$4b^3 - 6b^2 + 1 = 0$ $\quad 4b^2\left(b-\frac{1}{2}\right) - 4b\left(b-\frac{1}{2}\right) - 2\left(b-\frac{1}{2}\right) = 0$

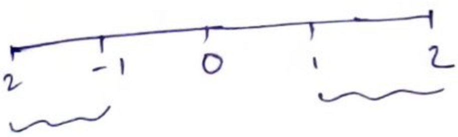
$b = 1/2 \rightarrow$ ans $\quad \Rightarrow \left(b-\frac{1}{2}\right)(4b^2 - 4b - 2) = 0$

$4b^2 - 4b - 2 = 0 \Rightarrow b = \frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}$

Tut - 3a -

1. $f(x) = \begin{cases} \frac{1}{4} & -2 < x < 2 \\ 0 & \text{elsewher} \end{cases}$

$P\{|X| > 1\}$ $\quad \int_{-2}^{-1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$



$= \left[\frac{x}{4}\right]_{-2}^{-1} + \left[\frac{x}{4}\right]_{1}^{2}$

$= \frac{-1}{4} + \frac{2}{4} + \frac{2}{4} - \frac{1}{4} = \frac{1}{2}$

2. $f(x) = \begin{cases} kxe^{x}, & x \le 0 \\ 0, & \text{elsewhere} \end{cases}$ $\quad \int_{-\infty}^{0} f(x)\,dx = 1$

$\int_{-\infty}^{0} kxe^{x}\,dx = 1$

$\Rightarrow k\int_{-\infty}^{0} \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{e^{x}}\,dx = 1$ $\quad \mathrm{I}\int \mathrm{II}\,dx - \int\left(\frac{d}{dx}\mathrm{I} \cdot \int \mathrm{II}\,dx\right)dx$

$\Rightarrow k\left[x\int_{-\infty}^{0} e^{x}\,dx - \int\left(\left(\frac{dx}{dx}\right)\int e^{x}\,dx\right)dx\right] = 1$

$\Rightarrow k\left[xe^{x} - \int(1.\,e^{x}\,dx)\,dx\right] = 1$

$\Rightarrow k\left[x e^{x} - e^{x}\right]_{-\infty}^{0} = 1$

$\Rightarrow k\left[-1 - \left(\frac{-\infty}{e^{\infty}} - \frac{1}{e^{\infty}}\right)\right] = 1$

$\Rightarrow -k = 1 \Rightarrow k = -1.$

3.

x	P(x)	y
-2	0.1	0
-1	$k = 1/15$	-1
0	0.2	0
1	$2k = 2/15$	3
2	0.3	8
3	$3k = 1/5$	15

$6k = 0.~~4~~$

$k = \frac{0.2}{3} = \frac{1}{15}$

y: -1 $\quad 0 \quad 3 \quad 8 \quad 15$

$P(y)$: $\frac{1}{15} \quad 0.3 \quad \frac{2}{15} \quad 0.3 \quad \frac{3}{15}$

$$\text{Mean} = \Sigma y_i p_i = 5.73$$

$$= \cancel{-\frac{1}{15} + \frac{6}{15} + 0.25 + 3} \qquad = -\frac{1}{15} + \frac{6}{15} + 0.24 + 3$$

$$= \cancel{0.33 + 0.25 + 3} \qquad =$$

$$= \cancel{3.58}$$

$$\text{Variance} = E(y^2) - (E(y))^2$$

$$E(y^2) = \Sigma x_i^2 p_i$$

$$= \frac{1}{15} + \frac{18}{15} + 19.2 + \frac{3}{15}(3 \times 15)$$

$$= 65.46 \qquad E(y) = 5.73$$

$$\text{Variance} = 65.46 - 32.8329$$

$$= 32.6331$$

4. $f(x) = kx(2-x)$

$$k\int_0^2 x(2-x)\,dx \Rightarrow k\int_0^2 (2x - x^2)\,dx$$

$$\Rightarrow k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$

$$\Rightarrow k\left[\left(4 - \frac{8}{3}\right) - 0\right]$$

$$\cancel{\Rightarrow \frac{8k}{3} = 1 \quad k = \frac{3}{8}} \qquad \frac{4k}{3} = 1$$

$$k = 3/4$$

Q: $f(x) = \int x\, e^{-x^2/2}$

Tut - 3a

4. $f(x) = kx(2-x)$ $\quad k = \cancel{\tfrac{3}{8}}\ 3/4$

$$\text{Mean} = \int_0^2 x \cdot \frac{3}{4} \cdot x(2-x)\, dx$$

$$= \frac{3}{4} \int_0^2 (2x^2 - x^3)\, dx$$

$$= \frac{3}{4} \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2$$

$$\cancel{= \frac{3}{4}\left(8 - \frac{16}{4}\right) = 8}$$

$$= \frac{3}{4} \left(\frac{16}{3} - \frac{16}{4} \right) = 1$$

$$F(x) = \int_0^x \frac{3}{4} x(2-x)\, dx$$

$$= \frac{3}{4} \left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3} \right]_0^x$$

$$= \frac{3}{4} x^2 - \frac{x^3}{3}$$

6. $f(x) = k(1+x) \qquad 2 < x < 5$

$$P(x<4) = \int_2^4 k(1+x)\,dx$$

$$\int_2^5 k(1+x)\,dx = 1$$

$$\Rightarrow \quad k\left[x + \frac{x^2}{2}\right]_2^5 = 1$$

$$\Rightarrow \quad k\left[5 + \frac{25}{2} - 2 - \frac{4}{2}\right] = 1$$

$$\Rightarrow \quad \frac{27}{2}k = 1 \qquad \Rightarrow \quad k = 2/27$$

$$P(x<4) = \int_2^4 \frac{2}{27}(1+x)\,dx$$

$$= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^4$$

$$= \frac{2}{27}\left[4 + \frac{16}{2} - 2 - \frac{4}{2}\right]$$

$$= \frac{16}{27}$$

7. $f(x) = k x^2 e^{-x} \quad , x \geq 0$

$$\int_0^\infty k x^2 e^{-x} = 1$$

$$\Rightarrow \quad k\int_0^\infty x^2 e^{-x} = 1$$

$$\Rightarrow k\left[x^2\int e^{-x}\,dx-\int\left(\frac{d x^2}{dx}\int e^{-x}\,dx\right)dx\right]=1$$

$$\Rightarrow k\left[\left[-x^2e^{-x}\right]_0^{\infty}-\int 2x\,(-e^{-x})\,dx\right]=1$$

$$\Rightarrow k\left[0+2\int x\,e^{-x}\,dx\right]=1$$

$$\Rightarrow 2k\left[x\int e^{-x}\,dx-\int\left(\frac{dx}{dx}\int e^{-x}\,dx\right)dx\right]=1$$

$$\Rightarrow 2k\left[\left[x\,e^{-x}\right]_0^{\infty}+\int e^{-x}\,dx\right]=1$$

$$\Rightarrow 2k\left[0-\left[e^{-x}\right]_0^{\infty}\right]=1$$

$$\Rightarrow 2k\,(0-1)=1$$

$$\Rightarrow 2k=1 \quad\Rightarrow\quad k=1/2$$

Q: If the pdf of a continuous RV is given by.

$$f(x) = ax, \quad 0 \le x \le 1$$
$$= a, \quad 1 \le x \le 2$$
$$= 3a - ax, \quad 2 \le x \le 3$$
$$= 0, \quad \text{elsewhere}$$

(i) Find a (ii) $P(X > 1.5)$ (iii) Find cdf

~~(i)~~ $\cancel{\int_0^1 ax\,dx = a\left[\frac{x^2}{2}\right]_0^1 = \frac{a}{2}}$

$\cancel{\frac{a}{2} = 1 \Rightarrow a = 2}$

(i) $$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx = 1$$

$$a\left[\frac{x^2}{2}\right]_0^1 + a\left[x\right]_1^2 + \left[3ax - \frac{ax^2}{2}\right]_2^3 = 1$$

$$\frac{a}{2} + a + 3a\left[9a - \frac{9a}{2} - 6a + \frac{4a}{2}\right] = 1$$

$$\frac{3a}{2} + 3a - \frac{5a}{2} = 1$$

$$3a - a = 1$$

$$2a = 1 \Rightarrow a = 1/2$$

(ii) $P(x > 1.5)$

$$\int_{1.5}^{2} a\,dx + \int_{2}^{3} (3a - ax)\,dx$$

$$\Rightarrow a[x]_{1.5}^{2} + \left[3ax - \frac{ax^2}{2}\right]_2^3$$

$$\Rightarrow \frac{a}{2} + 3a - \frac{5a}{2}$$

$$\Rightarrow a = \frac{1}{2}$$

(iii) when $x < 0$ $\quad F(x) = \int_0^0 f(x)\,dx = 0$

when $0 \le x < 1$ $\quad F(x) = \int_0^x ax\,dx = \frac{x^2}{4}$

when $1 \le x < 2$ $\quad \cancel{F(x) = \int_1^x a\,dx + \frac{x^2}{4} = a[x]_1^x + \frac{x^2}{4} = \frac{1}{2}(x-1) + \frac{x^2}{4}}$

$$F(x) = \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$$

$$= a\left[\frac{x^2}{2}\right]_0^1 + a[x]_1^x$$

$$= \frac{1}{4} + \frac{1}{2}(x-1)$$

$$= \frac{x}{2} - \frac{1}{4}$$

when $2 \le x < 3$, $\quad F(x) = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^x f(x)\,dx$

$$= \frac{a}{2} + a + 3ax - \frac{ax^2}{2} - 6a + \frac{4a}{2}$$

$$= -\frac{5a}{2} + 3ax - \frac{ax^2}{2}$$

$$= -\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4}$$