COMBINED CHANNEL ESTIMATION AND FREQUENCY CORRECTION FOR PACKET ORIENTED MOBILE COMMUNICATION SYSTEMS

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Abstract— **In this paper we show how the well known Least Square (LS) algorithm can be used for combined channel estimation and frequency correction in packet oriented mobile communication systems. The method is derived by investigation of the effect of a frequency offset, e.g., due to the instability of the local oscillators, on the LS estimates. The algorithm corrects frequency offsets resulting in phase shifts per symbol that are in the range of several degrees. Imperfect correlation properties of the applied synchronisation sequence (SYS) are automatically compensated by the algorithm.**

I. Introduction

Modern packet oriented mobile communication systems, as e.g. HIPERLAN, often use a synchronisation sequence (SYS) as a preamble in front of the actual data packet allowing (aside from achieving frame and bit synchronisation) an estimation of the channel coefficients or the training of an equalizer.
Assuming perfect frame synchronisation the estimation of the coefficients can be performed by the well known Least Square (LS) algorithm [1] for stationary or slowly varying channels even if the SYS has imperfect correlation properties. However, the need for cheap equipment may force manufacturers to allow tolerances for the frequency stability of the local oscillators in receivers and transmitters of typically up to 10 ppm. In this case even channels that can be considered to be time invariant during the transmission of one or a few bursts become time variant due to this frequency offset. Without a frequency offset compensation the quality of the channel estimation decreases essentially since the coefficients will change considerably during the observation window used for the estimation. Apart from this, for a coherent receiver the resulting frequency offset has to be compensated to an amount that is tolerable for the data detection. (The residual part of the frequency offset may be compensated by a phase tracker.) Otherwise the resulting cumulative phase offset severely corrupts the reference phase.
In this paper we present an algorithm based on LS estimation allowing both the estimation of the channel coefficients and of the frequency offset. Other work related to this subject can be found in, e.g., [2], [3], [4], [5], [6].

II. The effect of a frequency offset to the LS estimate

Consider a transmission system as depicted in figure 1.

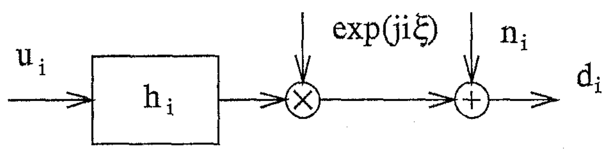

Fig. 1. Transmission system

ξ, u_i, h_i, and n_i denote the constant phase shift between adjacent receive symbols, the input data, the channel coefficients, and the noise which is assumed to be white Gaussian with zero mean and variance $\sigma^2 = N_0/2E_s$. Furthermore $j = \sqrt{-1}$ and typically $u_i \in \{-1, 1\}$.
The model assumes the frequency mismatch is at the receiver. A similar but different model results if the mismatch is at the transmitter, where the multiplier is located in front of the channel. However, this does not affect the estimation of the frequency offset and the estimated channel coefficients differ only by a phase shift.

The received signal d_i is given as:

$$d_i = \exp(ji\xi) \sum_{s=0}^{M-1} u_{i-s} h_s + n_i$$

where M denotes the number of channel coefficients. The LS algorithm can be written in matrix form as [1]:

$$\mathbf{\Phi h}^* = \mathbf{\Theta}, \tag{1}$$

with

$$\Phi_{t,k} = \sum_{i=M}^{N} u_{i-k} u_{i-t}^*, \qquad 0 \le t, k \le M-1, \tag{2}$$

$$\Theta_{-k} = \sum_{i=M}^{N} u_{i-k} d_i^*, \qquad 0 \le k \le M-1 \tag{3}$$

and $\mathbf{\Theta} = \left(\Theta_0, \Theta_{-1}, \ldots, \Theta_{-(M+1)}\right)^T$. N is the size of the observation window. In the following two observation windows are considered. The first starts with index i_0 where as the second starts with index i_1. Variables related to the first observation window are marked with a tilde while variables related to the second window are marked with a bar. For the LS algorithm the indices i_0 and i_1 are chosen in a way that the appropriate correlation matrices $\tilde{\mathbf{\Phi}}$ and $\bar{\mathbf{\Phi}}$ are invertible.

We now define the following matrices

$$\mathbf{A}^H = \begin{pmatrix} u_M & u_{M+1} & \cdots & u_N \\ u_{M-1} & u_M & \cdots & u_{N-1} \\ \vdots & \vdots & \vdots & \vdots \\ u_1 & u_2 & \cdots & u_{N-M+1} \end{pmatrix}$$

and

$$\mathbf{E} = \begin{pmatrix} \exp(jM\xi) & 0 & \cdots & 0 \\ 0 & \exp(j(M+1)\xi) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \exp(jN\xi) \end{pmatrix}.$$

Using these definitions one gets:

$$\begin{aligned} \mathbf{\Phi} &= \mathbf{A}^H \mathbf{A} \\ \mathbf{\Theta} &= \mathbf{A}^H \mathbf{E}^H \mathbf{A} \mathbf{h}^* + \mathbf{A}^H \mathbf{n}^* \end{aligned}$$

where $\mathbf{h}$ and $\mathbf{n}$ are the channel impulse response vector of length M and the noise vector of length $N - M + 1$, respectively, written as column vectors.

For small ξ the following approximation can be applied:

$$\begin{aligned} \mathbf{P} &= \mathbf{A}^H \mathbf{E}^H \mathbf{A} \approx \mathcal{K}/(N-M+1)\mathbf{\Phi} \\ &\approx \exp\left(\frac{N+M}{2}\xi\right) \mathbf{\Phi}. \end{aligned} \tag{4}$$

where

$$\begin{aligned} \mathcal{K} &= \exp(-jM\xi) \sum_{i=0}^{N-M} \exp(-ji\xi) \\ &= \frac{\exp(-j(N+1)\xi) - \exp(-jM\xi)}{\exp(-j\xi) - 1} \end{aligned}$$

Neglecting the noise term and using (4) we find:

$$\mathbf{\Theta} \approx \mathbf{\Phi h}^* \mathcal{K}/(N-M+1) \tag{5}$$

Channel estimation and correction of the frequency offset are derived from the following observations:

1. Choosing i_0 as the beginning of the first observation window one gets:

$$\tilde{\mathbf{\Theta}} = \tilde{\mathbf{\Phi}} \mathbf{h}^* \exp(-ji_0\xi)\mathcal{K}/(N-M+1) + n'$$

2. Choosing i_1 with $i_1 > i_0 + N$ as the beginning of the second observation window one gets:

$$\bar{\mathbf{\Theta}} = \bar{\mathbf{\Phi}} \mathbf{h}^* \exp(-ji_1\xi)\mathcal{K}/(N-M+1) + n'$$

A. *Estimation of the phase shift ξ*

From equation (1) for i_0 and i_1 it follows with $\bar{\mathbf{\Phi}}$ hermitian:

$$\begin{aligned} \bar{\mathbf{h}}^T \tilde{\mathbf{h}}^* &= \bar{\mathbf{\Theta}}^H \bar{\mathbf{\Phi}}^{-1} \tilde{\mathbf{\Phi}}^{-1} \tilde{\mathbf{\Theta}} \\ &\approx \mathbf{h}^T \exp(ji_1\xi)\mathcal{K}^*/(N-M+1)\bar{\mathbf{\Phi}}\bar{\mathbf{\Phi}}^{-1}\mathbf{h}^* \\ &\quad \exp(-ji_0\xi)\mathcal{K}/(N-M+1)\tilde{\mathbf{\Phi}}^{-1}\tilde{\mathbf{\Phi}} \end{aligned}$$

With $\frac{\mathcal{K}\mathcal{K}^*}{(N-M+1)^2} = \frac{1-\cos((N+M+1)\xi)}{(1-\cos(\xi))(N-M+1)^2} = f(\xi)$ and $|\mathbf{h}|^2 f(\xi) > 0$ the estimation $\hat{\xi}$ of the phase shift can be stated as:

$$\hat{\xi} = \frac{1}{i_1 - i_0} \arg\left\{\frac{\bar{\mathbf{h}}^T \tilde{\mathbf{h}}^*}{|\mathbf{h}|^2 f(\xi)}\right\} = \frac{1}{i_1 - i_0} \arg\left\{\bar{\mathbf{h}}^T \tilde{\mathbf{h}}^*\right\}. \tag{6}$$

The way of combining the angle estimates from the different coefficients can be interpreted as maximum ratio combining. The uniqueness of the solution for $\hat{\xi}$ requires that the value for the distance between the observation windows satisfies:

$$-\pi/\xi < i_1 - i_0 < \pi/\xi. \tag{7}$$

Nevertheless, it is possible to use bigger distances or to resolve larger frequency offsets by using more than two observation windows. If the condition in equation 7 is satisfied for any adjacent windows the sign of the offset can be determined and even an offset that results in a phase shift of more than 2π can properly be estimated by following the estimation values of the sequence of windows.

B. *Estimation of the channel coefficients h_s*

The channel coefficients can be derived as:

$$\mathbf{h} = \bar{\mathbf{h}} \exp(-ji_1\hat{\xi})(N - M + 1)/\mathcal{K}^* \qquad (8)$$

If $\bar{\mathbf{\Phi}} \approx (N - M + 1)\mathbf{I}_M$ it follows that:

$$\mathbf{h} = \bar{\mathbf{\Theta}}^* \exp(-ji_1\hat{\xi})/\mathcal{K}^* \qquad (9)$$

The estimate of equation (8) is better than the one of equation (9) as it takes into account non-ideal correlation properties of the SYS. On the other hand the complexity for the second approximation is slightly higher than for eqn. 9 since $\bar{\mathbf{h}}$ must be calculated by the LS algorithm. However, note that the matrix $\bar{\mathbf{\Phi}}$ does not depend on the receive vector and so necessary operations for the solution of the system of equations $\bar{\mathbf{\Phi}}\mathbf{h}^* = \bar{\mathbf{\Theta}}$ (Gauss-Jordan algorithm or matrix inversion) are known in advance and have only to be performed on $\bar{\mathbf{\Theta}}$. This means that one filter coefficient can be calculated with at most M multiplications and M additions from

$$\bar{h}_i^* = \sum_{s=0}^{M-1} \bar{\Phi}_{is}^{-1} \bar{\Theta}_s \qquad (10)$$

where the coefficients $\bar{\Phi}_{is}^{-1}$ can be determined from $\bar{\mathbf{\Phi}}$ in advance.

C. *An algorithm for frequency correction and channel estimation*

The algorithm can be stated as follows:

1. Precalculate $\bar{\mathbf{\Phi}}^{-1}$.
2. Calculate $\tilde{\mathbf{\Theta}}$ and $\bar{\mathbf{\Theta}}$ as given by the LS algorithm.
3. Use equation (6) to determine the phase shift $\hat{\xi}$.
4. Calculate $\mathcal{K}^*$ and $\mathbf{h}$ with the help of equation (8).

III. Complexity

For the estimation of the phase shift ξ it is sufficient to calculate $\tilde{\mathbf{\Theta}}$ and $\bar{\mathbf{\Theta}}$. As the SYS is known in advance the matrix $\tilde{\mathbf{\Phi}}^{-1}$ can be precalculated. Since usually $u_i \in \{-1, 1\}$ for the computation of $\tilde{\mathbf{\Theta}}$ and $\bar{\mathbf{\Theta}}$ only additions are necessary, i.e., the LS algorithm needs at most $2M^2$ multiplications to determine $\tilde{\mathbf{h}}$ and $\bar{\mathbf{h}}$. For the phase estimation one needs at most M additional multiplications and additions. Note that it is possible to estimate the frequency offset with less than M pairs of components.

For the channel estimation it is necessary to calculate one estimate of the channel coefficients $\bar{\mathbf{h}}$ from which an unrotated version may be determined with the help of equation (8). The absolute value of $\mathcal{K}$ is approximately equal to $N - M + 1$ so only an additional phase shift is necessary to estimate $\mathbf{h}$ from $\bar{\mathbf{h}}$. If the channel impulse response is used to adjust the coefficients of an equalizer it is even possible to omit this phase shift. One simply assumes the channel impulse response to be $\bar{\mathbf{h}}$ and locates the beginning of the phase shift due to the frequency offset to the middle of the observation window. This means in other words that the initial phase $((N + M)/2 \quad + i_1)\xi$ is included in the estimated channel impulse response and compensated by the equalizer. The advantage of this method is the following: By back rotating the input symbols from the channel the estimation error in ξ is accumulated for each symbol up to the end of the data block or to the point where an additional correction is performed. So the best strategy for minimizing this accumulated error is to start the back rotation as late as possible.

IV. Application

The applied SYS is a segment of length 450 bit from a m-sequence of length 2047 as specified in [7]. Figure 2 shows the center part of the KKF of a 50 bit segment from that SYS.

The results for a simulation with a window size of $N = 50$, a distance between the windows of 150 symbols and a set of 250 indoor channels measured at 5.2 GHz are shown in figure 3.

The signal to noise ratio was set to 15dB. A phase shift of 1^o between two adjacent symbols is assumed. The histogram shows the number of estimates that lie within ranges of width 0.1^o. It can be observed that the frequency offset is reduced in average by a factor bigger than 20 making it easy for a phase tracker to eliminate the residual offset. It should be noted that a decrease of the window size or the distance between the observation windows would lead to a broadening of the distribution of the estimates.

Figure 4 shows mean and standard deviation of the

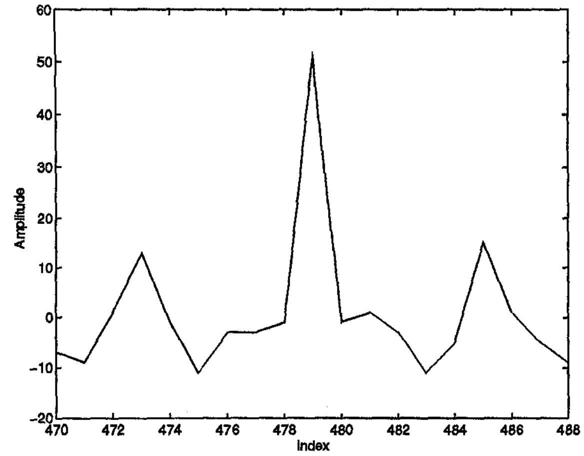

Fig. 2. KKF of the SYS and a segment of length 50

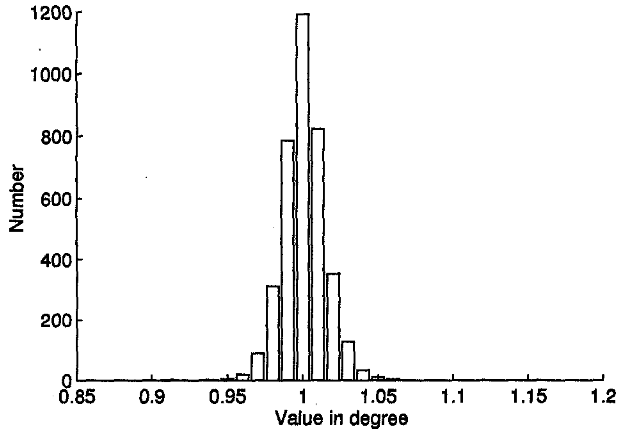

Fig. 3. $\hat{\xi}$

estimated frequency offset for three sets of measured indoor channels again at 5.2 GHz. The channels are all without LOS. MZ_1886_1 has a delay spread of 51 ns, MZ_1886_2 one of 61 ns, and MZ_1876 45 ns. The measurements were conducted in a typical office environment along a corridor, with the transmitter positioned in the next floor [8]. The LO's in receiver and transmitter are assumed to have an accuracy of 10 ppm which leads for a bandwidth of 23.5 MHz to a maximum shift of 1.54^o between adjacent symbols as specified for HIPERLAN [7]. All other parameters are similar to the ones above. Moreover, the performance in a simulation using an ETSI channel model with delay spread 40 ns is shown in the same figure.

In figure 5 the same is shown for an ETSI channel with a delay spread of 40 ns but now a frequency offset that leads to a phase shift of 5^o between adjacent symbols.

Note that the accuracy of the angle estimation is

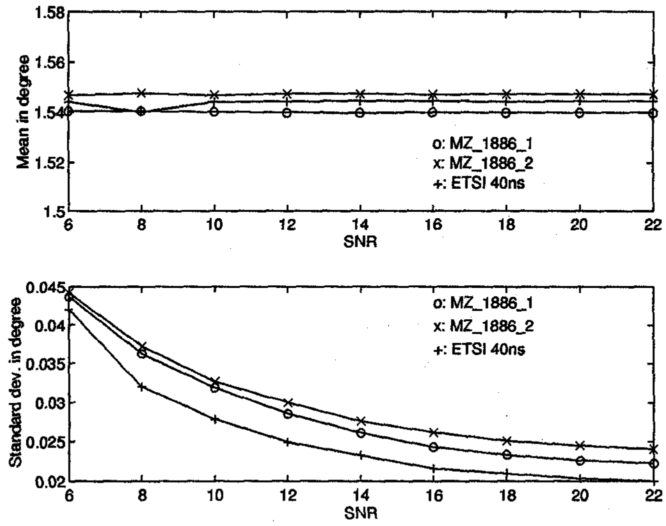

Fig. 4. Mean and standard deviation for the angle estimates

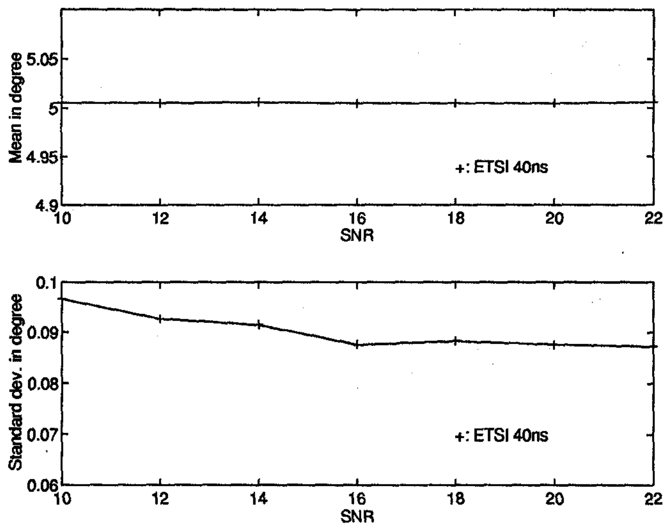

Fig. 5. Mean and standard deviation for the angle estimates

worse than that for 1.54 degrees but still acceptable for data detection.

V. Summary

In this talk we present an algorithm for estimating the channel coefficients and the frequency offset between transmitter and receiver for packet oriented mobile communication systems. We derive an algorithm that is based on a LS estimation and uses two or more observation windows. Results for the application of the channel and frequency offset estimation to a transmission system operating in a typical indoor environment are shown.

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