Fluids

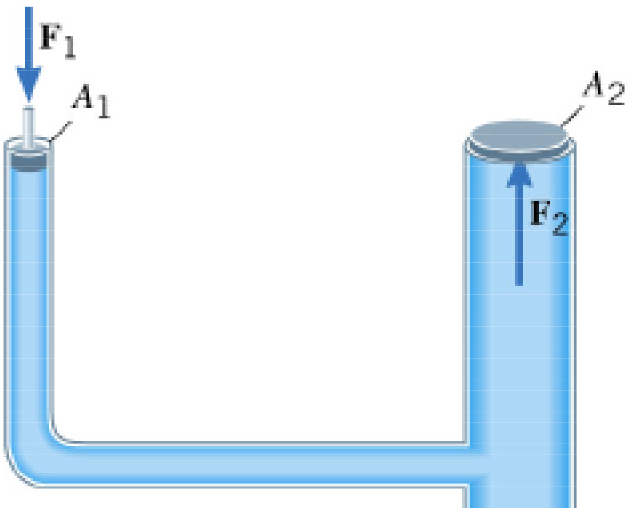
Fluids are relatively easier to deal with. We need to learn or understand 3 guiding definitions 2 of which you are already familiar with:

- Mass Density $\rho = \frac{mass}{Volume} \quad \frac{kg}{m^3}, \frac{gm}{cm^3}$
- Pressure $P = \frac{Force}{Area} \quad \frac{Nt}{m^2}, pascal, bar$
- Flow $Q = Area \cdot velocity \quad \frac{m^3}{s}$

The pressure of a fluid at any depth is simply equal to the weight of the liquid above that point. ***It doesn't matter how wide the channel is. Only the depth and density.***

$$P = \rho\, g\, h$$

The first principal is Pascal's law which states that pressure in any closed chamber is felt everywhere the same. This presumes that what ever you have done to change the pressure has stopped and the system is static or unchanging. (I want to distinguish this from the dynamic system we will be talking about later.)



If I push on piston A_1 with a force F_1 then I create a new pressure P felt everywhere through out the system. The force on the other piston is simply this pressure times the area that the pressure acts on.

$$\frac{F_1}{A_1} = P = \frac{F_2}{A_2}$$

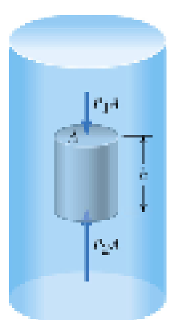
$$F_2 = A_2 \frac{F_1}{A_1}$$

The next rule is Archemedes' Principal. This is simply a statement that the bouyant force is equal to the weight of the displaced fluid. Keep in mind that we need to account for the relative changes in density between two materials.

$$F_B = P_2 A - P_1 A$$

$$P(y) = \rho\, g\, y$$

$$F_B = \rho\, g\, V$$



The shape doesn't matter, it is simply the weight of the displaced fluid.
A sphere with a radius of 0.3 m would have a volume of $\frac{4}{3}\pi(0.3m)^3 = 0.1m^3$
This implies that it would displace $\rho_{H_2O} V_{sphere} = 113\ kg$ *of water which weighs* 1100 Nt.
Assuming that the weight of the ball is negligable then this is, in fact, the Bouyant force.

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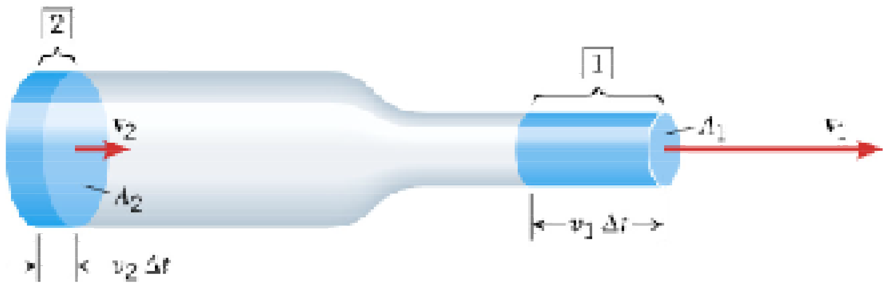
The last two ideas refer to systems that are changing. This is fluid *dynamics* as opposed to statics indicated above.

The first is the Continuity relation:



$$mass\ flow\ rate = \rho Q = \rho A v$$

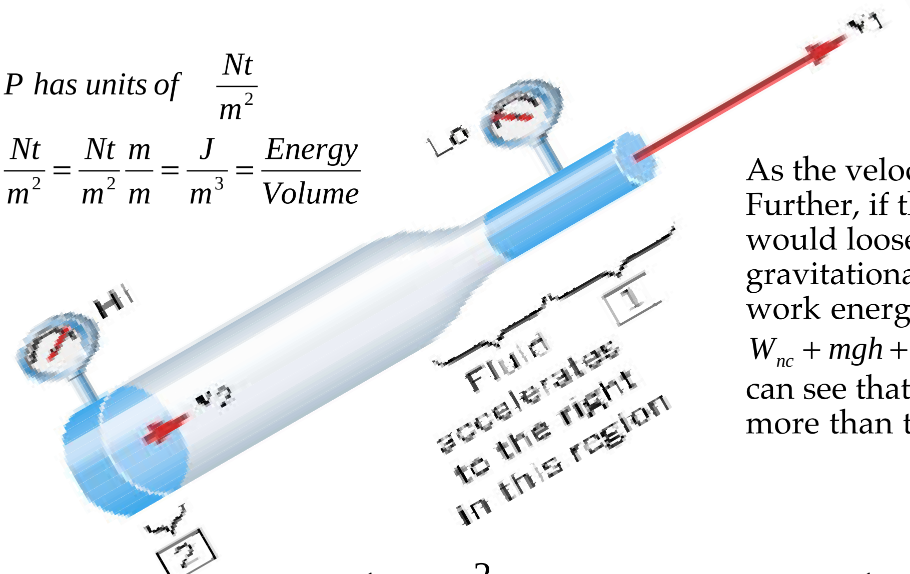
$$continuity: \quad \rho Q_1 = \rho Q_2$$

All this says is that if a tube gets smaller then the velocity of a fluid has to increase inversely proportional to area of the tube it flows through. Smaller diameter tube means faster flow because more stuff has to go through in a unit time. If you have watched a stream that narrows you have witnessed this phenomenon. Next time squeeze the end of a hoe and note how the velocity of the spray increases.

The final equation is the grand mother of them all. Bernoulli's equation which is, really, nothing more than the conservation of energy in a new cloak.

P has units of $\frac{Nt}{m^2}$

$$\frac{Nt}{m^2} = \frac{Nt}{m^2}\frac{m}{m} = \frac{J}{m^3} = \frac{Energy}{Volume}$$



As the velocity increases the pressure drops. Further, if the tube were to travel up a hill we would loose pressure because of the change in gravitational potential. By examining the simple work energy relation $W_{nc} + mgh + \frac{1}{2}mv^2 = E$ and divide by the volume we can see that the "internal" energy density is nothing more than the pressure.

$$P_1 + \tfrac{1}{2}\rho {v_1}^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho {v_2}^2 + \rho g y_2$$

So, if you know the change in the diameter of a pipe and the "head" or distance it rises or falls, you can calculate the change in pressure. Just writeout what you know and solve for what you don't.

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A force of 250 N is applied to a hydraulic jack piston that is 0.01 m in diameter. If the piston which supports the load has a diameter of 0.10 m, approximately how much mass can be lifted by the Ignore any difference in height between the pistons.

Ans: 2550 kg

A balloon inflated with helium gas (density = 0.2 kg/m^3) has a volume of 6×10^{-3} m^3. If the density of air is 1.3 kg/m^3, what is the buoyant force exerted on the balloon?

Ans: 0.08 N

An object weighs 15 N in air and 13 N when submerged in water. Determine the density of the object.

Ans: 7.5×10^3 kg/m^3

The density of the liquid flowing through the horizontal pipe in the drawing is 1500 kg/m^3. The speed of the fluid at point A is 5.5 m/s while at point B it is 8.0 m/s. What is the difference in pressure, $P_B - P_A$, between points B and A?

Ans: -2.5×10^4 Pa