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Cramer-Rao Lower Bound for Channel Estimation in a MUROS/VAMOS Downlink Transmission

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Abstract—**Voice over Adaptive Multi-user Channels on One Slot (VAMOS) is an extension of the Global System for Mobile Communications (GSM) standard, where two overlaid Gaussian minimum-shift keying (GMSK) signals are transmitted in the same time slot and on the same frequency. The overlaid signals are usually assigned different powers to combat slow fading and propagation loss. For channel estimation, specific training sequences are used for both signals. In the downlink, at each mobile station the channel coefficients and the sub-channel power imbalance ratio (SCPIR) have to be estimated. In this paper, the Cramer-Rao lower bound (CRB) for the training sequence based joint estimation of the SCPIR and the channel coefficients at the mobile station is derived and compared with the CRB for a conventional GSM transmission. The results are compared with the performance of channel estimation algorithms designed for VAMOS. It turns out that these algorithms perform very close to the CRB.**

I. Introduction

Due to still growing demands for voice services in Global System for Mobile Communications (GSM) networks, a new transmission mode called Voice over Adaptive Multi-user channels on One Slot (VAMOS) has been specified in 3GPP Release 9 [1]. With VAMOS, the capacity of existing GSM networks can be doubled and up to four half rate voice users can share one time slot. This extension of the GSM standard, which has been discussed by 3GPP in the work item Multiple Users Reusing One Slot (MUROS), requires efficient receivers in the mobile station to separate the users [2]. Prior to equalization, the channel coefficients have to be estimated. For VAMOS, where the signal of each user is transmitted on an orthogonal sub-channel (OSC), the sub-channel power imbalance ratio (SCPIR) also has to be estimated. A new set of training sequence codes (TSCs) [3], [4] has been introduced for the paired sub-channel (OSC-2). The new TSCs were designed for optimum autocorrelation properties and low cross-correlation with the set of legacy training sequence codes that are used for the first sub-channel (OSC-1) [1]. In the downlink, at each mobile station (MS) a superposition of both OSCs is received. Due to this fact it is possible to exploit the TSCs of both users to enhance the channel estimation performance [2].

The Cramer-Rao bound (CRB) is a lower bound for the variance of unbiased estimators [5]. The CRB for training sequence based channel estimation for GSM without VAMOS extension is well known, cf. [6]. As a performance bound the CRB for the joint estimation of the channel impulse response and the SCPIR indicates whether existing estimators perform already close to the optimum or there is still room for improvement of the algorithms. Furthermore, by comparing the CRB for training sequence based channel estimation in conventional (non VAMOS) GSM with the CRB for channel estimation in VAMOS, the degradation in channel estimation performance of VAMOS compared to the legacy system can be analyzed.

This paper is organized as follows. Section II introduces the system model of a VAMOS downlink transmission, and the state-of-the-art channel estimation algorithms for VAMOS are revisited. In Section III, the CRB for the joint estimation of the channel coefficients and the SCPIR is derived. Sections IV and V provide simulation results and conclusions, respectively.

II. System Model and Channel Estimation

A. System Model

In the considered scenario of a VAMOS downlink transmission [4], the base station transmits two user signals in the same time slot and on the same frequency resource. Both signals are Gaussian minimum-shift keying (GMSK) modulated, where the signal of the user on OSC-2 is rotated by $90°$ and scaled by a real valued factor $b > 0$ that is related to the SCPIR. After GMSK derotation at the receiver, the discrete-time received signal at time index k in equivalent complex baseband notation at one of the two involved MSs can be written as

$$r[k] = \sum_{\kappa=0}^{q_h} h[\kappa] a_1[k-\kappa] + j\ b \sum_{\kappa=0}^{q_h} h[\kappa] a_2[k-\kappa] + n[k]. \quad (1)$$

Here, the discrete-time channel impulse response $h[\kappa]$ of order q_h comprises the effects of GMSK modulation, the mobile channel from the base station (BS) to the considered user, receiver input filtering, and GMSK derotation at the receiver[1]. The channel impulse response $h[\kappa]$ is always assumed to be constant within a transmission burst, but varies randomly between bursts (block fading). $a_i[k]$ denotes the kth binary phase-shift keying (BPSK) training sequence symbol[2] of user $i \in \{1, 2\}$ with variance σ_a^2. Both TSCs are time-aligned.

[1]Knowledge of q_h is assumed. In GSM, $q_h \leq 4$ is valid in most cases. Thus, assuming $q_h = 4$ in system design is sufficient.

[2]GMSK modulation can be well approximated by filtered BPSK.

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The discrete-time additive white Gaussian noise (AWGN) of variance σ_n^2 is denoted by $n[k]$. In the following, it is assumed that $a_1[k]$ is the signal corresponding to the MS of interest.

B. Channel Estimation

For channel estimation one can exploit the fact that both user signals propagate through the same channel [2]. Therefore, the overall channel impulse response of OSC-2 is that of OSC-1 scaled by a factor b and rotated by 90° (factor j). If the received symbols corresponding to the TSCs of both users are collected in a vector $\mathbf{r}$, (1) can be rewritten as

$$\mathbf{r} = \mathbf{A}_1\mathbf{h} + \ b\mathbf{A}_2\mathbf{h} + \mathbf{n}, \tag{2}$$

where $\mathbf{A}_i$ represents the $(K - q_h) \times (q_h + 1)$ Toeplitz convolution matrix corresponding to the TSC symbols of user $i \in \{1, 2\}$, with training sequence length K, and $\mathbf{h} = [h[0]\ h[1]\ \ldots\ h[q_h]]^T$ ($(\cdot)^T$: transposition). Please note that the first q_h received symbols have to be discarded for channel estimation because they contain unknown data symbols. $\mathbf{n}$ is a vector with statistically independent AWGN entries. For an easier notation, the factor j in (1) has been absorbed in $\mathbf{A}_2$.

In the following, a short review of the joint maximum-likelihood (ML) estimation of $\mathbf{h}$ and b that has been already outlined in [2] will be given. The joint ML estimates for $\mathbf{h}$ and b result from minimizing the L_2-norm of the error vector $\mathbf{e} = \mathbf{r} - \mathbf{A}_1\hat{\mathbf{h}} - \hat{b}\mathbf{A}_2\hat{\mathbf{h}}$, where $\hat{\mathbf{h}}$ and $\hat{b}$ denote the estimated quantities. Differentiating $\mathbf{e}^H\mathbf{e}$ ($(\cdot)^H$: Hermitian transposition) with respect to $\hat{\mathbf{h}}^*$ ($(\cdot)^*$: complex conjugation) and $\hat{b}$, the following two conditions for the ML estimates of $\mathbf{h}$ and b are obtained after setting the derivatives to zero:

$$\hat{\mathbf{h}} = \left(\mathbf{V}^H\mathbf{V}\right)^{-1}\mathbf{V}^H\mathbf{r} \tag{3}$$

and

$$\begin{aligned}\hat{b} = \frac{1}{2}\left(\hat{\mathbf{h}}^H\ \mathbf{A}_2^H\ \mathbf{A}_2\ \hat{\mathbf{h}}\right)^{-1}\Big(\left(\hat{\mathbf{h}}^H\ \mathbf{A}_2^H\right)\left(\mathbf{r} - \mathbf{A}_1\ \hat{\mathbf{h}}\right)\\ + \left(\mathbf{r}^H - \hat{\mathbf{h}}^H\ \mathbf{A}_1^H\right)\left(\mathbf{A}_2\ \hat{\mathbf{h}}\right)\Big)\end{aligned} \tag{4}$$

with $\mathbf{V} = \mathbf{A}_1 + \hat{b}\mathbf{A}_2$. Eqs. (3) and (4) can also be interpreted as the ML estimate of the channel for given b and the ML estimate of b for given channel vector, respectively [2]. Both equations are coupled and it does not seem to be possible to obtain a closed-form solution for $\hat{\mathbf{h}}$ and $\hat{b}$. An iterative solution was proposed in [2], where $\hat{b}$ is blindly estimated and this result is used as an initial choice for $\hat{b}$ in (3). The resulting channel vector is then used for refining $\hat{b}$ via (4), etc., until convergence is reached. Due to the iterative refinement of the estimates, an unbiased estimation cannot be always ensured.

III. Cramer-Rao Lower Bound

For a real-valued parameter vector $\boldsymbol{\lambda}$, containing unknown parameters to be estimated from observations, the Fisher information matrix (FIM) is defined as [5]

$$\mathbf{J}(\boldsymbol{\lambda}) = \mathcal{E}_{\mathbf{r}|\boldsymbol{\lambda}}\left\{\left(\frac{\partial \ln p_{\mathbf{r}|\boldsymbol{\lambda}}}{\partial \boldsymbol{\lambda}}\right)\left(\frac{\partial \ln p_{\mathbf{r}|\boldsymbol{\lambda}}}{\partial \boldsymbol{\lambda}}\right)^T\right\}, \tag{5}$$

where $p_{\mathbf{r}|\boldsymbol{\lambda}}$ is the conditional probability density function of the vector with observations $\mathbf{r}$ (also called likelihood function) and $\mathcal{E}\{\cdot\}$ stands for the expectation operation. The Cramer-Rao bound is defined as

$$\mathrm{CRB}_{\boldsymbol{\lambda}} = \left(\mathbf{J}(\boldsymbol{\lambda})\right)^{-1}. \tag{6}$$

For an unbiased estimate $\hat{\boldsymbol{\lambda}}$ and the estimation error $\tilde{\boldsymbol{\lambda}} = \boldsymbol{\lambda} - \hat{\boldsymbol{\lambda}}$, the error covariance matrix $\mathbf{C}_{\tilde{\boldsymbol{\lambda}}} = \mathcal{E}\left\{\tilde{\boldsymbol{\lambda}}\tilde{\boldsymbol{\lambda}}^T\right\}$ satisfies

$$\mathbf{C}_{\tilde{\boldsymbol{\lambda}}} - \mathrm{CRB}_{\boldsymbol{\lambda}} \geq \mathbf{0}, \tag{7}$$

where $\mathbf{A} \geq \mathbf{0}$ means that matrix $\mathbf{A}$ is positive semidefinite [5].

A. Conventional GSM Transmission

The CRB for TSC based channel estimation for a conventional GSM transmission is given by [6]

$$\mathrm{CRB}_{\mathbf{h}} = \sigma_n^2(\mathbf{A}_1^H\mathbf{A}_1)^{-1}, \tag{8}$$

and only depends on the values of the symbols of the TSC and the noise variance. The lower bound for the mean-squared error (MSE) of an estimate of $\mathbf{h}$ is therefore

$$\mathrm{MSE}_{\mathbf{h}} \geq \sigma_n^2\ \mathrm{trace}\Big\{(\mathbf{A}_1^H\mathbf{A}_1)^{-1}\Big\}, \tag{9}$$

where $\mathrm{trace}\{\cdot\}$ stands for the trace of a matrix.

For optimum TSCs minimizing $\mathrm{MSE}_{\mathbf{h}}$, $\mathbf{A}_1^H\mathbf{A}_1$ is a scaled version of the identity matrix [6].

B. VAMOS Transmission

For the derivation of the CRB for a downlink VAMOS transmission, we rewrite (1) as

$$r[k] = (\mathbf{a}_1[k] + j\ b\ \mathbf{a}_2[k])\ \mathbf{h} + n[k], \tag{10}$$

with $k \in \{q_h+1,\ q_h+2,\ \ldots,\ K\}$, and the row vector $\mathbf{a}_i[k] = [a_i[k]\ a_i[k-1]\ \cdots\ a_i[k-q_h]]$ of user $i \in \{1, 2\}$ containing a section of the training sequence. This can be condensed to

$$r[k] = s[k] + n[k],$$

with the information bearing part of the received signal $s[k] = (\mathbf{a}_1[k] + j\ b\ \mathbf{a}_2[k])\ \mathbf{h}$.

The vector with parameters that need to be estimated is

$$\begin{aligned}\boldsymbol{\lambda} = \big[b\ \ \mathrm{Re}\{h[0]\}\ \mathrm{Re}\{h[1]\}\ \ldots\ \mathrm{Re}\{h[q_h]\}\\ \mathrm{Im}\{h[0]\}\ \mathrm{Im}\{h[1]\}\ \ldots\ \mathrm{Im}\{h[q_h]\}\big],\end{aligned}$$

where the channel impulse response has been separated into its real and imaginary parts, whereas b is per definition purely real-valued. The likelihood function of the received signal given the parameter vector $\boldsymbol{\lambda}$ [5] is obtained as

$$p_{\mathbf{r}|\boldsymbol{\lambda}}(\mathbf{r}|\boldsymbol{\lambda}) = \frac{1}{(\pi\sigma_n^2)^{K-q_h}}\exp\left(-\frac{1}{\sigma_n^2}\sum_{k=q_h+1}^{K}|r[k] - s[k|\boldsymbol{\lambda}]|^2\right).$$

The log-likelihood function can be expressed as

$$\ln\left(p_{\mathbf{r}|\boldsymbol{\lambda}}(\mathbf{r}|\boldsymbol{\lambda})\right) = -\frac{1}{\sigma_n^2}\sum_{k=q_h+1}^{K}\left(r[k]-s[k|\boldsymbol{\lambda}]\right)\left(r^*[k]-s^*[k|\boldsymbol{\lambda}]\right) -(K-q_h)\cdot\ln(\pi\sigma_n^2).$$

The derivative of the log-likelihood function with respect to λ_m, the mth element of vector $\boldsymbol{\lambda}$, can be calculated to

$$\frac{\partial}{\partial\lambda_m}\ln\left(p_{\mathbf{r}|\boldsymbol{\lambda}}(\mathbf{r}|\boldsymbol{\lambda})\right) = \frac{1}{\sigma_n^2}\sum_{k=q_h+1}^{K}\left(n^*[k]\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial\lambda_m}+n[k]\frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial\lambda_m}\right), \tag{11}$$

where $(r[k]-s[k|\boldsymbol{\lambda}])$ was substituted by $n[k]$.

The element in the mth row and nth column of the FIM $\mathbf{J}(\boldsymbol{\lambda})$ is defined by

$$[\mathbf{J}]_{mn} = J_{mn} = \mathcal{E}\left\{\frac{\partial\ln p_{\mathbf{r}|\boldsymbol{\lambda}}(\mathbf{r}|\boldsymbol{\lambda})}{\partial\lambda_m}\cdot\frac{\partial\ln p_{\mathbf{r}|\boldsymbol{\lambda}}(\mathbf{r}|\boldsymbol{\lambda})}{\partial\lambda_n}\right\}. \tag{12}$$

Inserting (11) into (12) yields

$$J_{mn} = \mathcal{E}\left\{\frac{1}{\sigma_n^4}\sum_{k=q_h+1}^{K}\sum_{\ell=q_h+1}^{K}\left(n^*[k]n^*[\ell]\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial\lambda_m}\cdot\frac{\partial s[\ell|\boldsymbol{\lambda}]}{\partial\lambda_n}+ n^*[k]n[\ell]\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial\lambda_m}\frac{\partial s^*[\ell|\boldsymbol{\lambda}]}{\partial\lambda_n}+n[k]n^*[\ell]\frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial\lambda_m}\frac{\partial s[\ell|\boldsymbol{\lambda}]}{\partial\lambda_n}+ n[k]n[\ell]\frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial\lambda_m}\frac{\partial s^*[\ell|\boldsymbol{\lambda}]}{\partial\lambda_n}\right)\right\}. \tag{13}$$

For $k\neq\ell$ the expected value is zero, therefore we only sum over values with $k=\ell$. Furthermore, for $k=\ell$, $\mathcal{E}\{n^2[k]\} = \mathcal{E}\{n^{*2}[k]\}=0$ and $\mathcal{E}\{n^*[k]n[k]\}=\mathcal{E}\{n[k]n^*[k]\}=\sigma_n^2$. Therefore, (13) can be simplified to

$$J_{mn} = \frac{1}{\sigma_n^2}\sum_{k=q_h+1}^{K}\left(\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial\lambda_m}\cdot\frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial\lambda_n} + \frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial\lambda_m}\cdot\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial\lambda_n}\right). \tag{14}$$

It is obvious from this expression, that $J_{mn}=J_{nm}$ always holds and therefore the FIM is a symmetric matrix. The partial derivatives are

$$\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial b} = j\ \mathbf{a}_2[k]\ \mathbf{h},$$
$$\frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial b} = -j\ \mathbf{a}_2[k]\ \mathbf{h}^*,$$
$$\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial\operatorname{Re}\{h[x]\}} = [\mathbf{a}_1[k]+j\ b\ \mathbf{a}_2[k]]_{(x+1)},$$
$$\frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial\operatorname{Re}\{h[x]\}} = [\mathbf{a}_1[k]-j\ b\ \mathbf{a}_2[k]]_{(x+1)},$$
$$\frac{\partial s[k|\boldsymbol{\lambda}]}{\partial\operatorname{Im}\{h[x]\}} = j\ [\mathbf{a}_1[k]+j\ b\ \mathbf{a}_2[k]]_{(x+1)},$$

and

$$\frac{\partial s^*[k|\boldsymbol{\lambda}]}{\partial\operatorname{Im}\{h[x]\}} = -j\ [\mathbf{a}_1[k]-j\ b\ \mathbf{a}_2[k]]_{(x+1)},$$

where $[\cdot]_{(x+1)}$ denotes the $(x+1)$th element of a vector, with $x\in\{0,\ 1,\ \ldots,\ q_h\}$.

The $(2q_h+3)\times(2q_h+3)$ FIM $\mathbf{J}$ can be separated into smaller blocks,

$$\mathbf{J} = \left[\begin{array}{c|c|c} J_1 & \mathbf{j}_2 & \mathbf{j}_3 \\ \hline \mathbf{j}_4 & \mathbf{J}_5 & \mathbf{J}_6 \\ \hline \mathbf{j}_7 & \mathbf{J}_8 & \mathbf{J}_9 \end{array}\right]. \tag{15}$$

With the partial derivatives the blocks of the FIM can be calculated. The scalar J_1 can be determined as

$$J_1 = \frac{2}{\sigma_n^2}\sum_{k=1+q_h}^{K}\mathbf{a}_2[k]\mathbf{h}\ \mathbf{a}_2[k]\mathbf{h}^*.$$

The $(x+1)$th elements ($x\in\{0,\ 1,\ \ldots,\ q_h\}$) of the $1\times(q_h+1)$ vectors $\mathbf{j}_2$ and $\mathbf{j}_3$ are

$$[\mathbf{j}_2]_{(x+1)} = \frac{2}{\sigma_n^2}\sum_{k=1+q_h}^{K}\mathbf{a}_2[k]\big(b\ [\mathbf{a}_2[k]]_{(x+1)}\operatorname{Re}\{\mathbf{h}\} -[\mathbf{a}_1[k]]_{(x+1)}\operatorname{Im}\{\mathbf{h}\}\big),$$

and

$$[\mathbf{j}_3]_{(x+1)} = \frac{2}{\sigma_n^2}\sum_{k=1+q_h}^{K}\mathbf{a}_2[k]\big([\mathbf{a}_1[k]]_{(x+1)}\operatorname{Re}\{\mathbf{h}\} +b\ [\mathbf{a}_2[k]]_{(x+1)}\operatorname{Im}\{\mathbf{h}\}\big),$$

respectively. Due to the symmetry properties of the FIM the $(q_h+1)\times 1$ vectors $\mathbf{j}_4$ and $\mathbf{j}_7$ are given by

$$\mathbf{j}_4 = \mathbf{j}_2^T \text{ and } \mathbf{j}_7 = \mathbf{j}_3^T.$$

The element in the $(x+1)$th row and the $(y+1)$th column ($x,\ y\in\{0,\ 1,\ \ldots,\ q_h\}$) of the $(q_h+1)\times(q_h+1)$ matrix $\mathbf{J}_5$ is given by

$$[\mathbf{J}_5]_{(x+1)(y+1)} = \frac{2}{\sigma_n^2}\sum_{k=1+q_h}^{K}\big([\mathbf{a}_1[k]]_{(x+1)}[\mathbf{a}_1[k]]_{(y+1)} +b^2\ [\mathbf{a}_2[k]]_{(x+1)}[\mathbf{a}_2[k]]_{(y+1)}\big),$$

and the elements of $\mathbf{J}_6$ are

$$[\mathbf{J}_6]_{(x+1)(y+1)} = \frac{2}{\sigma_n^2}\sum_{k=1+q_h}^{K}\big(-b\ [\mathbf{a}_1[k]]_{(x+1)}[\mathbf{a}_2[k]]_{(y+1)} +b\ [\mathbf{a}_2[k]]_{(x+1)}[\mathbf{a}_1[k]]_{(y+1)}\big).$$

Again due to symmetry $\mathbf{J}_8=\mathbf{J}_6^T$ holds. The elements of the $(q_h+1)\times(q_h+1)$ matrix $\mathbf{J}_9$ are

$$[\mathbf{J}_9]_{(x+1)(y+1)} = \frac{2}{\sigma_n^2}\sum_{k=1+q_h}^{K}\big([\mathbf{a}_1[k]]_{(x+1)}[\mathbf{a}_1[k]]_{(y+1)} + b^2\ [\mathbf{a}_2[k]]_{(x+1)}[\mathbf{a}_2[k]]_{(y+1)}\big),$$

i.e., $\mathbf{J}_5=\mathbf{J}_9$.

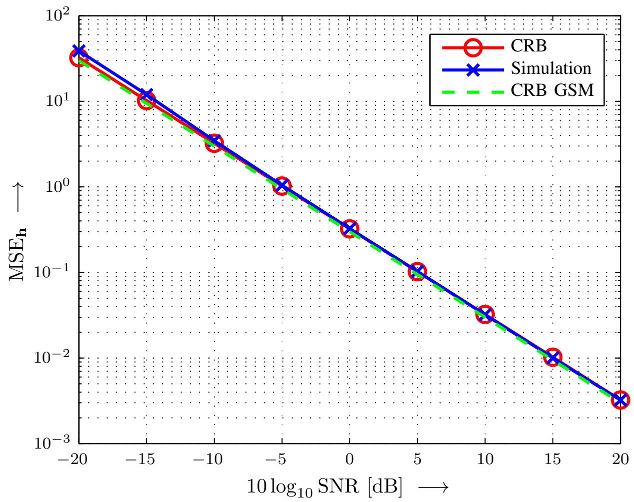

Fig. 1. MSE versus SNR for the estimation of the channel coefficients ($b = 1$).

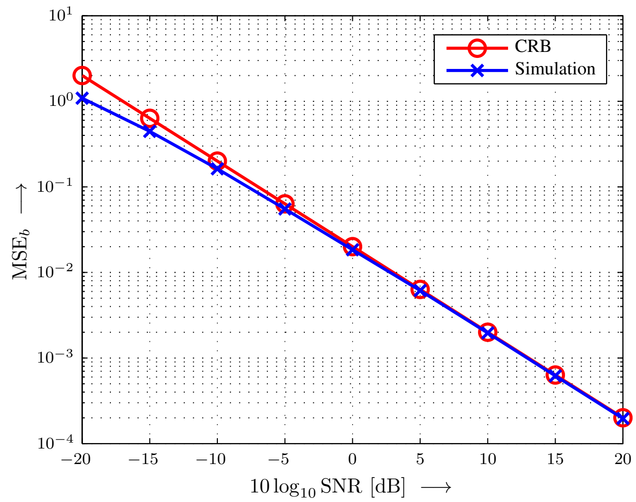

Fig. 2. MSE versus SNR for the estimation of the SCPIR ($b = 1$).

The lower bound for the MSE of the SCPIR estimation of a VAMOS transmission is now obtained as

$$\text{MSE}_b \geq [\mathbf{J}^{-1}]_{11}, \tag{16}$$

and the lower bound for the MSE for the estimation of the channel impulse response coefficients is

$$\text{MSE}_{\mathbf{h}} \geq \sum_{m=2}^{2q_h+3} [\mathbf{J}^{-1}]_{mm}. \tag{17}$$

In contrast to the CRB for the channel impulse response estimation in conventional GSM, now the performance does not only depend on the TSCs, but also on the actual channel impulse response and the SCPIR value. Due to the inversion of the FIM in (15), a closed-form solution for the MSE cannot be given.

IV. Simulation Results

In the following, numerical simulation results are presented in order to analyze the CRB for VAMOS in more detail. The MSEs of the joint channel and SCPIR estimation algorithm according to Section II-B are compared with the CRB for the joint estimation of the parameters. Thereby, the CRB is evaluated for deterministic parameters b and $\mathbf{h}$. For the calculation of the FIM the actual values of the parameters are used. For each simulation the SCPIR value is kept constant, while 5000 different channel impulse responses of order $q_h = 5$ are generated with a random complex normal distribution with variance $\sigma_h^2 = 1$. The resulting CRBs for the estimation of $\mathbf{h}$ and b, respectively, are averaged over the realizations of the channel impulse response. They are plotted for different values of the signal-to-noise ratio (SNR) of the composite signal of both users

$$\text{SNR} = (1 + b^2) \cdot \frac{\sigma_a^2}{\sigma_n^2}. \tag{18}$$

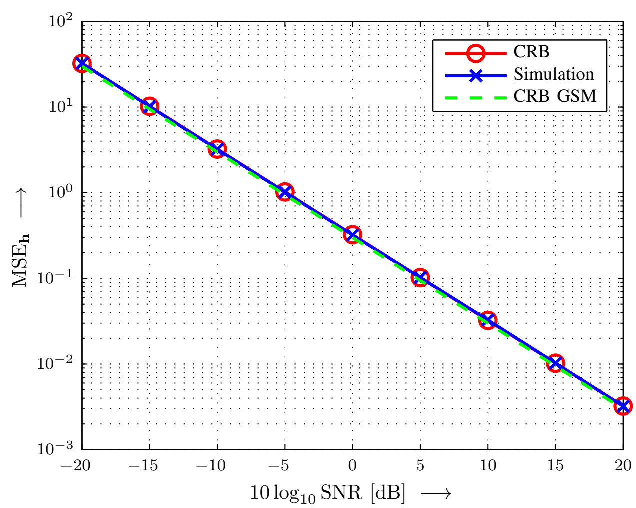

Fig. 3. MSE versus SNR for the estimation of the channel coefficients ($b = 1$). Estimation algorithm according to [7] has been used for the simulation.

Due to the fact that the channel estimation algorithm also exploits the signal of the orthogonal user, the factor $(1 + b^2)$ is incorporated in the SNR definition. Additionally, the CRB for channel estimation for a conventional GSM transmission ($\text{SNR} = \frac{\sigma_a^2}{\sigma_n^2}$) is given as a reference. This can be used to analyze the loss in channel estimation accuracy due to the necessity of estimating the SCPIR factor. TSC 0 of the TSC set for the conventional GSM system and the VAMOS TSC 0 [4] have been used for the simulations.

Figure 1 depicts the MSE for channel estimation for $b = 1$. For a broad range of SNR values the proposed estimator matches the CRB tightly and only for low SNRs a minor degradation in channel estimation accuracy is visible. A loss compared to a conventional GSM channel estimation is barely visible for this SCPIR value. For the estimation of b the results are depicted in Fig. 2. For moderate-to-high SNR, the bound and the simulation results match well, while for low SNR values the simulation results are slightly better than the CRB. This is because for low SNR, the estimation of b is not unbiased anymore.

Also a computationally less complex estimator [7] has been analyzed and the corresponding results for the channel coefficient estimation are shown in Fig. 3. This estimator also matches the lower bound very tightly in a broad range of SNRs. The results for the estimation of b are similar to the ones depicted in Fig. 2 and therefore not shown.

For $b \in \{1/2,\ 2\}$, Fig. 4 depicts the MSE for the channel estimation. For $b = 1/2$ the CRBs for VAMOS and GSM are even closer than for $b = 1$, while for $b = 2$ the bounds are further apart. For both values of b the channel estimation technique proposed in Section II-B matches the VAMOS CRB quite well. Hence, the proposed channel estimation technique is well suited for the use in a VAMOS system. For increasing b the gap between the CRBs for VAMOS and conventional GSM grows. Although a small b value is beneficial for the channel estimation performance of the user on OSC-1, this also results in a worse channel estimation performance for the orthogonal user on OSC-2. Fig. 5 shows the corresponding results for the SCPIR estimation. For an easier comparison the normalized mean-squared error (NMSE), $\mathrm{NMSE}_b = \mathrm{MSE}_b/b^2$, has been used. Due to the normalization the CRB for both values of b is identical. Again, for moderate-to-high SNR the bound and the simulation results match well. For low SNR values the simulation results are slightly better than the CRB for $b = 0.5$, while for $b = 2$ the performance of the estimation algorithm is noticeably better than the bound for low SNR because the estimation is not unbiased anymore in this regime[3].

V. Conclusions

The Cramer-Rao lower bound (CRB) for channel and SCPIR estimation in a VAMOS/MUROS GSM system has been derived in this paper. For channel estimation the CRB for VAMOS has been compared to the CRB for a conventional GSM system. It has been shown that only a minor loss in channel estimation accuracy occurs due to the additional estimation of the sub-channel power imbalance ratio. The mean-squared error of the channel estimation technique from [2] has been compared with the bound, showing that this channel estimation algorithm performs very close to the optimum. Also a second channel estimation technique [7] has been tested and found to perform equally well.

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[3] This is because also for low SNRs, the estimate of b tends to converge to a limited value.

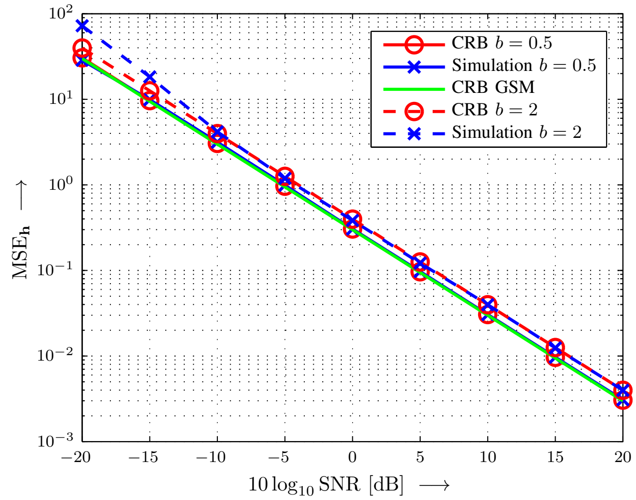

Fig. 4. MSE versus SNR for the estimation of the channel coefficients ($b \in \{1/2,\ 2\}$).

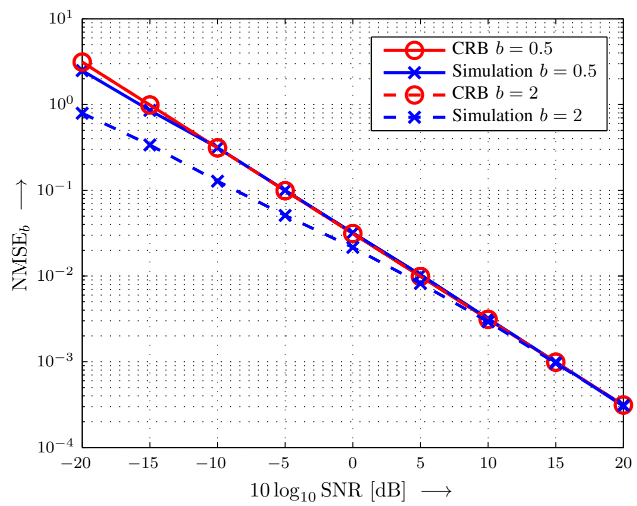

Fig. 5. NMSE versus SNR for the estimation of the SCPIR ($b \in \{1/2, 2\}$).

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