CS 540: Introduction to Artificial Intelligence
Homework Assignment # 11

Assigned: 12/11
Due: 12/18 before class

Hint: Many questions in this homework are variations of the Fall 17 final exam, for which you can find sample solutions on the course webpage. *Show your steps.* 100 points, equally spread.

1. (Search) Consider $n+1$ states. S_1 is the initial state, S_n is the goal state. S_0 is a dead-end state with no successors. For each state S_i, $i = 1, \ldots, n-1$, it has two successors: S_{i+1} and S_0. S_n also has no successors. There is no cycle check nor CLOSED list. How many goal-checks will be performed by depth first search? Assume everything being equal, state with small index is checked first. If a state is goal-checked multiple times, count it multiple times.

2. (Game) Consider a variant of the II-nim game. There are two piles, each pile has three sticks. A player can take one stick from a single pile; or take two sticks: one from each pile (when available). The player who takes the last stick wins. Let the game value be 1 if the first player wins. Show the game tree and give the game theoretical value at all nodes.

3. (Probability) There are two biased coins in my wallet: coin A has $P(Heads) = a$, coin B has $P(Heads) = b$. I took out one coin at random (with equal probability choosing A or B) and flipped it twice: the outcome was Head, Tail. What is the probability that the coin was A?

4. (PCA) You performed PCA in R^2. It turns out that the first principal component is $u_1 = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, and the second principal component is $u_2 = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. One of your data points has its *new* representation as $(1, 2)$. What was the original coordinates of the point?

5. (Resolution) Given the knowledge base

   (a) $A \vee B$

   (b) $C \Rightarrow A$

   use resolution to prove the query $A \vee B \vee C$.

6. (Clustering) There are six points in two-dimensional space: $a = (0, 0), b = (1, 0), c = (3, 1), d = (7, 7), e = (9, 9), f = (3, 6)$. Perform Hierarchical Agglomerative Clustering with single linkage and Euclidean distance. Complete the resulting clustering tree diagram (i.e., the dendrogram).

7. (Gradient descent) Let $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2$. We want to minimize the objective function $f(\mathbf{x}) = \sin((x_1 + x_2)\pi) + e^{x_1 - x_2} + x_1 x_2$. Let the stepsize $\eta = 0.1$. If we start at $\mathbf{x}^{(0)} = (1, 1)$, what is the next vector $\mathbf{x}^{(1)}$ produced by gradient descent?

8. (Sigmoid) Derive the derivative of the sigmoid function $\sigma(x) = \frac{1}{1+\exp(-x)}$.

9. (MDP) Consider state space $S = \{s_1, \ldots, s_n\}$ and action space $A = \{left, right\}$:

s_1	...	s_n

The actions move the agent one step in the corresponding direction, except when it is at an end: attempting to move beyond the end makes the agent stay in the current state. When the agent is in state s_{n-1}, taking the "right" action also gives it reward $r = 1$. All other state-action pairs have zero reward. Let γ be the discounting factor. What is the value $v(s_1)$ under the optimal policy?

10. (Q-learning) A robot initializes Q-learning by setting $q(s, a) = 1$ for all state s and action a. It has a learning rate α, and discounting factor γ. The robot senses that it is in state s_1 and decides to performs action a_1. For this action, the robot receives reward 100 and arrives at state s_2. After this one step of Q-learning, for all s, a pairs show their value $q(s, a)$.