Logic Assignment

K.N.Toosi University of Technology
Artificial Intelligence

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Part I

Practical Assignment

State whether the following formulae are **valid**, **satisfiable** or **unsatisfiable** and Justify your answers.

a) $(p \Rightarrow q) \vee (q \Rightarrow p)$: **valid**

b) $(\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p) \wedge (p \vee q)$: **valid**

c) $p \wedge (p \Rightarrow \neg q) \wedge q$: **unsatisfiable**

d) $(p \wedge (q \Rightarrow r)) \Leftrightarrow ((\neg p \vee q) \Rightarrow (p \wedge r))$: **valid**

e) $(p \Rightarrow q) \wedge (p \Rightarrow \neg q)$: **satisfiable**

Check the equivalence of the following expressions. Check the first one via truth tables:

a) $(p \Rightarrow q \vee r) \equiv (p \wedge q \Rightarrow r)$

b) $(p \Rightarrow q \vee r) \wedge (p \Rightarrow r) \equiv (q \Rightarrow r)$

Part II
Practical Assignment

Using **forward chaining**, determine whether the following knowledge base entails I. List the steps of the algorithm.

- $A \wedge C \rightarrow H$
- $A \wedge H \rightarrow I$
- $F \rightarrow G$
- $B \wedge D \rightarrow E$
- $A \wedge B \rightarrow D$
- C
- B
- A

Part III
Practical Assignment

Using the **DPLL** algorithm, determine whether the knowledge base entails E. Write down the steps of the algorithm.

- $A \wedge \neg B \Rightarrow (C \vee D \vee E)$
- $A \Rightarrow F \wedge \neg D$
- $F \Rightarrow G \wedge \neg B$
- $G \Rightarrow \neg C$
- A

Part IV
Practical Assignment

Convert the following FOL sentences to CNF:

- *All villagers are searching for a vampire.*
- *All tall vampires are seductive.*
- *No villager can hunt seductive vampires.*
- *Any villager who is searching for vampires and cannot hunt one goes insane.*

Using **resolution**, prove the following statement from the above:

- *If all vampires are tall, then all villagers go insane.*

Part V
Practical Assignment

Find the **most general unifier(MGU)** for the following pairs of terms, if it exists:

1. `Group(x, y, mother(x))`, `Group(Ali, Ali, Alice)`
2. `F(y, f(d, g(x)))`, `F(e, f(z, w))`
3. `Older(mother(x), x)`, `Older(mother(y), John)`
4. `Happy(G(x), A, F(x))`, `Happy(w, z, F(z))`

Part VI
Practical Assignment

Determine whether the following statements are true or false, and justify your answers:

- $P \vee Q \vee (P \Rightarrow Q)$ is always valid.
- If an inference procedure is sound, then it must be complete.
- The following is equivalent to the sentence "No dog bites its owner's child" in first-order logic:

$$\forall x\ \mathrm{dog}(x) \Rightarrow (\forall y\ \mathrm{child}(y, \mathrm{owner}(x)) \Rightarrow \neg\mathrm{bites}(x, y))$$

Part VII
Practical Assignment

Question 7

Using forward chaining, determine whether the following knowledge base entails $R(Avalue, Bvalue)$:

- $NJ(x) \land NT(y) \Rightarrow R(x, y)$
- $R(x, y) \land R(y, z) \Rightarrow R(x, z)$
- $NT(y) \land N(x, z) \Rightarrow R(y, z)$
- $NJ(x) \land FS(z) \Rightarrow N(x, z)$
- $NJ(Avalue)$
- $NT(Dvalue)$
- $FS(Bvalue)$

Part VIII
Implementation Assignment

Objective

In this assignment, you will implement a logic-based system that enables an agent to localize itself within a known maze environment. The agent begins with no knowledge of its initial location but is equipped with four sensors that detect the presence of walls to the North, South, East, and West. Through logical inference using these sensor readings and user-directed movement commands the agent gradually refines its estimate of its location.

Problem Description

The maze is defined in a text file, where each cell is either a wall (denoted by `1`) or a free space (denoted by `0`). At the start of the simulation, the agent assumes it could be located in any free cell. At each timestep, the user controls the agent's movement using the keyboard (arrow keys or WASD). After moving, the agent receives a 4-bit percept indicating whether a wall is present in each of the four cardinal directions. For example, the percept `1001` indicates walls to the North and West of the agent's current position. These percepts are translated into propositional logic sentences using predefined sensor axioms. The agent uses these logical statements combined with knowledge of the map and its movement history to update its belief state: the set of all positions consistent with the observations so far.

Implementation Requirements

Your program must:

- Load the maze layout from a text file.
- Allow the user to control the agent's movement via keyboard input.
- Translate each percept into logical sentences using sensor axioms.
- Maintain a knowledge base (KB) consisting of all logical sentences generated so far.
- Perform logical inference to determine whether a location is consistent with the KB (i.e., check entailment).
- Represent and update the agent's belief state based on entailment results.

The graphical interface must include two panels:

- **Left panel:** Displays the current set of possible agent locations (i.e., the belief state).
- **Right panel:** Shows the history of percepts and actions taken.

The simulation should run for a maximum of 10 time-steps. After each step, the program must highlight the agent's current belief state. At the end of 10 steps or earlier, if the agent localizes itself to a unique position the true location of the agent should be revealed and compared with the estimated belief state.

Deliverables

Submit a complete Python project that includes:

- The main program code.
- The input maze file.
- Any additional modules used for logic inference and visualization.
- A component that builds the knowledge base and performs entailment checks.

Additionally, include a brief report (1–2 pages) discussing your implementation. Explain your design decisions, describe how the knowledge base and inference mechanism are used to eliminate inconsistent positions, and reflect on any challenges you encountered.

Note

Any attempt to use AI tools for generating the code is strictly prohibited. Students will be asked to present and explain their code during a class session.