Probability Theory & Random Process

Random Experiment - An experiment is an random if it outcome cannot be predicted precisely.

Sample Space - The set of all possible outcomes of a random experiment.

one coin [$S = \{H, T\}$ or Ω]

two coin [$S = \{HH, HT, TT, TH\}$].

NOTE:- 1) A sample space may be finite, countably infinite or uncountable.

$$f : \mathbb{Z} \to \mathbb{Z}$$

$$f(x) = x^3 \text{ (one-one } f^n)$$

2) A finite or countably infinite sample space is called "discrete sample space"

3) An uncountable is called continuous sample space.

Types of Sample Space

1) Discrete
2) Continuous

Event - A event is simply a set of outcomes. In other words, an event is a subset of sample space S.

$$A \subseteq S$$

NOTE:- For a discrete sample space all the subsets are event.

Example:- If we toss a coin possible outcomes are Head and tail. Then the sample space $S = \{H, T\}$

Then the event are $\{H, T\}$, $\{H\}$, $\{T\}$ and $\{\phi\}$

If we roll a dice then the sample space $S = \{1, 2, 3, 4, 5, 6\}$. Then the events

A = An event of getting odd

B = An event of getting 6

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Picking the real number between -1 and 1.

$S = \{ s \in R \mid -1 \le s \le 1 \}$ or $[-1,1] \rightarrow$ Continuous sample space

Types of events

1) Exhaustive Event - A set of event is said to be exhaustive if it includes all the possible events.

2) Mutually Exclusive event ~ Two events A & B are said to be mutually exclusive event if they cannot occur together.

If two events are mutually exclusive then the probability of occuring either event it.

$$\boxed{P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)}$$

Independent event - Two events a said to be if probability of occuring or failure of one event does not affect the occuring or failure of another.

Otherwise, this event are said to be dependent.

$$\boxed{P(A \text{ and } B) = P(A \cap B) = P(A).\ P(B)}$$

Non-Mutually Exclusive event

Two events A and B are said to be mutually exclusive even if they can occur together.

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)}$$

Definitions of probability

1) Relative frequency Definition - Consider an experiment is repeated n times & let A & B are two events associated with E.

Let n_A and n_B be the number of time the event A and event B occured out of n times.

Then the relative frequency A in the n repeatation is defined as

$$\boxed{f_A = \frac{n_A}{n}}$$

Then the probability of event A is defined as

$$\boxed{P(A) = \lim_{n \to \infty} \frac{n_A}{n}}$$

The Classical Definition

Let the sample space S be the set of all possible distinct outcome then the probability of some events is equal to $\dfrac{\text{no. of ways an event can occur}}{\text{no. of outcomes in sample space}}$

Example:- A fair dice is rolled once. Then the what is the probability of getting 6

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{6\}$$

$$= \frac{1}{6}$$

- A fair coin is toss twiced

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$$= \frac{3}{8}$$

Probability Axioms

Given an event E in an sample space S which is either finite with n elements or countability infinite with infinite elements

$$\boxed{S = \bigcup_{i=1}^{N} E_i}$$

1) Probability of any event A is positive or zero it means probability of event A is greater than 0. Moraver probability of A lies b/w 0 & 1.

2) Probability of sure event is equal to 1.

$$1 > P(A) \geq 0$$

(i) $P(A) + P(\bar{A}) = 1$

$A \cup \bar{A} = S$, $P(A \cup \bar{A}) = 1$

Probability of an impossible event is 0.

$A \subset B$ then $P(A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Joint Probability

JP is likelyhood of more than one event occuring at the same time. In other words, if a sample space they consist of two events A & B which are not mutually exclusive then the probability then

$$\boxed{P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right) = P(B) \cdot P\left(\frac{A}{B}\right)}$$

If an event A and event B is mutually exclusive then the probability of event A & event B is zero.

Conditional Probability

The probability of event B under the condition that another event A has occurred is known as condition probability of B given A.

$P(B/A)$ = Conditional Probability of B given A

$$P(B/A) = \frac{\text{No. of favourable outcomes to A \& B}}{\text{No. of favourable outcomes of A.}}$$

$$= \frac{n(AB)}{n(A)} = \frac{\frac{n(AB)}{n}}{\frac{n(A)}{n}}$$

$$P(A \cap B) = P\left(\frac{B}{A}\right) \cdot P(A)$$

$$= P\left(\frac{A}{B}\right) \cdot P(B)$$

$$\boxed{P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}}$$

Example:- Consider an example rolling a fair dice find probability $\left(\frac{B}{A}\right)$ where A is an event of getting even number and B is an even of getting no. < 4

$$P(A) = \frac{\cancel{3}}{\cancel{6}} = \frac{1}{2} \quad , \quad P(B) = \frac{\cancel{3}}{\cancel{6}} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{6}$$

$$P\left(\frac{B}{A}\right) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{\cancel{6}_3} \times \cancel{2}^1 = \frac{1}{3}$$

$$\boxed{P\left(\frac{B}{A}\right) = \frac{1}{3}}$$

Ques A family has 2 child. It is known that atleast one of them is girl. What is the probability that both of child are girl.

A = event of atleast one girl
B = event of getting two girl

$S = \{GB, BG, GG, BB\}$

$P(A) = \{GG, BG, GB\} = \frac{3}{4}$

$P(B) = \{GG\} = \frac{1}{4}$

$P(A \cap B) = \frac{1}{4}$, $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3} = \frac{1}{3}$

Chain Rule of probability / Multiplication theorem

$$P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) = P(A_1)\, P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1 \cap A_2}\right) \ldots\ldots P\left(\frac{A_n}{A_1 \cap A_2 \ldots \cap A_{n-1}}\right)$$

$A \cap B \cap C = (A \cap B) \cap C$ [Associative]

$$P[A \cap B \cap C] = P[(A \cap B) \cap C]$$

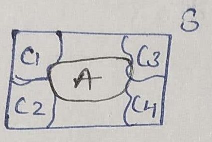
$$= P(A \cap B) \cdot P\left(\frac{C}{A \cap B}\right)$$

$$= P(A) \cdot P\left(\frac{B}{A}\right) \cdot P\left(\frac{C}{A \cap B}\right)$$

Total Prabability Theorem

Let events $C_1, C_2 \ldots C_n$ be the partitions of the sample space S where all the event have non-zero probability of occurence for any event A associated with S.

$$\boxed{P(A) = \sum_{K=0}^{n} P(C_K)\, P\left(\frac{A}{C_K}\right)}$$



$C_1 \cup C_2 \ldots\ldots \cup C_n = S$

$C_i \cap C_K = \phi$

S = all the partition

<u>Proof:</u> A is an event

$A = A \cap S$

$= A \cap (C_1 \cup C_2 \cup \ldots\ldots C_n)$

$= (A \cap C_1) \cup (A \cap C_2) \cup \ldots\ldots\ldots \cup (A \cap C_n)$

$\Rightarrow P(A) = P[(A \cap C_1) \cup [(A \cap C_2) \ldots\ldots \cup (A \cap C_n)]$ → Mutually Exclusive

$\boxed{P(A \cup B) = P(A) + P(B)}$

$= P[(A \cap C_1) + P(A \cap C_2) \ldots\ldots\ldots + P(A \cap C_n)]$

$= P(C_1) . P\left(\frac{A}{C_1}\right) + P(C_2) . P\left(\frac{A}{C_2}\right) \ldots\ldots\ldots + P(C_n) . P\left(\frac{A}{C_n}\right)$

Ques A person has undertaken a mining job. The prob. of completion of job on time with and without rain are 0.42 and 0.90 respectively. If the prob. that it will rain 0.45 then determine the prob. that the mining job will be completed on time.

Let the A be event that work will be completed and B is event that it rains

$P(B) = 0.45$, $P(B') = 1 - 0.45 = 0.55$

$P\left(\frac{A}{B}\right) = 0.42$

$P\left(\frac{A}{B'}\right) = 0.90$

$P(A) = P(B) . P\left(\frac{A}{B}\right) + P(B') P\left(\frac{A}{B'}\right)$

$= 0.45 \times 0.42 + 0.55 \times 0.90$

$= \cancel{1790} + 0.189 + 0.495$

$= 0.684$

0.45
0.42
090
170
0179 0
0.55
0.90
000
495

Baye's Theorem

Let E_1, E_2 ~~and~~ E_n be the set of event associated with a Sample space S where all the events have non-zero probability of occurrence and they form partition of S. Then A/Q to Baye's theorem

$$\boxed{P\left(\frac{E_1}{A}\right) = \frac{P(E_i)\, P(E_i / A)}{\sum_{k=0}^{n} P(E_k) P(A/E_k)}}$$

1) $P(E_i/A) = \frac{P(E_i \cap A)}{P(A)} \longrightarrow (1)$

2) $P(E_i \cap A) = P(E_i) P\left(\frac{A}{E_i}\right) \longrightarrow (2)$

3) By total probability theorem

$P(A) = \sum_{i=1}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right) \longrightarrow (3)$

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Substituting the value of $P(E_i \cap A)$ and $P(A)$ from (ii) and (iii) in equation (1)

$$= \frac{P(E_i)\, P\left(\frac{A}{E_i}\right)}{\sum_{K=0}^{n} P(E_i)\, P\left(\frac{A}{E_i}\right)}$$

Q) In a binary communication system a 0 and 1 transmitted with prob. 0.6 and 0.4 respectively due to error in communication system 0 becomes one with the prob of 0.1 and 1 becomes 0 with the prob. of 0.08. Determine the prob. of

(i) of receiving 1.

(ii) that a 1 was transmitted when received message was 1.

Let S be the sample space corresponding to binary communication. A be the event of transmitting 0 & B be the event of transmitting 1 & A_1 and B_1 be corresponding events of receiving 0 & 1 respectively

$P(A) = 0.6 \quad P(B) = 0.4$

$P\left(\frac{B_1}{A}\right) = 0.1 \quad P\left(\frac{B_1}{B}\right) = 1 - 0.08 = 0.92$

(i) $P(\text{receiving one}) = P(A) \cdot P\left(\frac{B_1}{A}\right) + P(B) \cdot P\left(\frac{B_1}{B}\right)$

$= 0.6 \times 0.1 + 0.4 \times 0.92$

$= 0.06 + 0.368 = \underline{\underline{0.428}}$

(ii) Using Baye's theorem

$$P\left(\frac{B}{B_1}\right) = \frac{P(B) \cdot P(B_1 | B)}{P(B) \cdot P(B_1 | B) + P(A) \cdot P\left(\frac{B_1}{A}\right)}$$

$$= \frac{0.4 \times 0.92}{0.4 \times 0.92 + 0.6 \times 0.1} = \frac{0.368}{0.428} = \underline{\underline{0.859}}$$

Ques(i) In an electronic laboratory there are identically looking capacitor A_1, A_2, A_3 in ratio 2:3:4. It is known that 1% of A_1, 1.5 of A_2 and 2% of A_3 are defective. What % of capacity in the lab are defective

(ii) If a capacitor picked at defective is found. What is the probability that it is from A_3

(i) $P(A_1) = \frac{2}{9}$, $P(A_2) = \frac{3}{9}$, $P(A_3) = \frac{4}{9}$

$$P\left(\frac{D}{A_1}\right) = \frac{1}{100} \quad P\left(\frac{D}{A_2}\right) = \frac{15}{1000} \quad P\left(\frac{D}{A_3}\right) = \frac{2}{100}$$

$$P(D) = P(A_1).P\left(\frac{D}{A_1}\right) + P(A_2).P\left(\frac{D}{A_2}\right) + P(A_3).P\left(\frac{D}{A_3}\right)$$

$$= \frac{2}{9} \times \frac{1}{100} + \frac{3}{9} \times \frac{15}{1000} + \frac{4}{9} \times \frac{2}{100}$$

$$= \frac{2 + 4.5 + 8}{900}$$

$$= \frac{14.5}{900} = 0.0161$$

(ii) To find $P\left(\frac{A_3}{D}\right)$

$$P\left(\frac{A_3}{D}\right) = \frac{P(A_3).P(D/A_3)}{P(A_1)P\left(\frac{D}{A_1}\right) + P(A_2)P\left(\frac{D}{A_2}\right) + P(A_3)P\left(\frac{D}{A_3}\right)}$$

$$= \frac{\frac{4}{9} \times \frac{2}{100}}{\frac{145}{9000}} = \frac{8}{\cancel{900}} \times \frac{\cancel{9000}}{145} = \frac{80}{145} = 0.55$$

Ques At a certain University of Guna 4% of men are over 6 ft tall & 1% of women are over 6ft tall. The total students population is divided in ratio 3:2 in favour of women. If a student is selected at random among all those over 6 ft. What will be the probability that the student is the women.

$$P(M) = \frac{2}{5} \qquad P(W) = \frac{3}{5}$$

Let event of over 6 ft is D

$$P\left(\frac{D}{M}\right) = \frac{4}{100} \qquad P\left(\frac{D}{W}\right) = \frac{1}{100}$$

$$P(D) = P(M)\,P\left(\frac{D}{M}\right) + P(W).P\left(\frac{D}{W}\right)$$

$$= \frac{2}{5} \times \frac{4}{100} + \frac{3}{5} \times \frac{1}{100}$$

$$= \frac{8}{500} + \frac{3}{500} = \frac{11}{500}$$

$$P\left(\frac{W}{D}\right) = \frac{P(W) \times P\left(\frac{D}{W}\right)}{P(W).\,P\left(\frac{D}{W}\right) + P(M)\,P\left(\frac{D}{M}\right)} = \frac{\frac{3}{5} \times \frac{1}{100}}{\frac{11}{500}}$$

$$= \frac{3}{\cancel{500}} \times \frac{\cancel{500}}{11} = \frac{3}{11}$$

Ques A factory production in a manufacturing are using three machine A, B and C. of the total outcome machine A is responsible for 25%, machine B 35% and machine C is rest. It is known that 5% of the outcome from A is defective 4% from machine B and 2% from machine C. What is the probability that the bolt came from

1) Machine A
2) Machine B
3) Machine C

Let the D is event of defected item

$P(A) = \frac{25}{100}$ $P(B) = \frac{35}{100}$ $P(C) = \frac{40}{100}$

$P\left(\frac{D}{A}\right) = \frac{5}{100}$ $P\left(\frac{D}{B}\right) = \frac{4}{100}$ $P\left(\frac{D}{C}\right) = \frac{2}{100}$

$$P(D) = P(A) \cdot P\left(\frac{D}{A}\right) + P(B) \cdot P\left(\frac{D}{B}\right) + P(C) \cdot P\left(\frac{D}{C}\right)$$

$$= \frac{25}{100} \times \frac{\cancel{5}\,1}{\cancel{100}\,20} + \frac{35}{100} \times \frac{\cancel{4}\,1}{\cancel{100}\,25} + \frac{40}{100} \times \frac{\cancel{2}\,1}{\cancel{100}\,50}$$

$$= \frac{25}{2000} + \frac{35}{2500} + \frac{40}{5000} = \frac{1}{10000}(125 + 140 + 80)$$

$$= \frac{1}{10000} \times 345 = \frac{345}{10000}$$

(i) Machine A

$$P\left(\frac{A}{D}\right) = \frac{P(A) \times P\left(\frac{D}{A}\right)}{P(A) \cdot P\left(\frac{D}{A}\right) + P(B) \cdot P\left(\frac{D}{B}\right) + P(C)\, P\left(\frac{D}{C}\right)}$$

$$= \frac{\frac{25}{100} \times \frac{5}{100}}{\frac{345}{10000}} = \frac{125}{345} = 0.362$$

(ii) Machine B

$$P\left(\frac{B}{D}\right) = \frac{P(B) \times P\left(\frac{D}{B}\right)}{P(A) \cdot P\left(\frac{D}{A}\right) - - -}$$

$$= \frac{\frac{35}{\cancel{100}} \times \frac{4}{100}}{345/\cancel{10000}} = \frac{140}{345} = 0.405$$

(iii) Machine C

$$P\left(\frac{C}{D}\right) = \frac{\frac{40}{100} \times \frac{2}{100}}{345/10000}$$

$$= \frac{80}{345} = 0.231$$

Ques A Bag contains 6 Red marble and 4 Black marble 2 marble are drawn without replacement from the Bag. What is the probability that both the marble are black

$P(R) = \frac{6}{10}$ $P(B) = \frac{4}{10}$

Let A is event of drawing 1^{st} marble it black

$P(A) = \frac{4}{10}$

Let B is the event of drawing 2^{nd} marble is black

$P\left(\frac{B}{A}\right) = \frac{3}{9}$

P

Ques A lot contains 10 good articles with 4 minor defect and 2 with major defect. Two articles are taken from a lot at random without replacement then find the probability.

(i) Both are good
(ii) Both have major defects
(iii) atleast one is good
(iv) Neither has major defect
(v) Neither is good
(vi) Exactly is good

Let A is event of good articles
Let B is event of minor defect
Let C is event of major defect

(i) P(Both are good) = $\dfrac{\text{No. of ways drawing 2 good articles}}{\text{Total no. of ways drawing 2 articles}}$

$$= \frac{{}^{10}C_2}{{}^{16}C_2}$$

$$\frac{n!}{r!(n-r)!} = \frac{10!}{2!\,8!} \times \frac{2!\,14!}{16!}$$

$$= \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{2}_{1} \times 1 \times \cancel{8!}} \times \frac{\cancel{2}^{1} \times 1 \times \cancel{14!}}{\cancel{16}_{8} \times 15 \times \cancel{14!}}$$

$$= \frac{\cancel{45}\,\cancel{9}\,3}{\cancel{120}\,\cancel{24}\,8} = \frac{3}{8}$$

(ii) P(Both have major defect) = $\dfrac{\text{No. of ways drawing articles with major defect}}{\text{Total no. of ways drawing 2 articles}}$

$$= \frac{{}^{2}C_2}{{}^{16}C_2} = \frac{\cancel{2!}}{0!\,\cancel{2!}} \times \frac{14!\,2!}{16!}$$

$$= \frac{\cancel{14!} \times \cancel{2} \times 1}{\cancel{16}_{8} \times 15 \times \cancel{14!}}$$

$$= \frac{1}{120}$$

(iii) P(atleast one is good) = P(exactly one is good or both are good)

$$= {}^{10}C_1 \times {}^{6}C_1 + {}^{10}C_2$$

$$= \frac{10!}{1!\,9!} \times \frac{6!}{5!\,1!} + \frac{10!}{8!\,2!}$$

$$= \frac{10 \times \cancel{9!}}{1 \times \cancel{9!}} \times \frac{6 \times \cancel{5!}}{\cancel{5!} \times 1!} + \frac{\overset{5}{\cancel{10}} \times 9 \times \cancel{8!}}{\cancel{8!} \times \underset{1}{\cancel{2}} \times 1}$$

$$= \frac{60+45}{{}^{16}C_2}$$

$$= \frac{60+45}{120} = \frac{\overset{7}{\cancel{105}}}{\underset{8}{\cancel{120}}} = \frac{7}{8}$$

(iv) Neither has major defect $= \frac{{}^{14}C_2}{{}^{16}C_2} = \frac{\overset{7}{\cancel{14}} \times 13}{\underset{8}{\cancel{16}} \times 15} = \frac{91}{120}$

(v) P(Neither is good) $= \frac{{}^{6}C_2}{{}^{16}C_2} = \frac{6 \times 5}{16 \times 15} = \frac{\overset{1}{\cancel{30}}}{\underset{8}{\cancel{240}}} = \frac{1}{8}$

(vi) P(Exactly is good) = P(one is good & one is defect)

$$= \frac{{}^{10}C_1 \times {}^{6}C_1}{{}^{16}C_2} = \frac{\cancel{60}}{\underset{2}{\cancel{120}}} = \frac{1}{2}$$

Ques In a shooting test the probability of hitting a target is 1/2 for A, $\frac{2}{3}$ for B and 3/4 for C. If all them fire at the target. Find the probability that

1) None of them hit the target
2) Atleast one of them hit the target

$P(A) = 1/2$, $P(B) = \frac{2}{3}$, $P(C) = 3/4$

$P(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$, $P(\bar{B}) = 1 - \frac{2}{3} = \frac{1}{3}$, $P(\bar{C}) = 1 - \frac{3}{4} = \frac{1}{4}$

1) P(None of them hit the target)

$P(\bar{A}\cap\bar{B}\cap\bar{C}) = P(\bar{A}).P(\bar{B}).P(\bar{C})$

$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4} = \frac{1}{24}$

2) $1-P(\bar{A}\cap\bar{B}\cap\bar{C}) = 1-\frac{1}{24} = \frac{23}{24}$

Ques A bag contains 10 white and 3 Black ball. Another bag contain 3 white & 5 black. Two balls are drawn at random from 1st bag & placed in 2nd bag and then 1 ball is taken at random from 2nd bag. What is the probability that it is a white ball.

$B_1 = 2$ white, $B_2 = 2$ Black, $B_3 = 1W, 1B$

$P(B_1) = \frac{^{10}C_2}{^{13}C_2} = \frac{10\times9\times8!}{2\times1\times8!}\times\frac{11!\times2\times1}{13\times12\times11!} = \frac{15}{26}$

$P(B_2) = \frac{^{3}C_2}{^{13}C_2} = \frac{3\times2\times1!}{2!\times1}\times\frac{11!\times2\times1}{13\times12\times11!} = \frac{1}{26}$

$P(B_3) = \frac{^{10}C_1\times{}^{3}C_1}{^{13}C_2} = \frac{10\times9!}{9!\times1}\times\frac{3\times2!}{2!\times1}\times\frac{2}{26}\frac{1}{78} = \frac{30}{78} = \frac{5}{13}$

Let A be the event of getting white ball

$P(A|B_1) = \frac{P(\text{getting a white ball})}{5W, 5B} = \frac{^5C_2}{^{10}C_2} = 5/10$

$P(A|B_2) = \frac{3}{10}$, $P(A|B_3) = \frac{4}{10}$

$P(A) = P(B_1)\times P\left(\frac{A}{B_1}\right) + P(B_2)P\left(\frac{A}{B_2}\right) + P(B_3)P\left(\frac{A}{B_3}\right)$

$= \frac{15}{26}\times\frac{5}{10} + \frac{1}{26}\times\frac{3}{10} + \frac{5}{13}\times\frac{4}{10}$

$= \frac{15}{52} + \frac{3}{260} + \frac{4}{26} = \frac{15\times5+3+40}{260} = \frac{118}{260} = \frac{59}{130} = 0.453$ Ans

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Random Variables

A RV is a function that assigns a real number $X(s)$ to every elements $s \in S$ where S is the sample space corresponding random experiment E.

$$f: S \rightarrow \mathbb{R}$$

→ Types of Random Variable

1 Discrete Random Variable
2 Continuous Random Variable

Discrete Random Variable (DRV)

If X is a RV which can take a finite number or countably infinite value of x, then X is called DRV.

When RV is discrete the possible value of X may be assumed as $x_1, x_2, \ldots x_n, \ldots$

Probability Function

If X is a random variable which can take values $x_1, x_2 \ldots x_n$ such that $P(X = x_i) = P_i$ then P_i is called the PF or Probability mass function provided it satisfied two property

(i) $P_i \geq 0$
(ii) $\sum_i P_i = 1$

Example:- If we toss a two coin simultaneously then sample space $S = \{HH, HT, TH, TT\}$

X = No. of Heads

$X(HH) = 2$
$X(HT) = X(TH) = 1$
$X(TT) = 0$

$P(\text{No Head}) = P(X=0) = \frac{1}{4}$

$P(\text{one head}) = P(X=1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(\text{two head}) = P(X=2) = \frac{1}{4}$

Ques If we are tossing a three coin & RV X represents the no. of tails then find the PMF along with RV.

S = HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

X :- No. of Tails

X(HHH) = 0

X(HHT) = 1

X(HTH) = X(THH) = 1

X(HTT) = X(THT) = X(TTH) = 2

X(TTT) = 3

P(No Tails) = $P(X=0) = \frac{1}{8}$

P(Two Tails) = $P(X=2) = \frac{3}{8}$

P(one Tails) = $P(X=1) = \frac{3}{8}$

P(three tail) = $P(X=3) = \frac{1}{8}$

Continuous Random Variable (CRV)

If X is a RV which can take all the values (Infinite no. of value) within the interval then it is known as CRV.

Probability Density Function (PDF)

If X is a CRV such that probability$\{x - \frac{1}{2}dx \le X \le x + \frac{1}{2}dx\}$ $= f(x)\,dx$ then $f(x)$ is called the (PDF). It satisfy two condition.

(i) $f(x) \ge 0,\ x \in R_x$

(ii) $\int_{R_x} f(x) = 1$

Mean $(E(x)) = \sum x . p_x$

Variance $= E(x^2) - [E(x)]^2$

$\hookrightarrow \sum x^2 . p(x)$

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Cummulative Distributive Function (CDF)

If X is a RV whether Discrete or Continuous then probability of $P(X \le x)$ is called the CDF and it is denoted as $F(x)$.

Continuous :-

$$\boxed{F(x) = \sum_{X \le x} p_i} \qquad \boxed{* \; F(x) = P(x \le x) = \int_0^x f(x)\,dx}$$

Ques A RV X has the following probability distribution

x	-2	-1	0	1	2	3
$P(x)$	0.1	K	0.2	2K	0.3	3K

Find (i) find K

(ii) Evaluate $P(X<2)$ and $P(-2<x\le 2)$

(iii) Find CDF of X.

(i) $P(x) = 0.1 + K + 0.2 + 2K + 0.3 + 3K = 1$

$0.6 + 6K = 1$

$6K = 1 - 0.6$

$6K = 0.4$

$K = \frac{0.\cancel{4}\,1}{\cancel{60}\,15}$

$K = \frac{1}{15}$

(ii)

x	-2	-1	0	1	2	3
$P(x)$	0.1	$\frac{1}{15}$	0.2	$\frac{2}{15}$	0.3	$\frac{1}{5}$

(Mutually Exclusive event)

$P(X = -2, -1, 0 \text{ or } 1)$

$= 0.1 + \frac{1}{15} + 0.2 + \frac{2}{15} = 0.3 + \frac{\cancel{3}\,1}{\cancel{15}\,5}$

$= \cancel{0.15}\ \frac{1.5+1}{5} = \frac{2.5}{5} = 0.5$

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$$P(X=-1,0,1) = \frac{1}{15} + 0.2 + \frac{2}{15}$$
$$= 0.2 + \frac{3}{15}$$
$$= 0.2 + \frac{1}{5}$$
$$= \frac{1+1}{5} = \frac{2}{5} = 0.4$$

(iii) $F(x) = 0 \quad x < -2$
$= 0.1 \quad -2 \leq x < -1$
$= 0.167 \quad -1 \leq x < 0$
$= 0.367 \quad 0 \leq x < 1$
$= 0.501 \quad 1 < x < 2$
$= 0.801 \quad 2 < x < 3$
$= 1 \quad 3 \leq x < 2$

11-08-25

Ques- A RV X has a following distribution:

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$
	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$3/10$	$1/100$	$\frac{2}{100}$	$\frac{17}{100}$

(i) Find K
(ii) $P(1.5 < X < 4.5 / x > 2)$
(iii) The smallest value of λ for which $P(x \leq \lambda) > \frac{1}{2}$

(i) $P(x) = 0 + K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K + 2K$
$\cancel{8K + 10K^2 + 2 = 0}$ $\quad 9K + 10K^2 = 1$
$\cancel{3K + 5K^2 + 1 = 0}$ $\quad 10K^2 + 9K - 1 = 0$
$10K^2 + 10K - K - 1 = 0$
$10K(K+1) - (K+1) = 0$
$(10K-1)(K+1) = 0$
$K = \frac{1}{10}, K = 1$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{P(1.5 < X < 4.5) \cap (X > 2)}{P(X > 2)}$$

$$= \frac{P(X=3) + P(X=4)}{\Sigma} = \frac{\frac{2}{10} + \frac{3}{10}}{\frac{7\cancel{0}}{10\cancel{0}}} = \frac{\frac{5}{\cancel{10}}}{\frac{7}{\cancel{10}}} = \frac{5}{7}$$

(iii) $P(x \le 0) = 0$ $\quad P(x \le 2) = \frac{3}{10}$

$P(x \le 1) = \frac{1}{10}$ $\quad P(x \le 3) = \frac{5}{10}$

$P(x \le 4) = \frac{8}{10}$

$\boxed{\lambda = 4}$ The smallest value of λ which holds the possible value of $\lambda = 4$.

Ques If $p(x) = \begin{cases} xe^{-x^2/2} & ; \; x \ge 0 \\ 0 & ; \; x < 0 \end{cases}$

(i) Show that this $p(x)$ is a PDF.

(ii) Find its Distribution F^n $P(x)$.

$$\int_0^\infty xe^{-x^2/2}\,dx$$

put $t = \frac{x^2}{2}$

$dt = x\,dx$

$$\int_0^\infty e^{-t}\,dt = -e^{-t}\Big|_0^\infty$$

$\cancel{\frac{-e^{-t}}{-t+1}}$ $\quad = 0 - (-1)$

$= 1$

(ii) $F(x) = P(X \leq x) = \int_0^x f(x)\,dx$

$F(x) = 0;\ x < 0$

$$F(x) = \int_0^x x e^{-x^2/2}\,dx;\ x \geq 0$$

put $t = \frac{x^2}{2}$

$dt = x\,dx$

$$\int_0^x e^{-t}\,dt = -e^{-t}\Big|_0^x = -e^{-x^2/2}\Big|_0^x$$

$$= 1 - e^{-x^2/2}$$

Q If a Density f^n of a Continuous RV X is given by

$f(x) = ax \quad 0 \leq x \leq 1 \quad x/2$

$= a \quad 1 \leq x \leq 2 \quad 1/2$

~~(i)~~ $= 3a - ax \quad 2 \leq x \leq 3 \quad 3 \times \frac{1}{2} - \frac{1}{2} \times a = \frac{3}{2} - \frac{a}{2} = \frac{3-a}{2}$

$= 0 \quad$ elsewhere

(i) Find the value of 'a'

(ii) Find cdf

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 3a - ax\,dx + \int$$

$$a\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^2 + a\left[3x - \frac{x^2}{2}\right]_2^3$$

$$a\left[\frac{1}{4}\right] + a[4-1] + a\left(\left(9 - \frac{9}{2}\right) - \left(6 - \frac{4}{2}\right)\right)$$

$$\frac{a}{4} + 3a + a\left(\left[\frac{9}{2}\right] - \left(\left[4a\right]\right)\right)$$

$$\frac{a}{4} + 3a + \frac{9a}{2} - 4a = \frac{2a + 24a + 36a - 32a}{8}$$

$$\frac{a + 12a + 18a - 16a}{4} = \frac{40}{8}a \quad \frac{30}{4}$$

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$0 \le x \le 1 \quad F(x) = \int_0^x f(x)\,dx$

$$= \int_0^x \frac{x}{2}\,dx = \frac{x^2}{4}$$

$$\int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$$

$$= \int_0^1 \frac{x}{2}\,dx + \int_1^x \frac{1}{2}\,dx = x^2$$

$$\int_0^1$$

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx$$

$$a\frac{x^2}{2}\Big|_0^1 + ax\Big|_1^2 + \left[\frac{-ax^2}{2} + 3ax\right]_2^3$$

$$\frac{a}{2} + 2a - a - \frac{9}{2}a + 9a + 2a - 6a = 1$$

$$\frac{a}{2} - \frac{9a}{2} + 6a = 1$$

$$2a = 1 \Rightarrow a = 1/2$$

Q A CRV X can assume any value b/w $x=2$ & $x=5$ has Density f^n given by $f(x) = K(1+x)$. Find the $P(X<4)$.

$$\int_2^5 K(1+x)$$

$$K\int_2^5 (1+x)\,dx$$

$$K\left[x+\frac{x^2}{2}\right]_2^5$$

$$\cancel{K\left[2+\frac{4}{2}-5-\frac{25}{2}\right]} \qquad K\left[5+\frac{25}{2}-2-\frac{4}{2}\right]$$

$$\cancel{K\left[\frac{4+4-10-25}{2}\right]} \qquad K\left[\frac{10+25-4-4}{2}\right]$$

$$\cancel{K\left[\frac{8-35}{2}\right]=1} \qquad K\left[\frac{35-8}{2}\right]$$

$$\cancel{K\left[\frac{27}{2}\right]=1} \qquad K\left[\frac{27}{2}\right]=1$$

$$\cancel{\frac{27K}{2}=1} \qquad K=\frac{2}{27}$$

$$\cancel{K=\frac{2}{27}}$$

$$f(x)=\frac{2}{27}(1+x)$$

$$P(2<X<4)=\int_2^4 \frac{2}{27}(1+x)\,dx$$

$$\frac{2}{27}\int_2^4 (1+x)\,dx = \frac{2}{27}\left[x+\frac{x^2}{2}\right]_2^4$$

$\frac{2}{27}\left[\cancel{2+\frac{4}{2}-4-\frac{16}{2}}\right]$

$e^{-\infty} \to \infty \to \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$e^{-x} \to 0 \to e^{-0} = \frac{1}{e^{0}} = \frac{1}{1} = 1$

$\frac{2}{27} \times \left[\frac{4+4-8-16}{2}\right]$

$= \cancel{\frac{2}{27} \times \frac{16}{2}} = 1$

$\frac{2}{27}\left[4+8-2-2\right]$

$= \frac{16}{27}$

$\frac{2}{27}\left[8\right] = \frac{16}{27}$

Q A CRV X has a PDF $f(x) = Kx^2e^{-x}, x \geq 0$

Find K.

$K = \frac{1}{2}$

$$f(x) = \int_0^{\infty} Kx^2e^{-x}\,dx = 1$$

$$K\int_0^{\infty} x^2e^{-x}\,dx$$

$$= x^2\int e^{-x}dx - \int \frac{d}{dx}(x^2)\int e^{-x}dx$$

$$= -x^2.e^{-x} - \int -2x.e^{-x}\,dx$$

$$= -x^2e^{-x} + \left[2x(e^{-x}) - \int 2e^{-x}\,dx\right]$$

$$\cancel{= -x^2e^{-x} + \left[2x(e^{-x}) + (2e^{-x}\right]_0^{\infty}}$$

$$= -x^2.e^{-x} + \left[2x(-e^{-x}) - \int 2(e^{-x})dx\right]$$

$$= -x^2.e^{-x} + \left[-2xe^{-x} + 2e^{-x}\right]$$

$$= -x^2.e^{-x} - 2xe^{-x} + 2e^{-x}$$

$$= e^{-x}\left[-x^2 - 2x + 2\right]_0^{\infty}$$

$$= \cancel{0[0} - (x^2 + 2x$$

$= -x^2e^{-x} + 2(x(-e^{-x}) - e^{-x})$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x}$

$= e^{-x}(-x^2 - 2x - 2)$

$= -e^{-x}(x^2 + 2x + 2)$

$K + (+2) = 1$

$2K = 1$

$\boxed{K = 1/2}$

Ans

Ques A CRV has a PDF $f(x) = 3x^2 \quad 0 \le x \le 1$ then find a & B. such that

(i) $P(X \le a) = P(X > a)$

(ii) $P(X > b) = 0.05$

(i)
$$\int_0^a 3x^2 dx = \int_a^1 3x^2 dx \qquad \text{(ii)}$$

$$3 \cdot \frac{x^3}{3}\Big|_0^a = 3 \frac{x^3}{3}\Big|_a^1$$

$$\cancel{3}\left[\frac{-a^3}{\cancel{3}}\right] = 3\left[\frac{a^3}{3} - \frac{1}{3}\right]$$

$$-a^3 = \frac{\cancel{3}(a^3-1)}{\cancel{3}}$$

$$-a^3 = a^3 - 1$$

$$-a^3 - a^3 = -1$$

$$\cancel{-}2a^3 = \cancel{-}1$$

$$a^3 = \frac{1}{2}$$

$$a = \sqrt[3]{1/2}$$

(ii)
$$\int_b^1 x^2 dx = \frac{x^3}{3}\Big|_b^1 = 0.05$$

$$0.05 = \frac{\cancel{b^3}}{\cancel{3}} - \frac{\cancel{1}}{\cancel{3}} \quad \frac{1}{3} - \frac{b^3}{3}$$

$$\Rightarrow 0.05 = \cancel{b^3 -} \frac{1-b^3}{3}$$

$$\cancel{0.05 + 1 = b^3}$$

$$\cancel{1.05 = b^3} \quad b^3 = 0.95$$

$$\cancel{b = \sqrt{1.45}} \quad b = \sqrt[3]{0.95}$$

$$0.15 = 1 - b^3$$

$$b^3 = 1 - 0.15$$

$$b^3 = 0.85$$

$$b = \sqrt[3]{0.85}$$

Ques A RV X has a PDF given by

$$f(x) = \begin{cases} Cxe^{-x}, & x > 0 \\ 0 & x \le 0 \end{cases}$$

(i) Find the value of C

(ii) Find its CDF

8.

$$\int_0^\infty Cxe^{-x}$$

$1 \mp e^{-x}(1+x)$

$\cancel{C\int(x e^{-x})\,dx}$

$\cancel{xe^{-x}\,dx - \int 1\,e^{-x}\,dx}$

$\cancel{xe^{-x} - e^{-x}}$

$\cancel{C\left[e^{-x}(x-1)\right]_0^\infty}$

$\cancel{C\left[-1-(0)\right] = 1}$

$\cancel{-C = 1}$

$$C\int(xe^{-x})\,dx$$

$$-xe^{-x} - \int 1(-e^{-x})\,dx$$

$C=1$

$$-xe^{-x} - e^{-x}\Big|_0^\infty$$

$$C\left[(-\infty e^{-\infty} - e^{-\infty}) - (-0\times e^{-0} - e^{-0})\right]$$

$$C\left[-(-1)\right] = 1$$

$\boxed{C=1}$ Ans

(ii) $F(x) = \int_0^x f(x)\,dx \;,\; x>0$

$$= \int_0^x xe^{-x}\,dx$$

$$= xe^{-x} - \int 1\cdot e^{-x}\,dx$$

$$= -xe^{-x} - e^{-x}\Big|_0^x$$

$\cancel{= e^{-x}(-x-1)}$

$\cancel{= -e^{-x}(1+x)}$

$$\left[-xe^{-x} - e^{-x} - (-1)\right]$$

$$-xe^{-x} - e^{-x} + 1$$

$$1 - e^{-x}(1+x)$$

Q. A PDF of RV X is given by

$$f(x) = \begin{cases} 2x, & 0 \le x < b \\ 0, & x > b \text{ / elsewhere} \end{cases}$$

$\int_{-\infty}^{\infty} f(x)\, dx$

$\int_{-\infty}^{0} \int_{0}^{b}$

For which value of b this PDF $f(x)$ is valid also find the CDF of RV X with about

$$\int_0^b (2x)\, dx$$

$$\cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_0^b = 1$$

$$\cancel{0} - b^2 = 1$$

$$b^2 = 1$$

$$b = 1$$

(i) $0 \le x < 1$

(ii) $x < 0$

(iii) $x \ge 1$

CDF: $F(x) = \int_0^x f(x)\, dx$

(i) $\int_0^x 2x\, dx = x^2$

(ii) $\int_{-\infty}^{x} 0\, dx = 0$

(iii) $\int_{-\infty}^{0} f(x)\, dx + \int_0^1 f(x)\, dx + \int_1^x f(x)\, dx$

$$0 + \int_0^1 2x\, dx + 0$$

$$= 2 \left.\frac{x^2}{2}\right|_0^1$$

$$= \cancel{2}\left[\frac{1}{\cancel{2}}\right]$$

$$= 1$$

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Ques A Random Variable X has a following probability distribution

x:	-2	-1	0	1	2	3
$p(x)$:	0.1	K	0.2	2K 0.3	0.3	3K

(i) Find $P((-2<X<3)/x<1)$ $\frac{1}{15}$ 0.2 $\frac{2}{15}$ $0.1+\frac{1}{15}+0.2$

(ii) Mean of x.

$p(x) = 0.1 + K + 0.2 + 0.3 + 2K + 3K$

$\Rightarrow 0.6 + 6K = 1$

$6K = 1 - 0.6$

$6K = 0.4$

$K = \frac{0.4}{60}$ $\frac{\cancel{60}}{\cancel{30}\ 15}$

$K = \frac{1}{15}$

$$P(-2<X<3/X<1) = \frac{P(-2<X<3)\cap(X<1)}{P(X<1)}$$

$$= \frac{\frac{0.1}{15}+\frac{0.2}{10}}{\frac{5.5}{15}} = \frac{\frac{10+30}{150}}{\frac{5.5}{15}} = \frac{\frac{4\cancel{0}}{15\cancel{0}}}{\frac{5.5}{\cancel{15}}}$$

$$= \frac{\cancel{40}}{\cancel{565}} = 0.72$$

(ii) Mean $(E(x)) = \Sigma x.p(x)$

$= -2\times0.1 + (-1)\times\frac{1}{15} + \frac{2}{15} + 0.6 + \frac{9}{15}$

$= -0.2 - \frac{1}{15} + \frac{2}{15} + 0.6 + \frac{9}{15}$

$= \frac{-3-1+2+9+9}{15} = \frac{20-4}{15} = \frac{\cancel{16}}{15} = \frac{16}{15}$

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Q X has the probability density function

$$f(x) = \begin{cases} Ke^{-3x} & , x \geq 0 \\ 0 & , \text{elsewhere} \end{cases}$$

Find the value of K and $P(0.5 \leq X \leq 1)$ also find the mean of x.

$$\int_0^\infty (Ke^{-3x})\,dx = -K e^{-3x}\Big|_0^\infty$$

$$= -\frac{K}{3}\left[e^{-3\infty} - e^{-3\times 0}\right]$$

$$= \cancel{+}\frac{K}{3}[0 + 1]$$

$$1 = \frac{K}{3}$$

$$\boxed{K = 3}$$

Now $P(0.5 \leq X \cancel{\leq} 1) = \int_{0.5}^{1} (3e^{-3x})\,dx$

$$= 3\int_{0.5}^{1} e^{-3x}$$

$$= -\frac{\cancel{3}}{\cancel{3}} e^{-3x}\Big|_{0.5}^{1}$$

$$= -e^{-3\times 1} - e^{+3\times 0.5}$$

$$= -e^{-3} - \cancel{e^{+1.5}}\ 1$$

$$\cancel{= 0.0149 - 4.481}\ 1$$

$$= -e^{-3} - 1$$

$$= -1 - \frac{1}{e^3}$$

Q Find the mean and variance of following prob. distributio.

x:	0	1	2	3	4	5	6	7
$p(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

$$K+2K+2K+3K+K^2+2K^2+7K^2+K=1$$

$$9K+10K^2-1=0$$

$$10K^2+10K-K-1=0$$

$$10K(K+1)-1(K+1)=0$$

$$(10K-1)(K+1)=0$$

$$K=\frac{1}{10},\ K=-1$$

3.01

$$\text{Mean}=\frac{1}{10}+\frac{4}{10}+\frac{6}{10}+\frac{12}{10}+\frac{5}{100}+\frac{12}{100}+\frac{49}{100}+\frac{7}{10}$$

$$=\frac{30}{10}+\frac{49+12+5}{100}$$

31.04

16.8

$$=\frac{30}{10}+\frac{66}{100}=\frac{3000+660}{1000}=\frac{3660}{1000}=3.66$$

1
17
49
6

$$= \cancel{140-12.6736=127.32}\quad \boxed{\text{Mean}=3.66}$$

x	x^2
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
	140

→ Calculate $E(X^2)=\Sigma x^2 \cdot p(x)$

$$E(X^2)=0^2\cdot 0+1\times\frac{1}{10}+4\times\frac{2}{10}+9\times\frac{2}{10}+16\times\frac{3}{10}+$$

$$25\times\frac{1}{100}+36\times\frac{2}{100}+49\left(\frac{7}{100}+\frac{1}{10}\right)$$

$$=0+\frac{1}{10}+\frac{8}{10}+\frac{18}{10}+\frac{48}{10}+\frac{25}{100}+\frac{72}{100}+\frac{343}{100}+\frac{49}{10}$$

$$=\frac{124}{10}+\frac{440}{100}=\frac{168}{100}=16.8$$

$$\boxed{E(X^2)=16.8}$$

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$$\text{Variance} = E(x^3) - (Ex)^2$$
$$= 16.8 - (3.66)^2$$
$$= 16.8 - 13.3956$$
$$= 3.4044$$

$$\boxed{\text{Variance} = 3.4044}$$