

1 Flow variables and units (SI)

Flow variables and their units:

1. Density ρ is kg/m³
2. Pressure p is Pa (is N/m² is J/m³)
3. Flow velocity v is m/s
4. Total energy per unit volume $E = \rho e = \rho \varepsilon_{\text{int}} + \rho v^2/2$ is J/m³
5. Total specific energy e is J/kg
6. Total specific internal energy ε is J/kg
7. Mass of the constituent species m is kg

2 Euler equations

For a two-dimensional flow of a single-species inviscid gas the compressible Euler equations governing such a flow are given by

$$\frac{\partial}{\partial t} \mathbf{u} + \frac{\partial}{\partial x} \mathbf{f}_x + \frac{\partial}{\partial y} \mathbf{f}_y = 0. \quad (1)$$

The vector of conservative variables $\mathbf{u} \in \mathbb{R}^4$ is given by

$$\mathbf{u} = (\rho, \rho v_x, \rho v_y, E)^T, \quad (2)$$

The inviscid fluxes are given by

$$\mathbf{f}_x = (\rho v_x, \rho v_x^2 + p, \rho v_x v_y, (E + p)v_x)^T, \quad (3)$$

$$\mathbf{f}_y = (\rho v_y, \rho v_x v_y, \rho v_y^2 + p, (E + p)v_y)^T. \quad (4)$$

Instead of E one can also write ρe .

Re-writing via scaled variables and reference quantities, we obtain

$$\frac{\partial}{\partial \hat{t}} \hat{\mathbf{u}} + \frac{\partial}{\partial \hat{x}} \hat{\mathbf{f}}_x + \frac{\partial}{\partial \hat{y}} \hat{\mathbf{f}}_y = 0, \quad (5)$$

$$\hat{\mathbf{u}} = \rho_{\text{ref}} (\hat{\rho}, v_{\text{ref}} \hat{\rho} \hat{v}_x, v_{\text{ref}} \hat{\rho} \hat{v}_y, p_{\text{ref}} / \rho_{\text{ref}} \hat{\rho} \hat{e}), \quad (6)$$

$$\hat{\mathbf{f}}_x = (\rho_{\text{ref}} v_{\text{ref}} \hat{\rho} \hat{v}_x, p_{\text{ref}} \hat{\rho} \hat{v}_x^2 + p_{\text{ref}} \hat{p}, p_{\text{ref}} \hat{\rho} \hat{v}_x \hat{v}_y, v_{\text{ref}} p_{\text{ref}} (\hat{\rho} \hat{e} + \hat{p}) \hat{v}_x), \quad (7)$$

$$\hat{\mathbf{f}}_y = (\rho_{\text{ref}} v_{\text{ref}} \hat{\rho} \hat{v}_y, p_{\text{ref}} \hat{\rho} \hat{v}_x \hat{v}_y, p_{\text{ref}} \hat{\rho} \hat{v}_y^2 + p_{\text{ref}} \hat{p}, v_{\text{ref}} p_{\text{ref}} (\hat{\rho} \hat{e} + \hat{p}) \hat{v}_y). \quad (8)$$

Accounting for the fact that $t_{\text{ref}} = L_{\text{ref}}/v_{\text{ref}}$, we can write:

$$\frac{\partial}{\partial \hat{t}} \hat{\mathbf{u}} + \frac{\partial}{\partial \hat{x}} \hat{\mathbf{f}}_x + \frac{\partial}{\partial \hat{y}} \hat{\mathbf{f}}_y = 0, \quad (9)$$

$$\hat{\mathbf{u}} = (\hat{\rho}, \hat{\rho} \hat{v}_x, \hat{\rho} \hat{v}_y, \hat{\rho} \hat{e}), \quad (10)$$

$$\hat{\mathbf{f}}_x = (\hat{\rho} \hat{v}_x, \hat{\rho} \hat{v}_x^2 + \hat{p}, \hat{\rho} \hat{v}_x \hat{v}_y, (\hat{\rho} \hat{e} + \hat{p}) \hat{v}_x), \quad (11)$$

$$\hat{\mathbf{f}}_y = (\hat{\rho} \hat{v}_y, \hat{\rho} \hat{v}_x \hat{v}_y, \hat{\rho} \hat{v}_y^2 + \hat{p}, (\hat{\rho} \hat{e} + \hat{p}) \hat{v}_y). \quad (12)$$

So the scaled Euler equations are identical to the non-scaled ones, as all reference quantities cancel out.

2.1 Scaling of specific heats

We have (for a single-component flow) $T = T_{ref}\hat{T} = m_{ref}p_{ref}/\rho_{ref}/k\hat{T}$, $\hat{T} = \hat{p}/\rho_{ref}$ (since at $T = T_{ref}$ and $n = n_{ref}$ the flow density is ρ_{ref} and the pressure is p_{ref}). Scaling of specific heats:

$$c_v(T) = \frac{\partial \varepsilon}{\partial T} = \frac{p_{ref}}{\rho_{ref}T_{ref}} \frac{\partial \hat{\varepsilon}}{\partial \hat{T}} = c_{v,ref}\hat{c}_v. \quad (13)$$

So $c_{v,ref} = p_{ref}/(\rho_{ref}T_{ref}) = k/m_{ref}$. Mayer's relation in scaled form then reads that $\hat{c}_p = \hat{c}_v + 1$.

3 Navier–Stokes equations