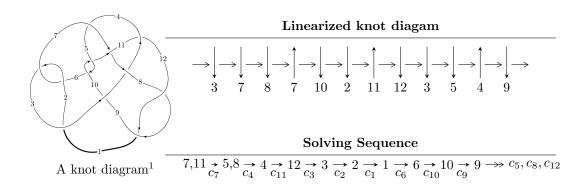
# $12n_{0609} (K12n_{0609})$



### Ideals for irreducible components<sup>2</sup> of $X_{par}$

$$\begin{split} I_1^u &= \langle 4.50523 \times 10^{31} u^{34} + 3.77516 \times 10^{31} u^{33} + \dots + 2.98297 \times 10^{32} b + 5.56578 \times 10^{32}, \\ &1.58581 \times 10^{33} u^{34} - 2.10588 \times 10^{32} u^{33} + \dots + 2.08808 \times 10^{33} a - 7.24786 \times 10^{33}, \ u^{35} - 7u^{33} + \dots - 15u - I_2^u \\ &= \langle -3u^{13} + 4u^{12} + 12u^{11} - 14u^{10} - 24u^9 + 33u^8 + 22u^7 - 41u^6 - 4u^5 + 33u^4 - 8u^3 - 16u^2 + b + 5u + 5, \\ &u^{13} - u^{12} - 4u^{11} + 3u^{10} + 8u^9 - 8u^8 - 8u^7 + 10u^6 + 3u^5 - 10u^4 + u^3 + 4u^2 + a - u - 2, \\ &u^{15} - u^{14} - 5u^{13} + 4u^{12} + 12u^{11} - 11u^{10} - 16u^9 + 18u^8 + 11u^7 - 20u^6 - 2u^5 + 14u^4 - 2u^3 - 6u^2 + u + 1 \rangle \end{split}$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 50 representations.

<sup>&</sup>lt;sup>1</sup>The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(http://www.layer8.co.uk/maths/draw/index.htm#Running-draw), where we modified some parts for our purpose(https://github.com/CATsTAILs/LinksPainter).

 $<sup>^2</sup>$  All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\begin{matrix} \text{I. } I_1^u = \\ \langle 4.51 \times 10^{31} u^{34} + 3.78 \times 10^{31} u^{33} + \dots + 2.98 \times 10^{32} b + 5.57 \times 10^{32}, \ 1.59 \times 10^{33} u^{34} - \\ 2.11 \times 10^{32} u^{33} + \dots + 2.09 \times 10^{33} a - 7.25 \times 10^{33}, \ u^{35} - 7u^{33} + \dots - 15u - 7 \rangle \end{matrix}$$

(i) Arc colorings

$$a_{7} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} -0.759458u^{34} + 0.100853u^{33} + \cdots - 4.69237u + 3.47107 \\ -0.151032u^{34} - 0.126557u^{33} + \cdots - 11.7344u - 1.86585 \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} 1 \\ -u^{2} \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} -0.608426u^{34} + 0.227410u^{33} + \cdots + 7.04204u + 5.33692 \\ -0.151032u^{34} - 0.126557u^{33} + \cdots - 11.7344u - 1.86585 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0.179494u^{34} - 0.738948u^{33} + \cdots - 15.5941u - 1.11702 \\ -0.545677u^{34} + 0.0968206u^{33} + \cdots - 9.38307u + 0.549020 \\ a_{3} = \begin{pmatrix} -1.02030u^{34} + 0.148917u^{33} + \cdots - 5.54020u + 5.06294 \\ -0.0217840u^{34} - 0.0983476u^{33} + \cdots - 7.67390u - 1.31640 \\ a_{2} = \begin{pmatrix} -1.04208u^{34} + 0.0505690u^{33} + \cdots - 13.2141u + 3.74654 \\ -0.0217840u^{34} - 0.0983476u^{33} + \cdots - 7.67390u - 1.31640 \\ a_{4} = \begin{pmatrix} 0.840961u^{34} - 1.49063u^{33} + \cdots - 33.0344u - 11.5939 \\ -0.517267u^{34} + 0.810261u^{33} + \cdots + 18.9933u + 12.5715 \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} 0.851085u^{34} - 0.116346u^{33} + \cdots - 25.8199u - 1.77087 \\ 0.0725660u^{34} - 0.436813u^{33} + \cdots + 17.0168u - 7.55024 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.456623u^{34} - 0.375284u^{33} + \cdots - 20.0850u + 2.76419 \\ -0.0904401u^{34} + 0.266844u^{33} + \cdots + 6.89220u + 3.33219 \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} -3.40594u^{34} + 1.42898u^{33} + \cdots - 12.4503u + 24.3578 \\ 1.40463u^{34} - 0.957154u^{33} + \cdots - 5.03678u - 8.25778 \end{pmatrix}$$

- (ii) Obstruction class = -1
- (iii) Cusp Shapes =  $1.81804u^{34} + 1.88122u^{33} + \cdots + 110.974u + 25.9634$

## (iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{35} + 62u^{34} + \dots + 4330075u + 134689$
$c_2, c_6$	$u^{35} + 2u^{34} + \dots + 4061u - 367$
$c_3$	$u^{35} + 2u^{34} + \dots - 7u + 1$
$c_4$	$u^{35} + 10u^{34} + \dots + 1712u + 311$
$c_5, c_{10}$	$u^{35} - u^{34} + \dots + 672u - 23$
$c_7$	$u^{35} - 7u^{33} + \dots - 15u + 7$
$c_8, c_{12}$	$u^{35} + 3u^{34} + \dots - 97u + 19$
<i>c</i> <sub>9</sub>	$u^{35} + 4u^{34} + \dots - 54708u + 6023$
$c_{11}$	$u^{35} - 5u^{34} + \dots + 422u - 13$

### (v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{35} - 206y^{34} + \dots + 2682621801371y - 18141126721$
$c_{2}, c_{6}$	$y^{35} - 62y^{34} + \dots + 4330075y - 134689$
$c_3$	$y^{35} + 2y^{34} + \dots + 63y - 1$
$c_4$	$y^{35} + 10y^{34} + \dots - 500008y - 96721$
$c_5, c_{10}$	$y^{35} - 3y^{34} + \dots + 454344y - 529$
$c_7$	$y^{35} - 14y^{34} + \dots + 1023y - 49$
$c_8, c_{12}$	$y^{35} - 55y^{34} + \dots + 11689y - 361$
$c_9$	$y^{35} - 164y^{34} + \dots - 17527853484y - 36276529$
$c_{11}$	$y^{35} + 9y^{34} + \dots + 217500y - 169$

## (vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = -0.675365 + 0.752140I		
a = 0.274011 + 1.179370I	-5.38269 - 0.91248I	-10.44886 + 2.53258I
b = -0.55916 + 1.59846I		
u = -0.675365 - 0.752140I		
a = 0.274011 - 1.179370I	-5.38269 + 0.91248I	-10.44886 - 2.53258I
b = -0.55916 - 1.59846I		
u = -0.928230		
a = -0.743205	-10.2373	-6.48310
b = -3.28718		
u = -0.780540 + 0.794419I		
a = 0.528114 + 0.428245I	-0.06975 - 2.50434I	-7.09658 + 4.10230I
b = -0.454633 + 0.505577I		
u = -0.780540 - 0.794419I		
a = 0.528114 - 0.428245I	-0.06975 + 2.50434I	-7.09658 - 4.10230I
b = -0.454633 - 0.505577I		
u = -1.170970 + 0.199419I		
a = 0.109806 - 0.892494I	2.93670 - 2.65045I	-2.92426 + 3.85701I
b = 0.256411 - 0.529286I		
u = -1.170970 - 0.199419I		
a = 0.109806 + 0.892494I	2.93670 + 2.65045I	-2.92426 - 3.85701I
b = 0.256411 + 0.529286I		
u = -1.027690 + 0.614606I		
a = -0.911917 - 0.413858I	-4.22355 - 4.34323I	-8.88159 + 4.48072I
b = 0.40767 - 1.44351I		
u = -1.027690 - 0.614606I		
a = -0.911917 + 0.413858I	-4.22355 + 4.34323I	-8.88159 - 4.48072I
b = 0.40767 + 1.44351I		
u = -0.790308		
a = 1.93557	-10.8736	0.541730
b = -2.52371		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = 0.779444 + 0.122722I		
a = 0.11921 + 1.82402I	4.75580 + 0.44115I	-1.38131 + 2.20530I
b = 0.692344 + 0.750731I		
u = 0.779444 - 0.122722I		
a = 0.11921 - 1.82402I	4.75580 - 0.44115I	-1.38131 - 2.20530I
b = 0.692344 - 0.750731I		
u = 1.097490 + 0.543646I		
a = -0.208642 + 0.776842I	1.07838 + 5.16629I	-6.84650 - 9.62882I
b = 0.97202 + 1.45913I		
u = 1.097490 - 0.543646I		
a = -0.208642 - 0.776842I	1.07838 - 5.16629I	-6.84650 + 9.62882I
b = 0.97202 - 1.45913I		
u = 0.879026 + 0.883079I		
a = -1.65073 + 0.15661I	-16.3550 + 2.1833I	-9.12494 - 2.49613I
b = -0.276490 + 0.900117I		
u = 0.879026 - 0.883079I		
a = -1.65073 - 0.15661I	-16.3550 - 2.1833I	-9.12494 + 2.49613I
b = -0.276490 - 0.900117I		
u = -1.004900 + 0.750402I		
a = 0.159109 - 0.878002I	0.76221 - 3.35277I	-9.00923 + 1.04834I
b = 0.886553 - 0.937980I		
u = -1.004900 - 0.750402I		
a = 0.159109 + 0.878002I	0.76221 + 3.35277I	-9.00923 - 1.04834I
b = 0.886553 + 0.937980I		
u = 0.980302 + 0.839353I		
a = -0.12512 - 1.44580I	-16.0274 + 4.2362I	-8.99278 - 2.47237I
b = -0.56077 - 1.91870I		
u = 0.980302 - 0.839353I		
a = -0.12512 + 1.44580I	-16.0274 - 4.2362I	-8.99278 + 2.47237I
b = -0.56077 + 1.91870I		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = 0.371475 + 0.598922I		
a = 0.882706 - 0.414519I	-1.039020 - 0.567157I	-9.02941 + 3.66448I
b = -0.207818 - 0.631352I		
u = 0.371475 - 0.598922I		
a = 0.882706 + 0.414519I	-1.039020 + 0.567157I	-9.02941 - 3.66448I
b = -0.207818 + 0.631352I		
u = 0.663303		
a = 1.17343	-1.52399	-5.20750
b = -0.625094		
u = 1.019370 + 0.891636I		
a = 0.374425 - 0.936128I	-4.62293 + 8.02466I	-8.99459 - 6.09889I
b = -0.81320 - 1.16561I		
u = 1.019370 - 0.891636I		
a = 0.374425 + 0.936128I	-4.62293 - 8.02466I	-8.99459 + 6.09889I
b = -0.81320 + 1.16561I		
u = -0.625529 + 1.237630I		
a = -1.149750 - 0.604247I	-17.6881 + 5.3333I	-10.32662 - 2.48689I
b = -0.498371 - 1.184570I		
u = -0.625529 - 1.237630I		
a = -1.149750 + 0.604247I	-17.6881 - 5.3333I	-10.32662 + 2.48689I
b = -0.498371 + 1.184570I		
u = 0.936546 + 1.042190I		
a = -0.390412 + 0.550295I	-5.01490 - 1.02895I	-12.04957 + 2.53411I
b = -0.114051 + 0.985066I		
u = 0.936546 - 1.042190I		
a = -0.390412 - 0.550295I	-5.01490 + 1.02895I	-12.04957 - 2.53411I
b = -0.114051 - 0.985066I		
u = 0.587494		
a = -0.0584535	-2.29968	3.86260
b = -1.70895		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = -1.22874 + 0.83593I		
a = 0.186277 + 1.290050I	-15.6932 - 12.6803I	-6.00000 + 5.76735I
b = -1.04714 + 1.80529I		
u = -1.22874 - 0.83593I		
a = 0.186277 - 1.290050I	-15.6932 + 12.6803I	-6.00000 - 5.76735I
b = -1.04714 - 1.80529I		
u = -0.364021 + 0.159376I		
a = -2.08592 + 2.84763I	-0.87711 - 2.42741I	-7.50647 + 0.56307I
b = 0.575909 + 0.656185I		
u = -0.364021 - 0.159376I		
a = -2.08592 - 2.84763I	-0.87711 + 2.42741I	-7.50647 - 0.56307I
b = 0.575909 - 0.656185I		
u = 2.09595		
a = -0.386813	-7.66684	0
b = -0.373598		

$$II. \\ I_2^u = \langle -3u^{13} + 4u^{12} + \dots + b + 5, \ u^{13} - u^{12} + \dots + a - 2, \ u^{15} - u^{14} + \dots + u + 1 \rangle$$

(i) Arc colorings

$$a_{7} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} -u^{13} + u^{12} + \dots + u + 2 \\ 3u^{13} - 4u^{12} + \dots - 5u - 5 \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} 1 \\ -u^{2} \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} -4u^{13} + 5u^{12} + \dots + 6u + 7 \\ 3u^{13} - 4u^{12} + \dots - 5u - 5 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -15u^{14} + 12u^{13} + \dots + 38u + 6 \\ 7u^{14} - 4u^{13} + \dots - 22u - 6 \end{pmatrix}$$

$$a_{3} = \begin{pmatrix} u^{14} - 5u^{13} + \dots + 5u + 6 \\ 3u^{13} - 3u^{12} + \dots - 4u - 5 \end{pmatrix}$$

$$a_{2} = \begin{pmatrix} u^{14} - 2u^{13} + \dots + u + 1 \\ 3u^{13} - 3u^{12} + \dots - 4u - 5 \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} 20u^{14} - 13u^{13} + \dots + 9u + 9 \\ -10u^{14} + 4u^{13} + \dots + 9u + 9 \\ 4u^{14} - 3u^{13} + \dots - 11u - 7 \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} -4u^{14} - 3u^{13} + \dots + 9u + 9 \\ 4u^{14} - 3u^{13} + \dots - 11u - 7 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -4u^{14} + 4u^{13} + \dots + 5u^{2} + 6u \\ 4u^{14} - 4u^{13} + \dots - 9u^{2} - 8u \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} 9u^{14} + 7u^{13} + \dots - 53u - 30 \\ -7u^{14} + 37u^{12} + \dots + 30u + 17 \end{pmatrix}$$

#### (ii) Obstruction class = 1

(iii) Cusp Shapes = 
$$-4u^{14} + 19u^{13} + 5u^{12} - 83u^{11} + 6u^{10} + 184u^9 - 78u^8 - 223u^7 + 161u^6 + 135u^5 - 187u^4 - 27u^3 + 108u^2 - 9u - 37$$

(iv) u-Polynomials at the component

$u^{15} - 15u^{14} + \dots + 5u - 1$
$u^{15} - 3u^{14} + \dots + 3u - 1$
$u^{15} + u^{14} + \dots + 3u + 1$
$u^{15} + u^{14} + \dots + 2u - 1$
$u^{15} + 6u^{13} + \dots - 2u - 1$
$u^{15} + 3u^{14} + \dots + 3u + 1$
$u^{15} - u^{14} + \dots + u + 1$
$u^{15} - 10u^{13} + \dots + u + 1$
$u^{15} - 5u^{14} + \dots - 14u + 1$
$u^{15} + 6u^{13} + \dots - 2u + 1$
$u^{15} - 2u^{13} + \dots - 11u^2 - 1$
$u^{15} - 10u^{13} + \dots + u - 1$

## (v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{15} - 59y^{14} + \dots + 5y - 1$
$c_2, c_6$	$y^{15} - 15y^{14} + \dots + 5y - 1$
$c_3$	$y^{15} + 5y^{14} + \dots + 17y - 1$
$c_4$	$y^{15} - 7y^{14} + \dots - 14y - 1$
$c_5,c_{10}$	$y^{15} + 12y^{14} + \dots - 6y - 1$
$c_7$	$y^{15} - 11y^{14} + \dots + 13y - 1$
$c_{8}, c_{12}$	$y^{15} - 20y^{14} + \dots - 9y - 1$
<i>c</i> <sub>9</sub>	$y^{15} - 53y^{14} + \dots + 38y - 1$
$c_{11}$	$y^{15} - 4y^{14} + \dots - 22y - 1$

# (vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = -0.945967 + 0.364350I		
a = 0.447371 - 1.296300I	0.121261 + 0.585920I	-6.11150 + 0.88466I
b = 0.54568 - 1.59008I		
u = -0.945967 - 0.364350I		
a = 0.447371 + 1.296300I	0.121261 - 0.585920I	-6.11150 - 0.88466I
b = 0.54568 + 1.59008I		
u = 0.910844 + 0.329867I		
a = 0.63588 + 1.36871I	4.62454 + 1.37648I	-3.28500 - 4.85991I
b = 0.434324 + 0.641040I		
u = 0.910844 - 0.329867I		
a = 0.63588 - 1.36871I	4.62454 - 1.37648I	-3.28500 + 4.85991I
b = 0.434324 - 0.641040I		
u = 0.483149 + 0.790685I		
a = 0.552124 + 0.303102I	-3.32772 - 1.16564I	-9.43576 + 1.29116I
b = -0.006540 - 0.521541I		
u = 0.483149 - 0.790685I		
a = 0.552124 - 0.303102I	-3.32772 + 1.16564I	-9.43576 - 1.29116I
b = -0.006540 + 0.521541I		
u = -0.858047 + 0.316539I		
a = 0.85127 - 1.27055I	-0.26649 - 3.45017I	-3.88329 + 5.28365I
b = -0.183732 + 0.291743I		
u = -0.858047 - 0.316539I		
a = 0.85127 + 1.27055I	-0.26649 + 3.45017I	-3.88329 - 5.28365I
b = -0.183732 - 0.291743I		
u = 1.047160 + 0.587752I		
a = 0.157821 + 0.780474I	-1.69704 + 6.28081I	-7.36459 - 5.95096I
b = 1.54439 + 1.24046I		
u = 1.047160 - 0.587752I		
a = 0.157821 - 0.780474I	-1.69704 - 6.28081I	-7.36459 + 5.95096I
b = 1.54439 - 1.24046I		

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = 0.686348		
a = 1.89062	-11.2981	-19.7640
b = -3.01568		
u = -1.189670 + 0.653681I		
a = 0.004952 - 0.642912I	1.40673 - 4.12593I	-4.27551 + 5.62522I
b = 0.747056 - 0.985367I		
u = -1.189670 - 0.653681I		
a = 0.004952 + 0.642912I	1.40673 + 4.12593I	-4.27551 - 5.62522I
b = 0.747056 + 0.985367I		
u = -0.428593		
a = 1.22503	-2.69518	-18.0200
b = -1.45207		
u = 1.84731		
a = -0.414496	-7.46852	5.49540
b = -0.694610		

## III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$ (u^{15} - 15u^{14} + \dots + 5u - 1)(u^{35} + 62u^{34} + \dots + 4330075u + 134689) $
$c_2$	$(u^{15} - 3u^{14} + \dots + 3u - 1)(u^{35} + 2u^{34} + \dots + 4061u - 367)$
$c_3$	$(u^{15} + u^{14} + \dots + 3u + 1)(u^{35} + 2u^{34} + \dots - 7u + 1)$
$c_4$	$(u^{15} + u^{14} + \dots + 2u - 1)(u^{35} + 10u^{34} + \dots + 1712u + 311)$
$c_5$	$(u^{15} + 6u^{13} + \dots - 2u - 1)(u^{35} - u^{34} + \dots + 672u - 23)$
$c_6$	$(u^{15} + 3u^{14} + \dots + 3u + 1)(u^{35} + 2u^{34} + \dots + 4061u - 367)$
$c_7$	$(u^{15} - u^{14} + \dots + u + 1)(u^{35} - 7u^{33} + \dots - 15u + 7)$
c <sub>8</sub>	$(u^{15} - 10u^{13} + \dots + u + 1)(u^{35} + 3u^{34} + \dots - 97u + 19)$
<i>c</i> <sub>9</sub>	$(u^{15} - 5u^{14} + \dots - 14u + 1)(u^{35} + 4u^{34} + \dots - 54708u + 6023)$
$c_{10}$	$(u^{15} + 6u^{13} + \dots - 2u + 1)(u^{35} - u^{34} + \dots + 672u - 23)$
$c_{11}$	$(u^{15} - 2u^{13} + \dots - 11u^2 - 1)(u^{35} - 5u^{34} + \dots + 422u - 13)$
$c_{12}$	$(u^{15} - 10u^{13} + \dots + u - 1)(u^{35} + 3u^{34} + \dots - 97u + 19)$ 16

## IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^{15} - 59y^{14} + \dots + 5y - 1)$ $\cdot (y^{35} - 206y^{34} + \dots + 2682621801371y - 18141126721)$
$c_2, c_6$	$(y^{15} - 15y^{14} + \dots + 5y - 1)(y^{35} - 62y^{34} + \dots + 4330075y - 134689)$
$c_3$	$(y^{15} + 5y^{14} + \dots + 17y - 1)(y^{35} + 2y^{34} + \dots + 63y - 1)$
C <sub>4</sub>	$(y^{15} - 7y^{14} + \dots - 14y - 1)(y^{35} + 10y^{34} + \dots - 500008y - 96721)$
$c_5, c_{10}$	$(y^{15} + 12y^{14} + \dots - 6y - 1)(y^{35} - 3y^{34} + \dots + 454344y - 529)$
<i>C</i> <sub>7</sub>	$(y^{15} - 11y^{14} + \dots + 13y - 1)(y^{35} - 14y^{34} + \dots + 1023y - 49)$
$c_8, c_{12}$	$(y^{15} - 20y^{14} + \dots - 9y - 1)(y^{35} - 55y^{34} + \dots + 11689y - 361)$
<i>c</i> 9	$(y^{15} - 53y^{14} + \dots + 38y - 1)$ $\cdot (y^{35} - 164y^{34} + \dots - 17527853484y - 36276529)$
$c_{11}$	$(y^{15} - 4y^{14} + \dots - 22y - 1)(y^{35} + 9y^{34} + \dots + 217500y - 169)$