31

PROBABILITY

In our daily life, we often used phrases such as 'It may rain today', or 'India may win the match' or ' I may be selected for this post.' These phrases involve an element of uncertainty. How can we measure this uncertainty ? A measure of this uncertainty is provided by a branch of Mathematics, called the theory of probability. Probability Theory is designed to measure the degree of uncertainty regarding the happening of a given event. The dictionary meaning of probability is ' likely though not certain to occur. Thus, when a coin is tossed, a head is likely to occur but may not occur. Similarly, when a die is thrown, it may or may not show the number 6.

In this lesson, we shall discuss some basic concepts of probability, addition and multiplication theorem of probability and applications of probability in our day to day life.

OBJECTIVES

After studying this lesson, you will be able to :

- define probability of occurance of an event;
- cite through examples that probability of occurance of an event is a non-negative fraction, not greater than one;
- use permutation and combinations in solving problems in probability;
- state and establish the addition theorems on probability and the conditions under which each holds;
- generalize the addition theorem of probability for mutually exclusive events;
- state the multiplication theorem on probability for any two events;
- state and prove the multiplication theorem on compound probability for independent events;
- solve problems on probability using addition and multiplication theorems;
- define conditional probability of an event; and
- solve problems involving conditional probability.

EXPECTED BACKGROUND KNOWLEDGE

- Knowledge of random experiments and events.

- The meaning of sample space.
- A standard deck of playing cards consists of 52 cards divided into 4 suits of 13 cards each : spades, hearts, diamonds, clubs and cards in each suit are - ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards and the other cards are called number cards.

31.1 EVENTS AND THEIR PROBABILITY

In the previous lesson, we have learnt whether an activity is a random experiment or not. The study of probability always refers to random experiments. Hence, from now onwards, the word experiment will be used for a random experiment only. In the preceeding lesson, we have defined different types of events such as equally likely, mutually exclusive, exhaustive, independent and dependent events and cited examples of the above mentioned events.

Here we are interested in the chance that a particular event will occur, when an experiment is performed. Let us consider some examples.

What are the chances of getting a ' Head' in tossing an unbiased coin ? There are only two equally likely outcomes, namely head and tail. In our day to day language, we say that the coin has chance 1 in 2 of showing up a head. In technical language, we say that the probability of getting a head is $\frac{1}{2}$.

Similarly, in the experiment of rolling a die, there are six equally likely outcomes 1, 2,3,4,5 or 6. The face with number '1' (say) has chance 1 in 6 of appearing on the top. Thus, we say that the probability of getting 1 is $\frac{1}{6}$.

In the above experiment, suppose we are interested in finding the probability of getting even number on the top, when a die is rolled. Clearly, the numbers possible are 2, 4 and 6 and the chance of getting an even number is 3 in 6. Thus, we say that the probability of getting an even number is $\frac{3}{6}$, i.e., $\frac{1}{2}$.

The above discussion suggests the following definition of probability.

If an experiment with '*n*' exhaustive, mutually exclusive and equally likely outcomes, *m* outcomes are favourable to the happening of an event A, the probability '*p*' of happening of A is given by

$$p = \mathrm{P}(\mathrm{A}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{m}{n} \qquad \text{....(i)}$$

Since the number of cases favourable to the non-happening of the event A are $n - m$, the probability 'q' that 'A' will not happen is given by

$$q = \frac{n-m}{n} = 1 - \frac{m}{n}$$

$$= 1 - p \quad \text{[Using (i)]}$$

$$\therefore \qquad p + q = 1.$$

Obviously, p as well as q are non-negative and cannot exceed unity.

i.e., $0 \le p \le 1, \quad 0 \le q \le 1$

Thus, the probability of occurrence of an event lies between 0 and 1[including 0 and 1].

Remarks

1. Probability 'p' of the happening of an event is known as the probability of success and the probability 'q' of the non-happening of the event as the probability of failure.
2. Probability of an impossible event is 0 and that of a sure event is 1

   if P (A) = 1, the event A is certainly going to happen and

   if P (A) = 0, the event is certainly not going to happen.
3. The number (m) of favourable outcomes to an event cannot be greater than the total number of outcomes (n).

Let us consider some examples

Example 31.1 A die is rolled once. Find the probability of getting a 5.

Solution : There are six possible ways in which a die can fall, out of these only one is favourable to the event.

$$\therefore \qquad P(5) = \frac{1}{6}.$$

Example 31.2 A coin is tossed once. What is the probability of the coin coming up with head ?

Solution : The coin can come up either 'head' (H) or a tail (T). Thus, the total possible outcomes are two and one is favourable to the event.

So, $$P(H) = \frac{1}{2}$$

Example 31. 3 A die is rolled once. What is the probability of getting a prime number ?

Solution : There are six possible outcomes in a single throw of a die. Out of these; 2, 3 and 5 are the favourable cases.

$$\therefore \qquad P(\text{Prime Number}) = \frac{3}{6} = \frac{1}{2}$$

Example 31.4 A die is rolled once. What is the probability of the number '7' coming up ? What is the probability of a number 'less than 7' coming up ?

Solution : There are six possible outcomes in a single throw of a die and there is no face of the die with mark 7.

$$\therefore \qquad P(\text{number } 7) = \frac{0}{6} = 0$$

[**Note :** That the probability of impossible event is zero]

As every face of a die is marked with a number less than 7,

$$\therefore \qquad P(\le 7) = \frac{6}{6} = 1$$

[**Note :** That the probability of an event that is certain to happen is 1]

Example 31.5 In a simultaneous toss of two coins, find the probability of

(i) getting 2 heads (ii) exactly 1 head

Solution : Here, the possible outcomes are

HH, HT, TH, TT.

i.e., Total number of possible outcomes = 4.

(i) Number of outcomes favourable to the event (2 heads) = 1 (i.e., HH).

$$\therefore \qquad P(\text{ 2 heads }) = \frac{1}{4}.$$

(ii) Now the event consisting of exactly one head has two favourable cases, namely HT and TH .

$$\therefore \qquad P(\text{ exactly one head }) = \frac{2}{4} = \frac{1}{2}.$$

Example 31.6 In a single throw of two dice, what is the probability that the sum is 9?

Solution : The number of possible outcomes is $6 \times 6 = 36$. We write them as given below :

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Now, how do we get a total of 9. We have :

$3 + 6 = 9$

$4 + 5 = 9$

$5 + 4 = 9$

$6 + 3 = 9$

In other words, the outcomes (3, 6), (4, 5), (5, 4) and (6, 3) are favourable to the said event, i.e., the number of favourable outcomes is 4.

Hence, $P(\text{a total of 9 }) = \frac{4}{36} = \frac{1}{9}$

Example 31.7 From a bag containing 10 red, 4 blue and 6 black balls, a ball is drawn at random. What is the probability of drawing

(i) a red ball ? (ii) a blue ball ? (iii) not a black ball ?

Solution : There are 20 balls in all. So, the total number of possible outcomes is 20. (Random drawing of balls ensure equally likely outcomes)

(i) Number of red balls = 10

$\therefore \qquad P(\text{a red ball}) = \frac{10}{20} = \frac{1}{2}$

(ii) Number of blue balls = 4

$\therefore \qquad P(\text{a blue ball}) = \frac{4}{20} = \frac{1}{5}$

(iii) Number of balls which are not black = 10 + 4

= 14

$\therefore \qquad P(\text{not a black ball}) = \frac{14}{20} = \frac{7}{10}$

Example 31.8 A card is drawn at random from a well shuffled deck of 52 cards. If A is the event of getting a queen and B is the event of getting a card bearing a number greater than 4 but less than 10, find P(A) and P (B).

Solution : Well shuffled pack of cards ensures equally likely outcomes.

$\therefore$ the total number of possible outcomes is 52.

(i) There are 4 queens in a pack of cards.

$\therefore \qquad P(A) = \frac{4}{52} = \frac{1}{13}$

(ii) The cards bearing a number greater than 4 but less than 10 are 5,6, 7,8 and 9.

Each card bearing any of the above number is of 4 suits diamond, spade, club or heart.

Thus, the number of favourable outcomes = 5 × 4 = 20

$\therefore \qquad = \frac{20}{52} = \frac{5}{13}$

Example 31.9 What is the chance that a leap year, selected at random, will contain 53 Sundays?

Solution : A leap year consists of 366 days consisting of 52 weeks and 2 extra days. These two extra days can occur in the following possible ways.

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

Out of the above seven possibilities, two outcomes, e.g., (i) and (vii), are favourable to the event

$$\therefore \qquad P\text{ (53 Sundays) } = \frac{2}{7}$$

CHECK YOUR PROGRESS 31.1

1. A die is rolled once. Find the probability of getting 3.
2. A coin is tossed once. What is the probability of getting the tail ?
3. What is the probability of the die coming up with a number greater than 3 ?
4. In a simultaneous toss of two coins, find the probability of getting ' at least' one tail.
5. From a bag containing 15 red and 10 blue balls, a ball is drawn 'at random'. What is the probability of drawing (i) a red ball ? (ii) a blue ball ?
6. If two dice are thrown, what is the probability that the sum is (i) 6 ? (ii) 8? (iii) 10? (iv) 12?
7. If two dice are thrown, what is the probability that the sum of the numbers on the two faces is divisible by 3 or by 4 ?
8. If two dice are thrown, what is the probability that the sum of the numbers on the two faces is greater than 10 ?
9. What is the probability of getting a red card from a well shuffled deck of 52 cards ?
10. If a card is selected from a well shuffled deck of 52 cards, what is the probability of drawing

    (i) a spade ? (ii) a king ? (iii) a king of spade ?
11. A pair of dice are thrown. Find the probability of getting

    (i) a sum as a prime number

    (ii) a doublet, i.e., the same number on both dice

    (iii) a multiple of 2 on one die and a multiple of 3 on the other.
12. Three coins are tossed simultaneously. Find the probability of getting

    (i) no head (ii) at least one head (iii) all heads

31.2. CALCULATION OF PROBABILITY USING COMBINATORICS (PERMUTATIONS AND COMBINATIONS)

In the preceding section, we calculated the probability of an event by listing down all the possible outcomes and the outcomes favourable to the event. This is possible when the number of outcomes is small, otherwise it becomes difficult and time consuming process. In general, we do not require the actual listing of the outcomes, but require only the total number of possible outcomes and the number of outcomes favourable to the event. In many cases, these can be found by applying the knowledge of permutations and combinations, which you have already studied.

Let us consider the following examples :

Example 31.10 A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue ?

Solution : Total number of balls = 3 + 6 + 7 = 16

Now, out of 16 balls, 2 can be drawn in ${}^{16}C_2$ ways.

$\therefore$ Exhaustive number of cases = ${}^{16}C_2 = \dfrac{16 \times 15}{2} = 120$

Out of 6 white balls, 1 ball can be drawn in ${}^{6}C_1$ ways and out of 7 blue balls, one can be drawn is ${}^{7}C_1$ ways. Since each of the former case is associated with each of the later case, therefore total number of favourable cases are ${}^{6}C_1 \times {}^{7}C_1 = 6 \times 7 = 42$.

$\therefore$ Required probability $= \dfrac{42}{120} = \dfrac{7}{20}$

Remarks

When two or more balls are drawn from a bag containing several balls, there are two ways in which these balls can be drawn.

(i) Without replacement : The ball first drawn is not put back in the bag, when the second ball is drawn. The third ball is also drawn without putting back the balls drawn earlier and so on. Obviously, the case of drawing the balls without replacement is the same as drawing them together.

(ii) With replacement : In this case, the ball drawn is put back in the bag before drawing the next ball. Here the number of balls in the bag remains the same, every time a ball is drawn.

In these types of problems, unless stated otherwise, we consider the problem of without replacement.

Example 31.11 Find the probability of getting both red balls, when from a bag containing 5 red and 4 black balls, two balls are drawn,

(i) with replacement.

(ii) without replacement.

Solution : (i) Total number of balls in the bag in both the draws = 5 + 4 = 9

Hence, by fundamental principle of counting, the total number of possible outcomes = 9 × 9 = 81.

Similarly, the number of favourable cases = 5 × 5 = 25.

Hence, probability (both red balls) $= \dfrac{25}{81}$.

(ii) Total number of possible outcomes is equal to the number of ways of selecting 2 balls out of 9 balls = ${}^{9}C_2$.

Number of favourable cases is equal to the number of ways of selecting

2 balls out of 5 red balls = ${}^{5}C_2$.

Hence, P (both red balls) = $\dfrac{{}^{5}C_2}{{}^{9}C_2} = \dfrac{\dfrac{5\times 4}{1\times 2}}{\dfrac{9\times 8}{1\times 2}} = \dfrac{5}{18}$

Example 31.12 Six cards are drawn at random from a pack of 52 cards. What is the probability that 3 will be red and 3 black?

Solution : Six cards can be drawn from the pack of 52 cards in ${}^{52}C_6$ ways.

i.e., Total number of possible outcomes = ${}^{52}C_6$

3 red cards can be drawn in ${}^{26}C_3$ ways and

3 black cards can be drawn in ${}^{26}C_3$ ways.

$\therefore$ Total number of favourable cases = ${}^{26}C_3 \times {}^{26}C_3$

Hence, the required probability = $\dfrac{{}^{26}C_3 \times {}^{26}C_3}{{}^{52}C_6} = \dfrac{13000}{39151}$

Example 31.13 Four persons are chosen at random from a group of 3 men, 2 women and 4 children. Show that the chance that exactly two of them will be children is $\dfrac{10}{21}$.

Solution : Total number of persons in the group = 3 + 2 + 4 = 9. Four persons are chosen at random. If two of the chosen persons are children, then the remaining two can be chosen from 5 persons (3 men + 2 women).

Number of ways in which 2 children can be selected from 4

children = ${}^{4}C_2 = \dfrac{4\times 3}{1\times 2} = 6$

Number of ways in which remaining of the two persons can be selected

from 5 persons = ${}^{5}C_2 = \dfrac{5\times 4}{1\times 2} = 10$

Total number of ways in which 4 persons can be selected out of

9 persons = ${}^{9}C_4 = \dfrac{9\times 8\times 7\times 6}{1\times 2\times 3\times 4} = 126$

Hence, the required probability = $\dfrac{{}^{4}C_2 \times {}^{5}C_2}{{}^{9}C_4} = \dfrac{6\times 10}{126} = \dfrac{10}{21}$

Example 31.14 Three cards are drawn from a well-shuffled pack of 52 cards. Find the probability that they are a king, a queen and a jack.

Solution : From a pack of 52 cards, 3 cards can be drawn in $^{52}C_3$ ways, all being equally likely.

$\therefore$ Exhaustive number of cases = $^{52}C_3$

A pack of cards contains 4 kings, 4 queens and 4 jacks .A king, a queen and a Jack can each be drawn in $^4C_1$ ways and since each way of drawing a king can be associated with each of the ways of drawing a queen and a jack, the total number of favourable cases = $^4C_1 \times {}^4C_1 \times {}^4C_1$

$$\therefore \quad \text{Required probability} = \frac{^4C_1 \times {}^4C_1 \times {}^4C_1}{^{52}C_3}$$

$$= \frac{4 \times 4 \times 4}{\frac{52 \times 51 \times 50}{1 \times 2 \times 3}}$$

$$= \frac{16}{5525}$$

Example 31.15 From 25 tickets, marked with the first 25 numerals, one is drawn at random. Find the probability that it is a multiple of 5.

Solution : Numbers (out of the first 25 numerals) which are multiples of 5 are 5, 10, 15, 20 and 25, i.e., 5 in all. Hence, required favourable cases are = 5.

$\therefore$ Required probability = $\frac{5}{25} = \frac{1}{5}$

Example 31.16 If the letters of the word 'REGULATIONS' be arranged at random, what is the probability that there will be exactly 4 letters between R and E?

Solution : The word 'REGULATIONS' consists of 11 letters. The two letters R and E can occupy $^{11}P_2$, i.e., 11×10=110 positions.

The number of ways in which there will be exactly 4 letters between R and E are enumerated below :

R is in the first place and E in the 6th place.
R is in the 2nd place and E in the 7th place
------- ------- ------- -------
------- ------- ------- -------
------- ------- ------- -------
R is in the 6th place and E is in the 11th place.

Since R and E can interchange their positions, the required number of favourable cases is 2 × 6 = 12.

$\therefore$ The required probability $= \dfrac{12}{110} = \dfrac{6}{55}$

Example 31.17 Out of (2n + 1) tickets consecutively numbered starting with 1, three are drawn at random. Find the chance that the numbers on them are in A.P.

Solution : Since out of (2n + 1) tickets, 3 tickets can be drawn in ${}^{2n+1}C_3$ ways,

Therefore, exhaustive number of cases = ${}^{2n+1}C_3$

$$= \frac{(2n+1)2n(2n-1)}{1\times 2\times 3}$$

$$= \frac{n\left(4n^2-1\right)}{3}$$

To find the favourable number of cases, we are to enumerate all the cases in which the number on the drawn tickets are in A.P. with common difference (say, d = 1, 2, 3, ..., n – 1, n)

If d = 1, possible cases are as follows :

$$\left.\begin{array}{l} 1,\ 2,\ 3 \\ 2,\ 3,\ 4 \\ : \\ 2n-1,\quad 2n,\ 2n+1 \end{array}\right\},\ \text{i.e., } (2n-1) \text{ cases in all.}$$

If d = 2, the possible cases are as follows :

$$\left.\begin{array}{l} 1,\ 3,\ 5 \\ 2,\ 4,\ 6 \\ : \\ 2n-3,\quad 2n-1,\ 2n+1 \end{array}\right\},\ (2n-3) \text{ cases in all.}$$

and so on

If d = n – 1, the possible cases are

$$\left.\begin{array}{l} 1,\ n,\quad 2n-1 \\ 2,\ n+1,\ 2n \\ 3,\ n+2,\quad 2n+1 \end{array}\right\},\ \text{i.e., 3 cases in all.}$$

If d = n, there is only one case, viz., 1, n + 1, 2n + 1

Hence, total number of favourable cases

$$= (2n-1) + (2n-3) + \ldots\ldots + 5 + 3 + 1$$
$$= 1 + 3 + 5 + \ldots\ldots. + (2n-1)$$

which is a series in A.P. with d = 2 and n terms.

$\therefore$ Number of favourable cases $= \frac{n}{2}[2 + (n-1)2]$

$$= \frac{n}{2}(2n) = n^2$$

Hence, required probability $= \dfrac{n^2}{\dfrac{n(4n^2-1)}{3}} = \dfrac{3n}{4n^2-1}$

CHECK YOUR PROGRESS 31.2

1. A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn at random are both white?

2. A bag contains 5 red and 8 blue balls. What is the probability that two balls drawn are red and blue ?

3. A bag contains 20 white and 30 black balls. Find the probability of getting 2 white balls, when two balls are drawn at random

   (a) with replacement (b) without replacement

4. Three cards are drawn from a well-shuffled pack of 52 cards. Find the probability that all the three cards are jacks.

5. Two cards are drawn from a well-shuffled pack of 52 cards. Show that the chances of drawing both aces is $\frac{1}{221}$.

6. In a group of 10 outstanding students in a school, there are 6 boys and 4 girls. Three students are to be selected out of these at random for a debate competition. Find the probability that

   (i) one is boy and two are girls.

   (ii) all are boys.

   (iii) all are girls.

7. Out of 21 tickets marked with numbers from 1 to 21, three are drawn at random. Find the probability that the numbers on them are in A.P.

8. Two cards are drawn at random from 8 cards numbered 1 to 8. What is the probability that the sum of the numbers is odd, if the cards are drawn together ?

9. A team of 5 players is to be selected from a group of 6 boys and 8 girls. If the selection is made randomly, find the probability that there are 2 boys and 3 girls in the team.

10. An integer is chosen at random from the first 200 positive integers.Find the probability that the integer is divisible by 6 or 8.

MODULE - VI
Statistics

Notes

31.3 EVENT RELATIONS

Let us consider the example of throwing a fair die. The sample space of this experiment is

$$S = \{1, 2, 3, 4, 5, 6\}$$

If A be the event of getting an even number, then the sample points 2, 4 and 6 are favourable to the event A.

The remaining sample points 1, 3 and 5 are not favourable to the event A. Therefore, these will occur when the event A will not occur.

In an experiment, the outcomes which are not favourable to the event A are called complement of A and defined as follows :

'The outcomes favourable to the complement of an event A consists of all those outcomes which are not favourable to the event A, and are denoted by 'not' A or by $\overline{A}$.

31.3.1 Event 'A or B'

Let us consider the example of throwing a die. A is an event of getting a multiple of 2 and B be another event of getting a multiple of 3.

The outcomes 2, 4 and 6 are favourable to the event A and the outcomes 3 and 6 are favourable to the event B.

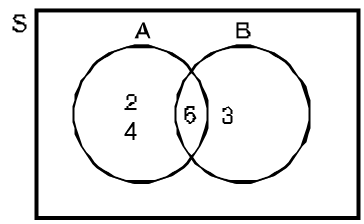

Fig. 31.1

The happening of event A or B is

$$A \cup B = \{2, 3, 4, 6\}$$

Again, if A be the event of getting an even number and B is another event of getting an odd number, then

$$A = \{2, 4, 6\}, \qquad B = \{1, 3, 5\}$$

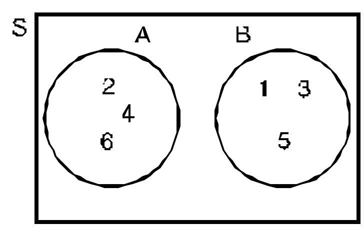

Fig. 31.2

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Here, it may be observed that if A and B are two events, then the event ' A or B ' ($A \cup B$) will consist of the outcomes which are either favourable to the event A or to the event B or to both the events.

Thus, the event 'A or B' occurs, if either A or B or both occur.

31.3.2 Event "A and B'

Recall the example of throwing a die in which A is the event of getting a multiple of 2 and B is the event of getting a multiple of 3. The outcomes favourable to A are 2, 4, 6 and the outcomes favourable to B are 3 , 6.

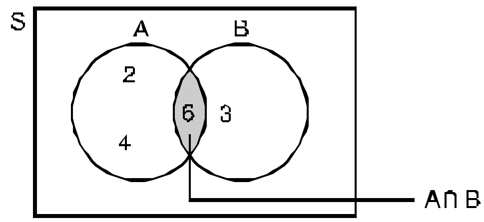

Fig. 31.3

Here, we observe that the outcome 6 is favourable to both the events A and B.

Draw a card from a well shuffled deck of 52 cards. A and B are two events defined as

A : a red card

B : a king

We know that there are 26 red cards and 4 kings in a deck of cards. Out of these 4 kings, two are red.

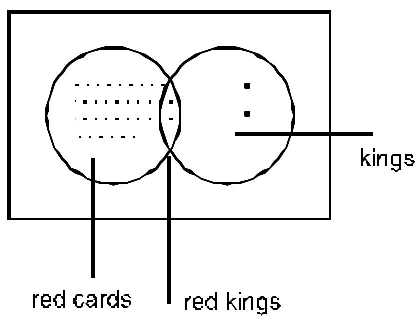

Fig. 31.4

Here, we see that the two red kings are favourable to both the events.

Hence, the event ' A and B' consists of all those outcomes which are favourable to both the events A and B. That is, the event 'A and B' occurs, when both the events A and B occur simultaneously. Symbolically, it is denoted as $A \cap B$.

31.4 ADDITIVE LAW OF PROBABILITY

Let A be the event of getting an odd number and B be the event of getting a prime number in a single throw of a die. What will be the probability that it is either an odd number or a prime number ?

In a single throw of a die, the sample space would be

$$S = \{ 1, 2, 3, 4, 5, 6 \}$$

The outcomes favourable to the events A and B are

$$A = \{ 1, 3, 5 \}$$

$B = \{ 2, 3, 5 \}$

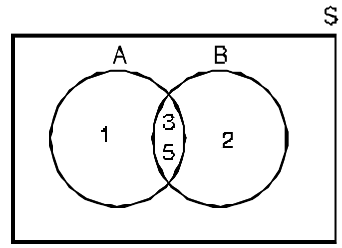

Fig. 31.5

The outcomes favourable to the event 'A or B' are

$$A \cup B = \{1, 2, 3, 5\}.$$

Thus, the probability of getting either an odd number or a prime number will be

$$P(A \text{ or } B) = \frac{4}{6} = \frac{2}{3}$$

To discover an alternate method, we can proceed as follows :

The outcomes favourable to the event A are 1, 3 and 5.

$$\therefore \qquad P(A) = \frac{3}{6}$$

Similarly, $\qquad P(B) = \frac{3}{6}$

The outcomes favourable to the event 'A and B' are 3 and 5.

$$\therefore \qquad P(A \text{ and } B) = \frac{2}{6}$$

Now, $\quad P(A) + P(B) - P(A \text{ and } B) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6}$

$$\frac{4}{6} = \frac{2}{3}$$

$$= P(A \text{ or } B)$$

Thus, we state the following law, called additive rule, which provides a technique for finding the probability of the union of two events, when they are not disjoint.

For any two events A and B of a sample space S,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

or $\qquad P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad$ .....(ii)

Example 31.18 A card is drawn from a well-shuffled deck of 52 cards. What is the prob ability that it is either a spade or a king ?

Solution : If a card is drawn at random from a well-shuffled deck of cards, the likelyhood of any of the 52 cards being drawn is the same. Obviously, the sample space consists of 52 sample points.

If A and B denote the events of drawing a 'spade card' and a 'king' respectively, then the event A consists of 13 sample points, whereas the event B consists of 4 sample points. Therefore,

$$P(A) = \frac{13}{52}, \qquad P(B) = \frac{4}{52}$$

The compound event $(A \cap B)$ consists of only one sample point, viz.; king of spade. So,

$$P(A \cap B) = \frac{1}{52}$$

Hence, the probability that the card drawn is either a spade or a king is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

Example 31.19 In an experiment with throwing 2 fair dice, consider the events

A : The sum of numbers on the faces is 8

B : Doubles are thrown.

What is the probability of getting A or B ?

Solution : In a throw of two dice, the sample space consists of $6 \times 6 = 36$ sample points.

The favourable outcomes to the event A (the sum of the numbers on the faces is 8) are

$$A = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

The favourable outcomes to the event B (Double means both dice have the same number) are

$$B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$A \cap B = \{(4,4)\}.$$

Now $P(A) = \frac{5}{36}$, $P(B) = \frac{6}{36}$, $P(A \cap B) = \frac{1}{36}$

Thus, the probability of A or B is

$$P(A \cup B) = \frac{5}{36} + \frac{6}{36} - \frac{1}{36}$$

$$= \frac{10}{36} = \frac{5}{18}$$

31.5 ADDITIVE LAW OF PROBABILITY FOR MUTUALLY EXCLUSIVE EVENTS

We know that the events A and B are mutually exclusive, if and only if they have no outcomes in common. That is, for mutually exclusive events,

$$P(A \text{ and } B) = 0$$

Substituting this value in the additive law of probability, we get the following law :

$$P(A \text{ or } B) = P(A) + P(B) \qquad \text{.....(iii)}$$

Example 31.20 In a single throw of two dice, find the probability of a total of 9 or 11.

Solution : Clearly, the events - a total of 9 and a total of 11 are mutually exclusive.

Now $\quad P(\text{a total of } 9) = P[(3, 6), (4, 5), (5, 4), (6, 3)] = \dfrac{4}{36}$

$\quad P(\text{a total of } 11) = P[(5, 6), (6, 5)] = \dfrac{2}{36}$

Thus, $\quad P(\text{a total of 9 or 11}) = \dfrac{4}{36} + \dfrac{2}{36}$

$$= \frac{1}{6}$$

Example 31.21 The probabilities that a student will receive an A, B , C or D grade are 0.30, 0.35, 0.20 and 0.15 respectively. What is the probability that a student will receive at least a B grade ?

Solution : The event at least a 'B' grade means that the student gets either a B grade or an A grade.

$\therefore \quad P(\text{at least B grade}) = P(\text{B grade}) + P(\text{A grade})$

$$= 0.35 + 0.30$$

$$= 0.65$$

Example 31.22 Prove that the probability of the non-occurrence of an event A is $1 - P(A)$.

i.e., $\quad P(\text{not } A) = 1 - P(A) \quad$ or, $\quad P(\bar{A}) = 1 - P(A)$.

Solution : We know that the probability of the sample space S in any experiment is 1.

Now, it is clear that if in an experiment an event A occurs, then the event $(\bar{A})$ cannot occur simultaneously, i.e., the two events are mutually exclusive.

Also, the sample points of the two mutually exclusive events together constitute the sample space S. That is,

$$A \cup \bar{A} = S$$

Thus, $\quad P\left(A \cup \bar{A}\right) = P(S)$

$\Rightarrow P(A) + P(\bar{A}) = 1$ ($\because$ A and $\bar{A}$ are mutually exclusive and S is sample space)

$\Rightarrow P\left(\bar{A}\right) = 1 - P(A)$,

which proves the result.

This is called the law of complementation.

Law of complimentation : $P\left(\bar{A}\right) = 1 - P(A)$

Example 31.23 Find the probability of the event getting at least 1 tail, if four coins are tossed once.

Solution : In tossing of 4 coins once, the sample space has 16 samples points.

$\therefore$ P (at least one tail) = P (1or 2 or 3 or 4 tails)

$= 1 - P(0 \text{ tail})$ (By law of complimentation)

$= 1 - P(H\,H\,H\,H)$

The outcome favourable to the event four heads is 1.

$$\therefore \quad P(H\,H\,H\,H) = \frac{1}{16}$$

Substituting this value in the above equation,

we get

$$P(\text{at least one tail}) = 1 - \frac{1}{16} = \frac{15}{16}$$

In many instances, the probability of an event may be expressed as odds - either odds in favour of an event or odds against an event.

If A is an event :

The odds in favour of A $= \dfrac{P(A)}{P(\bar{A})}$ or $P(A)$ to $P(\bar{A})$,

where P (A) is the probability of the event A and $P(\bar{A})$ is the probability of the event 'not A'.

Similarly, the odds against A are

$$\frac{P(\bar{A})}{P(A)} \text{ or } P(A) \text{ to } P(\bar{A}).$$

Example 31.24 The probability of the event that it will rain is 0.3. Find the odds in favour of rain and odds against rain.

Solution : Let A be the event that it will rain.

$$\therefore \quad P(A) = .3$$

By law of complementation,

$$P(\bar{A}) = 1 - .3 = .7 .$$

Now, the odds in favour of rain are $\dfrac{0.3}{0.7}$ or 3 to 7 (or 3 : 7).

The odds against rain are

$$\frac{0.7}{0.3} \text{ or } 7 \text{ to } 3.$$

When either the odds in favour of A or the odds against A are given, we can obtain the probability of that event by using the following formulae

If the odds in favour of A are *a* to *b*, then

$$P\ (A\ ) = \frac{a}{a+b}.$$

If the odds against A are a to b, then

$$P\ (A\ ) = \frac{b}{a+b}.$$

This can be proved very easily.

Suppose the odds in favour of A are *a* to *b*. Then, by the definition of odds,

$$\frac{P(A)}{P(\bar{A})} = \frac{a}{b}.$$

From the law of complimentation,

$$P\left(\bar{A}\right) = 1 - P\ (A)$$

Therefore, $\dfrac{P(A)}{1-P(A)} = \dfrac{a}{b}$ or $b\ P\ (A) = a - a\ P\ (A)$

or $(a + b)\ P\ (A) = a$ or $P\ (A) = \dfrac{a}{a+b}$

Similarly, we can prove that

$$P\ (A) = \frac{b}{a+b}$$

when the odds against A are b to a.

Example 31.25 Determine the probability of A for the given odds

(a) 3 to 1 in favour of A (b) 7 to 5 against A.

Solution : (a) $P\ (A) = \dfrac{3}{3+1} = \dfrac{3}{4}$ (b) $P\ (A) = \dfrac{5}{7+5} = \dfrac{5}{12}$

Example 31.26 If two dice are thrown, what is the probability that the sum is

(a) greater than 8 ? (b) neither 7 nor 11 ?

Solution : (a) If S denotes the sum on two dice, then we want $P(S > 8)$. The required event can happen in the following mutually exclusive ways :

(i) S = 9 (ii) S = 10, (iii) S = 11, and (iv) S = 12 .

Hence, by addition probability theorem for mutually exclusive events, we get

$$P(S > 8) = P(S = 9) + P(S = 10) + P(S = 11) + P(S = 12) \qquad ....(1)$$

In a throw of two dice, the sample space contains $6 \times 6 = 36$ points. The number of favourable cases can be enumerated as shown below :

S = 9 : (3, 6), (4, 5), (5, 4), (6, 3) , i.e., 4 sample points.

$\therefore \quad P(S=9) = \frac{4}{36}.$

S = 10 : (4, 6), (5, 5), (6, 4), , i.e., 3 sample points.

$\therefore \quad P(S=10) = \frac{3}{36}.$

S = 11 : (5, 6), (6, 5), i.e., 2 sample points.

$\therefore \quad P(S=11) = \frac{2}{36}.$

S = 12 : (6, 6), i.e., 1 sample point.

$\therefore \quad P(S=12) = \frac{1}{36}.$

Substituting these values in equation (1), we get

$$P(S>8) = \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36} = \frac{5}{18}.$$

(b) Let A and B denote the events of getting the sum 7 and 11 respectively on a pair of dice.

S = 7 : (1, 6), (2, 5), (3, 4), (4, 3) ,(5 , 2), (6, 1), i.e., 6 sample points.

$\therefore \quad P(S=7) = \frac{6}{36}$, or $P(A) = \frac{6}{36}.$

S = 11 : (5, 6) , (6, 5), i.e., 2 sample points.

$\therefore P(S=11) = \frac{2}{36}$, or $P(B) = \frac{2}{36}.$

Since A and B are disjoint events, therefore

$$P(\text{either} A \text{or} B) = P(A) + P(B)$$

$$= \frac{6}{36} + \frac{2}{36}$$

$$= \frac{8}{36}$$

Hence, by law of complementation,

P (neither 7 nor 11) = 1 – P (either 7 or 11)

$$= 1 - \frac{8}{36}$$

$$= \frac{28}{36}$$

$$= \frac{7}{9}.$$

Example 31.27 Are the following probability assignments consistent ? Justify your answer.

(a) $P(A) = P(B) = 0.6, \quad P(A \text{ and } B) = 0.05$

(b) $P(A) = 0.5, \quad P(B) = 0.4, \quad P(A \text{ and } B) = 0.1$

(c) $P(A) = 0.2, \quad P(B) = 0.7, \quad P(A \text{ and } B) = 0.4$

Solution : (a) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$= 0.6 + 0.6 - 0.05$$

$$= 1.15$$

Since $P(A \text{ or } B) > 1$ is not possible, hence the given probabilities are not consistent.

(b) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$= 0.5 + 0.4 - 0.1$$

$$= 0.8$$

which is less than 1.

As the number of outcomes favourable to event 'A and B' should always be less than or equal to those favourable to the event A,

Therefore, $P(A \text{ and } B) \le P(A)$

and similarly $P(A \text{ and } B) \le P(B)$

In this case, P (A and B) = 0.1, which is less than both P (A) = 0.5 and P (B) = 0.4. Hence, the assigned probabilities are consistent.

(c) In this case, P (A and B) = 0.4, which is more than P (A) = 0.2.

$$[\because P(A \text{ and } B) \le P(A)]$$

Hence, the assigned probabilities are not consistent.

Example 31.28 An urn contains 8 white balls and 2 green balls. A sample of three balls is selected at random. What is the probability that the sample contains at least one green ball ?

Solution : Urn contains 8 white balls and 2 green balls.

$\therefore$ Total number of balls in the urn = 10

Three balls can be drawn in ${}^{10}C_3$ ways = 120 ways.

Let A be the event " at least one green ball is selected".

Let us determine the number of different outcomes in A. These outcomes contain either one green ball or two green balls.

There are ${}^{2}C_1$ ways to select a green ball from 2 green balls and for this remaining two white balls can be selected in ${}^{8}C_2$ ways.

Hence, the number of outcomes favourable to one green ball

$$= {}^{2}C_1 \times {}^{8}C_2$$

$$= 2 \times 28 = 56$$

Similarly, the number of outcomes favourable to two green balls

$$= {}^{2}C_2 \times {}^{8}C_1 = 1 \times 8 = 8$$

Hence, the probability of at least one green ball is

P (at least one green ball)

$$= P \text{ (one green ball) } + P \text{ (two green balls)}$$

$$= \frac{56}{120} + \frac{8}{120}$$

$$= \frac{64}{120} = \frac{8}{15}$$

Example 31.29 Two balls are drawn at random with replacement from a bag containing 5 blue and 10 red balls. Find the probability that both the balls are either blue or red.

Solution : Let the event A consists of getting both blue balls and the event B is getting both red balls. Evidently A and B are mutually exclusive events.

By fundamental principle of counting, the number of outcomes favourable to $A = 5 \times 5 = 25$.

Similarly, the number of outcomes favourable to B = 10 × 10 = 100.

Total number of possible outcomes = 15 × 15 = 225.

$$\therefore \qquad P(A) = \frac{25}{225} = \frac{1}{9} \text{ and } P(B) = \frac{100}{225} = \frac{4}{9}.$$

Since the events A and B are mutually exclusive, therefore

$$P(A \text{ or } B) = P(A) + P(B)$$

$$= \frac{1}{9} + \frac{4}{9}$$

$$= \frac{5}{9}$$

Thus, P (both blue or both red balls) = $\frac{5}{9}$

CHECK YOUR PROGRESS 31.3

1. A card is drawn from a well-shuffled pack of cards. Find the probability that it is a queen or a card of heart.
2. In a single throw of two dice, find the probability of a total of 7 or 12.
3. The odds in favour of winning of Indian cricket team in 2010 world cup are 9 to 7. What is the probability that Indian team wins ?
4. The odds against the team A winning the league match are 5 to 7. What is the probability that the team A wins the league match.
5. Two dice are thrown. Getting two numbers whose sum is divisible by 4 or 5 is considered a success. Find the probability of success.
6. Two cards are drawn at random from a well-shuffled deck of 52 cards with replacement. What is the probability that both the cards are either black or red ?

7. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that the card is an ace or a black card.

8. Two dice are thrown once. Find the probability of getting a multiple of 3 on the first die or a total of 8.

9. (a) In a single throw of two dice, find the probability of a total of 5 or 7.

   (b) A and B are two mutually exclusive events such that P (A) = 0.3 and P (B) = 0.4. Calculate P (A or B).

10. A box contains 12 light bulbs of which 5 are defective. All the bulbs look alike and have equal probability of being chosen. Three bulbs are picked up at random. What is the probability that at least 2 are defective ?

11. Two dice are rolled once. Find the probability

    (a) that the numbers on the two dice are different,

    (b) that the total is at least 3.

12. A couple have three children. What is the probability that among the children, there will be at least one boy or at least one girl ?

13. Find the odds in favour and against each event for the given probability

    (a) P (A) = .7 (b) $P\,(A) = \dfrac{4}{5}$

14. Determine the probability of A for the given odds

    (a) 7 to 2 in favour of A (b) 10 to 7 against A.

15. If two dice are thrown, what is the probability that the sum is

    (a) greater than 4 and less than 9 ?

    (b) neither 5 nor 8 ?

16. Which of the following probability assignments are inconsistent ? Give reasons.

    (a) P (A) = 0.5 , P (B) = 0.3 , P (A and B) = 0.4

    (b) P (A) = P (B) = 0.4, P (A and B) = 0.2

    (c) P (A) = 0.85, P (B) = 0.8, P (A and B) = 0.61

17. Two balls are drawn at random from a bag containing 5 white and 10 green balls. Find the probability that the sample contains at least one white ball.

18. Two cards are drawn at random from a well-shuffled deck of 52 cards with replacement. What is the probability that both cards are of the same suit ?

31.6 SOME RESULTS ON PROBABILITY OF EVENTS

Result 1 : Probability of impossible event is zero.

Solution : Impossible event contains no sample points. Therefore, the certain event S and the impossible event ϕ are mutually exclusive.

Hence, $S \cup \phi = S$

$\Rightarrow \quad P\,(S \cup \phi) = P\,(S)$

or $\quad P(S) + P(\phi) = P(S)$

$\Rightarrow \quad P(\phi) = 0.$

Result 2 : Probability of the complementary event $\bar{A}$ of A is given by

$$P(\bar{A}) = 1 - P(A)$$

Solution : A and $\bar{A}$ are disjoint events. Also,

$$A \cup \bar{A} = S \quad \Rightarrow \quad P(A \cup \bar{A}) = P(S)$$

Using additive laws (ii) and (iii) , we get

$$P(A) + P(\bar{A}) = 1 \quad \Rightarrow \quad P(\bar{A}) = 1 - P(A).$$

Result 3 : Prove that $0 \le P(A) \le 1$, for any A in S.

Solution : We know that

$$A \subset S \quad \Rightarrow \quad P(A) \le P(S)$$

$\Rightarrow \quad P(A) \le 1.$ [Using additive law]

We know that $P(A) \ge 0.$

Hence, $\quad 0 \le P(A) \le 1.$

Result 4 : If A and B are any two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Solution :

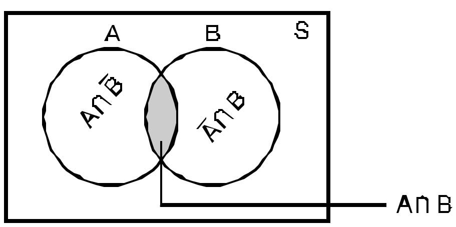

Fig. 31.6

From the above figure, we can write

$$A \cup B = A \cup (\bar{A} \cap B)$$

$\Rightarrow \quad P(A \cup B) = P[A \cup (\bar{A} \cap B)].$ ....(1)

Since the events A and $(\bar{A} \cap B)$ are disjoint, therefore law (iii) gives

$$P[A \cup (\bar{A} \cap B)] = P(A) + P(\bar{A} \cap B)$$

Substituting this value in (1) , we get

$$P(A \cup B) = P(A) + P(\bar{A} \cap B)$$

or $\quad P(A \cup B) = P(A) + [P(\bar{A} \cap B) + P(A \cap B)] - P(A \cap B)$ .....(2)

From Fig. 31.6, we see that

$$(\bar{A} \cap B) \cup (A \cap B) = B$$

$$\therefore \quad P[(\bar{A} \cap B) \cup (A \cap B)] = P(B).$$

Further, the events $(\bar{A} \cap B)$ and $(A \cap B)$ are disjoint, so from additive law, we get

$$P(\bar{A} \cap B) + P(A \cap B) = P(B)$$

Substituting this value in (2), we get

$$P(A \cup B) = P(A) + P(B) - P(A \cap B$$

Result 5 : If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Solution : From additive law, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{.....(1)}$$

Since A and B are mutually exclusive events,

Therefore $\quad A \cap B = \phi$

$$\Rightarrow \quad P(A \cap B) = P(\phi) = 0$$

Substituting this value in equation (1), we get

$$P(A \cup B) = P(A) + P(B),$$

which is additive law for mutually exclusive events.

31.7 MULTIPLICATION LAW OF PROBABILITY FOR INDEPENDENT EVENTS

Let us recall the definition of independent events.

Two events A and B are said to be independent, if the occurrence or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.

Can you think of some examples of independent events ?

The event of getting 'H' on first coin and the event of getting 'T' on the second coin in a simultaneous toss of two coins are independent events.

What about the event of getting 'H' on the first toss and event of getting 'T' on the second toss in two successive tosses of a coin ? They are also independent events.

Let us consider the event of 'drawing an ace' and the event of 'drawing a king' in two successive draws of a card from a well-shuffled deck of cards without replacement.

Are these independent events ?

No, these are not independent events, because we draw an ace in the first draw with probability $\frac{4}{52}$. Now, we do not replace the card and draw a king from the remaining 51 cards and this affect the probability of getting a king in the second draw, i.e., the probability of getting a king in the second draw without replacement will be $\frac{4}{51}$.

Note : If the cards are drawn with replacement, then the two events become independent.

Is there any rule by which we can say that the events are independent ?

How to find the probability of simultaneous occurrence of two independent events?

If A and B are independent events, then

$$P\ (A \text{ and } B) = P(A)\ .\ P\ (B)$$

or

$$P(A \cap B) = P(A).\ P\ (B)$$

Thus, the probability of simultaneous occurrence of two independent events is the product of their separate probabilities.

Note : The above law can be extended to more than two independent events, i.e.,

$$P(A \cap B \cap C...) = P(A) \cdot P(B) \cdot P(C)...$$

On the other hand, if the probability of the event 'A' and 'B' is equal to the product of the probabilities of the events A and B, then we say that the events A and B are independent.

Example 31.30 A die is tossed twice. Find the probability of a number greater than 4 on each throw.

Solution : Let us denote by A, the event 'a number greater than 4' on first throw. B be the event 'a number greater than 4' in the second throw. Clearly A and B are independent events.

In the first throw, there are two outcomes, namely, 5 and 6 favourable to the event A.

$\therefore$ $$P(A) = \frac{2}{6} = \frac{1}{3}$$

Similarly, $$P(B) = \frac{1}{3}$$

Hence, $P(A \text{ and } B) = P\ (A)\ .\ P\ (B)$

$$= \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}.$$

Example 31.31 Arun and Tarun appear for an interview for two vacancies. The probability of Arun's selection is $\frac{1}{3}$ and that of Tarun's selection is $\frac{1}{5}$. Find the probability that

(a) both of them will be selected.

(b) none of them is selected.

(c) at least one of them is selected.

(d) only one of them is selected.

Solution : Probability of Arun's selection $= P(A) = \frac{1}{3}$

Probability of Tarun's selection $= P(T) = \frac{1}{5}$

(a) P (both of them will be selected) = P (A) P (T)

$$= \frac{1}{3} \times \frac{1}{5}$$

$$= \frac{1}{15}$$

(b) P (none of them is selected)

$$= P\left(\bar{A}\right)P\left(\bar{T}\right)$$

$$= \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)$$

$$= \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

(c) P (at least one of them is selected)

$= 1 - P$ (None of them is selected)

$$= 1 - P\left(\bar{A}\right)P\left(\bar{T}\right)$$

$$= 1 - \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)$$

$$= 1 - \left(\frac{2}{3} \times \frac{4}{5}\right)$$

$$= 1 - \frac{8}{15} = \frac{7}{15}$$

(d) P (only one of of them is selected)

$$= P\left(A\right)P\left(\bar{T}\right) + P\left(\bar{A}\right) P(T)$$

$$= \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{5}$$

$$= \frac{6}{15} = \frac{2}{5}$$

Example 31.32 A problem in statistics is given to three students, whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that problem will be solved ?

Solution : Let p_1, p_2 and p_3 be the probabilities of three persons of solving the problem.

Here, $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{3}$ and $p_3 = \frac{1}{4}$.

The problem will be solved, if at least one of them solves the problem.

$\therefore$ P (at least one of them solves the problem)

$= 1 -$ P (None of them solves the problem) .....(1)

Now, the probability that none of them solves the problem will be

P (none of them solves the problem) $= (1 - p_1)(1 - p_2)(1 - p_3)$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Putting this value in (1), we get

P (at least one of them solves the problem) $= 1 - \frac{1}{4}$

$$= \frac{3}{4}$$

Hence, the probability that the problem will be solved is $\frac{3}{4}$.

Example 31.33 Two balls are drawn at random with replacement from a box containing 15 red and 10 white balls. Calculate the probability that

(a) both balls are red.

(b) first ball is red and the second is white.

(c) one of them is white and the other is red.

Solution :

(a) Let A be the event that first drawn ball is red and B be the event that the second ball drawn is red. Then as the balls drawn are with replacement,

therefore $P(A) = \frac{15}{25} = \frac{3}{5}, P(B) = \frac{3}{5}$

As A and B are independent events

therefore P (both red) = P (A and B)

$$= P(A) \times P(B)$$

$$= \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

(b) Let A : First ball drawn is red.

B : Second ball drawn is white.

$\therefore$ $P(A \text{ and } B) = P(A) \times P(B)$

$$= \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}.$$

(c) If WR denotes the event of getting a white ball in the first draw and a red ball in the second draw and the event RW of getting a red ball in the first draw and a white ball in the second draw. Then as 'RW' and WR' are mutually exclusive events, therefore

$\therefore$ P (a white and a red ball)

$$= P(WR \text{ or } RW)$$
$$= P(WR) + P(RW)$$
$$= P(W)\,P(R) + P(R)\,P(W)$$
$$= \frac{2}{5}\cdot\frac{3}{5} + \frac{3}{5}\cdot\frac{2}{5}$$
$$= \frac{6}{25} + \frac{6}{25} = \frac{12}{25}$$

Example 31.34 The odds against Manager X settling the wage dispute with the workers are 8 : 6 and odds in favour of manager Y settling the same dispute are 14 : 16.

(i) What is the chance that neither settles the dispute, if they both try independently of each other ?

(ii) What is the probability that the dispute will be settled ?

Solution : Let A be the event that the manager X will settle the dispute and B be the event that the manager Y will settle the dispute. Then, clearly

(i) $$P(A) = \frac{6}{14} = \frac{3}{7},\ P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{7} = \frac{4}{7}$$

$$P(B) = \frac{14}{30} = \frac{7}{15},\ P(\bar{B}) = 1 - \frac{14}{30} = \frac{16}{30} = \frac{8}{15}$$

The required probability that neither settles the dispute is given by

$$P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$$
$$= \frac{4}{7}\cdot\frac{8}{15} = \frac{32}{105}$$

(Since A, B are independent, therefore, $\bar{A}, \bar{B}$ also independent)

(ii) The dispute will be settled, if at least one of the managers X and Y settles the dispute. Hence, the required probability is given by

$$P(A \cup B) = P\,[\text{At least one of X and Y settles the dispute}]$$
$$= 1 - P\,[\text{None settles the dispute}]$$
$$= 1 - P(\bar{A} \cap \bar{B})$$
$$= 1 - \frac{32}{105} = \frac{73}{105}$$

Example 31.35 A dice is thrown 3 times. Getting a number '5 or 6' is a success. Find the probability of getting

(a) 3 successes (b) exactly 2 successes (c) at most 2 successes (d) at least 2 successes.

Solution : Let S denote the success in a trial and F denote the ' not success' i.e. failure. Therefore,

$$P(S) = \frac{2}{6} = \frac{1}{3}$$

$$P(F) = 1 - \frac{1}{3} = \frac{2}{3}$$

(a) As the trials are independent, by multiplication theorem for independent events,

$$P(SSS) = P(S)\ \ P(S)\ \ P(S)$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$$

$$P(SSF) = P(S)\,P(S)\,P(F)$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$$

Since the two successes can occur in ${}^3C_2$ ways

$\therefore$ P (exactly two successes) $= {}^3C_2 \times \frac{2}{27} = \frac{2}{9}$

(c) P (at most two successes) $= 1 - P(3\text{successes})$

$$= 1 - \frac{1}{27} = \frac{26}{27}$$

(d) P (at least two successes) = P (exactly 2 successes) + P (3 successes)

$$= \frac{2}{9} + \frac{1}{27} = \frac{7}{27}$$

Example 31.36 A card is drawn from a pack of 52 cards so that each card is equally likely to be selected. Which of the following events are independent ?

(i) A : the card drawn is a spade
B : the card drawn is an ace

(ii) A : the card drawn is black
B : the card drawn is a king

(iii) A : the card drawn is a king or a queen
B : the card drawn is a queen or a jack

Solution : (i) There are 13 cards of spade in a pack.

$$P(A) = \frac{13}{52} = \frac{1}{4}$$

There are four aces in the pack.

$\therefore$ $P(B) = \frac{4}{52} = \frac{1}{13}$

$A \cap B = \{$ an ace of spade $\}$

$\therefore \qquad P(A \cap B) = \frac{1}{52}$

Now $\qquad P(A)\ P(B) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$

Since $\qquad P(A \cap B) = P(A).\ P(B)$

Hence, the events A and B are independent.

(ii) There are 26 black cards in a pack.

$\therefore \qquad P(A) = \frac{26}{52} = \frac{1}{2}$

There are four kings in the pack.

$\therefore \qquad P(B) = \frac{4}{52} = \frac{1}{13}$

$A \cap B$ = { 2 black kings }

$\therefore \qquad P(A \cap B) = \frac{2}{52} = \frac{1}{26}$

Now, $\qquad P(A) \times P(B) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$

Since $\qquad P(A \cap B) = P(A)\ P(B)$

Hence, the events A and B are independent.

(iii) There are 4 kings and 4 queens in a pack of cards.

$\therefore$ Total number of outcomes favourable to the event A is 8.

$\therefore \qquad P(A) = \frac{8}{52} = \frac{2}{13}$

Similarly, $\qquad P(B) = \frac{2}{13}$

$A \cap B$ = { 4 queens }

$\therefore \qquad P(A \cap B) = \frac{4}{52} = \frac{1}{13}$

$\therefore \qquad P(A) \times P(B) = \frac{2}{13} \times \frac{2}{13} = \frac{4}{169}$

Here, $\qquad P(A \cap B) \neq P(A).P(B)$

Hence, the events A and B are not independent.

Example 31.37 Consider a group of 36 students. Suppose that A and B are two properties that each student either has or does not have. The events are

A : Student has blue eyes

B : Student is a male

Out of 36, there are 12 male and 24 female students and half of them in each has blue eyes. Are these events independent ?

Solution : With regard to the given two properties, i.e., either has or does not have, the 36 students are distributed as follows :

	Blue eyes A	Not blue eyes $(\bar{A})$	Total
Male (B)	6	6	12
Female $(\bar{B})$	12	12	24
Total	18	18	36

If we choose a student at random, the probabilities corresponding to the events A and B are

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

$$P(B) = \frac{12}{36} = \frac{1}{3}$$

$$P(A \cap B) = \frac{6}{36} = \frac{1}{6}$$

Also $\qquad P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Here, $\qquad P(A \cap B) = P(A) \cdot P(B)$

Hence, the events A and B are independent.

Example 31.38 Suppose that we toss a coin three times and record the sequence of heads and tails. Let A be the event ' at most one head occurs' and B the event ' both heads and tails occur'. Are these event independent ?

Solution : The sample space in tossing a coin three times will be

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$$

Also, $\qquad A \cap B = \{ TTH, THT, HTT \}$

$\therefore \qquad P(A) = \frac{4}{8} = \frac{1}{2}, \; P(B) = \frac{6}{8} = \frac{3}{4}, \quad P(A \cap B) = \frac{3}{8}$

Moreover, $\qquad P(A) \times P(B) = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

Which equals $P(A \cap B)$. Hence, A and B are independent.

CHECK YOUR PROGRESS 31.4

1. A husband and wife appear in an interview for two vacancies in the same department.

The probability of husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability that

(a) Only one of them will be selected ?

(b) Both of them will be selected ?

(c) None of them will be selected ?

(d) At least one of them will be selected ?

2. Probabilities of solving a specific problem independently by Raju and Soma are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(a) the problem is solved.

(b) exactly one of them solves the problem.

3. A die is rolled twice. Find the probability of a number greater than 3 on each throw.

4. Sita appears in the interview for two posts A and B, selection for which are independent. The probability of her selection for post A is $\frac{1}{5}$ and for post B is $\frac{1}{7}$. Find the probability that she is selected for

(a) both the posts

(b) at least one of the posts.

5. The probabilities of A, B and C solving a problem are $\frac{1}{3}, \frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

6. A draws two cards with replacement from a well-shuffled deck of cards and at the same time B throws a pair of dice. What is the probability that

(a) A gets both cards of the same suit and B gets a total of 6 ?

(b) A gets two jacks and B gets a doublet ?

7. Suppose it is 9 to 7 against a person A who is now 35 years of age living till he is 65 and 3:2 against a person B now 45 living till he is 75. Find the chance that at least one of these persons will be alive 30 years hence.

8. A bag contains 13 balls numbered from 1 to 13. Suppose an even number is considered a 'success'. Two balls are drawn with replacement, from the bag. Find the probability of getting

(a) Two successes (b) exactly one success

(c) at least one success (d) no success

9. One card is drawn from a well-shuffled deck of 52 cards so that each card is equally likely to be selected. Which of the following events are independent ?

(a) A : The drawn card is red B : The drawn card is a queen

(b) A : The drawn card is a heart B: The drawn card is a face card

31.8 CONDITIONAL PROBABILITY

Suppose that a fair die is thrown and the score noted. Let A be the event, the score is 'even'. Then

$$A = \{2, 4, 6\}$$

$$\therefore \qquad P(A) = \frac{3}{6} = \frac{1}{2}.$$

Now suppose we are told that the score is greater than 3. With this additional information what will be P (A) ?

Let B be the event, 'the score is greater than 3'. Then B is {4, 5, 6 }. When we say that B has occurred, the event 'the score is less than or equal to 3' is no longer possible. Hence the sample space has changed from 6 to 3 points only. Out of these three points 4, 5 and 6; 4 and 6 are even scores.

Thus, given that B has occurred, P (A) must be $\frac{2}{3}$.

Let us denote the probability of A given that B has already occurred by $P(A \mid B)$.

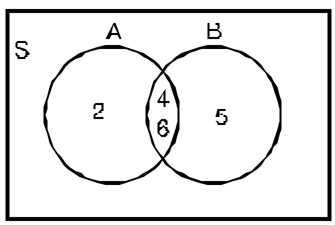

FIg. 31.7

Again, consider the experiment of drawing a single card from a deck of 52 cards. We are interested in the event A consisting of the outcome that a black ace is drawn.

Since we may assume that there are 52 equally likely possible outcomes and there are two black aces in the deck, so we have

$$P(A) = \frac{2}{52}.$$

However, suppose a card is drawn and we are informed that it is a spade. How should this information be used to reappraise the likelihood of the event A ?

Clearly, since the event B "A spade has been drawn " has occurred, the event "not spade" is no longer possible. Hence, the sample space has changed from 52 playing cards to 13 spade cards. The number of black aces that can be drawn has now been reduced to 1.

Therefore, we must compute the probability of event A relative to the new sample space B.

Let us analyze the situation more carefully.

The event A is " a black ace is drawn'. We have computed the probability of the event A knowing that B has occurred. This means that we are computing a probability relative to a new sample space B. That is, B is treated as the universal set. We should consider only that part of A which is included in B.

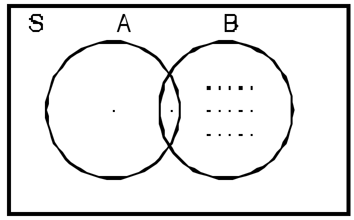

Fig. 31.8

Hence, we consider $A \cap B$ (see figure 31.8).

Thus, the probability of A. given B, is the ratio of the number of entries in $A \cap B$ to the number of entries in B. Since $n(A \cap B) = 1$ and $n(B) = 13$,

then $$P(A \mid B) = \frac{n(A \cap B)}{n(B)} = \frac{1}{13}$$

Notice that $$n(A \cap B) = 1 \Rightarrow P(A \cap B) = \frac{1}{52}$$

$$n(B) = 13 \Rightarrow P(B) = \frac{13}{52}$$

$\therefore$ $$P(A \mid B) = \frac{1}{13} = \frac{\frac{1}{52}}{\frac{13}{52}} = \frac{P(A \cap B)}{P(B)}.$$

This leads to the definition of conditional probability as given below :

Let A an B be two events defined on a sample space S. Let P(B) > 0, then the conditional probability of A, provided B has already occurred, is denoted by P(A|B) and mathematically written as :

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

Similarly, $$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0$$

The symbol P(A | B) is usually read as "the probability of A given B".

Example 31.39 Consider all families "with two children (not twins). Assume that all the elements of the sample space {BB, BG, GB,GG} are equally likely. (Here, for instance, BG denotes the birth sequence "boy girls"). Let A be the event {BB} and B be the event that 'at least one boy'. Calculate P (A | B).

Solution : Here, $A = \{BB\}$

$B = \{BB, BG, GB\}$

$A \cap B = \{BB\}$

$\therefore$ $$P(A \cap B) = \frac{1}{4}$$

$$P(B) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

Hence, $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

Example 31.40 Assume that a certain school contains equal number of female and male students. 5 % of the male population is football players. Find the probability that a randomly selected student is a football player male.

Solution : Let M = Male

F = Football player

We wish to calculate $P(M \cap F)$. From the given data,

$$P(M) = \frac{1}{2} \quad (\because \text{ School contains equal number of male and female students})$$

$$P(F|M) = 0.05$$

But from definition of conditional probability, we have

$$P(F \mid M) = \frac{P(M \cap F)}{P(M)}$$

$$\Rightarrow \qquad P(M \cap F) = P(M) \times P(F \mid M)$$

$$= \frac{1}{2} \times 0.05 = 0.025$$

Example 31.41 If A and B are two events, such that $P(A) = 0.8$, $P(B) = 0.6$, $P(A \cap B) = 0.5$, find the value of

(i) $P(A \cup B)$ (ii) $P(B \mid A)$ (iii) $P(A \mid B)$.

Solution :(i) $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.8 + 0.6 - 0.5 = 0.9$$

(ii) $$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.5}{0.8} = \frac{5}{8}$$

(iii) $$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.5}{0.6} = \frac{5}{6}$$

Example 31.42 Find the chance of drawing 2 white balls in succession from a bag containing 5 red and 7 white balls, the balls drawn not being replaced.

Solution : Let A be the event that ball drawn is white in the first draw. B be the event that ball drawn is white in the second draw.

$\therefore$ $$P(A \cap B) = P(A)P(B \mid A)$$

Here, $$P(A) = \frac{7}{12}, P(B \mid A) = \frac{6}{11}$$

$\therefore$ $$P(A \cap B) = \frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$$

Example 31.43 A coin is tossed until a head appears or until it has been tossed three times. Given that head does not occur on the first toss, what is the probability that coin is tossed three times ?

Solution : Here, it is given that head does not occur on the first toss. That is, we may get the head on the second toss or on the third toss or even no head.

Let B be the event, " no heads on first toss".

Then $$B = \{TH, TTH, TTT\}$$

These events are mutually exclusive.

$$P(B) = P(TH) + P(TTH) + P(TTT) \qquad .....(1)$$

Now $P(TH) = \frac{1}{4}$ ($\because$ This event has the sample space of four outcomes)

and $P(TTH) = P(TTT) = \frac{1}{8}$ ($\because$ This event has the sample space of eight outcomes)

Putting these values in (1), we get

$$P(B) = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

Let A be the event "coin is tossed three times".

Then $$A = \{TTH, TTT\}$$

$\therefore$ We have to find P (A | B) .

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Here, $$A \cap B = A$$

$\therefore$ $$P(A \mid B) = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

CHECK YOUR PROGRESS 31.5

1. A sequence of two cards is drawn at random (without replacement) from a well-shuffled deck of 52 cards. What is the probability that the first card is red and the second card is black ?
2. Consider a three child family for which the sample space is

   { BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG }

   Let A be the event " the family has exactly 2 boys " and B be the event " the first child is a boy". What is the probability that the family has 2 boys, given that first child is a boy ?
3. Two cards are drawn at random without replacement from a deck of 52 cards. What is the probability that the first card is a diamond and the second card is red ?
4. If A and B are events with P (A) = 0.4, P (B) = 0.2, $P(A \cap B) = 0.1$, find the probability of A given B. Also find P (B|A).
5. From a box containing 4 white balls, 3 yellow balls and 1 green ball, two balls are drawn one at a time without replacement. Find the probability that one white and one yellow ball is drawn.

31.9 THEOREMS ON MULTIPLICATION LAW OF PROBABILITY AND CONDITIONAL PROBABILITY.

Theorem 1 : For two events A and B,

$$P(A \cap B) = P(A).P(B \mid A),$$

and

$$P(A \cap B) = P(B).P(A \mid B),$$

where P (B|A) represents the conditional probability of occurrence of B, when the event A has already occurred and P (A|B) is the conditional probability of happening of A, given that B has already happened.

Proof : Let n (S) denote the total number of equally likely cases, n (A) denote the cases favourable to the event A, n (B) denote the cases favourable to B and $n(A \cap B)$ denote the cases favourable to both A and B.

$$\therefore \qquad P(A) = \frac{n(A)}{n(S)},$$

$$P(B) = \frac{n(B)}{n(S)}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \qquad .....(1)$$

For the conditional event A|B , the favourable outcomes must be one of the sample points of B, i.e., for the event A|B, the sample space is B and out of the n (B) sample points, $n(A \cap B)$ pertain to the occurrence of the event A, Hence,

$$P(A \mid B) = \frac{n(A \cap B)}{n(B)}$$

Rewriting (1), we get $P(A \cap B) = \frac{n(B)}{n(S)} \cdot \frac{n(A \cap B)}{n(B)} = P(B).P(A|B)$

Similarly, we can prove

$$P(A \cap B) = P(A).P(B \mid A)$$

Note : If A and B are independent events, then

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B)$$

$$P(A \cap B) = P(A).P(B)$$

Theorem 2 : Two events A and B of the sample space S are independent, if and only if

$$P(A \cap B) = P(A).P(B)$$

Proof : If A and B are independent events,

then $\quad P(A \mid B) = P(A)$

We know that $\quad P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \quad P(A \cap B) = P(A)P(B)$

Hence, if A and B are independent events, then the probability of 'A and B' is equal to the product of the probability of A and probability of B.

Conversely, if $P(A \cap B) = P(A).P(B)$, then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \text{ gives}$$

$$P(A \mid B) = \frac{P(A)P(B)}{P(B)} = P(A)$$

That is, A and B are independent events.

LET US SUM UP

- **Events Relation :** The complement of an event A consists of all those outcomes which are not favourable to the event A, and is denoted by 'not A' or by $\overline{A}$.
- **Event 'A or B' :** The event 'A or B' occurs if either A or B or both occur.
- **Event 'A and B' :** The event 'A and B' consists of all those outcomes which are favourable to both the events A and B.
- **Addition Law of Probability :** For any two events A and B of a sample space S

  $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- **Additive Law of Probability for Mutually Exclusive Events :** If A and B are two mutually exclusive events, then

$$P (A \text{ or } B) = P (A \cup B) = P (A) + P (B).$$

- **Odds in Favour of an Event :** If the odds for A are a to b, then

$$P(A) = \frac{a}{a + b}$$

If odds against A are a to b, then

$$P(A) = \frac{b}{a + b}$$

- Two events are mutually exclusive, if occurrence of one precludes the possibility of simultaneous occurrence of the other.
- Two events A and B are said to be independent, if the occurrence or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.
- If A and B are independent events, then

$$P (A \text{ and } B) = P (A). P (B)$$

or $$P(A \cap B) = P (A) . P (B)$$

- For two events A and B,

$$P(A \cap B) = P (A) P (B | A) \quad , \quad P (A) > 0$$

or $$P(A \cap B) = P (B) P (A | B) \quad , \quad P (B) > 0$$

where P (B|A) represents the conditional probability of occurrence of B, when the event A has already happened and P (A|B) represents the conditional probability of happening of A, given that B has already happened.

SUPPORTIVE WEB SITES

- *http://www.wikipedia.org*
- *http://mathworld.wolfram.com*

TERMINAL EXERCISE

1. In a simultaneous toss of four coins, what is the probability of getting
   (a) exactly three heads ? (b) at least three heads ?
   (c) atmost three heads ?
2. Two dice are thrown once. Find the probability of getting an odd number on the first die or a sum of seven.
3. An integer is chosen at random from first two hundred integers. What is the probability that the integer chosen is divisible by 6 or 8 ?
4. A bag contains 13 balls numbered from 1 to 13. A ball is drawn at random. What is the

probability that the number obtained it is divisible by either 2 or 3 ?

5. Find the probability of getting 2 or 3 heads, when a coin is tossed four times.

6. Are the following probability assignments consistent ? Justify your answer.

   (a) P (A) = 0.6, P (B) = 0.5 , P (A and B) = 0.4

   (b) P (A) = 0.2, P (B) = 0.3 , P (A and B) = 0.4

   (c) P (A) = P (B) = 0.7, P (A and B) = 0.2

7. A box contains 25 tickets numbered 1 to 25. Two tickets are drawn at random. What is the probability that the product of the numbers is even ?

8. A drawer contains 50 bolts and 150 nuts. Half of the bolts and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or is a bolt ?

9. A lady buys a dozen eggs, of which two turn out to be bad. She chose four eggs to scramble for breakfast. Find the chances that she chooses

   (a) all good eggs (b) three good and one bad eggs

   (c) two good and two bad eggs (d) at least one bad egg.

10. Two cards are drawn at random without replacement from a well-shuffled deck of 52 cards. Find the probability that the cards are both red or both kings.

11. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected at random from each group. Show the chances that three selected children consist of 1 girl and 2 boys is $\frac{13}{32}$.

12. A die is thrown twice. Find the probability of a prime number on each throw.

13. Kamal and Monika appears for an interview for two vacancies. The probability of Kamal's selection is $\frac{1}{3}$ and that of Monika's rejection is $\frac{4}{5}$. Find the probability that only one of them will be selected.

14. A bag contains 7 white, 5 black and 8 red balls. Four balls are drawn without replacement. Find the probability that all the balls are black.

15. For two events A and B, it is given that

$$P(A) = 0.4,\ P(B) = p \text{ and } P(A \cup B) = 0.6$$

   (a) Find p so that A and B are independent events.

   (b) For what value of p, of A and B are mutually exclusive ?

16. The odds against A speaking the truth are 3 : 2 and the odds against B speaking the truth are 5 : 3. In what percentage of cases are they likely to contradict each other on a identical issue ?

17. Let A and B be the events such that

$$P(\bar{A}) = \frac{1}{2},\ P(\bar{B}) = \frac{2}{3},\ P(A \cap B) = \frac{1}{4}.$$

   Compute P (A|B) and P (B|A).

CHECK YOUR PROGRESS 31.1

1. $\frac{1}{6}$ 2. $\frac{1}{2}$ 3. $\frac{1}{2}$ 4. $\frac{3}{4}$

5. (i) $\frac{3}{5}$ (ii) $\frac{2}{5}$

6. (i) $\frac{5}{36}$ (ii) $\frac{5}{36}$ (iii) $\frac{1}{12}$ (iv) $\frac{1}{36}$

7. $\frac{5}{9}$ 8. $\frac{1}{12}$ 9. $\frac{1}{2}$

10. (i) $\frac{1}{4}$ (ii) $\frac{1}{13}$ (iii) $\frac{1}{52}$

11. (i) $\frac{5}{12}$ (ii) $\frac{1}{6}$ (iii) $\frac{11}{36}$

12. (i) $\frac{1}{8}$ (ii) $\frac{7}{8}$ (iii) $\frac{1}{8}$

CHECK YOUR PROGRESS 31.2

1. $\frac{1}{8}$ 2. $\frac{20}{39}$ 3.(a) $\frac{4}{25}$ (b) $\frac{38}{245}$

4. $\frac{1}{5525}$ 6. (i) $\frac{3}{10}$ (ii) $\frac{1}{6}$ (iii) $\frac{1}{30}$

7. $\frac{10}{133}$ 8. $\frac{4}{7}$ 9. $\frac{60}{143}$ 10. $\frac{1}{4}$

CHECK YOUR PROGRESS 31.3

1. $\frac{4}{13}$ 2. $\frac{7}{36}$ 3. $\frac{9}{16}$ 4. $\frac{7}{12}$

5. $\frac{4}{9}$ 6. $\frac{1}{2}$ 7. $\frac{7}{13}$ 8. $\frac{5}{12}$

9. (a) $\frac{5}{18}$ (b) 0.7 10. $\frac{4}{11}$

11. (a) $\frac{5}{6}$ (b) $\frac{35}{36}$ 12. $\frac{3}{4}$

13. (a) The odds for A are 7 to 3 . The odds against A are 3 to 7

13. (b) The odds for A are 4 to 1 and The odds against A are 1 to 4

14. (a) $\frac{7}{9}$ (b) $\frac{7}{17}$ 15. (a) $\frac{5}{9}$ (b) $\frac{3}{4}$

16. (a), (c) 17. $\frac{4}{7}$ 18. $\frac{1}{4}$

CHECK YOUR PROGRESS 31.4

1. (a) $\frac{2}{7}$ (b) $\frac{1}{35}$ (c) $\frac{24}{35}$ (d) $\frac{11}{35}$

2. (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ 3 . $\frac{1}{4}$

4. (a) $\frac{1}{35}$ (b) $\frac{11}{35}$ 5. $\frac{1}{2}$

6. (a) $\frac{5}{144}$ (b) $\frac{1}{1014}$ 7. $\frac{53}{80}$

8. (a) $\frac{36}{169}$ (b) $\frac{84}{169}$ (c) $\frac{120}{169}$ (d) $\frac{49}{169}$

9. (a) Independent (b) Independent

CHECK YOUR PROGRESS 31.5

1. $\frac{13}{51}$ 2. $\frac{1}{2}$ 3. $\frac{25}{204}$

4. $\frac{1}{2}, \frac{1}{4}$ 5. $\frac{3}{7}$

TERMINAL EXERCISE

1. (a) $\frac{1}{4}$ (b) $\frac{5}{16}$ (c) $\frac{15}{16}$ 2. $\frac{7}{12}$

3. $\frac{1}{4}$ 4. $\frac{8}{13}$ 5. $\frac{5}{8}$

6. Only (a) is consistent

7. $\frac{456}{625}$ 8. $\frac{5}{8}$ 9. (a) $\frac{14}{33}$ (b) $\frac{16}{33}$

(c) $\frac{1}{11}$ (d) $\frac{19}{33}$

10. $\frac{55}{221}$ 12. $\frac{1}{4}$ 13. $\frac{2}{15}$ 14. $\frac{1}{969}$

15. $\frac{1}{3}$ 16. $\frac{19}{40}$ 17. $\frac{3}{4}, \frac{1}{2}$ respectively.