Rotational Kinematics

- I know how much you **loved** linear kinematics.
- We can do the same thing with objects that rotate.

Remember?

- Linear Kinematic Equations

$$\Delta s = v\ t + \tfrac{1}{2}\, a\, t^2$$

$$v_f = v_i + a\, t$$

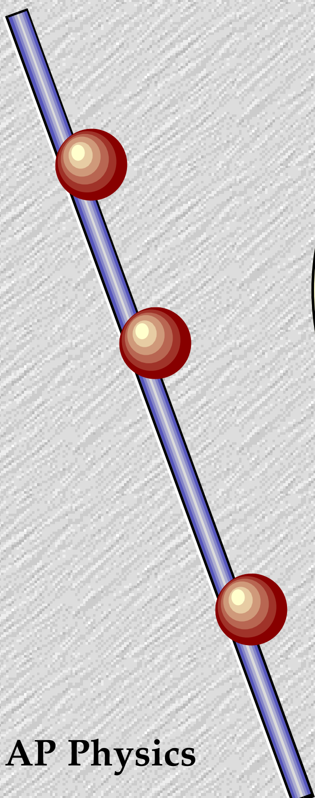
$$2\, a\, \Delta s = v_f^2 - v_i^2$$

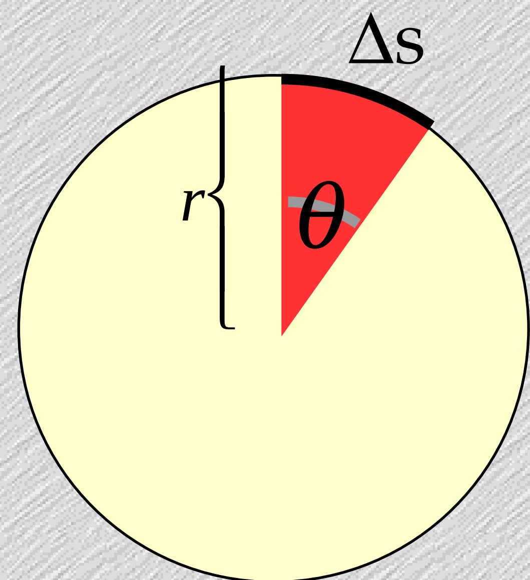
New Guys in Town

- Rotational Vocabulary

Connection between linear and rotation	
Rotational Motion	**Linear Motion**
θ (*theta*) rad	Δs (*delta ess*) m
ω (*omega*) rad/sec	v (*vee*) m/s
α (*alpha*) rad/sec^2	a (*aye*) m/s^2
I (*eye*) kg • m	**m (em) kg**

θ (*Theta*) and how we measure it.





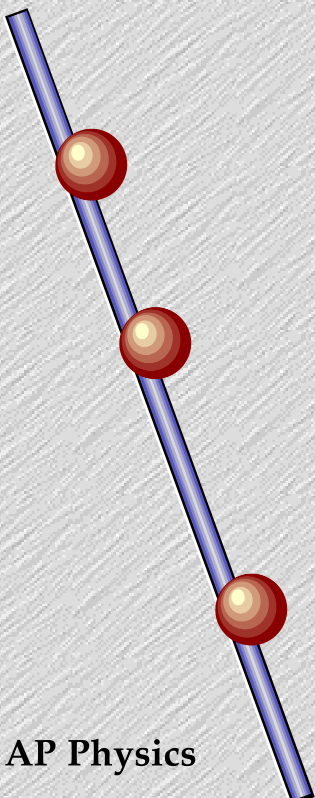
$$r\,\theta = \Delta s$$

$$\Delta s = C = 2\pi\,r$$
$$\Rightarrow\ \theta = 2\pi$$

Conversion:

$$360° = 2\pi$$

ω (*Omega*) - Measure of Speed



- Define Speed for linear systems:

$$v = \frac{\Delta s}{\Delta t}$$

- Define Speed for rotational systems:

$$\omega = \frac{\Delta \theta}{\Delta t}$$

- Common conversion:

$$1\, rpm = 2\pi \frac{1}{60\,\text{sec}} = 0.105\, {}^{rad}\!/_{s}$$

$$1\, {}^{rad}\!/_{s} = 9.55\, rpm$$

α (*Alpha*) - Angular Acceleration

- Define Acceleration for linear systems:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

- Define Acceleration for rotational systems:

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$$

- Example:
  - A wheel's angular rotation changes from 30rpm to 40rpm in 5 sec. What is the acceleration?

$$\frac{40\,rpm - 30\,rpm}{5s} = \frac{10\,rpm}{5s} = 0.2\,1/s^2$$

Kinematic Relationships

- Routine Expressions

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\theta_f - \theta_i = \Delta\theta = \omega\, t + \tfrac{1}{2}\alpha\, t^2$$

$$\omega_f^{\ 2} = \omega_i^{\ 2} + 2\,\alpha\,(\theta_f - \theta_i)$$

Examples

An airplane engine starts from rest; and 2 seconds later, it is rotating with an angular speed of 300 rev/min. If the angular acceleration is constant, how many revolutions does the propeller undergo during this time?

$$300rpm \Big/ 2s = 15.7 \Big/ s^2 = \alpha$$

$$\frac{(300rpm)^2}{2\,\alpha} = \Delta\theta = 5\,(2\pi) \quad or \quad 5\ rev$$

Connection to linear values

$$\boxed{\begin{aligned} \alpha\, r &= a \\ \omega\, r &= v \\ \theta\, r &= \Delta s \end{aligned}}$$

A 10*in* diameter circular saw blade rotates at a constant angular speed of 1100 rpm. What is the tangential speed of the tip of a saw tooth at the edge of the blade?

$$d = 10\,in = 0.25\,m$$

$$r = {}^{d}\!/_{2} = 0.13m$$

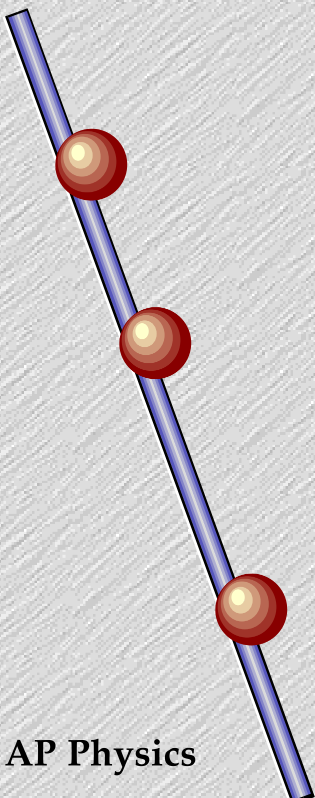
$$1100\,rpm = 115\,{}^{(rad)}\!/_{s}$$

$$r\,\omega = 0.13m\,115\,s^{-1} = 15\,{}^{m}\!/_{s}$$

Rotational and Linear Connection

Connection between linear and rotation		
Rotational Motion		**Linear Motion**
θ (*theta*) rad	$\theta\, r = \Delta s$	Δs (ex) m
ω (*omega*) rad/sec	$\omega\, r = v$	v (vee) m/s
α (*alpha*) rad/sec^2	$\alpha\, r = a$	a (*aye*) m/s^2

Rotational Kinematics



$$\omega = \frac{\theta_f - \theta_i}{t} = \frac{\Delta\theta}{t}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{\Delta\omega}{t}$$

$$\theta_f - \theta_i = \Delta\theta = \omega\, t + \tfrac{1}{2}\alpha\, t^2$$

$$\omega_f^{\;2} = \omega_i^{\;2} + 2\,\alpha\,(\theta_f - \theta_i)$$