

Tutorial - 2

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Q. If A and B are independent events then show that

- (i) A and \bar{B} are independent events
- (ii) \bar{A} and B " "
- (iii) \bar{A} and \bar{B} " "
- (iv) $P(B) = P(A) \cdot P(B/A) + P(\bar{A}) \cdot P(B/\bar{A})$, $A \neq \emptyset$
- (v) $P(A/B) + P(\bar{A}/\bar{B}) = 1$
- (vi) $P(A/B) = 1$ if $B \subseteq A$

Soln.

We are given that A and B are independent events

$$\text{then, } P(A \cap B) = P(A) \times P(B) \quad \text{--- (i)}$$

(i) T.P.T. A and \bar{B} are independent events
it can be proved if

L.H.S $P(A \cap \bar{B}) = P(A) \times P(\bar{B})$

$$\Rightarrow P(A) \cap P(\bar{B})$$

$$\Rightarrow P(A) \cap [P(1 - B)] (1 - P(B))$$

$$\Rightarrow P(A) - P(A \cap B)$$

As A and B are independent events, using (i)
this eqn. can be written as

$$\Rightarrow P(A) - P(A) \cdot P(B)$$

$$\Rightarrow P(A) (1 - P(B))$$

$$\Rightarrow P(A) P(\bar{B}) = \text{RHS} \quad \text{Hence proved}$$

(ii) \bar{A} and B are independent if

$$P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B)$$

$$P(\bar{A}) \cap P(B)$$

$$P(B) \cap P(1 - A)$$

$$P(B) - P(B) \cdot P(A)$$

$$P(B) (1 - P(A))$$

$$P(B) \cdot P(\bar{A}) = \text{RHS}$$

(iii) \bar{A} and \bar{B} are independent events if

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$$

$$= P(\bar{A}) \cdot P(B)$$

$$= P(1 - A) \cdot P(1 - B)$$

$$P(\bar{A} \cap \bar{B}) = P\{\overline{(A \cup B)}\} \text{ by De Morgan's Law}$$

$$= 1 - P(A \cup B)$$

$$= 1 - \{P(A) + P(B) - P(A \cap B)\}$$

$$= 1 - \{P(A) + P(B) - P(A) \cdot P(B)\} \text{ why?}$$

$$= 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$= (1 - P(A)) - P(B) \{1 - P(A)\}$$

$$= (1 - P(A))(1 - P(B))$$

$$= P(\bar{A}) \cdot P(\bar{B}) \quad \text{Hence proved.}$$

(iv) $P(B) = P(A) \cdot P(B/A) + P(\bar{A}) \cdot P(B/\bar{A})$

From Bayes' Rule
Theorem $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{P(A) \cdot P(B)}{P(B)} \quad \text{from (i)}$$

similarly, $P(B/A) = P(B)$ $= P(A)$

$\therefore P(B) = P(A) \cdot P(B) + \text{similarly}$

$$P(B/\bar{A}) \rightarrow \frac{P(B \cap \bar{A})}{P(\bar{A})} = \frac{P(B) \cdot P(\bar{A})}{P(\bar{A})}$$

$$= P(B)$$

RHS can be written as.

$$= P(A) \cdot P(B) + P(\bar{A}) \cdot P(B)$$

$$= P(B) \cdot (P(A) + P(\bar{A}))$$

$$= P(B) \times 1$$

$$= P(B)$$

$$(v) P(A/B) + P(\bar{A}/B) = 1$$

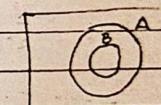
$$\text{LHS} = \frac{P(A \cap B)}{P(B)} + \frac{P(\bar{A} \cap B)}{P(B)} \quad \text{from Bayes' Th}$$

$$= \frac{P(A) \cdot P(B)}{P(B)} + \frac{P(\bar{A}) \cdot P(B)}{P(B)}$$

$$= P(A) + P(\bar{A})$$

$$= 1$$

(vi) $P(A/B) = 1 \text{ if } B \subseteq A$



$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Q2 If $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$

Then find (i) $P(A/B)$ (ii) $P(\bar{A}/B)$ (iii) $P(\bar{B}/A)$

$$(i) P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{3}{4} = \frac{3}{8} + \frac{5}{8} - P(A \cap B)$$

$$P(A \cap B) = \frac{3}{8} + \frac{5}{8} - \frac{3}{4} = \frac{1}{4}$$

$$\therefore P(A/B) = \frac{\frac{1}{4}}{\frac{5}{8}} = \frac{1 \times 8}{5 \times 4} = \frac{2}{5}$$

$$(ii) P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

$$= \frac{P(A \cup B)}{P(\bar{B})}$$

$$= \frac{P(A \cup B)}{P(1 - P(B))} = \frac{1 - P(A \cap B)}{1 - P(B)}$$

$$= \frac{1/4}{2/8} = \frac{2}{3}$$

$$(iii) P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{3}{4} = \frac{3}{8} + \frac{5}{8} - P(A \cap B)$$

$$\therefore P(A \cap B) = \frac{3}{4} + \frac{3}{8} + \frac{5}{8} = \frac{-6+3+5}{8} = \frac{1}{4}$$

$$\therefore P(B/A) = \frac{\frac{1}{4}}{\frac{3/8}{2/8}} = \frac{8/2}{3 \times 4} = \frac{2}{2}$$

Q3 A bolt is manufactured by 3 machines A, B, C. Machine A turns out twice as many items as B and machines B and C produce equal no. of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and 1 is chosen from the pile. What is the probability that it is defective?

Rephrased

Let, A = the event in which the item is produced by machine A and so on.

$$B = , C =$$

D = The event of the item being defective.

$$P(A) = \frac{1}{2}, P(B) = P(C) = \frac{1}{4}$$

$P(D/A) = P(\text{an item is defective given that A has produced it})$

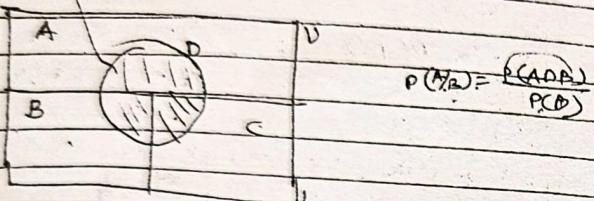
$$= \frac{2}{100} = P(D/B)$$

$$P(D/C) = \frac{4}{100}$$

$$P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$$

By theorem of total probability

$$\begin{aligned} P(D) &= P(A) \times P(D/A) + P(B) \times P(D/B) + P(C) \times P(D/C) \\ &= \frac{1}{2} \times \frac{2}{100} + \frac{1}{4} \times \frac{2}{100} + \frac{1}{4} \times \frac{4}{100} \\ &= \frac{1}{40} \end{aligned}$$



$$P(D/A) = \frac{P(D \cap A)}{P(A)}$$

$$\begin{aligned} P(D/A) &= \frac{P(D \cap A)}{P(A)}, \quad P(D/B) = \frac{P(D \cap B)}{P(B)} \quad \text{and} \\ P(D/C) &= \frac{P(D \cap C)}{P(C)} \end{aligned}$$

By theorem total prob. of defective

$$\begin{aligned} P(D) &= P(D \cap A) + P(D \cap B) + P(D \cap C) \\ &= P(A) \times P(D/A) + P(B) \times P(D/B) + P(C) \times P(D/C) \\ &= \frac{1}{2} \times \frac{2}{100} + \frac{1}{4} \times \frac{2}{100} + \frac{1}{4} \times \frac{4}{100} \\ &= \frac{1}{40} \end{aligned}$$

Q4

An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball?

Soln The two balls transferred may be both white or both black or 1 white and 1 black.

Let,

B_1 = Event of drawing 2 w balls from the 1st urn

B_2 = Event of drawing 2 b balls from the 1st urn

B_3 = Event of drawing 1 w and 1 b ball from the 1st urn

Clearly B_1, B_2, B_3 are exhaustive and mutually exclusive events

Let; A be = Event of drawing 1 ball from the 2nd urn after transfer.

$$\begin{aligned} P(B_1) &= \frac{10C_2}{13C_2}; \quad P(B_2) = \frac{3C_2}{13C_2}; \quad P(B_3) = \frac{10C_1 \times 3C_1}{13C_2} \\ &= \frac{15}{26} \quad ; \quad = \frac{1}{26} \quad = \frac{10}{26} \end{aligned}$$

$$P(A|B_1) = P(\text{drawing a white ball}/\text{2 white balls have been transferred})$$

$$\begin{aligned} &= P(\text{Drawing a white ball}/\text{urn 2 contains 5 white}) \\ &= \frac{5}{10} \end{aligned}$$

$$\text{similarly } P(A|B_2) = \frac{3}{10} \quad \& \quad P(A|B_3) = \frac{1}{10}$$

By theorem of total probability

$$P(A) = P(B_1) \times P(A|B_1) + P(B_2) \times P(A|B_2) + P(B_3) \times P(A|B_3)$$

Total Probability

$$P(A) = \frac{15}{50} \times \frac{5}{10} + \frac{1}{25} \times \frac{3}{10} + \frac{10}{25} \times \frac{4}{10}$$

$$= \frac{59}{120}$$

Q 5 In a certain city, 40% of the people consider themselves conservative (C), 35% consider themselves to be liberals (L), and 25% consider themselves to be independent (I). During a particular election, 45% of the conservative voted, 40% of the liberals voted and 60% of the independent voted.

Suppose that a person is randomly selected

- Find the probability that the person voted.
- If the person voted, find the probability that the voter is (i) conservative (ii) liberal (iii) independent.

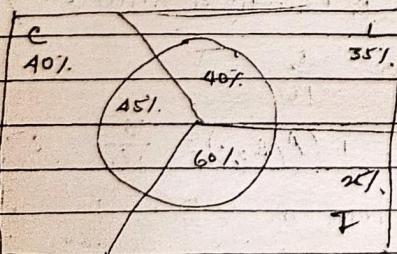
Soln. Let C - conservative people

L = Liberals

I = Independent

A = a selected person.

- (a) The probability that the selected person has voted = $P(A) = \frac{45}{100} \times \frac{40}{100} = 0.18$



(a) Let,

$P(C)$ is the prob. that a person selected at random is conservative = $\frac{40}{100}$

similarly $P(L) = \frac{35}{100}$ & $P(I) = \frac{25}{100}$

Let $P(V/C)$ is the prob. that a person voted is conservative = $\frac{45}{100}$

similarly $P(V/L) = \frac{40}{100}$ & $P(V/I) = \frac{60}{100}$

Now Prob. that a person has voted

$$P(V) = P(C) \times P(V/C) + P(L) \times P(V/L) + P(I) \times P(V/I)$$
$$= 0.4 \times 0.45 + 0.35 \times 0.4 + 0.25 \times 0.6$$

$$P(V) = 0.47$$

(b) (i) If a person has voted, and he is conservative = $P(C/V) = \frac{P(C \cap V)}{P(V)}$

$$= \frac{0.45}{0.47} = 0.95$$

(ii) similarly, $P(L/V) = \frac{P(L \cap V)}{P(V)} = \frac{0.4}{0.47} = 0.85$

$$P(I/V) = \frac{P(I \cap V)}{P(V)} = \frac{0.6}{0.47} = 1.27$$

$$P(C/V) = \frac{P(C) \times P(V/C)}{P(V)} = \frac{0.4 \times 0.45}{0.47} = 0.39$$

$$\text{Similarly } P(L/V) = \frac{P(L) \times P(V/L)}{P(V)}$$

$$= \frac{0.35 \times 0.4}{0.47} = 0.2978 \approx 0.30$$

$$\text{and } P(I/v) = \frac{P(I) \times P(v/I)}{P(v)}$$

$$= \frac{0.25 \times 0.6}{0.47} = 0.319$$

Q6 Three machines A, B, and C produce respectively 40%, 10%, and 50% of the items in a factory. The percentage of defective items produced by the machines are respectively 2%, 3%, and 4%. An item from the factory is selected at random.

- (a) Find the prob. that the item is defective
 (b) If the item is defective find the probability that the item was produced by (i) M/c A (ii) M/c B (iii) M/c C

Soln (a) Let B be the event that the part produced is defective and.

A_i be the event the relay is manufactured by the plant i ($i = 1, 2, 3$). The desired prob. P(B) is given by

$$P(B) = \sum_{i=1}^3 P(B/A_i) \cdot P(A_i)$$

$$P(B) = P(B/A_1) \cdot P(A_1) + P(B/A_2) \cdot P(A_2) + P(B/A_3) \cdot P(A_3)$$

$$= \frac{0.02 \times 0.4}{0.02 \times 0.4 + 0.03 \times 0.1 + 0.04 \times 0.5} = 0.031$$

(b) The derived prob. are

$$(i) P(A_1/B) \quad (ii) P(A_2/B) \text{ and } (iii) P(A_3/B)$$

$$(i) P(A_1/B) = \frac{P(B/A_1) \cdot P(A_1)}{P(B)} = \frac{0.02 \times 0.4}{0.021} = 0.258$$

$$P(A_2/B) = \frac{P(B/A_2) \cdot P(A_2)}{P(B)} = \frac{0.03 \times 0.1}{0.021} = 0.095$$

$$P(A_3/B) = \frac{P(B/A_3) \cdot P(A_3)}{P(B)} = \frac{0.04 \times 0.5}{0.021} = 0.64$$

Q7 Three boxes of the same appearance have the following proportion of balls.

I 2 black 1 white

II 1 black 2 white

III 2 black 2 white

One of the urn is selected and one ball is drawn. It turns out to be white. What is the probability of drawing white ball again, if the first is not replaced

Soln.

Let,
 A₁ = the even that the first ball is chosen from first

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Q1 The RV X has the following probability distn:

$$x : -2 \quad -1 \quad 0 \quad 1$$

$$P(x) : 0.4 \quad k \quad 0.2 \quad 0.3$$

Find k and mean value of X .

Soln. Since $\sum P(x) = 1$, $k + 0.9 = 1$
 $\therefore k = 0.1$

$$\text{Mean } E(X) = \sum x P(x)$$

$$= (-2 \times 0.4) + (-1 \times 0.1) + (0 \times 0.2) + (1 \times 0.3)$$

$$= -0.6$$

Q2 A shipment of 6 TV sets contain 2 defective sets. A hotel makes a random purchase of 3 of the sets. Find the prob. distri of X , if X is the no. of defective sets.

Soln. The no of defectives can be 0, 1 and 2.

$$\text{Prob. of purchasing 0 defective sets} = \frac{4C_3}{6C_3} = \frac{4}{20} = \frac{1}{5}$$

$$\text{Similarly } 1 \text{ defective set} = \frac{2C_1 \times 4C_2}{6C_3} = \frac{12}{20}$$

$$2 \text{ defective sets} = \frac{2C_2 \times 4C_1}{6C_3} = \frac{4}{20}$$

$$\begin{array}{cccc} X & 0 & 1 & 2 \\ P(x) & \frac{4}{20} & \frac{12}{20} & \frac{4}{20} \end{array}$$

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Q3 A random variable x may assume 4 values with probability $\frac{1+3x}{4}, \frac{1-x}{4}, \frac{1+2x}{4}, \frac{1-2x}{4}$. Find the condition on x so that these values represent the probability of x .

Soln. $P(X=x_i) = p_i = (1+3x)/4$
 $p_2 = (1-x)/4$
 $p_3 = (1+2x)/4$
 $p_4 = (1-2x)/4$

If the given probabilities represent a probability function, each $p_i \geq 0$ and $\sum p_i = 1$

In this problem, $p_1 + p_2 + p_3 + p_4 = 1$, for any x

But $p_1 \geq 0$, if $x \geq -\frac{1}{3}$

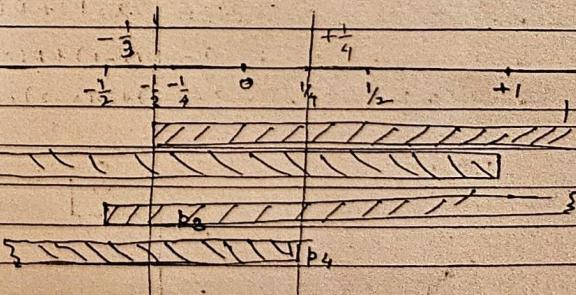
$p_2 \geq 0$, if $x \leq 1$

$p_3 \geq 0$ if $x \geq -\frac{1}{2}$

and $p_4 \geq 0$ if $x \leq +\frac{1}{4}$

Therefore the value of x for which a probability function is defined lie in the range

$$-\frac{1}{3} \leq x \leq \frac{1}{4}$$



Q4 If the random variable X takes the values 1, 2, 3, 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$ find the probability distribution and cumulative distribution function of X

Soln. Let $P(X=3) = 30K$.

$$\therefore 2P(X=1) = 30K, P(X=1) = 15K$$

$$\text{similarly } 3P(X=2) = 30K, P(X=2) = 10K$$

$$5P(X=4) = 30K, P(X=4) = 6K$$

$$\text{Since } \sum p_i = 1 \rightarrow 15K + 10K + 30K + 6K = 1$$

$$\therefore K = \frac{1}{61}$$

The probability distribution of X is given in the following table:

$X = 1$	1	2	3	4
p_i	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

The cdf $F(x)$ is defined as $F(x) = P(X \leq x)$

Accordingly the cdf for the above distribution is found as follows.

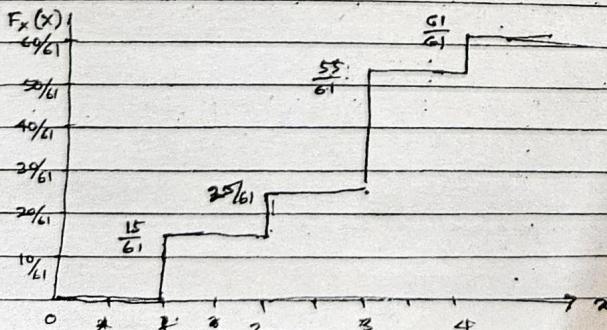
$$\text{when } x < 1, F(x) = 0$$

$$\therefore 1 \leq x \leq 2, F(x) = P(X=1) = \frac{15}{61}$$

$$2 \leq x \leq 3, F(x) = P(X=1) + P(X=2) = \frac{25}{61}$$

$$3 \leq x \leq 4, F(x) = P(X=1) + P(X=2) + P(X=3) = \frac{55}{61}$$

$$x \geq 4, F(x) = P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$$



Q5

A random variable X has the following probability distribution

$$x: -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$$

$$p(x): 0.1 \quad K \quad 0.2 \quad 2K \quad 0.3 \quad 3K$$

(a) Find K

(b) Evaluate $P(X < 2)$ and $P(-2 \leq X \leq 2)$

(c) Find the cdf of X and "cumulative distn funct"

(d) Evaluate the mean of X

Soln. (a) Since $\sum p(x) = 1, 6K + 0.6 = 1$

$$\therefore K = \frac{1}{15}$$

The probability distribution becomes

$$x: -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$$

$$p(x): \frac{1}{10} \quad \frac{1}{15} \quad \frac{1}{5} \quad \frac{3}{15} \quad \frac{3}{10} \quad \frac{1}{5}$$

$$(b) P(X < 2) = P(X = -2, -1, 0, 1)$$

$$= P(X = -2) + P(X = -1) + P(X = 0) + P(X = 1) \\ = \frac{1}{10} + \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{1}{2}$$

$$P(-2 \leq X \leq 2) = P(X = -1, 0 \text{ or } 1)$$

$$= P(X = -1) + P(X = 0) + P(X = 1) \\ = \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{2}{5}$$

(c) $F(x) = 0$, when $x < -2$

$$\frac{0+1}{10} = \frac{1}{10}, \text{ when } -2 \leq x \leq -1$$

$$\frac{1+1}{10} = \frac{1}{5}, \text{ when } -1 \leq x < 0$$

$$\frac{1+1+1}{10} = \frac{1}{3}, \text{ when } 0 \leq x < 1$$

$$\frac{1+1+2}{10} = \frac{1}{2}, \text{ when } 1 \leq x < 2$$

$$\frac{1+1+2+3}{10} = \frac{4}{5}, \text{ when } 2 \leq x < 3$$

$$\frac{1+1+2+3+1}{10} = 1, \text{ when } 2 \leq x \leq 3, 3 \leq x$$

(d) The mean of X is defined as $E(X) = \sum x_i p(x_i)$

$$\therefore \text{Mean of } X = \left(-2 \times \frac{1}{10}\right) + \left(-1 \times \frac{1}{15}\right) + \left(0 \times \frac{1}{5}\right) + \left(1 \times \frac{2}{15}\right) + \left(2 \times \frac{3}{10}\right) + \left(3 \times \frac{1}{5}\right)$$

$$= -\frac{1}{5} - \frac{1}{15} + 0 + \frac{2}{15} + \frac{3}{5} + \frac{3}{5} = \frac{16}{15} = 1.06$$

Q6 A random variable X has the following prob. distn

$$x: 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$

$$p(x): 0 \quad 2a \quad 2a \quad 2a \quad 3a \quad 2a^2 \quad 2a^2 \quad 7a^2 + a$$

(i) Find the value of a

(ii) Evaluate $P(1.5 < X < 4.5 | X > 2)$ and

(iii) The smallest value of K for which $P(X < K) > \frac{1}{2}$

$$\text{Soln. (i)} \quad \sum p(x) = 1$$

$$10a^2 + 9a = 1$$

$$\text{i.e. } (10a+1)(a+1) = 0$$

$$a = -\frac{1}{10} \text{ or } -1$$

The value of $a = -1$ makes some values of $p(x)$ negative which is meaningless.

$$\therefore a = \frac{1}{10}$$

The actual distribution is given below.

$$\begin{array}{ccccccc} x: & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ p(x): & \frac{2}{10} & \frac{2}{10} & \frac{1}{10} & \frac{3}{10} & \frac{1}{100} & \frac{2}{100} & \frac{17}{100} \end{array}$$

$$(ii) P(1.5 < X < 4.5 | X > 2) = P(A|B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P[(1.5 < X < 4.5) \cap (X > 2)]}{P(X > 2)} = \frac{P(2 < X < 4.5)}{P(X > 2)}$$

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$$P(X=2) + P(X=3) + P(X=4)$$

$$\sum_{x=3}^7 (x=r)$$

$$\frac{1}{10} + \frac{3}{10} + \frac{1}{100} + \frac{2}{100} + \frac{17}{100} = \frac{3/5}{3/5} = \frac{2}{3}$$

(iii) The smallest value of K for which $P(X < K) > \frac{1}{2}$

$$\text{By trials, } P(X \leq 0) = 0 < \frac{1}{2}$$

$$P(X \leq 1) = \frac{1}{10} < \frac{1}{2}$$

$$P(X \leq 2) = \frac{3}{10} < \frac{1}{2}$$

$$P(X \leq 3) = \frac{5}{10} \geq \frac{1}{2}$$

$$P(X \leq 4) = \frac{8}{10} \geq \frac{1}{2}$$

The smallest value of $K = 4$ where

$$P(X < K) > \frac{1}{2}$$

Q7 A discrete random variable X has following

$$x: 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

$$p(x): a \quad 3a \quad 5a \quad 7a \quad 9a \quad 11a \quad 13a \quad 17a$$

(i) Find the value of a (ii) $P(X < 5)$ (iii) Variance and distribution of X function of X

Soln.

To find find a

$$\sum P(x) = 1$$

$$86a = 1, a = \frac{1}{86}$$

∴ Prob. distribution can now be written as

$x:$	0	1	2	3	4	5	6	7	8
$p(x):$	$\frac{1}{86}$	$\frac{3}{86}$	$\frac{5}{86}$	$\frac{7}{86}$	$\frac{9}{86}$	$\frac{11}{86}$	$\frac{13}{86}$	$\frac{15}{86}$	$\frac{17}{86}$

$$(i) P(X < 3)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{1}{86} + \frac{3}{86} + \frac{5}{86} = \frac{1+3+5}{86} = \frac{9}{86}$$

$$\left(\sum_{i=0}^{\infty} (x_i^2 \cdot p(x_i)) - (\sum_{i=0}^{\infty} x_i \cdot p(x_i))^2 \right)^2$$

$$(iii) Variance. V(X) = E(X^2) - \{E(X)\}^2$$

$x:$	0	1	2	3	4	5	6	7	8
$p(x):$	$\frac{1}{86}$	$\frac{3}{86}$	$\frac{5}{86}$	$\frac{7}{86}$	$\frac{9}{86}$	$\frac{11}{86}$	$\frac{13}{86}$	$\frac{15}{86}$	$\frac{17}{86}$

$x \cdot p(x):$	0	$\frac{3}{86}$	$\frac{10}{86}$	$\frac{21}{86}$	$\frac{36}{86}$	$\frac{55}{86}$	$\frac{105}{86}$	$\frac{136}{86}$
$x^2 \cdot p(x):$	0	$\frac{3}{86}$	$\frac{10}{86}$	$\frac{21}{86}$	$\frac{36}{86}$	$\frac{55}{86}$	$\frac{105}{86}$	$\frac{136}{86}$

$x^2 \cdot p(x):$	0	$\frac{3}{86}$	$\frac{10}{86}$	$\frac{21}{86}$	$\frac{36}{86}$	$\frac{55}{86}$	$\frac{105}{86}$	$\frac{136}{86}$
$p(x)(x-\mu)^2$	0.3532	0.7169	0.2391	0.0499	1.2240	4.226		

$p(x)(x-\mu)^2$	0.3532	0.7169	0.2391	0.0499	1.2240	4.226
μ	5.5116	5.5116	5.5116	5.5116	5.5116	5.5116

$$E(x) = \frac{1}{n} \sum_{i=1}^n x_i \cdot p(x_i)$$

$$= 0 + \frac{3}{86} + \frac{10}{86} + \frac{21}{86} + \frac{36}{86} + \frac{55}{86} + \frac{105}{86} + \frac{136}{86}$$

$$E(x) = \frac{474}{86} = 5.5116 = \mu \dots E(x^2) = \frac{2976}{86} = 34.6 \dots$$

$$\text{Variance} = \sigma^2 = E[(X-\mu)^2] p(x)$$

$$\sigma^2 = 4.226$$

x	$p(x)$	$x^*p(x)$	$(x-\mu)$	$n/(1^2)(x-\mu)^2$
0	$\frac{1}{86}$	0.0000	-5.5116	0.3532
1	$\frac{3}{86}$	0.0349	-4.5116	0.7169
2	$\frac{5}{86}$	0.1163	-3.5116	0.2391
3	$\frac{7}{86}$	0.2442	-2.5116	0.0499
4	$\frac{9}{86}$	0.4186	-1.5116	1.2240
5	$\frac{11}{86}$	0.6395	-0.5116	4.226
6	$\frac{9}{43}$	1.2558	0.4884	
7	$\frac{15}{86}$	1.2209	1.4884	
8	$\frac{17}{86}$	1.5814	2.4884	
$\text{sum}(\mu) = 5.5116$		σ^2		

Q4 (Tut 3a)

- Q8 In a continuous distribution, the probability density is given by $f(x) = kx(2-x)$; $0 < x < 2$. Find (i) k , (ii) mean (iii) variance and (iv) distribution function.

Soln.

(i) If $f(x)$ is to be a pdf, $0 < x < 2$

$$\int_0^2 f(x) dx = 1$$

$$\int_0^2 kx(2-x) dx = 1$$

$$\int_0^2 (2kx - kx^2) dx = 1$$

$$k \left[\frac{2x^2}{2} - \frac{kx^3}{3} \right]_0^2 = 1$$

$$k \left[x^2 - \frac{x^3}{3} \right]_0^2 = 1$$

$$k \left[4 - \frac{8}{3} \right] \Rightarrow \frac{4}{3} k = 1 \Rightarrow k = \frac{3}{4}$$

(ii) Mean, (a)

$$f(x) = \frac{3}{4}x(2-x)$$

$$\text{Mean}(\mu) = \int_0^2 x f(x) dx$$

$$f(x) = \frac{3}{2}x - \frac{3}{4}x^2$$

$$= \int_0^2 x \left[\frac{3}{2}x - \frac{3}{4}x^2 \right] dx$$

$$= \int_0^2 \left(\frac{3}{2}x^2 - \frac{3}{4}x^3 \right) dx$$

$$= \left[\frac{3}{2}x^3 - \frac{3}{4}x^4 \right]_0^2$$

$$= \frac{10}{2}x^2 - \frac{3}{4}x^4$$

$$= 4 - 3$$

$$= 1$$

(iv) Distribution function

(iii) Variance (σ^2)

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$E(x) = \int x \cdot f(x) dx = 1$$

$$E(x^2) = \int_2^5 x^2 f(x) dx$$

$$E(x^2) = \int_0^2 x^2 \left(\frac{3}{2}x - \frac{3}{4}x^2 \right) dx$$

$$= \int_0^2 \left(\frac{3x^3}{2} - \frac{3x^4}{4} \right) dx$$

$$= \left[\frac{3}{2} \cdot \frac{x^4}{4} - \frac{3}{4} \cdot \frac{x^5}{5} \right]_0^2$$

$$= \frac{3}{2} \cdot \frac{16}{4} - \frac{3}{4} \cdot \frac{32}{5}$$

$$= 6 - \frac{24}{5} = \frac{30-24}{5} = \frac{6}{5}$$

$$\begin{aligned} Vx &= E(x^2) - (Ex)^2 \\ &= \frac{6}{5} - 1^2 \\ &= \frac{6}{5} - 1 \\ Vx &= \frac{1}{5} \end{aligned}$$

Q6 Tuf 3(a)

Q8 A continuous RV X that can assume any value between $x=2$ and $x=5$ has a density function given by $f(x) = k(1+x)$. Find $P(X < 4)$

Soln.

By the property of pdf.

$$\int f(x) dx = 1, \quad X \text{ takes values b/n 2 and 5}$$

Rx

$$\int_2^5 k(1+x) dx = 1$$

$$k \int_2^5 (1+x) dx = 1$$

$$= k \left[x + \frac{x^2}{2} \right]_2^5$$

$$= k \left[5 + \frac{25}{2} - \left(2 + \frac{4}{2} \right) \right] = k \frac{27}{2} = 1 \quad \text{or } k = \frac{2}{27}$$

\therefore The pdf $= f(x) = k(1+x)$

$$f(x) = \frac{2}{27}(1+x)$$

To find $P(X < 4)$

$$P(X < 4) = P(2 < x < 4) = \int_2^4 \frac{2}{27}(1+x) dx$$

$$= \frac{2}{27} \left[x + \frac{x^2}{2} \right]_2^4$$

$$= \frac{2}{27} \left[4 + \frac{16}{2} - \left(2 + \frac{4}{2} \right) \right] = \frac{2}{27} [4+8-2-2]$$

$$= \frac{16}{27}$$

Q7 Tuf 3(a)

Q10 A continuous RV X has a pdf $f(x) = kx^2 e^{-x}$. Find (i) k , (ii) mean and (iii) Variance

Soln.

By the property of pdf

$$\int_0^\infty kx^2 e^{-x} dx = 1 \quad | \int u \cdot v du = u \int v du - \int \frac{du}{dx} \cdot v dx$$

$$k \int_0^\infty x^2 e^{-x} dx = 1 \quad u = x^2, v = e^{-x}$$

$$k \left[x^2 \int e^{-x} dx - \int (2x \cdot \int e^{-x} dx) \right] = 1$$

$$k \left[-x^2 e^{-x} + 2 \int x \cdot e^{-x} dx \right] = 1 \quad u = x, v = e^{-x}$$

$$k \left[-x^2 e^{-x} + 2 \left[x e^{-x} - \int e^{-x} dx \right] \right] = 1$$

$$= k \left[-x^2 e^{-x} - 2x e^{-x} + 2 \int e^{-x} dx \right] = 1$$

$$= k \left[-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + C \right] = 1$$

$$= -k \left[e^{-x} (x^2 + 2x + 2) \right]_0^\infty - k[0-2] = 1 \therefore 2k = 1 \quad k = 1/2$$

substituting the value of k in pdf we get

$$f(x) = \frac{x^2 e^{-x}}{2}$$

(ii) Mean (μ)

$$\mu = E(X) = \int_0^\infty x f(x) dx$$

$$= \int_0^\infty \frac{x^3}{2} e^{-x} dx - \frac{1}{2} \int_0^\infty x^3 e^{-x} dx$$

we know that $\int u v dx = u \int v dx - \int \frac{du}{dx} \int v dx$

$$= \frac{1}{2} \int_0^\infty x^3 e^{-x} dx$$

$$= \frac{1}{2} \left[x^3 e^{-x} - \int 3x^2 (-e^{-x}) dx \right]_0^\infty$$

$$= \frac{1}{2} \left[-x^3 e^{-x} - 3x^2 e^{-x} + \int 6x e^{-x} dx \right]_0^\infty$$

$$= \frac{1}{2} \left[-x^3 e^{-x} - 3x^2 e^{-x} + 6x e^{-x} - \int 6(-e^{-x}) dx \right]_0^\infty$$

$$= \frac{1}{2} \left[-x^3 e^{-x} - 3x^2 e^{-x} - 6x e^{-x} - 6 e^{-x} \right]_0^\infty$$

$$= \frac{1}{2} [0 - 0 - 6] = 0 - \frac{1}{2} [-6]$$

$$= 3$$

(iii) Variance $\sigma^2 = E(X^2) - (E(X))^2 = V(X)$

$$E(X^2) = \int_0^\infty x^2 f(x) dx = \frac{1}{2} \int_0^\infty x^4 e^{-x} dx$$

$$= \frac{1}{2} \left[x^4 (-e^{-x}) - \int 4x^3 (-e^{-x}) dx \right]_0^\infty$$

$$= \frac{1}{2} \left[-x^4 e^{-x} - 4x^3 e^{-x} + \int 12x^2 e^{-x} dx \right]_0^\infty$$

$$= \frac{1}{2} \left[-x^4 e^{-x} - 4x^3 e^{-x} + 12x^2 (-e^{-x}) - \int 24x (-e^{-x}) dx \right]_0^\infty$$

$$= \frac{1}{2} \left[-x^4 e^{-x} - 4x^3 e^{-x} + 12x^2 e^{-x} - 24x e^{-x} + \int 24 e^{-x} dx \right]_0^\infty$$

$$E(X^2) = \frac{1}{2} \left[-x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24x e^{-x} - 24 e^{-x} \right]_0^\infty$$

$$= \frac{1}{2} [0 - (-24)] = 12$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= 12 - 3^2$$

$$= 3$$

Q.5 Tur 39

Q.11 If the density of a continuous RV X is given by

$$f(x) = ax \quad 0 \leq x \leq 1$$

$$= 3a - ax \quad 1 \leq x \leq 3$$

$$= 0 \quad \text{elsewhere.}$$

(i) find value of a

(ii) find cdf of X cumulative distribution function

Soln.

(i) Since $f(x)$ is a pdf, $\int_R f(x) dx = 1$

$$\text{i.e. } \int_0^3 f(x) dx = 1$$

$$\text{i.e. } \int_0^1 ax dx + \int_1^2 a dx + \int_2^3 (3a - ax) dx = 1$$

$$= \left[\frac{ax^2}{2} \right]_0^1 + \left[ax \right]_1^2 + \left[3ax - \frac{ax^2}{2} \right]_2^3$$

$$= \frac{a}{2} + a + \left[\left(9a - \frac{9a}{2} \right) - (6a - 2a) \right]$$

$$= \frac{a}{2} + a + \frac{9a}{2} - 4a = \frac{a + 2a + 9a - 8a}{2}$$

$$= \frac{4a}{2} = 2a = 1$$

$$a = \frac{1}{2}$$

$$f(x) = \begin{cases} \frac{x}{2} & 0 \leq x \leq 1 \\ \frac{1}{2} & 1 \leq x \leq 2 \\ \frac{3-x}{2} & 2 \leq x \leq 3 \end{cases}$$

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(ii) Cdf of x

$$\begin{aligned} F(x) &= E(X^2) - (E(X))^2 \\ &= \int_0^x z^2 f(z) dz - \left[\int_0^x f(z) dz \right]^2 \end{aligned}$$

$$E(X) = \int_0^x \int_0^z f(z) dz dz$$

(ii) $F(z) = P(X \leq z) = 0$, when $z < 0$

$$F(z) = \int_0^z \frac{z}{2} dz = \frac{z^2}{4}, \text{ when } 0 \leq z \leq 1$$

$$= \int_0^1 \frac{x}{2} dx + \int_1^2 \frac{1}{2} dx = \left[\frac{x^2}{4} \right]_0^1 + \left[\frac{x}{2} \right]_1^2 = \frac{1+x}{2}$$

$$= \frac{x}{2} - \frac{1}{2} \quad \text{when } 1 \leq z \leq 2$$

$$= \int_0^1 \frac{x}{2} dx + \int_1^2 \frac{1}{2} dx + \int_2^x \left(\frac{3-x}{2} - \frac{x}{2} \right) dx$$

$$= \left[\frac{x^2}{4} \right]_0^1 + \left[\frac{x}{2} \right]_1^2 + \left[\frac{3x}{2} - \frac{x^2}{4} \right]_2^x$$

$$= \frac{1}{4} + \left(\frac{2}{2} - \frac{1}{2} \right) + \left(\frac{3x}{2} - \frac{x^2}{4} \right) - \left(\frac{3x_2}{2} - \frac{x^2}{4} \right)$$

$$= \frac{1}{4} + \frac{1}{2} - 2 + \frac{3x}{2} - \frac{x^2}{4}$$

$$= \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4}, \text{ when } 2 \leq z \leq 3$$

$$= 1, \text{ when } z > 3$$

(iii) If x_1, x_2, x_3 are 3 independent observations of X , what is the probability that exactly one of these 3 is greater than 1.5?

Tut. 3(a)

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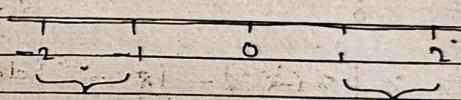
a). If pdf of a random variable X is given by

$$f(x) = \begin{cases} \frac{1}{4}, & -2 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{Find } P\{|X| > 1\}$$

Soln.

$$f(x) = \frac{1}{4}, \quad -2 \leq x \leq 2$$



$$\therefore P\{|X| > 1\} = P(-2 < X < -1) + P(1 < X < 2)$$

$$= \int_{-1}^2 \frac{1}{4} dx = \int_{-2}^2 \frac{1}{4} dx$$

$$= \left[\frac{x}{4} \right]_{-2}^2 + \left[\frac{x}{4} \right]_1^2$$

$$= \frac{1}{4} \left[-1 - (-2) + 2 - 1 \right] = \frac{1}{4} [-1 + 2 + 2 - 1]$$

$$= \frac{2}{4} = \frac{1}{2}$$

Q2 Find the value of k , if

$$f(x) = \begin{cases} kx^2, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

is the pdf of R.V. x

$$f(x) = kx^2 e^{-x}, \quad x > 0$$

then $\int_0^\infty f(x) dx = 1$

$$\int_0^\infty kx^2 e^{-x} dx = 1$$

$\int u du$

$$= u \int u du - \int \left(\frac{du}{dx} \right) \cdot u du$$

$$k \left[x(-e^{-x}) - \int (-e^{-x}) dx \right]_0^\infty$$

$$k \left[-xe^{-x} + x(-e^{-x}) - \int e^{-x} dx \right]_0^\infty$$

$$k \left[-xe^{-x} - xe^{-x} + x e^{-x} \right]_0^\infty$$

$$-k \left[xe^{-x} \right]_0^\infty = 1$$

~~- k~~

$$k \left[x(-e^{-x}) - \int -e^{-x} dx \right]_0^\infty$$

$$k \left[-xe^{-x} - e^{-x} \right]_0^\infty = 1$$

$$k \left[(0-0) - (0-1) \right] = 1$$

$$k = 1$$

$$\therefore f(x) = xe^{-x}$$

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Q3 The probability distribution of a R.V. is given below:

$$x: -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$$

$$P(x): 0.1 \quad k \quad 0.2 \quad 2k \quad 0.3 \quad 3k$$

Find $Y = X^2 + 2X$, find the probability distn, mean and variance of Y .

Soln:

$$x: -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$$

$$P(x): 0.1 \quad k \quad 0.2 \quad 2k \quad 0.3 \quad 3k$$

$$P(Y): \frac{1}{10} \quad \frac{1}{15} \quad \frac{2}{10} \quad \frac{2}{15} \quad \frac{3}{10} \quad \frac{3}{15}$$

$$Y = X^2 + 2X$$

$$\therefore Y: (-1 \times 1) \quad 0 \quad 3 \quad 8 \quad 15$$

$$P(Y) = \frac{1}{10} \quad \frac{3}{10} \quad \frac{2}{15} \quad \frac{3}{10} \quad \frac{3}{15}$$

$$P(Y=-1) = P(X^2 + 2X = -1) \Rightarrow X^2 + 2X + 1 = 0$$

$$(X+1)(X+1) = 0 \Rightarrow X = -1$$

$$\text{at } X = -1 \quad P(X) = \frac{1}{10} = P(Y)$$

$$P(Y=0) = P(X^2 + 2X = 0) \Rightarrow X^2 - 2X = 0$$

$$\text{at } X = -2 \quad P(X) = \frac{1}{10}$$

$$X(X+2) = 0$$

$$X(X-2) = 0$$

$$\text{i.e., } X = 0, X = 2$$

$$P(X=0) = \frac{3}{10}$$

$$P(X=2) = \frac{1}{10}$$

$$\therefore P(Y=0) = \frac{3}{10} + \frac{1}{10} = \frac{4}{10}$$

$$P(Y=3) \Rightarrow P(x^2+2x=3) \Rightarrow P(x^2+2x-3=0)$$

$$\text{or } x^2+2x-3 = 0 \Rightarrow x(x+3)-1(x+3)$$

$$(x+3)(x-1) = 0 \Rightarrow x = -3, 1$$

$$= P(X=-3) + P(X=1)$$

$$= 0 + \frac{2}{15} = \frac{2}{15}$$

$$P(Y=8) \Rightarrow P(x^2+2x=8) \Rightarrow P(x^2+2x-8=0)$$

$$x^2+4x-2x-8 = 0 \Rightarrow x(x+4)-2(x+4)$$

$$(x+4)(x-2) = 0 \Rightarrow x = -4, 2$$

$$= P(X=-4) + P(X=2)$$

$$= 0 + \frac{3}{10} = \frac{3}{10}$$

$$P(Y=15) \Rightarrow P(x^2+2x=15) \Rightarrow P(x^2+2x-15=0)$$

$$x^2+5x-3x-15 \Rightarrow x(x+5)-3(x+5)$$

$$(x+5)(x-3) \Rightarrow x = -5, +3$$

$$\Rightarrow P(X=-5) + P(X=3)$$

$$\Rightarrow 0 + \frac{3}{15}$$

\therefore The probability distribution of Y

	-1	0	3	8	15	Σ
P(Y)	$\frac{1}{15}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{3}{15}$	
$Y_i P(Y)$	$-\frac{1}{15}$	0	$\frac{3}{10}$	$\frac{24}{10}$	3	$\frac{81}{15}$
Y	1	0	9	64	225	
$Y^2 P(Y)$	$-\frac{1}{15}$	0	$\frac{9}{5}$	$\frac{96}{5}$	45	$\frac{981}{15}$

Mean of Y:

$$E(Y) = \sum_i Y_i P(Y)$$

$$= -1 \times \frac{1}{15} + 0 + 3 \times \frac{2}{10} + 8 \times \frac{3}{10} + 15 \times \frac{3}{15}$$

$$= -\frac{1}{15} + \frac{6}{10} + \frac{24}{10} + 3 = \frac{-2 + 18 + 72 + 90}{30}$$

$$= \frac{178}{30} = \frac{89}{15} = \frac{86}{15}$$

Variance of Y

$$V(Y) = E(Y^2) - \{E(Y)\}^2$$

$$E(Y^2) = \sum_i Y_i^2 P(Y_i)$$

$$E(Y^2) = \frac{1}{15} \times 1 + 0 + 9 \times \frac{2}{10} + 64 \times \frac{3}{10} + \frac{9}{15} \times 225 = \frac{891}{15}$$

$$V(Y) = \frac{891}{15} - \left(\frac{86}{15}\right)^2 = \frac{6944}{225}$$

$$= 30.86 - 32.54 = \frac{7534}{225}$$

Q6 A continuous random variable X, that can assume any value between $x=2$ and $x=5$ has a density function given by
 $f(x) = k(1+x)$

Find $P(X < 4)$

Soln.

$$\text{Pdf } f(x) = k(1+x).$$

$$\therefore \int f(x) dx = 1$$

$$\int_2^5 (k + kx) dx = k \left[x + \frac{x^2}{2} \right]_2^5 = 1$$

$$\Rightarrow k \left[5 + \frac{25}{2} - (2 + \frac{4}{2}) \right] = 1$$

$$\Rightarrow k \left[5 + \frac{21}{2} - 2 - 2 \right] = 1$$

$$\Rightarrow k \cdot \frac{25}{2} - \frac{27}{2} k = 1$$

$$k = \frac{2}{27}$$

$$P(Y=3) \Rightarrow P(x^2+2x=3) \Rightarrow P(x^2+2x-3=0)$$

$$x^2+3x-2x-3 = x(x+3)-1(x+3)$$

$$(x+3)(x-1) = 0 \Rightarrow x = -3, 1$$

$$= P(x=-3) + P(x=1)$$

$$= 0 + \frac{2}{15} = \frac{2}{15}$$

$$P(Y=8) \Rightarrow P(x^2+2x=8) \Rightarrow P(x^2+2x-8=0)$$

$$x^2+4x-2x-8 = 0 \quad x(x+4)-2(x+4)$$

$$(x+4)(x-2) = 0 \quad x = -4, 2$$

$$= P(x=-4) + P(x=2)$$

$$= 0 + \frac{3}{10} = \frac{3}{10}$$

$$P(Y=15) \Rightarrow P(x^2+2x=15) \Rightarrow P(x^2+2x-15=0)$$

$$x^2+5x-3x-15 \Rightarrow x(x+5)-3(x+5)$$

$$(x+5)(x-3) \Rightarrow x = -5, +3$$

$$\Rightarrow P(x=-5) + P(x=3)$$

$$\Rightarrow 0 + \frac{3}{15}$$

∴ The probability distribution of Y

Y	-1	0	3	8	15	\sum
P(Y)	$\frac{1}{15}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{3}{15}$	
$Y_i P(Y_i)$	$-\frac{1}{15}$	0	$\frac{3}{5}$	$\frac{24}{10}$	$\frac{3}{15}$	$\frac{86}{15}$
Y	-1	0	9	64	225	
$Y^2 \cdot P(Y)$	$-\frac{1}{15}$	0	$\frac{9}{5}$	$\frac{96}{5}$	$\frac{45}{15}$	$\frac{982}{15}$

$$E(Y) = \sum_i Y_i P(Y_i)$$

$$= -1 \times \frac{1}{15} + 0 + 3 \times \frac{2}{10} + 8 \times \frac{3}{10} + 15 \times \frac{3}{15}$$

$$= -\frac{1}{15} + \frac{6}{10} + \frac{24}{10} + 3 = \frac{-2 + 18 + 72 + 90}{30}$$

$$= \frac{178}{30} = \frac{89}{15} = \frac{86}{15}$$

Variance of Y

$$V(Y) = E(Y^2) - \{E(Y)\}^2$$

$$E(Y^2) = \sum_i Y_i^2 \cdot P(Y_i)$$

$$E(Y^2) = \frac{1}{15} \times 1 + 0 + 9 \times \frac{2}{10} + 64 \times \frac{3}{10} + \frac{9}{15} \times 225 = \frac{891}{15}$$

$$V(Y) = \frac{891}{15} - \left(\frac{86}{15}\right)^2 = \frac{891 - 7396}{225}$$

$$= 30.86 - 32.59 = \frac{7534}{225}$$

982

891

Q6 A continuous random variable X that can assume any value between $x=2$ and $x=5$ has a density function given by $f(x) = k(1+x)$.

Find $P(X < 4)$

Soln.

$$\text{Pdf } f(x) = k(1+x).$$

$$\therefore \int f(x) dx = 1$$

$$\int_2^5 (k + kx) dx = k \left[x + \frac{x^2}{2} \right]_2^5 = 1$$

$$\Rightarrow k \left[5 + \frac{25}{2} - \left(2 + \frac{4}{2} \right) \right] = 1$$

$$\Rightarrow k \left[5 + \frac{25}{2} - 2 - 2 \right] = 1$$

$$\Rightarrow k \left[\frac{25}{2} - 2 \right] = 1$$

$$k = \frac{2}{27}$$

$$\therefore \text{pdf} = \frac{2}{27} (1+x)$$

$$P(X < 4) = \int_2^4 \frac{2}{27} (1+x) dx$$

$$= \frac{2}{27} \left[x + \frac{x^2}{2} \right]_2^4$$

$$= \frac{2}{27} \left[4 + \frac{4^2}{2} - 2 - \frac{2^2}{2} \right]$$

$$= \frac{2}{27} [4 + 8 - 2 - 2]$$

$$= \frac{16}{27}$$