Single-Channel Blind Equalization for GSM Cellular Systems

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***Abstract*—Our work in this paper focuses on the study of blind equalization global system for mobile communications (GSM) systems using a single antenna. In order to utilize the well-known linear system model in conventional studies of blind equalization, an equivalent baseband quadrature amplitude modulation (QAM) approximation is used for the nonlinear GMSK signal in GSM systems. Since the GMSK signal in GSM has very little excess bandwidth to warrant oversampling, a derotation scheme is developed to create two subchannels for each received GMSK signal sampled at the baud rate. Linear approximation of the GMSK signal makes the traditional QAM blind equalization system model applicable for GSM. Derotation induces channel diversity without additional antenna and reduces the number of necessary radio frequency (RF) receivers (sensors) without increasing hardware or computational costs. Several second-order statistical and higher order statistical methods of blind equalization are adopted for GSM signals.**

***Index Terms*— Blind equalization, forward-link equalization, GSM equalization, nonlinear modulation.**

I. Introduction

In many digital communication systems signals are often transmitted through unknown linear channels which introduce undesirable linear distortion. To improve system performance it is important for receivers to remove the effect of linear channel distortions through equalization or sequence estimation. Because transmitted training signals in some practical systems may be too short for channel identification, blind channel identification/equalization can play a very important role in these systems.

Blind channel identification/equalization relies solely on the received channel output signal and *a priori* statistical knowledge of the desired channel input signal. Blind channel identification and equalization can exploit higher order statistics (HOS) of baud rate sampled channel output signals. When channel diversity (or excess bandwidth) is available, second-order statistics (SOS) can also lead to different approaches. The algorithm presented by Tong *et al.* [1] is one of the first subspace-based methods exploiting only SOS for fractionally sampled channel identification. Other notable works include the subchannel matching by Liu *et al.* [2] and its improvement by Hua [3], a MUSIC-like subspace method for channel estimation by Moulines *et al.* [4], and the linear prediction based approach presented by Slock [5] and refined by Abed-Meriam *et al.* [6] and Ding [7]. In essence, all SOS blind equalization methods rely on a single-input multiple-output (SIMO) linear system model. Multiple outputs are generated by additional sensors or by oversampling analog channels with excess bandwidth. In fact, it has also been shown in [12] that HOS blind equalizers can exhibit a much improved convergence property when applied to SIMO systems.

Although there are a number of SIMO blind equalization algorithms, they are designed and simulated for conventional linear quadrature amplitude modulation (QAM) systems. To the best of our knowledge, there has been no other reported work on the application of SIMO blind equalization in existing cellular wireless communication systems. Clearly, there is a very strong need to assess their applicability and effectiveness in practical wireless communication systems. Among several competing schemes, GSM is currently one of the most widely used wireless cellular communication systems in the world. Today, GSM systems occupy much of the European and Asian wireless digital cellular market. Its wide application and acceptance signify its reliability and simplicity in real-time operation. Although the study of blind equalization for GSM will not change the existing standard, it can be expected to influence the design of future wireless systems.

GSM is a time division multiple access (TDMA) system where each frequency band is shared by eight users, separated in time by their nonoverlapping time frames. Each frame consists of two 58-bit streams of real data transmission separated by a midamble of 26 training bits for channel equalization and receiver synchronization. Apparently, such a sizable midamble may be used for real data transmission if blind channel equalization can be successfully employed. In a recent work [22], single-input–single-output HOS blind equalization has already exhibited great potential for GSM. The main goal of this paper is to determine whether and how existing fractionally spaced blind equalization schemes may be applied to GSM and similar future systems.

Existing approaches to blind equalization and identification face two major obstacles in GSM systems. First, and foremost, GSM utilizes a nonlinear phase modulation scheme known as GMSK, while all existing blind algorithms assume a linear modulation system such as QAM. Second, even if the GMSK signal may be approximated by a linear modulation system, the modulated signal has very little excess bandwidth. As a result, SOS methods that require oversampling the modulated signal for channel diversity are no longer suitable. One natural solution to the second obstacle is to add more antenna elements

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[8]. This would increase the RF cost, however, and is often impractical for handsets.

In this paper, we study the application of blind channel equalization and identification for the phase-modulated GSM system without relying on additional antennae. Our goal is to investigate whether and how the mathematically elegant SIMO methods and other fractionally spaced blind equalization methods can be applied to the nonlinear modulated GMSK signals. As GMSK signals have little excess bandwidth, we are particularly interested in determining whether SOS methods relying on channel diversity can be adopted for baud rate sampled channel signals from a single antenna output. First, in order for the concept of linear blind equalization to be applicable, we derive an equivalent baseband QAM model for the GMSK signal used in GSM systems (Section III). To apply SIMO methods on the single channel output, we use a derotation scheme to create two subchannels for each received GMSK signal sampled at the baud rate (Section IV). We also derive the necessary and sufficient condition for the derotation-induced subchannels to share no common zeros in order to preserve channel diversity. Implementations of blind equalization for GMSK signals are presented in Section V, and simulation results of both linear and nonlinear blind equalization methods are given in Section VI.

II. Blind Equalization for QAM Systems

Blind equalization is typically designed only for QAM systems in which the received signal from a channel with impulse response $h(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k h(t-kT) + w(t), \qquad a_k \in \mathcal{A} \tag{2.1}$$

where T is the symbol baud period and $\mathcal{A}$ is the input constellation set. The channel input sequence $\{a_k\}$ is independently identically distributed (i.i.d.) while the noise $w(t)$ is stationary white and independent of channel input sequences a_k, but not necessarily Gaussian. Note that $h(t)$ is the composite channel impulse response that includes transmitter and receiver filters as well as the physical channel response. If J antennae are available, then $x(t), h(t)$, and $w(t)$ are all $J \times 1$ vectors.

The objective of blind channel identification and equalization is to identify the unknown channel responses $h(t)$ and recover the unknown sequence $\{a_k\}$ from the channel output $x(t)$ only. Based on statistical knowledge of the channel input $\{a_k\}$, the ultimate goal is to recover the input sequence $\{a_k\}$ from the received signal $x(t)$ with acceptable probability of error. It is upon this linear model that algorithms using HOS and SOS of the sampled channel output are designed and applied.

III. Linear Approximation of GSM Signals

The direct application of QAM blind equalization encounters an obstacle since GMSK is a phase modulation where the baseband signal is *nonlinear*

$$s(t) = \exp\left[j\frac{\pi}{2}\sum_{n=-\infty}^{\infty}\alpha_n \psi(t-nT)\right]$$

in which $\alpha_n = \pm 1$ is the binary data for transmission. Because existing blind equalization algorithms rely on linear system models, linear approximation of the GMSK signal becomes the necessary first step. Following the approach used in [9] and [10] for general GMSK signals, we now derive a linear QAM approximation of the GMSK signal used in GSM.

Consider the Gaussian pulse response given in the GSM standard [15]

$$g(t) = B\sqrt{\frac{2\pi}{\ln 2}}\exp\left[-\frac{2\pi^2 B^2 t^2}{\ln 2}\right]. \tag{3.1}$$

The impulse response of the pulse shaping filter after rectangular nonreturn to zero (NRZ) is $g(t)$. Denote the rectangular NRZ pulse as

$$\text{rect}(t/T) = \begin{cases} 1/T, & |t| \le T/2 \\ 0, & |t| > T/2. \end{cases} \tag{3.2}$$

The combined NRZ filter response is simply the convolution

$$c(t) = g(t) * \text{rect}(t/T).$$

Consequently, the continuous phase modulation (CPM) pulse is the integral

$$\psi(t) = \int_{-\infty}^{t} c(\tau - 2T)\, d\tau. \tag{3.3}$$

In GSM system, the GMSK parameter $BT = 0.3$ is chosen so that

$$\psi(t) \approx \begin{cases} 0, & t \le 0; \\ 1, & t \ge 4T. \end{cases}$$

Thus, we take the first approximation step

$$\begin{aligned} s(t) &= \exp\ j\left[\frac{\pi}{2}\sum_{n=-\infty}^{\infty}\alpha_n\psi(t-nT)\right] \\ &\approx \exp\left(j\frac{\pi}{2}\sum_{k=-\infty}^{n-4}\alpha_k\right)\prod_{k=n-3}^{n}\exp\left[j\frac{\pi}{2}\alpha_k\psi(t-kT)\right], \\ &\quad t \in [nT, (n+1)T). \end{aligned} \tag{3.4}$$

Since $\alpha_k = \pm 1$ we have

$$\begin{aligned} &\exp\left[j\frac{\pi}{2}\alpha_k\psi(t-kT)\right] \\ &\quad = \cos\left[\frac{\pi}{2}\alpha_k\psi(t-kT)\right] + j\sin\left[\frac{\pi}{2}\alpha_k\psi(t-kT)\right] \\ &\quad = \cos\left[\frac{\pi}{2}\psi(t-kT)\right] + j\alpha_k\sin\left[\frac{\pi}{2}\psi(t-kT)\right], \\ &\quad\ nT \le t < (n+1)T. \end{aligned}$$

We now must rely on a new definition for further simplification. Keep in mind that linear approximation of GMSK signal is not unique. Here we define a new symmetric function as

$$\beta(t) = \cos\left[\frac{\pi}{2}\,\text{sign}(t)\psi(t)\right]. \tag{3.5}$$

Based on the symmetry of the pulse $c(t)$, we have

$$\psi(4T - x) = \psi(4T) - \psi(x) = 1 - \psi(x). \tag{3.6}$$

Letting $x = 4T - t + kT$, we get

$$\begin{aligned}\sin\left[\frac{\pi}{2}\psi(t-kT)\right] &= \sin\left[\frac{\pi}{2}(1-\psi(4T-t+kT))\right]\\ &= \cos\left[\frac{\pi}{2}\psi(4T-t+kT)\right]\\ &= \beta(4T+kT-t) = \beta(t-4T-kT). \qquad (3.7)\end{aligned}$$

This allows a finite time approximation

$$\begin{aligned}s(t) \approx \exp\left(j\frac{\pi}{2}\sum_{k=-\infty}^{n-4}\alpha_k\right)\prod_{k=n-3}^{n}(\beta(t-kT)\\ + j\alpha_k\beta(t-kT-4T)). \qquad (3.8)\end{aligned}$$

The above equation becomes the basis of our linear approximation. It can be seen that there are 16 different terms in the product expansion of (3.8), each constituting a linear QAM signal. Hence, a GMSK signal can be approximated by 16 QAM signals. Among the 16 different linear pulses, however, only two pulses are significant while the others are nearly all zero. Retaining these two most significant pulses, the linear approximate model for GMSK with $BT = 0.3$ is

$$s(t) = \sum_{n=-\infty}^{\infty} a_{0,n}h_0(t-nT) + \sum_{n=-\infty}^{\infty} a_{1,n}h_1(t-nT) \quad (3.9)$$

where

$$\begin{aligned}a_{0,n} &\triangleq \exp\left(j\frac{\pi}{2}\sum_{k=-\infty}^{n}\alpha_k\right) = j\alpha_n a_{0,n-1}\\ &= -\alpha_n\alpha_{n-1}a_{0,n-2}\\ a_{1,n} &\triangleq j\alpha_n\exp\left(j\frac{\pi}{2}\sum_{k=-\infty}^{n-2}\alpha_k\right) = j\alpha_n a_{0,n-2}.\end{aligned}$$

In other words, the GMSK signal can be approximated with almost no error by the sum of two QAM signals with pulse shapes $h_0(t)$ and $h_1(t)$. These two pulses in the linear approximation are shown in Fig. 1 and are given by

$$\begin{aligned}h_0(t) &= \beta(t-T)\beta(t-2T)\beta(t-3T)\beta(t-4T),\\ &\qquad 0 \le t \le 5T;\\ h_1(t) &= \beta(t+T)\beta(t-T)\beta(t-2T)\beta(t-4T),\\ &\qquad 0 \le t \le 3T.\end{aligned}$$

This linear GMSK approximation is equivalent to a two-input–one-output system. The equalization of such a system requires the use of channel diversity from additional antennae or possibly from oversampling. Because the majority (99.5%) of signal energy in GMSK signal $s(t)$ is contained in the first pulse approximation $h_0(t)$, we can further simplify $s(t)$ into a single QAM transmission

$$s(t) \approx \sum_{n=-\infty}^{\infty} a_n h_0(t-nT), \qquad a_n = j\alpha_n a_{n-1}. \quad (3.10)$$

It can be noted that the approximation error may be viewed as an additive interference. Therefore, even in noiseless channels, the maximum signal-to-noise ratio (SNR) of this approximation is at 23 dB. With this linear QAM quatenary phase shift keying (QPSK) approximation, existing blind equalization methods can be applied.

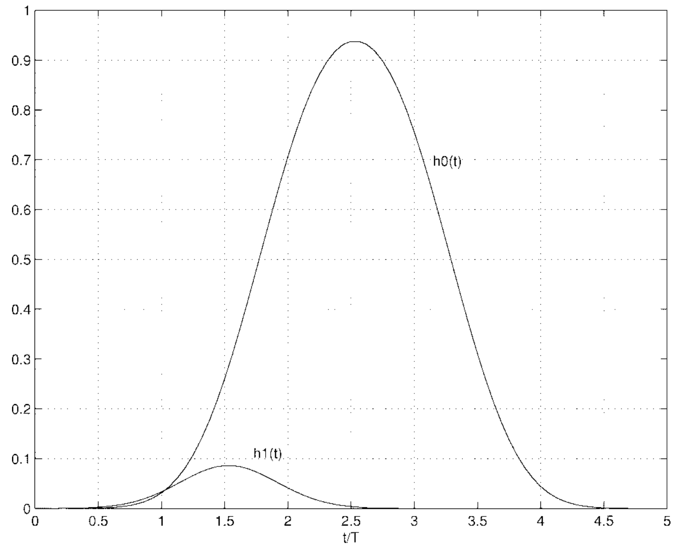

Fig. 1. Two pulseshapes in the GMSK linear approximation are shown. The power in $h_1(t)$ is 0.48% of the power in $h_0(t)$.

IV. Derotation to Generate Channel Diversity

For SOS- and HOS-based SIMO blind equalization algorithms without the benefit of additional physical antenna(s) the necessary channel diversity is often obtained by oversampling the received analog signal. Unfortunately, there is one critical obstacle associated with the use of these methods in GSM. Specifically, GMSK signals with $BT = 0.3$ and, subsequently, the QAM pulse $h_0(t)$ have little excess bandwidth beyond $1/2T$. Hence, sampling GSM channel output signal at a rate higher than $1/T$ will not generate reliable channel diversity. This point will be verified by simulation results given later.

Without sufficient excess signal bandwidth in GMSK signals one may create channel diversity by adding additional antennae. Unfortunately, additional antenna units require extra radio-frequency (RF) units and significantly increase hardware costs. Here, we show how to reduce hardware cost by processing the received data so that two channel outputs can be extracted from a single received GMSK signal.

Observe that $a_n = j\alpha_n a_{n-1}$ is a QPSK sequence. At any given time, however, a_n can only take on two values, rather than four. In other words, a_n is a pseudo-quatenary phase-shift keying (QPSK) and is realized by rotating a binary phase-shift keying (BPSK) signal. Hence

$$\begin{aligned}a_{n-i} &= j^{M+L-i}\left(\prod_{k=i}^{M+L-1}\alpha_{n-k}\right)a_{n-M-L}\\ &= j^{n-i}\left(\prod_{k=i}^{M+L-1}\alpha_{n-k}\right)\left(a_{n-M-L}j^{M+L-n}\right),\\ &\qquad i = 0, 1, \cdots, M+L-1.\end{aligned}$$

Without loss of generality, let $(a_{n-M-L}j^{M+L-n})$ be purely real. It then follows the definition of

$$\tilde{a}_{n-i}\left(\prod_{k=i}^{M+L-1}\alpha_{n-k}\right)\left(a_{n-M-L}j^{M+L-n}\right) = \pm 1 \qquad (4.1)$$

that

$$a_{n-i} = j^{n-i}\tilde{a}_{n-i}, \qquad i = 0, 1, \cdots, M+L-1. \tag{4.2}$$

For a physical channel impulse response $h_c(t)$ the combined linear approximation pulse is simply

$$h(t) = h_c(t) * h_0(t)$$

and the received GMSK signal is approximately

$$x(t) = \sum_{k=-\infty}^{\infty} s_k h(t-kT) + w(t) \tag{4.3}$$

where $w(t)$ is channel noise. The baud rate sampled discrete signals and responses are defined as

$$x_i \triangleq x(iT), \quad h_i \triangleq h(iT), \quad w_i \triangleq w(iT). \tag{4.4}$$

The channel output sequence is thus a stationary sequence

$$x_n = \sum_{k=-\infty}^{\infty} h_k a_{n-k} + w_n.$$

To extract channel diversity from the above single-channel system, signal preprocessing based on special characteristics of the pseudo-QPSK input is necessary. First, we recognize that

$$\begin{aligned} x_n &= \sum_{k=-\infty}^{\infty} h_k j^{n-k}\tilde{a}_{n-k} + w_n \\ &= j^n \sum_{k=-\infty}^{\infty} \left[h_k j^{-k}\right]\tilde{a}_{n-k} + w_n. \end{aligned} \tag{4.5}$$

As a result, we can obtain a new (derotated) sequence

$$\tilde{x}_n \triangleq x_n j^{-n} = \sum_{k=-\infty}^{\infty} \left[h_k j^{-k}\right]\tilde{a}_{n-k} + j^{-n}w_n. \tag{4.6}$$

Although this derotation has been noted before, for example, in [8], its significance in GSM blind equalization was not fully realized. In fact, derotation not only changes the GMSK detection problem into a simpler BPSK detection problem, it can also create a channel diversity useful in blind equalization. In an independent work [14] derotation was proposed for minimum shift-keying (MSK) signals.

Since $\{\tilde{a}_n\}$ is a real-valued sequence, we can induce two subchannel outputs from

$$\begin{aligned} x_{1,n} &\triangleq \mathrm{Re}\{\tilde{x}_n\} = \sum_{k=-\infty}^{\infty} \mathrm{Re}\left[h_k j^{-k}\right]\tilde{a}_{n-k} + \mathrm{Re}[j^{-n}w_n] \\ x_{2,n} &\triangleq \mathrm{Im}\{\tilde{x}_n\} = \sum_{k=-\infty}^{\infty} \mathrm{Im}\left[h_k j^{-k}\right]\tilde{a}_{n-k} + \mathrm{Im}[j^{-n}w_n]. \end{aligned} \tag{4.7}$$

Driven by the same BPSK data sequence $\{\tilde{a}_n\}$, two subchannel outputs can now be generated. Hence, we have shown that, even without oversampling and extra antenna(e), existing methods based on SIMO models can be applied.

Many SOS algorithms require that the two subchannel impulse responses

$$\{h_{1,i}\} = \left\{\mathrm{Re}\left(\mathrm{h_i j^{-i}}\right)\right\} \quad \text{and} \quad \{h_{2,i}\} = \left\{\mathrm{Im}\left(\mathrm{h_i j^{-i}}\right)\right\}$$

do not have common zeros. This condition is also necessary for the global convergence of HOS SIMO algorithms [12]. The sufficient and necessary conditions for the two derotated subchannels to have common zero are given by the following theorem, whose proof is given in the Appendix.

Theorem 4.1: For a given channel with sampled impulse response $[h_0, h_1, \cdots, h_L]$, the two derotated diversity channels will share common zeros if and only if its transfer function

$$H(z) = \sum_{n=0}^{L} h_n z^{-n}$$

has zero(s) symmetric to the imaginary axis.

V. GSM Equalization Using SIMO Algorithms

The following diagram describes how the received GMSK signal is processed prior to equalization. By using a single-user approximation of the GMSK signal, a single-input–two-output BPSK system has been shown to approximate the derotated GMSK signal very well. The root-raised cosine receiver filter will not color the sampled channel noise at the baud rate $1/T$. For optimum sampling phase at the baud rate timing extraction is needed.

A. Vector SIMO

Based on derotation, we can now adopt several effective blind SIMO equalization algorithms for GSM systems. Assume that $\{h(t)\}$ has finite support which spans $[0, LT]$. The sampled channel output signal vector of length M can be written as

$$\tilde{\boldsymbol{x}}[k] \triangleq \begin{bmatrix} \tilde{x}_k \\ \tilde{x}_{k-1} \\ \vdots \\ \tilde{x}_{k-M+1} \end{bmatrix} = \mathcal{H}\tilde{\boldsymbol{a}}[k] + \tilde{\boldsymbol{w}}[k] \tag{5.1}$$

where

$$\begin{aligned} \tilde{\boldsymbol{a}}[k] &\triangleq [\tilde{a}_k \quad \tilde{a}_{k-1} \quad \cdots \quad \tilde{a}_{k-L-M+1}]' \\ \tilde{\boldsymbol{w}}[k] &\triangleq [j^{-k}w_k \quad j^{-k+1}w_{k-1} \\ &\qquad \cdots \quad j^{-k+L+M-1}w_{k-L-M+1}]' \end{aligned}$$

and the $M \times (L+M)$ channel Toeplitz matrix is defined by

$$\mathcal{H} = \begin{bmatrix} h_0 & h_1 j^{-1} & \cdots & h_L j^{-L} & 0 & \cdots & 0 \\ 0 & h_0 & h_1 j^{-1} & \cdots & h_L j^{-L} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_0 & h_1 j^{-1} & \cdots & h_L j^{-L} \end{bmatrix}. \tag{5.2}$$

Since $\tilde{\boldsymbol{a}}[k]$ is binary real, we have

$$\begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{x}}[k]\} \\ \mathrm{Im}\{\tilde{\boldsymbol{x}}[k]\} \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{\mathcal{H}\} \\ \mathrm{Im}\{\mathcal{H}\} \end{bmatrix} \tilde{\boldsymbol{a}}[k] + \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{w}}[k]\} \\ \mathrm{Im}\{\tilde{\boldsymbol{w}}[k]\} \end{bmatrix}. \tag{5.3}$$

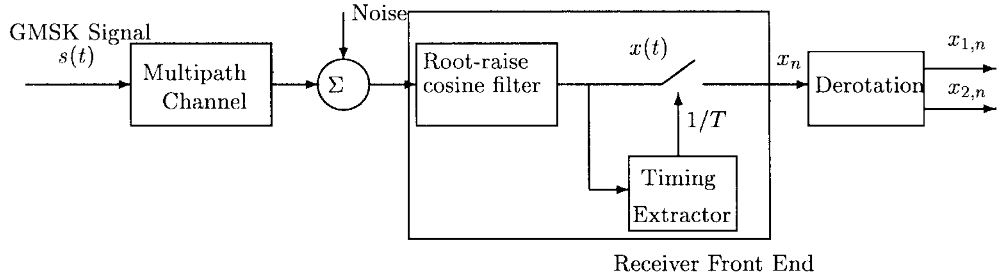

Fig. 2. Blind equalizers based on timing recovery and GSM derotation.

We hence arrive at the familiar equation in SIMO models

$$\boldsymbol{x}[k] = \boldsymbol{H}\tilde{\boldsymbol{a}}[k] + \boldsymbol{w}[k] \tag{5.4}$$

where

$$\boldsymbol{x}[k] \triangleq \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{x}}[k]\} \\ \mathrm{Im}\{\tilde{\boldsymbol{x}}[k]\} \end{bmatrix} \tag{5.5}$$

$$\boldsymbol{H} \triangleq \begin{bmatrix} \mathrm{Re}\{\mathcal{H}\} \\ \mathrm{Im}\{\mathcal{H}\} \end{bmatrix} \tag{5.6}$$

and $\boldsymbol{w}[k]$ approximates error and channel noise

$$\boldsymbol{w}[k] \triangleq \begin{bmatrix} \mathrm{Re}\{\tilde{\boldsymbol{w}}[k]\} \\ \mathrm{Im}\{\tilde{\boldsymbol{w}}[k]\} \end{bmatrix}. \tag{5.7}$$

$\boldsymbol{H}$ will have full column rank if $\{h_{1,i}\}$ and $\{h_{2,i}\}$ have no common zeros.

B. SOS Equalization Algorithms

SOS methods and some HOS methods [12] require that channel diversity be available in the form of excess bandwidth or excess antenna. SOS channel identification based on the SIMO channel model of (5.4) was first addressed by Tong *et al.* [1]. A subspace method (SSM) was later developed by Moulines *et al.* [4] which utilizes the block Toeplitz structure of the matrix $\boldsymbol{H}$ for its estimation. Recently, an out-product decomposition algorithm (OPDA) [7] was proposed by one of the authors. Based on the covariance matrix of $\boldsymbol{x}[k]$, an outer product of the unknown channel vector can be estimated to yield a more accurate channel $\boldsymbol{H}$. Because the received data vector $\boldsymbol{x}[k]$ after derotation is identical to the SIMO data vector [1], [4], existing SOS methods based on SIMO models, such as oversampling, can now be directly applied to the blind equalization of GSM signals without additional antennae/sensors.

C. SIMO HOS Linear Equalizer

Based on HOS, the baud rate sampled superexponential algorithm (SEA) was first proposed by Shalvi and Weinstein [11]. Its fractionally spaced counterpart was presented by Ding [12] as an SIMO algorithm and is shown to converge globally when SIMO subchannels have no common zeros. Following notations in [11] and [12], the SIMO superexponential algorithm can be implemented on cumulants of the equalizer output

$$y_k = \theta^H \boldsymbol{x}[k]. \tag{5.8}$$

First, define a fourth-order cumulant

$$\begin{aligned} \mathrm{cum}(y_1; y_2; y_3; y_4) &\triangleq E\{y_1 y_2 y_3 y_4\} - E\{y_1 y_2\}E\{y_3 y_4\} \\ &\quad - E\{y_1 y_3\}E\{y_2 y_4\} - E\{y_1 y_4\}\{y_2 y_3\}. \end{aligned} \tag{5.9}$$

Let superscript $^{\#}$ denote the matrix pseudo-inverse. The SEA is a simple iteration based on

$$\theta_w = R_{\boldsymbol{x}}[0]^{\#}\,\mathrm{cum}\left(y_k[i]; y_k[i]; y_k[i]^*; X_k^*\right) \tag{5.10}$$

and

$$\theta[k+1] = \frac{1}{\sqrt{\theta_w^H R_{\boldsymbol{x}}[0]\theta_w}}\theta_w. \tag{5.11}$$

Similarly, the SIMO implementation of the constant modulus algorithm (CMA) has been shown to converge globally [13], unlike its single-channel counterpart. Given the channel diversity generated by derotation of baud rate sampled GMSK signals, the CMA can also be applied for direct linear equalization.

VI. Simulations

A. Setup

We now present simulation results of GSM blind equalization. A true GMSK signal is generated and sent over multipath channels. The receiver anti-aliasing filter was selected as a root-raised cosine pulse with rolloff factor 0.1 over frequency $1/(2T)$. Equalizer output bit error rate (BER) is used as the performance measure averaged over 200 randomly selected COST207 mobile channels [16] from bad urban (BU), hilly terrain (HT), and typical urban (TU) environments. The mobile channel is assumed to have 100 potential multipaths and the vehicle speed is set to 80 km/h. For simplicity, channel fading over one short data frame is assumed to be minimum and channel impulse response is considered to be time invariant in each data frame.

The analog GMSK signal is approximated by 16 times oversampling. The baud rate sampling phase is obtained by selecting the strongest subsequence from the 16 times over sampled receiver data. For convenience, the sampled signal sequence is normalized to unit power. For $2/T$ oversampling, the receiver filter remains the same while the sampling phase is arbitrarily chosen.

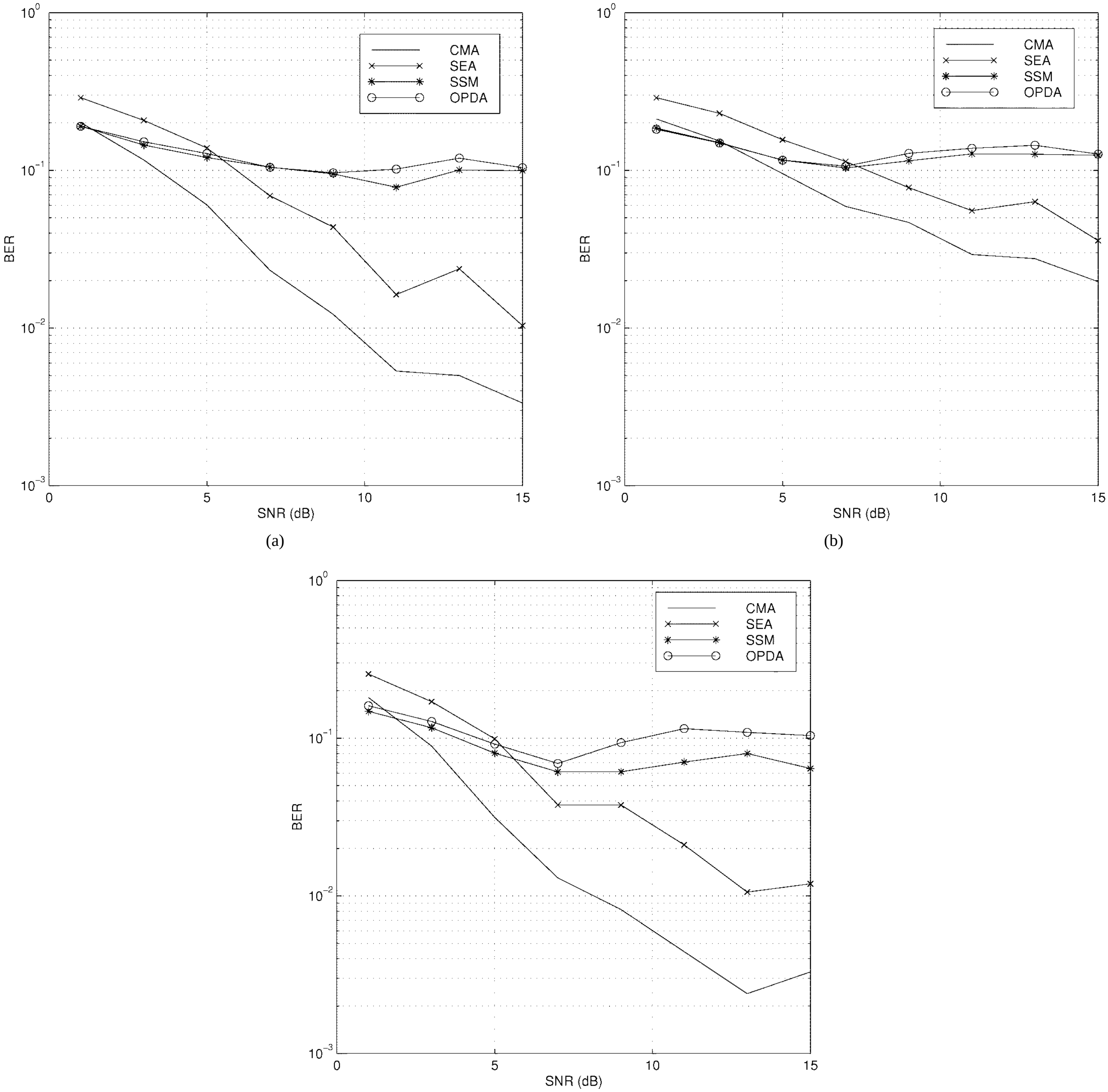

Fig. 3. BER of linear equalizers using MDL channel-order estimation. (a) BU environment. (b) HT environment. (c) TU environment.

B. GSM Linear Blind Equalization

Four blind linear equalizers are selected for performance comparison. BER's are generated for SSM, OPDA, SEA, and CMA. Two SIMO second-order statistical methods, SSM and OPDA, are chosen. For SSM and OPDA $M = 10$ bauds of data are selected for the algorithms. The channel length is estimated by applying the information theoretic criterion MDL [17] to estimate the signal subspace dimension in $R\boldsymbol{x}[0]$ and is also fixed *a priori* according to the environment as 3(BU), 4(TU), and 6(HT), respectively.

Given the estimate of $\boldsymbol{H}$, multiple equalization outputs with different delays can be generated from $\boldsymbol{H}^{\#}\ \boldsymbol{x}[k]$. A simple criterion should be used to select the final equalization output from multiple sequences. In our simulation of baud rate sampled systems we simply select the output sequence with the strongest power.

For the SIMO SEA and the SIMO CMA, two equalizer filters of length seven are chosen for the two subchannel outputs and are initialized to be all zero except for the unity center tap of the second equalizer filter. Because of the short data burst in each frame, CMA was implemented

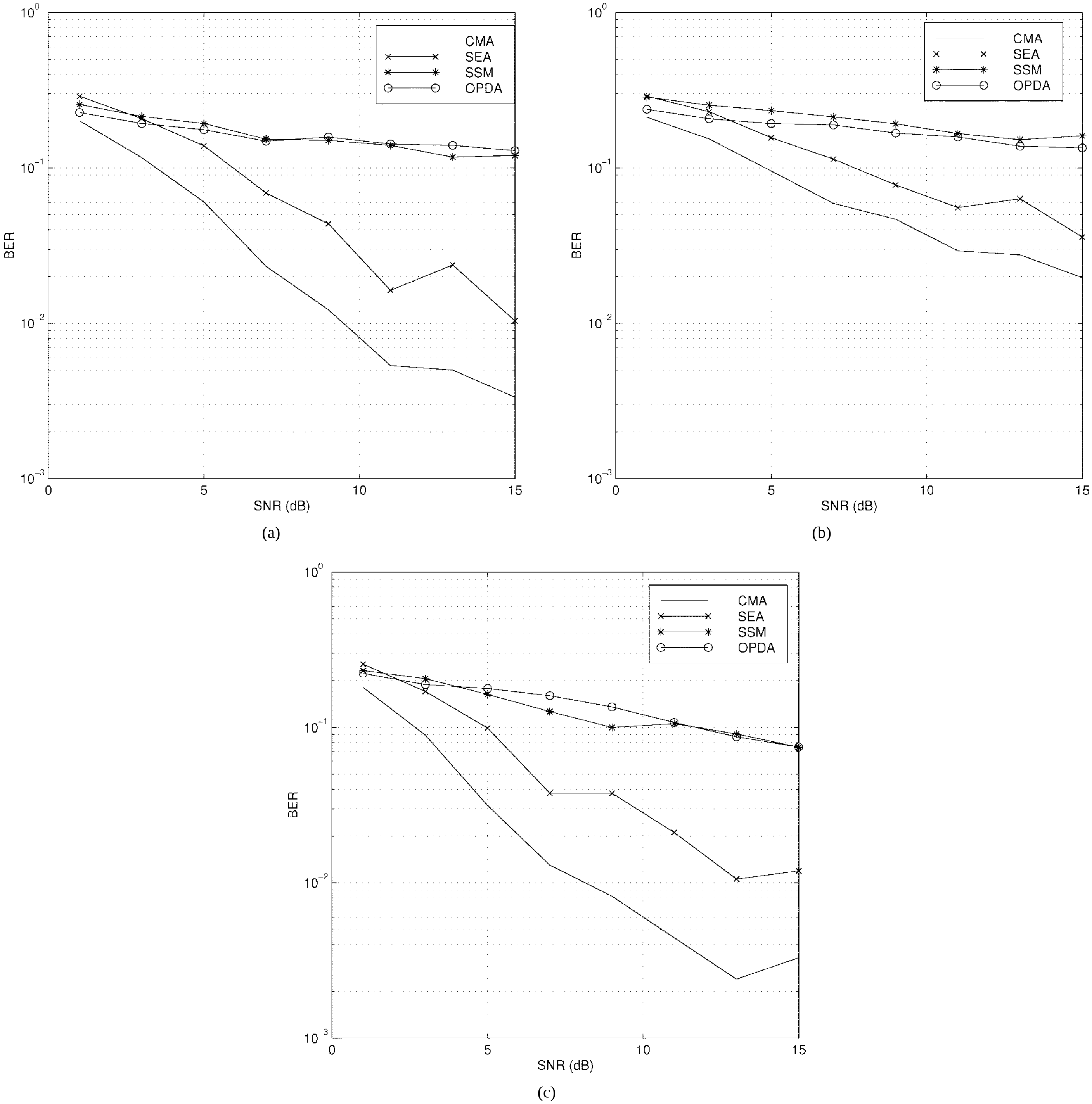

Fig. 4. BER of linear equalizers using fixed channel-order assumption. (a) BU environment. (b) HT environment. (c) TU environment.

to iteratively reuse the single received signal frame. For satisfactory convergence, the number of iteration for SEA is set to ten while the number of iteration for CMA is set to 20.

The resulting raw BER's are computed for all four linear equalizers and are shown in Figs. 3 and 4.

C. Nonlinear Equalization via MLSE

Based on blind channel identification, the Viterbi algorithm is used for maximum likelihood sequence estimation (MLSE) of GSM raw input data. Blind channel estimates from OPDA and SSM are used. In training, channel impulse response is obtained from least squares estimation using the midamble training data of 26 bits. In least squares channel estimation we assume that the fading channel has no absolute delay, while the receiver filter delay and the linear approximation delays are known. The overall channel length is $L = 8$.

As in linear equalization, for blind algorithms the channel length is obtained from MDL estimation and from a fixed set of values, according to the channel type. A maximum of six dominant channel taps are used in the Viterbi algorithm by windowing the estimated channel responses.

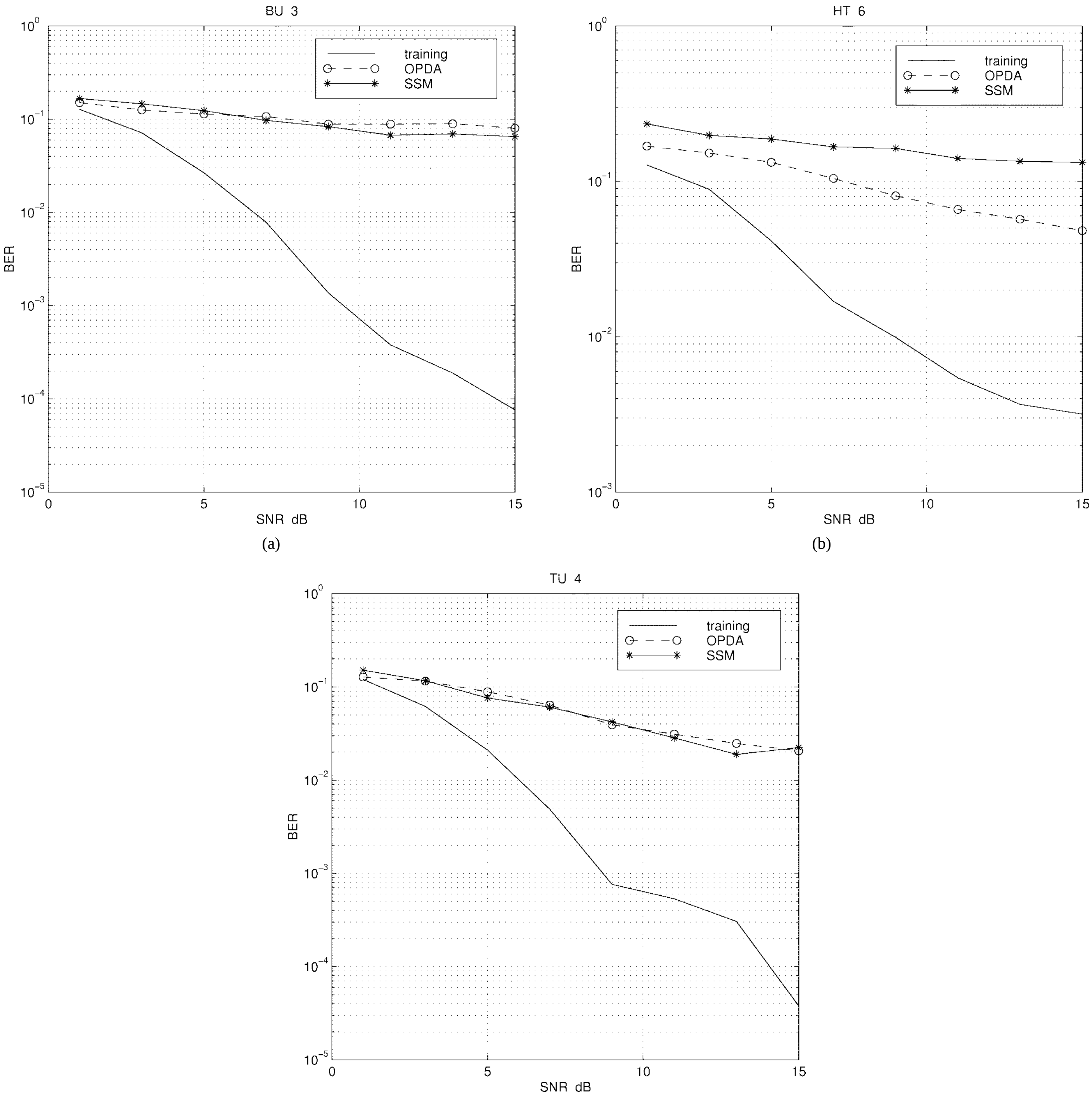

Fig. 5. BER of MLSE with T sampling and fixed channel length. (a) BU environment. (b) HT environment. (c) TU environment.

The BER from MLSE for the following four cases are given in Figs. 5–8:

- fixed channel length with baud rate sampling;
- fixed channel length with $T/2$ sampling;
- estimated channel length from MDL with baud rate sampling;
- estimated channel length from MDL with $T/2$ sampling.

The results for $T/2$ oversampled GSM signals are presented to illustrate the effect of oversampling. Because oversampling preserves the signal information, the sampling phase is arbitrarily chosen, unlike baud rate sampling. For complex GMSK signals with our anti-aliasing receiver filter, the E_b/N_0 is equivalent to the SNR of the digital system.

D. Results and Discussions

Linear Equalizers: From the simulation results of linear equalizers it is clear that the BER's of SOS blind identification methods OPDA and SSM are generally worse than direct

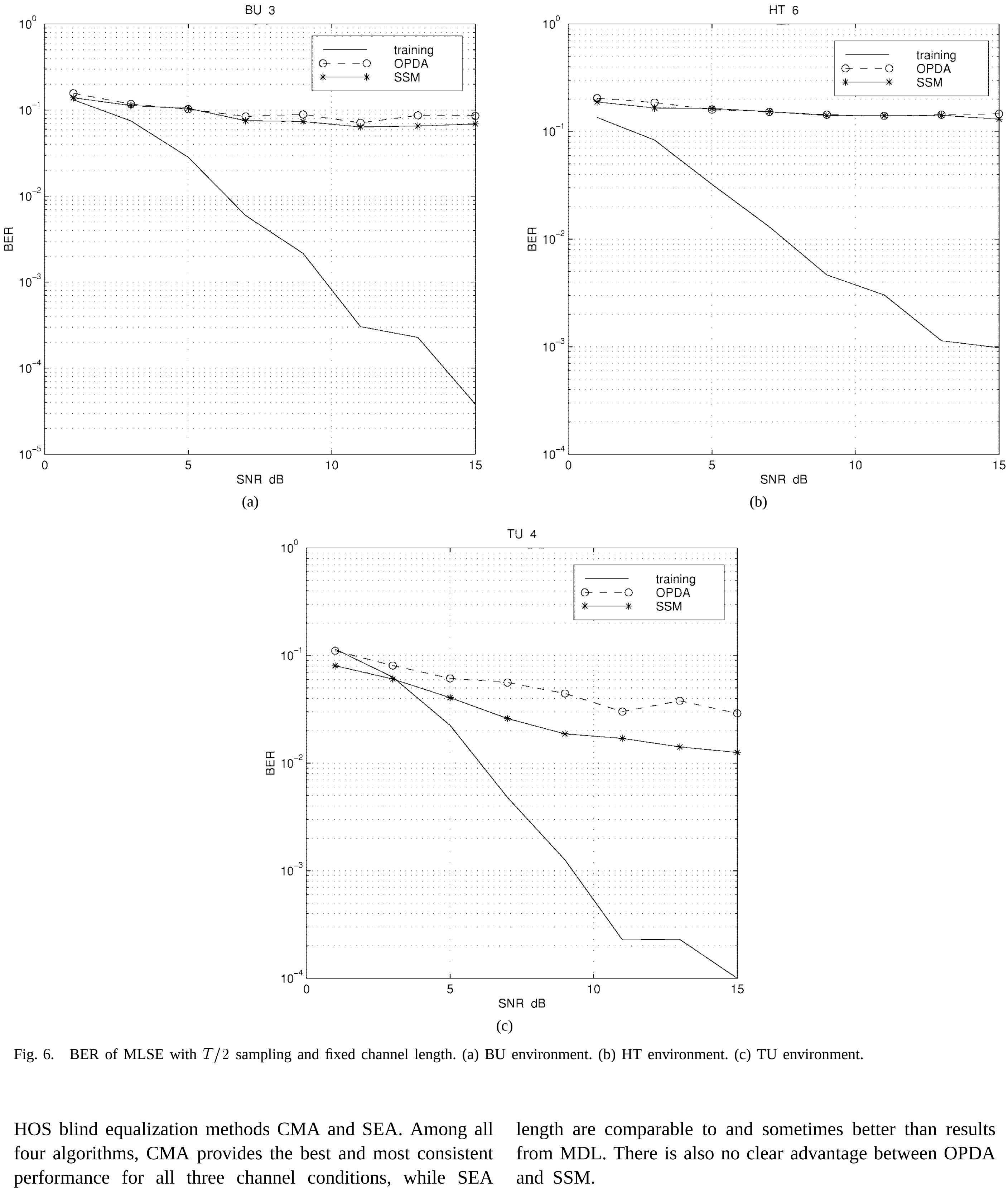

Fig. 6. BER of MLSE with $T/2$ sampling and fixed channel length. (a) BU environment. (b) HT environment. (c) TU environment.

HOS blind equalization methods CMA and SEA. Among all four algorithms, CMA provides the best and most consistent performance for all three channel conditions, while SEA also generates acceptable BER's. On the other hand, linear equalization based on SOS blind identification is unable to deliver acceptable BER's for GSM wireless systems.

It should be noted that channel-order estimation based on MDL only affects the performance of OPDA and SSM for a low SNR. For higher channel SNR, results using fixed channel length are comparable to and sometimes better than results from MDL. There is also no clear advantage between OPDA and SSM.

It should be noted, however, that identification-based linear equalizers can, in fact, generate much better performances in simulation. Among multiple equalizer outputs of $\boldsymbol{H}^{\#}\,\boldsymbol{x}[k]$ the best output sequence yields a much lower BER. Unfortunately, our lack of knowledge on the best output sequence forces us to make a simple choice in selecting the sequence with the largest

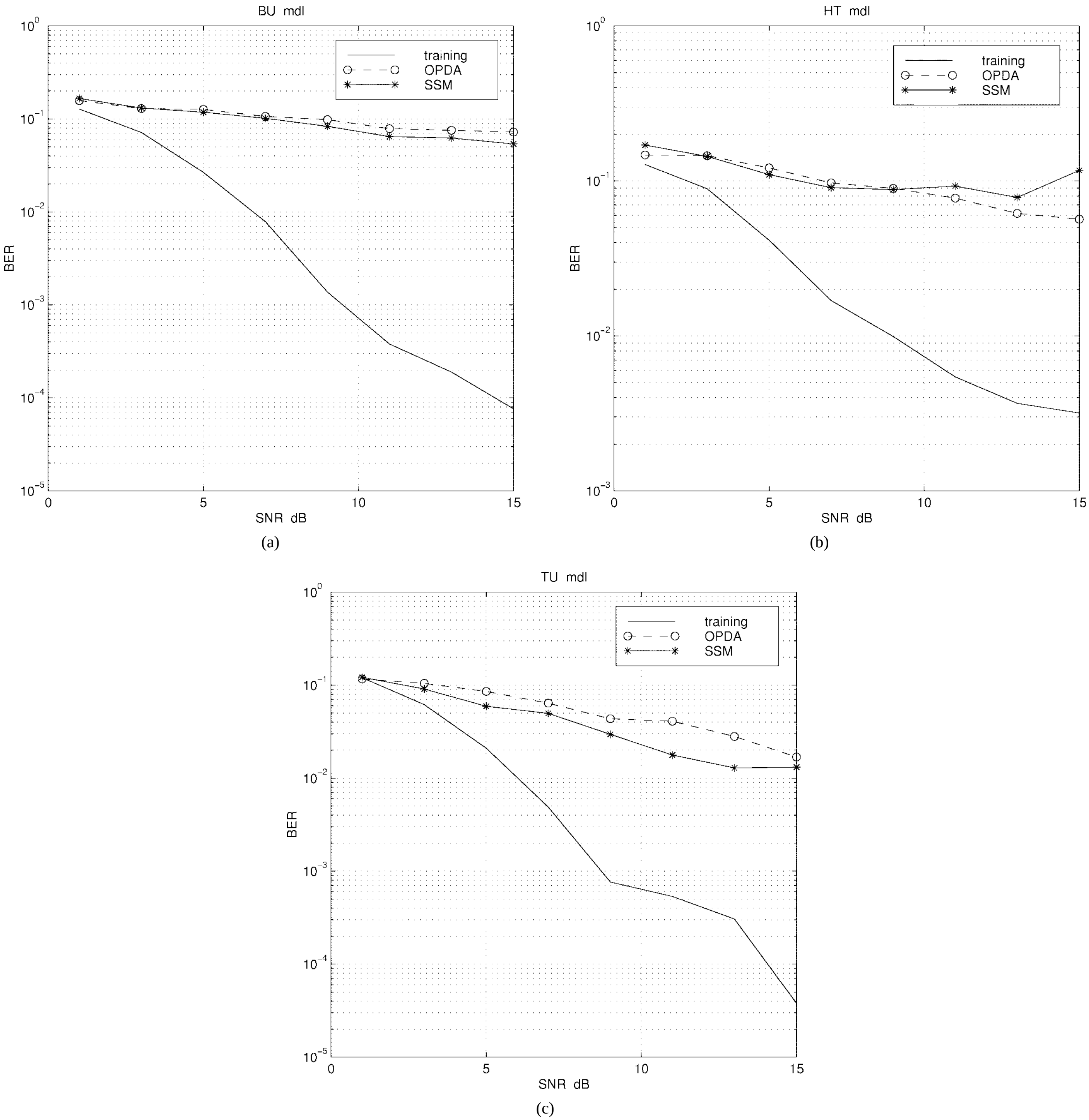

Fig. 7. BER of MLSE with T sampling and MDL channel length estimation. (a) BU environment. (b) HT environment. (c) TU environment.

average power. On the other hand, direct linear equalizers such as CMA and SEA do not require channel order estimates and output selection and are more robust.

Nonlinear Equalizers: When results from blind channel identification are used in Viterbi algorithms for MLSE , the GSM output BER is improved. Comparative results between blind algorithms based on MDL channel length estimation and fixed channel length estimation show that MLD estimation generally leads to a lower BER. In particular, for HT channels with longer channel delay spread, MDL estimation is superior. It is also apparent that oversampling only provides no obvious improvement in BER for blind identification methods, as expected. In fact, even doubling the receiver filter bandwidth failed to improve the performance of the MLSE.

It should be noted, however, that except for the more benign TU channels, SOS blind channel identification does not generate BER's comparable to those in systems relying on training. The BER is low enough, however, to encourage further studies on means to improve blind channel estimation for short GSM data bursts.

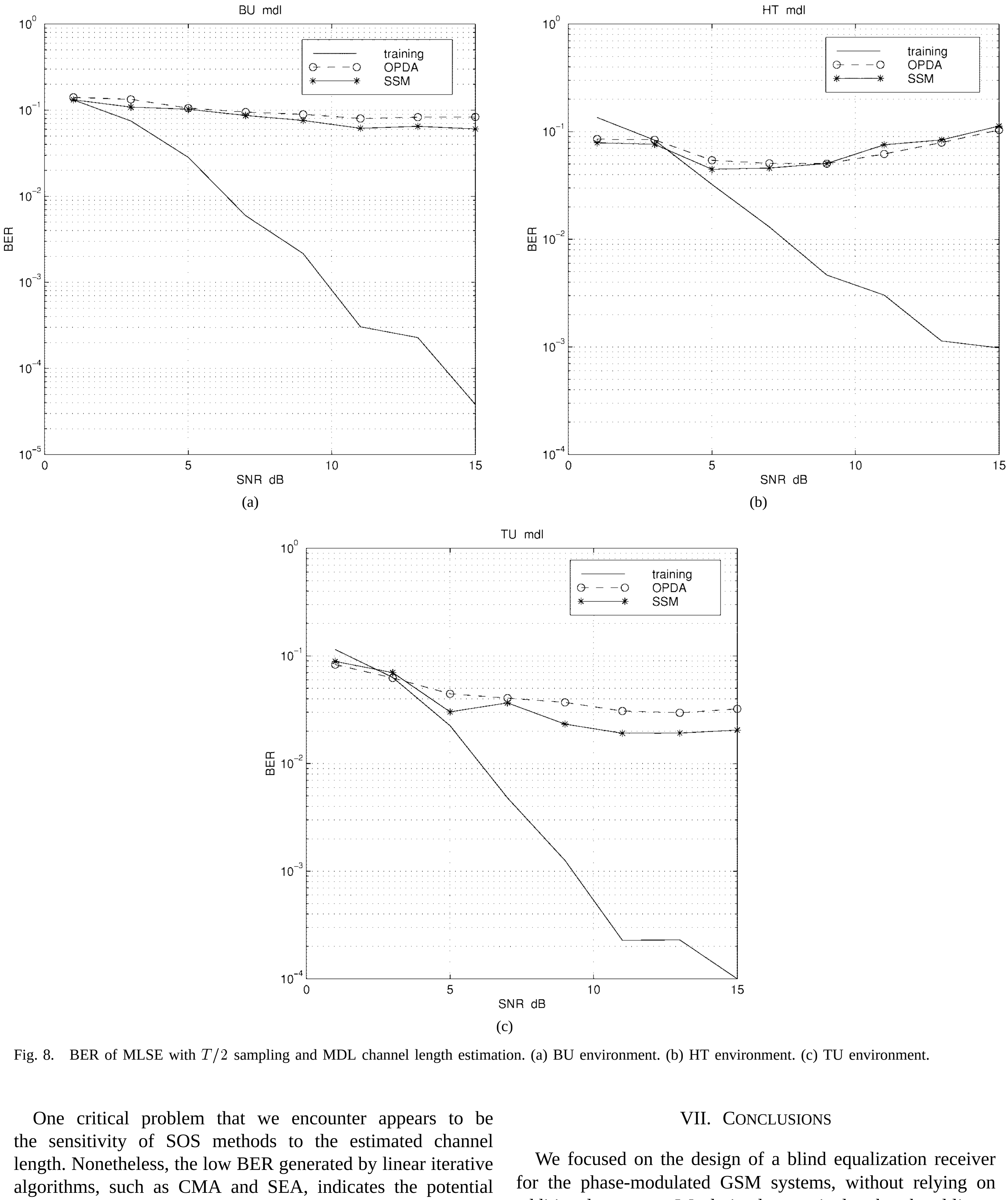

Fig. 8. BER of MLSE with $T/2$ sampling and MDL channel length estimation. (a) BU environment. (b) HT environment. (c) TU environment.

One critical problem that we encounter appears to be the sensitivity of SOS methods to the estimated channel length. Nonetheless, the low BER generated by linear iterative algorithms, such as CMA and SEA, indicates the potential for improved blind channel estimation methods that are more robust when both HOS and SOS information is exploited. Our linear equalization results suggest that for GSM systems with moderate to high SNR, some HOS methods can still perform very effectively, even when the frame of data (142 bits) available for equalization is very short.

VII. Conclusions

We focused on the design of a blind equalization receiver for the phase-modulated GSM systems, without relying on additional antennae. We derived an equivalent baseband linear QAM model for the GMSK signal used in GSM systems. More importantly, we were able to use a derotation scheme to create two subchannels for each GMSK signal, even though the GMSK signal has very little excess bandwidth. We effectively adopted SOS and HOS blind equalization algorithms for

GMSK signals by using the SIMO system model. Based on the two critical steps of linear GMSK approximation and derotation, effective simulation results using simple SOS and HOS algorithms demonstrated the great potential of blind equalization algorithms in GSM wireless systems. Our results also exposed the weaknesses of some existing SOS blind channel identification algorithms when applied in GSM systems.

APPENDIX
EXISTENCE OF COMMON ZEROS BETWEEN DEROTATED DIVERSITY CHANNELS

Proof of Theorem 4.1: We show that for a given channel with sampled impulse response $[h_0, h_1, \cdots, h_L]$ the two derotated diversity channels will share common zeros if and only if its transfer function

$$H(z) = \sum_{n=0}^{L} h_n z^{-n}$$

has zero(s) symmetric to the imaginary axis.

Let the transfer functions of the two derotated diversity channels be

$$H_1(z) = \sum_{n=0}^{L} \text{ Real } \{h_n j^{-n}\} z^{-n} \qquad \text{(A.1)}$$

$$H_2(z) = \sum_{n=0}^{L} \text{ Imag } \{h_n j^{-n}\} z^{-n}. \qquad \text{(A.2)}$$

Clearly

$$\begin{aligned} H_1(z) + jH_2(z) &= \sum_{n=0}^{L} (\text{Real}\{h_n j^{-n}\} \\ &\quad + j \cdot \text{Imag}\{h_n j^{-n}\}) z^{-n} \\ &= \sum_{n=0}^{L} h_n j^{-n} z^{-n} = H(jz) \end{aligned}$$

Notice that polynomials $H_1(z)$ and $H_2(z)$ are both real. As a result, there are two different cases to consider.

- $H_1(z)$ and $H_2(z)$ share a real zero z_0:
  Because

  $$H_1(z_0) + jH_2(z_0) = H(jz_0) = 0 \qquad \text{(A.3)}$$

  jz_0 must be a zero of $H(z)$ if z_0 is a common zero of $H_1(z)$ and $H_2(z)$. Conversely, if jz_0 is a zero of $H(z)$, then

  $$H_1(z_0) = \text{Real}\{H(jz_0)\} = 0$$

  and

  $$H_2(z_0) = \text{Imag}\{H(jz_0)\} = 0.$$

  In other words, $H_1(z_0)$ and $H_2(z)$ share a real common zero z_0.

- $H_1(z)$ and $H_2(z)$ share a pair of conjugate zeros $a \pm jb$: Since a and b are real, we have

  $$\begin{aligned} H(-b+ja) &= H(j(a+jb)) = H_1(a+jb) \\ &\quad + jH_2(a+jb) = 0 \\ H(b+ja) &= H(j(a-jb)) = H_1(a-jb) \\ &\quad + jH_2(a-jb) = 0. \end{aligned} \qquad \text{(A.4)}$$

  This implies that $\pm b + ja$ are zeros of $H(z)$ if $a \pm jb$ are common zeros of $H_1(z)$ and $H_2(z)$. Conversely, if $\pm b + ja$ are zeros of $H(z)$ notice that

  $$H_1^*(z) = H_1(z^*) \quad \text{and} \quad H_2^*(z) = H_2(z^*).$$

  As a result, we have

  $$\begin{aligned} H(-b+ja) + H(b+ja) &= \text{Real}\{H_1(a+jb)\} \\ &\quad + j \text{ Real } \{H_2(a+jb)\} = 0 \end{aligned} \qquad \text{(A.5)}$$

  $$\begin{aligned} H(-b+ja) - H(b+ja) &= \text{Imag } \{H_1(a+jb)\} \\ &\quad + j \text{ Imag } \{H_2(a+jb)\} = 0 \end{aligned} \qquad \text{(A.6)}$$

  Hence, we have $H_1(a+jb) = 0$ and $H_2(a+jb) = 0$. Because both polynomials $H_1(z)$ and $H_2(z)$ are real, $a \pm jb$ are a conjugate pair of their common zeros.

To summarize, the two derotated diversity channels have common zero(s) if and only if $H(z)$ has zero(s) symmetric to the imaginary axis.

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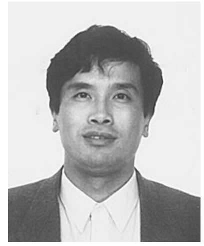

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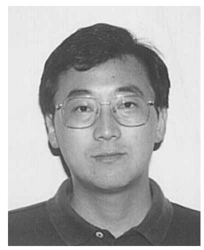

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