CS 540-1: Introduction to Articial Intelligence
Homework Assignment # 7
Assigned: 10/30
Due: 11/8 before class

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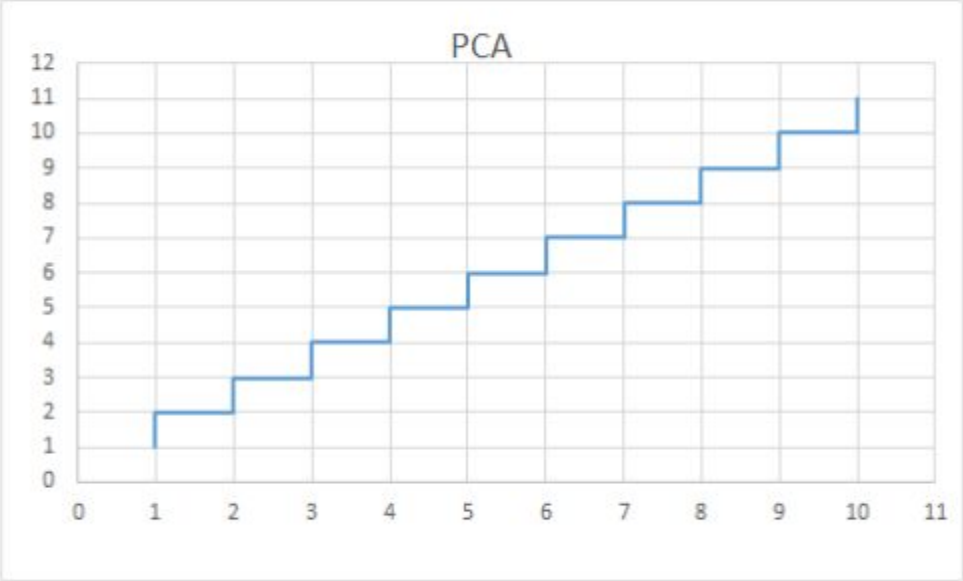
Question 1: PCA [25 points]

Consider a data set with 20 two-dimensional points of the form (i, i),(i, i + 1) for i = 1, . . . , 10.

1. Given the points (i,i), (i, i+1) for i = 1, ..., 10. Below represent the data points



Mean of x-coordinates

$X(mean) = (1 + 2 + ... + 10 + 1 + 2 + ... + 10) / 20 =$ **5.5**

Mean of y-coordinates

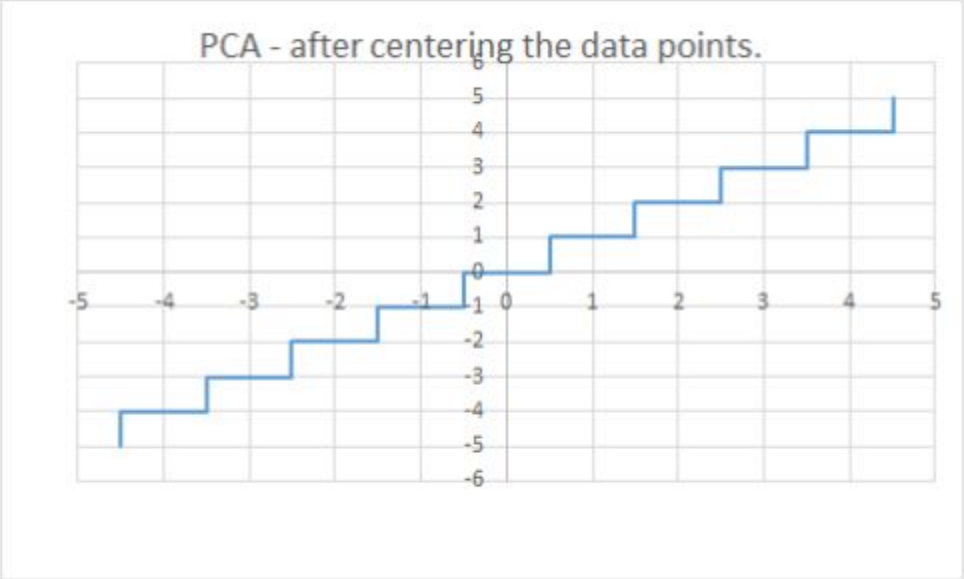
Y(mean) = (1 + 2 + ... + 10 + 2 + 3 + ... + 11) / 20 = **6**

Now, for computing the centered data points, we will shift all points using

X(i-centered)= X(i)− X(mean)

Y(i-centered)= Y(i)− Y(mean), where (Xi, Yi) are the i-th points in the given space.

Using above, the X(centered) data points can be represented as follow:



2. S = $(1/(n-1))X^T X$

For our centered data, X(centered) as computed above can be used.
n = 20, Upon computation we get below Covariance matrix.
= 1/(20-1) *
(165.000000 165.000000
165.000000 170.000000)

= (8.68421052 8.68421052
8.68421052 8.9473683)

3. Using eigen computation, eigenvalues and eigenvectors of covariance matrix are:

Eigenvalue $\lambda_1 = 0.131$
Eigenvector V_1 : [-0.712
-0.702]

Eigenvalue $\lambda_2 = 17.501$
Eigenvector V_2 : [0.702
- 0.712]

4. Compute the new two-dimensional coordinates of CENTERED points { corresponding to the old (i; i) and (i; i+1) { by projecting them onto v1; v2. (Hint: if you only keep the rst dimension, it would be a maximum spread projection of the original data set.)

Now, projection of a point X on a vector V is given by, X(centered) V^T

And, projection of any given points on these two vectors would be (X(centered) ${V_1}^T$, X(centered) ${V_2}^T$)

Using above for all the centered points, we can compute the projected values to be the following matrix.

original (x,y)	**projected x**	**projected y**
(1,1)	-6.71999989	-0.3026573662
(2,2)	-5.305826903	-0.3133701832
(3,3)	-3.891653917	-0.3240830001
(4,4)	-2.47748093	-0.334795817
(5,5)	-1.063307944	-0.3455086339
(6,6)	0.3508650424	-0.3562214508
(7,7)	1.765038029	-0.3669342678
(8,8)	3.179211015	-0.3776470847
(9,9)	4.593384002	-0.3883599016
(10,10)	6.007556988	-0.3990727185
(1,2)	-6.007556988	0.3990727185
(2,3)	-4.593384002	0.3883599016
(3,4)	-3.179211015	0.3776470847
(4,5)	-1.765038029	0.3669342678
(5,6)	-0.3508650424	0.3562214508
(6,7)	1.063307944	0.3455086339
(7,8)	2.47748093	0.334795817
(8,9)	3.891653917	0.3240830001
(9,10)	5.305826903	0.3133701832
(10,11)	6.71999989	0.3026573662

Question 2: Resolution [25 points]

Given the knowledge base

$$p \Rightarrow (q \Rightarrow r)$$

use resolution to prove the query

$$(p \wedge q) \Rightarrow (q \Rightarrow r)$$

Be sure to show what you convert to CNF and how (do not skip steps), and how you perform each resolution step.

Solution:

Knowledge Base, converting to CNF form: $p \Rightarrow (q \Rightarrow r)$

$p \Rightarrow (q \Rightarrow r)$
$\sim p \vee (q \Rightarrow r)$ // Removing implication
$\sim p \vee \sim q \vee r$ // Removing implication

Query (β), converting to CNF form: $(p \wedge q) \Rightarrow (q \Rightarrow r)$

$(p \wedge q) \Rightarrow (q \Rightarrow r)$
$\sim(p \wedge q) \vee (q \Rightarrow r)$ // Removing implication
$\sim(p \wedge q) \vee \sim q \vee r$ // Removing implication
$\sim p \vee \sim q \vee r$ //Moving negation inwards
Now, for resolution we add negation of Query (**β**),
So, $\sim\beta = \sim(\sim p \vee \sim q \vee r)$
$p \wedge q \wedge \sim r$ // Moving negation inwards

The components we have for to check resolution are:
A: $\sim p \vee \sim q \vee r$ // from KB
B1: p // from query
B2: q // from query
B3: $\sim r$ // from query

We can clearly see that conjunction of all above statements gives empty.
$\sim p \vee \sim q \vee r \wedge p \wedge q \wedge \sim r$
$(\sim p \wedge p) \vee (\sim q \wedge q) \vee (r \wedge \sim r)$
Everything will be nullified, thus proves the resolution. Thus we can say KB entails query.

Question 3: Hierarchical Clustering [25 points]

Consider the following six major cities. In the US: Madison, Seattle, Boston; and in Canada: Vancouver, Winnipeg, Montreal. For the purpose of this question ignore the curvature of the Earth, and compute the Euclidean distance. Suppose the cities are located at the following coordinates:
city coordinate
Madison (-89, 43)
Seattle (-122, 48)
Boston (-71, 42)
Vancouver (-123, 49)
Winnipeg (-97, 50)
Montreal (-74, 46)

1. Use hierarchical clustering with complete linkage to produce TWO clusters by hand. Specifically, show the following in each iteration: (1) the closest pair of clusters; (2) the distance between them as defined by complete linkage; (3) all clusters at the end of that iteration.

Iteration 1:

Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[Vancouver (-123, 49), Seattle (-122, 48)]	$\sqrt{\lvert -123-(-122)\rvert^2 + \lvert 49-48\rvert^2} = \sqrt{1+1} = \sqrt{2} = 1.414$

Iteration 2:

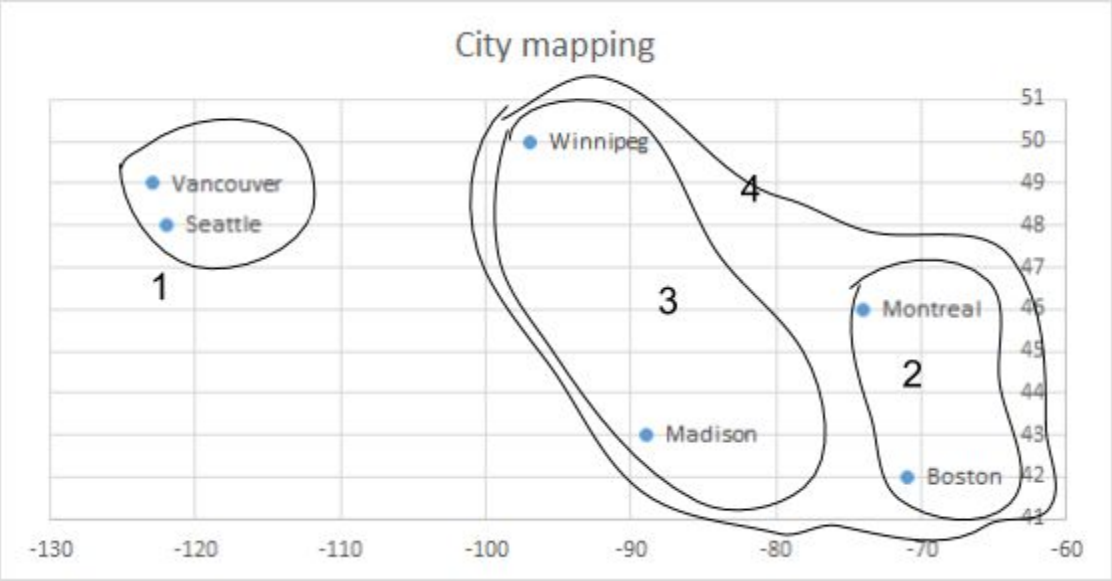
Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[Montreal (-74, 46), Boston (-71, 42)]	$\sqrt{\lvert -74-(-71)\rvert^2 + \lvert 46-42\rvert^2} = \sqrt{9+16} = 5$

Iteration 3:

Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[Winnipeg (-97, 50), Madison (-89, 43)]	$\sqrt{\lvert -89-(-97)\rvert^2 + \lvert 50-43\rvert^2} = \sqrt{64+49} = \sqrt{113} = 10.63$

Iteration 4:

Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[[Winnipeg (-97, 50), Madison (-89, 43)] , [Montreal (-74, 46), Boston (-71, 42)]]	$\sqrt{\lvert -97-(-71)\rvert^{2} + \lvert 50-42\rvert^{2}} = \sqrt{676+64}$ $= \sqrt{740} = 27.2$



After fourth iteration it forms two different clusters.

1. One cluster is formed which is shown in Iteration-1 Output.
   Output: [Vancouver (-123, 49), Seattle (-122, 48)]
2. Another cluster is shown in Iteration-4 output.
   Output: [[Winnipeg (-97, 50), Madison (-89, 43)] , [Montreal (-74, 46), Boston (-71, 42)]]

2. Now repeat the above question, but with the following constraint: at no point should a US city and a Canadian city be put in the same cluster. Equivalently, whenever the complete linkage between two clusters is due to two cities in different countries, treat the two clusters as infinity apart, regardless of what other cities are in those two clusters. Show the same (1)(2)(3) as above in each iteration.

Iteration 1:

Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[Madison (-89, 43), Boston (-71, 42)]	$\sqrt{\lvert -89-(-71)\rvert^{2} + \lvert 43-42\rvert^{2}} = \sqrt{324+1}$ $= \sqrt{325} = 18.03$

Iteration 2:

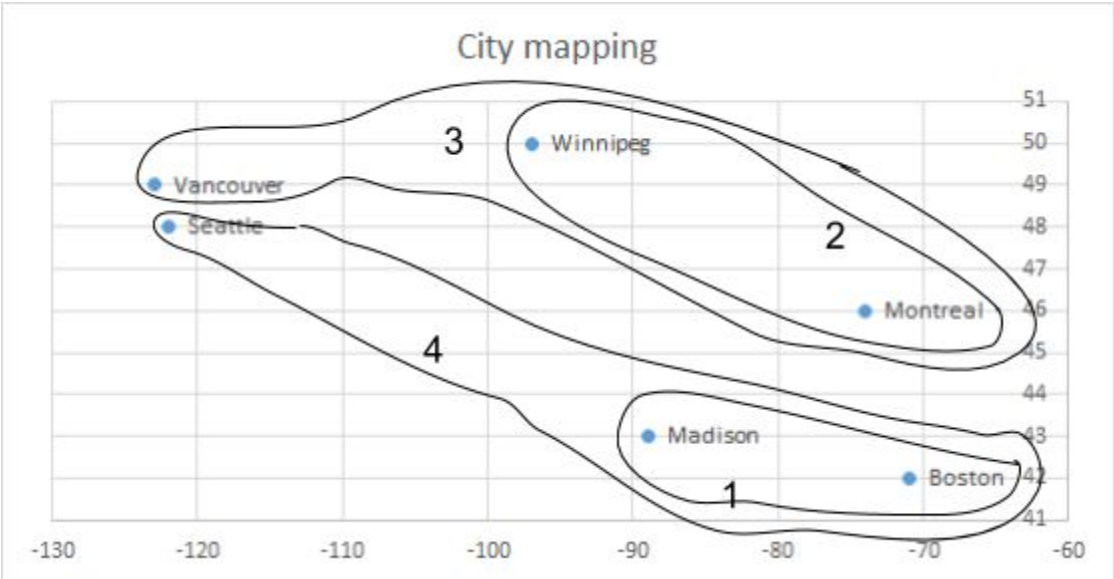
Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[Winnipeg (-97, 50), Montreal (-74, 46)]	$\sqrt{\lvert -97-(-74)\rvert^{2}+\lvert 50-46\rvert^{2}}$ $\sqrt{529+16}=\sqrt{545}=23.345$

Iteration 3:

Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[Vancouver (-123, 49), [Winnipeg (-97, 50), Montreal (-74, 46)]]	$\sqrt{\lvert -123-(-74)\rvert^{2}+\lvert 49-46\rvert^{2}}$ $\sqrt{2401+9}=\sqrt{2410}=49.092$

Iteration 4:

Closest Pair Of Cluster	Euclidean Distance (Complete Linkage)
[[Madison (-89, 43), Boston (-71, 42)] , Seattle (-122, 48)]	$\sqrt{\lvert -122-(-71)\rvert^{2}+\lvert 48-42\rvert^{2}}$ $\sqrt{2601+36}=\sqrt{2637}=51.352$



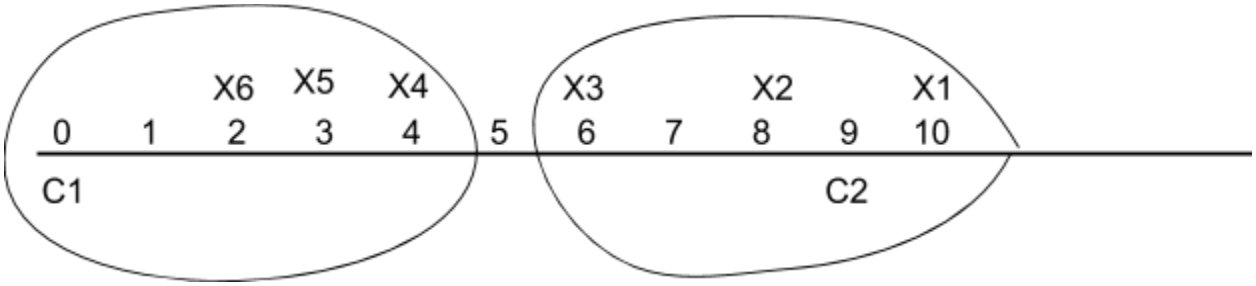
After fourth iteration it forms two different clusters.

1. One cluster is formed which is shown in Iteration-3 Output.
   Output: [Vancouver (-123, 49), [Winnipeg (-97, 50), Montreal (-74, 46)]]
2. Another cluster is shown in Iteration-4 Output.
   Output: [[Madison (-89, 43), Boston (-71, 42)] , Seattle (-122, 48)]

Question 4: K-means Clustering [25 points]

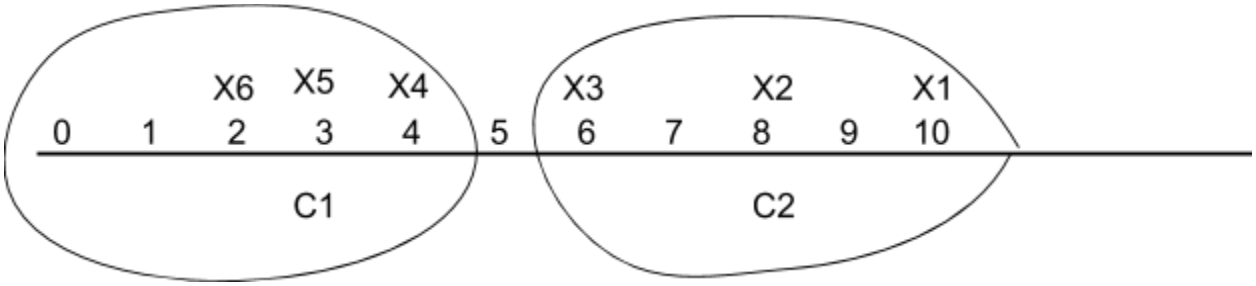
Given the following six items in 1D: x1 = 10; x2 = 8; x3 = 6; x4 = 4; x5 = 3; x6 = 2, perform k-means clustering to obtain k = 2 clusters by hand. Specifically,

1. Start from initial cluster centers c1 = 0; c2 = 9. Show your steps for all iterations: (1) the cluster assignments y1; : : : ; y6; (2) the updated cluster centers at the end of that iteration; (3) the energy at the end of that iteration.

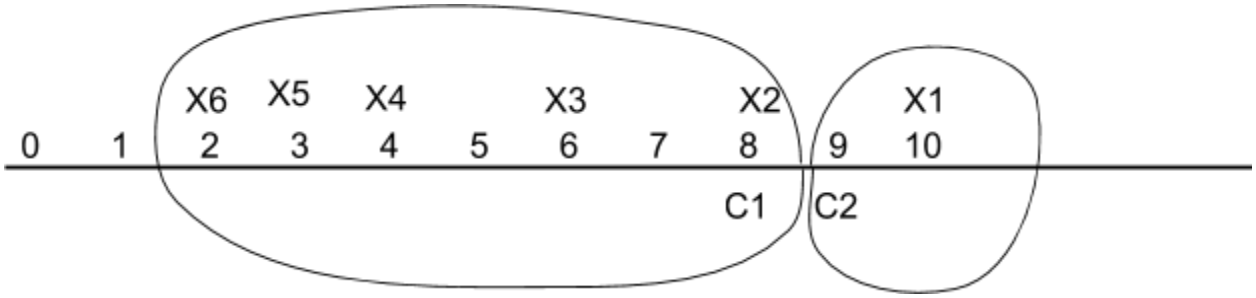


$C1 = (2 + 3 + 4) \div (3) = 3$

$C2 = (6 + 8 + 10) \div (3) = 8$

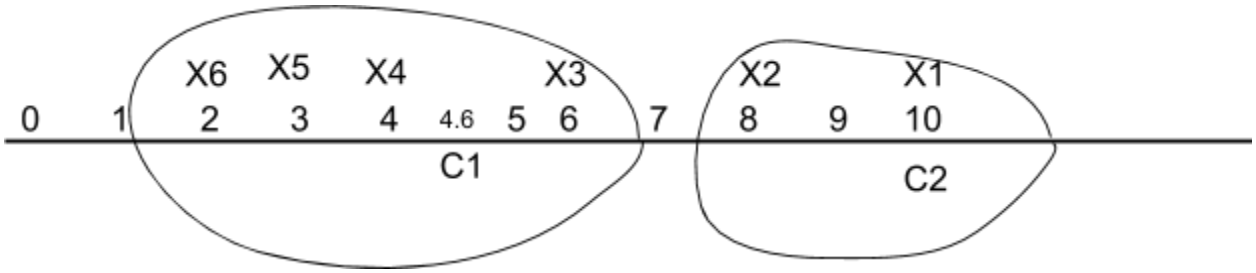


2. Repeat the above but start from initial cluster centers c1 = 8; c2 = 9.



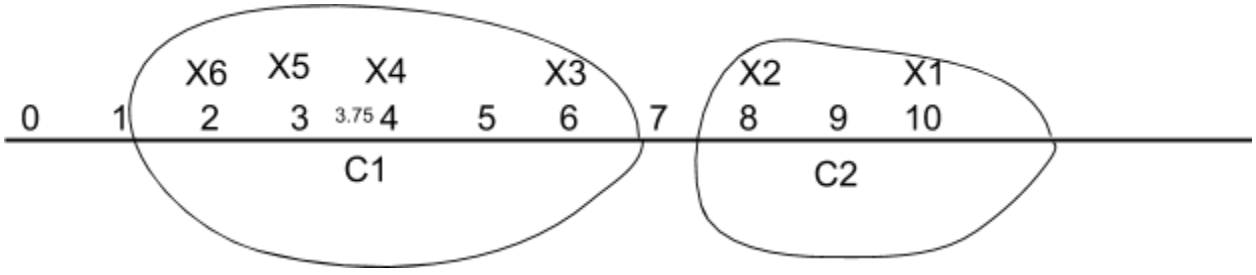
$C1 = (2 + 3 + 4 + 6 + 8) \div (5) = 4.6$

$C2 = (10) \div (1) = 10$



$C1 = (2 + 3 + 4 + 6) \div (4) = 3.75$

$C2 = (8 + 10) \div (2) = 9$



3. Which k-means solution is better? Why?

First is better. All the points are closer to centroid (this shows less distortion or energy value) when compared to the cluster solution resulted in second one. Less distortion of points in the resulted clustering makes first better than second.

Reason for respective solution: Choosing cluster centroids apart will make clustering quickly compared to the having to choose the centroids close to each other. The reason is that with centroid at some extreme will includes points whose variance is quite more having to recenter the centroid multiple times.