Simple Harmonic Motion

- Things that wiggle.
- Things that wiggle with a definable force.

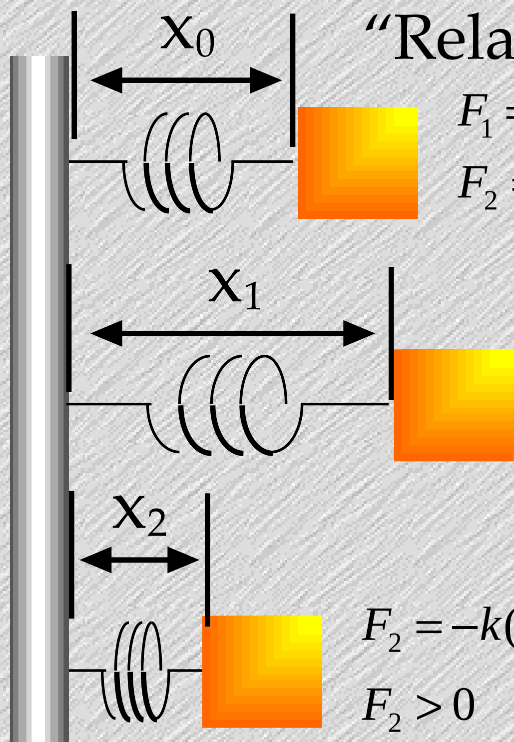
$$F = -k \cdot x$$

$$ma = -k \cdot x$$

$$m\frac{\Delta v}{\Delta t} = -k \cdot x \quad m\frac{\Delta^2 x}{\Delta t^2} = -k \cdot x$$

$F = -k \cdot x$ Pulling on a Spring

"Relaxed" or "zero" position



$F_1 = -k(x_0 - x_0)$

$F_2 = 0$

$F_1 = -k(x_1 - x_1)$

$F_2 < 0 \quad F \longleftarrow$

Measure from the relaxed position.

$F_2 = -k(x_2 - x_0)$

$F_2 > 0 \quad F \longrightarrow$

How They Wiggle

$$F = -k \cdot x$$

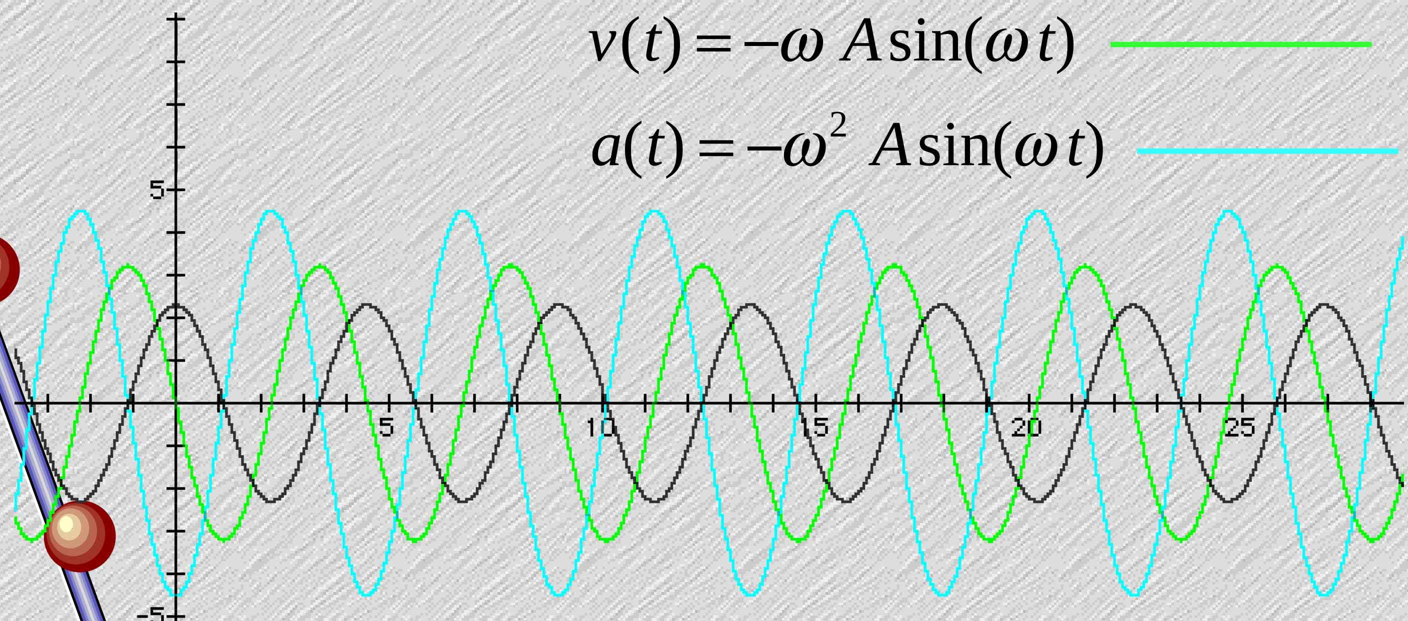
$$ma = -k \cdot x$$

$$m\frac{\Delta^2 x}{\Delta t^2} = -k \cdot x$$

$$x(t) = A\cos(\omega t)$$

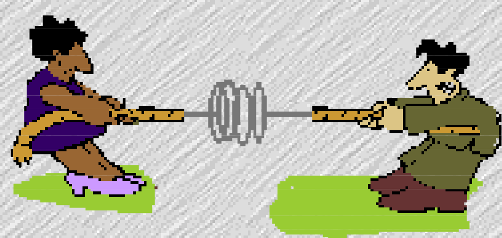
$$v(t) = -\omega A\sin(\omega t)$$

$$a(t) = -\omega^2 A\sin(\omega t)$$



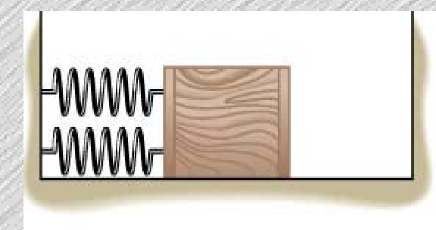
Question

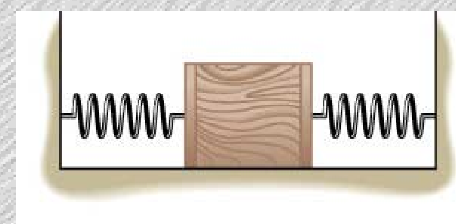
- Two people pull on a spring attached to a wall. Then they detach the spring from the wall and pull on it from opposite ends with the same force.



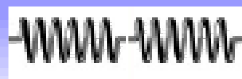
Question

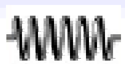
- Consider the following:





- If I were to pull the masses and let go which box experiences the greater net force?

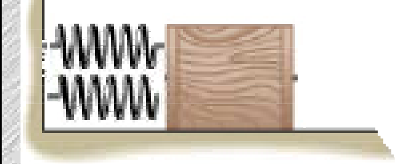
 k

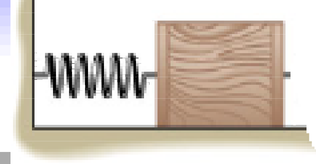
 2 k

Spring **K**onstants

- If I cut a spring in half I double its spring constant.
  - The argument goes something like this: When I cut the spring in half, the same force will only pull the spring (Δx) 1/2 as much. The ratio of F to Δx is therefore doubled, the spring constant is doubled.

k x 2 = 2 k

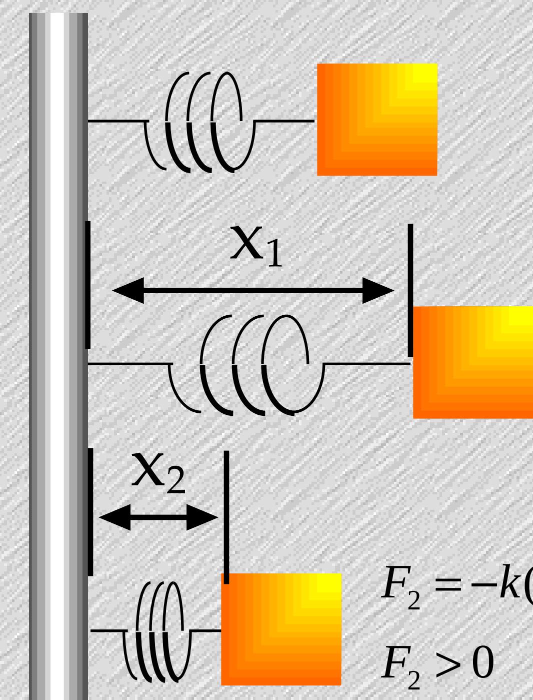




Spring **K**onstants

- If I double the number of springs connected to a mass, I effectively double the spring constant.
  - This is rationalized this way: lets hang the mass from a ceiling with 2 identical springs of constant *k*. The force pulling on the object is thus distributed over 2 springs. Each spring sees 1/2 the force for a given total displacement. This is equivalent to 1 spring with 2*k*.

$U_s = -\frac{1}{2} k\, x^2$ Energy



$F_1 = -k(x_1 - x_0)$

$F_2 < 0 \quad F \longleftarrow$

$F_2 = -k(x_2 - x_0)$

$F_2 > 0 \quad F \longrightarrow$

Measure from the relaxed position.

Resonance

- Things that wiggle when driven by a another wiggle.

$$F = -k \cdot x + a\cos(\omega_0 t)$$

$$ma = -k \cdot x + a\cos(\omega_0 t)$$

$$m\frac{\Delta v}{\Delta t} = -k \cdot x + a\cos(\omega_0 t)$$

$$m\frac{\Delta^2 x}{\Delta t^2} = -k \cdot x + a\cos(\omega_0 t)$$

Solution is complicated but the effect is clear. Things get wild when $\omega_0 = \omega$