$$\text{Variance} = E(X^2) - (EX)^2$$
$$= 16.8 - (3.66)^2$$
$$= 16.8 - 13.3956$$
$$= 3.4044$$

$$\boxed{\text{Variance} = 3.4044}$$

11-8-25

Two-Dimension Random Variable (2D RV):-

- Let S be the sample space associated with a Random Experiment (E).
- Let X and Y be two functions each assigning a real number to each outcome, then ordered pair X, Y is called two-dimension RV.
- If the possible values of this ordered pair (X, Y) are finite or countably infinite then the (X, Y) is known as <u>2D DRV.</u>
- If (X, Y) can assume all values in a specified region R in (X, Y) plane then these is known as <u>2D CRV.</u>

Probability function of (X, Y)

If (X, Y) is a 2D DRV such that $P(X = x_i, Y = y_j) = P_{ij}$ then P_{ij} is probability function or simply probability mass function, if it satisfied two conditions:-

$$\boxed{\begin{array}{l} (i)\ P_{ij} \geq 0 \ \forall i, j \\ (ii)\ \sum_i \sum_j P_{ij} = 1 \end{array}}$$

Then triplet $\{x_i, y_j, p_{ij}\}$ is known as joint Probability distribution.

Joint Probability Density Function :-

If (X, Y) is a 2D continuous random variable such that $P(x-\frac{1}{2}dx \le X \le x+\frac{1}{2}dx$ and $y-\frac{1}{2}dy \le Y \le y+\frac{1}{2}dy)$

$= f(x,y)\,dx\,dy$

Then this f^n $f(x,y)$ is the <u>pdf</u> of (X, Y). If it satisfied two conditions :-

(i) $f(x,y) \ge 0 \quad \forall\, x, y \in R$

(ii) $\iint_R f(x,y)\,dx\,dy = 1 \qquad R = \text{Region}$

Cummulative Distribution Function (CDF) :-

If (X, Y) is a 2D RV (whether discrete or continuous) then $F(x,y) = P(X \le x$ and $Y \le y)$ is called CDF of RV X, Y.

- For Distrete Random Variable : $F(x,y) = \sum\limits_{y_j \le y} \sum\limits_{x_i \le x} P_{ij}$

- For Continuous Random Variable : $F(x,y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(x,y)\,dx\,dy$

<u>Ques</u> 3 balls are drawn at random w/o replacement from a box containing 2 white balls, 3 red balls and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red ball drawn. Find the joint probability distribution of X, Y.

Totall ~~balls~~ = 9 balls $\begin{cases} 2W \\ 3R \\ 4B \end{cases}$

$\frac{n!}{(n-r)!r!}$

(iii) 1/7

(iv)

$X : 0, 1, 2$

$Y : 0, 1, 2, 3$

	0	1	2	3
0	1/21	3/14	1/7	1/84
1	1/7	2/7	1/14	0
2	1/21	1/28	0	0

(i) $P(X=0 \,\&\, Y=0)$

$$\frac{^4C_3}{^9C_3} = \frac{4!}{1!3!} \times \frac{6!3!}{9!} = \frac{4\times \cancel{3!}}{1\times \cancel{3!}} \times \frac{\cancel{6!}\times 3\times 2\times 1}{9\times 8\times 7\times \cancel{6!}} = \frac{4\times 6}{504} = \frac{\cancel{24}}{\cancel{504}} = \frac{1}{21}$$

(ii) $P(X=0 \,\&\, Y=1)$

$$\frac{^3C_1 \times {}^4C_2}{^9C_3} = \frac{3!}{1!2!} \times \frac{4!}{2!2!} \times \frac{\cancel{6!}\times \cancel{3}\times \cancel{2}\times 1}{\cancel{9}\times \cancel{8}\times 7\times \cancel{6!}} = \frac{3\times \cancel{2!}}{1\times \cancel{2!}} \times \frac{\cancel{4}\times 3\times \cancel{2!}}{\cancel{2!}\times \cancel{2}\times 1} \times \frac{1}{84} = \frac{\cancel{18}}{\cancel{84}} = 3/14$$

(iii) $P(X=0 \,\&\, Y=2)$

$$\frac{^3C_2 \times {}^4C_1}{^9C_3} = \frac{3!}{1!2!} \times \frac{4!}{3!1!} \times \frac{1}{84} = \frac{3\times \cancel{2!}}{\cancel{2!}} \times \frac{4\times \cancel{3!}}{\cancel{3!}} = \frac{\cancel{12}}{\cancel{84}} = \frac{1}{7}$$

(iv) $P(X=0 \,\&\, Y=3)$

$$\frac{^3C_3 \times {}^4C_0}{^9C_3} = \frac{\cancel{3!}}{0!\cancel{3!}} \times \frac{\cancel{4!}}{\cancel{4!}0!} \times \frac{1}{84} = \frac{1}{84}$$

(v) $P(X=1 \,\&\, Y=0)$

$$\frac{^2C_1 \times {}^4C_2}{^9C_3} = \frac{2!}{1!1!} \times \frac{4!}{2!2!} \times \frac{1}{84} = \frac{2\times 1!}{1\times 1} \times \frac{\cancel{4}\times 3\times \cancel{2!}}{\cancel{2!}\times \cancel{2}\times 1} \times \frac{1}{84} = \frac{\cancel{12}}{\cancel{84}} = \frac{1}{7}$$

(vi) $P(X=1 \,\&\, Y=1)$

$$\frac{^2C_1 \times {}^3C_1 \times {}^4C_1}{^9C_3} = \frac{2!}{1!1!} \times \frac{3!}{2!1!} \times \frac{4!}{3!1!} \times \frac{1}{84}$$

$$= \frac{\cancel{2}\times 1}{1} \times \frac{\cancel{3}\times \cancel{2!}}{\cancel{2!}} \times \frac{\cancel{4}\times \cancel{3!}}{\cancel{3!}\times 1} \times \frac{1}{\cancel{84}} = \frac{2}{7}$$

$\cancel{42}$ $\cancel{21}$ 7

(vii) $P(X=1 \,\&\, Y=2)$

$$\frac{{}^2C_1 \times {}^3C_2}{{}^9C_3} = \frac{\cancel{2}^1 \times \cancel{3}^1 \times 1}{\cancel{84}\ \cancel{42}\ 14} = \frac{1}{14}$$

(viii) $P(X=1 \,\&\, Y=3) = 0$ because it is impossible, since ~~2C1 x~~ only 2 white balls exist → all probabilities → 0

(ix) $P(X=2 \,\&\, Y=0)$

$$\frac{{}^2C_2 \times {}^4C_1}{{}^9C_3} = \frac{\cancel{2!}}{\cancel{2!}} \times \frac{\cancel{4}\cancel{2}^1}{\cancel{84}\ \cancel{42}\ 21} = \frac{1}{21}$$

(x) $P(X=2 \,\&\, Y=1)$

$$\frac{{}^2C_2 \times {}^3C_1}{{}^9C_3} = \frac{\cancel{3}^1}{\cancel{84}\ 28} = \frac{1}{28}$$

21/08/25

Marginal Prob. Distribution

$$P(X=x_i) = P(X=x_i, Y=y_1) \text{ or } P(X=x_i, Y=y_2) \text{ or } \ldots$$
$$= P_{i1} + P_{i2} + P_{i3} + \ldots$$
$$= \sum_j P_{ij}$$

is known as MPD of i

Then this $\sum_j P_{ij}$ and denoted as P_{i*}. Similarly $P(Y=y_j) = \sum_i P_{ij}$ and denoted as P_{*j} is known as marginal prob distribution of j.

Conditional Prob. Distribution

$$P(X=x_i / Y=y_j) = \frac{P(X=x_i, Y=y_j)}{P(Y=y_j)} = \frac{P_{ij}}{P_{*j}}$$

This is known Conditional Prob. of X given $Y=y_j$.

→ Conditional Prob of Y given $X=x_i$

$$P(Y=y_j / X=x_i) = \frac{P(Y=y_j, X=x_i)}{P(X=x_i)} = \frac{P_{ij}}{P_{i*}}$$

Ques From the 2D Prob Distribution of (X, Y). Find

(i) $P(X \leq 1)$

(ii) $P(Y \leq 3)$

(iii) $P(X \leq 1, Y \leq 3)$

(iv) $P(X \leq 1 / Y \leq 3)$

(v) $P(Y \leq 3 / X \leq 1)$

X/Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

(i) $P(X \leq 1)$

$P(X=0) + P(X=1)$

$\sum_{j=1}^{6} \{P(X=0, y=j) + \sum_{j=1}^{6} (P(X=1, y=j))$

$= 0 + 0 + \frac{1}{32} + \frac{2}{32} + \frac{2}{32} + \frac{3}{32} + \frac{1}{16} + \frac{1}{16} + \frac{4}{8}$

$= \frac{\cancel{8}^{1}}{\cancel{32}_{4}} + \frac{\cancel{2}^{1}}{\cancel{16}_{8}} + \frac{\cancel{4}^{1}}{\cancel{8}_{2}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{2} = \frac{2+1+4}{8} = \frac{7}{8}$

(ii) $P(Y \leq 3)$

$P(Y=1) + P(Y=2) + P(Y=3)$

$= 0 + \frac{1}{16} + \frac{1}{32} + 0 + \frac{1}{16} + \frac{1}{32} + \frac{1}{32} + \frac{1}{8} + \frac{1}{64}$

$= \frac{\cancel{2}^{1}}{\cancel{16}_{8}} + \frac{3}{32} + \frac{1}{8} + \frac{1}{64}$

$= \frac{1}{4} + \frac{3}{32} + \frac{1}{64} = \frac{16+6+1}{64} = \frac{23}{64}$

(iii) $P(X \leq 1, Y \leq 3)$

$= \sum_{j}^{3} (P(X=1, y=j))$

$P(X=0, y \leq 3) + P(X=1, y \leq 3)$

$P(X=0, y=1) + P(X=0, Y=2) + P(X=0, Y=3) + P(X=1, y=1) +$
$P(X=1, Y=2) + P(X=1, Y=3)$

$= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} = \frac{1+4+4}{32} = \frac{9}{32}$

(iv) $P(X \leq 1 / Y \leq 3) = \frac{P_{ij}}{P_{*j}} = \frac{9}{\cancel{32}} \times \frac{\cancel{64}^{2}}{23} = \frac{18}{23}$

(v) $P(Y \leq 3 / X < 1) = \frac{P_{ij}}{P_{i*}} = \frac{9}{\cancel{32}_{4}} \times \frac{\cancel{8}^{1}}{7} = \frac{9}{28}$

Q. For the CRV its PDF defined as $f(x) = kx^2 e^{-3x}, x \geq 0$ find the value of K also find its CDF

$$\int_0^\infty Kx^2 e^{-3x}$$

$$K\int_0^\infty x^2 e^{-3x}$$

$$-\frac{x^2 e^{-3x}}{3} - \int_0^\infty 2x \frac{-e^{-3x}}{3} dx$$

$$-\frac{x^2 e^{-3x}}{3} + \frac{2}{3}\int_0^\infty x e^{-3x} dx$$

$$-\frac{x^2 e^{-3x}}{3} + \frac{2}{3}\left[\frac{-x e^{-3x}}{3} + \int_0^\infty \frac{e^{-3x}}{3} dx\right]$$

$$-\frac{x^2 e^{-3x}}{3} + \frac{2}{3}\left[\frac{-x e^{-3x}}{3} + \frac{1}{3}\left(\frac{-e^{-3x}}{3}\right)\right]$$

$$K\left[\frac{-x^2 e^{-3x}}{3} - \frac{2}{9} x e^{-3x} - \frac{2}{27} e^{-3x}\right]_0^\infty = 1$$

$$K\left[\frac{2}{27}\right] = 1 \quad , \quad K = \frac{27}{2}$$

CDF: $\frac{27}{2}\int x^2 e^{-3x} dx$

$$F(x) = \frac{27}{2}\left[\frac{-x^2 e^{-3x}}{3} - \int 2x \frac{-e^{-3x}}{3}\right]$$

$$= \frac{27}{2}\left[\frac{-x^2 e^{-3x}}{3} + \frac{2}{3}\int x e^{-3x} dx\right]$$

$$= \frac{27}{2}\left[\frac{-x^2 e^{-3x}}{3} + \frac{2}{3}\left[\frac{-x e^{-3x}}{3} + \int \frac{e^{-3x}}{3} dx\right]\right]$$

$$= \frac{27}{2}\left[\frac{-x^2 e^{-3x}}{3} + \frac{2}{3}\left[\frac{-x e^{-3x}}{3} - \frac{e^{-3x}}{9}\right]\right]$$

$$= \frac{27}{2}\left[\frac{-x^2 e^{-3x}}{3} - \frac{2}{9} x e^{-3x} - \frac{2}{27} e^{-3x}\right]_0^x$$

$$= \left(\frac{-9}{2} x^2 e^{-3x} - 3x e^{-3x} - e^{-3x}\right) - \left(0 - 0 - \frac{2}{27}\right)$$

$$= \frac{-9}{2} x^2 e^{-3x} - 3x e^{-3x} - e^{-3x} + 1$$

$$\boxed{= 1 - \frac{9}{2} x^2 e^{-3x} - 3x e^{-3x} - e^{-3x}}$$

Ans

Ques A RV X has a probability density function given by $f(x) = \begin{cases} \frac{K}{1+x^2} & -\infty < x < \infty \\ 0 & \text{otherwise} \end{cases}$

$\frac{\pi}{2} + \frac{\pi}{2}$

$\frac{\pi^2}{2}$

Determine the value of K & its distribution f^n.
Also find $P(X \geq 0)$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x$

$$\int_{-\infty}^{\infty} \frac{K}{1+x^2}\,dx$$

$$K\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

$$K \int_{-\infty}^{\infty} [\tan^{-1}x]_{-\infty}^{\infty}$$

$$K[\tan^{-1}\infty - \tan^{-1}-\infty]$$

$$K\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 1$$

$$K\left[\frac{\cancel{2}\pi}{\cancel{2}}\right] = 1$$

$$K\pi = 1$$

$$K = \frac{1}{\pi}$$

$$P(X \geq 0) \rightarrow \frac{1}{\pi}\int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$

$$\frac{1}{\pi}[\tan^{-1}x]_0^{\infty}$$

$$\frac{1}{\pi}\left[[\tan^{-1}x]_0^{\infty} \cancel{- \tan^{-1}0}\right]$$

$$\frac{1}{\pi}(\tan^{-1}\infty - \tan^{-1}0)$$

$$\frac{1}{\cancel{\pi}} \times \frac{\cancel{\pi}}{2} = 1/2$$

Ques-1 Three coins are tossed simultaneously. Find the mean and variance corresponding to RV X where X represents the no. of Head.

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

$P(X=0) = \frac{1}{8} \Rightarrow 1$

$P(X=1) = \frac{3}{8} \Rightarrow 3$

$P(X=2) = \frac{3}{8} \Rightarrow 3$

$P(X=3) = \frac{1}{8} \Rightarrow 1$

$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{\cancel{8}}{\cancel{8}} = 1$$

X : No. of Heads

$X(HHH) = 3$

$X(HTH) = 2 = X(HHT) = X(THH)$

$X(HTT) = 1 = X(TTH) = X(THT)$

$X(TTT) = 0$

x	0	1	2	3
$p(x)$	1/8	3/8	3/8	1/8

Mean:

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + \frac{3}{8} \times 2 + \frac{1}{8} \times 3$$

$$= \frac{\cancel{1}}{\cancel{8}} + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{\cancel{20}^{5}}{\cancel{8}_{2}} = \frac{5}{2} = 2.5$$

$$= \frac{1\cancel{3}}{\cancel{8}} = \frac{\cancel{12}^{3}}{\cancel{8}_{2}} = \frac{3}{2} = 1.5$$

Variance $E(x^2) - (E(x))^2$ $\quad E(x^2) = \sum x^2 p(x)$

$$E(x^2) = 0\times\frac{1}{8} + 1\times\frac{3}{8} + 4\times\frac{3}{8} + 9\times\frac{1}{8}$$

$$= \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{24}{8} = 3$$

$$V = 3 - (1.5)^2$$

$-\infty$ to $-a$
$-a$ to 0
0 to a
a to ∞

$$= 3 - 2.25$$
$$= 0.75$$

Ques The PDF of a continuous RV is given by

$$f(x) = \begin{cases} \frac{bx}{a} + b, & -a \le x \le 0 \\ -\frac{bx}{a} + b, & 0 \le x \le a \end{cases}$$

a & b are constants

(i) find the relation b/w a & b Ans $ab = 1$

(ii) find the CDF of $f(x)$.

(i) $$\int_{-\infty}^{\infty} f(x)\,dx = 1 \qquad (-\infty, \infty)$$

$$\int_{-a}^{0}\left(\frac{bx}{a} + b\right)dx + \int_{0}^{a}\left(-\frac{bx}{a} + b\right)dx = 1$$

$$\Rightarrow \int_{-a}^{0} \frac{bx}{a}\,dx + \int_{-a}^{0} b.dx + \int_{0}^{a}\left(-\frac{bx}{a}\right)dx + \int_{0}^{a} b.dx$$

$\frac{bx^2}{a\,2}$

$$\int_{-a}^{0} \frac{bx+ab}{a}.dx + \int_{0}^{a}\frac{-bx+ab}{a}.dx = 1$$

$$\frac{b}{a}\int_{-a}^{0}(x+a)\,dx + \frac{b}{a}\int_{0}^{a}(-x+a)\,dx = 1$$

$= \cancel{\left[\frac{b}{a}\frac{x^2}{2}\right]_{-a}^{0} + \frac{b}{a}\left[\frac{-x^3}{2}\right]_{0}^{a}} \Rightarrow \left[\frac{x^2 b}{2a}\right]_{-a}^{0} - \cancel{\left[\frac{x^2 b}{2a}\right]_{0}^{a}}$ cdf=

$= 0 - \left[\frac{a^2 b}{2a}\right] - \cancel{\left[\frac{a^2 b}{2a}\right]} \Rightarrow \frac{-\cancel{2}a^{\cancel{2}}b}{\cancel{2a}} = 1$

$\Rightarrow ab = 1$

$\frac{b}{a}\left[\frac{x^2}{2}\right]_{-a}^{0} + b[x]_{-a}^{0} + \left(\frac{-b}{a}\right)\left[\frac{x^2}{2}\right]_{0}^{a} + b[x]_{0}^{a}$

$-\frac{b}{a}\left[\frac{a^2}{2}\right] + b[0+a] + \frac{(-b)}{a}\left[\frac{a^2}{2}\right] + ab$

$-\frac{a^{\cancel{2}}b}{2\cancel{a}} + ab - \frac{a^{\cancel{2}}b}{2\cancel{a}} + ab \Rightarrow \frac{-ab}{2} - \frac{ab}{2} + 2ab = \frac{-2ab}{2} + 2ab$

$= \frac{-2ab + 4ab}{2} = \frac{\cancel{2}ab}{\cancel{2}}$

$\boxed{\therefore \text{Relation } ab = 1}$

(ii) Case 1: $F(x) = 0 \quad x < -a$

(ii) Case 2: $-a \le x \le 0$

$F(x) = 0 + \int_{0}^{x}\left(\frac{bx}{a} + b\right)dx$

$= 0 + \frac{b}{a}\left[\frac{x^2}{2}\right]_{0}^{x} + b[x]_{0}^{x}$

$= \frac{bx^2}{2a} + bx$

(iii) Case 3: $0 \le x < a$

$F(x) = 0 + \int_{-a}^{0}\left(\frac{bx}{a} + b\right) + \int_{0}^{x}\frac{(-bx)}{a} + b$

$= -\frac{ab}{2} + ab - \frac{b}{a}\frac{x^2}{2} + bx = \frac{ab}{2} - \frac{bx^2}{a2} + bx$

(iv) $x > a$

$$= 0 + \int_{-a}^{0} \left(\frac{bx}{a} + b\right) dx + \int_{0}^{a} \left(\frac{-bx}{a} + b\right) dx + \int_{0}^{x} 0\, dx$$

$$= 0 + \frac{(-\cancel{a}^2 b)}{2\cancel{a}} + ab + \frac{(-a\cancel{a} b)}{2\cancel{a}} + ab + 0$$

$$= 2ab - \frac{2ab}{2} = \frac{4ab - 2ab}{2} = \frac{\cancel{2}ab}{\cancel{2}} = ab$$

$\boxed{ab = 1}$ we find that relation b/w a and b is $\boxed{ab = 1}$

Ques A RV has an experimental PDF given by

$f(x) = ae^{-b|x|}$ where a & b are constant. Find

(i) relation b/w a & b

(ii) CDF corresponding to this $f(x)$ & plot it

$$f(x) = \begin{cases} ae^{bx} & -\infty \le x < 0 \\ ae^{-bx} & 0 \le x < \infty \end{cases}$$

Part I $\Rightarrow \int_{0}^{-\infty} ae^{bx}\,dx$

$$a\int_{0}^{-\infty} e^{bx}dx \Rightarrow \frac{ae^{bx}}{b}\Big|_{-\infty}^{0}$$

$$\frac{a}{b}(1-0) = \frac{a}{b}$$

Part II: $\int_{0}^{\infty} ae^{-bx}\,dx = -\frac{a}{b}e^{-bx}\Big|_{0}^{\infty}$

$$= -\frac{a}{b}[0-1] = \frac{a}{b}$$

$$\int_{-\infty}^{\infty} f(x)dx = \frac{a}{b} + \frac{a}{b} = \frac{2a}{b}$$

$\therefore \frac{2a}{b} = 1 \Rightarrow a = \frac{b}{2}$

(ii) CDF

Case 1: $x < 0$

$$F(x) = \int_0^x ae^{-bx}\,dx$$

$$a\int_0^x e^{-bx}.dx$$

$$= \frac{a}{b}\left[e^{-bx}\right]_0^x \Rightarrow$$

<u>Ques</u>

$$f(x) = \begin{cases} \frac{2}{9}(x-1) & 1 \le x \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Find the mean & Variance of the CRV whose PDF is given by

$$\int_1^4 \frac{2}{9}(x-1) = \int_1^4 \frac{2}{9}x.\,dx + \int_1^4 \frac{2}{9}\,dx$$

$$= \frac{2}{9}\left[\frac{x^2}{2}\right]_1^4 + \frac{2}{9}\,x\Big|_1^4$$

$$=$$

$$= \int_{-\infty}^{1} + x \cdot \frac{2}{9}(x-1)dx + \int_1^4 x.\frac{2}{9}(x-1)dx + \int_4^{\infty} x.\frac{2}{9}(x-1)dx$$

$$\frac{2}{9}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{-\infty}^{1} + \frac{2}{9}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{-\infty}^{1} + \frac{2}{9}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{4}^{\infty}$$

2.965
0.166

0.5 Mean 0.8

$$= \int_1^4 x \frac{2}{9}(x-1)\,dx = \frac{2}{9}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^4$$

$$= \frac{2}{9}\left[\left[\frac{64}{3} - \frac{16}{2}\right] - \left[\frac{1}{3} - \frac{1}{2}\right]\right]$$

$$= \frac{2}{9}\left[\left[\frac{128-48}{6}\right] - \left[\frac{2-3}{6}\right]\right]$$

$$= \frac{2}{9}\left[\left[\frac{80}{6}\right] - \left[\frac{-1}{6}\right]\right]$$

$$= \frac{160}{54} + \frac{2}{54} = \cancel{3.12}\ \frac{162}{54} = 3$$

$$\boxed{\text{Mean} = 3}$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$\frac{2}{9}\int_1^4 x^2(x-1)\,dx$$

$$\frac{2}{9}\left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^4$$

$$\frac{2}{9}\left[\left[\frac{256}{4} - \frac{64}{3}\right] - \left[\frac{1}{4} - \frac{1}{3}\right]\right]$$

$$\Rightarrow \frac{2}{9}\left[\frac{768-256}{12}\right] - \frac{2}{9}\left[\frac{3-4}{12}\right]$$

$$\Rightarrow \frac{\cancel{2}^{1}}{9} \times \frac{512}{\cancel{12}_{6}} + \frac{2}{108}$$

$$= \frac{512}{54} + \frac{\cancel{2}^{1}}{\cancel{108}_{54}} = \frac{513}{54} = 9.5$$

$$E(x^2) = 9.5$$

$$= 9.5 - 9$$

$$\boxed{\text{Variance} = 0.5}$$

Ques find all statistical of CRV whose PDF is given by

$$f(x) = \begin{cases} \frac{1}{2a} & -a \le x \le a \\ 0 & \text{elsewhere} \end{cases}$$

Mean :- $= \int_{-a}^{a} \frac{1}{2a} \cdot x \, dx$

$= \cancel{\frac{x}{2a}\Big|_{-a}^{a}} = \cancel{\frac{a}{2a} + \frac{a}{2a}}$

$\frac{1}{2a}\left[\frac{x^2}{2}\right]_{-a}^{a} = \frac{1}{2a}\left[\frac{a^2}{2} - \frac{a^2}{2}\right] = 0$

Variance :- $\frac{1}{2a}\int_{-a}^{a} x^2 \, dx$

$\frac{1}{2a}\left[\frac{x^3}{3}\right]_{-a}^{a}$

$\frac{1}{2a}\left[\frac{a^3}{3} + \frac{a^3}{3}\right] = \frac{1}{\cancel{2a}}\left[\frac{\cancel{2}a^{\cancel{3}}}{3}\right] = \frac{a^2}{3}$

$= \frac{a^2}{3} - 0 = \frac{a^2}{3}$

Q An Eng. Company advertise a job in three newspapers A, B, C. It is know that these paper attracts B.Tech student in ratio 2:3:1. The prob. that the B.Tech student see and reply the job in these three newpapers are 0.002, 0.001 and 0.005. Assume that the B.tech student sees only one advertisement.

(i) If the Eng. company receives only one reply to its advertisement Calculate the prob. that the applicant has same the job advertisement in place A, B and C.

(ii) If the company receives two reply what is the prob. that both the applicant ~~shows~~ saw the job advertise in paper A.

~~P(A) = 0.002~~

~~P(B) = 0.001~~

~~P(C) = 0.005~~

$P(A) = \frac{\cancel{2}}{\cancel{6}} = \frac{1}{3}$

$P(B) = \frac{\cancel{3}}{\cancel{6}} = \frac{1}{2}$

$P(C) = \frac{1}{6}$

(i)

$P\left(\frac{R}{A}\right) = 0.002$

$P\left(\frac{R}{B}\right) = 0.001$

$P\left(\frac{R}{C}\right) = 0.005$

$$P\left(\frac{A}{R}\right) = \frac{P(A).P\left(\frac{R}{A}\right)}{P(A).P\left(\frac{R}{A}\right) + P(B).P\left(\frac{R}{B}\right) + P(C).P\left(\frac{R}{C}\right)}$$

$= P(A).P\left(\frac{R}{A}\right) + P(B).P\left(\frac{R}{B}\right) + P(C).P\left(\frac{R}{C}\right)$

$= \frac{1}{3} \times 0.002 + \frac{1}{2} \times 0.001 + \frac{1}{6} \times 0.005$

$= \frac{0.002}{3} + \frac{0.001}{2} + \frac{0.005}{6}$

$\frac{0.004 + 0.003 + 0.005}{6}$

$= \frac{0.012}{6000} = \frac{1}{500}$

$$= \frac{\frac{1}{3} \times 0.002}{\frac{1}{500}}$$

$= \frac{\cancel{2}}{\cancel{3000}} \times \frac{\cancel{1}}{\cancel{500}}$

$= \frac{1}{3}$

$$P\left(\frac{B}{R}\right) = \frac{P(B) \times P\left(\frac{R}{B}\right)}{\frac{1}{500}} = \frac{\frac{1}{2} \times 0.001}{500} = \frac{1}{2000} \times 500 = \frac{1}{4}$$

$$P\left(\frac{C}{R}\right) = \frac{P(C) \cdot P\left(\frac{R}{C}\right)}{P(A) \cdot P\left(\frac{R}{A}\right) + P(B) \cdot P\left(\frac{R}{B}\right) + P(C)\, P\left(\frac{R}{C}\right)}$$

$$= \frac{\frac{1}{6} \times 0.005}{\frac{1}{3} \times 0.002 + \frac{1}{2} \times 0.001 + \frac{1}{6} \times 0.005}$$

$$= \frac{\frac{0.005}{6}}{\frac{0.002}{3} + \frac{0.001}{2} + \frac{0.005}{6}} = \frac{\frac{0.005}{6}}{\frac{0.004 + 0.003 + 0.005}{6}}$$

$$= \frac{0.005}{6000} \times \frac{6000}{0.012}$$

$$= \frac{5}{12}$$

(ii)

Ques Machine A & B produce 10% and 90% of the production of a company of the motors industry from this experience it is know that the prob. that produces A defective component 0.01, while The prob. that B produces a defective component 0.05. If a component is selected at random from the production A found to be defective. Find the prob. that it was made by
(i) Machine A (ii) Machine B

$$P(A) = \frac{10}{100} , \quad P(B) = \frac{90}{100}$$

$$P\left(\frac{D}{A}\right) = 0.01 \qquad P\left(\frac{D}{B}\right) = 0.05$$

(i) $$P\left(\frac{A}{D}\right) = \frac{\frac{1\cancel{0}}{100} \times \frac{0.01}{10\cancel{0}}}{\frac{10}{100} \times 0.01 + \frac{9\cancel{0}}{100} \times \frac{0.0\cancel{5}}{\cancel{10}\cancel{0}_{2}}} = \frac{\frac{1}{1000}}{\frac{1}{1000} + \frac{9}{200}} = \frac{\frac{1}{1000}}{\frac{200 + 9000}{200000}}$$

$$= \frac{1}{1\cancel{000}} \times \frac{2\cancel{00000}}{92\cancel{00}} = \frac{2}{92} = 0.0217$$

(ii) $$P\left(\frac{B}{D}\right) = \frac{\frac{9\cancel{0}}{100} \times \frac{0.05}{10\cancel{0}}}{\frac{92\cancel{00}}{2000\cancel{00}}} = \frac{45}{1\cancel{000}} \times \frac{2\cancel{000}}{92} = 0.978$$

Ques $f(x) = \begin{cases} K(x-4), & 0 \le x \le 4 \\ 0, & \text{elsewhere} \end{cases}$

Find the mean and variance for the CRV, whose probability is given by $f(x) =$

Also find $P(X \geq 2)$

$$\int_0^4 K(x-4)\,dx$$

$$K\int_0^4 (x-4)\,dx$$

$$K\left[\left[\frac{x^2}{2}\right]_0^4 - 4\left[x\right]_0^4\right]$$

$$K\left[\frac{16}{2} - \frac{0}{2}\right] - 4\left[4-0\right]$$

$$8K - 16K = 1$$

$$-8K = 1$$

$$K = \frac{-1}{8}$$

Mean:- $\int_0^4 \frac{-1}{8} x(x-4)$

$$\frac{-1}{8}\int_0^4 x^2 - 4x$$

$$\frac{-1}{8}\left[\left(\frac{x^3}{3}\right)_0^4 - 4\left(\frac{x^2}{2}\right)_0^4\right]$$

$$\frac{-1}{8}\left[\frac{64}{3} - 4\left(\frac{\overset{8}{\cancel{16}}}{\cancel{2}}\right)\right]$$

$$\frac{-1}{8}\left[\frac{64}{3} - 32\right]$$

$$\frac{-1}{8}\left[\frac{64-96}{3}\right] = \frac{\cancel{1}}{\cancel{8}} \times \frac{\overset{4}{\cancel{+32}}}{3}$$

$$= 4/3$$

Variance $= -\frac{1}{8}\int_0^4 x^2(x-4)\,dx$

1/4

$= -\frac{1}{8}\left[\left[\frac{x^4}{4}\right]_0^4 - 4\left[\frac{x^3}{3}\right]_0^4\right]$

$= -\frac{1}{8}\left[\frac{256}{4} - \frac{256}{3}\right]$

$= -\frac{1}{8}\left(\frac{768 - \overset{1024}{\cancel{256}}}{12}\right) = \cancel{\frac{1}{8}} \times \cancel{\frac{\overset{64}{512}}{12}} = \cancel{5}.33$

$= -\frac{1}{\cancel{8}_{1}} \times -\frac{\overset{\cancel{128}\ \cancel{64}\ 8}{\cancel{256}}}{\cancel{12}\ \cancel{6}\ 3} = \frac{8}{3} = 2.666$

→ $P(x \geq 2)$

$\int_2^4 -\frac{1}{8}(x-4)$

$-\frac{1}{8}\left[\frac{x^2}{2} - 4x\right]_2^4$

$-\frac{1}{8}\left[\left[\frac{\overset{8}{\cancel{16}}}{\cancel{2}_{1}} - \frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right] - 4\left[4-2\right]\right]$

$= -\frac{1}{8}\left[[6] - [8]\right]$

$= \frac{1}{\cancel{8}_{4}} \times \cancel{2}^{1} = \frac{1}{4}$

Ques A discrete Random variable X probability f^n is given as $P(0) = 0$, $P(1) = K^3$, $P(2) = K^2$, $P(3) = -K$

$2K \quad K \quad 3K$

$\frac{2}{6} \quad \frac{1}{6} \quad \frac{3}{6}$

(i) find value of K

(ii) $P(0 \le X \le 2)$

(iii) $P(X > 1)$

(iv) CDF

$$P(0) + P(1) + P(2) + P(3) = 1$$

$$0 + K^3 + K^2 + (-K) = 1$$

$$K^3 + K^2 - K = 1$$

$$K(K^2 + K - 1) = 1$$

$$K^3 + K^2 - K = 1$$

$$K^2(K+1) - K - 1 = 1$$

$$K^2(K+1) - 1(K+1) = 1$$

$$(K^2 - 1)(K+1) = 1$$

$$K^2 = 1 \qquad K = -1$$

$$K = \sqrt{1}$$

$$K = \pm 1$$

(ii) $P(0 \le X \le 2)$

<u>Ques</u> A pair of dice is thrown find the probability that the sum is 10 or greater. $\frac{3}{11}$

$\frac{6}{36}$

(i) If five appears on first dice. $\frac{1}{3}$

(ii) five appears on atleast one of dice. $\frac{3}{11}$

S = { (1,1) (1 2) (1 3) (1 4) (1 5) (1 6)
(2 1) (2 2) (2 3) (2 4) (2 5) (2 6)
(3 1) (3 2) (3 3) (3 4) (3 5) (3 6)
(4 1) (4 2) (4 3) (4 4) (4 5) (4 6)
(5 1) (5 2) (5 3) (5 4) (5 5) (5 6)
(6 1) (6 2) (6 3) (6 4) (6 5) (6, 6) }

~~(i) P(A) = { (5,5) (5,6) } = $\frac{2}{36}$ { (5,6), (6,5) (6,6) } = $\frac{5}{36}$~~
~~P(B) = { (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) }~~

Total outcomes = 36

~~$P\left(\frac{A}{B}\right)$ =~~ Possible sum ≥ 10

10 → (4,6) (5,5) (6,4) → 3 outcomes
11 → (5,6) (6,5) → 2 outcomes
12 → (6,6) → 1 outcomes

$P(\text{sum} \geq 10) = \frac{6}{36} = \frac{1}{6}$

(i) $P(\text{sum} \geq 10 \mid \text{first dice} = 5) = \frac{2}{6} = \frac{1}{3}$
{ (5,5, (5,6) }

(ii) Outcomes where atleast one is 5 = 11
(sum ≥ 10) = (5,5) (5,6) (6,5)
= 3

$P(\text{sum} \geq 10 \mid \text{At least one dice} = 5) = \frac{3}{11}$

<u>Ques</u> Two dice are rolled find the CDF of the RV X which is getting sum on 2 dice.

- $P(X=2) = \frac{1}{36}$
- $P(X=3) = \frac{2}{36}$
- $P(X=4) = \frac{3}{36}$
- $P(X=5) = \frac{4}{36}$

• $P(X=6)=\frac{5}{36}$

• $P(X=7)=\frac{6}{36}$

• $P(X=8)=\frac{5}{36}$

• $P(X=9)=\frac{4}{36}$

• $P(X=10)=\frac{3}{36}$

• $P(X=11)=\frac{2}{36}$

• $P(X=12)=\frac{1}{36}$

CDF $F(x)=P(X\leq x)$

$F(x)=0, \quad x<2$

$F = \frac{1}{36}, \quad 2\leq x<3$

$= \frac{3}{36}, \quad 3\leq x<4$

$= \frac{6}{36}, \quad 4\leq x<5$

$= \frac{10}{36}, \quad 5\leq x<6$

$= \frac{15}{36}, \quad 6\leq x<7$

$= \frac{21}{36}, \quad 7\leq x<8$

$= \frac{26}{36}, \quad 8\leq x<9$

$= \frac{30}{36}, \quad 9\leq x<10$

$= \frac{33}{36}, \quad 10\leq x<11$

$= \frac{35}{36}, \quad 11\leq x<12$

$= 1, \quad x\geq 12$