Both receivers show a performance superior to flat fading until τ exceeds 15 μs. It is also worth noting that VE1 is superior under all conditions, but the difference in performance is not dramatic.

As the vehicle speed increases, the channel can no longer be considered burst-stationary. Thus the initial channel estimation no longer represents the real conditions, and errors will grow as the end of each information block is approached. As a consequence, the receiver will eventually register errors even under high signal strength conditions. This effect is demonstrated in Fig.9 using algorithm VE2 and a two-path channel with $\tau = 10\mu s$. The initial channel conditions are still independent from burst to burst, but the channel is now allowed to vary with the classical 'bath-tub' power spectral density. The curves shown indicate some performance degradation even for relatively low velocities (e.g. 2 dB loss at 10^{-3} for v=100 km/h), but it is particularly significant that large levels of irreducible BER's are observed for higher speeds. More comprehensive tests have shown that for a two-path model, the irreducible BER can be kept under 10^{-3} for all values of τ (including the flat fading case) provided the vehicle speed remains below 100 km/h. Higher speed values (common for instance in high-speed trains) lead to degradation, and would require a continuously adaptive equaliser to achieve acceptable operation.

COMPLEX CHANNEL MODELS

A two-path model can provide an excellent indication of equaliser performance, and constitutes a rigorous test when the two paths have equal magnitudes (or equal average power). However, it is interesting to test the equaliser using more complex channel models, particularly if they represent 'typical' channel impulse response profiles for specific geographical areas. A number of these have in fact been proposed by COST-207 [1, annex 3] for receiver testing, and some results are here shown (Fig.10) for illustration. These were obtained with algorithm VE1, but very similar curves have also been obtained using VE2. In the simulation, all of these channel models have been represented by taking a 12-path model (with independent Rayleigh fading imposed on each path) with average powers for each path set by sampling the continuous channel impulse response.

The worst performance is that for the 'hilly terrain' model. This may be described as a smeared two-path model with a very long delay (over 16 μs), and hence the equaliser may not fully compensate for all the resulting intersymbol interference. An improved performance is obtained for the 'rural area' model, which is similar to a single-path, flat fading channel, but with a little dispersion. The resulting BER curve is in fact almost identical to that of the single-path case in Fig.7.

Interestingly, an even better performance is achieved with the 'typical urban' model whose channel impulse response is distributed over a width of 2T. This result is again an instance of the 'diversity' effect already referred to above, which the equaliser can make use of as long as the dispersion is not excessive. Finally, the last case is that of a specially designed equaliser 'test' model, which has six equally spaced signal paths with the same average power. Such a model should have a high 'diversity' gain, and in fact it exhibits the lowest BER at low values of Eb/No. However, the relative delay between the first and the sixth path is equal to 16 μs, and thus some of the signal dispersion may not be equalised. This effect becomes noticeable at high Eb/No, as the signal elements outside the equaliser window effectively function as co-channel interference to the receiver, and the curve inflects suggesting the onset of an irreducible BER. This model, although artificial, gives information not only about the equalisation capability (at low Eb/No), but also on the width of the channel impulse response that the equaliser can cope with (at high Eb/No).

CONCLUSIONS

Two equaliser structures based on the VA algorithm have been presented and shown to provide good performance in the mobile radio environment, within the GSM system framework. Both can operate adequately in environments with delay spreads up to 14-16 μs. Further, they can cope with vehicle speeds up to 100 km/h without undue performance degradation.

The differences in performance between the two algorithms presented (VE1 and VE2) have been found to be relatively small under most conditions tested. Since VE2 is a simple sub-optimal algorithm without a matched filter, it could be used to provide some complexity reduction in practical implementations.

Finally, the equaliser performance in a Rayleigh fading multipath channel has been found to depend mainly on two factors - the energy distribution inside the equaliser window, and the fraction of energy outside that window. This suggests that, for this application, equalisers may be characterised using relatively simple tests. Results obtained with more complex channel models are as expected from the simple two-path testing and therefore support this conclusion.

ACKNOWLEDGEMENTS

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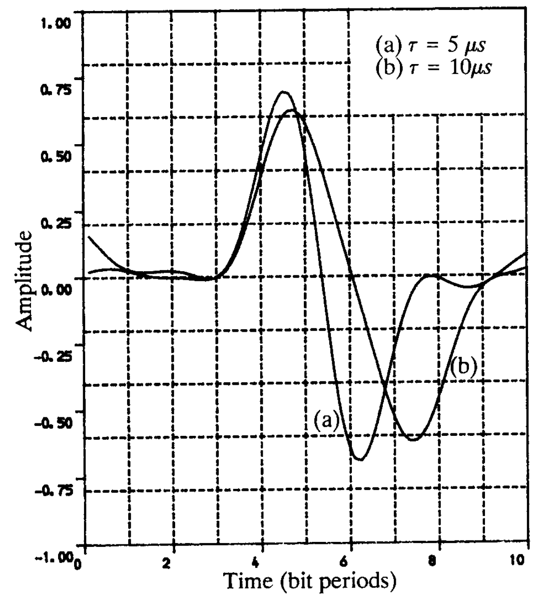

Figure 2.Real Part of Channel Estimates

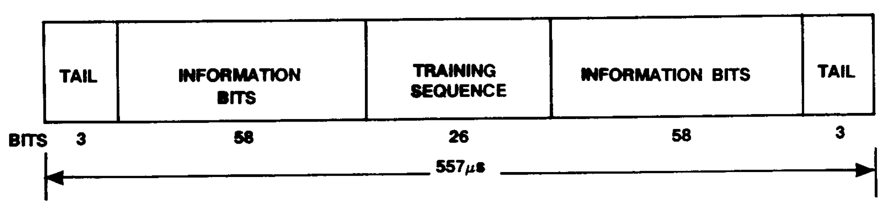

Figure 1.Slot Structure for the GSM Burst

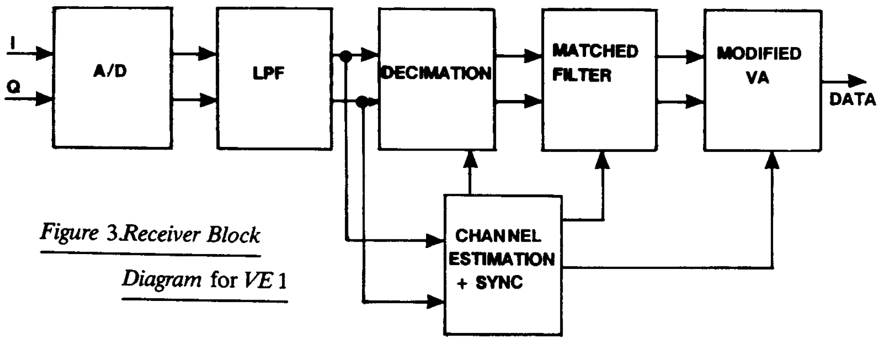

Figure 3.*Receiver Block Diagram* for *VE* 1

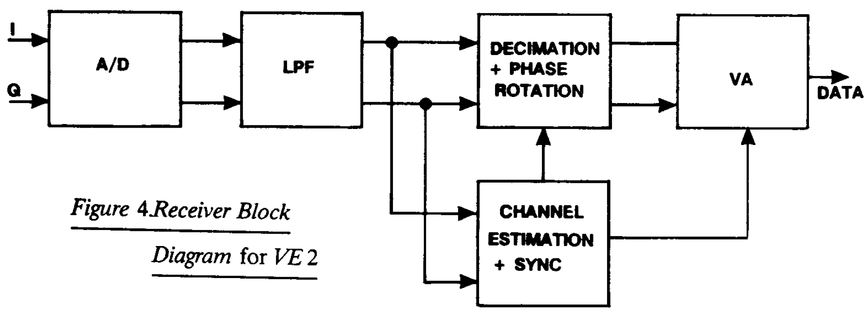

Figure 4.*Receiver Block Diagram* for *VE* 2

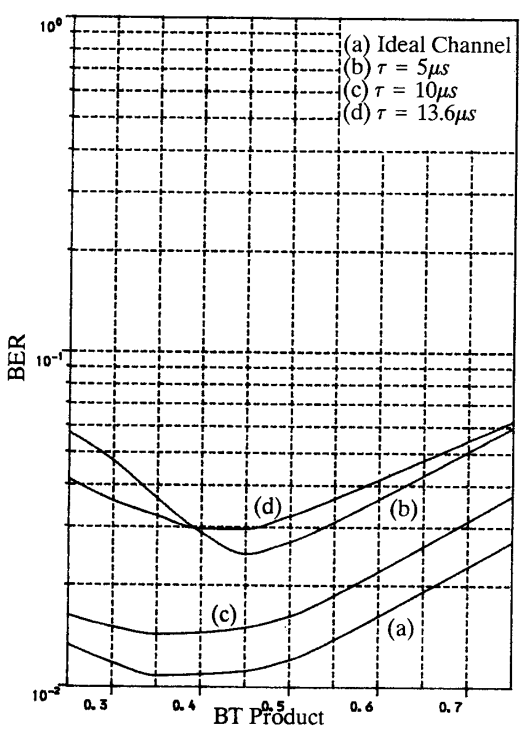

Figure 5.*Effect of Low −pass Filter Bandwidth* (*VE* 1)

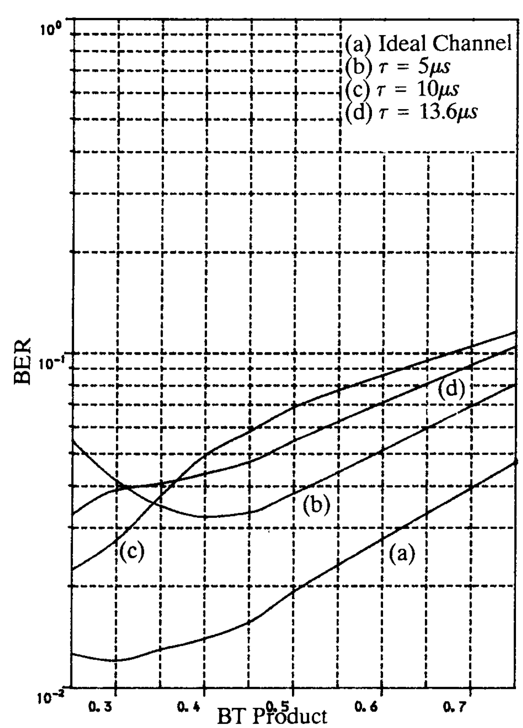

Figure 6.*Effect of Low −pass Filter Bandwidth* (*VE* 2)

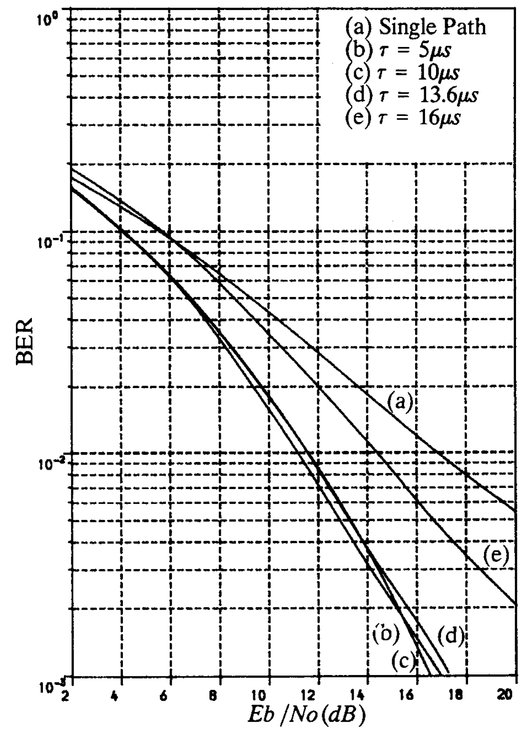

Figure 7.*BER* for *Burst Static Fading* (*VE* 1)

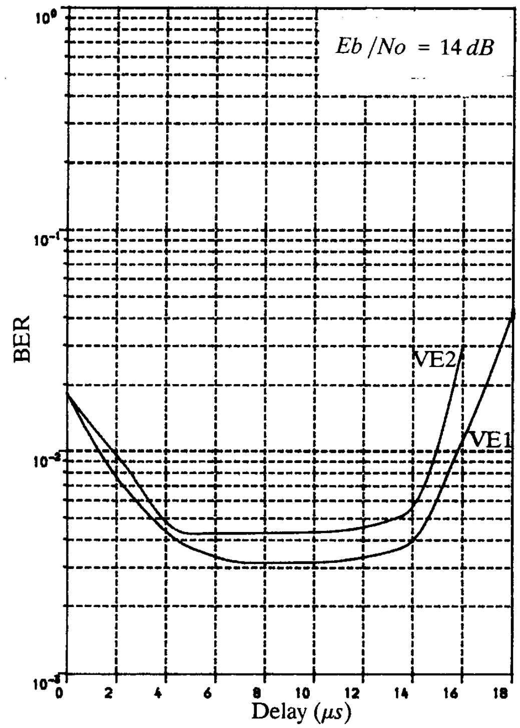

Figure 8.*Variation of BER with Second Path Delay* for *VE* 1 and *VE*

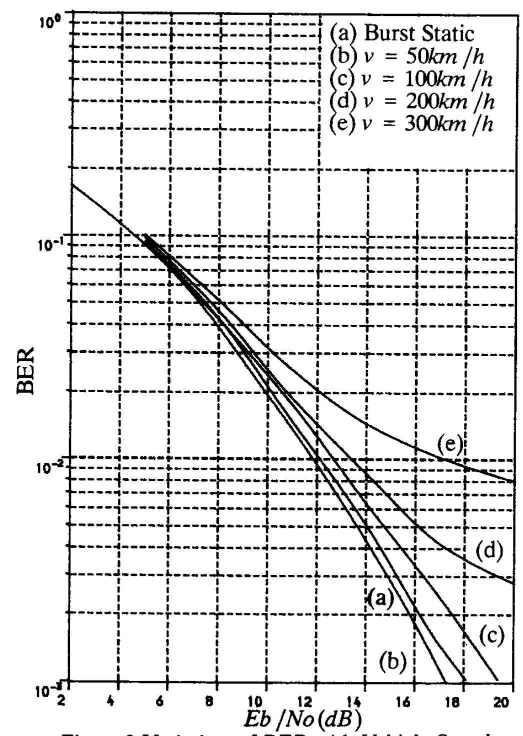

Figure 9.*Variation of BER with Vehicle Speed* for *VE* 2, *Second Path Delay* = 10μs

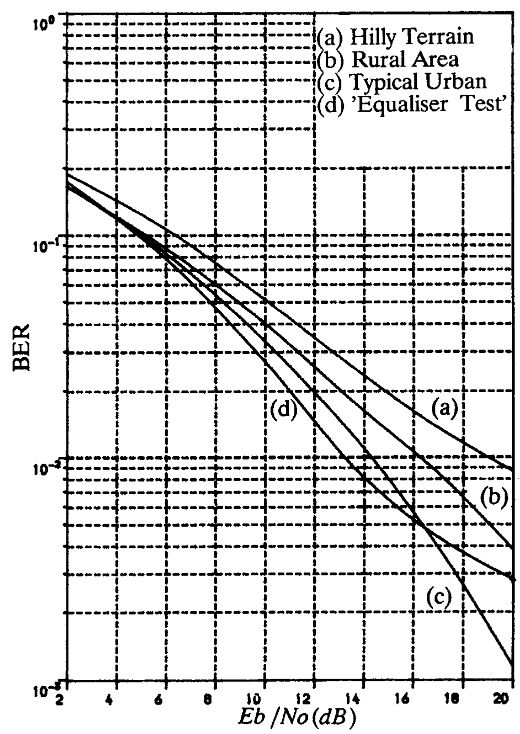

Figure 10.*BER* for *Various Channel Models* (*VE* 1)