# An Iterative Interference Canceller for Serially Concatenated Continuous Phase Modulation

Michael Anderson†, Mark C. Reed\*, Gerard Borg†
† Research School of Physical Sciences and Engineering
Australian National University
Canberra, ACT 0200, Australia
Email:{michael.anderson,gerard.borg}@anu.edu.au
\* National ICT Australia, The Australian National University
Canberra, ACT 0200, Australia
Email: mark.reed@nicta.com.au

Abstract—It is well known that continuous phase modulation (CPM) serially concatenated with convolutional codes facilitates powerful error correction. CPM also has the advantage of being bandwidth efficient and compatible with non-linear amplifiers. The downfall of CPM systems is their poor performance over ISI channels. We propose an iterative receiver incorporating an interference canceller (IC) for serially concatenated continuous phase modulation (SCCPM) over known ISI channels. We present results for our scheme based on a common ISI channel and show that the performance is comparable to that of Serially Concatenated CPM (SCCPM) on an AWGN channel with no ISI.

#### I. INTRODUCTION

Continuous phase modulation (CPM) is a commonly used modulation scheme in both commercial and military wireless systems due to its bandwidth efficiency and constant envelope. When transmitted over frequency-selective channels, CPM signals will be subject to inter-symbol-interference (ISI), which degrades the performance of the system. The optimum receiver for CPM signals transmitted over channels with memory is a matched filter bank to produce sufficient statistics, followed by MLSE or MAP (maximum a-posteriori) decoding. The number of states required for encoding in such receivers is exponential with the sum of the inherent CPM memory, and the memory of the channel. An alternate iterative approach to optimal equalization is to design a receiver filter under the minimum mean square error (MMSE) criteria [1]. This method can also be computationally expensive because it involves inverting large matrices.

Serially concatenated continuous phase modulation (SC-CPM) was introduced in [2] as a way of combining the memory of convolutional codes with the memory of CPM to provide excellent error correction capabilities. This is possible because if the modulation index of CPM is rational and irreducible, it can be described by a trellis using the titled phase representation [3][4]. The system can then be treated as a serially concatenated convolutional code (SCCC), and the iterative decoding techniques of [5] can be used. We propose a method of turbo equalization for SCCPM based on the interference canceller model in [6]. The motivation for this, is to preserve the excellent performance of SCCPM with a scheme that has lower

complexity than the optimal and MMSE methods. The fundamental idea behind interference cancellation is to use data estimates from the turbo decoder to cancel ISI provided that some information about the channel is available at the receiver.

Several methods for equalizing CPM have been proposed recently. In [7] frequency domain equalization is applied to CPM under severe ISI using the Gram-Schmidt procedure to obtain a discrete signal model for the IFFT/FFT processing. In [8] equalized energy detection of CPM is examined, where the equalization is combined with the design of the receiver filters under the minimum mean squared error (MMSE) criteria. Neither of these papers treat coded CPM. Results for equalizing coded MSK, again using a filter designed under the MMSE criteria, are presented in [9].

This remainder of this paper is arranged as follows. Section II describes the system model, including the CPM model, channel model and the SCCPM interference canceller. Section III presents simulation results for a given ISI channel with full and partial-response SCCPM schemes, and compares against SCCPM with no ISI. Finally, Section IV concludes the paper.

## II. SYSTEM MODEL

### A. CPM model

If the CPM modulation index, h=K/P, is rational and irreducible, then the system can be represented by a trellis. Furthermore, Rimoldi derived a method for describing CPM modulations as the concatenation of a continuous-phase encoder (CPE) and a memoryless modulator (MM) [4]. The complex envelope of a CPM signal with symbol interval T and energy E is given by

$$s(\tau + nT) = \sqrt{\frac{2E}{T}} e^{j\overline{\psi}(\tau + nT)} \tag{1}$$

where  $\overline{\psi}$  is the titled-phase and is given by

$$\overline{\psi}(\tau + nT) = \left[2\pi h \left[\sum_{i=0}^{n-L} u_i \text{mod} P\right] + 4\pi h \sum_{i=0}^{L-1} u_{n-i} q(\tau + iT) + W(\tau)\right] \text{mod} 2\pi \qquad 0 \le \tau < T$$
(2)

where  $W(\tau)$  is independent of the information symbol, and given by

$$W(\tau) = \pi h(M-1)\frac{\tau}{T} - 2\pi h(M-1)\sum_{i=0}^{L-1} q(\tau+iT) + \pi h(M-1)(L-1)$$
(3)

In (2),  $u_i$  is from an M-ary information set,  $u_i \in \{0, 1, \ldots, M-1\}$ , and q(t) is the integral of a normalized frequency pulse that is non-zero for L symbol intervals. The LREC family of pulses is defined by

$$q(t) = \begin{cases} 0 & t < 0\\ \frac{t}{2LT} & 0 \le t \le LT\\ 1/2 & t > LT \end{cases} \tag{4}$$

while the LRC family is defined as

$$q(t) = \begin{cases} 0 & t < 0\\ \frac{t}{2LT} - \frac{(\sin\frac{2\pi t}{LT})}{4\pi} & 0 \le t \le LT\\ 1/2 & t > LT \end{cases}$$
 (5)

The function q(t) is always normalized to 1/2 for t>LT. When L=1 the system is a full-response and when it L>1 it is partial-response.

# B. Channel model

The channel model we use in this work is the discrete symbol-spaced tapped-delay-line (TDL). We assume herein that the channel is known and non-time-varying. The  $k^{\rm th}$  sample of the received over-sampled baseband signal can then be represented by

$$y_k = \sum_{j=0}^{N-1} s_{k-j} h_j + z_k \tag{6}$$

where N is the length of the channel in terms of samples,  $s_{k-j}$  are the CPM complex baseband samples,  $h_j$  are the discrete channel coefficients, and  $z_k$  is a zero-mean complex Gaussian noise sample with double sided power spectral density  $N_0/2$ . Since we assume a symbol-spaced channel, most values of  $h_j$  are zero. The channel considered in this paper is the well known Proakis channel (a) from [10]. The symbol-spaced coefficients are  $\{0.04, -0.05, 0.07, -0.21, -0.5, 0.72, 0.36, 0, 0.21, 0.03, 0.07\}$ .

The proposed receiver is shown in Fig. 1. It can be seen that the encoder is a standard SCCPM encoder. Since we are assuming the channel is known at the receiver, the filter  $h^*(-t)$  is matched perfectly. The SCCPM decoder operates as the

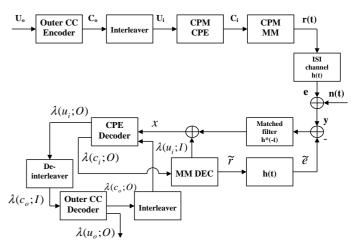


Fig. 1. Proposed system model.

SCCC described in [5], such that  $\lambda(u;)$  and  $\lambda(c;)$  are the loglikelihood ratios (LLRs) of the information and coded symbols respectively. The information  $\lambda(u_i;O)$  and  $\lambda(c_o;O)$  is extrinsic information which becomes a-priori information for the next decoder. On the final iteration, a hard decision is made on  $\lambda(u_o;O)$ .

In [5] the output from the inner decoder,  $\lambda(c_i;O)$ , is not used. We propose using this output as a data estimate to cancel ISI. The information  $\lambda(c_i;O)$  consists of  $PM^L-1$  symbol LLRs, given that the codeword LLRs are defined as

$$\lambda(c_q;) = \log \frac{P(c_q;)}{P(c_0;)} \tag{7}$$

where  $c_0$  is an arbitrary reference symbol. From these LLRs we obtain the normalised set of probabilities  $P(c_n)$ . The decoder now also consists of a memoryless modulator which we label "MM DEC". We refer to it as this because it is designed to mimic the function of the memoryless modulator in the encoder. Each symbol interval it takes the set of  $Q = PM^L$  normalised probabilities and produces the discrete output defined by

$$\underline{\tilde{r}} = \sum_{q=1}^{Q} P(c_q;) \underline{r_q}$$
(8)

where  $\underline{r_q}$  is the oversampled qth complex baseband CPM signal. It can be seen that by using the soft outputs of the CPM decoder, an "average" symbol is obtained. We use the underline notation to specify a discrete signal. Once we have average symbols for an entire block length, we can obtain an estimate of the signal corrupted by ISI as  $\underline{\tilde{e}} = \underline{\tilde{r}} \otimes \underline{h}$ , where  $\otimes$  is the convolution operator. This signal, which is initialised to zero on the first iteration, is then subtracted from the received signal, the result is matched filtered, and  $\underline{\tilde{r}}$  is added back in. Although not explicitly shown in Fig. 1, a bank of matched filters precedes the CPE decoder to produce sufficient statistics for decoding. The input to the matched filter bank is given by

$$\underline{x} = \underline{\tilde{r}} + \underline{h} \otimes (y - \underline{\tilde{e}}) \tag{9}$$

from which a set of channel LLRs,  $\lambda(c_i; I)$ , is obtained.

Iterative interference cancellation is typically performed by taking a data estimate from the last of a set of concatenated decoders. In the case of CPM, this would require a hard decision to be made on  $\lambda(c_o;O)$  for re-encoding by a CPE encoder and a memoryless modulator in the decoder. This hard decision would result the loss of valuable information. The fact that a CPE would also be required in the decoder would increase the complexity of the system. The consequence of taking a data estimate from the inner decoder in this case, is that the decoder will be half an iteration behind a typical interference canceller. It should also be noted that the feedback term  $\lambda(c_o;O)$  is the full a-posteriori information as opposed to extrinsic information. It was found that using the full information produced better results. This agrees with the concepts presented in [11].

## III. RESULTS

## A. Full-response system

An MSK system with parameters M=2, h=1/2 and L=1 was first considered. The REC pulse shape given in (4) was used. The outer code was a G[5,7] rate 1/2 non-recursive convolutional code. The log-MAP algorithm was used in both the CPE and outer CC decoder, with both trellises being fully terminated. The interleavers were psuedo-random. The block size used was 256 which corresponds to an interleaver size of 512. Simulation results for this CPM scheme over Proakis channel (a) are shown in Fig. 2.

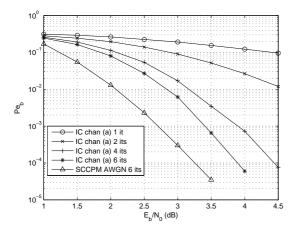


Fig. 2. Performance of the IC receiver for MSK over iterations.

In Fig. 2, IC stands for the interference cancelling receiver in Fig. 1. The SCCPM result represents the same MSK scheme with no ISI. A clear improvement in performance can be seen over iterations of the IC. It was found that after more than six iterations there was only a very slight improvement in performance. At a BER of  $10^{-4}$  the performance of the IC is approximately only  $0.6~{\rm dB}$  worse than the result with no ISI.

The complexity of this receiver is highly dependent on the number of matched filters required for the particular CPM scheme. This is because the filtering must be done on each iteration of the receiver compared to SCCPM where  $\lambda(c_i;I)$  would be calculated once and then stored. This must therefore be considered when evaluating the performance of the receiver

for a given scheme. The optimum number of matched filters is  $M^L$  since each set of P signals differs only by a phase rotation. This means that only 2 matched filters are required for this full-response scheme. A significant amount of work has been done on deriving suboptimal sets of matched filters [2] [3]. In [2] such a set is derived so that, for example, the number of matched filters required for an octal 3LRC h=1/2 scheme is reduced from 512 to 6. This ensures that the complexity of the IC receiver can remain manageable by sacrificing performance, even for high order CPM schemes.

#### B. Partial-response system

The partial-response system considered had the parameters  $M=2,\,h=2/3,\,L=2$  and uses the LRC pulse shape. Simulation results for this system over channel (a) are shown in Fig. 3. Again the result with no ISI is shown for comparison pur-

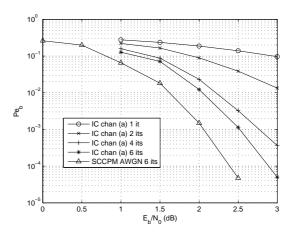


Fig. 3. Performance of the IC receiver for M=2, h=2/3 2LRC SCCPM over iterations.

poses. It can be seen that the overall performance of this CPM system is better than the full-response scheme. In this case, at a BER of  $10^{-4}$ , the performance of the IC is approximately 0.5 dB worse than the result with no ISI. The optimal number of matched filters for this CPM scheme is 4.

# IV. CONCLUSION

An iterative interference canceller has been proposed for SC-CPM systems. It uses the LLRs of the CPE codes to obtain an average CPM signal that is then used to cancel ISI. Simulation results have been presented for a full-response and partial-response system for a known ISI channel. The performance of the IC was similar for both schemes, and in both cases was found to approach AWGN performance over iterations. This means that the proposed receiver is a good alternative to higher complexity equalization methods for some channels.

#### ACKNOWLEDGEMENTS

M.C. Reed is with National ICT Australia and affiliated with the Australian National University. National ICT Australia is funded through the Australian Government's Backing Australia's Ability initiative and in part through the Australian Research Council.

## REFERENCES

- M. Tüchler, A. Singer, and R. Koetter, "Minimum mean squared error equalization using a priori information," *IEEE. Trans. on Signal Process*ing, vol. 50, pp. 673–683, March 2002.
- [2] P. Moqvist, "Serially concatenated systems: An iterative decoding approach with application to continuous phase modulataion," Master's thesis, Chalmers University of Technology, (Göteborg, Sweden), 1999.
- [3] J. B. Anderson, T. Aulin, and C.-E. Sundberg, *Digital Phase Modulation*. Plenum Press, 1986.
- [4] B. E. Rimoldi, "A decomposition approach to CPM," *IEEE Trans. Inform. Theory*, vol. 34, pp. 260–270, March 1988.
- [5] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, "Serial concatenation of interleaved codes: Performance analysis, design, and iterative decoding," *IEEE Trans. Inform. Theory*, vol. 44, pp. 909–926, May 1998.
- [6] M. C. Reed, Iterative Receiver Techniques for Coded Multiple Access

- Communication Systems. PhD thesis, The University of South Australia, Australia, Oct. 1999.
- [7] J. Tan and G. L. Stüber, "Frequency-domain equalization for continuous phase modulation," in *Proc. IEEE 15th Int. Symp. on PIMRC*, pp. 2267– 2272, Sept. 2004.
- [8] X. Huang and Y. Li, "Simple noncoherent CPM receivers by PAM decomposition and MMSE equalization," in *Proc. IEEE 14th Int. Symp. on PIMRC*, pp. 707–711, Sept. 2003.
- Composition and MMSE equalization, in *Proc. IEEE 14th Int. Symp. on PIMRC*, pp. 707–711, Sept. 2003.
  [9] L. Feng and M. Soleymani, "A sub-optimal receiving scheme for SCCC with coded MSK over ISI channels," in *Canadian Conference on Electrical and Computer Engineering*, pp. 77–80, May 2003.
- 10] J. G. Proakis, Digital Communications. McGraw-Hill, 3rd ed., 1995.
- [11] F. Vogelbruch and S. Haar, "Improved soft ISI cancellation for turbo equalization using full soft output channel decoder's information," in *Proc. IEEE Globecom*, pp. 1736–1740, Dec. 2003.