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Ali Habibi (S'64–M'70) received the B.S. degree from West Virginia University, Morgantown, and the M.S. degree from Cornell University, Ithica, N. Y., and the Ph.D. degree from Purdue University, Lafayette, Ind., in 1965, 1967, and 1970 respectively, all in electrical engineering. While at Cornell University he was a Research Assistant in the System Theory Group, and from 1967 to 1970 he was a Research Instructor at Purdue University. From 1970 to 1971 he was employed by Bell Laboratories, Holmdel, N. J., working in the Telephone and Picturephone Planning Center. Since 1971 he has been with the University of Southern California, Los Angeles, Calif., as an Assistant Professor of Electrical Engineering.

In 1972 Dr. Habibi received Leonard Abraham Prize Paper award from the IEEE Communications Society for his contribution to the technical literature in communications. He is a member of Eta Kappa Nu, Tau Beta Pi, Sigma Tau Sigma, and Sigma Xi.

Adaptive Maximum-Likelihood Receiver for Carrier-Modulated Data-Transmission Systems

GOTTFRIED UNGERBOECK

Abstract—**A new look is taken at maximum-likelihood sequence estimation in the presence of intersymbol interference. A uniform receiver structure for linear carrier-modulated data-transmission systems is derived which for decision making uses a modified version of the Viterbi algorithm. The algorithm operates directly on the output signal of a complex matched filter and, in contrast to the original algorithm, requires no squaring operations; only multiplications by discrete pulse-amplitude values are needed. Decoding of redundantly coded sequences is included in the consideration. The reason and limits for the superior error performance of the receiver over a conventional receiver employing zero-forcing equalization and symbol-by-symbol decision making are explained. An adjustment algorithm for jointly approximating the matched filter by a transversal filter, estimating intersymbol interference present at the transversal filter output, and controlling the demodulating carrier phase and the sample timing, is presented.**

I. INTRODUCTION

THE design of an optimum receiver for synchronous data-transmission systems that employ linear carrier-modulation techniques [1] is discussed. All forms of digita amplitude modulation (AM), phase modulation (PM) and combinations thereof, are covered in a unified manner Without further mention in this paper, the results are alsc applicable to baseband transmission.

In synchronous data-transmission systems intersymbo interference (ISI) and noise, along with errors in th demodulating carrier phase and the sample timing, ar the primary impediments to reliable data reception [1] The goal of this paper is to present a receiver structur that deals with all these effects in an optimum way an an adaptive manner. In deriving the receiver the concep of maximum-likelihood (ML) sequence estimation [2], [3 will be applied. This assures that the receiver is optimun in the sense of sequence-error probability, provided tha data sequences have equal *a priori* probability.

The modulation schemes considered in this paper ca be viewed in the framework of digital quadrature ampli tude modulation (QAM) [1]. They can therefore be repre sented by an equivalent linear baseband model that differ from a real baseband system only by the fact that signal and channel responses are complex functions [2], [4], [5]

Conventional receivers for synchronous data signal

Paper approved by the Associate Editor for Communication Theory of the IEEE Communications Society for publication without oral presentation. Manuscript received May 18, 1973; revised December 21, 1973.
The author is with the IBM Zurich Research Laboratory, Rüschlikon, Switzerland.

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comprise a linear receiver filter or equalizer, a symbol-rate sampler, and a quantizer for establishing symbol-by-symbol decisions. A decoder, possibly with error-detection and/or error-correction capability, may follow. The purpose of the receiver filter is to eliminate intersymbol interference while maintaining a high signal-to-noise ratio (SNR). It has been observed by several authors [1], [6]–[9] that for various performance criteria the optimum linear receiver filter can be factored as a matched filter (MF) and a transversal filter with tap spacings equal to the symbol interval. The MF establishes an optimum SNR irrespective of the residual ISI at its output. The transversal filter then eliminates or at least reduces intersymbol interference at the expense of diminishing the SNR.

If symbols of a data sequence are correlated by some coding law, a better way than making symbol-by-symbol decisions is to base decisions on the entire sequence received. The same argument holds true if data sequences are disturbed by ISI. The correlation introduced by ISI between successive sample values is of discrete nature as in the case of coding, in the sense that a data symbol can be disturbed by adjacent data symbols only in a finite number of ways. ISI can even be viewed as an unintended form of partial response coding [1]. Receivers that perform sequence decisions or in some other way exploit the discreteness of ISI exhibit highly nonlinear structures. Decision feedback equalization [10], [11] represents the earliest step in this direction. Later, several other nonlinear receiver structures were described [12]–[17]. In view of the present state of the art, many of these approaches can be regarded as attempts to avoid, by nonlinear processing methods, noise enhancement which would otherwise occur if ISI were eliminated by linear filtering.

A new nonlinear receiver structure was introduced by Forney [18]. The receiver consists of a "whitened" MF (i.e., an MF followed by a transversal filter that whitens the noise), a symbol-rate sampler, and a recursive nonlinear processor that employs the Viterbi algorithm in order to perform ML sequence decisions. The Viterbi algorithm was originally invented for decoding of convolutional codes [19]. Soon thereafter the algorithm was shown to yield ML sequence decisions and that it could be regarded as a specific form of dynamic programming [20]–[22]. Its applicability to receivers for channels with intersymbol interference and correlative level coding was noticed by Omura [23] and Kobayashi [24]–[26]. A survey on the Viterbi algorithm is given by Forney [27]. Very recently, adaptive versions of Forney's receiver have been proposed [28], [29], and its combination with decision-feedback equalization has been suggested [30].

In Forney's [18] receiver, whitening of the noise is essential because the Viterbi algorithm requires that noise components of successive samples be statistically independent. In this paper a receiver similar to that of Forney will be described. The receiver employs a modified Viterbi algorithm that operates directly on the MF output without whitening the noise. In a different form the algorithm has already been used for estimating and subtracting ISI terms from disturbed binary data sequences [17]. Here the algorithm is restated and extended so that it performs ML decisions for complex-valued multilevel sequences. Apparently, Mackechnie [31] has independently found the same algorithm.

In Section II we define the modulation scheme. The general structure of the maximum likelihood receiver (MLR) is outlined in Section III. In Section IV we derive the modified Viterbi algorithm. The error performance of the MLR is discussed in Section V and compared with the error performance of the conventional receiver. Finally, a fully adaptive version of the MLR is presented in Section VI.

II. MODULATION SCHEME

We consider a synchronous linear carrier-modulated data-transmission system with coherent demodulation of the general form shown in Fig. 1. By combining in-phase and quadrature components into complex-valued signals (indicated by heavy lines in Fig. 1), all linear carrier-modulation schemes can be treated in a concise and uniform manner [2], [4], [5]. Prior to modulation with carrier frequency ω_c, the receiver establishes a pulse-amplitude modulated (PAM) signal of the form

$$x(t) = \sum_n a_n f(t - nT) \tag{1}$$

where the sequence $\{a_n\}$ represents the data symbols, T is the symbol spacing, and $f(t)$ denotes the transmitted baseband signal element. Generally, $\{a_n\}$ and $f(t)$ may be complex (but usually only one of them is; see Table I).[1] The data symbols are selected from a finite alphabet and may possibly succeed one another only in accordance with some redundant coding rule.

Assuming a linear dispersive transmission medium with impulse response $g_c(t)$ and additive noise $w_c(t)$, the receiver will observe the real signal

$$y_c(t) = g_c(t) * \mathrm{Re}\,\{\sqrt{2}x(t)\exp(j\omega_c t)\} + w_c(t) \tag{2}$$

where $*$ denotes convolution. One side of the spectrum of $y_c(t)$ is redundant and can therefore be eliminated without loss of information; the remaining part must be transposed back in the baseband. In Fig. 1 we adhere to the conventional approach of demodulating by transposing first and then eliminating components around twice the carrier frequency. The demodulated signal thus becomes

$$y(t) = \sum_n a_n h(t - nT) + w(t) \tag{3}$$

where

$$\begin{aligned} h(t) &= [g_c(t)\exp(-j\omega_c t - j\varphi_c)] * f(t) \\ &= g(t) * f(t) \end{aligned} \tag{4}$$

[1] A more general class of PAM signals is conceivable where $f(t)$ depends on n or a_n.

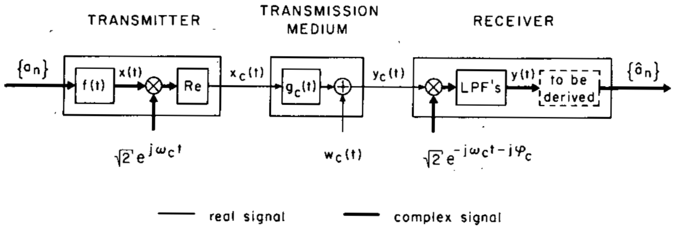

Fig. 1. General linear carrier-modulated data-transmission system.

TABLE I

Modulation Scheme	$\{a_n\}$	$f(t)$
DSB-AM	real	real
SSB, VSB-AM	real	complex[a]
PM	$\mid$ complex $\mid = 1$	real
AM-PM	complex	real

[a] SBB: $f(t) = f_1(t) \pm j\mathcal{H}\{f_1(t)\}$, where $\mathcal{H}$ is the Hilbert transform [1].

and

$$w(t) = \sqrt{2}w_c(t) \exp(-j\omega_c t - j\varphi_c). \tag{5}$$

In (5) the effect of low-pass filtering the transposed noise is neglected since it affects only noise components outside the signal bandwidth of interest. Our channel model does not include frequency offset and phase jitter. It is understood that the demodulating carrier phase φ_c accounts for these effects.

III. STRUCTURE OF THE MAXIMUM-LIKELIHOOD RECEIVER

The objective of the receiver is to estimate $\{a_n\}$ from a given signal $y(t)$. Let the receiver observe $y(t)$ within a time interval I which is supposed to be long enough so that the precise conditions at the boundaries of I are insignificant for the total observation. Let $\{\alpha_n\}$ be a hypothetical sequence of pulse amplitudes transmitted during I. The MLR by its definition [2], [3] determines as the best estimate of $\{a_n\}$ the sequence $\{\alpha_n\} = \{\hat{a}_n\}$ that maximizes the likelihood function $p[y(t), t \in I \mid \{\alpha_n\}]$.

In the following paragraphs the shape of the signal element $h(t)$ and the exact timing of received signal elements are assumed to be known. The noise of the transmission medium is supposed to be stationary Gaussian noise with zero mean and autocorrelation function $W_c(\tau)$. From (5) the autocorrelation function of $w(t)$ is obtained as

$$W(\tau) = E[\bar{w}(t)w(t+\tau)] = \bar{W}(-\tau)$$
$$= 2W_c(\tau) \exp(-j\omega_c\tau). \tag{6}$$

For example, if $w_c(t)$ is white Gaussian noise (WGN) with double-sided spectral density N_0, then $W_c(\tau) = N_0\delta(\tau)$ and $W(\tau) = 2N_0\delta(\tau)$. In view of this important case it is appropriate to introduce

$$W(\tau) = 2N_0K(\tau), \quad \text{WGN}: K(\tau) = \delta(\tau). \tag{7}$$

If $\{\alpha_n\}$ were the actual sequence of the pulse amplitudes transmitted during I, then

$$w(t \mid \{\alpha_n\}) = y(t) - \sum_{nT \in I} \alpha_n h(t - nT), \qquad t \in I \tag{8}$$

must be the realization of the noise signal $w(t)$. Hence, owing to the Gaussian-noise assumption, the likelihood function becomes (apart from a constant of proportionality) [32]

$$p[y(t), t \in I \mid \{\alpha_n\}] = p[w(t \mid \{\alpha_n\})]$$
$$\sim \exp\left\{-\frac{1}{4N_0}\int_I\int_I \bar{w}(t_1 \mid \{\alpha_n\})K^{-1}(t_1 - t_2) \cdot w(t_2 \mid \{\alpha_n\})\, dt_1\, dt_2\right\} \tag{9}$$

where $K^{-1}(\tau)$ is the inverse of $K(\tau)$

$$K(\tau) * K^{-1}(\tau) = \delta(\tau). \tag{10}$$

The correctness of (9) for the complex-signal case is proven in Appendix I. Substituting (8) into (9) and considering only terms that depend on $\{\alpha_n\}$, yields

$$p[y(t), t \in I \mid \{\alpha_n\}]$$
$$\sim \exp\left\{\frac{1}{4N_0}\left[\sum_{nT \in I} 2\,\mathrm{Re}\,(\bar{\alpha}_n z_n) - \sum_{iT \in I}\sum_{kT \in I} \bar{\alpha}_i s_{i-k} \alpha_k\right]\right\} \tag{11}$$

where

$$z_n = \int_I\int_I \bar{h}(t_1 - nT)K^{-1}(t_1 - t_2)y(t_2)\, dt_1\, dt_2 \tag{12}$$

$$s_l = \int_I\int_I \bar{h}(t_1 - iT)K^{-1}(t_1 - t_2)h(t_2 - kT)\, dt_1\, dt_2$$
$$= \bar{s}_{-l}, \qquad l = k - i. \tag{13}$$

The quantities z_n and s_l can be interpreted as sample values taken at the output of a complex MF with impulse response function[2]

$$g_{\mathrm{MF}}(t) = \bar{h}(-t) * K^{-1}(t). \tag{14}$$

The derivation presented is mathematically weak in that it assumes $K^{-1}(t)$ exists. This is not the case if the spectral power density of the noise becomes zero somewhere along the frequency axis. The difficulty can be avoided by defining z_n and s_l in terms of the reproducing-kernel Hilbert space (RKHS) approach [33], [34]. Here it is sufficient to consider the frequency-domain equivalent of (14) given by

$$G_{\mathrm{MF}}(f) = \bar{H}(f)/K(f) \tag{15}$$

where $G_{\mathrm{MF}}(f)$, $H(f)$, and $K(f)$ are the Fourier transforms of $g_{\mathrm{MF}}(t)$, $h(t)$, and $K(t)$, respectively. It follows from (15) that $g_{\mathrm{MF}}(t)$ exists if the spectral power density

[2] Here we are not concerned with realizability.

of the noise does not vanish within the frequency band where signal energy is received. Only in this case have we a truly probabilistic (nonsingular) receiver problem. It can easily be shown that

$$z_n = g_{MF}(t) * y(t)\,|_{t=nT} = \sum_l a_{n-l}s_l + r_n \qquad (16)$$

$$s_l = g_{MF}(t) * h(t)\,|_{t=lT} = \bar{s}_{-l} \qquad (17)$$

and that the covariance of the noise samples r_n reads

$$R_l = E(\bar{r}_n r_{n+l}) = 2N_0 s_l. \qquad (18)$$

Since the noise of the transmission medium does not exhibit distinct properties relative to the demodulating carrier phase, the following relations must hold:

$$E[\mathrm{Re}\,(r_n)\,\mathrm{Re}\,(r_{n+l})] = E[\mathrm{Im}\,(r_n)\,\mathrm{Im}\,(r_{n+l})] = N_0\,\mathrm{Re}\,(s_l) \qquad (19)$$

$$E[\mathrm{Re}\,(r_n)\,\mathrm{Im}\,(r_{n+l})] = -E[\mathrm{Im}\,(r_n)\,\mathrm{Re}\,(r_{n+l})] = N_0\,\mathrm{Im}\,(s_l). \qquad (20)$$

The similarity of s_l and R_l expressed by (18) implies that $s_0 \geq |s_l|$ and that the Fourier transform of the sampled signal element $\{s_l\}$ is a real nonnegative function

$$S^*(f) = \sum_l s_l \exp(-j2\pi flT) \geq 0 \qquad (21)$$

with period $1/T$. Clearly, the MF performs a complete phase equalization, but does not necessarily eliminate ISI (ISI: $s_l \neq 0$ for $l \neq 0$). The main effect of the MF is that it maximizes the SNR, which we define as

$$S/N_{MF} = \frac{\text{instantaneous peak power of a single signal element}}{\text{average power of the real part of the noise}} = \frac{s_0^2}{E\{\mathrm{Re}\,(r_0)^2\}} = \frac{s_0}{N_0}. \qquad (22)$$

The part of the receiver which in Fig. 1 was left open can now be specified, as indicated in Fig. 2. From (16) it comprises a MF and a symbol-rate sampling device sampling at times nT. It follows a processor, called maximum-likelihood sequence estimator (MLSE), that determines as the most likely sequence transmitted the sequence $\{\alpha_n\} = \{\hat{a}_n\}$ that maximizes the likelihood function given by (11), or equivalently, that assigns the maximum value to the metric

$$J_I(\{\alpha_I\}) = \sum_{nT\in I} 2\,\mathrm{Re}\,(\bar{\alpha}_n z_n) - \sum_{iT\in I}\sum_{kT\in I} \bar{\alpha}_i s_{i-k}\alpha_k. \qquad (23)$$

The values of s_l are assumed to be known. The sequence $\{z_n\}$ contains all relevant information available about $\{a_n\}$ and hence forms a so-called set of sufficient statistics [2], [3]. The main difficulty in finding $\{\hat{a}_n\}$ lies in the fact that $\{\hat{a}_n\}$ must only be sought among discrete sequences

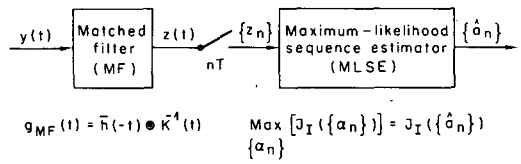

Fig. 2. MLR structure.

$\{\alpha_n\}$ which comply with the coding rule. The exact solution to this discrete maximization problem is presented in Section IV.

Solving the problem approximately by first determining the nondiscrete sequence $\{\alpha_n\} = \{z_{Ln}\}$ that maximizes (23) and then quantizing the elements of $\{z_{Ln}\}$ in independent symbol-by-symbol fashion, leads to the optimum conventional receiver [5]. Applying the familiar calculus of variation to (23), one finds that $\{z_{Ln}\}$ is obtained from $\{z_n\}$ by a linear transversal filter which, having the transfer function $1/S^*(f)$, eliminates ISI. The series arrangement of the MF and the transversal filter is known as the optimum linear equalizer, which for zero ISI yields maximum SNR [6], [8]. Calculating the SNR at the transversal filter output in a manner equivalent to (22), gives

$$S/N_L = \frac{S/N_{MF}}{T^2 \int_0^{1/T} S^*(f)\,df \int_0^{1/T} 1/S^*(f)\,df} \leq S/N_{MF}. \qquad (24)$$

This reflects the obvious fact that, since the MF provides the absolutely largest SNR, elimination of ISI by a subsequent filter must diminish the SNR. Equation (24) indicates, however, that a significant loss will occur only if somewhere along the frequency axis $S^*(f)$ dips considerably below the average value.

For systems that transmit only real pulse amplitudes, i.e., double-sideband amplitude modulation (DSB-AM), vestigial-sideband amplitude modulation (VSB-AM), and single-sideband amplitude modulation (SSB-AM), it follows from (23) that only the real output of the MF is relevant. In those cases $S^*(f)$ should be replaced by $\frac{1}{2}[S^*(-f) + S^*(f)]$ without further mention throughout the paper.

IV. MAXIMUM-LIKELIHOOD SEQUENCE ESTIMATION

In this section the exact solution to the discrete maximization problem of (23) is presented. The MLSE algorithm that will be derived determines the most likely sequence $\{\hat{a}_n\}$ among sequences $\{\alpha_n\}$ that satisfy the coding rule. Clearly, the straightforward approach of computing $J_I(\{\alpha_n\})$ for all sequences allowed, and selecting the sequence that yields the maximum value, is impracticable in view of the length and number of possible messages. Instead, by applying the principles of dynamic programing [22], we shall be able to conceive a nonlinear recursive algorithm that performs the same selection with greatly reduced computational effort. The MLSE algorithm thus obtained represents a modified version of the well-known Viterbi algorithm [18]-[21], [23]-[30].

The algorithm is obtained, observing $s_l = \bar{s}_{-l}$, by first realizing from (23) that $J_I(\{\alpha_n\})$ can iteratively be computed by the recursive relation

$$J_n(\cdots,\alpha_{n-1},\alpha_n) = J_{n-1}(\cdots,\alpha_{n-1}) + \mathrm{Re}\,[\bar{\alpha}_n(2z_n - s_0\alpha_n - 2\sum_{k\leq n-1} s_{n-k}\alpha_k)]. \tag{25}$$

Conditions concerning the boundaries of I are not needed since once we have a recursive relationship, the length of I becomes unimportant. We now assume that at the MF output ISI from a particular signal element is limited to L preceding and L following sampling instants:

$$s_l = 0, \qquad |l| > L. \tag{26}$$

Changing indices we obtain from (25) and (26)

$$J_n(\cdots,\alpha_{n-1},\alpha_n) = J_{n-1}(\cdots,\alpha_{n-1}) + \mathrm{Re}\,[\bar{\alpha}_n(2z_n - s_0\alpha_n - 2\sum_{l=1}^{L} s_l\alpha_{n-l})]. \tag{27}$$

We recall that sequences may be coded. For most transmission codes a state representation is appropriate. Let μ_j be the state of the coder after α_j has been transmitted. The coding state μ_j determines which sequences can be further transmitted. Given μ_j and an allowable sequence $\alpha_{j+1},\alpha_{j+2},\cdots,\alpha_{j+k}$, the state μ_{j+k} is uniquely determined as

$$\mu_j\colon \alpha_{j+1},\alpha_{j+2},\cdots,\alpha_{j+k} \longrightarrow \mu_{j+k}. \tag{28}$$

The sequence of states $\{\mu_j\}$ is Markovian, in the sense that $\Pr(\mu_j \mid \mu_{j-1},\mu_{j-2},\cdots) = \Pr(\mu_j \mid \mu_{j-1})$.

Let us now consider the metric

$$\begin{aligned}\tilde{J}_n(\mu_{n-L}\colon \alpha_{n-L+1},\cdots,\alpha_n) &= \tilde{J}_n(\sigma_n)\\ &= \max_{\{\cdots,\alpha_{n-L-1},\alpha_{n-L}\}\to\mu_{n-L}} \{J_n(\cdots,\alpha_{n-L},\alpha_{n-L+1},\cdots,\alpha_n)\}\end{aligned} \tag{29}$$

where the maximum is taken over all allowable sequences $\{\cdots,\alpha_{n-L-1},\alpha_{n-L}\}$ that put the coder into the state μ_{n-L}. In accordance with the VA literature, $\tilde{J}_n$ is called survivor metric. There exist as many survivor metrics as there are survivor states

$$\sigma_n \triangleq \mu_{n-L}\colon \alpha_{n-L+1},\cdots,\alpha_n. \tag{30}$$

Clearly, the succession of states $\{\sigma_n\}$ is again Markovian. Associated with each σ_n is a unique path history, namely, the sequence $\{\cdots,\alpha_{n-L-1},\alpha_{n-L}\}$, which in (29) yields the maximum. With respect to σ_n this sequence is credited ML among all other sequences. It is not difficult to see that the further one looks back from time $n - L$, the less will a path history depend on the specific σ_n to which it belongs. One can therefore expect that all σ_n will have a common path history up to some time $n - L - m$; m being a nonnegative random variable. Obviously, the common portion of the path histories concurs with the most likely sequence $\{\hat{\alpha}_n\}$ for which we are looking.

The final step in deriving the MLSE algorithm is to apply the maximum operation defined by (29) to (27). Introducing the notation of a survivor metric also on the right-hand side, we obtain

$$\tilde{J}_n(\sigma_n) = 2\,\mathrm{Re}\,(\bar{\alpha}_n z_n) + \max_{\{\sigma_{n-1}\}\to\sigma_n} \{\tilde{J}_{n-1}(\sigma_{n-1}) - F(\sigma_{n-1},\sigma_n)\} \tag{31}$$

where the maximum is taken over all states $\{\sigma_{n-1}\}$ that have σ_n as a possible successor state, and

$$F(\sigma_{n-1},\sigma_n) = \bar{\alpha}_n s_0 \alpha_n + 2\,\mathrm{Re}\,(\bar{\alpha}_n \sum_{l=1}^{L} s_l\alpha_{n-l}). \tag{32}$$

Verifying (31), the reader will observe that L is just the minimum number of pulse amplitudes that must be associated with σ_n. Thus, L takes on the role of a constraint length inherent to ISI. Equation (31) enables us to calculate survivor metrics and path histories in recursive fashion. The path history of a particular σ_n is obtained by extending the path history of the σ_{n-1}, which in (31) yields the maximum by the α_{n-L} associated with the selected σ_{n-1}. At each sampling instant n, survivor metrics and path histories must be calculated for all possible states σ_n. Instead of expressing path histories in terms of pulse amplitudes α_{n-L}, they could also be represented in any other one-to-one related terms.

This concludes the essential part in the derivation of the MLSE algorithm. The algorithm can be extended to provide maximum *a posteriori* probability (MAP) decisions, as is shown in Appendix II. However, for reasons given there, the performance improvement which thereby can be attained will usually be insignificant.

The algorithm is identical to the original Viterbi algorithm if there is no ISI at the MF output, i.e., $F(\sigma_{n-1},\sigma_n) = \bar{\alpha}_n s_0 \alpha_n$. In the presence of ISI the algorithm differs from the original Viterbi algorithm in that it operates directly on the MF output where noise samples are correlated according to (18). For the original Viterbi algorithm statistical independence of the noise samples is essential. Forney [18] proposed to decorrelate the MF output noise by a transversal filter which thereby reduces the number of nonzero sample values of the signal element from $2L + 1$ to $L + 1$. The constraint length inherent to ISI is therefore the same, namely, L, for both the Viterbi algorithm as used by Forney and its modified version presented in this section.[3] Clearly, this indicates some fundamental lower

[3] G. D. Forney, private communication.

limit to the complexity of MLSE. Yet the modified Viterbi algorithm offers computational advantages in that the large number of squaring operations needed for the original Viterbi algorithm [29] are no longer required. Only the simple multiplications by discrete pulse amplitude values occurring in Re $(\bar{a}_n z_n)$ must be executed in real time. It was observed by Price [35] that Forney's whitened MF is identical to the optimum linear input section of a decision-feedback receiver. In Section VI we shall see that basically the same principle can be applied in order to realize the MF and the whitened MF in adaptive form. Hence in this respect the two algorithms are about equal.

We shall now illustrate the algorithm by a specific example. Let us consider a simple binary run-length-limited code with $a_n \in \{0,1\}$ and runs no longer than two of the same symbol. The state-transition diagram of this code is shown in Fig. 3(a). According to (30), with $L = 1$ the following survivor states and allowed transitions between them are obtained:

$$\sigma^1 \triangleq (\mu^1{:}1) \rightarrow (\mu^3{:}\{0,1\}) \triangleq \{\sigma^4,\sigma^5\}$$

$$\sigma^2 \triangleq (\mu^2{:}0) \rightarrow (\mu^1{:}1) \triangleq \sigma^1$$

$$\sigma^3 \triangleq (\mu^2{:}1) \rightarrow (\mu^3{:}\{0,1\}) \triangleq \{\sigma^4,\sigma^5\}$$

$$\sigma^4 \triangleq (\mu^3{:}0) \rightarrow (\mu^2{:}\{0,1\} \triangleq \{\sigma^2,\sigma^3\}$$

$$\sigma^5 \triangleq (\mu^3{:}1) \rightarrow (\mu^4{:}0) \triangleq \sigma^6$$

$$\sigma^6 \triangleq (\mu^4{:}0) \rightarrow (\mu^2{:}\{0,1\}) \triangleq \{\sigma^2,\sigma^3\}.$$

The corresponding state-transition diagram is depicted in Fig. 3(b). By introducing the time parameter explicitly, we obtain Fig. 4, in which allowed transitions are indicated by dashed lines (the so-called trellis picture [20]). The solid lines, as an example, represent the path histories of the six possible states σ_n and demonstrate their tendency to merge at some time $n - L - m$ into a common path. As the algorithm is used to compute the path histories of the states σ_{n+i}, $i = 1,2,\cdots$, new path-history branches appear on the right, whereas certain existing branches are not continued further and disappear. In this way, with some random time lag $L + m$, $m \geq 0$, a common path history develops from left to right.

In order to obtain the output sequence $\{\hat{a}_n\}$ only the last, say, M, pulse amplitudes of each path history have to be stored. M should be chosen such that the probability for $m > M$ is negligible compared with the projected error probability of the ideal system (infinite M). Then, at time n, the α_{n-L-M} of all path histories will with high probability be identical; hence any one of them can be taken as $\hat{a}_{n-L-M}$. The path histories can now be shortened by the α_{n-L-M}. Thus the path histories are kept at length M, and a constant delay through the MLSE of $L + M$ symbol intervals results. For decoding of convolutional codes the value of M has been discussed in the literature [20], [36], [37]. At the present time no such results are available for the ISI case.

A refined way of selecting $\hat{a}_{n-L-M}$, which allows for a

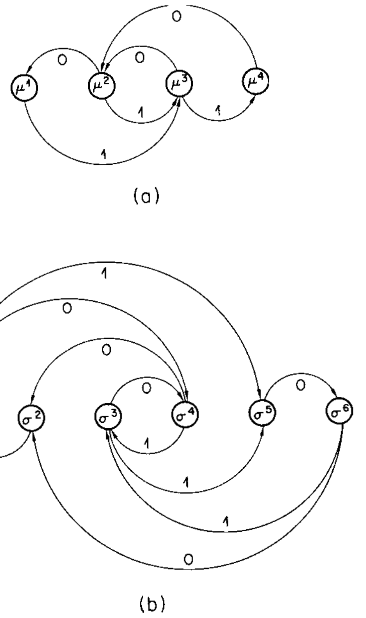

Fig. 3. (a) State-transition diagram of binary ($a_n = \{0,1\}$) run-length-limited code with runs ≤ 2. (b) State-transition diagram of corresponding survivor states for $L = 1$.

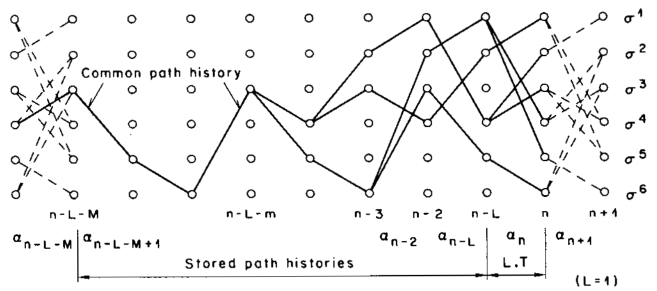

Fig. 4. Time-explicit representation of Fig. 3(b) (dashed lines) and illustration of path histories at time n (solid lines).

reduction of M, is to take α_{n-L-M} for the path history corresponding to the largest survivor metric. From (31) it is clear that without countermeasures the survivor metrics would steadily increase in value. A suitable method of confining the survivor metrics to a finite range is to subtract the largest $\tilde{J}_{n-1}$ from all $\tilde{J}_n$ after each iteration.

V. ERROR PERFORMANCE

Since the receiver of this paper realizes the same decision rule as Forney's receiver [18], it is not surprising that identical error performance will be found. In this section, following closely Forney's approach, we present a short derivation of the error-event probability for the modified Viterbi-algorithm case. The influence of ISI present at the MF output on the error performance of the MLR is discussed, and bounds for essentially no influence are given in explicit form. The results are compared with the error performance of the optimum conventional receiver.

We recall that $\{a_n\}$ represents the data sequence transmitted, whereas $\{\hat{a}_n\}$ denotes the sequence estimated by the receiver. Then

$$\{e_n\} = \{\hat{a}_n\} - \{a_n\} \tag{33}$$

is the error sequence. Since consecutive symbol errors are generally not independent of each other, the concept of sequence error events must be used. Hence, as error events we consider short sequences of symbol errors that intuitively are short compared with the mean time between them and that occur independently of each other. Presuming stationarity, the beginning of a specific error event $\mathcal{E}$ can arbitrarily be aligned with time 0:

$$\mathcal{E}: \{e_n\} = \cdots,0,0,e_0,e_1,\cdots,e_H,0,0,\cdots; \qquad |e_0|, |e_H| \geq \delta_0, \quad H \geq 0. \tag{34}$$

Here δ_0 denotes the minimum symbol error distance

$$\delta_0 = \min_{i \neq k} \{ | a^{(i)} - a^{(k)} | \}. \tag{35}$$

We are not concerned with the meaning of error events in terms of erroneous bits of information.

Let E be the set of events $\mathcal{E}$ permitted by the transmission code. For a distinct event $\mathcal{E}$ to happen, two subevents must occur:

- $\mathcal{E}_1$: $\{a_n\}$ is such that $\{a_n\} + \{e_n\}$ is an allowable data sequence;
- $\mathcal{E}_2$: the noise terms are such that $\{a_n\} + \{e_n\}$ has ML (within the observation interval).

It is useful to define beyond that the subevent

- $\mathcal{E}_2'$: the noise terms are such that $\{a_n\} + \{e_n\}$ has greater likelihood than $\{a_n\}$, but not necessarily ML.

Then we have

$$\Pr(\mathcal{E}) = \Pr(\mathcal{E}_1) \Pr(\mathcal{E}_2 \mid \mathcal{E}_1) \leq \Pr(\mathcal{E}_1) \Pr(\mathcal{E}_2' \mid \mathcal{E}_1) \tag{36}$$

where $\Pr(\mathcal{E}_1)$ depends only on the coding scheme. Events $\mathcal{E}_1$ are generally not mutually exclusive. Note that in (36) conditioning of $\mathcal{E}_2$ and $\mathcal{E}_2'$ on $\mathcal{E}_1$ tightens the given bound, since prescribing $\mathcal{E}_1$ reduces the number of other events that could, when $\mathcal{E}_2'$ occurs, still have greater likelihood, so that $\mathcal{E}_2$ would not be satisfied. From (23) we conclude that $\Pr(\mathcal{E}_2' \mid \mathcal{E}_1)$ is the probability that

$$J_I(\{a_n\}) < J_I(\{a_n\} + \{e_n\}). \tag{37}$$

By substituting (16) into (23), and observing (33) and $s_l = \bar{s}_{-l}$, (37) becomes

$$\delta^2(\mathcal{E}) \triangleq \frac{1}{s_0} \sum_{i=0}^{H} \sum_{k=0}^{H} \bar{e}_i s_{i-k} e_k < \frac{2}{s_0} \operatorname{Re} [\sum_{i=0}^{H} \bar{e}_i r_i]. \tag{38}$$

We call $\delta(\mathcal{E})$ the distance of $\mathcal{E}$. The right-hand side of (38) is a normally distributed random variable with zero mean and, from (18), (19), and (20), variance

$$\operatorname{var} \left\{ \frac{2}{s_0} \operatorname{Re} [\sum_{i=0}^{H} \bar{e}_i r_i] \right\} = \frac{4N_0}{s_0} \delta^2(\mathcal{E}). \tag{39}$$

Hence, observing (22), the probability of (37) being satisfied is given by

$$\Pr(\mathcal{E}_2' \mid \mathcal{E}_1) = Q[(S/N_{\mathrm{MF}})^{1/2} \delta(\mathcal{E})/2] \tag{40}$$

where

$$Q(x) = \frac{1}{(2\pi)^{1/2}} \int_x^\infty \exp(-y^2/2)\, dy \simeq \frac{1}{x(2\pi)^{1/2}} \exp(-x^2/2), \qquad x > 3.5. \tag{41}$$

We continue as indicated by Forney [18]. Let $E(\delta)$ be the subset of E containing all events $\mathcal{E}$ with distance $\delta(\mathcal{E}) = \delta$. Let Δ be the set of the possible values of δ. From (36) and (40) the probability that any error event $\mathcal{E}$ occurs becomes and is upper bounded by

$$\Pr(E) = \sum_{\mathcal{E} \epsilon E} \Pr(\mathcal{E}) \leq \sum_{\delta \epsilon \Delta} Q((S/N_{\mathrm{MF}})^{1/2}\delta/2) \sum_{\mathcal{E} \epsilon E(\delta)} \Pr(\mathcal{E}_1). \tag{42}$$

Owing to the steep decrease of $Q(x)$, the right-hand side of (42) will already at moderate SNR be dominated by the term involving the smallest value in Δ, denoted by $\delta_{\min}$. Likewise, the bound given by (36) becomes tight for all $\mathcal{E} \in E(\delta_{\min})$, as then $\mathcal{E}_2'$ very likely implies $\mathcal{E}_2$. Consequently, as the SNR is increased, $\Pr(E)$ approaches asymptotically

$$\Pr(E) \simeq Q((S/N_{\mathrm{MF}})^{1/2}\delta_{\min}/2) \sum_{\mathcal{E} \epsilon E(\delta_{\min})} \Pr(\mathcal{E}_1) \tag{43}$$

where

$$\delta_{\min}{}^2 = \min_{\mathcal{E} \epsilon E} \left\{ \frac{1}{s_0} \sum_{i=0}^{H} \sum_{k=0}^{H} \bar{e}_i s_{i-k} e_k \right\}. \tag{44}$$

Equations (43) and (44) differ only in notation from Forney's original finding.

In the following this result should be discussed in more detail. Specifically, we are interested in the influence of ISI on the value of $\delta_{\min}$. Using Parseval's theorem, from (21) and (38), $\delta^2(\mathcal{E})$ can be rewritten in the form

$$\delta^2(\mathcal{E}) = \frac{T}{s_0} \int_0^{1/T} S^*(f) E^*(f)\, df \tag{45}$$

where $E^*(f)$ is the energy density spectrum of the error sequence $\{e_n\}$,

$$E^*(f) = \sum_{i=0}^{H} \sum_{k=0}^{H} \bar{e}_i \exp[j2\pi f(i-k)T] e_k \geq 0. \tag{46}$$

If $S^*(f)$ were constant (no ISI at the MF output), (45) becomes

$$\delta^2(\mathcal{E}) = T \int_0^{1/T} E^*(f)\, df = \sum_{i=0}^{H} |e_i|^2 \geq w_H(\mathcal{E}) \delta_0^2 \tag{47}$$

where $w_H \geq 1$ denotes the number of nonzero symbol errors of $\mathcal{E}$. In this case $\delta_{\min}$ would simply be the smallest Euclidian distance between any two allowed data sequences. In a noncoded system, where pulse amplitudes of a given alphabet may occur in arbitrary succession, single error events ($w_H = 1$) with minimum distance $\delta_{\min} = \delta_0$ would be the dominating error events. The value of $\delta_{\min}$ can be increased by redundant sequence coding, e.g., by convolutional encoding [36]. If $S^*(f)$ is not constant, ISI at the MF output distorts the space in which error-event distances are measured. Depending on the weighting of $E^*(f)$ by $S^*(f)$, error-event distances can become smaller or larger. By sequence coding one can prevent that error sequences are allowed which have spectral peaks where $S^*(f)$ is small. This is precisely what is accomplished by correlative-level (partial-response) coding [25]. Clearly, if $S^*(f)$ vanishes on one side of $f = 0$, as in SSB or VSB systems, only (real) data sequences with symmetric error-sequence spectra $E^*(f) = E^*(-f)$ can be transmitted.

We now limit our attention to noncoded systems. As long as ISI does not exceed limits discussed further in the following paragraphs, we have $\delta_{\min} = \delta_0$. From (43) the probability of occurrence of the then dominating single error events becomes

$$\Pr(E) \simeq Q((S/N_{\mathrm{MF}})^{1/2}\delta_0/2) \cdot \sum_{|e_0|=\delta_0} \Pr(e_0 \text{ allowed}), \qquad \delta_{\min} = \delta_0. \tag{48}$$

For comparison, the error performance of the optimum conventional receiver is given by

$$\Pr(E) \simeq Q((S/N_L)^{1/2}\delta_0/2) \sum_{|e_0|=\delta_0} \Pr(e_0 \text{ allowed}), \qquad S/N_L \le S/N_{\mathrm{MF}}. \tag{49}$$

We note that in (48) ISI at the MF output has essentially no influence on the error performance of the MLR, whereas in (49) ISI affects the error performance of the conventional receiver through the loss of SNR expressed by (24). The evaluation of Pr (E) is shown by Fig. 5 for a specific octal AM–PM scheme.

In order to determine the degree of ISI up to which (48) holds, we must look for multiple error events ($w_H \ge 2$) with distance smaller than δ_0. Such error events would then be more probable than the minimum single error events. A first condition for the nonexistence of such events can be derived from (45) and the inequality expressed in (47). Noting that

$$\delta^2(\mathcal{E}) \ge \frac{1}{s_0} \min\{S^*(f)\} w_H(\mathcal{E})\delta_0^2 \tag{50}$$

it follows that if

$$\min\{S^*(f)\} w_H(\mathcal{E}) \ge s_0 \triangleq T \int_0^{1/T} S^*(f)\, df \tag{51}$$

is satisfied, no event $\mathcal{E}$ can have smaller distance than δ_0. Hence, a sufficient but not necessary condition for the nonexistence of multiple error events with distance smaller

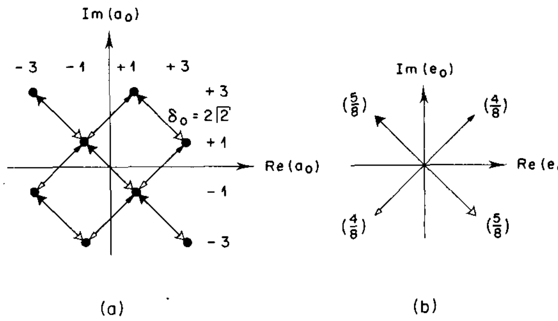

Fig. 5. (a) Minimum symbol error distance in a specific octal pulse-amplitude alphabet. (b) Minimum symbol errors and probabilities that they may occur (assuming that pulse amplitudes are transmitted with equal probability).
$\sum_{|e_0|=\delta_0} \Pr(e_0 \text{ allowed}) = 2(\frac{4}{8} + \frac{5}{8}) = \frac{9}{4}$, $\quad \Pr(E) \simeq \frac{9}{4} Q((2 \cdot S/N)^{1/2})$.

than δ_0 is that $S^*(f)$ dips nowhere more than 6 dB [$20 \log (w_H = 2)$] below average value.

A second generally less restrictive condition is that

$$\sum_{l \ne 0} |s_l| \le s_0 \tag{52}$$

which is the familiar condition for peak distortion at the MF output being smaller than unity. In order to prove this sufficient but again not necessary condition for the nonexistence of error events $\mathcal{E}$ with distance smaller than δ_0, one should first realize from (34) and the Schwarz inequality that

$$\left| \sum_{k=0}^{H} \bar{e}_{l+k} e_k \right| \le \sum_{k=0}^{H} |e_k|^2 - \delta_0^2, \qquad l \ne 0. \tag{53}$$

Condition (52) can then be verified by transforming and bounding $\delta^2(\mathcal{E})$ as follows:

$$\begin{aligned} \delta^2(\mathcal{E}) &= \frac{1}{s_0} \sum_l s_l \sum_{k=0}^{H} \bar{e}_{l+k} e_k \\ &\ge \sum_{k=0}^{H} |e_k|^2 - \frac{1}{s_0} \sum_{l \ne 0} |s_l| \left| \sum_{k=0}^{H} \bar{e}_{l+k} e_k \right| \\ &\ge \delta_0^2 + \left(\sum_{k=0}^{H} |e_k|^2 - \delta_0^2\right)\left(1 - \frac{1}{s_0} \sum_{l \ne 0} |s_l|\right) \\ &\ge \delta_0^2 \qquad [\text{holds, if (52) is true}]. \end{aligned} \tag{54}$$

Comparing (52) with the definition of $S^*(f)$ in (21) reveals that at distinct frequencies, $S^*(f)$ may approach zero level without this significantly affecting the error performance of the MLR. This was first observed by Kobayashi [25] for ML decoding of $(2m - 1)$-ary correlative-level encoded signals. A signal of this kind can just as well be interpreted as a noncoded m-ary signal with intentionally introduced ISI, which causes $S^*(f)$ to become zero (usually) at $f = 0$ and/or $1/2T$. For the MLR the two concepts are equivalent. A conventional receiver, however, can interpret such signals only as $(2m - 1)$-ary coded sequences and thereby loses in the limit 3 dB, unless

(vs. partial response precoding)

error correcting schemes are used. But even then the loss can only partly be compensated, since the hard decisions made by the symbol-by-symbol decision circuit of the conventional receiver cause an irreversible loss of information.

VI. AUTOMATIC RECEIVER ADAPTATION

So far the exact signal and timing characteristics have been assumed to be known. However, in a realistic case the MLR must at least be able to extract the carrier phase and sample timing from the signal received. Beyond that, automatic adjustment of the MF will often be desirable or necessary. In this section we present an algorithm that simultaneously adjusts the demodulating carrier phase and the sample timing, approximates the MF by a transversal filter, and estimates ISI present at the approximated MF output. The algorithm works in decision-directed mode in much the same way as described by Kobayashi [5] and Qureshi and Newhall [29].

In the proposed fully adaptive MLR the MF is approximated by a transversal filter, similar to the familiar adaptive equalizers described by Lucky *et al.* [1], [38], [39] and others [5], [40]–[45]. An analog implementation will be assumed. Assuming $N+1$ taps equally spaced by τ_p seconds with tap gains g_i, $0 \leq i \leq N$, the output signal of the transversal filter at the nth sampling instant $nT + \tau_s$, where τ_s denotes the sampling phase, becomes

$$\hat{z}_n = \sum_{i=0}^{N} g_i y(nT + \tau_s - i\tau_p, \varphi_c) \triangleq \sum_{i=0}^{N} g_i y_{ni}(\tau_s, \varphi_c). \quad (55)$$

Note that according to (4) and (5) we have $y(t,\varphi_c) \triangleq y(t,\varphi_c = 0) \exp(-j\varphi_c)$, where φ_c is the demodulating carrier phase. The ideal MF characteristic can be accomplished if $1/2\tau_p$ exceeds or at least is equal to the bandwidth of $y(t)$, and $N\tau_p$ corresponds to the duration of the signal element $h(t)$.

For the derivation of the adjustment algorithm we make the usual assumption that the transmitted data sequence $\{a_n\}$ is known. The decision delay of the Viterbi algorithm will be taken into account later when we devise the final adaptive MLR structure. In Section III we have seen that the MF is rigorously defined by the fact that it minimizes the noise power relative to the instantaneous peak power of the signal element. Let $\hat{s}_l$, $|l| \leq L$, be estimated values of the sample values of the signal element at the transversal filter output. Then

$$\hat{r}_n = \hat{z}_n - \sum_{l=-L}^{L} \hat{s}_l a_{n-l} \quad (56)$$

represents the estimated noise component of $\hat{z}_n$. In order to adjust the parameters g_i, $\hat{s}_l$, τ_s, and φ_c, the variance of $\hat{r}_n$,

$$\begin{aligned} \operatorname{var}(\hat{r}_n) &= \sum_{i=0}^{N}\sum_{k=0}^{N} \bar{g}_i E[\bar{y}_{ni}(\tau_s,\varphi_c) y_{nk}(\tau_s,\varphi_c)] g_k \\ &\quad - 2\operatorname{Re}\{\sum_{i=0}^{N}\sum_{l=-L}^{L} \bar{g}_i E[\bar{y}_{ni}(\tau_s,\varphi_c) a_{n-l}]\hat{s}_l\} \\ &\quad + \sum_{i=-L}^{L}\sum_{k=-L}^{L} \bar{\hat{s}}_i E[\bar{a}_{n-i} a_{n-k}]\hat{s}_k \end{aligned} \quad (57)$$

must be minimized as a function of these parameters, with $\hat{s}_0$ held constant. Differentiating (57) and applying the Robbins–Monro stochastic approximation method [46] leads to the stochastic steepest-descent algorithm comprising the following recursive relations:

$$g_i^{(n+1)} = g_i^{(n)} - \alpha_g^{(n)} \hat{r}_n \bar{y}_{ni}, \qquad 0 \leq i \leq N \quad (58)$$

$$\hat{s}_l^{(n+1)} = \hat{s}_l^{(n)} + \alpha_s^{(n)} \hat{r}_n \bar{a}_{n-l}, \qquad 1 \leq |l| \leq L \quad (59)$$

$$\tau_s^{(n+1)} = \tau_s^{(n)} - \alpha_\tau^{(n)} \operatorname{Re}(\hat{r}_n \bar{\dot{\hat{z}}}_n), \quad (60)$$

$$\varphi_c^{(n+1)} = \varphi_c^{(n)} + \alpha_\varphi^{(n)} \operatorname{Im}(\hat{r}_n \bar{\hat{z}}_n). \quad (61)$$

The step-size gains α_g, α_s, α_τ, and α_φ must be positive and may depend on n. In (60) $\dot{\hat{z}}_n$ denotes the time derivative of the transversal filter output at the nth sampling instant. As the algorithm adjusts the transversal filter as MF, the values $\hat{s}_l$ approach the values s_l required by the MLSE algorithm.

Equations (58) and (59) differ from the corresponding equations of an adaptive decision-feedback equalizer, or whitened MF, only by the fact that here at the transversal filter output L preceding *and* L trailing ISI terms are considered. The algorithm will force ISI outside this interval to zero. The true MF characteristic may often require a large value of L. However, since the complexity of the MLSE algorithm increases exponentially with L, for the choice of L a compromise suggests itself [29]. In many practical cases already with values $L = 1$ or $L = 2$, a good approximation of the ideal MF characteristic will be obtained. The potential advantages of MLSE can thus be exploited to a commensurate degree at a still manageable receiver complexity.

Introducing the symmetry condition $\hat{s}_{-l} = \bar{\hat{s}}_l$ into (56), we obtain instead of (59)

$$\hat{s}_l^{(n+1)} = \hat{s}_l^{(n)} + \alpha_s^{(n)} (\bar{\hat{r}}_n a_{n+l} + \hat{r}_n \bar{a}_{n-l}), \qquad 1 \leq l \leq L. \quad (62)$$

This modification has the desirable effect of forcing the transversal filter to produce at its output a symmetric signal element even if L and the transversal filter parameters are not fully adequate to achieve therewith the ideal MF characteristic.

Equations (60) and (61) have been reported by Kobayashi [5]. They describe the operation of two first-order phase-locked loops. Theoretically, if by (58) the (complex) tap gains are adapted, the adjustment of τ_s and φ_c appears to be not really necessary. In practice, however, these phases must be controlled in order to compensate carrier and sampling frequency offsets. In case of considerable offset one might even add second-order terms to (60) and (61).

The structure of the proposed MLR is seen in Fig. 6. It is basically a combination of the approaches of Kobayashi [5] and Qureshi and Newhall [29], except that here the transversal filter approximates a true MF with ISI at the transversal filter not being predefined. The receiver operates in decision-directed mode with the MLSE exhibiting a decision delay of $M + L$ symbol intervals (see

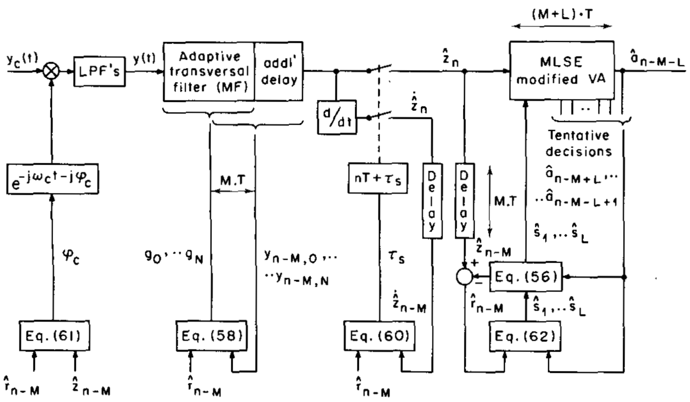

Fig. 6. Adaptive MLR.

Section IV). In order to shorten the feedback delay, tentative decisions taken from the path history with largest survivor metric are employed in the feedback paths as suggested by Qureshi [30]. With this approach, delays of M symbol intervals must be used in the forward paths. Not shown in Fig. 6 is the possibility of incorporating decision-feedback cancellation of further trailing ISI in the receiver [30].

In the remainder of this section we discuss topics related to convergence and convexity of the adjustment algorithm. It must be assumed that already safe enough decisions are available. To begin with, τ_s and φ_c are considered as given constant values. With sufficiently small step-size gains α_g and α_s, convergence from arbitrary initial settings $g_i^{(0)}$ and $\hat{s}_l^{(0)}$ towards globally optimum settings $g_i^*(\tau_s,\varphi_c)$ and $\hat{s}_l^*(\tau_s)$ is assured by the quadratic positive-definite nature of var $(\hat{r}_n)$ in g_i and $\hat{s}_l$. For adaptive zero-forcing equalizers (equivalent to $L = 0$) it has been shown [45] that fastest convergence takes place if α_g corresponds to $1/[N \times$ average power of $y(t)]$. The additional variability introduced by (59) and eventually (60) and (61), and the delay of M symbol intervals necessary for decision-directed operation, suggest that α_g must here be somewhat smaller than the preceding value.

In Appendix III we calculate $g_i^*(\tau_s,\varphi_c)$ and $\hat{s}_l^*(\tau_s)$, thus showing that with adequate values of N, L, τ_p, and τ_s the transversal filter indeed assumes the desired MF characteristic. We further evaluate the minimum value of var $(\hat{r}_n)$ as a function of τ_s. On the whole, convexity is found within an interval comparable to the length of the transversal filter delay line; yet, depending on the value of τ_p, there can be some small ripple. If $1/2\tau_p$ does not exceed the bandwidth of $y(t)$ this ripple can even become quite pronounced. In any case, the form of var $(\hat{r}_n)$ will guarantee that τ_s does not drift away when all quantities g_i, $\hat{s}_l$, τ_s, and φ_c are adjusted simultaneously.

Another case of interest relates to carrier and timing synchronization in an otherwise already optimized and fixed receiver. In Appendix III var $(\hat{r}_n)$ is calculated as a function of τ_s and φ_c for fixed values $g_i^*(\tau_s^*,\varphi_c^*)$ and $\hat{s}_l^*(\tau_s^*)$, τ_s^* and φ_c^* being the timing phase and carrier phase to which the optimum receiver settings correspond. It is shown that var $(\hat{r}_n)$ is convex in a region around τ_s^* and φ_c^*, which depends mainly on the form of the signal element at the MF output. With the receiver working in decision-directed mode and disregarding phase ambiguity, τ_s and φ_c must in principle be close to the optimum settings. Convergence towards τ_s^* and φ_c^* should therefore generally not be a problem.

VII. SUMMARY AND CONCLUSIONS

A uniform fully adaptive receiver structure has been derived for synchronous data-transmission systems that employ linear carrier-modulation techniques. The structure realizes the ML sequence rule. In the receiver, first an information reduction to a set of sufficient statistics takes place by the demodulation, matched filtering, and symbol-rate sampling process. Sequence estimation is performed by a modified Viterbi algorithm that exhibits the same performance characteristics as the original scheme. The algorithm represents an attractive design alternative due to the fact that squaring operations are no longer needed. Besides add and compare operations, only a few simple multiplications by discrete pulse-amplitude values must be performed in real time. In addition to performance gains realized by the MLSE principle, one may expect that the approximation of the MF will generally require fewer filter taps than are needed for the zero-forcing equalizer of a conventional receiver. The proposed adaptation scheme permits compromise solutions between the conventional receiver and the ideal ML receiver.

The choice of the decoding delay of MLSE in the presence of ISI and the dynamics of the presented adjustment algorithm have not been discussed in detail. Also the issues of effective QAM coding and joint transmitter–receiver design have not been addressed. These could be fruitful areas for further research. For example, how should the transmitter filter be designed for a given channel characteristic in order to attain with (52) as secondary condition maximum SNR at the matched filter output? The specific implementation of ML receivers will be another interesting topic. Recent progress in circuit technology will allow here for much more complex designs than we are still used to.

APPENDIX I

PROOF OF (9)

Owing to the one-to-one relation between $w_c(t)$ and $w(t)$ expressed by (5) we have [32]

$$p[w_c(t), t \in I] = p[w(t), t \in I] \sim \exp\left\{-\frac{1}{2}\int_I\int_I w_c(t_1)\,W_c^{-1}(t_1 - t_2)\,w_c(t_2)\,dt_1\,dt_2\right\}. \tag{A1}$$

It follows from (6), (7), and (10) that

$$W_c^{-1}(\tau) = \frac{1}{N_0}K^{-1}(\tau)\exp(+j\omega_c\tau). \tag{A2}$$

Substituting (A2) into (A1) and observing (5) we obtain

$$p[w(t),t \in I] \sim \exp\left\{-\frac{1}{4N_0}\int_I\int_I \bar{w}(t_1)K^{-1}(t_1-t_2)w(t_2)\,dt_1\,dt_2\right\}. \tag{A3}$$

APPENDIX II

EXTENSION OF THE MLSE ALGORITHM TO THE MAP RULE

We proceed as indicated by Forney [27]. To satisfy the MAP rule the algorithm has to determine the sequence $\{\alpha_n\}$ which maximizes

$$\Pr[\{\alpha_n\} \mid y(t),t \in I] \sim p[y(t),t \in I \mid \{\alpha_n\}]\Pr[\{\alpha_n\}]. \tag{A4}$$

Since there is a one-to-one correspondence between $\{\alpha_n\}$ and the sequence of survivor states $\{\sigma_n\}$, and since $\{\sigma_n\}$ is Markovian, we have

$$\Pr[\{\alpha_n\}] = \Pr[\{\sigma_n\}] \sim \prod_{nT\epsilon I} \Pr(\sigma_n \mid \sigma_{n-1}). \tag{A5}$$

It follows from (30) that

$$\Pr(\sigma_n \mid \sigma_{n-1}) = \Pr(\mu_n \mid u_{n-1}). \tag{A6}$$

Taking the logarithm of (A4) and observing (A5), it is seen that transitions (σ_{n-1},σ_n) are to be weighted by $\ln[\Pr(\sigma_n \mid \sigma_{n-1})]$. In this way the MAP version of (31) becomes

$$\tilde{J}_n(\sigma_n) = 2\,\mathrm{Re}(\bar{\alpha}_n z_n) + \max_{\{\sigma_{n-1}\}\to\sigma_n}\{\tilde{J}_{n-1}(\sigma_{n-1}) - F(\sigma_{n-1},\sigma_n) + 4N_0\ln[\Pr(\sigma_n \mid \sigma_{n-1})]\}. \tag{A7}$$

The factor $4N_0$ follows from (11) and, according to (22), is inversely proportional to the SNR at the MF output. The ML rule and the MAP rule are therefore equivalent for infinite SNR. The MAP rule can offer a significant advantage only at very low SNR's and when a code is used that leads to considerable differences among the conditional probabilities $\Pr(\sigma_n \mid \sigma_{n-1})$.

APPENDIX III

First, we show that minimizing $\mathrm{var}(\hat{r}_n)$ with $\hat{s}_0$ held constant indeed adjusts the transversal filter as MF, and that thereby the values of s_l are provided to a sufficient degree of approximation. Second, we study the convexity of $\mathrm{var}(\hat{r}_n)$ relative to the sampling phase τ_s. Assuming sampling instant $n = 0$, we drop the index n in the following calculations.

To begin with, the sampling phase τ_s and the demodulating carrier phase φ_c are considered as given constant values. It follows from (57) that the optimum tap gains g_i^* and values $\hat{s}_l^*$ which minimize $\mathrm{var}(\hat{r})$ must satisfy

$$\sum_{k=0}^{N} E(\bar{y}_i y_k)g_k^* - \sum_{l=-L}^{L} E(\bar{y}_i a_{-l})\hat{s}_l^* = 0, \qquad 0 \le i \le N \tag{A8}$$

and

$$\sum_{i=0}^{N} E(\bar{a}_{-l} y_i)g_i^* - \sum_{k=-L}^{L} E(\bar{a}_{-l}a_{-k})\hat{s}_k^* = \begin{cases} 0, & 1 \le |l| \le L \\ -\lambda, & l = 0 \end{cases} \tag{A9}$$

where λ acts as Lagrangian multiplier. Substitution o (A8) and (A9) into (57) yields

$$\mathrm{var}[\hat{r} \mid \{g_i^*(\tau_s,\varphi_c)\},\{\hat{s}_l^*(\tau_s)\}] = \mathrm{var}(\hat{r} \mid \tau_s) = \lambda. \tag{A10}$$

From (3) and (55) we have

$$y_i(\tau_s) = \sum_k a_k h(\tau_s - i\tau_p - kT) + w(\tau_s - i\tau_p). \tag{A11}$$

With power series notations

$$h(t,D) = \sum_k h(t+kT)D^k \tag{A12}$$

$$A(D) = \sum_k E(\bar{a}_0 a_k)D^k \tag{A13}$$

$$\hat{s}^*(D) = \sum_{l=-L}^{L} \hat{s}_l^* D^l \tag{A14}$$

(A8) and (A9) can be rewritten in the form

$$\{\bar{h}(\tau_s - i\tau_p,D^{-1})\sum_{k=0}^{N} A(D)h(\tau_s - k\tau_p,D)g_k^*\}_{D0} + \sum_{k=0}^{N} W[(i-k)\tau_p]g_k^* = \{\bar{h}(\tau_s - i\tau_p,D^{-1})A(D)\hat{s}^*(D)\}_{D0}, \qquad 0 \le i \le N \tag{A15}$$

and

$$\{\sum_{k=0}^{N} A(D)h(\tau_s - k\tau_p,D)g_k^*\}_{Dl} = \{A(D)\hat{s}^*(D) - \lambda\}_{Dl}, \qquad |l| \le L. \tag{A16}$$

Here $\{\cdot\}_{Dl}$ indicates the coefficient belonging to D^l. Witl L not being too restricted, and $w_{ik} = W[(i-k)\tau_p]$ substitution of (A16) into (A15) yields in good approx imation

$$\sum_{k=0}^{N} w_{ik}g_k^* \simeq \lambda\bar{h}(\tau_s - i\tau_p), \qquad 0 \le i \le N. \tag{A17}$$

Let w_{ik}^{-1} be the elements of the inverse of the $(N+1)\times$

$(N+1)$ matrix with elements w_{ik}. Then from (A17),

$$g_k^* \simeq \lambda \sum_{i=0}^{N} \bar{h}(\tau_s - i\tau_p) w_{ki}^{-1}, \qquad 0 \leq k \leq N. \quad \text{(A18)}$$

Substituting (A18) into (A16), and observing (A12) and (A14), we obtain

$$\hat{s}_l^* \simeq \lambda \sum_{i=0}^{N} \sum_{k=0}^{N} \bar{h}(\tau_s - i\tau_p) w_{ki}^{-1} h(\tau_s - k\tau_p + lT) + \lambda \{A^{-1}(D)\}_{Dl}, \qquad |l| \leq L. \quad \text{(A19)}$$

For moderate SNR we can neglect the last term in (A19) which involves minor approximation. Comparison of (A18) with (14) and of (A19) with (17) exhibits that with adequate values of N, τ_p, L, and τ_s, the desired adjustment of the adaptive MLR will be achieved.

We investigate now the convexity of var $(\hat{r})$ relative to τ_s. Considering (A10) and determining λ from (A19) with $l = 0$, we find

$$\text{var}\,(\hat{r} \mid \tau_s) \simeq \frac{\hat{s}_0}{\sum_{i=0}^{N} \sum_{k=0}^{N} \bar{h}(\tau_s - i\tau_p) w_{ki}^{-1} h(\tau_s - k\tau_p)}. \quad \text{(A20)}$$

The denominator of (A20) expresses the weighted energy of the values of $h(t)$ seen at time $t = \tau_s$ at the transversal filter taps. We assume that the length of the transversal filter delay line $N\tau_p$ exceeds or at least corresponds to the duration of $h(t)$. Var $(\hat{r} \mid \tau_s)$ will then, on the whole, be convex within an interval comparable to $N\tau_p$. Unless the tap spacing τ_p is very small, however, there will be some ripple within this region.

Suppose now that for some τ_s^* and φ_c^* the optimum values g_i^* and $\hat{s}_l^*$ are given. In attempting to resynchronize the receiver, only τ_s and φ_c should be readjusted. Let $\Delta\tau_s = \tau_s - \tau_s^*$ and $\Delta\varphi_c = \varphi_c - \varphi_c^*$. In (57) only the second term depends noticeably on $\Delta\tau_s$ and $\Delta\varphi_c$. Using (A11) and

$$\sum_{i=0}^{N} g_i^*(\tau_s^*, \varphi_c^*) h(\tau_s - i\tau_p + lT, \varphi_c) \simeq s(\Delta\tau_s + lT) \exp(-j\Delta\varphi_c) \quad \text{(A21)}$$

where $s(t)$ denotes the time-continuous signal element at the MF output $[s(lT) = s_l]$, (57) becomes

$$\text{var}\,(\hat{r} \mid \Delta\tau_s, \Delta\varphi_c) \simeq \text{first term} + \text{third term} - 2\,\text{Re}\,\Big\{ \sum_{l=-L}^{L} \sum_{k} \bar{s}(lT) E(\bar{a}_k a_l) \cdot s(\Delta\tau_s + kT) \exp(-j\Delta\varphi_c) \Big\}. \quad \text{(A22)}$$

Minimizing (A22) with respect to $\Delta\varphi_c$ yields

$$\text{var}\,(\hat{r} \mid \Delta\tau_s) \simeq \text{first term} + \text{third term} - 2 \,\Big| \sum_{l=-L}^{L} \sum_{k} \bar{s}(lT) E(\bar{a}_k a_l) s(\Delta\tau_s + kT) \Big|. \quad \text{(A23)}$$

The form of (A23) permits local minima to occur within short distance from τ_s^*. However, with the receiver working in decision-directed mode, $|\Delta\tau_s| < T/2$ can be assumed, and hence convergence towards τ_s^* and φ_c^* can hardly be a problem.

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Gottfried Ungerboeck was born in Vienna, Austria, in 1940. He received the Dipl.Ing. degree in telecommunications from the Technische Hochschule, Vienna, in 1964 and the Ph.D. degree from the Swiss Federal Institute of Technology, Zurich, Switzerland, in 1970.

After graduating from the Technische Hochschule, he joined the technical staff of the Wiener Schwachstromwerke, Vienna. In 1967 he became a member of the IBM Zurich Research Laboratory, Rüschlikon, Switzerland. He worked in speech processing, data switching, and data transmission theory. His present interests cover signal processing and detection and estimation theory.

(Fredricsson's work didn't appear till 7/74 IT.