

Ideals for irreducible components² of X_{par}

$$\begin{split} I_1^u &= \langle -u^3 - u^2 + 2b - 3u + 1, \ a + 1, \ u^4 + 4u^2 - 2u + 1 \rangle \\ I_2^u &= \langle b - 1, \ 2a + u - 1, \ u^2 + u + 2 \rangle \\ I_3^u &= \langle b - u, \ a + 1, \ u^2 + 1 \rangle \end{split}$$

* 3 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 8 representations.

¹The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(http://www.layer8.co.uk/maths/draw/index.htm#Running-draw), where we modified some parts for our purpose(https://github.com/CATsTAILs/LinksPainter).

² All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.
$$I_1^u = \langle -u^3 - u^2 + 2b - 3u + 1, \ a + 1, \ u^4 + 4u^2 - 2u + 1 \rangle$$

(i) Arc colorings

$$a_{3} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{7} = \begin{pmatrix} 1 \\ -u^{2} \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} \frac{1}{2}u^{3} + \frac{1}{2}u^{2} + \frac{3}{2}u - \frac{1}{2} \end{pmatrix}$$

$$a_{2} = \begin{pmatrix} -u \\ u^{3} + u \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} \frac{1}{2}u^{3} + \frac{1}{2}u^{2} + \frac{3}{2}u + \frac{1}{2} \\ \frac{1}{2}u^{3} + \frac{1}{2}u^{2} + \frac{1}{2}u + \frac{1}{2} \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} \frac{1}{2}u^{3} + \frac{1}{2}u^{2} + \frac{1}{2}u + \frac{1}{2} \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} \frac{1}{2}u^{3} - \frac{1}{2}u^{2} + \frac{3}{2}u - \frac{1}{2} \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} -u^{2} - 1 \\ u^{2} \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} -u^{2} - 1 \\ u^{2} \end{pmatrix}$$

- (ii) Obstruction class = -1
- (iii) Cusp Shapes = $-4u^3 14u + 8$

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|----------------------------|--------------------------------|
| c_1, c_5 | $u^4 + 3u^3 + 5u^2 + 3u + 2$ |
| c_2, c_3, c_6 c_8, c_9 | $u^4 + 4u^2 - 2u + 1$ |
| C_4 | $u^4 - u^3 + 11u^2 - 11u + 4$ |
| | $u^4 + 8u^3 + 18u^2 + 4u + 1$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|----------------------------|------------------------------------|
| c_1, c_5 | $y^4 + y^3 + 11y^2 + 11y + 4$ |
| c_2, c_3, c_6 c_8, c_9 | $y^4 + 8y^3 + 18y^2 + 4y + 1$ |
| C_4 | $y^4 + 21y^3 + 107y^2 - 33y + 16$ |
| | $y^4 - 28y^3 + 262y^2 + 20y + 1$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_1^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---------------------------|---------------------------------------|---------------------|
| u = 0.264316 + 0.422125I | | |
| a = -1.00000 | 0.426736 + 1.175630I | 4.79089 - 5.96277I |
| b = -0.219104 + 0.751390I | | |
| u = 0.264316 - 0.422125I | | |
| a = -1.00000 | 0.426736 - 1.175630I | 4.79089 + 5.96277I |
| b = -0.219104 - 0.751390I | | |
| u = -0.26432 + 1.99036I | | |
| a = -1.00000 | -16.8761 - 4.7517I | -0.79089 + 2.00586I |
| b = -1.28090 - 1.27441I | | |
| u = -0.26432 - 1.99036I | | |
| a = -1.00000 | -16.8761 + 4.7517I | -0.79089 - 2.00586I |
| b = -1.28090 + 1.27441I | | |

II.
$$I_2^u = \langle b-1, 2a+u-1, u^2+u+2 \rangle$$

(i) Arc colorings

$$a_{3} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{7} = \begin{pmatrix} 1 \\ u+2 \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} -\frac{1}{2}u + \frac{1}{2} \\ 1 \end{pmatrix}$$

$$a_{2} = \begin{pmatrix} -u \\ 2 \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} \frac{1}{2}u + \frac{1}{2} \\ -1 \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} \frac{1}{2}u + \frac{3}{2} \\ -1 \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} -\frac{1}{2}u + \frac{1}{2} \\ -1 \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} u+1 \\ -u-2 \end{pmatrix}$$

$$a_{8} = \begin{pmatrix} u+1 \\ -u-2 \end{pmatrix}$$

- (ii) Obstruction class = -1
- (iii) Cusp Shapes = -2

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|----------------------------|--------------------------------|
| c_1, c_5 | $(u-1)^2$ |
| c_2, c_3, c_6 c_8, c_9 | $u^2 + u + 2$ |
| c_4 | $(u+1)^2$ |
| C ₇ | $u^2 + 3u + 4$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|----------------------------|------------------------------------|
| c_1, c_4, c_5 | $(y-1)^2$ |
| $c_2, c_3, c_6 \ c_8, c_9$ | $y^2 + 3y + 4$ |
| c ₇ | $y^2 - y + 16$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_2^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|--------------------------|---------------------------------------|------------|
| u = -0.50000 + 1.32288I | | |
| a = 0.750000 - 0.661438I | -4.93480 | -2.00000 |
| b = 1.00000 | | |
| u = -0.50000 - 1.32288I | | |
| a = 0.750000 + 0.661438I | -4.93480 | -2.00000 |
| b = 1.00000 | | |

III.
$$I_3^u = \langle b-u, a+1, u^2+1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u+1\\1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ u+1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- (ii) Obstruction class = 1
- (iii) Cusp Shapes = 0

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|---------------------------------------|--------------------------------|
| c_1, c_2, c_3 c_5, c_6, c_8 c_9 | $u^2 + 1$ |
| c_4, c_7 | $(u+1)^2$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|---|------------------------------------|
| c_1, c_2, c_3 c_5, c_6, c_8 c_9 | $(y+1)^2$ |
| c_4, c_7 | $(y-1)^2$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_3^u | | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|----------------------|------------|---------------------------------------|------------|
| u = | 1.000000I | | |
| a = -1.00000 | | -1.64493 | 0 |
| b = | 1.000000I | | |
| u = | -1.000000I | | |
| a = -1.00000 | | -1.64493 | 0 |
| b = | -1.000000I | | |

IV. u-Polynomials

| Crossings | u-Polynomials at each crossing |
|-------------------------------|--|
| c_1, c_5 | $(u-1)^2(u^2+1)(u^4+3u^3+5u^2+3u+2)$ |
| c_2, c_3, c_6 c_8, c_9 | $(u^2+1)(u^2+u+2)(u^4+4u^2-2u+1)$ |
| c_4 | $(u+1)^4(u^4-u^3+11u^2-11u+4)$ |
| | $(u+1)^2(u^2+3u+4)(u^4+8u^3+18u^2+4u+1)$ |

V. Riley Polynomials

| Crossings | Riley Polynomials at each crossing |
|-------------------------------|---|
| c_1,c_5 | $(y-1)^{2}(y+1)^{2}(y^{4}+y^{3}+11y^{2}+11y+4)$ |
| c_2, c_3, c_6 c_8, c_9 | $(y+1)^2(y^2+3y+4)(y^4+8y^3+18y^2+4y+1)$ |
| c_4 | $(y-1)^4(y^4+21y^3+107y^2-33y+16)$ |
| | $(y-1)^2(y^2-y+16)(y^4-28y^3+262y^2+20y+1)$ |