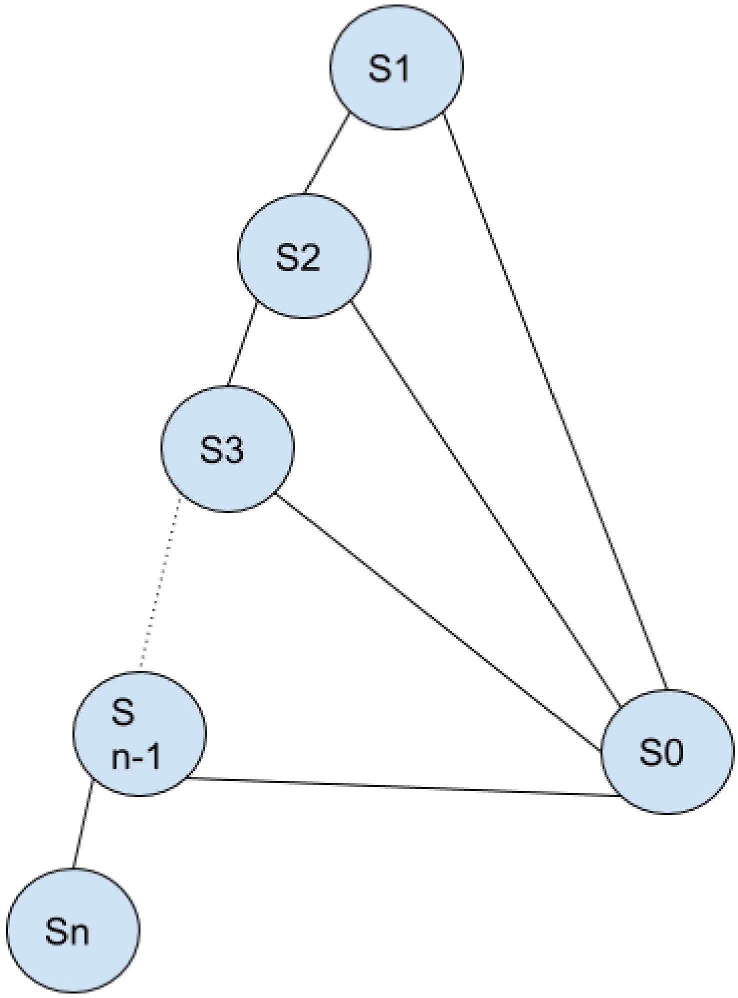
A 1.



Given the state connection to be, as shown above, we can now run DFS on the same.

Initial State = S1
Pop from stack = S1 ……… 1 Goal Check
Add successors of S1 -> S0, S2

Pop from stack = S0 ……… 1 Goal Check
No successors of S0
Pop from stack = S2 ……… 1 Goal Check
Add successors of S2 -> S0, S3

Pop from stack = S0 ……… 1 Goal Check
No successors of S0
Pop from stack = S3 ……… 1 Goal Check
Add successors of S3 -> S0, S4

…..
…..

…..
…..

Pop from stack = S0 ……… 1 Goal Check
No successors of S0
Pop from stack = S_{n-1} ……… 1 Goal Check
Add successors of S_{n-1} -> S0, S_n

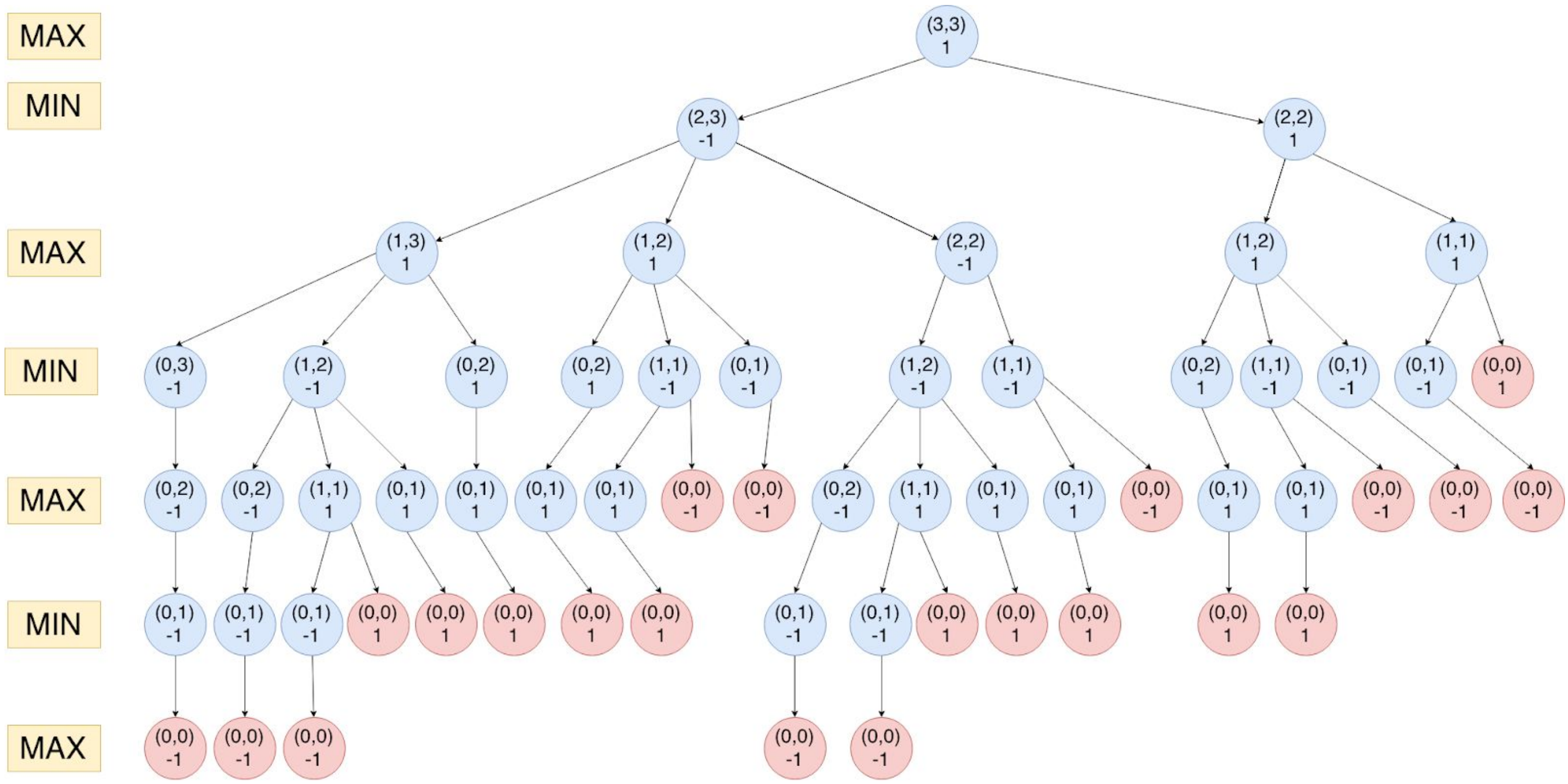
Pop from stack = S0 ……… 1 Goal Check
No successors of S0
Pop from stack = S_n ……… 1 Goal Check
Goal Found

Thus, the goal checks are made for
S1, S0, S2, S0, S3, …. S0, S_{n-1}, S0, S_n

Total no of goal checks = 1 + 2 + 2 + ….. (n-2+1)times … + 2
Total no of goal checks = 1 + 2(n-1)
Total no of goal checks = 1 + 2n - 2

Total no of goal checks = 2n - 1

A 2.



A 3.

P(H|A) = a
P(H|B) = b
P(A) = 0.5
P(B) = 0.5

Event (E) = H, T

To calculate: P(A|E) = P(E|A) / P(E)

now, P(E) = P(E,A) + P(E,B)
=> P(E) = P(E|A).P(A) + P(E|B).P(B)
=> P(E) = a.(1-a).0.5 + b.(1-b).0.5 {given the toin cosses are independent}

=> P(A|E) = $\frac{a.(1-a).0.5}{a.(1-a).0.5 + b.(1-b).0.5}$

=> P(A|E) = $\frac{a(1-a)}{a(1-a) + b(1-b)}$

A 4.

$u_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T$

$u_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)^T$

Projected data point = (1, 2)
Let us assume, the original point is (x_1, x_2)

=> The projected data point = $(u_1^T x, u_2^T x)$

=> $u_1^T x = 1$, and $u_2^T x = 2$
=> $\frac{x_1}{\sqrt{2}} + \frac{x_2}{\sqrt{2}} = 1$ …….. (1)
,and $-\frac{x_1}{\sqrt{2}} + \frac{x_2}{\sqrt{2}} = 2$ ……. (2)

adding (1) and (2)
=> $\frac{x_2}{\sqrt{2}} + \frac{x_2}{\sqrt{2}} = 3$
=> $2\frac{x_2}{\sqrt{2}} = 3$
=> $x_2 = \frac{3}{\sqrt{2}}$

subtracting (2) from (1)

=> $\frac{x_1}{\sqrt{2}} + \frac{x_1}{\sqrt{2}} = -1$
=> $2\frac{x_1}{\sqrt{2}} = -1$
=> $x_1 = \frac{-1}{\sqrt{2}}$

hence, the original point is $(\frac{-1}{\sqrt{2}}, \frac{3}{\sqrt{2}})$

A 6.

	a(0,0)	b(1,0)	c(3,1)	d(7,7)	e(9,9)	f(3,6)
a(0,0)		1	3.162278	9.899495	12.727922	6.708204
b(1,0)			2.236068	9.219544	12.041595	6.324555
c(3,1)				7.211103	10	5
d(7,7)					2.828427	4.123106
e(9,9)						6.708204
f(3,6)						

Cluster 1: (a+b)

	a(0,0)+b(1,0)	c(3,1)	d(7,7)	e(9,9)	f(3,6)
a(0,0)+b(1,0)		2.236068	9.219544	12.041595	6.324555
c(3,1)			7.211103	10	5
d(7,7)				2.828427	4.123106
e(9,9)					6.708204
f(3,6)					

Cluster 1: (a+b)
Cluster 2: (c + (a+b))

	a(0,0)+b(1,0)+c(3,1)	d(7,7)	e(9,9)	f(3,6)
a(0,0)+b(1,0)+c(3,1)		7.211103	10	5

d(7,7)			2.828427	4.123106
e(9,9)				6.708204
f(3,6)				

Cluster 1: (a+b)
Cluster 2: (c + (a+b))
Cluster 3: (d+e)

	a(0,0)+b(1,0)+c(3,1)	d(7,7)+e(9,9)	f(3,6)
a(0,0)+b(1,0)+c(3,1)		7.211103	5
d(7,7)+e(9,9)			4.123106
f(3,6)			

Cluster 1: (a+b)
Cluster 2: (c + (a+b))
Cluster 3: (d+e)
Cluster 4: (f + (d+e))

	a(0,0)+b(1,0)+c(3,1)	d(7,7)+f(3,6)+e(9,9)
a(0,0)+b(1,0)+c(3,1)		5
d(7,7)+f(3,6)+e(9,9)		

Cluster 1: (a+b)
Cluster 2: (c + (a+b))
Cluster 3: (d+f)
Cluster 4: (e + (d+f))
Cluster 5: ((c + (a+b)) + (e + (d+f)))

A 7.

Let x = (x1, x2) ∈ R2

objective function f(x) = $sin((x1 + x2)\pi) + e^{x1-x2} + x1x2$
stepsize η = 0.1
$x^{(0)}$ = (1, 1)
$x^{(1)}$ = ?

From gradient descent, we know

$x^{(1)} = x^{(0)} - \eta \nabla f(x^{(0)})$

$\nabla f(x) = \nabla(sin((x1 + x2)\pi) + e^{x1-x2} + x1x2)$

$= (\partial(sin((x1 + x2)\pi) + e^{x1-x2} + x1x2)/\partial x1, \partial(sin((x1 + x2)\pi) + e^{x1-x2} + x1x2)/\partial x2)$

$= (\pi.cos((x1 + x2)\pi) + e^{x1-x2} + x2, \pi.cos((x1 + x2)\pi) - e^{x1-x2} + x1)$

now, $\nabla f(x^{(0)}) = (\pi.cos((1 + 1)\pi) + e^{1-1} + 1, \pi.cos((1 + 1)\pi) - e^{1-1} + 1)$

$= (\pi.cos(2\pi) + e^{0} + 1, \pi.cos(2\pi) - e^{0} + 1)$

$= (\pi + 1 + 1, \pi - 1 + 1)$

$= (\pi + 2, \pi)$

$x^{(1)} = (1,1) - 0.1 * \nabla f(x^{(0)})$

$= (1,1) - 0.1 * (\pi + 2, \pi)$

$= (1,1) - (0.1 * \pi + 0.2, 0.1 * \pi)$

$= (0.8 - 0.1 * \pi, 1 - 0.1 * \pi)$

$x^{(1)}$ **= (0.8 - 0.1*** π **,1 - 0.1*** π **)**

A 8.

$\sigma(x) = \frac{1}{1+exp(-x)}$

$\sigma'(x) = \frac{d}{dx} \frac{1}{1+exp(-x)}$

Using the quotient and chain rule, we can differentiate above.

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$$

Hence,

$\sigma'(x) = \frac{(1+exp(-x)) \frac{d}{dx} 1 - 1 \frac{d}{dx} 1+exp(-x)}{(1+exp(-x))^2}$

$\sigma'(x) = \frac{0 - 1(-1)exp(-x)}{(1+exp(-x))^2}$

$\sigma'(x) = \frac{exp(-x)}{(1+exp(-x))^2}$

$\sigma'(x) = \frac{1 + exp(-x) - 1}{(1+exp(-x))^2}$

σ'(x) = $\frac{1 + exp(-x)}{(1+exp(-x))^2} - \frac{1}{(1+exp(-x))^2}$

σ'(x) = $\frac{1}{1+exp(-x)} - (\frac{1}{1+exp(-x)})^2$

σ'(x) = $\sigma(x) - \sigma(x)^2$

σ'(x) = $\sigma(x)(1 - \sigma(x))$

A 9.

The optimal policy for our case is
$\pi(s_1)$ = right
$\pi(s_2)$ = right
$\pi(s_3)$ = right
...
...
...
…
$\pi(s_{n-1})$ = right
$\pi(s_n)$ = left

Now,
$v(s_1) = \gamma^0.R0 + \gamma^1.R1 + \gamma^2.R2 + \gamma^3.R3 + \ldots\ldots$
$v(s_1) = \gamma^0.0 + \gamma^1.0 + \gamma^2.0 + \gamma^3.0 + \ldots\ldots \gamma^{n-3}.0 + \gamma^{n-2}.1 + \gamma^{n-1}.0 + \gamma^n.1 + \gamma^{n+1}.0 + \gamma^{n+2}.1 + \ldots.$
$v(s_1) = \gamma^{n-2}.1 + \gamma^n.1 + \gamma^{n+2}.1 + \ldots.$
$v(s_1)$ = $\frac{\gamma^{n-2}}{1 - \gamma^2}$

A 10.

Given that: q(s, a) = 1 ∀ s,a
Learning rate = α
Discounting factor = γ

s1, a1 -> s2
R(s1, a1) = 100

From bellman's equation,

q'(s1, a1) = α[R + γ.max(q(s', a'))] + (1 - α).q(s1,a1)
q'(s1, a1) = α[100 + γ.max(q(s2, a'))] + (1 - α).1

now, q(s2, a') = 1 ∀ a
=> max(q(s2, a')) = 1

=> $q'(s1, a1) = \boldsymbol{\alpha}[100 + \boldsymbol{\gamma}.1] + (1 - \boldsymbol{\alpha}).1$

$q'(s1, a1) = \boldsymbol{\alpha}[100 + \boldsymbol{\gamma}] + (1 - \boldsymbol{\alpha})$

$q'(s1, a1) = \boldsymbol{\alpha}[99 + \boldsymbol{\gamma}] + 1$

Rest of the Q-table remains intact.

Thus, the updated values will be

$q(s, a) = 1 \;\forall\; s,a - \{s1, A1\}$

$q(s1, a1) = \boldsymbol{\alpha}[99 + \boldsymbol{\gamma}] + 1$