Data-Aided Frequency Estimation for Burst Digital Transmission

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***Abstract*—We propose a new algorithm for carrier frequency estimation in burst-mode phase shift keying (PSK) transmissions. The algorithm is data-aided and clock-aided and has a feedforward structure that is easy to implement in digital form. Its estimation range is large, about $\pm 20\%$ of the symbol rate and its accuracy is close to the Cramer–Rao bound (CRB) for a signal-to-noise ratio (SNR) as low as 0 dB. Comparisons with earlier methods are discussed.**

I. Introduction

BURST transmission of digital data is employed in several applications such as satellite time-division multiple-access (TDMA) systems and terrestrial mobile cellular radio. In many cases, a preamble of known symbols is appended to the beginning of each burst for carrier and clock recovery. Data-aided (DA) algorithms are normally employed to attain good performance with short preambles. Even so, synchronization is difficult, especially at low signal-to-noise ratio (SNR) values that are typical with coded modulations.

Application of maximum likelihood (ML) estimation criteria indicates that the optimal solution to this problem amounts to locating the peak of a periodogram. In many instances, however, this task is too time-consuming and simpler methods are preferable. References [1]–[6] give a good sample of the results in this area. In this paper, we focus on algorithms proposed by Lovell and Williamson (L&W) [2], Fitz [3], and Luise and Reggiannini (L&R) [4], as they are simpler to implement than the ML estimator and still attain the Cramer–Rao bound (CRB) at low/intermediate SNR.

These algorithms have a rather different performance. A first difference is concerned with their *threshold,* i.e., E_b/N_0 channel at which mean-square errors start to depart from the CRB. As we shall see, with preamble lengths of about 100 symbols (which are typical for local area networks (LAN's) interconnection by satellite), threshold occurs below 0 dB in Fitz and L&R schemes, as opposed to 12 dB or more in L&W's. The relevance of this difference is easily understood bearing in mind that operation below threshold is plagued by large errors. When a large error occurs, the whole data packet gets lost. Thus, operation above threshold is mandatory in burst transmissions.

A second difference is in their *estimation range* $\pm\Delta f$, as normalized to the symbol rate $1/T$. In the Fitz and L&R algorithms, the product $\Delta f \times T$ may be increased at the expense of estimation accuracy whereas in the L&W algorithm $\Delta f \times T$ is large and independent of the accuracy.

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We propose a new method that has some advantages over previous schemes. Its threshold is much lower than in the L&W algorithm and, for a given estimation accuracy, its acquisition range is larger than in the Fitz and L&R algorithms.

II. Problem Formulation

We assume M-ary PSK modulation and additive white Gaussian noise (AWGN) with two-sided power spectral density (PSD) $N_0/2$. Filtering is evenly split between transmitter and receiver and the overall channel response is Nyquist. Timing is ideal but the local frequency reference at the receiver is in error by f_d Hz. This quantity is small compared with the symbol rate. Also, the ratio E_s/N_0 is much greater than unity. The above restrictions are only temporary. In the simulations discussed later they are relaxed and, in particular, frequency errors as large as 20% of $1/T$ are considered.

Filtering the received waveform in a matched filter and sampling at proper times yields

$$x(k) = c_k e^{j(2\pi f_d kT+\theta)} + n(k) \tag{1}$$

where $\{c_i\}$ are unit-amplitude symbols, θ is the carrier phase, and $\{n(k)\}$ are complex-valued independent and identically distributed (i.i.d.) Gaussian random variables (rv's) with zero mean and variance $1/(E_s/N_0)$. It is apparent from (1) that $x(k)$ depends on $\{c_i\}$. However, because of the property $c_k c_k^* = 1$ (c_k^* means complex conjugate of c_k), the dependence can be removed by multiplying $x(k)$ by c_k^*. In fact, letting $z(k) \triangleq x(k)c_k^*$, we have

$$z(k) = e^{j(2\pi f_d kT+\theta)} + n(k)c_k^* \tag{2}$$

where $n(k)c_k^*$ is statistically equivalent to $n(k)$. Thus, $z(k)$ may be viewed as a complex sinusoid embedded in white Gaussian noise and the problem is to derive an estimator of f_d based on the observation of a few consecutive samples $\{z(k), 0 \le k \le L_0 - 1\}$. This is just the problem treated in [1]–[5]. A different solution is now considered.

III. Estimation Algorithm

The proposed estimator exploits the sample correlations

$$R(m) = \frac{1}{L_0 - m} \sum_{k=m}^{L_0-1} z(k) z^*(k-m), \qquad 1 \le m \le N \tag{3}$$

where N is a design parameter not greater than $L_0/2$. To explain the method let us rewrite (2) in the form $z(k) = e^{j(2\pi f_d kT+\theta)}[1+\tilde{n}(k)]$, with $\tilde{n}(k) \triangleq n(k)c_k^* e^{-j(2\pi f_d kT+\theta)}$.

Note that $\tilde{n}(k)$ has the same statistics as $n(k)$. Next, substituting into (3) yields

$$R(m) = e^{j2\pi m f_d T}[1 + \gamma(m)] \tag{4}$$

with

$$\gamma(m) \triangleq \frac{1}{L_0 - m} \sum_{k=m}^{L_0-1} [\tilde{n}(k) + \tilde{n}^*(k-m) + \tilde{n}(k)\tilde{n}^*(k-m)]. \tag{5}$$

For $E_s/N_0 \gg 1$, the amplitude of $\tilde{n}(k)$ is less than unity with high probability and the last term in (5) can be neglected. Also, letting $\gamma(k) \triangleq \gamma_R(k) + j\gamma_I(k)$ and denoting with $\arg\{R(m)\}$ the *principal value* of the argument of $R(m)$, from (4) we get

$$\arg\{R(m)\} \approx [2\pi m f_d T + \gamma_I(m)]_{2\pi}, \qquad 1 \le m \le N \tag{6}$$

where $[x]_{2\pi}$ is the value of x reduced to the interval $[-\pi, \pi)$. This equation establishes a relation between f_d and $\arg\{R(m)\}$. Unfortunately, it is highly nonlinear and awkward to handle. An unwrapping process would be needed to transform it into a linear equation but this would be difficult at low E_s/N_0. In the sequel, we follow another route which parallels Kay's approach in [6].

The idea is to relate f_d to the phase increments $\phi(m) \triangleq [\arg\{R(m)\} - \arg\{R(m-1)\}]_{2\pi}$ rather than to $\arg\{R(m)\}$ itself. In fact, exploiting the general formula $[[x]_{2\pi} - [y]_{2\pi}]_{2\pi} = [x-y]_{2\pi}$ with $x \triangleq 2\pi m f_d T + \gamma_I(m)$ and $y \triangleq 2\pi(m-1)f_d T + \gamma_I(m-1)$ it can be shown that

$$\phi(m) \approx 2\pi f_d T + \gamma_I(m) - \gamma_I(m-1), \qquad 1 \le m \le N \tag{7}$$

provided that $|2\pi f_d T| < \pi$ and the rv's $\gamma_I(m)$ are sufficiently small. Clearly, (7) relates f_d to $\phi(m)$ in a linear fashion and the problem is to estimate a constant, $2\pi f_d T$, from noisy measurements $\{\phi(m)\}$. In particular, the ML estimator of f_d can be derived with the methods indicated in [6] and the result is

$$\hat{f}_d = \frac{1}{2\pi T} \sum_{m=1}^{N} w(m) \times [\arg\{R(m)\} - \arg\{R(m-1)\}]_{2\pi} \tag{8}$$

where $w(m)$ is a smoothing function given by

$$w(m) \triangleq \frac{3[(L_0-m)(L_0-m+1) - N(L_0-N)]}{N(4N^2 - 6NL_0 + 3L_0^2 - 1)}. \tag{9}$$

Inspection of (3) reveals that the computation of $\arg\{R(m)\}$ requires $N(2L_0 - N - 1)/2$ complex multiplications. Depending on the parameters L_0 and N, this may render (8) as complex as a full fledged ML estimator. Things can be made easier, however, noting that (at high E_s/N_0) the amplitude of $z(k)$ is close to unity [see (2)] and $R(m)$ may be approximated as [see (3)]

$$\overline{R}(m) = \frac{1}{L_0 - m} \sum_{k=m}^{L_0-1} \exp\{j[\arg\{z(k)\} - \arg\{z(k-m)\}]\}, \qquad 1 \le m \le N. \tag{10}$$

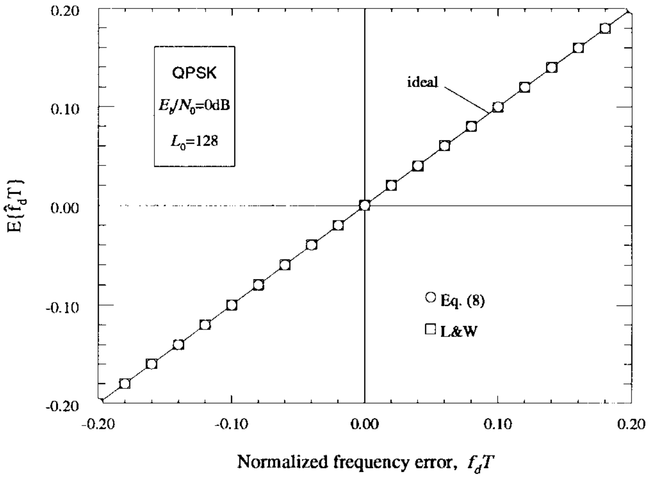

Fig. 1. Average estimates as obtained with (8) and L&W algorithms.

In this equation, there are no multiplications and $\arg\{z(k)\}$ can be taken from a ROM. All the simulations discussed later have been run by replacing $R(m)$ with $\overline{R}(m)$ in (8).

Important issues about (8) include: i) whether it is biased or not; ii) how its variance compares with the CRB; and iii) how its performance compares with that of other algorithms. These questions are addressed in the next section. Before proceeding we recall the structure of the L&W, Fitz, and L&R algorithms.

The L&W estimator has the form

$$\hat{f}_d = \frac{1}{2\pi T} \arg\left\{ \sum_{k=1}^{L_0-1} w(k) \times \exp\{j[\arg\{z(k)\} - \arg\{z(k-1)\}]\} \right\} \tag{11}$$

with the smoothing function $w(k) \triangleq 6k(L_0-k)/[L_0(L_0^2-1)]$. The Fitz estimator reads

$$\hat{f}_d = \frac{2}{\pi T N(N+1)} \sum_{m=1}^{N} \arg\{R(m)\}. \tag{12}$$

With this algorithm, the product $\Delta f \times T$ is less than $1/(2N)$. Finally, the L&R estimator reads

$$\hat{f}_d = \frac{1}{\pi T(N+1)} \arg\left\{ \sum_{m=1}^{N} R(m) \right\} \tag{13}$$

and the product $\Delta f \times T$ is less than $1/N$.

IV. Computer Simulations

Computer simulations have been run to compare (8) with the above algorithms. The following hypotheses have been made: i) modulation is quaternary phase shift keying (QPSK) with 50% roll-off; ii) carrier frequency errors are within $\pm 20\%$ of $1/T$; and iii) the estimation interval is of $L_0 = 128$ symbols. Fig. 1 illustrates average estimates, $E\{\hat{f}_d T\}$, versus $f_d T$ as obtained with (8) and the L&W algorithm. The ideal line

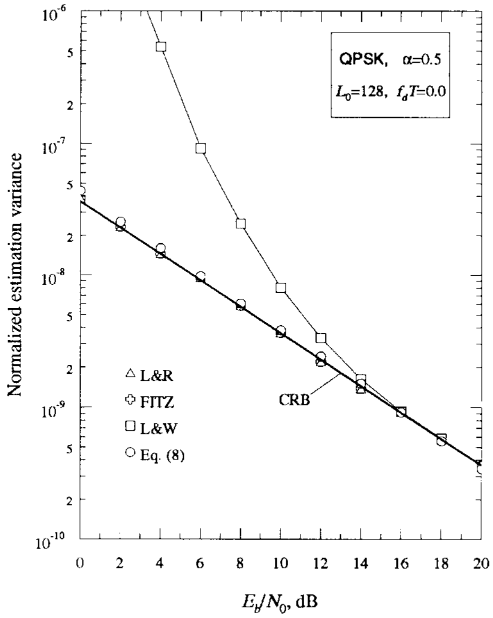

Fig. 2. Estimation variance with the four algorithms.

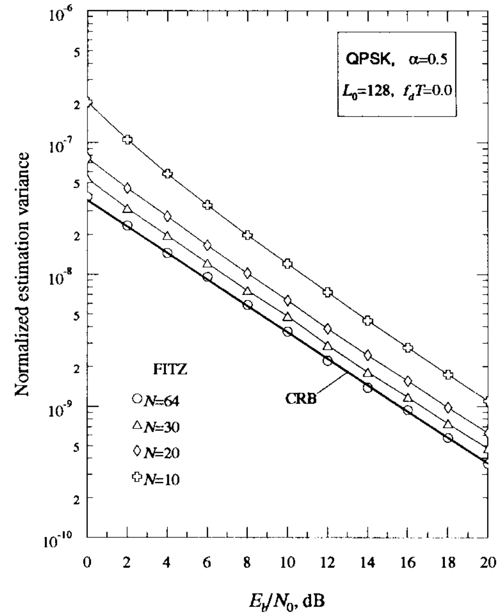

Fig. 4. Estimation variance with the Fitz algorithm.

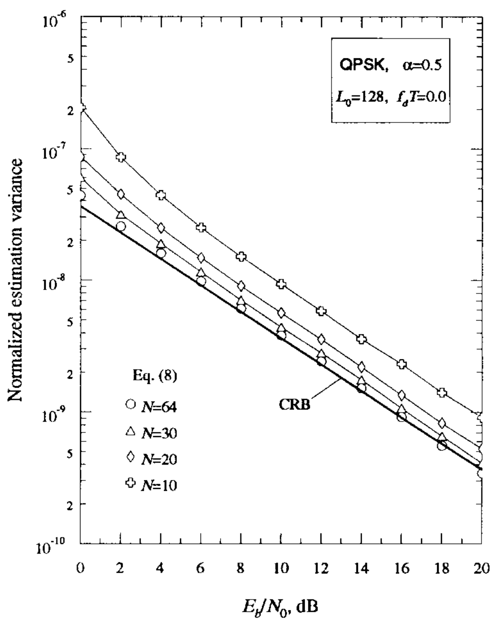

Fig. 3. Estimation variance with the proposed algorithm.

is indicated as a reference. Both algorithms appear to be unbiased over the full range $|f_dT| \leq 0.2$. Corresponding data for the Fitz and L&R estimators are not shown due to space limitations. Anyway, these estimators are found to be unbiased over $|f_dT| \leq 1/(2N)$ and $|f_dT| \leq 1/N$, respectively. For example, to cover a range $|f_dT| \leq 0.1$, the parameter N must be limited to five in the Fitz case and ten in the L&R case.

Fig. 2 compares the variance $\mathrm{Var}\{\hat{f}_dT\}$ for the four algorithms. Parameter N is set at 64 wherever appropriate. The CRB is also shown as a baseline. Apart from the L&W algorithm, which has a rather high threshold, all the others are virtually stuck to CRB up to $E_b/N_0 = 0$ dB.

Further results for $\mathrm{Var}\{\hat{f}_dT\}$ are illustrated in Figs. 3–4 for (8) and the Fitz algorithm. As is intuitively clear, estimation accuracy degrades as N decreases. Conversely, the computational load diminishes. Thus, accuracy and computational complexity may be exchanged. Accuracy may also be exchanged with estimation range with the Fitz algorithm. This is not the case with (8) because its estimation range is rather large and independent of N.

V. Conclusions

We have proposed a new algorithm for carrier frequency estimation in burst-mode transmissions. The algorithm is data-aided and clock-aided and is suitable for PSK signaling. Its estimation range is about ±20% of $1/T$. Its accuracy achieves the CRB at E_b/N_0 values close to 0 dB. This same accuracy can be obtained with other methods but with a reduced estimation range.

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