Channel Estimation for EGPRS Modems

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Abstract

In this paper, we examine applicability of different channel estimation techniques for EGPRS modems. Correlation and Least Squares (LS) techniques are analyzed and compared. Afterwards the LS technique is presented as the candidate for EGPRS. Finally, possible modifications of the LS algorithm are discussed.

1. Introduction

Modern TDMA systems, such as IS-136, GSM and EGPRS (Enhanced Global Packet Radio System) cellular systems, reserve a segment of each transmission burst for a known training sequence. The modems use the sequence to perform data aided synchronization. An important part of the synchronization task is to estimate the channel model, which has a direct influence on the equalizer performance. Legacy GSM modems are usually based on a Correlation technique [1] for this purpose. EGPRS modems utilize different constellation scheme and operate under different SNR conditions than GSM. Hence, applicability of common channel estimation techniques for EGPRS (ETSI GSM Phase 2+, Release 99, standard) modems should be reexamined.

In this paper we propose to adopt the Least Squares technique (LS) [2] and its modifications such as Weighted Least Squares (WLS), for channel estimation in EGPRS cellular systems, and demonstrate its superiority over the conventional Correlation technique.

The outline of the paper is as follows: We begin with a background (Section 2). A description of the Correlation technique is provided in Section 3. Its applicability for EGPRS modems is analyzed. A variation of the Correlation technique which provides uncorrelated estimation noise samples for each of the channel taps is also presented.

In Section 4, the LS method is presented as a candidate for EGPRS. We provide a mathematical formulation of the algorithm, and derive an expression for the estimation error in this case. A comparison between the two techniques is presented with simulation results (Section 5). Some important aspects, such as estimated channel length and estimated error, are analyzed. An example of the estimation error covariance matrix is given for each technique using a training sequence from the GSM standard [3]. Also, a situation where the LS and the Correlation techniques coincide is described. Finally, possible modifications for the LS algorithm, such as Weighted Least Squares, WLS, are discussed.

2. Background

An EGPRS communication system has the same burst structure and training sequences as in GSM Voice applications (see figure 1).

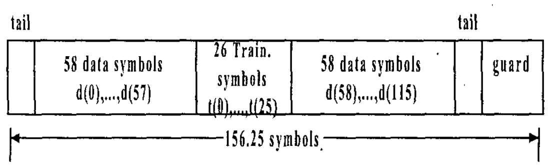

Figure 1: GSM/EGPRS burst

There are 8 possible training sequences defined in the standard. A GMSK modulation was adopted for GSM, which might be considered as $\pi/2$ rotated BPSK. EGPRS utilizes $3\pi/8$ rotated 8-PSK modulation in good link conditions and GMSK for the hard ones [4]. Since the training sequences are the same for both modulations, only the BPSK subset of 8-PSK constellation is used for the training sequence, i.e. the training sequence gets values of +/-1. The effects of pulse shaping, channel impairments and receive filter on the transmitted data can be modeled as a symbol-spaced FIR channel. Moreover, the equivalent channel is assumed to be time constant along the training sequence. Hence, the received sequence obtained by transmitting the training sequence, $\{t(n)\}$, through the equivalent channel, $h(n)$, is

$$\underline{r} = \mathbf{T}\underline{h} + \underline{n} \qquad (1)$$

$$\mathbf{T}=\begin{bmatrix} t(L-1) & t(L-2) & \dots & t(0) \\ t(L) & t(L-1) & \dots & t(1) \\ . & . & . & . \\ t(N) & t(N-1) & \dots & t(N-L+1) \end{bmatrix} \quad (1.a)$$

where the training sequence is rearranged in a matrix form **T** to represent the convolution operation, $\underline{r}$ is the output sample vector and $\underline{n}$ is an additive noise vector.

Throughout the paper the following notations are being used: bold capital letters are used for matrixes; vectors are underlined and assumed to be column vectors. Assume that the channel length is L and the training sequence length is N (26 symbols in our case). Thus we have only N-L+1 samples resulting from the convolution of the known training sequence with the channel impulse response which can be used for the channel estimation operation. For most practical cases the channel response can be modeled by $L \le 7$, therefore, at least 20 samples may be utilized in the channel estimation process.

3. THE CORRELATION TECHNIQUE

The transmitted training sequences are periodic with a period of 16. Each one consists of 16 symbols, preceded and followed by 5 symbols from other periods. The 16 symbol sequences are chosen to have good cyclic correlation properties. Hence, correlation of the 16 central symbols with the entire training sequence has a central peak surrounded by five zeros from each side. This property is used to estimate the channel impulse response by correlating the received samples with 16 symbols of the training sequence, or in a matrix notation

$$\hat{\underline{h}}=\mathbf{C}\underline{r} \quad (2)$$

where

$$\mathbf{C}=\frac{1}{16}\begin{bmatrix} t(5) & \dots & t(20) & 0 & & \dots & 0 \\ 0 & t(5) & \dots & t(20) & 0 & \dots & 0 \\ . & . & . & . & . & . & . \\ 0 & \dots & 0 & t(5) & & \dots & t(20) \end{bmatrix} \quad (3)$$

$\hat{\underline{h}}$ is the channel estimate and $\underline{r}$ is a vector of 16+L-1 input samples. In order to find the noise term of the channel estimation method, we substitute (1) into (2)

$$\hat{\underline{h}}=\mathbf{C}\underline{r}=\mathbf{CT}\underline{h}+\mathbf{C}\underline{n}=\underline{h}+\underline{v} \quad (4)$$

where $\underline{v}$ is the estimation noise or estimation error. Note that **CT** is equal to unity matrix due to good correlation properties of the training sequence. Assuming the input noise, $\underline{n}$, is white with variance N_0, we can find an expression for the estimation noise covariance matrix

$$\mathbf{R}_V=E\left[\underline{v}\underline{v}^T\right]=E\left[\mathbf{C}\underline{n}\underline{n}^H\mathbf{C}^H\right]=N_0\mathbf{CC}^T \quad (5)$$

As can be seen from (3) and (5), the variance of each element of $\underline{v}$ is only $\frac{1}{16}N_0$, as the input noise is averaged over exactly 16 samples. In addition, since $\mathbf{R}_V$ is not a diagonal matrix, the estimation noise samples are correlated. Moreover, due to the structure of the training sequence, the estimated channel response is corrupted by ISI for channels with L>6.

As was mentioned above, correlation of the central 16 symbols with the entire training sequence has one peak surrounded by zeros. One can easily verify, that each 16 symbols replica of the training sequence has similar correlation properties. Figure 2 shows the correlation of $\{t(i)\}_{i=4}^{19}$ with the entire sequence. The peak of the correlation is preceded by 4 zeros and followed by 6 zeros. It means that the correlation of the chosen replica with each other is zero. This property holds for each 16-symbols replica of the training sequence due to the training sequence structure.

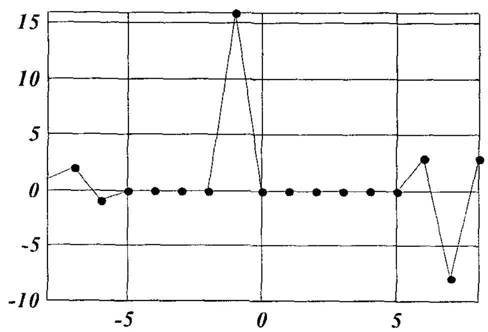

Figure 2: correlation of $\{t(i)\}_{i=4}^{19}$ with the entire sequence

We may use this fact to modify the correlation technique as follows: Instead of sliding 16 training sequence symbols over the input samples, we can choose 16 input samples and correlate them with shifted replicas of the training sequence. The matrix notation for this technique is similar to the first one

$$\hat{\underline{h}}=\mathbf{C}_1\underline{r} \quad (6)$$

where

$$\mathbf{C}_1=\frac{1}{16}\begin{bmatrix} t(5) & \dots & t(20) \\ t(4) & \dots & t(19) \\ . & . & . \\ t(5-L+1) & \dots & t(21-L) \end{bmatrix} \quad (7)$$

where $\underline{r}$ is a vector of 16 input samples. The estimation noise variance, in this case, can be calculated similar to (5), thus

$$\mathbf{R}_v = E\left[\underline{v}\underline{v}^T\right] = E\left[\mathbf{C}_l \underline{n}\underline{n}^H \mathbf{C}_l^H\right] = N_0 \mathbf{C}_l \mathbf{C}_l^T = \frac{N_0}{16}\mathbf{I} \quad (8)$$

where $\mathbf{C}_l\mathbf{C}_l^T$ is equal to the unity matrix as mentioned above. This method has all the drawbacks of the previous one except that the estimation noise is uncorrelated (see (8)). We will show later that the last method is a particular case of the LS technique.

4. The Least Squares Technique

Given a channel, $\underline{\tilde{h}}$, the input samples estimations, $\underline{\hat{r}}$, are given by

$$\underline{\hat{r}} = \mathbf{T}\underline{\tilde{h}} \quad (9)$$

In the LS technique, the channel estimation is found by minimization of the Euclidian distance between the received input samples and their estimations based on a specific channel, $\underline{\tilde{h}}$, as follows:

$$\underline{\hat{h}} = \arg\min_{\underline{h}\in C^L}\left\{D\left(\underline{\tilde{h}}\right)\right\} \quad (10)$$

where the distance is

$$D\left(\underline{\tilde{h}}\right) = \left(\underline{r}-\underline{\hat{r}}\right)^H\left(\underline{r}-\underline{\hat{r}}\right) = \left(\underline{r}-\mathbf{T}\underline{\tilde{h}}\right)^H\left(\underline{r}-\mathbf{T}\underline{\tilde{h}}\right) \quad (11)$$

This problem has a well-known solution

$$\underline{\hat{h}} = \left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\underline{r} \quad (12)$$

By substitution of (1) into (12), we get

$$\begin{aligned}\underline{\hat{h}} &= \left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{T}\underline{h} + \left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\underline{n} = \\ &= \underline{h} + \underline{v}\end{aligned} \quad (13)$$

The variance of the estimation noise in

$$\begin{aligned}&\mathbf{R}_v = E\left[\underline{v}\underline{v}^T\right] = \\ &E\left[\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\underline{n}\underline{n}^H\mathbf{T}\left[\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\right]^H\right] = \\ &N_0\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{I}\mathbf{T}\left[\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\right]^H = N_0\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\end{aligned}$$

(14)

(As $\mathbf{T}^H\mathbf{T}$ is a conjugate-symmetric matrix, hence its inverse is also a conjugate-symmetric.)

As was explained in section 2, up to N-L+1 samples may be utilized in the channel estimation process. For the case of 7 tap long channel and 20 samples long input vector, simulations show that variance of the estimation noise elements is less than $\frac{1}{19}N_0$, which means that the LS method averages the input noise over all available input samples. Moreover, the LS technique does not make any assumptions regarding the training sequence correlation. Hence it is not limited to the channels of up to 6 tap length.

5. Comparison between the LS and the Correlation Techniques

Consider the second variation of the correlation technique ((6) and (7) above). As one can see, $\mathbf{C}_l$ is equal to $\frac{1}{16}\mathbf{T}^T$. Note that $\mathbf{T}^T$=$\mathbf{T}^H$ since $\mathbf{T}$ is a real matrix. Due to the correlation properties of the training sequence we have:

$$\mathbf{T}^H\mathbf{T} = 16\mathbf{I} \quad (15)$$

Hence, the LS estimation based on 16 input samples can be rewritten as

$$\underline{\hat{h}} = \left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\underline{r} = \frac{1}{16}\mathbf{I}\mathbf{T}^H\underline{r} = \mathbf{C}_l\underline{r} \quad (16)$$

In this particular case, the correlation technique and the LS technique are equivalent. However, the LS technique can utilize more input samples, thus, it provides estimation noise which is lower at about 1dB than the correlation technique.

Moreover, EGPRS 8-PSK modems are designed to operate at higher SNRs than GMSK modems. Hence, modem noise sources, such as residual ISI, should be kept low. Simulations show that for some propagation scenarios, such as Hilly Terrain channels, 6-tap channel model may cause the residual ISI to be as high as 20dB below the signal, which is not sufficient. Consider utilizing the correlation technique on a 7-tap long channel. The model matrix, $\mathbf{T}$ ($\mathbf{T}$ and $\mathbf{C}$ as defined in (1) and (3) respectively), takes the following form:

$$\mathbf{T} = \begin{bmatrix} t(5) & \dots & t(0) & d_l(0) \\ t(6) & \dots & t(1) & t(0) \\ . & . & . & . \\ d_r(0) & t(25) & \dots & t(20) \end{bmatrix} \quad (18)$$

where $d_l(0)$ is the first data symbol on the left hand side of the training sequence and $d_r(0)$ is the first one on the right hand side. The channel estimate is

$$\underline{\hat{h}} = \mathbf{C}\mathbf{T}\underline{h} = \mathbf{K}\underline{h} = \begin{bmatrix} 1 & & \dots & & k_{06} \\ & 1 & & 0 & \\ . & . & . & . & . \\ & 0 & & 1 & \\ k_{60} & & \dots & & 1 \end{bmatrix}\underline{h} \quad (19)$$

where

$$k_{06} = \frac{1}{16}\left(t(5)d_l(0) + \sum_{k=6}^{20} t(k)t(k-6)\right) \quad (20)$$

and

$$k_{60} = \frac{1}{16}\left(t(20)d_r(0) + \sum_{k=5}^{19} t(k)t(k+6)\right) \quad (21)$$

The matrix $\mathbf{K}$ has ones on the main diagonal and zeros elsewhere except two corners, k_{60} and k_{06}. It means that the first tap of the estimation is corrupted by interference with the last one and vice versa.

$$\hat{h}(0) = h(0) + k_{06}h(6) \tag{22.a}$$

and

$$\hat{h}(6) = h(6) + k_{60}h(0) \tag{22.b}$$

The interference term may estimated using the correlation properties and the possible data symbol values

$$k_{ij} = \frac{1}{16}(\pm 1 + \pm d), \quad (i,j) = (0,6),(6,0) \tag{23}$$

where d is a data symbol. Under most propagation conditions, the ray with the shortest delay to receiver, has the strongest signal energy. It usually makes the first tap of the equivalent symbol-spaced channel to be significant. Hence the last channel estimation tap may suffer from large interference, as can be seen in (22.b). Note that this ISI problem in the outer taps does not exist in the LS technique.

Another advantage of LS technique is its ability to incorporate side information, such as noise spectrum or pulse shaping.

Khayrallah et al, [5], suggested to use pulse shaping information in the following manner:

The equivalent channel, $\underline{h}$, can be viewed as a convolution of known filter (receive filter, say), $\underline{a}$, and unknown filter, $\underline{b}$, as follows

$$\underline{h} = \mathbf{A}\underline{b} \tag{24}$$

where element of the filter $\underline{a}$ are rearranged as matrix $\mathbf{A}$ to represent convolution. Then equation (12) takes a form

$$\hat{\underline{b}} = \left(\mathbf{A}^H\mathbf{T}^H\mathbf{T}\mathbf{A}\right)^{-1}\mathbf{A}^H\mathbf{T}^H\underline{r} \tag{25}$$

and the channel estimation is obtained by substituting (25) into (24).

Since EGPRS carrier spacing is narrower than the symbol rate, choosing a narrowband receive filter is mandatory. It means that the input noise after the receive filter will be correlated. This property may be utilized by applying Weighted LS approach (WLS). Assume that the input noise has covariance matrix $\mathbf{R}$. Then the optimal channel estimate in MMSE sense is

$$\hat{\underline{h}} = \left(\mathbf{T}^H\mathbf{R}^{-1}\mathbf{T}\right)^{-1}\mathbf{T}^H\mathbf{R}^{-1}\underline{r} \tag{26}$$

In the case of gaussian noise, it is also the Maximum Likelihood estimation technique.

The two approaches described above in (25) and (26) can be combined by adding $\mathbf{R}^{-1}$ to (25) to yield

$$\hat{\underline{b}} = \left(\mathbf{A}^H\mathbf{T}^H\mathbf{R}^{-1}\mathbf{T}\mathbf{A}\right)^{-1}\mathbf{A}^H\mathbf{T}^H\mathbf{R}^{-1}\underline{r} \tag{27}$$

The figure below shows benefits of the LS algorithm with respect to the Correlation one. A Hilly Terrain channel was simulated for mobile terminal moving at 100 km/h, carrier frequency is in the 900 MHz frequency band. The modem incorporates reduced complexity Viterbi equalizer and LS or Correlation channel estimation. Block Error Rates at the modem output are shown in figure 3 for the two methods as the function of the SNR.

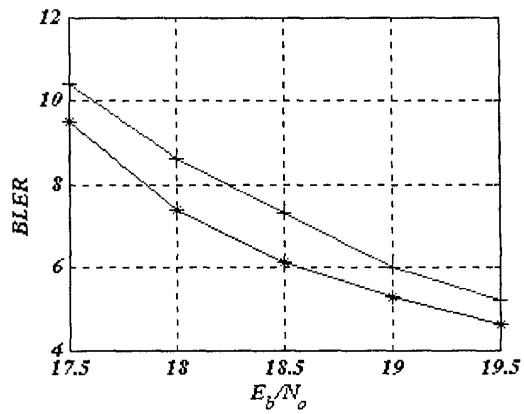

Figure 3: LS (*) versus Correlation (+), BLER performance over HT100 channel

As one can see, the LS outperforms the Correlation technique by 0.5dB. Hence, using of LS technique improves modem.

References

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