Exercise 1
3D Computer Vision

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1 Introduction

This report presents the results of the theoretical and practical parts of Exercise 1.

2 Theory

2.1 Properties of Rotation Matrices

(a) The task is showing the fact that the rows and columns of R are orthonomal (orthogonal and of length 1).
Let's begin with the rotation matrix around the x-axis.
Rotation by ϕ aroud x axis:

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix}. \tag{1}$$

We can prove that the rows and columns of R are orthogonal by proving the equations $R^{-1} = R^T$ and $det(R) = 1$. Let's calculate ${R_x}^{-1} = \frac{(R_x^*)^T}{det(R_x)}$, we need to calculate $det(R_x)$ using minors M.

$det(R_x) = (using\ first\ row) = 1 * (cos\phi * cos\phi - (-sin\phi * sin\phi)) - 0 + 0 = \cos^2\phi + \sin^2\phi = 1$

Find transposed matrix R_x^T:

$$R_x^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix}$$

Find minor matrix M:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

Find adjugate matrix R_x^*:

$$R_x^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

Thus, we can find the value of ${R_x}^{-1}$

$${R_x}^{-1} = \frac{(R_x^*)^T}{det(R_x)} = \frac{\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix}}{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix} = R^T$$

Q.E.D.
Now we will prove that the length of any row or column is equal to 1.
Row 1 $r_1 = (1, 0, 0)$. $|r_1| = \sqrt{1^2 + 0^2 + 0^2} = 1$
Row 2 $r_2 = (0, cos\phi, -sin\phi)$. $|r_2| = \sqrt{0^2 + (\cos\phi)^2 + (-\sin\phi)^2} = \sqrt{\cos^2\phi + \sin^2\phi} = 1$
Row 3 $r_3 = (0, sin\phi, cos\phi)$. $|r_3| = \sqrt{0^2 + (\sin\phi)^2 + (\cos\phi)^2} = \sqrt{\sin^2\phi + \cos^2\phi} = 1$
Column 1 $c_1 = (1, 0, 0)$. $|c_1| = \sqrt{1^2 + 0^2 + 0^2} = 1$

Column 2 $c_2 = (0, cos\phi, sin\phi)$. $|c_2| = \sqrt{0^2 + (\cos\phi)^2 + (\sin\phi)^2} = \sqrt{\cos^2\phi + \sin^2\phi} = 1$
Column 3 $c_3 = (0, -sin\phi, cos\phi)$. $|c_3| = \sqrt{0^2 + (-\sin\phi)^2 + (\cos\phi)^2} = \sqrt{\sin^2\phi + \cos^2\phi} = 1$

Thus, since rows and columns of matrix R_x are orthogonal and their length is equal to 1, we prove that they are orthonormal.

Now consider the rotation matrix around the y-axis.
Rotation by θ aroud y axis:

$$R_y = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}. \tag{2}$$

We can prove that the rows and columns of R are orthogonal by proving the equations $R^{-1} = R^T$ and $det(R) = 1$. Let's calculate ${R_y}^{-1} = \frac{(R_y^*)^T}{det(R_y)}$, we need to calculate $det(R_y)$ using minors M.

$det(R_y) = (using\ second\ row) = 0 + 1 * (cos\theta * cos\theta - (-sin\theta * sin\theta)) - 0 = \cos^2\theta + \sin^2\theta = 1$
Find transposed matrix R_y^T:

$$R_y^T = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}$$

Find minor matrix M:

$$M = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Find adjugate matrix R_y^*:

$$R_y^* = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}$$

Thus, we can find the value of ${R_y}^{-1}$

$${R_y}^{-1} = \frac{(R_y^*)^T}{det(R_y)} = \frac{\begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix}}{1} = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} = R^T$$

Q.E.D.
Now we will prove that the length of any row or column is equal to 1.
Row 1 $r_1 = (\cos\theta, 0, \sin\ theta)$. $|r_1| = \sqrt{(\cos\theta)^2 + 0^2 + (\sin\theta)^2} = 1$
Row 2 $r_2 = (0, 1, 0)$. $|r_2| = \sqrt{0^2 + 1^2 + 0^2} = 1$
Row 3 $r_3 = (-\sin\theta, 0, \cos\theta)$. $|r_3| = \sqrt{(-\sin\theta)^2 + 0^2 + (\cos\theta)^2} = \sqrt{\sin^2\theta + \cos^2\theta} = 1$
Column 1 $c_1 = (\cos\theta, 0, -\sin\ theta)$. $|c_1| = \sqrt{(\cos\theta)^2 + 0^2 + (-\sin\theta)^2} = \sqrt{\cos^2\theta + \sin^2\theta} = 1$
Column 2 $c_2 = (0, 1, 0)$. $|c_2| = \sqrt{0^2 + 1^2 + 0^2} = 1$
Column 3 $c_3 = (\sin\theta, 0, \cos\theta)$. $|c_3| = \sqrt{(\sin\theta)^2 + 0^2 + (\cos\theta)^2} = \sqrt{\sin^2\theta + \cos^2\theta} = 1$
Thus, since rows and columns of matrix R_y are orthogonal and their length is equal to 1, we prove that they are orthonormal.

And in the end, consider the rotation matrix around the z-axis.
Rotation by ψ aroud z axis:

$$R_z = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

We can prove that the rows and columns of R are orthogonal by proving the equations $R^{-1} = R^T$ and $det(R) = 1$. Let's calculate ${R_z}^{-1} = \frac{(R_z^*)^T}{det(R_z)}$, we need to calculate $det(R_z)$ using minors M.

$det(R_z) = (using\ third\ row) = 0 - 0 + 1 * (cos\psi * cos\psi - (-sin\psi * sin\psi)) = \cos^2\psi + \sin^2\psi = 1$
Find transposed matrix R_z^T:

$$R_z^T = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find minor matrix M:

$$M = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Find adjugate matrix R_y^*:

$$R_z^* = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus, we can find the value of ${R_z}^{-1}$

$${R_z}^{-1} = \frac{(R_z^*)^T}{det(R_z)} = \frac{\begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}}{1} = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = R^T$$

Q.E.D.
Now we will prove that the length of any row or column is equal to 1.
Row 1 $r_1 = (\cos\psi, -\sin\psi, 0)$. $|r_1| = \sqrt{(\cos\psi)^2 + (-\sin\psi)^2 + 0^2} = \sqrt{\cos^2\psi + \sin^2\psi} = 1$
Row 2 $r_2 = (\sin\psi, \cos\psi, 0)$. $|r_3| = \sqrt{(\sin\psi)^2 + (\cos\psi)^2 + 0^2} = \sqrt{\sin^2\psi + \cos^2\psi} = 1$
Row 3 $r_3 = r_2 = (0, 0, 1)$. $|r_2| = \sqrt{0^2 + 0^2 + 1^2} = 1$
Column 1 $c_1 = (\cos\psi, \sin\psi, 0)$. $|c_1| = \sqrt{(\cos\psi)^2 + (\sin\psi)^2 + 0^2} = \sqrt{\cos^2\psi + \sin^2\psi} = 1$
Column 2 $c_2 = (-\sin\psi, \cos\psi, 0)$. $|c_2| = \sqrt{(-\sin\psi)^2 + (\cos\psi)^2 + 0^2} = \sqrt{\sin^2\psi + \cos^2\psi} = 1$
Column 3 $c_3 = (0, 0, 1)$. $|c_3| = \sqrt{0^2 + 0^2 + 1^2} = 1$
Thus, since rows and columns of matrix R_y are orthogonal and their length is equal to 1, we prove that they are orthonormal.

(b) Prove that properties $R^{-1} = R^T$ and $det(R) = 1$ also hold true for $R = R_z(\psi)R_y(\theta)R_x(\phi)$

Let's calculate R as the multiplication of the matrixes [1].:

$$R = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix} =$$
$$\begin{bmatrix} \cos\psi\cos\theta & -\sin\psi & \cos\psi\sin\theta \\ \cos\theta\sin\psi & \cos\psi & \sin\theta\sin\psi \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix} = \tag{4}$$
$$\begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ \cos\theta\sin\psi & \cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

In this case, a transposed matrix R^T looks like:

$$R^T = \begin{bmatrix} \cos\psi\cos\theta & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi & \cos\theta\sin\phi \\ \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi & \cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi & \cos\theta\cos\phi \end{bmatrix}$$

Let's calculate $R^{-1} = \frac{R^{*T}}{det(R)}$, we need to calculate $det(R)$ using minors M.

$$det(R) = (using\ third\ row) = -\sin\theta M_1^3 - \cos\theta\sin\phi M_2^3 + \cos\theta\cos\phi M_3^3 \tag{5}$$

Calculate minors:

$$\begin{gathered} M_1^3 = \begin{bmatrix} -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ \cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi \end{bmatrix} = \\ = (-\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi)(-\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi) - \\ -(\sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi)(\cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi) = \\ = \sin\psi\cos\phi\cos\psi\sin\phi - \sin^2\psi\cos^2\phi\sin\theta - \cos^2\psi\sin^2\psi\sin\theta + cos\psi\sin^2\theta\sin\phi\sin\psi\cos\phi - \\ -\sin\psi\sin\phi\cos\psi\cos\phi - \sin^2\psi\sin^2\phi\sin\theta - \cos^2\psi\cos^2\phi\sin\theta + \cos\psi\sin^2\theta\cos\phi\sin\psi\sin\phi = \\ = -\sin\theta(\sin^2\psi\cos^2\phi + \cos^2\psi\sin^2\phi + \sin^2\psi\sin^2\phi + \cos^2\psi\cos^2\phi) = \\ = -\sin\theta(\sin^2\psi(\cos^2\phi + \sin^2\phi) + \cos^2\psi(\sin^2\phi + \cos^2\phi)) = -\sin\theta(\sin^2\psi + \cos^2\psi) = -\sin\theta \end{gathered} \tag{6}$$

$$M_2^3 = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ \cos\theta\sin\psi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi \end{bmatrix} =$$

$$\begin{gathered} = (\cos\psi\cos\theta)(-\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi) - (\sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi)(\cos\theta\sin\psi) = \\ = \cos\psi\cos\theta\sin\theta\sin\psi\cos\phi - \cos^2\psi\cos\theta\sin\phi - \cos\theta\sin^2\psi\sin\phi - \cos\theta\sin\psi\cos\psi\sin\theta\cos\phi = \\ = -\cos\theta\sin\phi(\cos^2\psi + \sin^2\psi) = -\cos\theta\sin\phi \end{gathered} \tag{7}$$

$$M_3^3 = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi \\ \cos\theta\sin\psi & \cos\psi\sin\phi + \sin\theta\sin\psi\sin\phi \end{bmatrix} =$$

$$\begin{gathered} = (\cos\psi\cos\theta)(\cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi) - (-\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi)(\cos\theta\sin\psi) = \\ = \cos^2\psi\cos\theta\cos\phi + \cos\psi\cos\theta\sin\theta\sin\psi\sin\phi - \cos\theta\sin\psi\cos\psi\sin\theta\sin\phi + \cos\theta\sin^2\psi\cos\phi = \\ = \cos\theta\cos\phi(\cos^2\psi + \sin^2\psi) = \cos\theta\cos\phi \end{gathered} \tag{8}$$

Let us substitute the obtained values of the minors 6, 7 , 8 into expression 5:

$$\begin{gathered} det(R) = (using\ third\ row) = -\sin\theta M_1^3 - \cos\theta\sin\phi M_2^3 + \cos\theta\cos\phi M_3^3 = \\ = -\sin\theta(-\sin\theta) - \cos\theta\sin\phi(-\cos\theta\sin\phi) + \cos\theta\cos\phi\cos\theta\cos\phi = \\ = \sin^2\theta + \cos^2\theta\sin^2\phi + \cos^2\theta\cos^2\phi = \\ = \sin^2\theta + \cos^2\theta(\sin^2\phi + \cos^2\phi) = \sin^2\theta + \cos^2\theta = 1 \end{gathered} \tag{9}$$

Thus, we have proved that the determinant of matrix 4 is equal to 1. Find minor matrix M:

$$M = \begin{bmatrix} \cos\psi\cos\theta & \sin\psi\cos\phi - \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ -\cos\theta\sin\psi & \cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi & \cos\psi\sin\phi - \sin\theta\sin\psi\cos\phi \\ -\sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Find adjugate matrix R^{*T}:

$$R^{*T} = \begin{bmatrix} \cos\psi\cos\theta & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi & \cos\theta\sin\phi \\ \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi & \cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi & \cos\theta\cos\phi \end{bmatrix}$$

And now we can calculate $R^{-1} = \frac{R^{*T}}{det(R)}$

$$R^{-1} = \frac{\begin{bmatrix} \cos\psi\cos\theta & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi & \cos\theta\sin\phi \\ \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi & \cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi & \cos\theta\cos\phi \end{bmatrix}}{1} =$$

$$\begin{bmatrix} \cos\psi\cos\theta & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \cos\psi\cos\phi + \sin\theta\sin\psi\sin\phi & \cos\theta\sin\phi \\ \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi & \cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi & \cos\theta\cos\phi \end{bmatrix}$$

Thus, we have proved that $R^{-1} = R^T$ So we prove that properties $R^{-1} = R^T$ and $det(R) = 1$ also hold true for $R = R_z(\psi)R_y(\theta)R_x(\phi)$ [1]

*(c)Name the geometric interpretation of the determinant of a square 3 * 3 matrix? Why does a rotation matrix have to have determinant 1*

Let's calculate the determinant of a square 3 * 3 matrix X:

$$det(X) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}) =$$

$$a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}$$

Now we look at the formula of the volume of parallelepiped which is used vectors:

$$V = |\vec{a}\vec{b}\vec{c}| = det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}) =$$

$$a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31},$$

where $\vec{a} = (a_{11}, a_{12}, a_{13}), \vec{b} = (a_{21}, a_{22}, a_{23}), \vec{c} = (a_{31}, a_{32}, a_{33})$
We can see what the determinant of a 3x3 square matrix is the volume of the parallelepiped built on $\vec{a}, \vec{b}, \vec{c}$ vectors, which is shown in Figure 1.

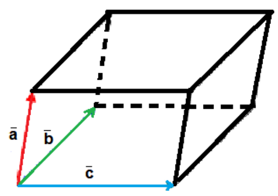

Figure 1: Parallelepiped built on three vectors.

Since each column and row represents the coordinates of a unit vector, the length of the vectors defined by the rows and columns of the rotation matrix is 1. The determinant of the rotation matrix is +1 for a right-handed frame of reference and -1 for a left-handed one. Thus, if the determinant value is -1, the image will be inverted for a right-handed frame of reference.

3 Transformation Chain

Let's define the coordinates of the point in the camera coordinat system and in the world coordinat system(the values do not match). Let $M = (x_w, y_w, z_w)$ - point in the world coordinates and $M = (x_c, y_c, z_c)$ - point in the camera coordinates and $t = (O_c, O_w)$ is translation vector between origins. The extrinsic parameters of camera are contained in the rotation matrix R and the translation vector t, they describe the camera position. Figure 2 is presented for better understanding.

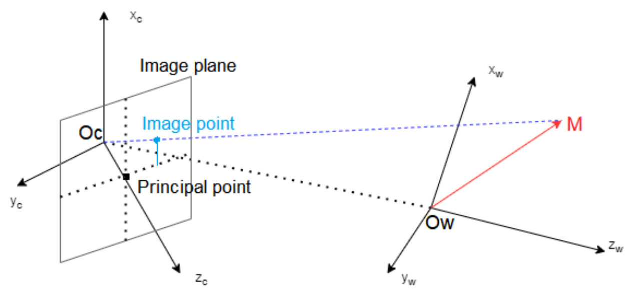

Figure 2: W to C.

We can represent points using this equality:

$$\begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} = t + R \begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix}$$

We can transform equation in the form below using homogeneous coordinates [1].:

$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = \begin{pmatrix} R & t \\ 0_3^T & 1 \end{pmatrix} \begin{pmatrix} x_w \\ y_w \\ z_w \\ 1 \end{pmatrix}$$

Now we can get a point in the 3D camera coordinate system, project the point onto the image plane and calculate its position on the image. (Figure 2)

We represent image point (x_i, y_i):
$x_i = f\frac{x_c}{z_c}$
$y_i = f\frac{y_c}{z_c}$
We can present this equations in matrix form:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}$$

In this case, perspective projection becomes linear in homogeneous coordinates:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix}$$

Obviously, the optical center (principal point) (x_0, y_0) of the camera may not matched with the center of the image coordinate system, and there may also be a skew. We use principal point (x_0, y_0), s - skew parameter and $\alpha_x = fk_x$, $\alpha_y = fk_y$ - scaling factors, so we can get the calibration matrix K, its contains the intrinsic camera parameters:

$$K = \begin{pmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

We can represent formula:

$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} = \begin{pmatrix} \alpha_x & s & x_0 & 0 \\ 0 & \alpha_y & y_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix} = K(I_3|0_3) \begin{pmatrix} x_c \\ y_c \\ z_c \\ 1 \end{pmatrix}$$

So we can find the pixel coordinates (u, v): $x_{pix} = \frac{u'}{w'}$; $y_{pix} = \frac{v'}{w'}$

4 Implementation

Results of program work: - projected points without correction of the distortion in red

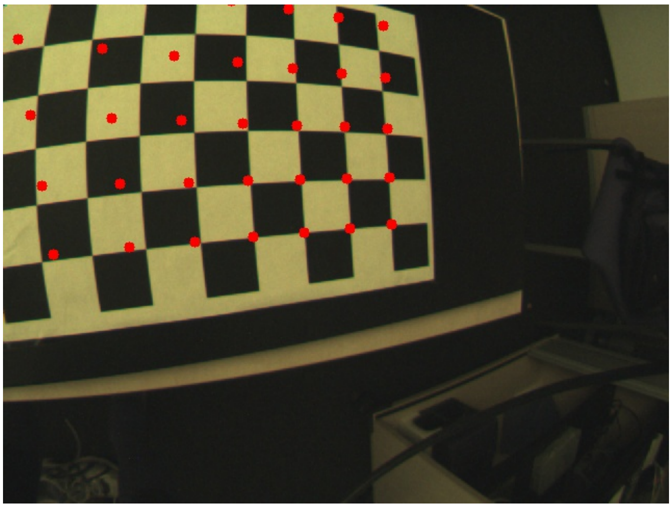

Figure 3: Projected points without correction of the distortion

- projected points with correction of the radial distortion (k1, k2 and k5) in green

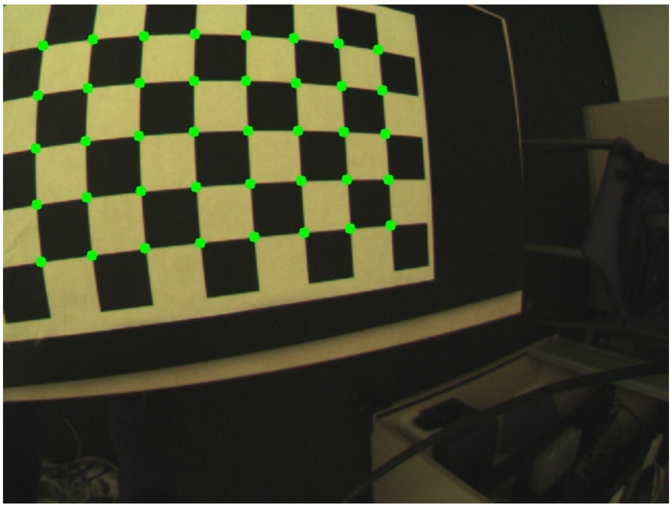

Figure 4: Projected points with correction of the radial distortion (k1, k2 and k5)

References

[1] Stricker D. Prof. *Camera Model and Calibration, 3D Computer Vision, Kaiserslautern University.*