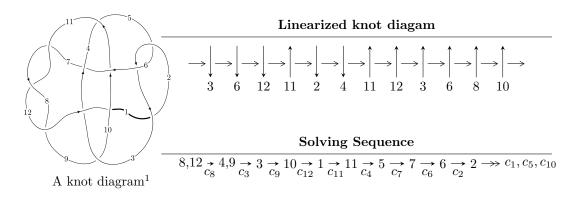
# $12n_{0439} \ (K12n_{0439})$



#### Ideals for irreducible components<sup>2</sup> of $X_{par}$

$$\begin{split} I_1^u &= \langle -51u^8 + 4u^7 + 487u^6 + 424u^5 - 1094u^4 - 281u^3 + 1891u^2 + 551b + 812u + 165, \\ & 267u^8 + 919u^7 + 173u^6 - 2317u^5 + 185u^4 + 5555u^3 + 2060u^2 + 1102a - 2436u - 799, \\ & u^9 + 5u^8 + 7u^7 - 3u^6 - 7u^5 + 17u^4 + 32u^3 + 18u^2 + 7u + 2 \rangle \\ I_2^u &= \langle u^7 - 2u^6 - u^5 + 5u^4 - 2u^3 - 5u^2 + b + 2u + 2, \quad -u^7 + 3u^6 - u^5 - 4u^4 + 3u^3 + 3u^2 + 3a - u - 4, \\ & u^8 - 3u^7 + u^6 + 7u^5 - 9u^4 - 3u^3 + 10u^2 - 2u - 3 \rangle \\ I_3^u &= \langle b + 2a - 1, \ 4a^2 - 6a + 7, \ u - 2 \rangle \end{split}$$

$$I_1^v = \langle a, b + v, v^2 - v + 1 \rangle$$

\* 4 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 21 representations.

<sup>&</sup>lt;sup>1</sup>The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(http://www.layer8.co.uk/maths/draw/index.htm#Running-draw), where we modified some parts for our purpose(https://github.com/CATsTAILs/LinksPainter).

 $<sup>^2</sup>$  All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I. 
$$I_1^u = \langle -51u^8 + 4u^7 + \dots + 551b + 165, \ 267u^8 + 919u^7 + \dots + 1102a - 799, \ u^9 + 5u^8 + \dots + 7u + 2 \rangle$$

$$a_{8} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} -0.242287u^{8} - 0.833938u^{7} + \dots + 2.21053u + 0.725045 \\ 0.0925590u^{8} - 0.00725953u^{7} + \dots - 1.47368u - 0.299456 \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} 1 \\ -u^{2} \end{pmatrix}$$

$$a_{3} = \begin{pmatrix} -0.242287u^{8} - 0.833938u^{7} + \dots + 2.21053u + 0.725045 \\ 0.441016u^{8} + 1.25953u^{7} + \dots + 0.684211u + 0.455535 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0.0735027u^{8} + 0.376588u^{7} + \dots - 0.0526316u + 0.409256 \\ -0.0326679u^{8} - 0.0562613u^{7} + \dots + 0.578947u - 0.0707804 \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} -0.0408348u^{8} - 0.320327u^{7} + \dots - 0.526316u - 0.338475 \\ -0.0907441u^{8} + 0.399274u^{7} + \dots + 2.05263u + 0.470054 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} 0.227768u^{8} + 0.697822u^{7} + \dots + 3.15789u + 0.910163 \\ -0.377495u^{8} - 1.53902u^{7} + \dots - 2.42105u - 0.484574 \end{pmatrix}$$

$$a_{7} = \begin{pmatrix} -u^{2} + 1 \\ u^{2} \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} 0.0825771u^{8} + 0.336661u^{7} + \dots - 1.15789u + 0.262250 \\ -0.00907441u^{8} + 0.0399274u^{7} + \dots + 1.10526u + 0.147005 \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} -0.0735027u^{8} - 0.376588u^{7} + \dots + 1.05263u + 0.590744 \\ -0.0762250u^{8} - 0.464610u^{7} + \dots - 0.315789u - 0.165154 \end{pmatrix}$$

#### (ii) Obstruction class = -1

(iii) Cusp Shapes 
$$= \frac{1779}{551}u^8 + \frac{8482}{551}u^7 + \frac{9752}{551}u^6 - \frac{10058}{551}u^5 - \frac{11137}{551}u^4 + \frac{36963}{551}u^3 + \frac{46798}{551}u^2 + \frac{379}{19}u + \frac{5232}{551}u^3 + \frac{11137}{551}u^3 +$$

Crossings	u-Polynomials at each crossing	
$c_1$	$u^9 + 6u^8 + \dots + 1105u + 16$	
$c_2, c_5$	$u^9 + 6u^8 + 15u^7 + 19u^6 + 39u^5 + 96u^4 + 137u^3 + 93u^2 + 19u - 4$	
$c_3, c_6$	$u^9 - u^8 + 6u^7 + 6u^6 + 39u^5 + 60u^4 + 40u^3 + 13u^2 + u - 1$	
$c_4, c_9$	$u^9 + u^8 + 7u^7 + 9u^6 + 14u^5 + 11u^4 + 8u^3 + u^2 - u - 1$	
$c_7, c_8, c_{11}$	$u^9 - 5u^8 + 7u^7 + 3u^6 - 7u^5 - 17u^4 + 32u^3 - 18u^2 + 7u - 2$	
$c_{10}, c_{12}$	$u^9 + 6u^8 + 14u^7 - 5u^6 + 66u^5 - 202u^4 + 132u^3 + u^2 - 10u - 1$	

Crossings	Riley Polynomials at each crossing	
$c_1$	$y^9 + 114y^8 + \dots + 1135425y - 256$	
$c_2, c_5$	$y^9 - 6y^8 + \dots + 1105y - 16$	
$c_3, c_6$	$y^9 + 11y^8 + \dots + 27y - 1$	
$c_4, c_9$	$y^9 + 13y^8 + 59y^7 + 109y^6 + 106y^5 + 73y^4 + 32y^3 + 5y^2 + 3y - 1$	
$c_7, c_8, c_{11}$	$y^9 - 11y^8 + \dots - 23y - 4$	
$c_{10}, c_{12}$	$y^9 - 8y^8 + \dots + 102y - 1$	

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = 1.18169 + 0.81660I		
a = -1.13356 - 0.92427I	4.72372 - 1.44557I	2.79413 + 0.05589I
b = 0.538744 + 0.723261I		
u = 1.18169 - 0.81660I		
a = -1.13356 + 0.92427I	4.72372 + 1.44557I	2.79413 - 0.05589I
b = 0.538744 - 0.723261I		
u = -0.557684		
a = -0.337455	0.803810	12.4850
b = -0.425472		
u = -1.42962 + 0.20941I		
a = 0.119321 + 0.349200I	3.18435 - 3.04209I	1.27965 + 3.40109I
b = -0.023607 - 1.115360I		
u = -1.42962 - 0.20941I		
a = 0.119321 - 0.349200I	3.18435 + 3.04209I	1.27965 - 3.40109I
b = -0.023607 + 1.115360I		
u = -0.058623 + 0.424215I		
a = 0.74163 + 1.41569I	-1.48784 + 0.34537I	-5.00907 - 1.24859I
b = 0.476502 - 0.460767I		
u = -0.058623 - 0.424215I		
a = 0.74163 - 1.41569I	-1.48784 - 0.34537I	-5.00907 + 1.24859I
b = 0.476502 + 0.460767I		
u = -1.91461 + 0.93499I		
a = -0.308657 - 1.376960I	13.7395 - 8.2460I	2.69287 + 2.56767I
b = -0.27890 + 2.28182I		
u = -1.91461 - 0.93499I		
a = -0.308657 + 1.376960I	13.7395 + 8.2460I	2.69287 - 2.56767I
b = -0.27890 - 2.28182I		

II. 
$$I_2^u = \langle u^7 - 2u^6 - u^5 + 5u^4 - 2u^3 - 5u^2 + b + 2u + 2, -u^7 + 3u^6 + \dots + 3a - 4, u^8 - 3u^7 + \dots - 2u - 3 \rangle$$

$$a_{8} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} \frac{1}{3}u^{7} - u^{6} + \dots + \frac{1}{3}u + \frac{4}{3} \\ -u^{7} + 2u^{6} + u^{5} - 5u^{4} + 2u^{3} + 5u^{2} - 2u - 2 \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} 1 \\ -u^{2} \end{pmatrix}$$

$$a_{3} = \begin{pmatrix} \frac{1}{3}u^{7} - u^{6} + \dots + \frac{1}{3}u + \frac{4}{3} \\ -u^{7} + 3u^{6} - u^{5} - 5u^{4} + 5u^{3} + 3u^{2} - 3u - 2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} \frac{2}{3}u^{7} - u^{6} + \dots + \frac{2}{3}u + \frac{8}{3} \\ -u^{7} + u^{6} + 2u^{5} - 4u^{4} - u^{3} + 5u^{2} + u - 1 \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} \frac{1}{3}u^{7} - \frac{2}{3}u^{5} + \dots - \frac{5}{3}u - \frac{5}{3} \\ 2u^{7} - 5u^{6} - 2u^{5} + 14u^{4} - 8u^{3} - 15u^{2} + 11u + 7 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} -\frac{2}{3}u^{7} + u^{6} + \dots - \frac{11}{3}u - \frac{5}{3} \\ -u^{5} + 2u^{4} - 3u^{2} + 2u + 1 \end{pmatrix}$$

$$a_{7} = \begin{pmatrix} -u^{2} + 1 \\ u^{2} \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} -\frac{1}{3}u^{7} + u^{6} + \dots - \frac{7}{3}u + \frac{2}{3} \\ u^{7} - 2u^{6} - u^{5} + 6u^{4} - 3u^{3} - 5u^{2} + 3u + 2 \end{pmatrix}$$

$$a_{2} = \begin{pmatrix} \frac{2}{3}u^{7} - u^{6} + \dots + \frac{5}{3}u + \frac{5}{3} \\ -u^{3} + u^{2} - 1 \end{pmatrix}$$

- (ii) Obstruction class = 1
- (iii) Cusp Shapes =  $3u^7 7u^6 + 3u^5 + 10u^4 12u^3 2u^2 + 3u + 3u^4 + 3$

Crossings	u-Polynomials at each crossing		
$c_1$	$u^8 - 10u^7 + 39u^6 - 81u^5 + 117u^4 - 122u^3 + 63u^2 - 11u + 1$		
$c_2$	$u^8 + 4u^7 + 3u^6 - 7u^5 - 13u^4 - 4u^3 + 7u^2 + 5u + 1$		
$c_3, c_6$	$u^8 + 2u^7 - 2u^5 - 3u^4 - 3u^3 + 3u^2 + 4u + 1$		
$c_4,c_9$	$u^8 + 7u^6 + 10u^4 - 5u^3 - u^2 - 2u - 1$		
$c_5$	$u^8 - 4u^7 + 3u^6 + 7u^5 - 13u^4 + 4u^3 + 7u^2 - 5u + 1$		
$c_7, c_8$	$u^8 - 3u^7 + u^6 + 7u^5 - 9u^4 - 3u^3 + 10u^2 - 2u - 3$		
$c_{10}, c_{12}$	$u^8 + u^7 + 3u^6 + 2u^5 - 6u^4 + 6u^3 - 8u^2 + 5u - 1$		
$c_{11}$	$u^8 + 3u^7 + u^6 - 7u^5 - 9u^4 + 3u^3 + 10u^2 + 2u - 3$		

Crossings	Riley Polynomials at each crossing		
$c_1$	$y^8 - 22y^7 + 135y^6 + 251y^5 - 1379y^4 - 1846y^3 + 1519y^2 + 5y + 1$		
$c_2, c_5$	$y^8 - 10y^7 + 39y^6 - 81y^5 + 117y^4 - 122y^3 + 63y^2 - 11y + 1$		
$c_3, c_6$	$y^8 - 4y^7 + 2y^6 + 14y^5 - 17y^4 - 11y^3 + 27y^2 - 10y + 1$		
$c_4, c_9$	$y^8 + 14y^7 + 69y^6 + 138y^5 + 84y^4 - 59y^3 - 39y^2 - 2y + 1$		
$c_7, c_8, c_{11}$	$y^8 - 7y^7 + 25y^6 - 65y^5 + 125y^4 - 167y^3 + 142y^2 - 64y + 9$		
$c_{10}, c_{12}$	$y^8 + 5y^7 - 7y^6 - 68y^5 - 48y^4 + 34y^3 + 16y^2 - 9y + 1$		

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = -1.143550 + 0.105994I		
a = 0.027839 + 1.059490I	4.97207 - 3.05412I	5.35335 + 5.43549I
b = -0.039265 - 0.565787I		
u = -1.143550 - 0.105994I		
a = 0.027839 - 1.059490I	4.97207 + 3.05412I	5.35335 - 5.43549I
b = -0.039265 + 0.565787I		
u = 1.084730 + 0.492548I		
a = 0.753232 - 0.582528I	-12.04710 + 1.95234I	-1.37368 - 3.45942I
b = 0.13996 + 2.26543I		
u = 1.084730 - 0.492548I		
a = 0.753232 + 0.582528I	-12.04710 - 1.95234I	-1.37368 + 3.45942I
b = 0.13996 - 2.26543I		
u = 1.03265 + 1.04538I		
a = -0.527623 + 0.897663I	-5.86984 + 3.80835I	2.98023 - 3.30420I
b = -0.31449 - 1.42869I		
u = 1.03265 - 1.04538I		
a = -0.527623 - 0.897663I	-5.86984 - 3.80835I	2.98023 + 3.30420I
b = -0.31449 + 1.42869I		
u = -0.483343		
a = 1.10069	-0.371792	2.79670
b = -0.358657		
u = 1.53567		
a = -0.274253	6.52231	9.28350
b = 0.786231		

III. 
$$I_3^u = \langle b+2a-1, \ 4a^2-6a+7, \ u-2 \rangle$$

1) Arc colorings
$$a_{8} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$a_{4} = \begin{pmatrix} a \\ -2a+1 \end{pmatrix}$$

$$a_{9} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$a_{3} = \begin{pmatrix} a \\ -6a+1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 2a - \frac{5}{2} \\ -10a+16 \end{pmatrix}$$

$$a_{1} = \begin{pmatrix} -8a - \frac{3}{2} \\ 54a - 8 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$a_{5} = \begin{pmatrix} -3a+4 \\ 2a-3 \end{pmatrix}$$

$$a_{7} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$a_{6} = \begin{pmatrix} 0.5 \\ -2a \end{pmatrix}$$

$$a_{2} = \begin{pmatrix} a - \frac{3}{2} \\ 1 \end{pmatrix}$$

- (ii) Obstruction class = -1
- (iii) Cusp Shapes = 3

Crossings	u-Polynomials at each crossing
$c_1$	$(u+9)^2$
$c_2, c_5$	$(u-3)^2$
$c_3, c_6$	$u^2 - 3u + 7$
$c_4, c_9$	$u^2 - u + 5$
$c_7, c_8, c_{11}$	$(u+2)^2$
$c_{10}, c_{12}$	$u^2 - 4u + 23$

Crossings	Riley Polynomials at each crossing
$c_1$	$(y-81)^2$
$c_{2}, c_{5}$	$(y-9)^2$
$c_{3}, c_{6}$	$y^2 + 5y + 49$
$c_4, c_9$	$y^2 + 9y + 25$
$c_7, c_8, c_{11}$	$(y-4)^2$
$c_{10}, c_{12}$	$y^2 + 30y + 529$

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
u = 2.00000		
a = 0.750000 + 1.089730I	-9.86960	3.00000
b = -0.50000 - 2.17945I		
u = 2.00000		
a = 0.750000 - 1.089730I	-9.86960	3.00000
b = -0.50000 + 2.17945I		

IV. 
$$I_1^v = \langle a, b+v, v^2-v+1 \rangle$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0 \\ -v \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -v \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} v+1\\ -v+1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ v - 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ -v \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -v+1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

- (ii) Obstruction class = 1
- (iii) Cusp Shapes = 3

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_{10}$ $c_{12}$	$(u-1)^2$
$c_3, c_4, c_6$ $c_9$	$u^2 + u + 1$
<i>C</i> <sub>5</sub>	$(u+1)^2$
$c_7, c_8, c_{11}$	$u^2$

Crossings	Riley Polynomials at each crossing		
$c_1, c_2, c_5 \\ c_{10}, c_{12}$	$(y-1)^2$		
$c_3, c_4, c_6$ $c_9$	$y^2 + y + 1$		
$c_7, c_8, c_{11}$	$y^2$		

Solutions to $I_1^v$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
v = 0.500000 + 0.866025I		
a = 0	0	3.00000
b = -0.500000 - 0.866025I		
v = 0.500000 - 0.866025I	0	3.00000
a = 0 $b = -0.500000 + 0.866025I$	U	3.00000

#### V. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u-1)^{2}(u+9)^{2}$ $\cdot (u^{8}-10u^{7}+39u^{6}-81u^{5}+117u^{4}-122u^{3}+63u^{2}-11u+1)$ $\cdot (u^{9}+6u^{8}+\cdots+1105u+16)$
$c_2$	$((u-3)^2)(u-1)^2(u^8+4u^7+\cdots+5u+1)$ $\cdot (u^9+6u^8+15u^7+19u^6+39u^5+96u^4+137u^3+93u^2+19u-4)$
$c_3, c_6$	$(u^{2} - 3u + 7)(u^{2} + u + 1)(u^{8} + 2u^{7} + \dots + 4u + 1)$ $\cdot (u^{9} - u^{8} + 6u^{7} + 6u^{6} + 39u^{5} + 60u^{4} + 40u^{3} + 13u^{2} + u - 1)$
$c_4, c_9$	$(u^{2} - u + 5)(u^{2} + u + 1)(u^{8} + 7u^{6} + 10u^{4} - 5u^{3} - u^{2} - 2u - 1)$ $\cdot (u^{9} + u^{8} + 7u^{7} + 9u^{6} + 14u^{5} + 11u^{4} + 8u^{3} + u^{2} - u - 1)$
$c_5$	$((u-3)^2)(u+1)^2(u^8-4u^7+\cdots-5u+1)$ $\cdot (u^9+6u^8+15u^7+19u^6+39u^5+96u^4+137u^3+93u^2+19u-4)$
$c_7, c_8$	$u^{2}(u+2)^{2}(u^{8}-3u^{7}+u^{6}+7u^{5}-9u^{4}-3u^{3}+10u^{2}-2u-3)$ $\cdot (u^{9}-5u^{8}+7u^{7}+3u^{6}-7u^{5}-17u^{4}+32u^{3}-18u^{2}+7u-2)$
$c_{10}, c_{12}$	$((u-1)^2)(u^2 - 4u + 23)(u^8 + u^7 + \dots + 5u - 1)$ $\cdot (u^9 + 6u^8 + 14u^7 - 5u^6 + 66u^5 - 202u^4 + 132u^3 + u^2 - 10u - 1)$
$c_{11}$	$u^{2}(u+2)^{2}(u^{8}+3u^{7}+u^{6}-7u^{5}-9u^{4}+3u^{3}+10u^{2}+2u-3)$ $\cdot (u^{9}-5u^{8}+7u^{7}+3u^{6}-7u^{5}-17u^{4}+32u^{3}-18u^{2}+7u-2)$

## VI. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y-81)^{2}(y-1)^{2}$ $\cdot (y^{8}-22y^{7}+135y^{6}+251y^{5}-1379y^{4}-1846y^{3}+1519y^{2}+5y+1)$ $\cdot (y^{9}+114y^{8}+\cdots+1135425y-256)$
$c_2, c_5$	$(y-9)^{2}(y-1)^{2}$ $\cdot (y^{8}-10y^{7}+39y^{6}-81y^{5}+117y^{4}-122y^{3}+63y^{2}-11y+1)$ $\cdot (y^{9}-6y^{8}+\cdots+1105y-16)$
$c_3, c_6$	$(y^{2} + y + 1)(y^{2} + 5y + 49)$ $\cdot (y^{8} - 4y^{7} + 2y^{6} + 14y^{5} - 17y^{4} - 11y^{3} + 27y^{2} - 10y + 1)$ $\cdot (y^{9} + 11y^{8} + \dots + 27y - 1)$
$c_4, c_9$	$(y^{2} + y + 1)(y^{2} + 9y + 25)$ $\cdot (y^{8} + 14y^{7} + 69y^{6} + 138y^{5} + 84y^{4} - 59y^{3} - 39y^{2} - 2y + 1)$ $\cdot (y^{9} + 13y^{8} + 59y^{7} + 109y^{6} + 106y^{5} + 73y^{4} + 32y^{3} + 5y^{2} + 3y - 1)$
$c_7, c_8, c_{11}$	$y^{2}(y-4)^{2}$ $\cdot (y^{8}-7y^{7}+25y^{6}-65y^{5}+125y^{4}-167y^{3}+142y^{2}-64y+9)$ $\cdot (y^{9}-11y^{8}+\cdots-23y-4)$
$c_{10}, c_{12}$	$(y-1)^{2}(y^{2} + 30y + 529)$ $\cdot (y^{8} + 5y^{7} - 7y^{6} - 68y^{5} - 48y^{4} + 34y^{3} + 16y^{2} - 9y + 1)$ $\cdot (y^{9} - 8y^{8} + \dots + 102y - 1)$