Springs & Pendulums

All simple harmonic motion begins with the most elementary of force laws:

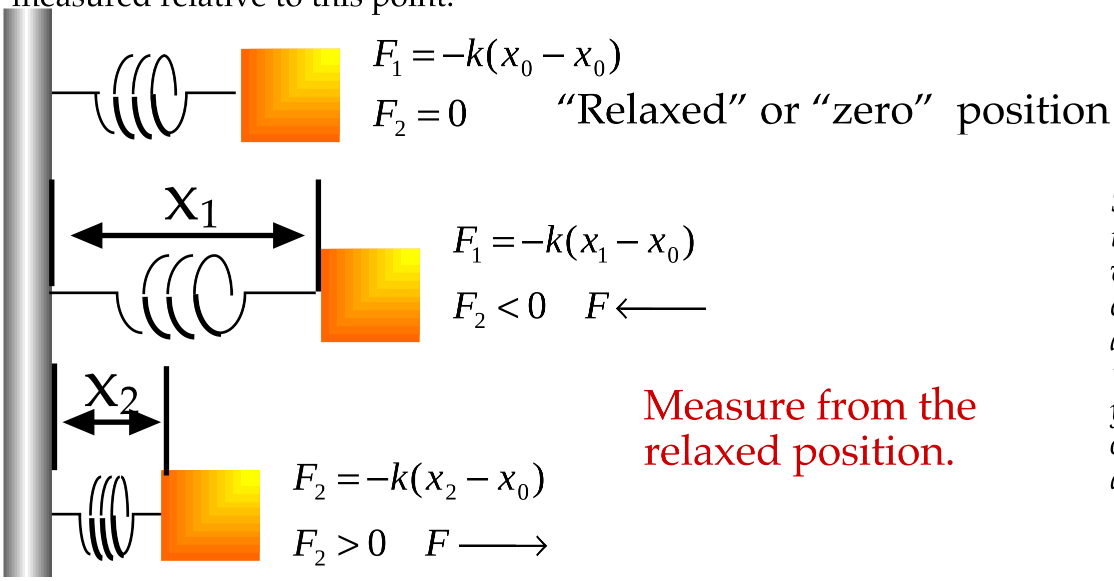
$$F = -k \cdot \Delta x$$

Remembering Newton's Second Law (you o remember the second law, don't you?) we can analyze the motion:

$$ma = -k \cdot \Delta x$$

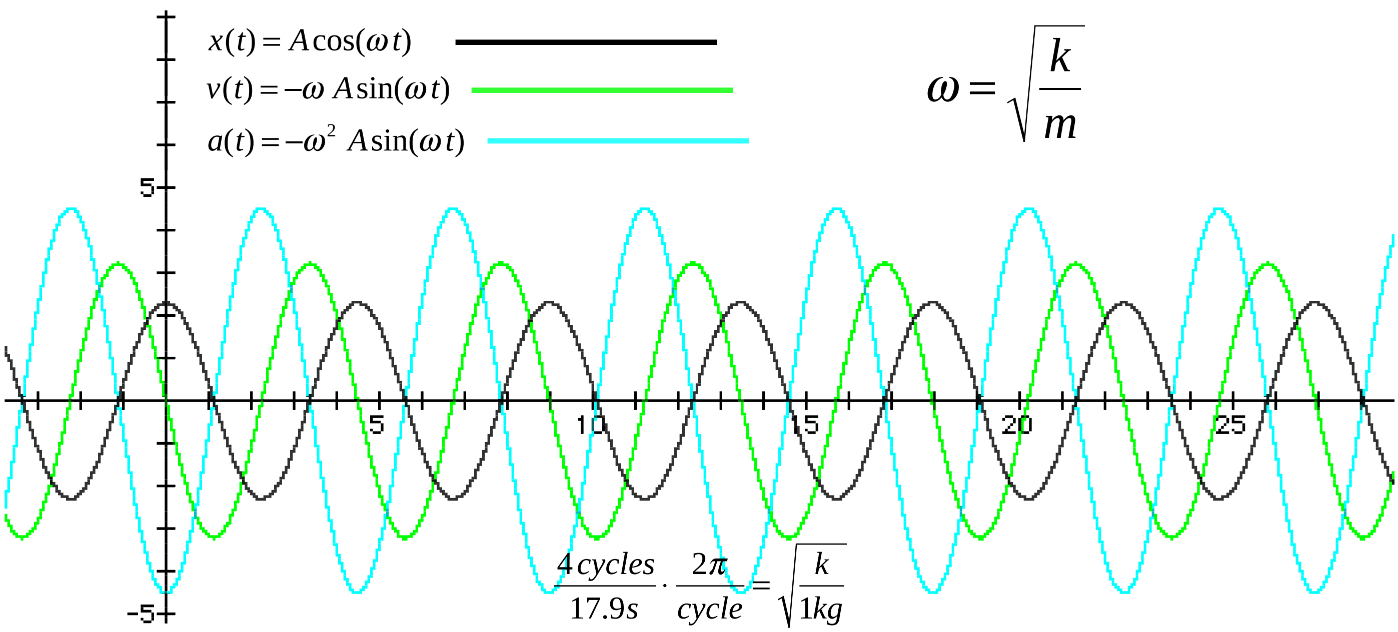
$$m\frac{\Delta v}{\Delta t} = -k \cdot \Delta x \quad m\frac{\Delta^2 x}{\Delta t^2} = -k \cdot \Delta x$$

It is beyond the scope of this course to solve this difference equation suffice it to say that this is solvable. The solution is a simple sinusoid. We usually start by assuming that at time t=0 the object is at its zero point. The zero point is the place where a mass attached to a spring will eventually come to rest. All displacements, x, are measured relative to this point.



Springs are characterized by their "spring" constant (k) which is determined by its construction and materials. If a spring is described as a "Hooke's Law" spring then you are being told that it obeys the simple force law described above.

Given all these ideas and concepts you would see the motion described as follows. Assuming that a mass is attached to a spring and drawn away from the zero point by an amount of 2.3 cm you could expect to see the following traces of the object as a function of time.



using the above graph we can figure out the spring constant if we knew the mass of the object attached to the spring. Lets assume it is 1 kg. The frequency of any "Hooke's Law" spring is given above we can figure that the spring constant is roughly 2 Nt/m

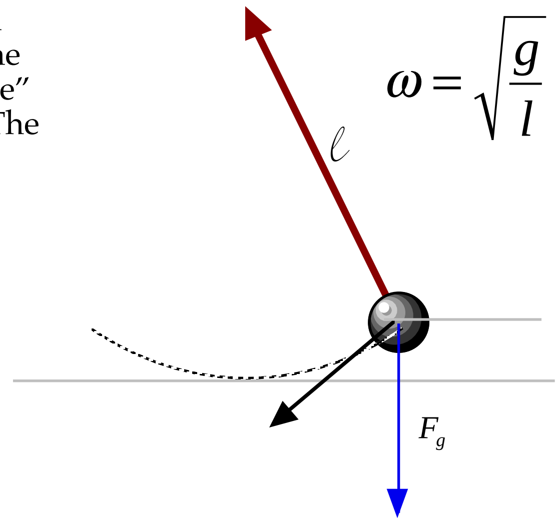
Springs & Pendulums

Pendulums are similar to springs but their restoring force is due to gravity. Their frequency is dependent only on the length of the suspending string and the acceleration due to gravity.

The result of the force of gravity on the plumb bob and the tension in the string gives rise to a "restoring force" that looks different than a spring. The restoring force is $F = -m\,g\,\sin(\theta)$

For small θ the sin(θ) becomes θ so the formula looks like the more familiar form shown above:

$$F = -m\,g\,\theta$$



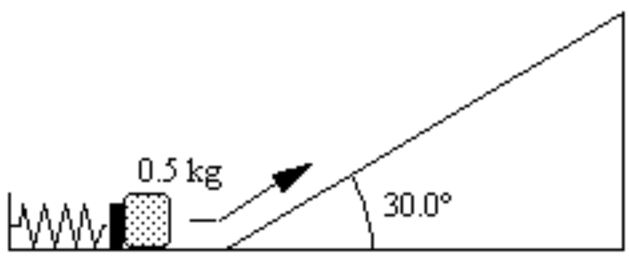
In summary…

	Springs	Pendulums
Force Law	$F = -k \cdot \Delta x$	$F = -m\,g\,\theta$
Frequency	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{l}}$
Amplitude	$A = \Delta x$	$A = \ell \Delta\theta$
Time Function	$x(t) = A\cos(\omega t)$ $v(t) = -\omega\,A\sin(\omega t)$ $a(t) = -\omega^2\,A\sin(\omega t)$	$s(t) = A\cos(\omega t)$ $v(t) = -\omega\,A\sin(\omega t)$ $a(t) = -\omega^2\,A\sin(\omega t)$
Maximum Speed	$\omega\,A$	$\omega\,A$
Maximum Acceleration	$\omega^2\,A$	$\omega^2\,A$
Energy	$E_s = \frac{1}{2}k\,\Delta x^2$	$E_p = mgh$ $= mg\ell\sin(\theta)\Delta\theta \approx mgl\Delta\theta^2$

Springs & Pendulums

A spring with constant k = 40.0 N/m is at the base of a frictionless, 30.0° inclined plane. A 0.50 kg block is pressed against the spring, compressing it 0.20 m from its equilibrium position. The block is then released. If the block is not attached to the spring, how far along the incline will it travel before it stops?

Ans: 0.32 m



An iron ball hangs from a 24-m steel cable and is used in the demolition of a building at a location where the acceleration due to gravity is 9.9 m/s^2. The ball is swung outward from its equilibrium position for a distance of 4.5 m. Assuming the system behaves as a simple pendulum, find the maximum speed of the ball during its swing.

Ans. 2.9 m/s

The acceleration of a certain simple harmonic oscillator is given by a = —(15.8 m/s^2) cos (2.51t). What is the amplitude of the simple harmonic motion?

Ans: 2.51 m