Why T.F. Does a Tortoise & hare meet?? (the genius of modulo)

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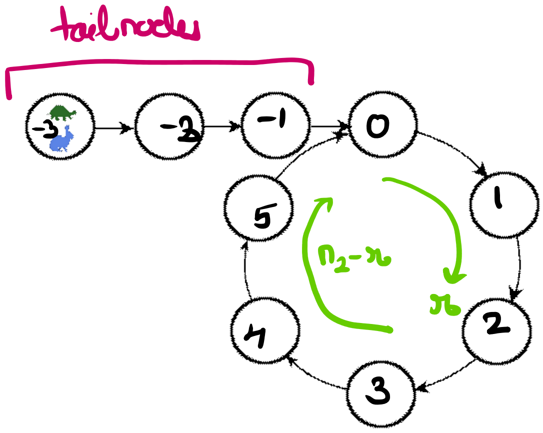
☆ Prerequisite



① Say we have n_1 tail nodes
& n_2 cycle nodes

② In n_1 steps, tor → at cycle node 0.
hare → $k \times n_2 + x^{th}$ node in cycle

③ In $n_2 - x$ steps, tor → $(n_2 - x)^{th}$ node.
hare → $x + (n_2 - x) \times 2^{th}$ n
$\Rightarrow (2n_2 - x)^{th}$ node

④ Sum of steps: $(n_1 + n_2 - x)$
$= (k \times n_2 + x) + (2 \times n_2 - x)$
$= (k-2) n_2$

$\therefore$ $n_1 + n_2 - x$ steps will lead to 0^{th} node of cycle.

☆ Proof of how this works.

from the 0^{th} node we move x steps to reach node i
$$(0 + x) \% n_2 = i$$
① → $\Rightarrow x \% n_2 = i$

from x^{th} node, the hare moves $2 \times x$ steps to reach node i.
$$((n_2 \times k + x) + 2x) \% n_2 = i$$
② → $\Rightarrow (x + 2x) \% n_2 = i$
$\Rightarrow x \% n_2 + (2x) \% n_2 = i$
$\Rightarrow x \% n_2 + 2 \times x \% n_2 = i$

* from 1

$\Rightarrow (x \% n_2 + 2 * i) \% n_2 = i$

$\Rightarrow (x + i) \% n_2 = 0$

$\boxed{\Rightarrow i = n_2 - x}$

But since $n_1 + n_2 - x = 0$, We know n_1 steps from node i in cyclic nodes will lead to the 0^{th} node.