Tutorial - 2

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Q.1 If A and B are independent events then show that

(i) A and $\bar{B}$ are independent events

(ii) $\bar{A}$ and B " " "

(iii) $\bar{A}$ and $\bar{B}$ " " "

(iv) $P(B) = P(A) \cdot P(B/A) + P\bar{A} \cdot P(B/\bar{A})$, $A \neq 0$

(v) $P(A/B) + P(\bar{A}/B) = 1$

(vi) $P(A/B) = 1$ if $B \subseteq A$

Soln.

We are given that A and B are independent events

then, $P(A \cap B) = P(A) \times P(B)$ —①

(i) T.P.T. A and $\bar{B}$ are independent events

it can be proved if

$P(A \cap \bar{B}) = P(A) \times P(\bar{B})$

Take L.H.S

$\Rightarrow P(A) \cap P(\bar{B})$

$\Rightarrow P(A) \cap \cancel{P(1-B)} (1 - P(B))$

$\Rightarrow P(A) - P(A \cap B)$

As A and B are independent events, using -①

this eqn. can be written as

$\Rightarrow P(A) - P(A) \cdot P(B)$

$\Rightarrow P(A)(1 - P(B))$

$\Rightarrow P(A) P(\bar{B}) = RHS$ Hence proved

(ii) $\bar{A}$ and B are independent events, if

$P(\bar{A} \cap B) = P(\bar{A}) \cdot P(B)$

$P(\bar{A}) \cap P(B)$

$P(B) \cap P(1-A)$

$P(B) - P(B) \cdot P(A)$

$P(B)(1 - P(A))$

$P(B) \cdot P(\bar{A}) = RHS$

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(iii) $\bar{A}$ and $\bar{B}$ are independent events if

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

~~$P(\bar{A}) \cap P(\bar{B})$~~

~~$P(1-A) \cap P(1-B)$~~

$P(\bar{A} \cap \bar{B}) = P\{(\overline{A \cup B})'\}$ by DeMorgan's Law

$= 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - \{P(A) + P(B) - P(A) \cdot P(B)\}$ using ①

$= 1 - P(A) - P(B) + P(A) \cdot P(B)$

$= (1 - P(A)) - P(B)\{1 - P(A)\}$

$= (1 - P(A))(1 - P(B))$

$= P(\bar{A}) \cdot P(\bar{B})$ Hence proved.

(iv) $P(B) = P(A) \cdot P(B/A) + P(\bar{A}) \cdot P(B/\bar{A})$

From Bay's ~~theorem~~ Rule $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{P(A) \cdot P(B)}{P(B)}$ from ①

$= P(A)$

similarly, $P(B/A) = P(B)$

$\therefore$ ~~$P(B) = P(A) \cdot P(B)$~~. similarly

$P(B/\bar{A}) = \dfrac{P(B \cap \bar{A})}{P(\bar{A})} = \dfrac{P(B) \cdot P(\bar{A})}{P(\bar{A})}$

$= P(B)$

$\therefore$ RHS can be written as.

$= P(A)\, P(B) + P(\bar{A}) \cdot P(B)$

$= P(B)\,(P(A) + P(\bar{A}))$

$= P(B) \times 1$

$= P(B)$

(iv) $P(A/B) + P(\bar{A}/B) = 1$

LHS

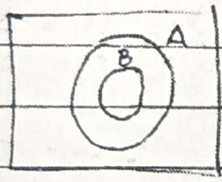
$$= \frac{P(A \cap B)}{P(B)} + \frac{P(\bar{A} \cap B)}{P(B)}$$ (from Bay's The

$$= \frac{P(A) \cdot P(B)}{P(B)} + \frac{P(\bar{A}) \cdot P(B)}{P(B)}$$

$$= P(A) + P(\bar{A})$$

$$= 1$$

(vi) $P(A/B) = 1$ if $B \subseteq A$



$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Q2 If $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$

Then find (i) $P(A/B)$ (ii) $P(\bar{A}/\bar{B})$ (iii) $P(B/A)$

(i) $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{3}{4} = \frac{3}{8} + \frac{5}{8} - P(A \cap B)$

$P(A \cap B) = \frac{3}{8} + \frac{5}{8} - \frac{3}{4} = \frac{1}{4}$

$\therefore P(A/B) = \dfrac{1/4}{5/8} = \frac{1}{\cancel{4}} \times \frac{\cancel{8}\,2}{5} = \frac{2}{5}$

(ii) $P(\bar{A}/\bar{B}) = \dfrac{P(\overline{A \cap B})}{P(\bar{B})}$

$= \dfrac{P(\overline{A \cup B})}{P(\bar{B})}$

$= \dfrac{P(\overline{A \cup B})}{1 - P(B)} = \dfrac{1 - P(A \cup B)}{1 - P(B)}$

$= \dfrac{1/4}{3/8} = \frac{2}{3}$

(iii) $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{3}{4} = \frac{3}{8} + \frac{5}{8} - P(A \cap B)$

$\therefore P(A \cap B) = -\frac{3}{4} + \frac{3}{8} + \frac{5}{8} = \frac{-6+3+5}{8} = \frac{1}{4}$

$\therefore P(B/A) = \frac{1/4}{3/8} = \frac{\cancel{8}\,2}{3 \times \cancel{4}} = \frac{2}{3}$

Q3 A bolt is manufactured by 3 machines A, B, C. A turns out twice as many items as B and machines B and C produce equal no. of item 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and 1 is chosen from the pile. What is the probability that it is defective?

Soln.

Let; A = the event in which the item is produced by machine A and so on.

B = , C =

D = The event of the item being defective.

$P(A) = \frac{1}{2}$, $P(B) = P(C) = \frac{1}{4}$

$P(D/A) = P($an item is defective given that A has produced it$)$

$= \frac{2}{100} = P(D/B)$

$P(D/C) = \frac{4}{100}$

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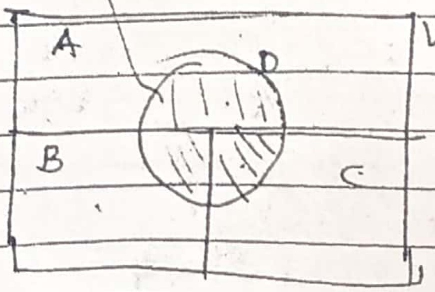
$P(D) = P(D\cap A) + P(D\cap B) + P(D\cap C)$

By theorem of total probability

$P(D) = P(A)\times P(D/A) + P(B)\times P(D/B) + P(C)\times P(D/C)$

$= \frac{1}{2}\times\frac{2}{100} + \frac{1}{4}\times\frac{2}{100} + \frac{1}{4}\times\frac{4}{100}$

$= \frac{1}{40}$



$P(A/B) = \frac{P(A\cap B)}{P(B)}$

$P(D/A) = \frac{P(D\cap A)}{P(A)}$, $P(D/B) = \frac{P(D\cap B)}{P(B)}$ and

$P(D/C) = \frac{P(D\cap C)}{P C}$

By theorem total prob. of defective

$P(D) = P(D\cap A) + P(D\cap B) + P(D\cap C)$

$= P(A)\times P(D/A) + P(B)\, P(D/B) + P(C)\times P(D/C)$

$= \frac{1}{2}\times\frac{2}{100} + \frac{1}{4}\times\frac{2}{100} + \frac{1}{4}\times\frac{4}{100}$

$= 1/40.$

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Q4 An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball?

Soln The two balls transferred may be both white or both black or 1 white and 1 black.

Let,

B_1 = Event of drawing 2 W balls from the 1st urn

B_2 = " " " 2 B " " "

B_3 = " " " 1W & 1B " " "

Clearly B_1, B_2, B_3 are exhaustive and mutually exclusive events

Let; A = Event of drawing 1 ball from the 2nd urn after transfer.

$$P(B_1) = \frac{^{10}C_2}{^{13}C_2} ; \quad P(B_2) = \frac{^{3}C_2}{^{13}C_2} ; \quad P(B_3) = \frac{^{10}C_1 \times {^{3}C_1}}{^{13}C_2}$$

$$= \frac{15}{26} \qquad = \frac{1}{26} \qquad = \frac{10}{26}$$

$P(A/B_1)$ = P(drawing a white ball / 2 white balls have been transferred)

= P(Drawing a white ball / urn 2 contains 5W+5B)

$= \frac{5}{10}$

similarly $P(A/B_2) = \frac{3}{10}$ & $P(A/B_3) = \frac{4}{10}$

By theorem of total probability

$$P(A) = P(B_1) \times P(A/B_1) + P(B_2) \times P(A/B_2) + P(B_3) \times P(A/B_3)$$

∴ Total Probability

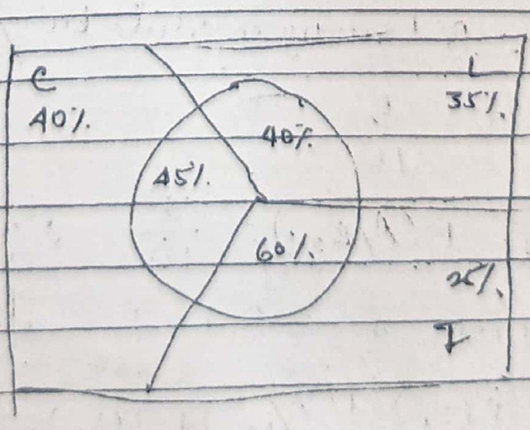
$$P(A) = \cancel{P(E_1)\times}\ \frac{15}{26}\times\frac{5}{10} + \frac{1}{26}\times\frac{3}{10} + \frac{10}{26}\times\frac{4}{10}$$

$$= \frac{59}{130}$$

Q 5 In a certain city, 40% of the people consider themselves conservative (C), 35% consider them selves to be liberals (L), and 25% consider themselves to be independent (I). During a particular election, 45% of the conservative voted, 40% of the liberals voted and 60% of the independent voted. Suppose that a person is randomly selected
(a) Find the probability that the person voted.
(b) If the person voted, find the probability that the voter is (i) conservative (ii) Liberal (iii) Independent.

Soln. Let C = Conservative people
L = Liberals
I = Independent
A = a selected person.

~~(a) The probability~~ that the selected person has ~~voted = P(...)~~ ~~45 C1 / 400 C1~~ = ~~0.45~~



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(a) Let,

P(C) is the prob. that a person selected at random is conservative $= \frac{40}{100}$

Similarly $P(L) = \frac{35}{100}$ & $P(I) = \frac{25}{100}$

Let $P(V/C)$ is the prob. that a person voted is conservative $= \frac{45}{100}$

Similarly $P(V/L) = \frac{40}{100}$ & $P(V/I) = \frac{60}{100}$

Now Prob that a person has voted

$$P(V) = P(C) \times P(V/C) + P(L) \times P(V/L) + P(I) \cdot P(V/I)$$

$$= 0.4 \times 0.45 + 0.35 \times 0.4 + 0.25 \times 0.6$$

$$P(V) = 0.47$$

(b) (i) If a person has voted, and he is conservative $= P(C/V) = \frac{P(C \cap V)}{P(V)}$

$$~~= \frac{0.45}{0.47} = 0.95~~$$

~~(ii) Similarly, $P(L/V) = \frac{P(L \cap V)}{P(V)} = \frac{0.4}{0.47} = 0.85$~~

~~$L(I/V) = \frac{P(I \cap V)}{P(V)} = \frac{0.6}{0.47} = 1.27$~~

$$P(C/V) = \frac{P(C) \times P(V/C)}{P(V)} = \frac{0.4 \times 0.45}{0.47} = 0.38$$

Similarly $P(L/V) = \frac{P(L) \times P(V/L)}{P(V)}$

$$= \frac{0.35 \times 0.4}{0.47} = 0.2978 \simeq 0.30$$

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and $P(I/V) = \frac{P(I) \times P(V/I)}{P(V)}$

$= \frac{0.25 \times 0.6}{0.47} = 0.319$

Q6 Three machines A, B, and C produce respectively 40%, 10% and 50% of the items in a factory. The percentage of defective items produced by the machines are respectively 2%, 3% and 4%. An item from the factory is selected ~~at random~~ random.
(a) Find the prob. that the item is defective
(b) If the item is defective find the probability that the item was produced by (i) M/c A (ii) M/c B (iii) M/c C

Soln (a) Let B be the event that the part produced is defective and.
A_i be the event the relay is manufactured by the plant i ($i = 1, 2, 3$). The desired prob. $P(B)$ is given by

$$P(B) = \sum_{i=1}^{3} P(B/A_i) \cdot P(A_i)$$

$$P(B) = P(B/A_1) P(A_1) + P(B/A_2) \cdot P(A_2) + P(B/A_3) P(A_3)$$

$= ~~0.02 \times 0.25 + 0.95 \times 0.2 +~~$

$(0.02)(0.4) + (0.03) \times (0.10) + 0.04 \times 0.5$

$= 0.031$

.008
.003
.02
.031

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(b) The desired prob. are

(i) $P(A_1/B)$ (ii) $P(A_2/B)$ and (iii) $P(A_3/B)$

(i) $P(A_1/B) = \frac{P(B/A_1).P(A_1)}{P(B)} = \frac{0.02\times0.4}{0.031} = 0.258$

$P(A_2/B) = \frac{P(B/A_2).P(A_2)}{P(B)} = \frac{0.03\times0.10}{0.031} = 0.096$

$P(A_3/B) = \frac{P(B/A_3).P(A_3)}{P(B)} = \frac{0.04\times0.5}{0.031} = 0.64$

Q7 Three boxes of the same appearance have the following proportion of balls.

I	2 black	1 white
II	1 black	2 white
III	2 black	2 white

One of the urn is selected and one ball is drawn; It turns out to be white. What is the probability of drawing white ball again, if the first is not replaced

Soln.

Let,

A_1 = the even that the first white ball is drawn chosen from first

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Q1 The RV X has the following probability distrⁿ

x :	-2	-1	0	1
$P(x)$:	0.4	k	0.2	0.3

Find k and mean value of X.

Soln. Since $\sum P(x) = 1$, $k + 0.9 = 1$

$\therefore k = 0.1$

Mean $E(X) = \sum X \, P(x)$

$= (-2 \times 0.4) + (-1 \times 0.1) + (0 \times 0.2) + (1 \times 0.3)$

$= -0.6$

Q2 A shipment of 6 TV sets contain 2 defective sets. A hotel makes a random purchase of 3 of the sets. Find the prob. distri of X, if X is the no. of defective sets.

Soln. The no of defectives can be 0, ~~02~~ 1 and 2

Prob. of purchasing 0 defective sets $= \dfrac{{}^4C_3}{{}^6C_3} = \dfrac{4}{20} = \dfrac{1}{5}$

Similarly " " 1 " " $= \dfrac{{}^2C_1 \times {}^4C_2}{{}^6C_3} = \dfrac{12}{20}$

" " 2 " " $= \dfrac{{}^2C_2 \times {}^4C_1}{{}^6C_3} = \dfrac{4}{20}$

X	0	1	2
$P(x)$	$4/20$	$12/20$	$4/20$

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Q3 A random variable X may assume 4 values with probability $\frac{1+3x}{4}, \frac{1-x}{4}, \frac{1+2x}{4}, \frac{1-4x}{4}$. Find the condition on x so that these values represent the probability of X

Soln.

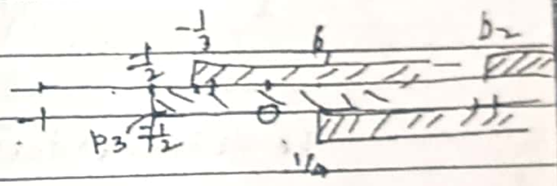
$$P(X=x_1) = p_1 = (1+3x)/4$$
$$p_2 = (1-x)/4$$
$$p_3 = (1+2x)/4$$
$$p_4 = (1-4x)/4$$

If the given probabilities represent a probability function, each $p_i \geq 0$ and $\sum_i p_i = 1$

In this problem, $p_1 + p_2 + p_3 + p_4 = 1$ for any x

But $p_1 \geq 0$, if $x \geq -\frac{1}{3}$

$p_2 \geq 0$, if $x \leq 1$

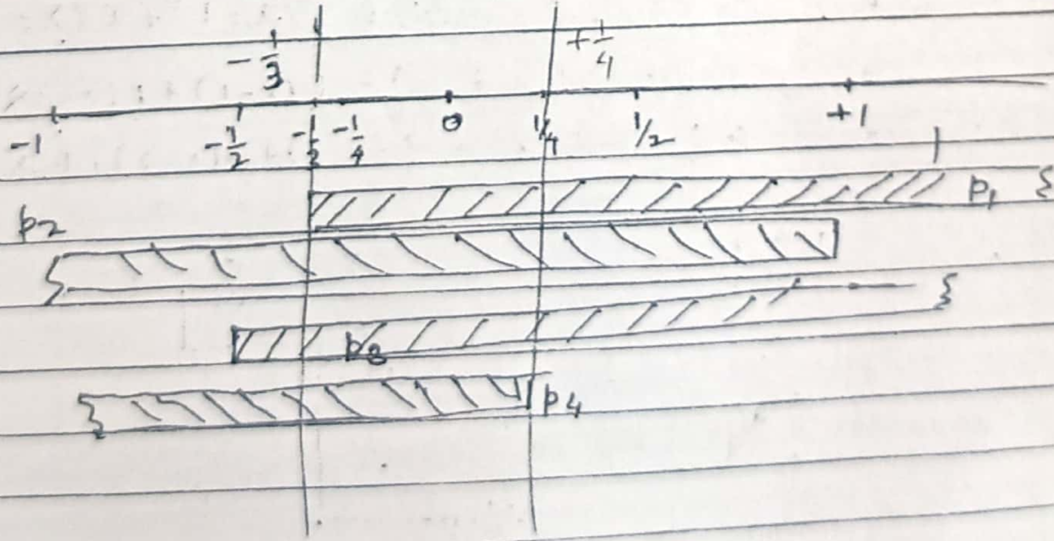


$p_3 \geq 0$ if $x \geq -\frac{1}{2}$

and $p_4 \geq 0$ if $x \leq +\frac{1}{4}$

Therefore the value of x for which a probability function is defined lie in the range

$$-\frac{1}{3} \leq x \leq \frac{1}{4}$$



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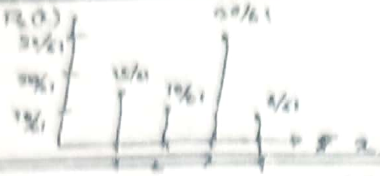


Q4 If the random variable X takes the values 1, 2, 3 & 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, find the probability distribution and cumulative distribution function of X

Soln. Let $P(X=3) = 30K$

$\because 2P(X=1) = 30K, \quad P(X=1) = 15K$

simil^y $3P(X=2) = 30K, \quad P(X=2) = 10K$

$5P(X=4) = 30K, \quad P(X=4) = 6K$

Since $\sum p_i = 1, \quad 15K + 10K + 30K + 6K = 1$

$\therefore K = 1/61$

The probability distribution of X is given in the following table:

$X = x_i$	1	2	3	4
p_i	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

The cdf $F(x)$ is defined as $F(x) = P(X \le x)$

Accordingly the cdf for the above distribution is found as follows.

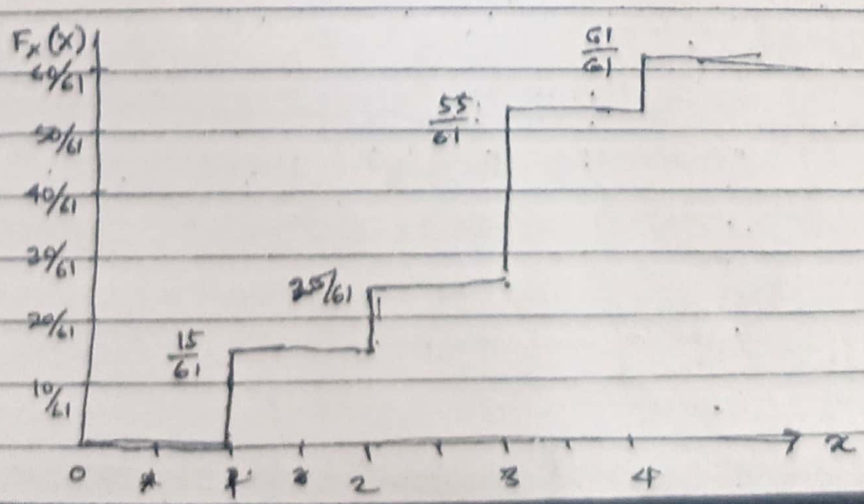
when $x < 1, \quad F(x) = 0$

" $1 \le x < 2, \quad F(x) = P(X=1) = \frac{15}{61}$

" $2 \le x < 3, \quad F(x) = P(X=1) + P(X=2) = \frac{25}{61}$

" $3 \le x < 4 \quad F(x) = P(X=1) + P(X=2) + P(X=3) = \frac{55}{61}$

" $x \ge 4, \quad F(x) = P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$



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Q5 A random variable X has the following probability distribution

x:	-2	-1	0	1	2	3
$p(x)$:	0.1	K	0.2	$2K$	0.3	$3K$

(a) Find K

(b) Evaluate $P(X<2)$ and $P(-2<X<2)$,

(c) Find the cdf of X and → cumulative distri^n funct^n

(d) Evaluate the mean of X

Soln (a) Since $\sum P(x) = 1$, $6K+0.6=1$

$\therefore K = \frac{1}{15}$

$\therefore$ The probability distribution becomes.

x:	-2	-1	0	1	2	3
$p(x)$:	$1/10$	$1/15$	$1/5$	$2/15$	$3/10$	$1/5$

(b) $P(X<2) = P(X=-2,-1,0,1)$

$= P(X=-2)+P(X=-1)+P(X=0)+P(X=1)$

$= \frac{1}{10}+\frac{1}{15}+\frac{1}{5}+\frac{2}{15} = \frac{1}{2}$.

$P(-2<X<2) = P(X=-1, 0 \text{ or } 1)$

$= P(X=-1)+P(X=0)+P(X=1)$

$= \frac{1}{15}+\frac{1}{5}+\frac{2}{15} = \frac{2}{5}$

(c) $F(x)=0$, when $x<-2$

$0+\frac{1}{10} = \frac{1}{10}$, when $-2 \le x \le -1$

$\frac{3+2}{30} = \frac{1}{10}+\frac{1}{15} = \frac{1}{6}$, when $-1 \le x < 0$

$\frac{5+6}{30} = \frac{1}{6}+\frac{1}{5} = \frac{11}{30}$, when $0 \le x < 1$

$\frac{11+4}{30} = \frac{11}{30}+\frac{2}{15} = 1/2$, when $1 \le x < 2$

$\frac{5+3}{10}$ $\frac{1}{2}+\frac{3}{10} = \frac{4}{5}$, when $2 \le x < 3$

$\frac{4}{5}+\frac{1}{5} = 1$, when ~~$2 \le x \le 3$~~ $3 \le x$

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(d) The mean of X is defined as $E(X) = \sum x\, p(x)$

$\therefore$ Mean of $X = \left(-2\times\frac{1}{10}\right) + \left(-1\times\frac{1}{15}\right) + \left(0\times\frac{1}{5}\right) + \left(1\times\frac{2}{15}\right) + \left(2\times\frac{3}{10}\right) + \left(3\times\frac{1}{5}\right)$

$= -\frac{1}{5} - \frac{1}{15} + 0 + \frac{2}{15} + \frac{3}{5} + \frac{3}{5} = \frac{16}{15} \cdot 1.06$

Q6 A random variable X has the following prob. distn.

x :	0	1	2	3	4	5	6	7
$P(x)$:	0	2λ	2λ	λ	3λ	λ^2	$2\lambda^2$	$7\lambda^2+\lambda$

(i) Find the value of λ

(ii) Evaluate $P(1.5 < X < 4.5 \mid X > 2)$ and

(iii) The smallest value of K for which $P(X \le K) > \frac{1}{2}$

Soln. (i) $\sum p(x) = 1$

$\therefore \quad 10\lambda^2 + 9\lambda = 1$

i.e. $(10\lambda - 1)(\lambda + 1) = 0$

$\lambda = \frac{1}{10}$ or -1

The value of $\lambda = -1$ makes some values of $p(x)$ negative which is meaningless

$\therefore \quad \lambda = 1/10$

The actual distribution is given below.

x ;	0	1	2	3	4	5	6	7
$p(x)$:	0	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

(ii) $P(1.5 < X < 4.5 / X > 2) = P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{P[(1.5 < X < 4.5) \cap (X > 2)]}{P(X > 2)} = \frac{P(2 < X < 4.5)}{P(X > 2)}$

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$$= \frac{\cancel{P(X=2)} + P(X=3) + P(X=4)}{\sum_{r=3}^{7} (X=r)}$$

$$= \frac{\frac{8}{10} + \frac{1}{10} + \frac{3}{10}}{\frac{1}{10} + \frac{3}{10} + \frac{1}{100} + \frac{8}{100} + \frac{17}{100}} = \frac{2/5}{3/5} = \frac{2}{3}$$

(iii) The smallest value of k for which $P(X \le K) > \frac{1}{2}$

By trials, $P(X \le 0) = 0 \; < \frac{1}{2}$

$P(X \le 1) = 1/10 \; < 1/2$

$P(X \le 2) = 3/10 \; < 1/2$

$P(X \le 3) = 5/10 \; = 1/2$

$P(X \le 4) = 8/10 \; > 1/2$

$\therefore$ The smallest value of $K = 4$ whe

$P(X \le k) > 1/2$

Q7 A discrete random variable X has following

x:	0	1	2	3	4	5	6	7	8
$P(x)$:	a	3a	5a	7a	9a	11a	18a	15a	17a

(i) Find the value of a (ii) $P(X < 3)$ (iii) variance and distribution ~~of x~~ function of X

Soln. To ~~fi~~ find a

$\Sigma P(x) = 1$

$86 a = 1, \quad a = \frac{1}{86}$

$\therefore$ Prob. distribution can now be written as

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x:	0	1	2	3	4	5	6	7	8
$P(x)$:	$\frac{1}{86}$	$\frac{3}{86}$	$\frac{5}{86}$	$\frac{7}{86}$	$\frac{9}{86}$	$\frac{11}{86}$	$\frac{18}{86}$	$\frac{15}{86}$	$\frac{17}{86}$

(ii) $P(X < 3)$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{1}{86} + \frac{3}{86} + \frac{5}{86} = \frac{1+3+5}{86} = \frac{9}{86}$$

$$\sum_{i=0}^{n} (x^2 \cdot p(x)) - (x \cdot p x)^2$$

(iii) Variance $\quad V(X) = E(X^2) - \{E(X)\}^2$

x:	0	1	2	3	4	5	6	7	8	Σ
$P(x)$:	$1/86$	$3/86$	$5/86$	$7/86$	$9/86$	$11/86$	$18/86$	$15/86$	$17/86$	
$x.p(x)$:	0	$3/86$	$10/86$	$21/86$	$36/86$	$55/86$	$108/86$	$\frac{105}{86}$	$136/86$	
$x^2 =$	0	1	4	9	16	25	36	49	64	
$x^2 \cancel{x} p(x) =$	0	$3/86$	$\frac{20}{86}$	$63/86$	$\frac{144}{86}$	$\frac{275}{86}$	$\frac{648}{86}$	$\frac{735}{86}$	$\frac{1088}{86}$	$\frac{2976}{86}$
$p(x)(x-\mu)^2$	0.3532	0.7101	0.7169	0.5135	0.2391	0.0335	0.0499	0.3864	1.2240	4.226

$$E(x) = \sum_{i=1}^{n} x_i \cdot p(x_i)$$

μ

$$= 0 + \frac{3}{86} + \frac{10}{86} + \frac{21}{86} + \frac{36}{86} + \frac{55}{86} + \frac{108}{86} + \frac{105}{86} + \frac{1088}{86}$$

$E(x) = \frac{474}{86} = 5.5116 = \mu$ . . $\quad Ex^2 \; \frac{2976}{86} = 34.60$

Variance $= \sigma^2 = E[(X-\mu)^2]\, p(x)$

$\sigma^2 = \{4.226\}$

x	p(x)	x*p(x)	(x-μ)	p(x)*(x-μ)^2
0	1/86	0.0000	-5.5116	0.3532
1	3/86	0.0349	-4.5116	0.7101
2	5/86	0.1163	-3.5116	0.7169
3	7/86	0.2442	-2.5116	[illegible]
4	9/86	0.4186	-1.5116	0.2391
5	11/86	0.6395	-0.5116	[illegible]
6	9/43	1.2558	0.4884	[illegible]
7	15/86	1.2209	1.4884	[illegible]
8	17/86	1.5814	2.4884	
	sum(μ)=	5.5116	σ^2	

Q 4 (Tut 3a)

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~~Q8~~ In a continuous distribution, the probability density is given by $f(x) = kx(2-x)$; $0 < x < 2$

Find (i) k, (ii) mean (iii) Variance and ~~std~~ (iv) distribution function

Soln. (i) If $f(x)$ is to be a pdf, $0 < x < 2$

$$\int_0^2 f(x)\,dx = 1$$

$$\int_0^2 kx(2-x)\,dx = 1$$

$$\int_0^2 (2kx - kx^2)\,dx = 1$$

$$k\left[\frac{2x^2}{2} - \frac{\cancel{k}x^3}{3}\right]_0^2$$

$$k\left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$k\left[4 - \frac{8}{3}\right] \Rightarrow \frac{4}{3}k = 1 \Rightarrow \boxed{k = 3/4}$$

(ii) Mean, (μ)

$$\text{Mean}(\mu) = \int_0^2 x\,f(x)\,dx$$

$$= \int_0^2 x\left[\frac{3}{2}x - \frac{3}{4}x^2\right]dx$$

$$= \int_0^2 \left(\frac{3x^2}{2} - \frac{3x^3}{4}\right)dx$$

$$= \left[\frac{3\times x^3}{2\times 3} - \frac{3}{4}\times\frac{x^4}{4}\right]_0^2$$

$$= \frac{\cancel{3}\times 8}{2\times\cancel{3}} - \frac{3}{4}\times\frac{16}{4}$$

$$= 4 - 3$$

$$= 1$$

$\therefore f(x) = \frac{3}{4}x(2-x)$

$f(x) = \frac{3}{2}x - \frac{3}{4}x^2$

(iv) Distri. function

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(iii) Variance (σ^2)

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$E(x) = \int_{R} x \cdot f(x) dx. = 1$$

$$E(x^2) = \int_{R} x^2 f(x) \cdot dx$$

$$E(x^2) = \int_0^2 x^2 \left(\frac{3}{2}x - \frac{3}{4}x^2\right) \cdot dx$$

$$= \int_0^2 \left(\frac{3x^3}{2} - \frac{3x^4}{4}\right) dx$$

$$= \left[\frac{3}{2} \cdot \frac{x^4}{4} - \frac{3}{4} \cdot \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{2} \cdot \frac{16}{4} - \frac{3}{4} \cdot \frac{32}{5}$$

$$= 6 - \frac{24}{5} = \frac{30-24}{5} = \frac{6}{5}$$

$$VX = EX^2 - (EX)^2$$

$$= \frac{6}{5} - 1^2$$

$$= \frac{6}{5} - 1$$

$$\boxed{VX = \frac{1}{5}}$$

Q6 Tut 3(a)

~~Q9~~ A continuous RV X that can assume any value between $x = 2$ and $x = 5$ has a density function given by. $f(x) = k(1+x)$. Find $P(X<4)$

Soln. By the property of pdf.

$$\int_{Rx} f(x) \cdot dx = 1, \quad X \text{ takes values b/n 2 and 5}$$

$$\therefore \int_2^5 k(1+x)\, dx = 1$$

$$k \int_2^5 (1+x)\, dx = 1$$

$$= k\left[x + \frac{x^2}{2}\right]_2^5$$

$$= k\left[5 + \frac{5^2}{2} - \left(2 + \frac{2^2}{2}\right)\right] = k\,\frac{27}{2} = 1 \quad \text{or} \quad \boxed{k = \frac{2}{27}}$$

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$\therefore$ The pdf = $f(x) = k(1+x)$

$$f x = \frac{2}{27}(1+x)$$

To find $P(X<4)$

$$P(x<4) = P(2<x<4) = \int_2^4 \frac{2}{27}(1+x)\,dx$$

$$= \frac{2}{27}\left[x+\frac{x^2}{2}\right]_2^4$$

$$= \frac{2}{27}\left[4+\frac{4^2}{2}-\left(2+\frac{2^2}{2}\right)\right] = \frac{2}{27}\left[4+8-2-2\right]$$

$$= \frac{16}{27}$$

Q.7 Tut 3(a)
~~Q10~~ A continuous RV X has a pdf $f(x) = kx^2 e^{-x}$
Find (i) k (ii) mean and (iii) Variance

Soln. By the property of pdf

$$\int_0^\infty kx^2 e^{-x}\,dx = 1 \qquad \int u\cdot v\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\cdot\int v\,dx\right)$$

$$k\int_0^\infty x^2 \cdot e^{-x}\,dx = 1 \qquad u = x^2,\ v = e^{-x}$$

$$k\left[x^2\int e^{-x}dx - \int\left(2x\cdot\int -e^{-x}dx\right)\right] = 1$$

$$k\left[-x^2\cdot e^{-x} + 2\int x\cdot e^{-x}dx\right] = 1 \qquad u = x,\ v = e^{-x}$$

$$k\left[-x^2e^{-x} + 2\left[-xe^{-x} - \int -e^{-x}dx\right]\right] = 1$$

$$= k\left[-x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}dx\right] = 1$$

$$= k\left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c\right] = 1$$

$$= -k\left[e^{-x}(x^2+2x+2)\right]_0^\infty - k\left[0-2\right] = 1 \quad \therefore 2k = 1$$

$$k = 1/2$$

substituting the value of k. in pdf we get

$$f(x) = \frac{x^2 e^{-x}}{2}$$

(ii) Mean (μ)

$$\mu = E(X) = \int_0^\infty x.f(x)\,dx.$$

$$= \int_0^\infty \frac{x^3.e^{-x}}{2}\,dx. = \frac{1}{2}\int_0^\infty x^3.e^{-x}$$

we know that $\int u.v\,dx = u\int v\,dx - \int \frac{du}{dx}.\int v\,dx$

$$= \frac{1}{2}\int_0^\infty x^3 e^{-x}\,dx.$$

$$= \frac{1}{2}\left[x^3.(-e^{-x}) - \int 3x^2.(-e^{-x})\,dx\right]_0^\infty$$

$$= \frac{1}{2}\left[-x^3 e^{-x} - 3x^2.e^{-x} + \int 6x.e^{-x}\,dx\right]_0^\infty$$

$$= \frac{1}{2}\left[-x^3 e^{-x} - 3x^2 e^{-x} + 6x(-e^{-x}) - \int 6.(-e^{-x})\,dx\right]_0^\infty$$

$$= \frac{1}{2}\left[-x^3 e^{-x} - 3x^2 e^{-x} - 6x.e^{-x} - 6e^{-x}\right]_0^\infty$$

$$= 0 - \frac{1}{2}[-6]$$

$$= 3$$

(iii) Variance $\sigma^2 = E(X^2) - (E(X))^2 = V(X)$

$$E(X^2) = \int_0^\infty x^2.f(x)\,dx = \frac{1}{2}\int_0^\infty x^4 e^{-x}\,dx.$$

$$= \frac{1}{2}\left[x^4(-e^{-x}) - \int 4x^3.(-e^{-x})\,dx\right]_0^\infty$$

$$= \frac{1}{2}\left[-x^4 e^{-x} - 4x^3 e^{-x} + \int 12x^2.e^{-x}\,dx\right]_0^\infty$$

$$= \frac{1}{2}\left[-x^4 e^{-x} - 4x^3 e^{-x} + 12x^2(-e^{-x}) - \int 24x(-e^{-x})\,dx\right]_0^\infty$$

$$= \frac{1}{2}\left[-x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24xe^{-x} + \int 24.e^{-x}\,dx\right]_0^\infty$$

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$$E(X^2) = \frac{1}{2}\left[-x^4e^{-x} - 4x^3e^{-x} - 12x^2e^{-x} - 24xe^{-x} - 24e^{-x}\right]_0^\infty$$

$$= \frac{1}{2}\left[0 - (-24)\right]$$

$$= 12$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= 12 - 3^2$$

$$= 3.$$

Q.5 Tut 3a

~~Q.15~~ If the density of a continuous RV X is given by

$$f(x) = ax \quad 0 \le x \le 1$$
$$= a \quad 1 \le x \le 2$$
$$= 3a - ax \quad 2 \le x \le 3$$
$$= 0 \quad \text{elsewhere.}$$

(i) find value of a

(ii) find cdf of X — cumulative distribution function

Soln. (i) Since $f(x)$ is a pdf, $\int_{R_x} f(x)\,dx = 1$

i.e. $\int_0^3 f(x)\,dx = 1$

i.e. $\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx = 1$

$$= \left[\frac{ax^2}{2}\right]_0^1 + \left[ax\right]_1^2 + \left[3ax - \frac{ax^2}{2}\right]_2^3$$

$$= \frac{a}{2} + a + \left[\left(9a - \frac{9a}{2}\right) - (6a - 2a)\right]$$

$$= \frac{a}{2} + a + \frac{9a}{2} - 4a = \frac{a + 2a + 9a - 8a}{2}$$

$$= \frac{4a}{2} = 2a = 1$$

$$a = \frac{1}{2}$$

$f(x) = \frac{x}{2}$

$= \frac{1}{2}, \quad 1 \le x \le 2$

$= \frac{3}{2} - \frac{x}{2}, \quad 2 \le x \le 3$

$= 0$ elsewhere

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(ii) cdf of x

$V(x) = E(x^2) - (E(x))^2$

$= \int_0^3 x^2 f(x)dx - \left[\int_0^3 x f(x)dx\right]^2$

$E(x) = \int_0$

(ii) $F(x) = P(X \le x) = 0$, when $x < 0$

$F(x) = \int_0^x \frac{x}{2} dx = \frac{x^2}{4}$, when $0 \le x \le 1$

$= \int_0^1 \frac{x}{2} dx + \int_1^x \frac{1}{2} dx = \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^x = \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$

$= \frac{x}{2} - \frac{1}{4}$ when $1 \le x \le 2$

$= \int_0^1 \frac{x}{2} dx + \int_1^2 \frac{1}{2} dx + \int_2^x \left(\frac{3}{2} - \frac{x}{2}\right) dx$

$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^2 + \left[\frac{3x}{2} - \frac{x^2}{4}\right]_2^x$

$= \frac{1}{4} + \left(\frac{2}{2} - \frac{1}{2}\right) + \left(\frac{3x}{2} - \frac{x^2}{4}\right) - \left(\frac{3 \times 2}{2} - \frac{2^2}{4}\right)$

$= \frac{1}{4} + \frac{1}{2} - 2 + \frac{3x}{2} - \frac{x^2}{4}$

$= \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4}$, when $2 \le x \le 3$

$= 1$, when $x > 3$

(iii) If x_1, x_2 and x_3 are 3 independent observations of X, what is the probability that exactly one of these 3 is greater than 1.5

Tut. 3(a)

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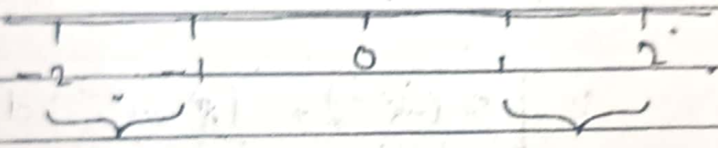
Q1. If pdf of a random variable X is given by

$$f(x)\begin{cases}1/4, & -2<x<2\\ 0 & \text{elsewhere.}\end{cases}$$

Find $P\{|X|>1\}$

Soln. $f(x) = \frac{1}{4}, \quad -2<x<2$



$$\therefore P\{|X|>1\} = P(-2<X<-1) + P(1<x<2)$$

$$= \int_{-2}^{-1} \frac{1}{4}\,dx + \int_{1}^{2} \frac{1}{4}\,dx$$

$$= \left[\frac{x}{4}\right]_{-2}^{-1} + \left[\frac{x}{4}\right]_{1}^{2}$$

$$\frac{1}{4}\left[-1-(-2)+2-1\right] = \frac{1}{4}\left[-1+2+2-1\right]$$

$$= \frac{2}{4} = \frac{1}{2}$$

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Q2 Find the value of k, if

$$f(x) = \begin{cases} kxe^{-x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$ is the pdf of a R.V. X.

$$f(x) = kxe^{-x}, \quad x > 0$$

then $$\int_0^{\infty} f(x)\,dx = 1$$

$$\int_0^{\infty} k.xe^{-x}\,dx = 1$$

$\int u.v\,dx = u\int v\,dx - \int \left(\frac{du}{dx}\right).\int v\,dx$

$$k\left[x(-e^{-x}) - \int \cancel{x}(-e^{-x})\,dx\right]_0^{\infty}$$

$$k\left[-xe^{-x} + x(-e^{-x}) - \int e^{-x}\,dx\right]_0^{\infty}$$

$$k\left[-xe^{-x} - xe^{-x} + xe^{-x}\right]_0^{\infty}$$

$$-k\left[xe^{-x}\right]_0^{\infty} = 1$$

$$-\cancel{k}\left[\right.$$

$$k\left[x(-e^{-x}) - \int -e^{-x}\,dx.\right]_0^{\infty}$$

$$k\left[-xe^{-x} - e^{-x}\right]_0^{\infty} = 1$$

$$k\left[(0-0) - (0-1)\right] = 1$$

$$k = 1$$

$$\therefore \quad f(x) = xe^{-x}$$

Q3 The probability distribution of a R.V. is given below.

x:	-2	-1	0	1	2	3
$P(x)$:	0.1	k	0.2	$2k$	0.3	$3k$

Find $Y = X^2 + 2X$, find the probability distrⁿ, mean and variance of Y.

Soln:

x:	-2	-1	0	1	2	3
$P(x)$:	0.1	k	0.2	$2k$	0.3	$3k$
$P(x)$:	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{2}{10}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{3}{15}$
$Y = X^2+2X$:	0	-1	0	3	8	15

$\therefore$

Y:	-1	0	3	8	15
$P(Y)$:	$\frac{1}{15}$	$\frac{3}{10}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{3}{15}$

$P(Y=-1) = P(x^2+2x=-1) \Rightarrow x^2+2x+1 = 0$

$(x+1)(x+1) = 0 \Rightarrow x = -$

at $x=-1$ $P(x) = \frac{1}{15} = P(Y=$

$P(Y=0) = P(x^2+2x = 0) \Rightarrow$ ~~$x^2 = -2x \Rightarrow x = -2$~~

~~at $x=-2$ $P(x) = \frac{1}{10}$~~

~~$x(x^2+2x)=0$~~

$x(x+2) = 0$

i.e. $x = 0$, $x = -2$

$P(X=0) = \frac{2}{10}$

$P(X=-2) = \frac{1}{10}$

$\therefore P(Y=0) = \frac{2}{10} + \frac{1}{10} = \frac{3}{10}$

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$P(Y=3) \Rightarrow P(x^2+2x=3) \Rightarrow P(x^2+2x-3=0)$

$\cancel{P(}\ x^2+3x-x-3 = x(x+3)-1(x+3)$

$(x+3)(x-1)=0 \Rightarrow x=-3, 1$

$= P(X=-3)+P(X=1)$

$= 0 + \frac{2}{15} = \frac{2}{15}$

$P(Y=8) \Rightarrow P(x^2+2x=8) \Rightarrow P(x^2+2x-8=0)$

$x^2+4x-2x-8=0 \quad x(x+4)-2(x+4)$

$(x+4)(x-2)=0 \quad x=-4, 2$

$= P(X=-4)+P(X=2)$

$= 0+\frac{3}{10} = \frac{3}{10}$

$P(Y=15) \Rightarrow P(x^2+2x=15) \Rightarrow P(x^2+2x-15=0)$

$x^2+5x-3x-15 \Rightarrow x(x+5)-3(x+5)$

$(x+5)(x-3) \Rightarrow x=-5, +3$

$\Rightarrow P(X=-5)+P(X=3)$

$\Rightarrow 0+\frac{3}{15}$

$\therefore$ The probability distr^n of Y

						Σ
Y :	-1	0	3	8	15	
$P(Y)$:	$\frac{1}{15}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{3}{15}$	
$Y \times P(Y)$:	$-1/15$	0	$3/5$	$24/10$	3	$86/15$
Y^2 :	1	0	9	64	225	
$Y^2 . P(Y)$:	$-\frac{1}{15}$	0	$\frac{9}{5}$	$\frac{96}{5}$	45	$\frac{982}{15}$

Mean of Y :

$$E(Y) = \sum_i Y_i \cancel{\xi} P(Y_i)$$

$$= -1\times\frac{1}{15} + 0 + 3\times\frac{2}{10} + 8\times\frac{3}{10} + 15\times\frac{3}{15}$$

$$= -\frac{1}{15} + \frac{6}{10} + \frac{24}{10} + 3 = \frac{-2+\cancel{18}^{12}+72+90}{30}$$

$$= \frac{\cancel{178}\ 172}{30} = \frac{\cancel{89}}{15} \quad \frac{86}{15}$$

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Variance of Y

$$V(Y) = E(Y^2) - \{E(Y)\}^2$$

$$E(Y^2) = \sum_i Y_i^2 \cdot P(Y_i)$$

$$E(Y^2) = \frac{1}{15}\times 1 + 0 + 9\times\frac{2}{10} + 64\times\frac{3}{10} + \frac{3}{15}\times 225 = \frac{\cancel{899}\ 982}{15}$$

$$V(Y) = \frac{\cancel{991}\ 982}{15} - \left(\frac{\cancel{89}\ 86}{15}\right)^2 = \frac{\cancel{6944}}{\cancel{225}}$$

$$= \cancel{30.86}\ 32.54 = \frac{7334}{225}$$

Q. 6 A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by

$$f(x) = k(1+x).$$

Find $P(X<4)$

Soln.

Pdf $f(x) = k(1+x)$.

$$\therefore \int f(x)\,dx = 1$$

$$\int_2^5 (k + kx)\,dx = k\left[x + \frac{x^2}{2}\right]_2^5 = 1$$

$$\Rightarrow k\left[5 + \frac{5^2}{2} - \left(2 + \frac{2^2}{2}\right)\right] = 1$$

$$\Rightarrow k\left[5 + \frac{25}{2} - 2 - 2\right] = 1$$

$$\Rightarrow \cancel{k + \frac{25}{2}}\ \frac{27}{2}k = 1$$

$$k = \frac{2}{27}$$

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$\therefore \quad pdf = \frac{2}{27}(1+x)$

$$P(X<4) = \int_2^4 \frac{2}{27}(1+x)\,dx$$

$$= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^4$$

$$= \frac{2}{27}\left[4 + \frac{4^2}{2} - 2 - \frac{2^2}{2}\right]$$

$$= \frac{2}{27}\left[4 + 8 - 2 - 2\right]$$

$$= \frac{16}{27}$$

$\rightarrow$

$x^2+2x-y=0$

$P(y<7)$

$P(x^2+2x<y)$

$P(x$