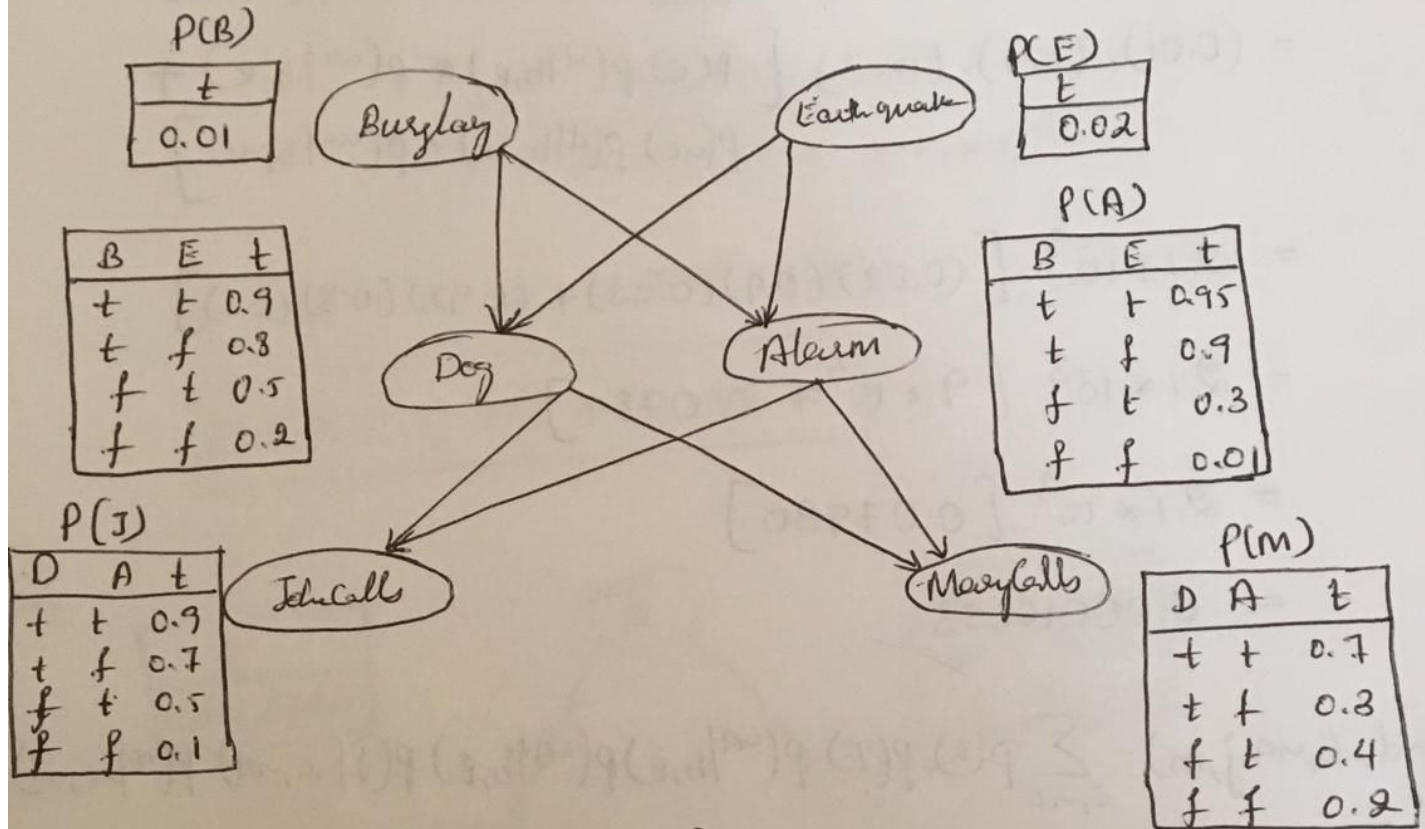


Homework - 2

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①



Compute $p(d|b, na, j, m)$.

$$\begin{aligned}\Rightarrow p(d|b, na, j, m) &= \frac{p(d, b, na, j, m)}{p(b, na, j, m)} \\ &= \frac{p(d, b, na, j, m)}{p(d, b, na, j, m) + p(\neg d, b, na, j, m)}\end{aligned}$$

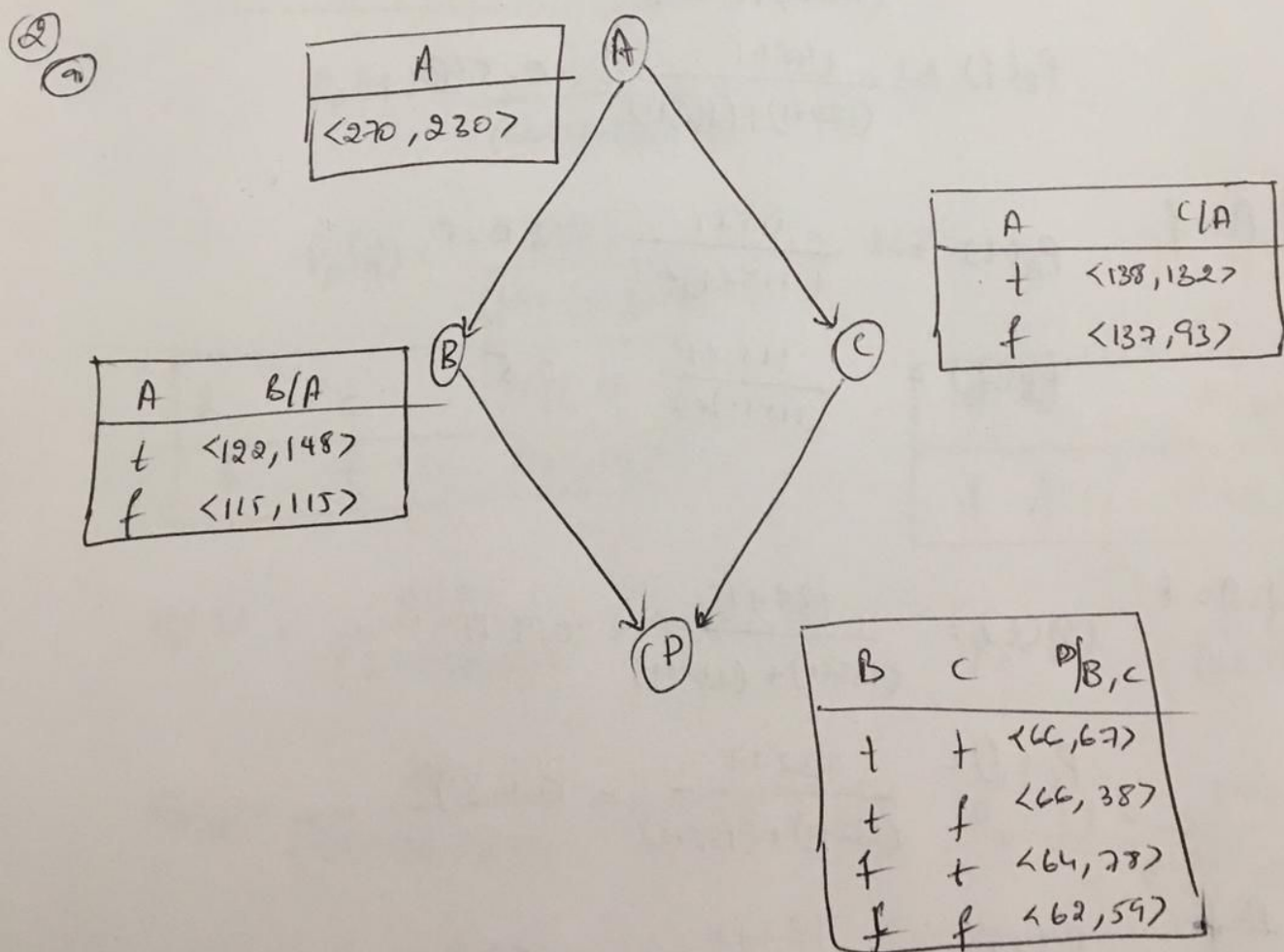
$$\begin{aligned}
P(d, b, na, j, m) &= \sum_{e, ne} P(b) P(E) P(d/b, E) P(na/b, E) P(j/na, d) P(m/na, d) \\
&= P(b) \cdot P(j/na, d) P(m/na, d) \sum_{e, ne} P(E) \cdot P(d/b, E) \cdot P(na/b, E) \\
&= (0.01) \cdot (0.7) \cdot (0.3) \left[P(e) P(d/b, e) \cdot P(na/b, e) + \right. \\
&\quad \left. P(ne) P(d/b, ne) \cdot P(na/b, ne) \right] \\
&= 2.1 \times 10^{-3} \left[(0.02)(0.9)(0.05) + (0.98)(0.8)(0.1) \right] \\
&= 2.1 \times 10^{-3} \left[9 \times 10^{-4} + 0.0784 \right] \\
&= 2.1 \times 10^{-3} \left[0.07930 \right] \\
&= 0.00016653
\end{aligned}$$

$$\begin{aligned}
P(nd, b, na, j, m) &= \sum_{e, ne} P(b) P(E) P(nd/b, E) P(na/b, E) P(j/na, nd) P(m/na, nd) \\
&= P(b) P(j/na, nd) P(m/na, nd) \sum_{e, ne} P(E) P(nd/b, E) P(na/b, E) \\
&= P(b) P(j/na, nd) P(m/na, nd) \left[P(e) P(nd/b, e) P(na/b, e) + \right. \\
&\quad \left. P(ne) P(nd/b, ne) P(na/b, ne) \right] \\
&= (0.01)(0.1)(0.2) \left[(0.02) \cdot (0.1) \cdot (0.05) + \right. \\
&\quad \left. (0.98) \cdot (0.2) \cdot (0.1) \right] \\
&= 2 \times 10^{-4} \left[1 \times 10^{-4} + 0.0196 \right] \\
&= 2 \times 10^{-4} \left[0.0196 \right] \\
&= 0.0394 \times 10^{-4} = 3.94 \times 10^{-6}
\end{aligned}$$

Now,

$$\begin{aligned}
 P(d|b, uq, j, m) &= \frac{P(d, b, uq, j, m)}{P(d, b, uq, j, m) + P(u, b, uq, j, m)} \\
 &= \frac{0.00016653}{0.00016653 + 3.94 \times 10^{-8}} \\
 &= 0.9768874
 \end{aligned}$$

$$P(d|b, uq, j, m) = 0.977$$



Computing Joint probability

[A] $\langle 270, 230 \rangle$

$$P_A(t) = \frac{270+1}{(270+1)+(230+1)} = 0.54$$

$$P_A(f) = \frac{230+1}{(270+1)+(230+1)} = 0.46$$

[B] If $A = t$

$$P_B(t) = \frac{122+1}{(122+1)+(148+1)} = 0.452$$

$$P_B(f) = \frac{148+1}{(122+1)+(148+1)} = 0.548$$

If $A = f$

$$P_B(t) = \frac{115+1}{(115+1) \times 2} = 0.5$$

$$P_B(f) = \frac{115+1}{(115+1) \times 2} = 0.5$$

[C]

If $A = t$

$$P_C(t) = \frac{138+1}{(138+1)+(132+1)} = 0.511$$

$$P_C(f) = \frac{132+1}{(132+1)+(138+1)} = 0.489$$

If $A = f$

$$P_C(t) = \frac{137+1}{(137+1)+(93+1)} = 0.595$$

$$P_C(f) = \frac{93+1}{(137+1)+(93+1)} = 0.405$$

D

B	C	D/B, C
t	+	<66, 67>

$$P_D(t) = \frac{66+1}{(66+1)+(67+1)} = 0.496$$

$$P_D(f) = \frac{67+1}{(66+1)+(67+1)} = 0.504$$

B	C	D/B, C
t	f	<66, 38>

$$P_D(t) = \frac{66+1}{(66+1)+(38+1)} = 0.632$$

$$P_D(f) = \frac{38+1}{(66+1)+(38+1)} = 0.368$$

B	C	D/B, C
f	+	<64, 78>

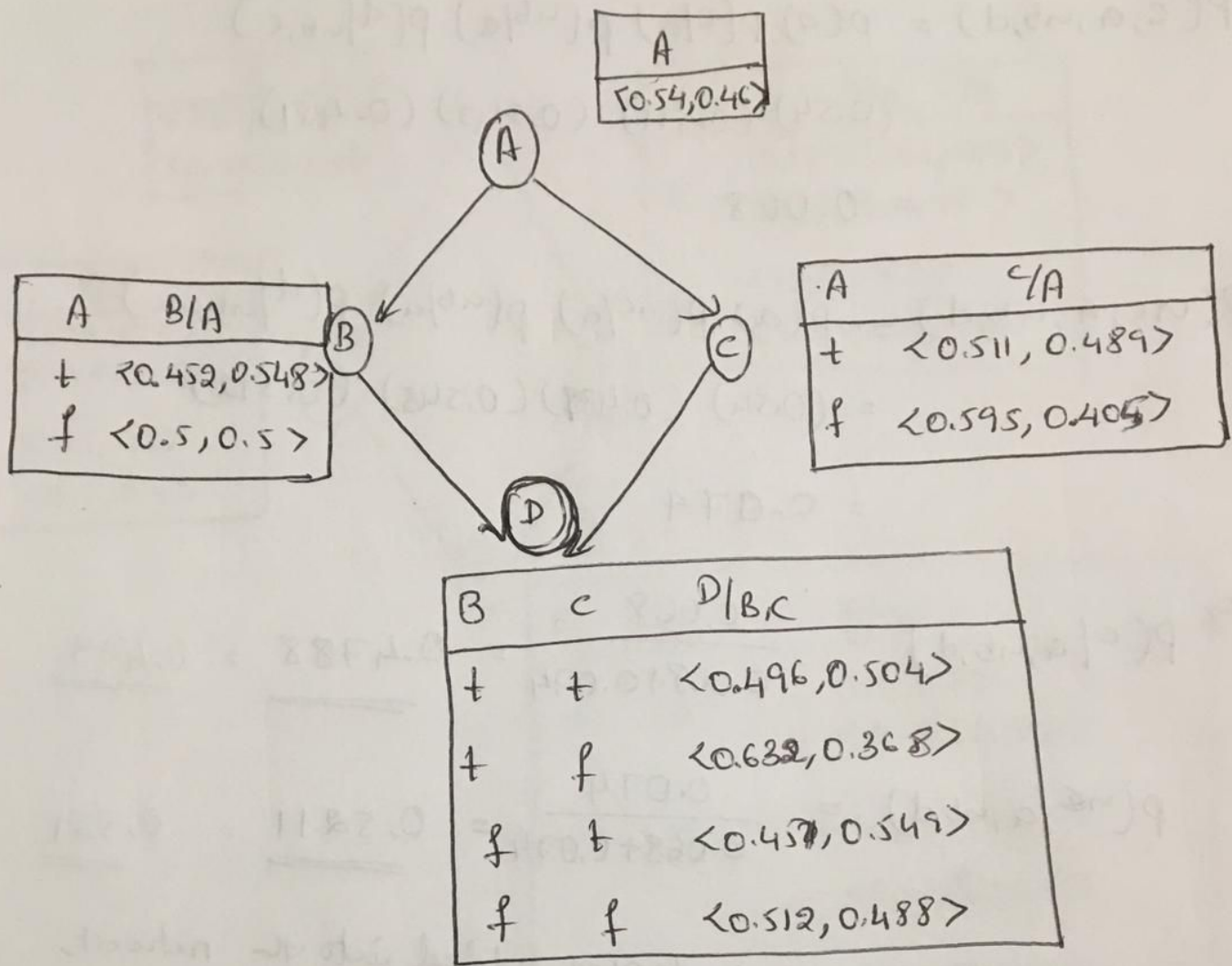
$$P_D(t) = \frac{64+1}{(64+1)+(78+1)} = 0.451$$

$$P_D(f) = \frac{78+1}{(64+1)+(78+1)} = 0.549$$

B	C	D/B, C
f	f	<62, 59>

$$P_D(t) = \frac{62+1}{(62+1)+(59+1)} = 0.512$$

$$P_D(f) = \frac{59+1}{(62+1)+(59+1)} = 0.488$$



- ② Given a new instance with $A = \text{true}$, $B = \text{false}$, $d = \text{true}$
 estimate $C = ?$

Let's ~~prob~~ calculate $P(C|a, \neg b, d)$ with existing conditional probability table

$$\begin{aligned}
 P(C|a, \neg b, d) &= \frac{P(C, a, \neg b, d)}{P(a, \neg b, d)} \\
 &= \frac{P(C, a, \neg b, d)}{P(C, a, \neg b, d) + P(\neg C, a, \neg b, d)}
 \end{aligned}$$

$$\begin{aligned}
 P(c, a, nb, d) &= P(a) P(c|a) P(nb|a) P(d|nb, c) \\
 &= (0.54) (0.511) (0.548) (0.451) \\
 &= 0.068
 \end{aligned}$$

$$\begin{aligned}
 P(nc, a, nb, d) &= P(a) P(nc|a) P(nb|a) P(d|nb, nc) \\
 &= (0.54) (0.489) (0.548) (0.512) \\
 &= 0.074
 \end{aligned}$$

$$\text{So, } P(c|a, nb, d) = \frac{0.068}{0.068 + 0.074} = \underline{\underline{0.4788}} = \underline{\underline{0.479}}$$

$$P(nc|a, nb, d) = \frac{0.074}{0.068 + 0.074} = \underline{\underline{0.5211}} = \underline{\underline{0.521}}$$

So, with 10 new instances added into the network,

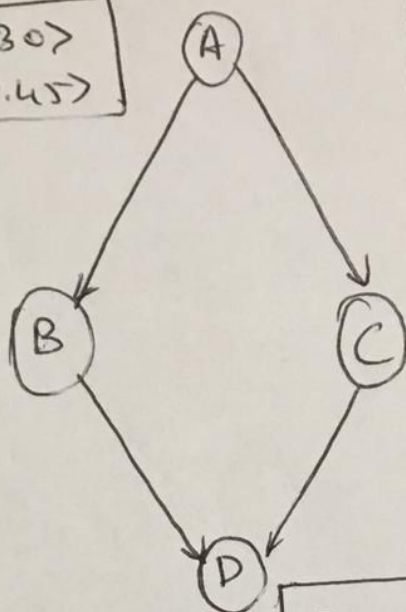
$$P_c(t) = 0.4788 \times 10 = 4.788$$

$$P_c(f) = 0.5211 \times 10 = 5.211$$

are the new shared added to the existing counts.

Hence the new conditional probability table after 1-iteration of E-M will be as shown below:

A
$\langle 280, 230 \rangle$
$\langle 0.55, 0.45 \rangle$

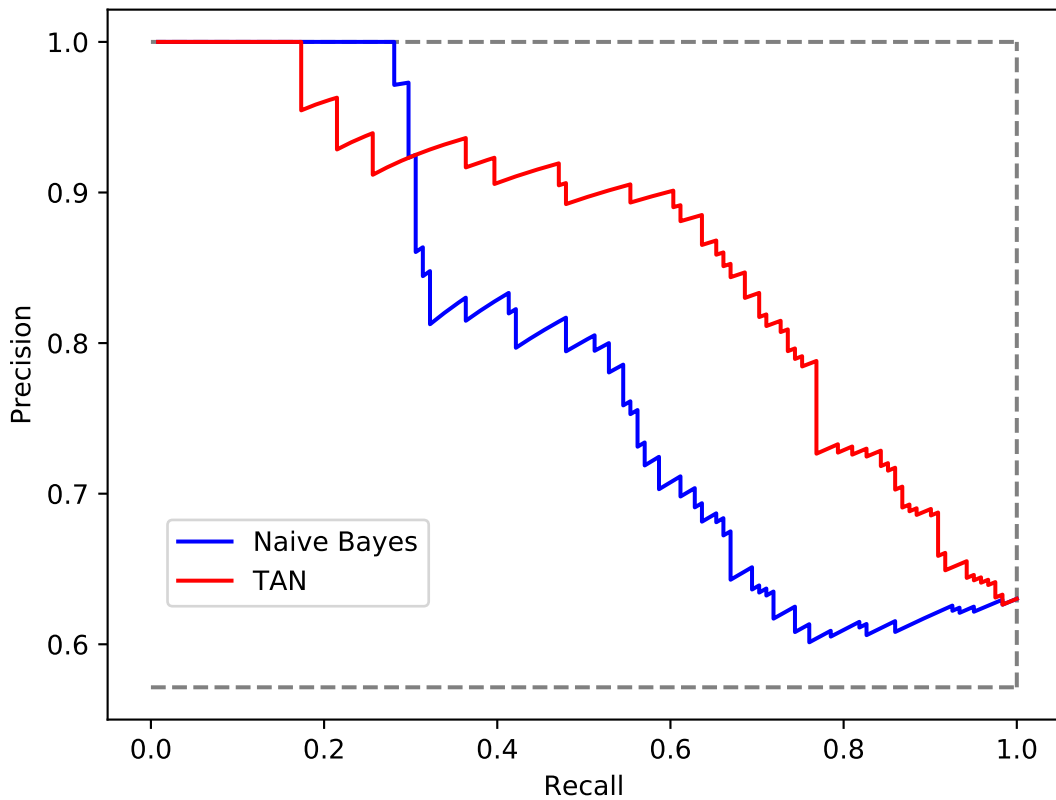


A	A/C C/A
t	$\langle 142.793, 137.207 \rangle$
	$\langle 0.51, 0.490 \rangle$
f	$\langle 137, 93 \rangle$
	$\langle 0.595, 0.404 \rangle$

A	B/A
t	$\langle 122, 158 \rangle$
	$\langle 0.436, 0.564 \rangle$
f	$\langle 115, 115 \rangle$
	$\langle 0.5, 0.5 \rangle$

B	C	D/B,C
t	t	$\langle 66, 67 \rangle$
		$\langle 0.496, 0.503 \rangle$
t	f	$\langle 66, 38 \rangle$
		$\langle 0.63, 0.368 \rangle$
f	t	$\langle 68.793, 78 \rangle$
		$\langle 0.440, 0.530 \rangle$
f	f	$\langle 67.207, 59 \rangle$
		$\langle 0.530, 0.470 \rangle$

PR Curve



Part: 2

Precision-Recall curve:

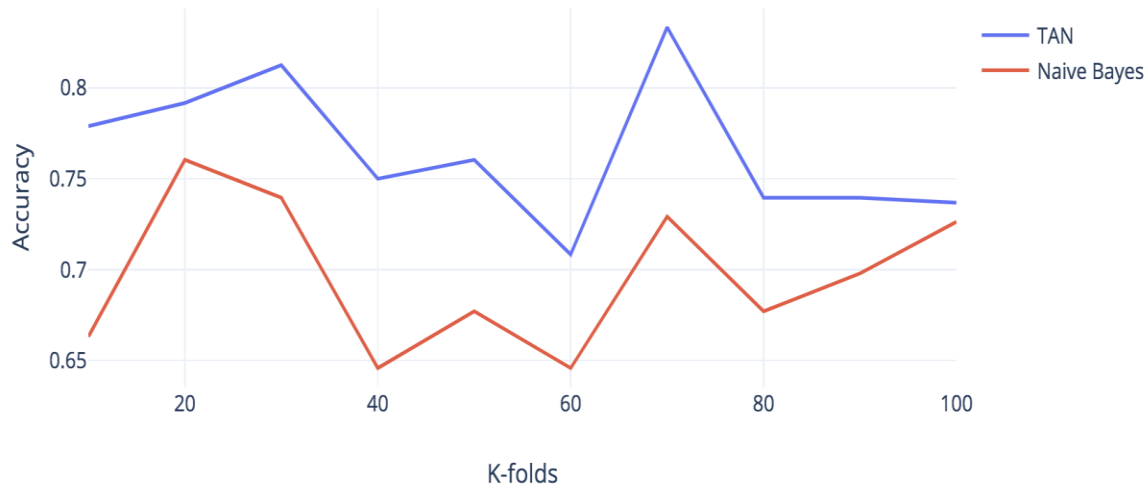
We know that, a model with higher precision relates to low false positive rate and a model with higher recall relates to low false negative rate. In an ideal case it is expected that a model has good recall as false negatives are more dangerous than false positive (Ex: If an health check report is false negative, the patient may go untreated, which is about question of one's life). So, a model with higher recall & precision is always the best (not that precision being worst is fine, a good precision would still be expected).

After observing the P-R curve for both Naive-bayes & TAN, we can clearly see that TAN offered higher recall than Naive-bayes. Precision of TAN is comparatively better than Naive-bayes while we can see recall by TAN is substantially higher than Naive-bayes. Hence TAN model is better and has more predictive power.

Thus a very high recall and better precision offered by TAN (compared with Naive-bayes) makes it a more powerful predictive model.

Part-3

Comparing Accuracy of Naive Bayes and TAN with k-fold validation



Calculated value:

Sample Mean : -0.06888157894736842

SD : 0.032494820478905485

t statistic : -6.703304560300331

P-value : 8.820443574104684e-05

Given Threshold: 0.05

In a statistical hypothesis test, a p-value would help us **to determine the significance of the result**. P-value is used to show if there is any significant difference between the systems. This offers a means to reject points to provide smallest level of significance at which the null hypothesis would be rejected. (NULL hypothesis: the hypothesis that there is no significant difference between specified populations). A small p-value (less than 0.05) suggests that null hypothesis is to be rejected while a large p-value (greater than 0.05) denotes that null hypothesis is to be accepted due to lack of counter proposition against it.

Here we see that the p-value is much smaller than 0.05. Hence **P-value helps us to show that there is a significant difference between the system.**