GMSK LINEARIZATION AND STRUCTURED CHANNEL ESTIMATE FOR GSM SIGNALS

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Abstract

GMSK is a spectrum-efficient modulation scheme, and it is adopted as the modulation standard of GSM systems. However, because of its phase modulation, Gaussian filtering, and partial response signaling properties, GMSK is not a linear modulation. In this paper, we present a linear approximation of GMSK signals at T/2-fractional spacing. The partial response channel with a memory length of 3 symbol periods, which is the pulse-shaping function inherent in the GMSK modulation, can be clearly identified in this linear approximation. Further more, we use this partial response channel information in the estimation of the wireless channel, which includes the physical propagation channel and the modulation pulse-shaping function. Conventional channel estimators, *e.g.*, correlator or least-squares estimator approaches, do not exploit the information of the pulse-shaping function which is known and available. Our proposed structured channel estimate is more accurate since we incorporate the knowledge of the channel structure in the estimation procedure. We compared BER(bit-error-probability) performance of a GSM system with structured and unstructured channel estimate, and simulation results show improvement of a Viterbi equalizer in different test channels when a structured channel estimate is used.

I INTRODUCTION

GMSK (Gaussian-filtered Minimum Shift Keying) has been adopted as the modulation scheme for GSM systems because of its spectral and power efficiency. Compared to MSK, the Gaussian pulse-shaping filter employed in GMSK can considerably reduce the side-lobe levels in the transmitted spectrum. Further more, GMSK is power-efficient because of the phase modulation and its constant envelope. However, because of the phase modulation and Gaussian filtering, GMSK is not a linear modulation. It is often preferable to have a linear modulation because of the simplicity of generating signals, deriving algorithms, and analyzing performance. Recently, [1, 2] describe a linear representation of general constant amplitude phase modulations by superposition of amplitude modulated pulses (AMP). In this paper, we show a simple and accurate linear approximation of GMSK in the discrete time domain. The pulse-shaping function $g(t)$ with a memory length of 3 symbol periods inherent in GMSK can be clearly identified from this linear representation.

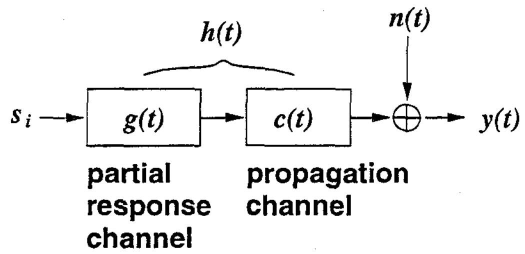

Figure 1: The composite channel response $h(t)$ for equalizers.

In GSM systems, because of its short symbol period ($3.7\mu s$) compared to the multipath delay spread, equalizers are required for the receiver to function properly. Due to the partial response signaling in GMSK, the composite channel response $h(t)$ for the equalizer is a cascade of the partial response and the radio propagation channel, as shown in Fig. 1. A correlator or least-squares approach using the training sequence and the sampled channel output $y(k)$ can be used to estimate the equivalent FIR channel $h(k)$ assuming no knowledge about the pulse-shaping function $g(t)$. However, a complete information of $g(t)$ is available from the linear representation of GMSK signals. Therefore, the channel estimate can be improved by using the known structure $g(t)$ within $h(t)$. It is also shown in [3] that $h(t)$ lies in the subspace spanned by $g(t)$, and thus less unknowns need to be estimated compared to the unstructured method.

II GMSK LINEARIZATION

A GMSK modulated signal $x(t)$ can be expressed as [4]:

$$x(t) = exp(j\varphi(t)) \tag{1}$$

with the following definitions:

$$\varphi(t) = \varphi_0 + \sum_i k_i \Phi(t - iT) \tag{2}$$

$$k_i = s_i s_{i-1}, s_i \in \{1, -1\} \tag{3}$$

$$\Phi(xT) = \frac{\pi}{2}(G(x - \frac{1}{2}) - G(x - \frac{3}{2})) \tag{4}$$

$$G(x) = x\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} exp(\frac{-t^2}{2\sigma^2})dt + \frac{\sigma}{\sqrt{2\pi}} exp(\frac{-x^2}{2\sigma^2}) \tag{5}$$

where φ_0 is the unknown initial phase, T is the symbol period, s_i is the information carrying data bit, and $\sigma=\sqrt{ln(2)}/(2\pi 0.3)$. Data bits are first differentially encoded as described in Eq. (3), and then pass through the Gaussian filter as shown in Eq. (4)-(5) and then phase modulated. Because of a series of non-linear operation, GMSK is not a linear modulation scheme. Due to this non-linearity, it is not easy to analyze its performance and derive equalization algorithms for GMSK. MSK (Minimum-Shift-Keying) is often used as an approximation of GMSK because of its linearity and its equivalence to OQPSK [5]. However, this linear MSK approximation has a shorter partial response channel and higher sidelobes in the transmitted spectrum.

We proceed with the GMSK linearization by expanding Eq. (1) using the definition in Eq. (2)-(5). Considering a T_s timing offset at the sampling point and zero initial phase φ_0, which will be absorbed into the channel, the modulated signal $x(t)$ oversampled at T/2 can be written as:

$$x(kT + T_s) \approx exp(j \sum_{i=-\infty}^{k} k_i \Phi((k-i)T + T_s))$$

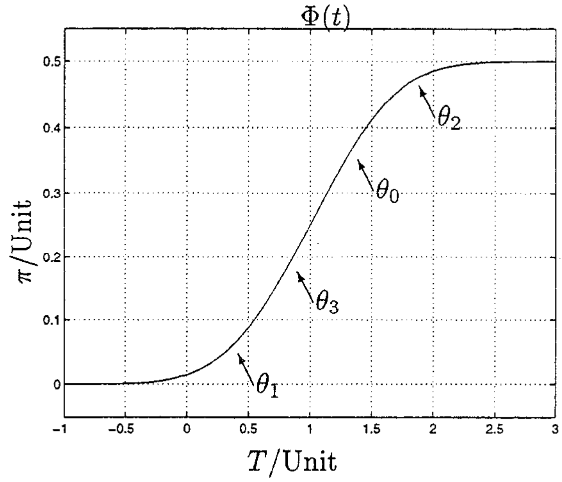

Figure 2: $[\theta_1, \theta_3, \theta_0, \theta_2]$ sampled at $[T_s, T/2 + T_s, T + T_s, 3T/2 + T_s]$, respectively.

$$= j^{k-1}cos(\theta_0)cos(\theta_1)s_{k-2} + j^k(sin(\theta_0)cos(\theta_1)s_{k-1} + cos(\theta_0)sin(\theta_1)s_{k-2}s_{k-1}s_k) + j^{k+1}sin(\theta_0)sin(\theta_1)s_k \tag{6}$$

$$x(kT + \frac{1}{2}T + T_s) \approx exp(j \sum_{i=-\infty}^{k} k_i \Phi((k-i)T + \frac{1}{2}T + T_s))$$

$$= j^{k-1}cos(\theta_2)cos(\theta_3)s_{k-2} + j^k(sin(\theta_2)cos(\theta_3)s_{k-1} + cos(\theta_2)sin(\theta_3)s_{k-2}s_{k-1}s_k) + j^{k+1}sin(\theta_2)sin(\theta_3)s_k \tag{7}$$

where $0 \le T_s \le T/2$ without loss of generality, and $[\theta_0, \theta_1, \theta_2, \theta_3]$ are defined in Fig. 2 and can be written as:

$$\theta_i = \begin{cases} \Phi((1-i)T + T_s) & i = 0, 1 \\ \Phi((\frac{7}{2} - i)T + T_s) & i = 2, 3 \end{cases} \tag{8}$$

Equation (6) and (7) contains a nonlinear term $s_{k-2}s_{k-1}s_k$ with coefficient $cos(\theta_i)sin(\theta_{i+1})$, which is smaller than $sin(\theta_i)cos(\theta_{i+1})$ by at least one order of magnitude within the range of consideration for θ_i's. We will ignore this non-linear term and thus obtain a linear approximation $\hat{x}(t)$ of the original GMSK signal $x(t)$:

$$\begin{bmatrix} \hat{x}(kT + T_s) \\ \hat{x}(kT + \frac{1}{2}T + T_s) \end{bmatrix} = j^k \mathbf{A} \begin{bmatrix} s_k \\ s_{k-1} \\ s_{k-2} \end{bmatrix} \tag{9}$$

where

$$\mathbf{A} = \begin{bmatrix} jsin(\theta_0)sin(\theta_1) & sin(\theta_0)cos(\theta_1) & j^{-1}cos(\theta_0)cos(\theta_1) \\ jsin(\theta_2)sin(\theta_3) & sin(\theta_2)cos(\theta_3) & j^{-1}cos(\theta_2)cos(\theta_3) \end{bmatrix} \tag{10}$$

Equation (9) and (10) show that even and odd samples of the oversampled $x(t)$ have different 3-tap partial response channels. A comparison between a sample GMSK waveform and its linear approximation described in Eq. (9) is shown in Fig. 3. Recently, [1, 2, 6]

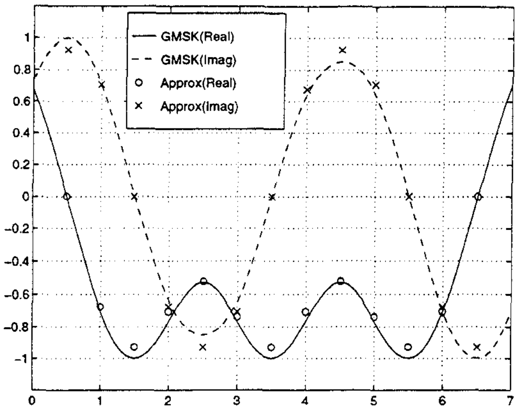

Figure 3: A sample GMSK waveform and its linear approximations.

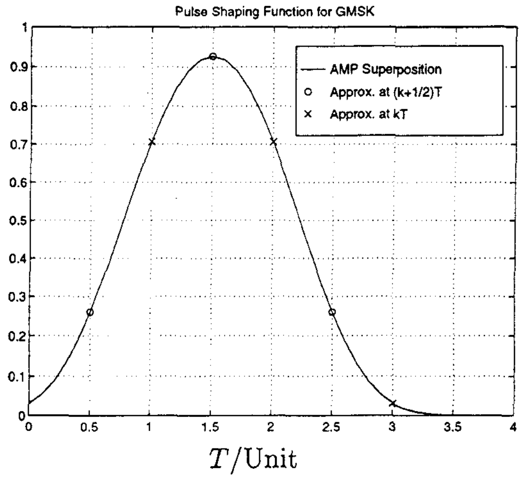

Figure 4: Comparison between pulse-shaping functions of the AMP superposition and the linear approximation.

show an approximation of CPM signals by superposition of finite amplitude modulated pulses (AMP). We compare the pulse-shaping function in Eq. (9) with the one used in [6], and it shows exact agreement in Fig. 4.

III STRUCTURED CHANNEL ESTIMATE

In GSM systems, there is a 26-bit training sequence embedded in the center of a data burst, which can be used for channel estimation. The most common approach is a least-squares solution assuming no structures in the channel. However, the composite channel, which includes the physical propagation channel and the pulse-shaping function, is partially known in GMSK from the information of the partial response channel as shown in Eq. (9) and (10). Therefore, a channel estimate can be improved by incorporating the pulse-shaping function knowledge into the estimation procedure.

Suppose $s_k, s_{k+1}, \cdots, s_{k+v}$ are the training symbols in a data burst, and the span of the propagation channel $c(t)$ is NT. The corresponding received signal is

$$\begin{aligned} y(t) &= \hat{x}(t) * c(t) \\ &= \int_0^{NT} \hat{x}(t-\tau)c(\tau)d\tau \\ &\approx \frac{T}{2}\sum_{i=0}^{2N} \hat{x}(t-\frac{i}{2}T)c(\frac{i}{2}T) \end{aligned} \tag{11}$$

Therefore, the sampled output y_k and $y_{k+\frac{1}{2}}$ are

$$\begin{aligned} y_k &= y(kT+T_s) \\ &= \sum_{i=0}^{2N} \hat{x}((k-\frac{i}{2})T+T_s)c(\frac{i}{2}T)\frac{T}{2} \\ &= j^k[s_k s_{k-1}\cdots s_{k-N-2}]\mathbf{G_1}[c_0 c_{\frac{1}{2}} c_1 \cdots c_N]^T \\ &= j^k[s_k s_{k-1}\cdots s_{k-N-2}]\mathbf{h}_1 \end{aligned} \tag{12}$$

$$\begin{aligned} y_{k+\frac{1}{2}} &= y(kT+\frac{1}{2}T+T_s) \\ &= \sum_{i=0}^{2N} \hat{x}((k-\frac{i-1}{2})T+T_s)c(\frac{i}{2}T)\frac{T}{2} \\ &= j^k[s_k s_{k-1}\cdots s_{k-N-2}]\mathbf{G_2}[c_0 c_{\frac{1}{2}} c_1 \cdots c_N]^T \\ &= j^k[s_k s_{k-1}\cdots s_{k-N-2}]\mathbf{h}_2 \end{aligned} \tag{13}$$

where c_k denotes $\frac{T}{2}c(kT)$, $\mathbf{h}_1$ and $\mathbf{h}_2$ are the composite channels for even and odd samples, and $\mathbf{G_1}$ and $\mathbf{G_2}$ whose sizes are $(N+3)\times(2N+1)$, *e.g.*, when N=1, are

$$\mathbf{G_1} = \begin{bmatrix} j sin(\theta_0)sin(\theta_1) & 0 & 0 \\ sin(\theta_0)cos(\theta_1) & sin(\theta_2)sin(\theta_3) & sin(\theta_0)sin(\theta_1) \\ j^{-1}cos(\theta_0)cos(\theta_1) & j^{-1}sin(\theta_2)cos(\theta_3) & j^{-1}sin(\theta_0)cos(\theta_1) \\ 0 & j^{-2}cos(\theta_2)cos(\theta_3) & j^{-2}cos(\theta_0)cos(\theta_1) \end{bmatrix}$$

$$\mathbf{G_2} = \begin{bmatrix} j sin(\theta_2)sin(\theta_3) & j sin(\theta_0)sin(\theta_1) & 0 \\ sin(\theta_2)cos(\theta_3) & sin(\theta_0)cos(\theta_1) & sin(\theta_2)sin(\theta_3) \\ j^{-1}cos(\theta_2)cos(\theta_3) & j^{-1}cos(\theta_0)cos(\theta_1) & j^{-1}sin(\theta_2)cos(\theta_3) \\ 0 & 0 & j^{-2}cos(\theta_2)cos(\theta_3) \end{bmatrix}$$

The pulse-shaping matrices $\mathbf{G_1}$ and $\mathbf{G_2}$, which contain the information from the pulse-shaping function, are the known structures within the composite channels

$\mathbf{h}_1$ and $\mathbf{h}_2$. Therefore, they should be incorporated into the channel estimation.

Equation (12) and (13) show that y_k and $y_{k+\frac{1}{2}}$ are rotated by 90 degrees at each sampling time because of the term j^k. A simple derotation technique described in [7], which derotates the sampled signals by -90 degrees at each sampling instant, can remove the j^k term. From now on, we assume all sampled signals have been derotated, and the j^k term in Eq. (12) and (13) can be removed. Therefore, the channel estimation can be formulated as the following two least-squares problems:

1. Unstructured Channel Estimate:

$$\begin{aligned} \begin{bmatrix} \mathbf{y}_e \\ \mathbf{y}_o \end{bmatrix} &= \begin{bmatrix} \mathbf{S} & 0 \\ 0 & \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} = \mathcal{S} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} && (14) \\ \begin{bmatrix} \hat{\mathbf{h}}_1 \\ \hat{\mathbf{h}}_2 \end{bmatrix} &= (\mathcal{S}^H \mathcal{S})^{-1} \mathcal{S}^H \begin{bmatrix} \mathbf{y}_e \\ \mathbf{y}_o \end{bmatrix} && (15) \end{aligned}$$

2. Structured Channel Estimate:

$$\begin{aligned} \begin{bmatrix} \mathbf{y}_e \\ \mathbf{y}_o \end{bmatrix} &= \begin{bmatrix} \mathbf{S} & 0 \\ 0 & \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix} \mathbf{c} = \mathcal{S}\mathbf{G}\mathbf{c} && (16) \\ \hat{\mathbf{c}} &= ((\mathcal{S}\mathbf{G})^H (\mathcal{S}\mathbf{G}))^{-1} (\mathcal{S}\mathbf{G})^H \begin{bmatrix} \mathbf{y}_e \\ \mathbf{y}_o \end{bmatrix} && (17) \\ \begin{bmatrix} \hat{\mathbf{h}}_1 \\ \hat{\mathbf{h}}_2 \end{bmatrix} &= \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix} \hat{\mathbf{c}} && (18) \end{aligned}$$

where $\mathbf{y}_e = [y_{k+N+2}, y_{k+N+3}, \cdots, y_{k+v}]^T$ and $\mathbf{y}_o = [y_{k+N+\frac{5}{2}}, y_{k+N+\frac{7}{2}}, \cdots, y_{k+v+\frac{1}{2}}]^T$ are even and odd samples of the channel output. $\mathbf{S}$ is a toeplitz matrix formed by the training symbols with $[s_{k+N+2}, s_{k+1}, \cdots, s_{k+v}]^T$ and $[s_{k+N+2}, s_{k+N+1}, \cdots, s_k]$ as its first column and row. In general, if P is the oversampling factor, the size of $\mathbf{G}$ is $P(N+3) \times (PN+1)$. Therefore, $\mathbf{G}$ is a tall matrix and the matrix inverse in Eq. (17) exists. In the structured channel estimate, not only the information from the pulse-shaping matrix $\mathbf{G}$ is used, but also the number of unknowns to be estimated in Eq. (17) reduces to $(PN+1)$ compared to $P(N+3)$ in Eq. (15).

IV SIMULATION RESULTS

To evaluate the accuracy of the linear approximation of GMSK, and the performance of the structured channel estimation, we simulated a GSM system based on [8] with a two sensor antenna array with T/2 oversampling. A multiple channel, T/2 fractionally spaced Viterbi equalizer [9, 10] is used at the receiver. Figure 5 is the BER curves for the receiver in a TU (Typical Urban) 12-ray channel [8] with 1.35T delay spread, and the distribution of angles of arrival of each path is uniform between 0 to 2π. Figure 5 shows at 10^{-3} BER, the improvement is about 1.2 dB for a structured method using N=2. In order to investigate the effect of propagation channel models on the performance of the structured method, we tested two other channels with 5 independent equi-power Rayleigh fading multipaths, and delay at [0, 0.5T, 1T, 1.5T, 2T] and [0, 0.125T, 0.25T, 0.375T, 0.5T], respectively. We choose N to be 2 and 1 according to the delay spread of each channel. Figure 6 shows 1 dB and 2 dB improvement at 10^{-3} BER for 2T and 0.5T delay spread channels, respectively. Sim-

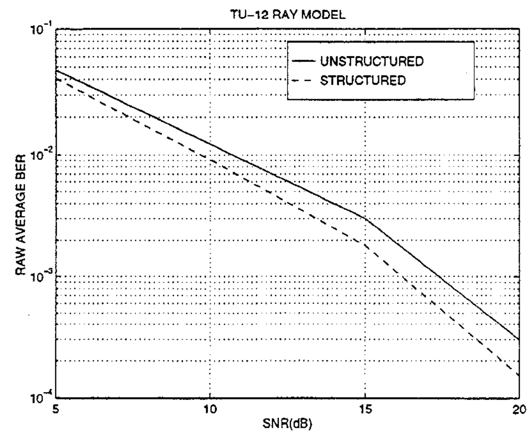

Figure 5: BER comparison between structured and unstructured method for TU-12 ray channel model.

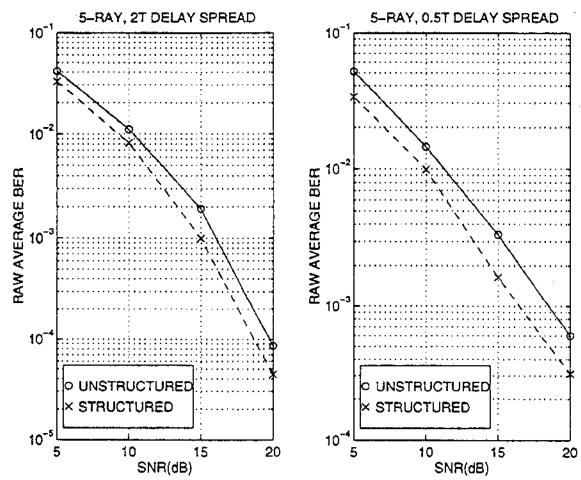

Figure 6: BER comparison between structured and unstructured method for 5 ray test channel model.

ulation results suggest that the structured method performs better when the channel delay spread is smaller. In general, the structured method reduces the number of unknowns by a ratio of $(PN+1)/(PN+PL)$, where L is the length of the partial response channel. A large L or a small N is desired to decrease the ratio. Thus, in a IS-54/136 system, because the raised cosine pulse shape is relatively longer than the channel delay spread, the improvement of a structured method is greater [3]. A large oversampling factor P can also reduce the ratio. However, the pulse-shaping matrices $\mathbf{G}_1$ and $\mathbf{G}_2$ become numerically ill conditioned for the matrix inverse in Eq. (17) as P increases.

V CONCLUSION

In this paper, we present a simple linear approximate of GMSK modulations at T/2 fractional spacing. In general, the entire waveform can be constructed at any sampling point by changing the timing offset T_s and its corresponding θ_i's. Once the sampling point is determined, we can pre-calculate the 3-tap pulse-shaping function and use it for the structured channel estimate. Compared to the unstructured method, the structured method exploits the known pulse-shaping function contained in the channel to be estimated, and thus it reduces the number of unknowns and estimation error.

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