PYQs → 2024

Ques-1 $4!$ $^4C_2$

$= \frac{4!}{2!2!} = \frac{\cancel{4}^{2} \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2}_{1} \times 1} = 6$

D = Defective
G = good

DD GG
GG D

1	DDGG	→ 2nd test
2	DGDG	→ 3rd test
3	DGGD	→ 4rd test
4	GDDG	→ 3rd test
5	GDGD	→ 4th test
6	GGDD	→ 4th

(a) $\frac{1}{6}$

(b) $\frac{\cancel{2}}{\cancel{6}} = \frac{1}{3}$

(c) $\frac{3}{6} = \frac{1}{2}$

Ques2. ~~$P(T_0) = 0.95$~~

(a) P(received 0 | transmit 0) = 0.95
P(received 1 | transmit 0) = 0.05
P(received 1 | transmit 1) = 0.90
P(received 0 | transmit 1) = 0.10

P(transmit 0) = 0.4
P(transmit 1) = 0.6

(a) P(1 received) = P(transmit 0) P(receive 1 | transmit 0) + P(transmit 1) P(receive 1 | transmit 1)

$= 0.40 \times 0.05 + 0.6 \times 0.90$

$= 0.02 + 0.54$

$= 0.56$

(b) $P(t-1 / r-1) = \dfrac{0.6 \times 0.90}{P(\text{receive } 1)}$

$= \dfrac{0.6 \times 0.90}{0.56}$

$= \dfrac{0.54}{0.56} = 0.964$

Ques-3

x :	-2	-1	0	1	2	3
P(x) :	0.1	K	0.2	2K	0.3	3K

$P(x) = 0.1 + K + 0.2 + 2K + 0.3 + 3K = 1$

$0.6 + 6K = 1$

$6K = 1 - 0.6$

$6K = 0.4$

$K = \dfrac{\cancel{4}0}{\cancel{60}} \; \cancel{30}\; 15$

$K = \dfrac{1}{15}$

x :	-2	-1	0	1	2	3
P(x) :	0.1	$\frac{1}{15}$	0.2	$\frac{2}{15}$	0.3	$\frac{3}{15}$

$$P(X<2) = 0.1 + \frac{1}{15} + 0.2 + \frac{2}{15}$$

$$= \frac{0.3}{10} + \frac{3}{15}$$

$$= \frac{45+30}{150}$$

$$= \frac{75}{150} = 0.5$$

$$P(-2<X<2) = \frac{1}{15} + 0.2 + \frac{2}{15}$$

$$= \frac{3}{15} + \frac{0.2}{0.5}$$

$$= \frac{15+15}{75}$$

$$= \frac{30}{75}$$

$$= 0.4$$

CDF: 0 $\quad x < -2$

0.1 $\quad -2 \le x < -1$

$\underline{0.167}$ $\quad \frac{1}{15} + \frac{0.1}{10} = \frac{10+15}{150}$ $\quad -1 \le x < 0$

$$= \frac{25}{150} = \frac{1}{6}$$

$0.167 + 0.2 = \underline{0.367}$ $\quad 0 \le x < 1$

0.3

$0.367 + \frac{2}{15} = 0.5$ $\quad 1 \le x < 2$

$0.5 + 0.3 = 0.8$ $\quad 2 \le x < 3$

$0.8 + 0.2 = 1$ $\quad 3 \le x < 2$

(c) Mean $= -0.2 - \frac{1}{15} + 0 + \frac{2}{15} + 0.6 + \frac{9}{15}$

$= -0.2 - \frac{1}{15} + \frac{2}{15} + 0.6 + \frac{9}{15}$

$= 0.4 - \frac{1}{15} + \frac{11}{15}$

$\cancel{= 0.4 - \frac{12}{15}}$ $\quad \frac{6 - 1 + 11}{15} = \frac{17-1}{15}$

$= \cancel{\frac{6-12}{15}}$ $\quad = \frac{16}{15}$

$= \cancel{\frac{-6}{15}}$ $\quad = 1.067$

Variance $= E[X^2] - (E(X))^2$

$E[X^2] = 4 \times 0.1 + \frac{1}{15} + \frac{2}{15} + 0.12 + \frac{27}{15}$

$= 0.4 + \frac{1}{15} + \frac{2}{15} + 0.12 + \frac{27}{15}$

$= \frac{6 + 1 + 2 + 1.8 + 27}{15} = \frac{37.8}{15} = \cancel{2.52}\ \frac{54}{15} = 3.6$

0.8

$0.367 + \frac{2}{15} = 0.5$ $\quad 1 \leq x < 2$

$0.5 + 0.3$
$= 0.8$ $\quad 2 \leq x < 3$

$0.8 + 0.2 = 1$ $\quad 3 \leq x < 2$

(c) Mean $= -0.2 - \frac{1}{15} + 0 + \frac{2}{15} + 0.6 + \frac{9}{15}$

$= -0.2 - \frac{1}{15} + \frac{2}{15} + 0.6 + \frac{9}{15}$

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$\cancel{= 0.4 - \frac{12}{15}}$ $\qquad \frac{6 - 1 + 11}{15} = \frac{17 - 1}{15}$

$= \cancel{\frac{6 - 12}{15}}$ $\qquad = \frac{16}{15}$

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Variance $= E[X^2] - (E(X))^2$

$E[X^2] = 4 \times 0.1 + \frac{1}{15} + \frac{2}{15} + 0.12 + \frac{27}{15}$

$= 0.4 + \frac{1}{15} + \frac{2}{15} + 0.12 + \frac{27}{15}$

$= \frac{6 + 1 + 2 + 1.8 + 27}{15} = \frac{37.8}{15} = \frac{\cancel{2.52}\, 54}{15} = 3.6$

$= 3.6 - (1.067)^2$
$= 3.6 - 1.138$
$= 2.462$

Ques-4 $P(D) = 0.80$ → Probability train departs on time

$P(A) = 0.82$ → —— " —— arrives on time

$P(D \cap A) = 0.75$

(a) $P\left(\frac{A}{D}\right) = \frac{P(A \cap D)}{P(D)}$

$= \frac{0.75}{0.80} = 0.9375$

(b) $P\left(\frac{D}{A}\right) = \frac{P(D \cap A)}{P(A)} = \frac{0.75}{0.82} = 0.9146$

PYQs (2023)

Ques-1 Total persons = 950
Hindi speaks = 750 = P(H)
English speaks = 460 = P(E)

(a) $P(H \cup E) = P(H) + P(E) - P(H \cap E)$
$950 = 750 + 460 - P(H \cap E)$
$950 = 1210 - P(H \cap E)$
$950 - 1210 = -P(H \cap E)$
$P(H \cap E) = 260$

~~Ques-2~~ (b) $P(H \text{ only}) = P(H) - P(H \cap E)$
$= 750 - 260$
$= 490$

(c) $P(E \text{ only}) = P(E) - P(H \cap E)$
$= 460 - 260$
$= 200$

Ques-2 Box 1 : 3 red, 2 white → (B_1)
Box 2 : 4 red, 5 white → (B_2)
Box 3 : 2 red, 4 white → (B_3)

$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$

R: "drawn ball is red"

$P\left(\frac{R}{B_1}\right) = \frac{\text{Number of red balls}}{\text{Total balls}}$
$= \frac{3}{5}$

$P\left(\frac{R}{B_2}\right) = \frac{4}{9}$, $P\left(\frac{R}{B_3}\right) = \frac{\cancel{2}}{\cancel{6}} = \frac{1}{3}$

$P(R) = P(B_1) \times P\left(\frac{R}{B_1}\right) + P(B_2) \times P\left(\frac{R}{B_2}\right) + P(B_3) \times P\left(\frac{R}{B_3}\right)$

$= \frac{1}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{4}{9} + \frac{1}{3} \times \frac{1}{3}$

$= \frac{1}{3}\left[\frac{3}{5} + \frac{4}{9} + \frac{1}{3}\right]$

$= \frac{\cancel{1}}{3}\left[\frac{27+20+15}{45}\right]$

$$\begin{array}{r|l} 3 & 5,9,3 \\ 3 & 5,3,1 \\ 5 & 5,1,1 \end{array}$$

$= \frac{1}{3} \times \frac{62}{45} = \frac{62}{135} = 0.459$

$P\left(\frac{B_2}{R}\right) = \frac{P(B_2) \times P\left(\frac{R}{B_2}\right)}{\cancel{0.4}\ P(R)}$

$= \frac{\frac{1}{3} \times \frac{4}{9}}{\frac{62}{135}} = \frac{4}{\cancel{27}_{\cancel{9}}} \times \frac{\cancel{135}^{\cancel{45}\ \cancel{15}\ 5}}{62} = \frac{20}{62}$

$= 0.322$

Ques-3

$$\int_0^{\infty} \frac{1}{4} e^{-x/4}\, dx$$

$$\cancel{4} \times \frac{1}{\cancel{4}} [-e^{-x/4}]$$

$$[-e^{-x/4}]_0^{\infty}$$

$$[-e^{-\infty/4} - (-e^{-0/4})]$$

$$[0 - (-1)]$$

$$1$$

PDF is valid.

(a) Probability that life is b/w 2 and 4 years

$$\int_2^4 \frac{1}{4} e^{-x/4}\, dx$$

$e^{-1/2} = 0.6065$

$e^{-1} = 0.367$

$$-\frac{1}{\cancel{4}} e^{-x/4} \times \cancel{4} \Big|_2^4$$

$$[-e^{-x/4}]_2^4$$

$$[-e^{-4/4} - (-e^{-2/4}))]$$

$$[-e^{-1} + e^{-1/2}]$$

$P(2 \leq X < 4) = e^{-1/2} - e^{-1}$

$= 0.6065 - 0.367$

$= 0.2386$

(b) CDF :

$x \geq 0 \quad F(x) = P(X \leq x) = \int_0^x \frac{1}{4} e^{-x/4} \, dx$

$= \frac{-1}{\cancel{4}} \times \cancel{4} e^{-x/4} \Big|_0^x$

$= -e^{-x/4} - (-e^{0})$

$= -e^{-x/4} + 1$

$= 1 - e^{-x/4}$

$$F(x) = \begin{cases} 0, & x < 0 \\ 1-e^{-x/4} & x \geq 0 \end{cases}$$

~~Doubt~~

(c) $P(X=4) = 0$

Probability its lasts at least 4 years

i.e., $P(X \geq 4)$

$F(x) = P(X \leq x) = 1 - e^{-x/4}, x \geq 0$

$P(X \geq 4) = 1 - P(X \leq 4)$

$F(4) = \cancel{P(X \leq 4)} = 1 - e^{-1}$

$P(X < 4) = 1 - 0.3679$

(atleast4 for years)

$= 0.6321$

$P(X \geq 4) = 1 - F(4) P(X<4)$

$= 1 - 0.6321$

$= 0.3679$

PYQs (2022)

Ques-3
(a)

$$\int_0^1 \frac{3}{2}(1-x^2)\,dx$$

$$\frac{3}{2}\int_0^1 (1-x^2)\,dx$$

$$\frac{3}{2}\left[x-\frac{x^3}{3}\right]_0^1$$

$$\frac{3}{2}\left[\left(1-\frac{1}{3}\right)-(0)\right]$$

$$\frac{\cancel{3}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}} = 1$$

(b) CDF: $\int_0^x \frac{3}{2}(1-x^2)\,dx$

$$\frac{3}{2}\left[x-\frac{x^3}{3}\right]$$

$$\frac{\cancel{3}}{2}\left[\frac{3x-x^3}{\cancel{3}}\right]$$

$$\frac{3x-x^3}{2}$$

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{3x-x^3}{2} & 0 \le x < 1 \\ 1 & x > 1 \end{cases}$$

$\frac{x}{5}$ $\quad a \quad e^{-x/5} \quad -5e^{-x/5} \quad \frac{-5e^{-x/5}}{-1/5}$

(c)

$$\int_{0.25}^{0.75} \frac{3}{2}(1-x^2)\,dx$$

$$\cancel{\frac{3}{2}}\left[\frac{3x-x^3}{2}\right|_{0.25}^{0.75}$$

$$\left[\frac{3\times 0.75-(0.75)^3}{2}\right]-\left[\frac{3\times 0.25-(0.25)^3}{2}\right]$$

$$\left[\frac{2.25-0.42}{2}\right]-\left[\frac{0.75-0.015}{2}\right]$$

$$\frac{1.83}{2}-\frac{0.74}{2}=\frac{1.09}{2}=0.54$$

(d) Mean $E(X) = \int_0^1 x\cdot\frac{3}{2}(1-x^2)\,dx$

$$=\frac{3}{2}\int_0^1 (x-x^3)\,dx$$

$$=\frac{3}{2}\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1$$

$$=\frac{3}{2}\left[\left(\frac{1}{2}-\frac{1}{4}\right)-(0)\right]$$

$= \frac{3}{2} \times \left(4 - \frac{3}{8}\right)$

$= \frac{3}{2} \times \frac{\cancel{2}1}{\cancel{8}4}$

$= \frac{3}{8}$

$= 0.375$

Variance

$\frac{3}{2}\int_0^1 x^2(1-x^2)\,dx$

$\frac{3}{2}\int_0^1 (x^2 - x^4)\,dx$

$\frac{3}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$

$\frac{3}{2}\left[\left(\frac{1}{3} - \frac{1}{5}\right) - (0)\right]$

$\frac{\cancel{3}1}{2} \times \frac{5-3}{\cancel{15}5} = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{5} = \frac{1}{5}$.

$\text{Variance} = E(X^2) - (E(X))^2$

$= \frac{1}{5} - \left(\frac{3}{8}\right)^2$

$= \frac{1}{5} - \frac{9}{64} = \frac{64-45}{64 \times 5} = \frac{19}{320} = 0.0059$

PDF graph:

$$f(x) = \frac{3}{2}(1-x^2), \quad 0 \le x \le 1$$

$x = 0$

$$f(0) = \frac{3}{2}(1-0) = 1.5$$

$$f(1) = \frac{3}{2}(1-1) = 0$$

So graph ends at (1,0)

Mid points.

At $x = 0.5$

$$f(0.5) = \frac{3}{2}(1 - 0.25).$$

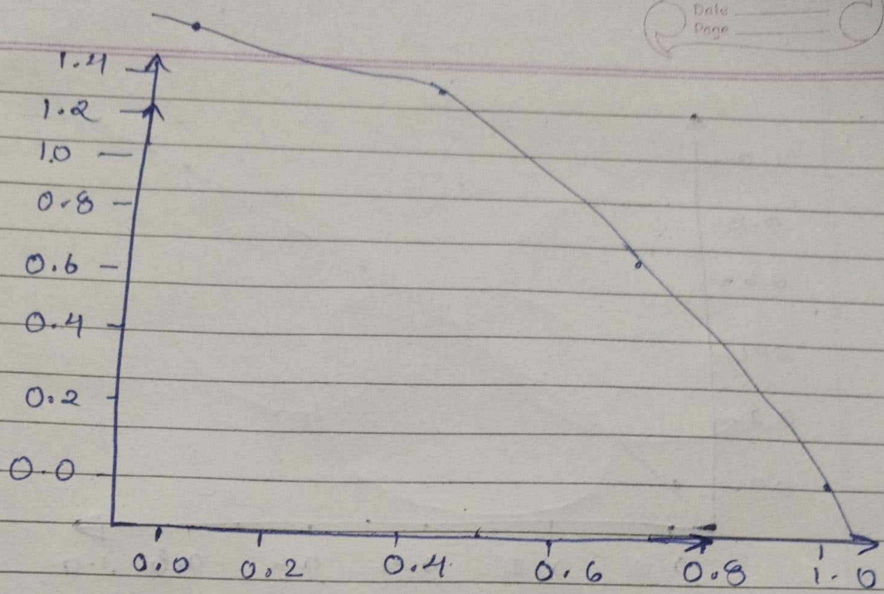
$$= \frac{3}{2} \times 0.75 = 1.125$$

$$f(0.75) = \frac{3}{2} \times (1 - 0.5625)$$

$$= \frac{3}{2} \times 0.4375$$

$$= 0.656$$

(0, 1.5) (0.5, 1.125) (0.75, 0.656) → (1,0)



CDF Graph:

$$F(x) = \frac{3x - x^3}{2} \quad 0 \le x \le 1$$

$x = 0$, $F(x) = 0$

$x = 1$, $\frac{3 \times 1 - (1)^3}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$

$x = 0.5$ $\quad \frac{3 \times 0.5 - 0.125}{2}$

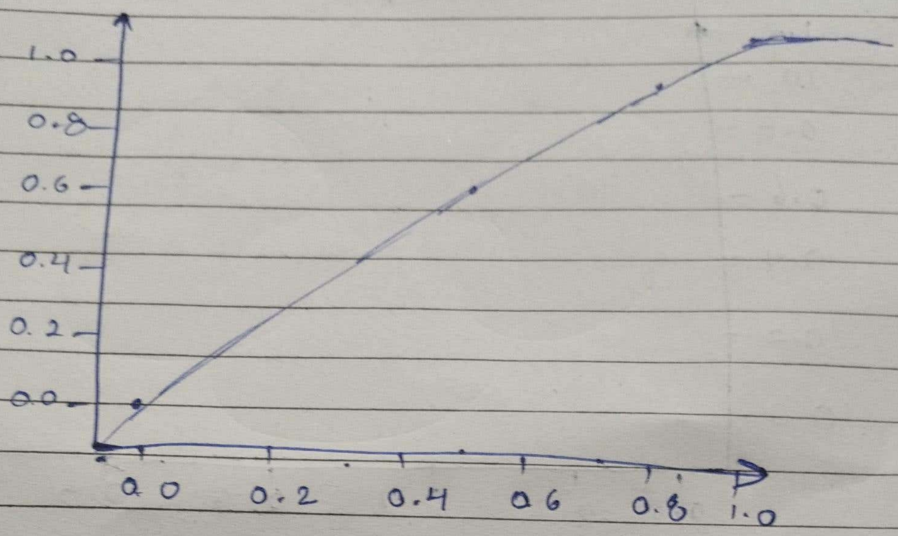
$= \frac{1.5 - 0.125}{2} = 0.6875$

$x = 0.75$ $\quad \frac{3 \times 0.75 - (0.75)^3}{2}$

$= \frac{2.25 - 0.4219}{2}$

$= 0.914$

$(0,0)$ $(1,1)$



Ques-2

$P(C) = \frac{40}{100}$

$P(L) = \frac{35}{100}$

$P(I) = \frac{25}{100}$

$P\left(\frac{V}{C}\right) = \frac{45}{100}$

$P\left(\frac{V}{L}\right) = \frac{40}{100}$

$P\left(\frac{V}{C}\right) = \frac{60}{100}$

$$P(V) = \frac{40}{100} \times \frac{45}{100} + \frac{35}{100} \times \frac{40}{100} + \frac{25}{100} \times \frac{60}{100}$$

$$= \frac{1}{10000}(180+140+150)$$

$$= \frac{1}{100\cancel{0}} \times 47\cancel{0}$$

$$= 0.47$$

(i) $P\left(\frac{C}{V}\right) = 0.382$

(ii) $P\left(\frac{L}{V}\right) = 0.219$

(iii) $P\left(\frac{I}{V}\right)$ 0.317

$$P\left(\frac{C}{V}\right) = \frac{\frac{40}{100} \times \frac{45}{100}}{0.47} = \frac{\frac{18\cancel{0}}{100\cancel{0}}}{0.47} = \frac{18}{1\cancel{0}} \times \frac{1\cancel{0}}{047}$$

$$= \frac{18}{47} = 0.382$$