

2(c) exercise 7.1.
$$\begin{pmatrix} L_{00}^T \lambda_{10} e_L & 0 \\ 0 & 1 & 0 \\ 0 & v_{12} e_F & U_{22}^T \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{matrix} L_{00}^T x_0 + \lambda_{10} e_L \cdot x_1 = 0. & - (1) \\ x_1 = 1 \\ x_1 \cdot v_{12} e_F + U_{22}^T x_2 = 0. & - (2) \end{matrix}$$

$(1) L_{00}^T x_0 + \lambda_{10} e_L = 0. \Rightarrow x_0 = L_{00} \cdot (-\lambda_{10}) e_L = -\lambda_{10} \cdot L_{00} \cdot e_L = -\lambda_{10} \begin{pmatrix} \lambda_{m-2, m-1} \\ \lambda_{m-1, m-1} \end{pmatrix}$

$(2) v_{12} e_F + U_{22}^T x_2 = 0. \Rightarrow x_2 = U_{22} \cdot (-v_{12}) e_F = -v_{12} U_{22} \cdot e_F \Rightarrow -v_{12} \cdot \begin{pmatrix} U_{00} \\ U_{10} \\ 0 \end{pmatrix}$

$$\Rightarrow X = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 \\ -\lambda_{10} \cdot \lambda_{m-2, m-1} \\ -\lambda_{10} \cdot \lambda_{m-1, m-1} \\ 1 \\ -v_{12} \cdot U_{00} \\ -v_{12} \cdot U_{10} \\ 0 \end{bmatrix}$$

* Cost.

- The cost of $-\lambda_{10} \cdot L_{00} \cdot e_L$ is given L is bidiagonal, $O(4)$ for each row in L (eg. $(\lambda_{10} \cdot 0 + \lambda_{11} \cdot 0) \cdot \lambda_{10}$) at most.
- So, it costs $4m$ at most to get $-\lambda_{10} \cdot L_{00} \cdot e_L$.
- The logic applies to $-v_{12} \cdot U_{22} \cdot e_F$.
- So, the total cost of getting X is $\sim O(n)$