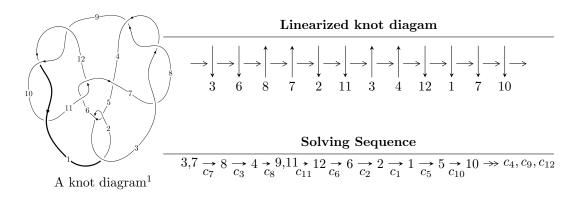
## $12n_{0345} \ (K12n_{0345})$



#### Ideals for irreducible components<sup>2</sup> of $X_{par}$

$$\begin{split} I_1^u &= \langle -4.30590 \times 10^{29} u^{38} + 1.40716 \times 10^{30} u^{37} + \dots + 3.25262 \times 10^{28} b - 5.80343 \times 10^{30}, \\ &- 1.92426 \times 10^{30} u^{38} + 6.35210 \times 10^{30} u^{37} + \dots + 3.25262 \times 10^{28} a - 2.53835 \times 10^{31}, \\ &u^{39} - 3u^{38} + \dots + 36u + 4 \rangle \\ I_2^u &= \langle -au + b - 2a - 1, \ 2a^2 - au + 2a + 2u - 3, \ u^2 - 2 \rangle \end{split}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 45 representations.

<sup>&</sup>lt;sup>1</sup>The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(http://www.layer8.co.uk/maths/draw/index.htm#Running-draw), where we modified some parts for our purpose(https://github.com/CATsTAILs/LinksPainter).

<sup>&</sup>lt;sup>2</sup> All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I. 
$$I_1^u = \langle -4.31 \times 10^{29} u^{38} + 1.41 \times 10^{30} u^{37} + \dots + 3.25 \times 10^{28} b - 5.80 \times 10^{30}, \ -1.92 \times 10^{30} u^{38} + 6.35 \times 10^{30} u^{37} + \dots + 3.25 \times 10^{28} a - 2.54 \times 10^{31}, \ u^{39} - 3 u^{38} + \dots + 36 u + 4 \rangle$$

(i) Arc colorings

$$a_{3} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{7} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} 1 \\ -u^{2} \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} u \\ -u^{3} + u \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} 59.1604u^{38} - 195.292u^{37} + \dots + 4436.04u + 780.403 \\ 13.2382u^{38} - 43.2623u^{37} + \dots + 993.748u + 178.423 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 45.9221u^{38} - 152.030u^{37} + \dots + 993.748u + 178.423 \\ 13.2382u^{38} - 43.2623u^{37} + \dots + 993.748u + 178.423 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 45.9221u^{38} - 152.030u^{37} + \dots + 3442.29u + 601.980 \\ 13.2382u^{38} - 43.2623u^{37} + \dots + 993.748u + 178.423 \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} 51.1260u^{38} - 169.150u^{37} + \dots + 3814.68u + 661.729 \\ 23.7145u^{38} - 78.2230u^{37} + \dots + 1784.77u + 313.995 \end{pmatrix}$$

$$a_{2} = \begin{pmatrix} -1.07361u^{38} + 3.79195u^{37} + \dots + 41.8527u + 1.03040 \\ 26.3378u^{38} - 87.1349u^{37} + \dots + 1988.06u + 348.765 \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} -1.07361u^{38} + 3.79195u^{37} + \dots + 1988.06u + 348.765 \\ 26.7461u^{38} - 88.2756u^{37} + \dots + 2004.32u + 351.049 \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} -u^{3} + 2u \\ -u^{3} + u \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} -u^{3} + 2u \\ -u^{3} + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 7.08403u^{38} - 23.4120u^{37} + \dots + 528.155u + 95.4828 \\ -26.7461u^{38} + 88.2756u^{37} + \dots + 2004.32u - 351.049 \end{pmatrix}$$

- (ii) Obstruction class = -1
- (iii) Cusp Shapes =  $-56.2608u^{38} + 183.733u^{37} + \cdots 4395.07u 821.737$

### (iv) u-Polynomials at the component

| Crossings             | u-Polynomials at each crossing            |
|-----------------------|---|
| $c_1$                 | $u^{39} + 15u^{38} + \dots + 5679u + 81$  |
| $c_2, c_5$            | $u^{39} + 3u^{38} + \dots - 45u + 9$      |
| $c_3, c_7, c_8$       | $u^{39} - 3u^{38} + \dots + 36u + 4$      |
| <i>c</i> <sub>4</sub> | $u^{39} + 9u^{38} + \dots - 12340u - 380$ |
| $c_6,c_{11}$          | $u^{39} - 2u^{38} + \dots - 10u + 1$      |
| $c_9, c_{10}, c_{12}$ | $u^{39} - 4u^{38} + \dots - 6u - 1$       |

### (v) Riley Polynomials at the component

| Crossings             | Riley Polynomials at each crossing                |
|-----------------------|---|
| $c_1$                 | $y^{39} + 25y^{38} + \dots + 19679031y - 6561$    |
| $c_2, c_5$            | $y^{39} - 15y^{38} + \dots + 5679y - 81$          |
| $c_3, c_7, c_8$       | $y^{39} - 49y^{38} + \dots + 560y - 16$           |
| $c_4$                 | $y^{39} - 109y^{38} + \dots + 28757360y - 144400$ |
| $c_6, c_{11}$         | $y^{39} - 6y^{38} + \dots + 46y - 1$              |
| $c_9, c_{10}, c_{12}$ | $y^{39} - 30y^{38} + \dots + 38y - 1$             |

## (vi) Complex Volumes and Cusp Shapes

| Solutions to $I_1^u$      | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape          |
|---------------------------|---------------------------------------|---------------------|
| u = -0.278366 + 0.955359I |                                       |                     |
| a = -0.591023 + 0.828865I | -3.35402 + 4.18126I                   | -7.78277 - 6.99514I |
| b = -0.623478 + 0.614458I |                                       |                     |
| u = -0.278366 - 0.955359I |                                       |                     |
| a = -0.591023 - 0.828865I | -3.35402 - 4.18126I                   | -7.78277 + 6.99514I |
| b = -0.623478 - 0.614458I |                                       |                     |
| u = -0.887627 + 0.471243I |                                       |                     |
| a = -1.50907 + 0.17598I   | 2.35149 - 5.07575I                    | -1.77054 + 6.40036I |
| b = -0.886394 - 0.797483I |                                       |                     |
| u = -0.887627 - 0.471243I |                                       |                     |
| a = -1.50907 - 0.17598I   | 2.35149 + 5.07575I                    | -1.77054 - 6.40036I |
| b = -0.886394 + 0.797483I |                                       |                     |
| u = -0.957964             |                                       |                     |
| a = -0.302923             | -8.12538                              | -10.0210            |
| b = 1.25836               |                                       |                     |
| u = 1.065650 + 0.085902I  |                                       |                     |
| a = -0.718491 + 0.083392I | 2.77568 - 0.03264I                    | 0                   |
| b = -0.515563 + 0.600875I |                                       |                     |
| u = 1.065650 - 0.085902I  |                                       |                     |
| a = -0.718491 - 0.083392I | 2.77568 + 0.03264I                    | 0                   |
| b = -0.515563 - 0.600875I |                                       |                     |
| u = -0.796295 + 0.736026I |                                       |                     |
| a = 1.42339 + 0.01369I    | -1.74573 - 9.69427I                   | 0. + 8.10788I       |
| b = 0.950413 + 0.884348I  |                                       |                     |
| u = -0.796295 - 0.736026I |                                       |                     |
| a = 1.42339 - 0.01369I    | -1.74573 + 9.69427I                   | 0 8.10788I          |
| b = 0.950413 - 0.884348I  |                                       |                     |
| u = 0.851097 + 0.768811I  |                                       |                     |
| a = 0.636558 - 0.270946I  | 0.64343 + 2.93674I                    | 0                   |
| b = 0.379215 - 0.675835I  |                                       |                     |

| Solutions to $I_1^u$      | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape          |
|---------------------------|---------------------------------------|---------------------|
| u = 0.851097 - 0.768811I  |                                       |                     |
| a = 0.636558 + 0.270946I  | 0.64343 - 2.93674I                    | 0                   |
| b = 0.379215 + 0.675835I  |                                       |                     |
| u = 0.756459 + 0.321115I  |                                       |                     |
| a = 0.848073 + 0.323450I  | -1.16384 + 3.37058I                   | -4.85431 - 4.93903I |
| b = 0.687338 + 0.927457I  |                                       |                     |
| u = 0.756459 - 0.321115I  |                                       |                     |
| a = 0.848073 - 0.323450I  | -1.16384 - 3.37058I                   | -4.85431 + 4.93903I |
| b = 0.687338 - 0.927457I  |                                       |                     |
| u = -0.729654 + 0.114009I |                                       |                     |
| a = 1.94616 - 0.30625I    | -0.720527 - 0.677496I                 | -4.45492 + 3.52872I |
| b = 0.877049 + 0.529169I  |                                       |                     |
| u = -0.729654 - 0.114009I |                                       |                     |
| a = 1.94616 + 0.30625I    | -0.720527 + 0.677496I                 | -4.45492 - 3.52872I |
| b = 0.877049 - 0.529169I  |                                       |                     |
| u = 1.36027               |                                       |                     |
| a = -1.02723              | 3.15342                               | 0                   |
| b = -0.392618             |                                       |                     |
| u = 0.025858 + 0.596227I  |                                       |                     |
| a = 1.28935 - 0.78910I    | -0.39053 + 1.36769I                   | -3.78544 - 4.37417I |
| b = 0.476410 - 0.516668I  |                                       |                     |
| u = 0.025858 - 0.596227I  |                                       |                     |
| a = 1.28935 + 0.78910I    | -0.39053 - 1.36769I                   | -3.78544 + 4.37417I |
| b = 0.476410 + 0.516668I  |                                       |                     |
| u = -1.42742              |                                       |                     |
| a = -10.9292              | 1.67196                               | 0                   |
| b = -0.157920             |                                       |                     |
| u = 1.47236               |                                       |                     |
| a = 0.751761              | -4.21706                              | 0                   |
| b = 1.74885               |                                       |                     |

| Solutions to $I_1^u$      | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape         |
|---------------------------|---------------------------------------|--------------------|
| u = 0.111960 + 0.411759I  |                                       |                    |
| a = -4.18092 - 0.61496I   | -3.05808 - 0.73206I                   | -13.3386 - 6.5570I |
| b = -0.394904 + 0.488828I |                                       |                    |
| u = 0.111960 - 0.411759I  |                                       |                    |
| a = -4.18092 + 0.61496I   | -3.05808 + 0.73206I                   | -13.3386 + 6.5570I |
| b = -0.394904 - 0.488828I |                                       |                    |
| u = 1.64569 + 0.02330I    |                                       |                    |
| a = -0.940304 - 0.261465I | 7.65877 + 1.14141I                    | 0                  |
| b = -1.33024 + 0.90609I   |                                       |                    |
| u = 1.64569 - 0.02330I    |                                       |                    |
| a = -0.940304 + 0.261465I | 7.65877 - 1.14141I                    | 0                  |
| b = -1.33024 - 0.90609I   |                                       |                    |
| u = -1.64517 + 0.08077I   |                                       |                    |
| a = -0.403452 - 0.032549I | 7.20723 - 4.84674I                    | 0                  |
| b = -0.98617 + 1.32010I   |                                       |                    |
| u = -1.64517 - 0.08077I   |                                       |                    |
| a = -0.403452 + 0.032549I | 7.20723 + 4.84674I                    | 0                  |
| b = -0.98617 - 1.32010I   |                                       |                    |
| u = 1.65274 + 0.23220I    |                                       |                    |
| a = -1.008000 - 0.471693I | 6.4748 + 13.4066I                     | 0                  |
| b = -1.18235 + 1.09377I   |                                       |                    |
| u = 1.65274 - 0.23220I    |                                       |                    |
| a = -1.008000 + 0.471693I | 6.4748 - 13.4066I                     | 0                  |
| b = -1.18235 - 1.09377I   |                                       |                    |
| u = 1.66262 + 0.24913I    |                                       |                    |
| a = -0.241455 - 0.016965I | 2.81425 + 0.97653I                    | 0                  |
| b = -0.136254 + 0.574075I |                                       |                    |
| u = 1.66262 - 0.24913I    |                                       |                    |
| a = -0.241455 + 0.016965I | 2.81425 - 0.97653I                    | 0                  |
| b = -0.136254 - 0.574075I |                                       |                    |

| Solutions to $I_1^u$      | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---------------------------|---------------------------------------|------------|
| u = -0.315278             |                                       |            |
| a = 4.23572               | -2.39339                              | 9.22220    |
| b = -0.575388             |                                       |            |
| u = 1.67924 + 0.13585I    |                                       |            |
| a = 0.987276 + 0.371552I  | 11.24360 + 7.46891I                   | 0          |
| b = 1.25098 - 1.02158I    |                                       |            |
| u = 1.67924 - 0.13585I    |                                       |            |
| a = 0.987276 - 0.371552I  | 11.24360 - 7.46891I                   | 0          |
| b = 1.25098 + 1.02158I    |                                       |            |
| u = -0.301204             |                                       |            |
| a = -2.36999              | -10.3072                              | 10.7730    |
| b = -1.68053              |                                       |            |
| u = -1.69258 + 0.19776I   |                                       |            |
| a = -0.675700 + 0.148591I | 9.38751 - 6.54529I                    | 0          |
| b = -0.787702 - 1.170530I |                                       |            |
| u = -1.69258 - 0.19776I   |                                       |            |
| a = -0.675700 - 0.148591I | 9.38751 + 6.54529I                    | 0          |
| b = -0.787702 + 1.170530I |                                       |            |
| u = -1.70589 + 0.04672I   |                                       |            |
| a = 0.538540 - 0.089371I  | 12.52260 - 0.71174I                   | 0          |
| b = 0.85440 + 1.24646I    |                                       |            |
| u = -1.70589 - 0.04672I   |                                       |            |
| a = 0.538540 + 0.089371I  | 12.52260 + 0.71174I                   | 0          |
| b = 0.85440 - 1.24646I    |                                       |            |
| u = -0.262239             |                                       |            |
| a = 2.84003               | -1.18388                              | -7.92960   |
| b = 0.533756              |                                       |            |

II. 
$$I_2^u = \langle -au + b - 2a - 1, \ 2a^2 - au + 2a + 2u - 3, \ u^2 - 2 \rangle$$

(i) Arc colorings

$$a_{3} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{7} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} u \\ -u \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} a \\ au + 2a + 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -au - a - 1 \\ au + 2a + 1 \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} \frac{1}{2}u \\ -au - 2a - 2 \end{pmatrix}$$

$$a_{2} = \begin{pmatrix} \frac{1}{2}u \\ -au - 2a + u - 2 \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} 1 \\ -au - 2a - 2 \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} au + 2a + \frac{1}{2}u \\ -au - 2a - 2 \end{pmatrix}$$

- (ii) Obstruction class = 1
- (iii) Cusp Shapes = -8

### (iv) u-Polynomials at the component

| Crossings             | u-Polynomials at each crossing |
|-----------------------|--------------------------------|
| $c_1, c_5$            | $(u-1)^4$                      |
| $c_2$                 | $(u+1)^4$                      |
| $c_3, c_4, c_7$ $c_8$ | $(u^2-2)^2$                    |
| $c_6, c_{12}$         | $(u^2 - u - 1)^2$              |
| $c_9, c_{10}, c_{11}$ | $(u^2+u-1)^2$                  |

# (v) Riley Polynomials at the component

| Crossings                           | Riley Polynomials at each crossing |  |  |
|-------------------------------------|------------------------------------|--|--|
| $c_1, c_2, c_5$                     | $(y-1)^4$                          |  |  |
| $c_3, c_4, c_7$ $c_8$               | $(y-2)^4$                          |  |  |
| $c_6, c_9, c_{10}$ $c_{11}, c_{12}$ | $(y^2 - 3y + 1)^2$                 |  |  |

## (vi) Complex Volumes and Cusp Shapes

| Solutions to $I_2^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|----------------------|---------------------------------------|------------|
| u = 1.41421          |                                       |            |
| a = -0.473911        | 2.30291                               | -8.00000   |
| b = -0.618034        |                                       |            |
| u = 1.41421          |                                       |            |
| a = 0.181018         | -5.59278                              | -8.00000   |
| b = 1.61803          |                                       |            |
| u = -1.41421         |                                       |            |
| a = 1.05505          | -5.59278                              | -8.00000   |
| b = 1.61803          |                                       |            |
| u = -1.41421         |                                       |            |
| a = -2.76216         | 2.30291                               | -8.00000   |
| b = -0.618034        |                                       |            |

III. 
$$I_1^v = \langle a, \ b+v+2, \ v^2+3v+1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ -v - 2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} v+2 \\ -v-2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -v - 3 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} v - 1 \\ v + 3 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ v+3 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -v - 2 \\ v + 3 \end{pmatrix}$$

- (ii) Obstruction class = 1
- (iii) Cusp Shapes = -26

### (iv) u-Polynomials at the component

| Crossings             | u-Polynomials at each crossing |  |  |
|-----------------------|--------------------------------|--|--|
| $c_1, c_2$            | $(u-1)^2$                      |  |  |
| $c_3, c_4, c_7$ $c_8$ | $u^2$                          |  |  |
| <i>C</i> <sub>5</sub> | $(u+1)^2$                      |  |  |
| $c_6, c_9, c_{10}$    | $u^2 + u - 1$                  |  |  |
| $c_{11}, c_{12}$      | $u^2 - u - 1$                  |  |  |

## (v) Riley Polynomials at the component

| Crossings                           | Riley Polynomials at each crossing |
|-------------------------------------|------------------------------------|
| $c_1, c_2, c_5$                     | $(y-1)^2$                          |
| $c_3, c_4, c_7$ $c_8$               | $y^2$                              |
| $c_6, c_9, c_{10}$ $c_{11}, c_{12}$ | $y^2 - 3y + 1$                     |

## (vi) Complex Volumes and Cusp Shapes

| Solutions to $I_1^v$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|----------------------|---------------------------------------|------------|
| v = -0.381966        |                                       |            |
| a = 0                | -10.5276                              | -26.0000   |
| b = -1.61803         |                                       |            |
| v = -2.61803         |                                       |            |
| a = 0                | -2.63189                              | -26.0000   |
| b = 0.618034         |                                       |            |

IV. u-Polynomials

| Crossings             | u-Polynomials at each crossing                                       |
|-----------------------|--|
| $c_1$                 | $((u-1)^6)(u^{39} + 15u^{38} + \dots + 5679u + 81)$                  |
| $c_2$                 | $((u-1)^2)(u+1)^4(u^{39}+3u^{38}+\cdots-45u+9)$                      |
| $c_3, c_7, c_8$       | $u^{2}(u^{2}-2)^{2}(u^{39}-3u^{38}+\cdots+36u+4)$                    |
| $c_4$                 | $u^{2}(u^{2}-2)^{2}(u^{39}+9u^{38}+\cdots-12340u-380)$               |
| <i>C</i> <sub>5</sub> | $((u-1)^4)(u+1)^2(u^{39}+3u^{38}+\cdots-45u+9)$                      |
| <i>C</i> <sub>6</sub> | $((u^2 - u - 1)^2)(u^2 + u - 1)(u^{39} - 2u^{38} + \dots - 10u + 1)$ |
| $c_9, c_{10}$         | $((u^2 + u - 1)^3)(u^{39} - 4u^{38} + \dots - 6u - 1)$               |
| $c_{11}$              | $(u^2 - u - 1)(u^2 + u - 1)^2(u^{39} - 2u^{38} + \dots - 10u + 1)$   |
| $c_{12}$              | $((u^2 - u - 1)^3)(u^{39} - 4u^{38} + \dots - 6u - 1)$               |

### V. Riley Polynomials

| Crossings             | Riley Polynomials at each crossing                                    |
|-----------------------|---|
| $c_1$                 | $((y-1)^6)(y^{39} + 25y^{38} + \dots + 19679031y - 6561)$             |
| $c_{2}, c_{5}$        | $((y-1)^6)(y^{39} - 15y^{38} + \dots + 5679y - 81)$                   |
| $c_3, c_7, c_8$       | $y^{2}(y-2)^{4}(y^{39}-49y^{38}+\cdots+560y-16)$                      |
| $c_4$                 | $y^{2}(y-2)^{4}(y^{39}-109y^{38}+\cdots+2.87574\times10^{7}y-144400)$ |
| $c_6, c_{11}$         | $((y^2 - 3y + 1)^3)(y^{39} - 6y^{38} + \dots + 46y - 1)$              |
| $c_9, c_{10}, c_{12}$ | $((y^2 - 3y + 1)^3)(y^{39} - 30y^{38} + \dots + 38y - 1)$             |