Channels and Channel Models

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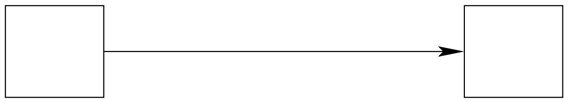
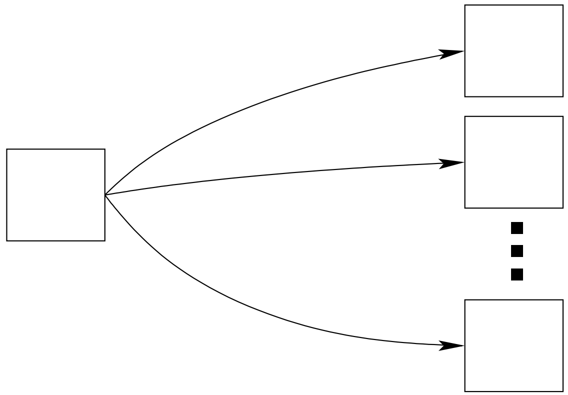
Channel types: single-user / multi-user

Depending on the *topology* of the channel, we distinguish single-user channels and multi-user channels:

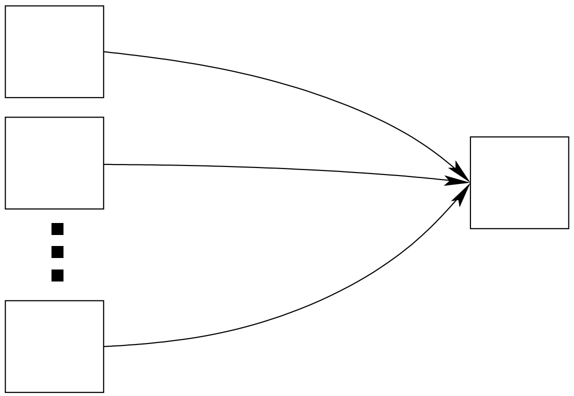
Single-user channel



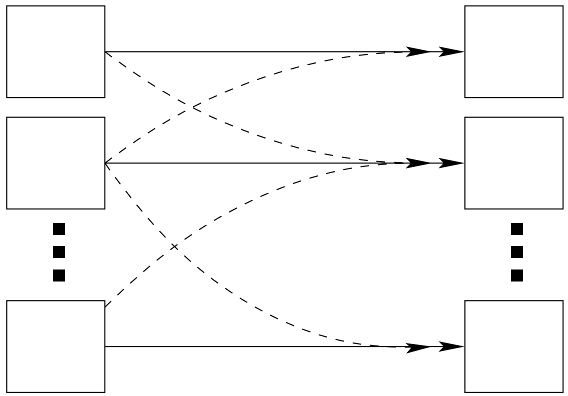
Broadcast channel



Multiple-access channel



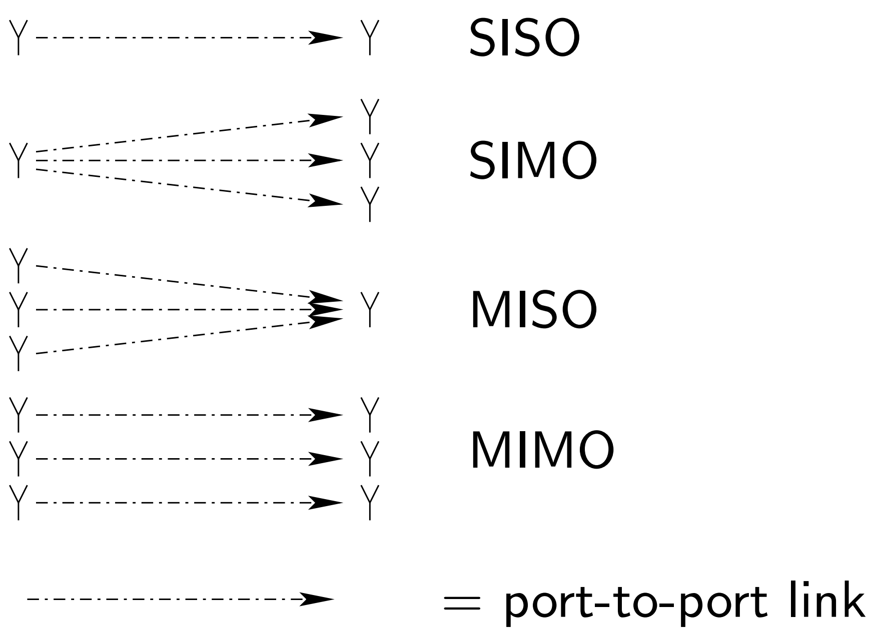
Interference channel



Channel types: SISO, MIMO, SIMO, MISO

Depending on the *number of ports* of a *user-to-user link*, we distinguish

- Single-input single-output (SISO) channel
- Single-input multi-output (SIMO) channel
- Multi-input single-output (MISO) channel
- Multi-input multi-output (MIMO) channel

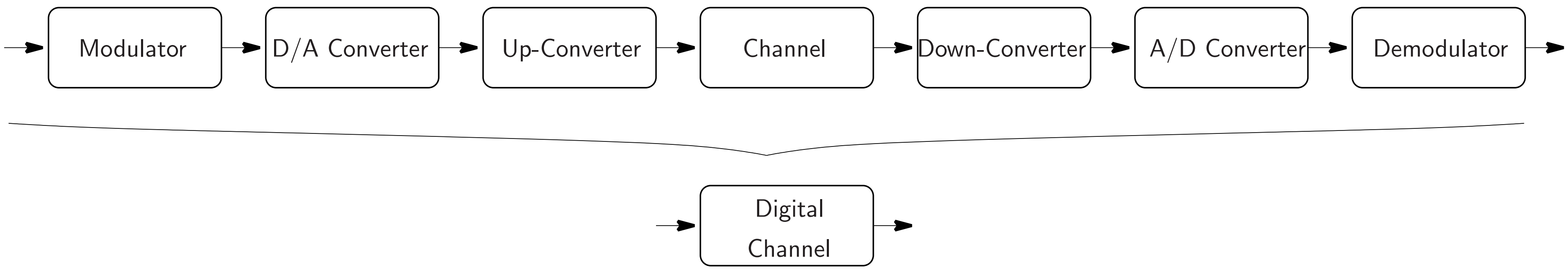


Examples:

- Multi-antenna systems
- Multi-pair cables

Channel types: information theoretic view

Sometimes, all the blocks like modulation, demodulation, up-conversion, (physical) channel, down-conversion, etc. are modelled as a single entity called *digital channel*:



- "digital" refers to the quantisation in amplitude (the set of output symbols is finite)
- digital channel is described by transition probabilities $p(y_k|x_l)$, i.e., the conditional probabilities that y_k is detected given that x_l was transmitted

Channel types: classification according to medium

Depending on the medium, we distinguish

- guided channels
  - *wire* (e.g.: copper twisted-pairs in the access network)
  - cable (e.g.: coax cables used in cable networks)
  - fibre (e.g.: optical fibres in backbone networks)
  - microwave guides (e.g.: feeder "pipes" for high-power RF transmitters, radar)
- unguided channels
  - *wireless channel*
  - underwater acoustic channel

Channel properties

The transmitted waveforms may experience effects like

- reflection
- absorption
- attenuation (scaling in amplitude)
- dispersion (spreading) in time
- refraction (bending due to variation of the media's refraction index)
- diffraction (scattered re-radiation, caused by an edge or an object whose size is in the order of the wave length)

Channel properties cont'd

The net effect of every channel can be described by

- modification of the signal
- addition of noise

sequence notation: $r(n) = h(n) * s(n) + w(n)$ matrix notation: $\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w}$

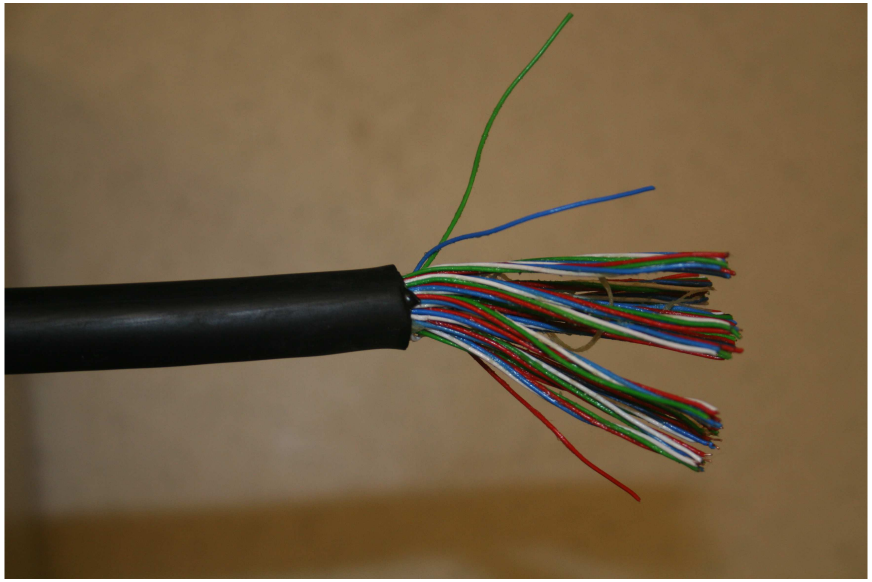
Depending on the channel properties, a channel can be

- linear / non-linear channels
- time-invariant / time-variant (fading) channels
- frequency-flat / frequency-selective (time-dispersive) channels

The additive noise can be

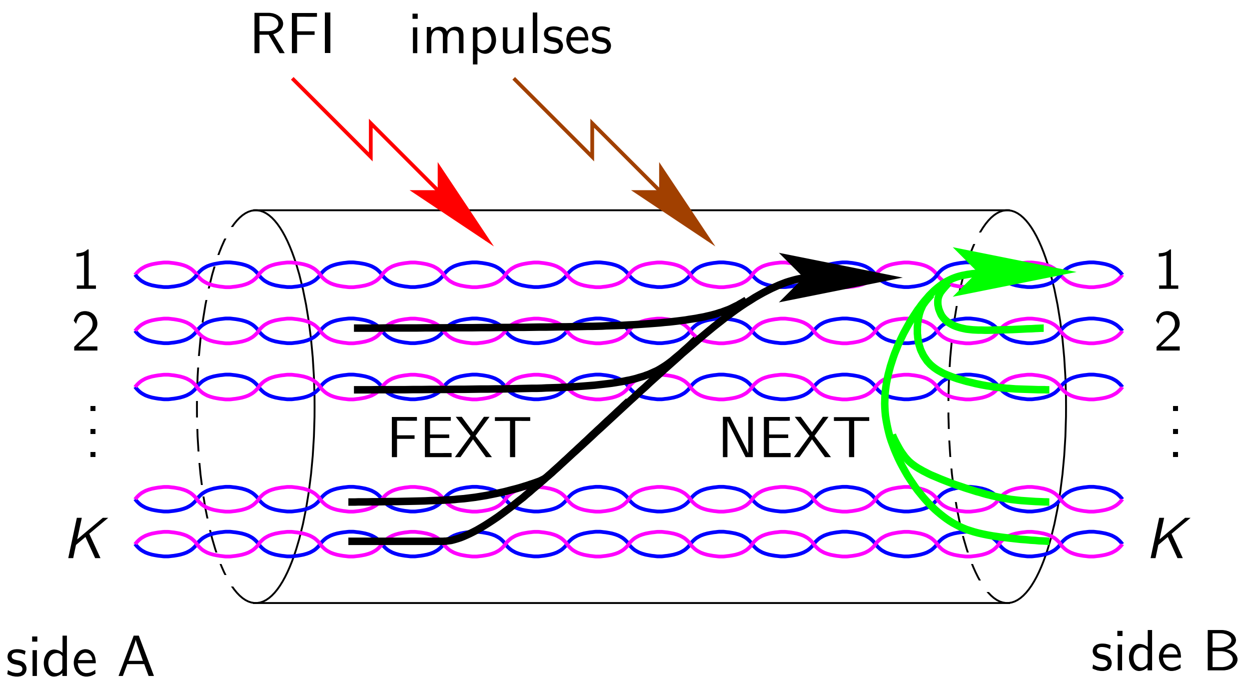
- Gaussian / non-Gaussian
- correlated in time/frequency, spatially (in MIMO system), over users (in multi-user systems)

Wireline channel: physical mechanisms/effects



- essentially time-invariant, frequency-selective attenuation, or equivalently, dispersion in time
- crosstalk: electromagnetic coupling among wire pairs (also called loops) in a cable
- extrinsic noise/interference (impulse noise, radio frequency interference)
- background noise (thermal noise, front-end noise)

Wireline channel: physical mechanisms/effects cont'd



- Far-end crosstalk (FEXT)
- Near-end crosstalk (NEXT)
- Impulse noise
- Radio frequency interference (RFI)

Wireline channel: modelling as LTI system

Assuming proper termination, the insertion loss can be modelled as LTI system:

$$H_{\text{loop}}(f,d) = e^{-\frac{d}{d_0}(k_1\sqrt{f}+k_2 f)} e^{-j\frac{d}{d_0}k_3 f}, \tag{47}$$

where

- f is the frequency in Hz
- d is the length of the loop in m
- k_1, k_2, k_3 are constants depending on the diameter of the wire; exemplary values for 0.5mm loop:
  $k_1 = 3.8 \cdot 10^{-3}$, $k_2 = -0.541 \cdot 10^{-8}$, $k_3 = 4.883 \cdot 10^{-5}$

Wireline channel: modelling as LTI system

Assuming proper termination, the NEXT coupling and FEXT coupling can be modelled via LTI systems:

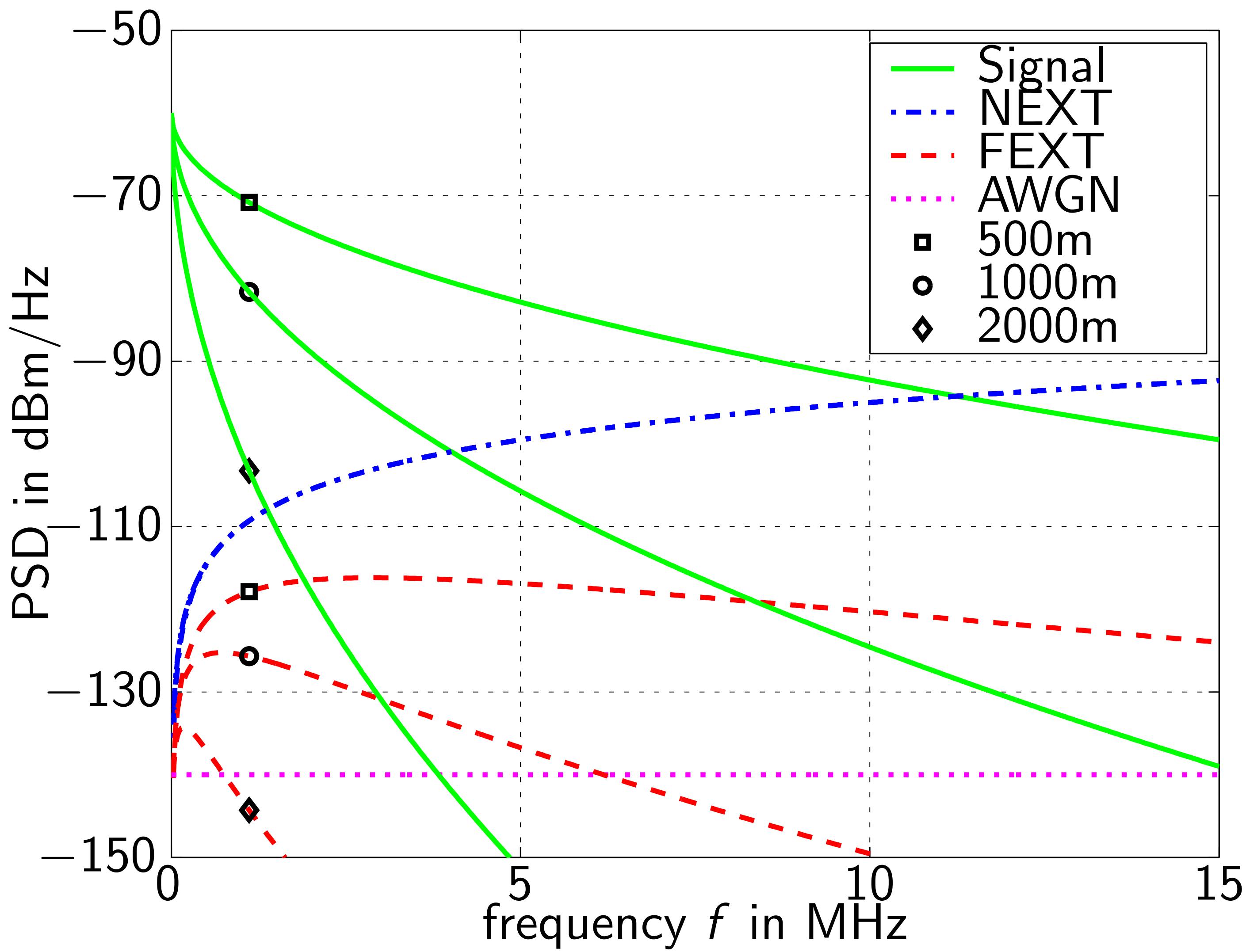
$$H_{\mathrm{FEXT}}(f,d) = k_{\mathrm{f}} \frac{f}{f_0} \sqrt{\frac{d}{d_0}} \left| H_{\mathrm{loop}}(f,d) \right|, \; k_{\mathrm{f}} = 10^{-45/20}, f_0 = 1\,\mathrm{MHz}, d_0 = 1\,\mathrm{km} \tag{48}$$

$$H_{\mathrm{NEXT}}(f,d) = k_{\mathrm{n}} \left(\frac{f}{f_0} \right)^{\frac{3}{4}} \sqrt{1 - \left| H_{\mathrm{loop}}(f,d) \right|^4}, \; k_{\mathrm{n}} = 10^{-50/20}, f_0 = 1\,\mathrm{MHz} \tag{49}$$

where

- f is the frequency in Hz
- d is the coupling length of the loops in m

Wireline channel: receive PSDs



Transmit signal PSD: flat −60 dBm/Hz

Wireless channel: physical mechanisms/effects

- Fixed terminals
  - Path loss
  - Background noise
- Mobile terminal(s)
  - Path loss
  - Background noise
  - Doppler effect
  - Time-varying impulse response
    - $\longrightarrow$ dispersion in frequency
    - $\longrightarrow$ receive signal amplitude fluctuations (fading)
  - Dispersion in time, or equivalently, frequency selectivity

Obstacle-free transmission: path loss

The receive signal power is given by

$$P_{\mathrm{r}} = P_{\mathrm{t}} G_{\mathrm{t}} G_{\mathrm{r}} L_{\mathrm{p}}. \tag{50}$$

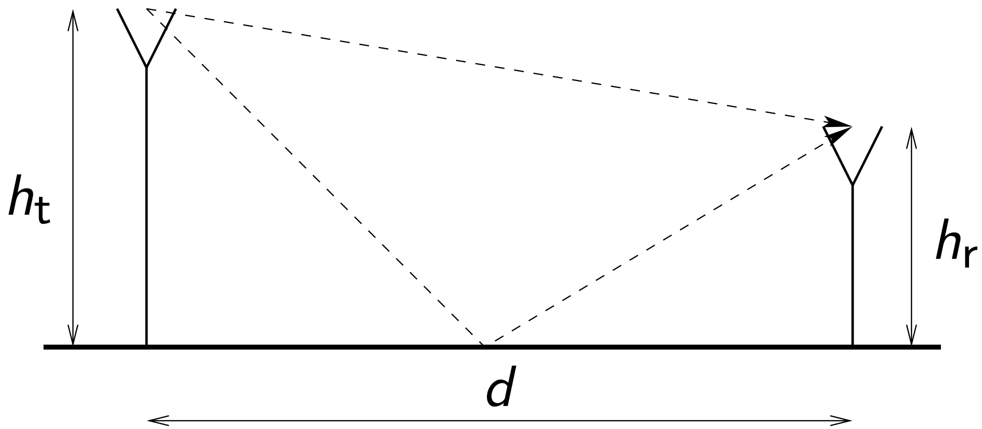
- P_{t} is the transmit power
- G_{t} is the transmit antenna gain (ratio of the received power compared to the power an isotropic antenna would receive; for a dish antenna with effective area A, the antenna gain is roughly $G \approx 4\pi A/\lambda^2$),
- G_{r} is the receive antenna gain and
- L_{p} is the free-space path loss, given by

$$L_{\mathrm{p}} = \left(\frac{\lambda}{4\pi d}\right)^2 \tag{51}$$

  - d distance
  - λ wavelength

Presence of obstacles: ray tracing

Simple two-ray model



$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4} \qquad (d^2 \gg h_t h_r) \tag{52}$$

- h_t height of transmit antenna
- h_r height of receive antenna

Simplified path loss model

$$P_{\mathrm{r}} = P_{\mathrm{t}} G_{\mathrm{t}} G_{\mathrm{r}} \underbrace{P_{\mathrm{r}}(d_0) / P_t}_{K} \left(\frac{d_0}{d} \right)^{\gamma} \tag{53}$$

- d_0 reference distance
- K ratio of receive and transmit power for d_0
- path loss exponent γ, depends on wavelength and environment, typically in the range $2 - 8$ for 1 GHz

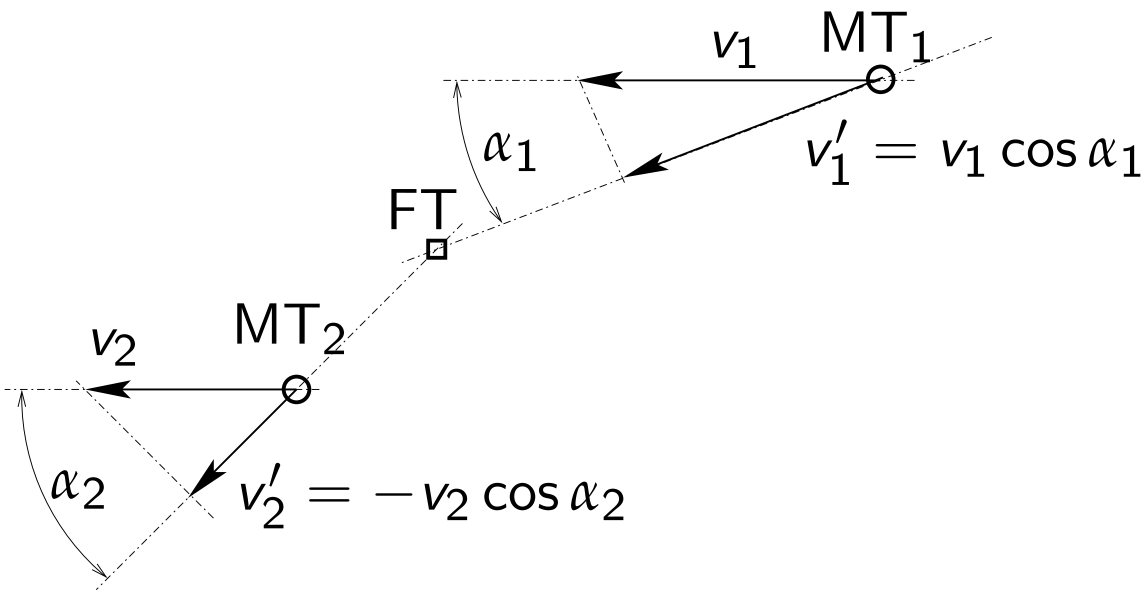
Mobile terminal(s)

- Most often, only one of the terminals is moving, which we call the mobile terminal (MT). The other one, the fixed terminal (FT), does not move
- Due to reciprocity, it does not matter whether we observe downlink (FT $\rightarrow$ MT) or uplink (MT $\rightarrow$ FT)

Mobile terminal(s): Doppler effect

When the FT transmits a signal with frequency $f = c/\lambda$, the MT receives this signal at frequency $f + \nu = f + v'/\lambda$, where v' is the relative velocity of the MT with respect to the FT.

- Note that v' is a signed quantity



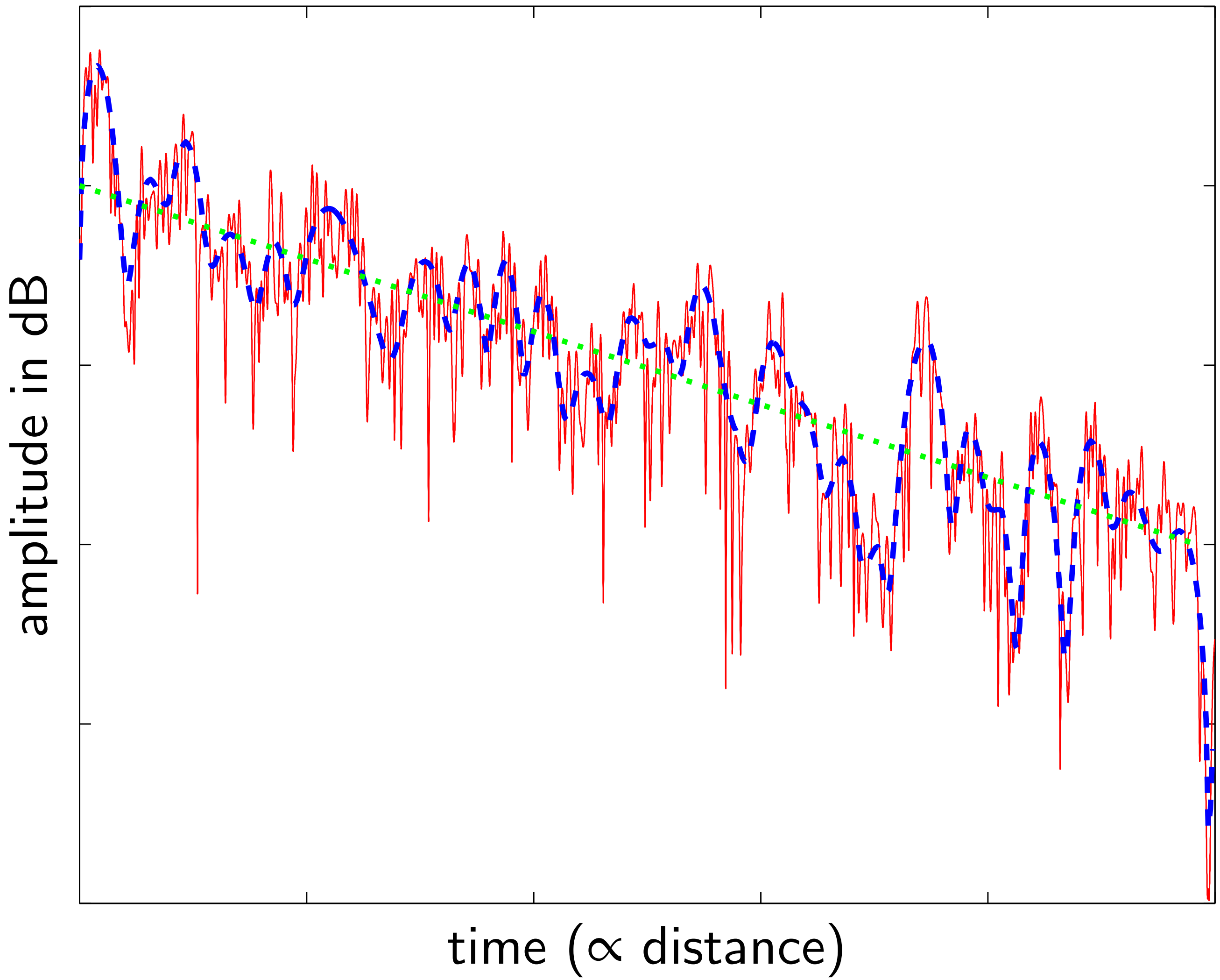
- ν is called the Doppler shift (example: $\nu \approx 83\,\mathrm{Hz}$ for $100\,\mathrm{km/h}$ and $900\,\mathrm{MHz}$)

Mobile terminal(s): characterisation of effects

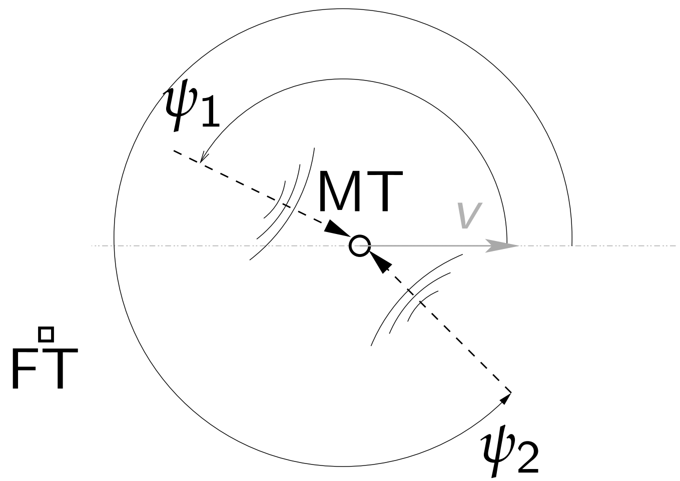
- Dispersion in time
  - transmitted beam is reflected and scattered along the way $\longrightarrow$ *multi-path propagation*
  - often, there is neither a direct beam from the FT to the MT nor a stationary reflection (both of which are referred to as *line of sight (LOS) components*)
  - if the beams arrive with different delays, *time-dispersion* of the transmitted signal occurs
- Dispersion in frequency
  - if either the MT or scatterers are moving, each received beam has a different relative velocity with respect to the MT $\longrightarrow$ *frequency-dispersion* of the transmitted signal occurs
  - motion is not the exclusive cause of frequency dispersion; more generally, frequency dispersion is caused by a time-varying channel impulse response
- Fluctuations in amplitude (fading)

Characterisation of amplitude fluctuations (fading)

- path loss (dotted green curve)
- large-scale (macroscopic) fading (dashed blue curve)
- small-scale (microscopic) fading (solid red curve)



Small-scale fading: Rayleigh distribution, Rice distribution



- many waves arrive from arbitrary directions
- the amplitudes $A_I \sim \mathcal{N}(m_I, \sigma^2)$ and $A_Q \sim \mathcal{N}(m_Q, \sigma^2)$ of inphase and quadrature receive component, respectively, are independent and Gaussian distributed (central limit theorem)
- no LOS component: $m_I = m_Q = 0 \rightarrow$ amplitude $U = \sqrt{A_I^2 + A_Q^2}$ has Rayleigh distribution
- with LOS component: $m_I \neq 0$ and/or $m_Q \neq 0 \rightarrow$ amplitude U has Rice distribution

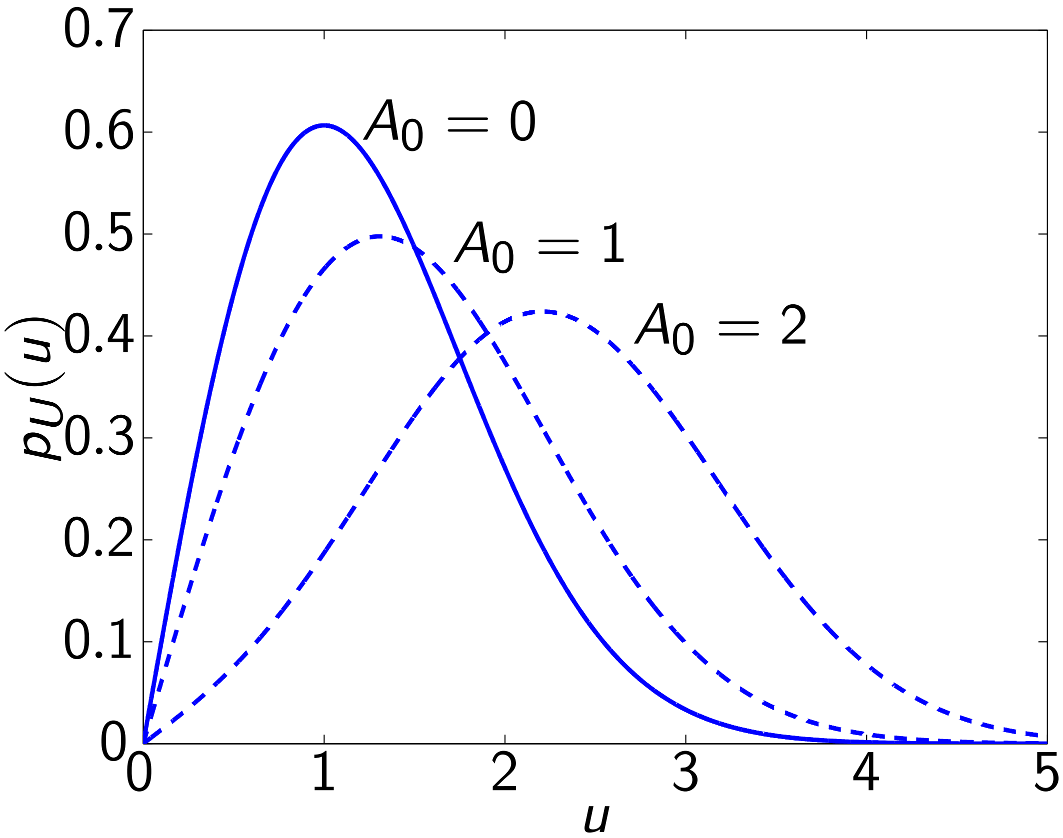
Small-scale fading: Rayleigh/Rice distribution cont'd

Rayleigh distribution:

$$p_U(u) = \frac{u}{\sigma^2} e^{-\frac{u^2}{2\sigma^2}}, \qquad u \geq 0. \tag{54}$$

Rice distribution:

$$p_U(u) = \frac{u}{\sigma^2} e^{-\frac{u^2+s^2}{2\sigma^2}} I_0\left(\frac{us}{\sigma^2}\right), \qquad u \geq 0; \qquad s = \sqrt{m_I^2 + m_Q^2} = A_0. \tag{55}$$



Large-scale fading

- Models the channel property changes caused by movement of the MT
- Characterises the mean value of the small-scale fading model
- The log-normal distribution has been found to yield a good match with measurements

The mean value in dB γ_{dB} is Gaussian distributed

$$p_{\gamma_{\text{dB}}}(\gamma_{\text{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{dB}}} e^{-\frac{(\gamma_{\text{dB}} - m_{\text{dB}})^2}{2(\sigma_{\text{dB}})^2}}, \tag{56}$$

where σ_{dB}, the standard deviation of γ_{dB}, is typically in the range of 6-12 dB.
Then the distribution of $\gamma = 10^{\gamma_{\text{dB}}/20}$ is given by

$$p_{\gamma}(\gamma) = \frac{20}{\gamma\sqrt{2\pi}\sigma_{\text{dB}} \ln 10} e^{-\frac{(20\log_{10}\gamma - m_{\text{dB}})^2}{2(\sigma_{\text{dB}})^2}}. \tag{57}$$

Charaterisation of dispersion in time and frequency

Deterministic analysis

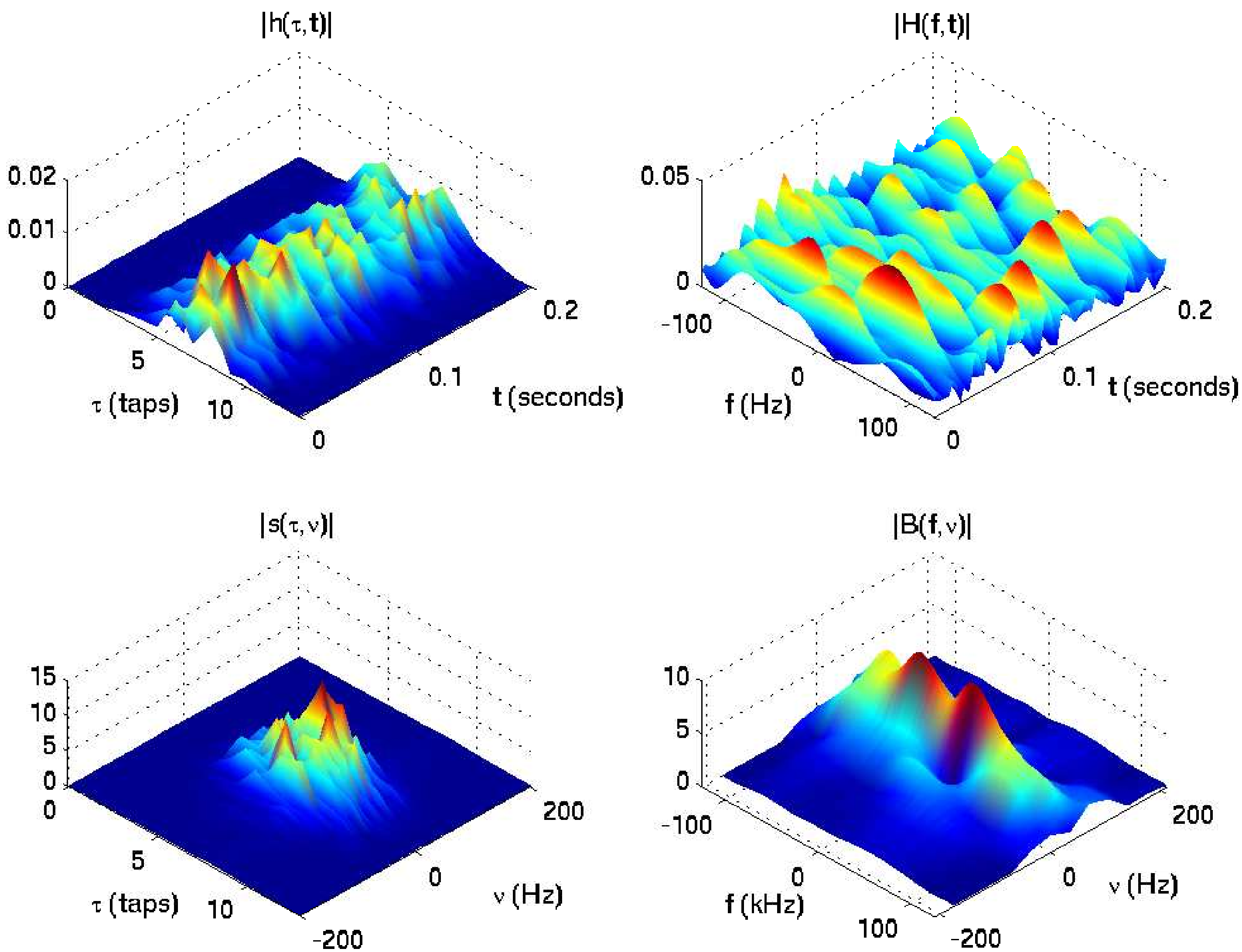
- channel is modelled as linear time-variant (LTV) system, described by a time-variant impulse response $h(\tau, t)$
- time-variant frequency response $H(f, t) = \mathcal{F}_{\tau}\{h(\tau, t)\}$
- delay Doppler spreading function $s(\tau, \nu) = \mathcal{F}_{t}\{h(\tau, t)\}$
- output Doppler spreading function $B(f, \nu) = \mathcal{F}_{t}\{H(f, t)\}$

Charaterisation of dispersion in time and frequency

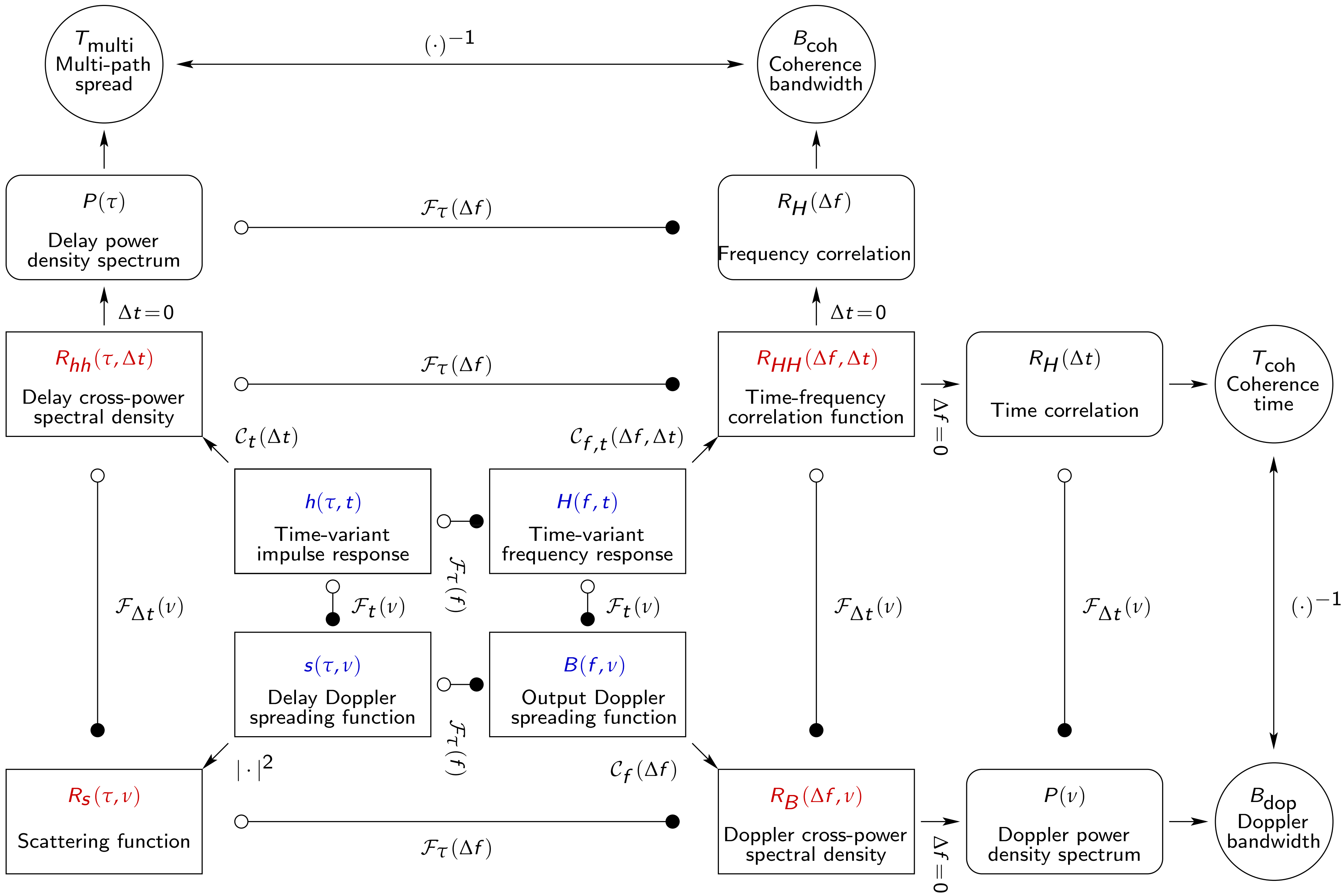
Stochastic analysis

- auto-correlation function $R_{hh}(\tau_1, \tau_2, t_1, t_2) = \mathrm{E}\left\{h^*(\tau_1, t_1)h(\tau_2, t_2)\right\}$ of the impulse response
  - wide-sense stationarity (WSS) assumption: $R_{hh}(\tau_1, \tau_2, t_1, t_2)$ depends only on the time difference $\Delta t = t_2 - t_1$
  - uncorrelated scattering (US) assumption: scatterers act independently $\rightarrow$ it is sufficient to observe $R_{hh}(\tau, t_1, t_2)$
  - WSS + US $\rightarrow$ WSSUS assumption: it is sufficient to observe the *delay cross-power spectral density* $R_{hh}(\tau, \Delta t)$
- *time-frequency correlation function* $R_{HH}(\Delta f, \Delta t) = \mathcal{F}_\tau\{R_{hh}(\tau, \Delta t)\}$
- *scattering function* $R_s(\tau, \nu) = \mathcal{F}_{\Delta t}\{R_{hh}(\tau, \Delta t)\}$
- *Doppler cross-power spectral density* $R_B(\Delta f, \nu) = \mathcal{F}_{\Delta t}\{R_{HH}(\Delta f, \Delta t)\}$

Exemplary system functions



Summary of wireless channel characterisation measures



Charaterisation of dispersion in time and frequency

Two functions commonly used in practice:

1. delay power density spectrum (*power delay profile*)
   $P(\tau) = R_{hh}(\tau, \Delta t)|_{\Delta t=0}$ specifies time-dispersion (or equivalently, frequency-selectivity) characteristic
2. Doppler power density spectrum (*Doppler spectrum*)
   $P(\nu) = R_B(\Delta f, \nu)|_{\Delta f=0}$ specifies frequency dispersion, or equivalently, the correlation of realisations observed over time of a given coefficient of the tapped delay line filter

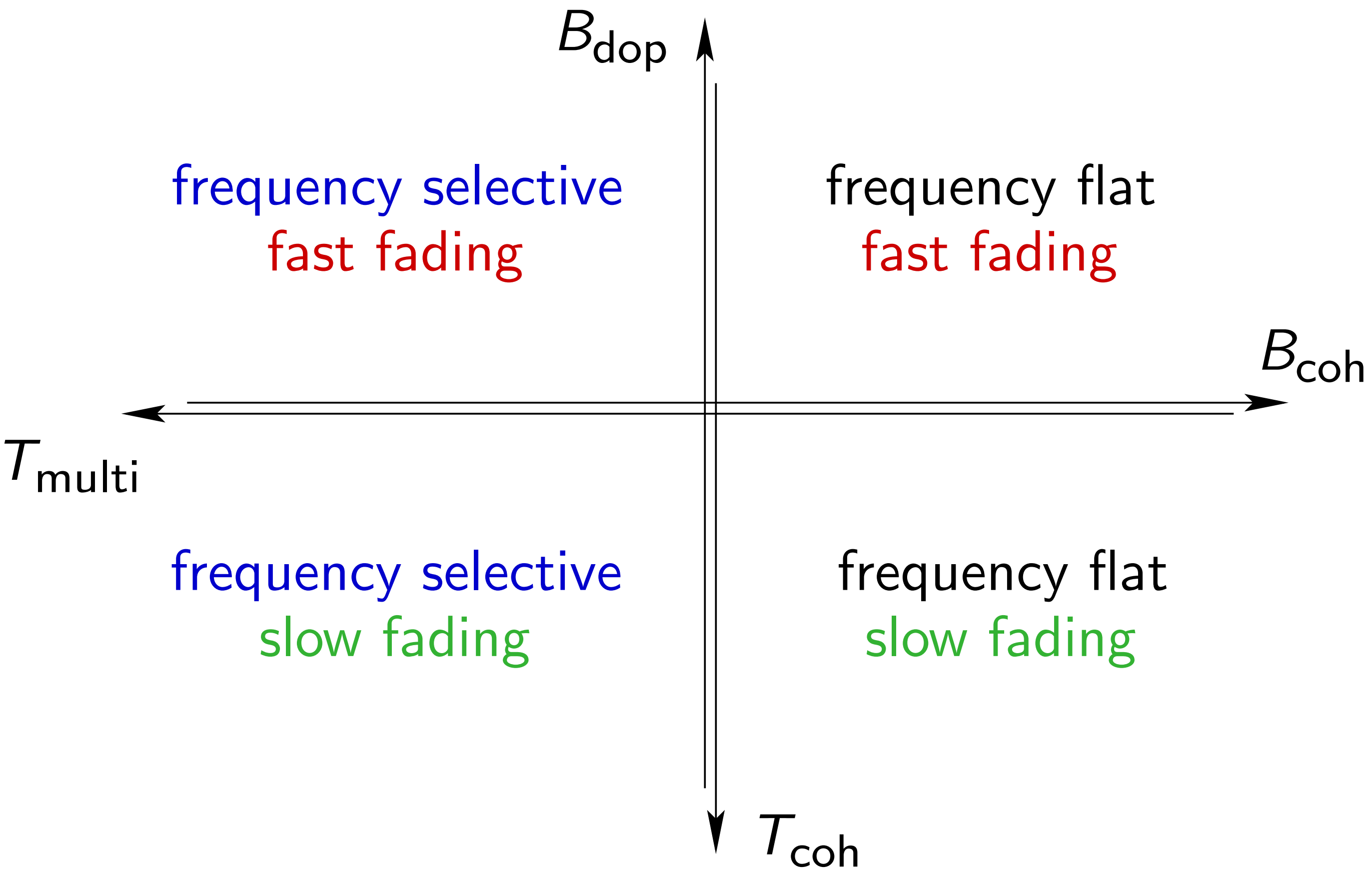
Charaterisation of dispersion in time and frequency

Two scalars commonly used in practice:

1. The *multi-path spread* T_{multi} specifies the approximate support of the power delay profile $P(\tau)$, or equivalently, the approximate length of the channel impulse response. Dual measure: *coherence bandwidth* $B_{\text{coh}} \approx 1/T_{\text{multi}}$.
2. The *coherence time* T_{coh} specifies the approximate support of the time correlation function $R_H(\Delta t)$, or equivalently, the time during which the impulse response remains constant. Dual measure: *Doppler bandwidth* $B_{dop} \approx 1/T_{\text{coh}}$.

Assessment of wireless channels

The parameters T_{coh} and T_{multi} of a wireless channel have to be seen in context with symbol period T_{sym} of the system.



Ergodicity

- useful description of a linear channel with additive noise:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

- $\mathbf{s} \in \mathbb{C}^S$: channel input. $\mathbf{r} \in \mathbb{C}^R$: channel output. $\mathbf{H} \in \mathbb{C}^{R \times S}$: channel matrix. $\mathbf{n} \in \mathbb{C}^R$: additive noise.
- *Ergodic* channel

$$\mathbf{r}_n = \mathbf{H}_n \mathbf{s}_n + \mathbf{n}_n.$$

  $\mathbf{H}_n$ are realizations of a random process. Transmitted symbol/codeword $\mathbf{s}_n, n = 0, 1, \ldots, N$; $N \gg$ "sees" all channel states. Valid for fast fading channels.
- *Nonergodic* channel: Consider the model

$$\mathbf{r}_n = \mathbf{H}\mathbf{s}_n + \mathbf{n}_n,$$

  Here, $\mathbf{H}$ is constant over the symbol/codeword $\mathbf{s}_n, n = 0, 1, \ldots, N$; $N \gg$. Transmitted symbol/codeword "sees" only one state ($\mathbf{H}$). Valid for slowly fading channels.

Block fading

- Interleavers spreads out codewords in time and/or frequency.
- Long interleaver can thus turn a nonergodic channel (where each codesymbol of a codeword sees one channel state only) into an ergodic channel (where each codesymbol of a codeword sees a different channel state)
- Block fading characterizes the situation in between those two extremes. If the interleaver is not long enough, blocks of codesymbols see the same channel state.

Summary

1. Classification of channels
   - single-user, multi-user
   - SISO, MIMO, MISO, SIMO
   - "digital" channels (BSC, DMC)
   - physical medium (copper, coax, fiber, air/space)
2. Wireline channel
   - essentially time-invariant, strongly frequency selective
3. Wireless channel
   - fixed terminals
     - static view on attenuation (link budget) is sufficient, limited by background noise
   - mobile terminal(s)
     - time-variant (frequency-dispersive), time-dispersive (frequency selective)
   - ergodicity