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Assignment 7

Algorithm Analysis, and Recursion

Date Due: July 20, 2018, 10pm **Total Marks: ??**

General Instructions

- **This assignment is individual work.** You may discuss questions and problems with anyone, but the work you hand in for this assignment must be your own work.
- **Assignments are being checked for plagiarism.** We are using state-of-the-art software to compare every pair of student submissions.
- Each question indicates what to hand in. You must give your document the name we prescribe for each question, usually in the form aNqM, meaning Assignment N, Question M.
- Make sure your name and student number appear at the top of every document you hand in. These conventions assist the markers in their work. Failure to follow these conventions will result in needless effort by the markers, and a deduction of grades for you.
- Do not submit folders, or zip files, even if you think it will help.
- Programs must be written in Python 3.
- **Assignments must be submitted to Moodle.** There is a link on the course webpage that shows you how to do this.
- **Moodle will not let you submit work after the assignment deadline.** It is advisable to hand in each answer that you are happy with as you go. You can always revise and resubmit as many times as you like before the deadline; only your most recent submission will be graded.
- Read the purpose of each question. Read the Evaluation section of each question.

Questions 1-4

Questions 1-4 are written questions, and you are asked to submit a single document containing the answers to all 4 questions in a file called `a7.txt`. You can submit a text file, as indicated, but PDF or RTF formats are acceptable. The rule of thumb is that you use a common file format. If the marker cannot open your file, you will get no marks.

Question 0 (5 points):

Purpose: To force the use of Version Control in Assignment 7

Degree of Difficulty: Easy

You are expected to practice using Version Control for Assignment 7. This is a tool that you need to be required to use, even when you don't need it, so that when you do need it, you are already familiar with it. Do the following steps.

1. Create a new PyCharm project for Assignment 7.
2. Use `Enable Version Control Integration...` to initialize Git for your project.
3. Download the Python and text files provided for you with the Assignment, and add them to your project.
4. Before you do any coding or start any other questions, make an initial commit.
5. As you work on each question, use Version Control frequently at various times when you have implemented an initial design, fixed a bug, completed a question, or want to try something different. Make your most professional attempt to use the software appropriately.
6. When you are finished your assignment, open the terminal in your Assignment 6 project folder, and enter the command: `git --no-pager log` (double dash before the word 'no'). The easiest way to do this is to use PyCharm, locate PyCharm's `Terminal` panel at the bottom of the PyCharm window, and type your command-line work there.

   Note: You might have trouble with this if you are using Windows. Hopefully you are using the department's network filesystem to store your files. If so, you can log into a non-Windows computer (Linux or Mac) and do this. Just open a command-line, `cd` to your A6 folder, and run `git --no-pager log` there. If you did all your work in this folder, git will be able to see it even if you did your work on Windows. Git's information is out of sight, but in your folder.

   Note: If you are working at home on Windows, Google for how to make git available on your command-line window. You basically have to tell the command-line app where the git app is.

You may need to work in the lab for this; Git is installed there.

What to Hand In

After completing and submitting your work for Questions 5-7, open a command-line window in your Assignment 7 project folder. Run the following command in the terminal: `git --no-pager log` (double dash before the word 'no'). Git will output the full contents of your interactions with Git in the console. Copy/-paste this into a text file named `a7-git.log`.

If you are working on several different computers, you may copy/paste output from all of them, and submit them as a single file. It's not the way to use git, but it is the way students work on assignments.

Be sure to include your name, NSID, student number, course number and laboratory section at the top of all documents.

Evaluation

- 5 marks: The log file shows that you used Git as part of your work for Assignment 7. For full marks, your log file contains
  - Meaningful commit messages.
  - At least two commits per programming question for a total of at least 6 commits.

Question 1 (5 points):

Purpose: To practice using the big-O notation.

Degree of Difficulty: Easy

Suppose you had 5 algorithms, and you analyzed each one to determine the number of steps it required, and expressed the number of steps as a function of a size parameter, as follows:

1. $f_1(n) = n^{32} + 2^n$
2. $f_2(n) = \frac{n^4}{950} + 8052n\log(n)$
3. $f_3(n) = 50000n^2 + 2n!$
4. $f_4(n) = 3\log(n) + \frac{2n^2(n-1)}{2} + 4n^3\log(n)$
5. $f_5(n) = 2n^4 + n^{4.123}$ + 12n

For each of the given functions, express it using big-O. For example, if $f(n) = 17n^4 + 42$ then we would write $f(n) = O(n^4)$; you could also write $f(n) \in O(n^4)$ showing that the function is in the category $O(n^4)$.

Justifications for your answers are not necessary. Just apply the rules and state the answers.

What to hand in

Include your answers in a file called `a7.txt`, though PDF and RTF files are acceptable. Clearly identify the question number and each part. If you are submitting a text file, you can write exponents such as n^2 like this: `n^2`.

Evaluation

1 mark for every correct answer. Answers that do not use the big-O notation will not be considered correct.

Question 2 (4 points):

Purpose: To practice analyzing algorithms to assess their run time complexity.

Degree of Difficulty: Easy

Analyze the following pseudo-code, and answer the questions below. Each question asks you to analyze the code under the assumption of a cost for the function doSomething(). For these questions you don't need to provide a justification.

1. Consider the following loop:

```
i = 0
while i < n:
    doSomething(...)   # see below!
    i = i + 1
```

   What is the worst case time complexity (Big-O) of this loop if `doSomething()` is a function requiring $O(1)$ steps?

2. Consider the following loop:

```
i = 0
while i < n:
    doSomething(...)   # see below!
    i = i + 2
```

   What is the worst case time complexity (Big-O) of this loop if `doSomething()` is a function requiring $O(n)$ steps?

3. Consider the following loop:

```
i = 1
while i < n:
    doSomething(...)   # see below!
    i = i * 2
```

   What is the worst case time complexity (Big-O) of this loop if `doSomething()` is a function requiring $O(m)$ steps? Note: treat m and n as independent input-size parameters.

4. Consider the following loop:

```
i = n
while i > 0:
    doSomething(...)   # see below!
    i = i - 1
```

   What is the worst case time complexity (Big-O) of this loop if `doSomething()` is a function requiring $O(m!)$ steps? Note: treat m and n as independent input-size parameters.

What to hand in

Include your answer in the `a7.txt` document. Clearly mark your work using the question number. If you are submitting a text file, you can write exponents such as n^2 like this: `n^2`.

Evaluation

- 1 mark for each correct result using big-O notation. No justification needed.

Question 3 (9 points):

Purpose: To practice analyzing algorithms to assess their run time complexity.

Degree of Difficulty: Easy

(a) (3 points) Analyze the following Python script, and determine the worst-case time complexity. Justify your work with a brief explanation.

```
for i in range(n):
    j = 0
    while j < n:
        print(j - i)
        j = j + 1
```

(b) (3 points) Analyze the following Python script, and determine the worst-case time complexity. Justify your work with a brief explanation.

```
for i in range(len(alist)):
    j = 0
    while j < i:
        alist[j] = alist[j] - alist[i]
        j = j + 1
```

(c) (3 points) Analyze the following Python script, and determine the worst-case time complexity. Justify your work with a brief explanation.

```
1 for i in range(len(alist)):
2     j = 1
3     while j < len(alist):
4         alist[j] = alist[j] - alist[i]
5         j = j * 2
```

What to hand in

Include your answer in the `a7.txt` document. Clearly identify your work using the question number. If you are submitting a text file, you can write exponents such as n^2 like this: `n^2`.

Evaluation

- 1 marks for each correct result using big-O notation;
- 2 marks for each correct justification.

Question 4 (6 points):

Purpose: To practice analyzing algorithms to assess their run time complexity.

Degree of Difficulty: Easy

Phoenix Wright, ace attorney, has just setup a new computer filing system to keep track of his court cases. He had his friend Larry Butz write the code (below) for the filing system. Larry claims his code is super fast, and runs in O(1) time. Is this statement at all correct? What about in the worst CASE? Best CASE? (no need for average case). JUSTICE-ify your answer. Carefully read the evaluation criteria of the question.

```
1  def file_case(case, closed_cases):
2      """
3      Purpose:
4          Checks whether the given case has been closed,
5          and if so puts it into the closed_cases.
6          If the case has not yet been closed, a search
7          is done to return any previously closed and related cases.
8      Pre:
9          case: a dictionary containing 3 fields -
10             status = a string. Either "Open" or "Closed"
11             keywords = list of strings. May match tags of other cases.
12                 Can be any arbitrary length.
13             crime = string. What the single alleged crime is.
14                 E.g. "murder", "burglary", etc.
15         closed_cases: list of cases (see case above)
16     Post: case is appended to closed_cases is if the status
17         of case is "Closed"
18     Return:  Empty list if case's status was "Closed",
19         otherwise a list of related cases
20     """
21     if case["status"] == "Closed":
22         # Case is closed. Just add it to our list of closed cases
23         closed_cases.append(case)
24         return []
25     # Case is not closed, do a search to return any related closed cases
26     related_cases = []
27     for search_term in case["keywords"]:
28         for closed_case in closed_cases:
29             if search_term == closed_case["crime"]:
30                 related_cases.append(closed_case)
31     return related_cases
```

What to hand in

Include your answer in the `a7.txt` document. Clearly identify your work using the question number.

Evaluation

- 2 marks: You correctly identified the best and worst case in Big-O, and whether Larry sucks.
- 2 mark: You identified any size input size parameters.
- 2 marks: Your JUSTICE-ifications and analysis are correct (accusing people and OBJECTIONS! are encouraged).

Question 5 (8 points):

Purpose: To practice simple recursion on integers.

Degree of Difficulty: Easy

(a) (2 points) The Fibonacci sequence is a well-known sequence of integers that follows a pattern that can be seen to occur in many different areas of nature. The sequence looks like

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots.$$

That is, the sequence starts with 0 followed by 1, and then every number to follow is the sum of the previous two numbers. The Fibonacci numbers can be expressed as a mathematical function as follows:

$$f(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ f(n-1) + f(n-2) & \text{if } n > 1 \end{cases}$$

Translate this function into Python, and test it. The function must be recursive.
Your function should only accept a non-negative n integer as input, and return the nth Fibonacci number. No other parameters are allowed. It should not display anything.

(b) (2 points) The Moosonacci sequence is a less well-known sequence of integers that follows a pattern that is rarely seen to occur in nature. The sequence looks like this:

$$0, 1, 2, 3, 6, 11, 20, 37, 68, 125 \ldots$$

That is, the sequence starts with 0 followed by 1, and then 2; then every number to follow is the sum of the previous *three* numbers. For example:

- $m(3) = 3 = 0 + 1 + 2$
- $m(4) = 6 = 1 + 2 + 3$
- $m(5) = 11 = 2 + 3 + 6$

Write a recursive Python function to calculate the nth number in the Moosonacci sequence. As with the Fibonacci sequence, we'll start the sequence with $m(0) = 0$.
Your function should only accept a non-negative n integer as input, and return the nth Moosonacci number. No other parameters are allowed. It should not display anything.

(c) (4 points) Design a recursive Python function named `substr` that takes as input a string s, a target character c, and a replacement character r, that returns a new string with every occurrence of the character t replaced by the character r. For example:

```
>>> substr('l', 'x', 'Hello, world!')
'Hexxo, worxd!'
>>> substr('o', 'i', 'Hello, world!')
'Helli, wirld!'
>>> substr('z', 'q', 'Hello, world!')
'Hello, world!'
```

If the target does not appear in the string, the returned string is identical to the original string.
Addendum: Your function should accept two single character strings and an arbitrary string as input, and return a new string with the substitutions made. It should not display anything.

Non-credit activities

You should of course test your functions before you submit. There is no credit for testing in this question, so the issue is to test enough that you are confident without wasting your time. Use the debugger and step through these functions for a few small values of n (try a base case and a non-base case). Try to identify the stack frames in the debugging window, and notice how each function call gets its own set of variables.

What to Hand In

A file called `a7q5.py` containing:

- Your recursive functions.

Be sure to include your name, NSID, student number, course number and laboratory section at the top of all documents.

Evaluation

This is just a warm up, and the functions are very simple.

- 2 marks: Fibonacci function. Full marks if it is recursive, zero marks otherwise.
- 2 marks: Moosonacci function. Full marks if it is recursive, zero marks otherwise.
- 4 marks: `subst` function. Full marks if it is recursive, and if it works, zero marks otherwise.

Question 6 (18 points):

Purpose: Practical recursion using the Node ADT.

Degree of Difficulty: Moderate

Below are three recursive functions that work on node-chains (**not Linked Lists**), described below. You MUST implement them using the node ADT, and you MUST use recursion. We will impose very strict rules on implementing these functions which will hopefully show you new ways to think about recursion. When adding new parameters, consider making them optional. Keep in mind that the node ADT is recursively designed, since the `next` field refers to another node-chain (possibly empty).

You will implement the following 3 functions:

(a) (6 points) `average(chain, ...)`: Assuming the given node-chain only contains numbers, return the average/mean of all data values in the chain. For a completely empty chain, the average() should return 0. For this question you are not allowed to use any data collections (lists, stacks, queues). Instead, you will recursively pass any needed information using PARAMETERS (notice the ... in `average(chain, ...)`). You may add as many parameters as you like, so long as they are not data collections.

To test this function, create the following test cases:

- An empty chain.
- A chain with one node.
- A chain with several nodes.

(b) (6 points) `reverse(chain, ...)`: The order of the data values in the node-chain is reversed. Your function should reverse the node chain, and return the reference to the first node in the chain. For this question you are not allowed to use any data collections (lists, stacks, queues). Instead, you will recursively pass any needed information using PARAMETERS (notice the ... in `reverse(chain, ...)`). You may add as many parameters as you like, so long as they are not data collections.

To test this function, create the following test cases:

- An empty chain.
- A chain with one node.
- A chain with several nodes.

(c) (6 points) `copy(chain)`: A new node-chain is created, with the same values, in the same order, but it's a separate distinct chain. Adding or removing something from the copy must not affect the original chain. Your function should copy the node chain, and return the reference to the first node in the new chain. For this one, DO NOT ADD ANY EXTRA PARAMETERS. None are needed.

To test this function, create the following test cases:

- An empty chain.
- A chain with one node.
- A chain with one several nodes.

Be sure to check that you have two distinct chains with the same values!

What to Hand In

A file called `a7q6.py` containing:

- Your recursive functions for `average(chain, ...)`, `reverse(chain, ...)`, `copy(chain)`.
- A test-script, including the cases above, and any other tests you consider important.

Be sure to include your name, NSID, student number, course number and laboratory section at the top of all documents.

Evaluation

- (3 marks each) Your `subst(chain, t, r)`, `reverse(chain)`, `copy(chain)` functions are recursive, and correctly implement the intended behaviour.
- (3 marks each) You implemented the test cases given above.

Question 7 (15 points):

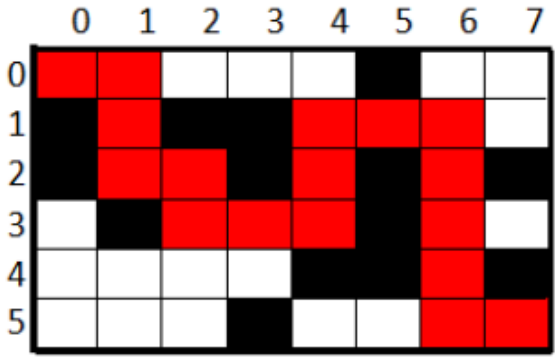
Purpose:

Degree of Difficulty: Moderate to Tricky

Problem:

A maze is a rectangular arrangement of blocks (walls) and open spaces (paths). A maze may have a path through it from a given start location to a given end location. You can usually move only one cell at a time. From each cell you choose your next move. Moves can only be into unblocked cells. In a restricted maze (which we are describing here) you can not cross your existing path: i.e., once you visit a particular cell, you can't visit it again (for all intents and purposes, a visited cell is blocked to future attempts to move).

For example, this maze:



Has a path: `(0,0),(0,1)(1,1)(2,1)(2,2)(3,2)(3,3)(3,4)(2,4)(1,4)(1,5)(1,6)(2,6)(3,6)(4,6)(5,6)(5,7)` from start=`(0,0)` to goal=(5,7).

Solution:

Implement a Python function `MazeSolver(m,s,g)` to determine if a path exists within the maze, `m`, from the start location `s` to the end location `g`. The function must be RECURSIVE. Some constraints on your solution:

- Your maze is to be represented as a Python list of lists (recall the magic square and sudoku requirements from an earlier assignment).
- Start and goal locations are to be represented as Python tuples `(x,y)` where `x` and `y` represent the index values into the list of the cell location.
- Input to your application will be a file of text containing `'0'` for an open cell and `'1'` for a blocked cell. For example the above maze will be in a file as:

```
0 0 0 0 0 1 0 0
1 0 1 1 0 0 0 0
1 0 0 1 0 1 0 1
0 1 0 0 0 1 0 0
0 0 0 0 1 1 0 1
0 0 1 0 0 0 0 0
```

- Your output must include:
  - The maze with the path showing if a path exists (mark the path with `'P'`)

- Your function returns the boolean value `True` if a path exists, `False` otherwise

The recursive definition of the `MazeSolver()` is given by:

$$MazeSolver(m_0, s_0, g)$$
$$= \begin{cases} T, output\ maze & if\ current\ location\ is\ available\ and = g \\ F & if\ current\ location\ is\ unavailable\ or\ unreachable \\ MazeSolver(m_i, s_i, g) & where\ m_i\ is\ the\ maze\ with\ current\ cell\ blocked\ and\ s_i\ is\ a\ reachable\ cell\ from\ s_0 \end{cases}$$

(NOTE: The recursive nature of the solution is that each time the path consumes a location, the new maze is a smaller version of the original, i.e. it contains fewer open cells to try.)

Reachable cells are defined as any cell in one of the four cardinal directions (north, west, south, east), except where:

- You can not go west of the left most cell of any row,
- You can not go east of the right most cell of any row,
- You can not go north of the top most cell of any row,
- You can not go south of the bottom most cell of any row,
- You can not occupy a cell already occupied by your current path.

Examples

You are given a series of mazes to solve.

- `maze1.txt` Start (0,3) Goal (4,5)
- `maze2.txt` Start (0,0) Goal (8,9)
- `maze3.txt` Start (3,0) Goal (23,30)

The three mazes provided may or may not have a path from start to goal. You can also create your own for testing purposes.

What to Hand In

- Your code for `MazeSolver()` as `MazeSolver.py`
- Screen capture shots of the output generated by your code when the maze has a path from start to finish.

Be sure to include your name, NSID, student number, course number and laboratory section at the top of all documents.

Evaluation

- 3 marks: Your script correctly reads in the maze data from a text file, and produces a list of lists using the data.
- 3 marks: You're using tuples to represent position, including start and goal.
- 9 marks: Your mazeSolver function is recursive, and correctly determines and displays a path from start to goal, if one exists.

Extra work:

Modify your code so that it will find ALL paths that meet the requirements.